Introduction to MaRTIn

Massive Recursive Tensor Integration

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Landscape

- All-purpose tools:
  - MaRTIn – two-loop; based on FORM
  - FeynCalc – one(two)-loop; based on mathematica
  - PackageX (no longer actively supported)
  - FeynArts + FormCalc + LoopTools (one-loop; numerical evaluation)
- Special tools:
  - Diagram generation: qgraf
  - IBP reduction: LiteRed, FIRE, Reduze, Kira
  - IBP & packages for special problems: MINCER, MATAD, FORCER, FMFT
- + ...

Scope of MaRTIn

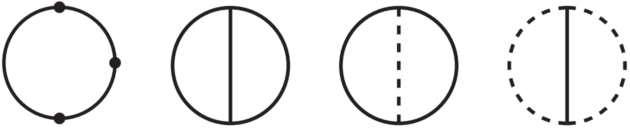
- Fully automated:
  - Main code written in FORM
  - python3 wrapper code
  - Calculational tasks organized via a Makefile
- The "physics" (current version):
  - Geared towards calculating anomalous dimensions and matching conditions
  - Any massive/massless two-loop vacuum diagram (relativistic propagators)
    [Davydychev, Tausk Nucl.Phys.B 397 (1993) 123; Bobeth et al. hep-ph/9910220]
  - Infrared rearrangement and different schemes for γ_5
  - Expand around $d = 4 - 2\epsilon$ and $d = 3 - 2\epsilon$
  - User can provide new models

Installation

- Download from this public gitlab repository
- Requisites:
  - `FORM`, `qgraf`, `python`, . . .
- Setup:
  - Source code directory (e.g. `/home/username/martin/`)
  - Copy `.martin.conf` into home directory
  - Working directory should contain the subdirectories
    - `models` (contains the model files, e.g. SM)
    - `problems` (contains info about the process to be calculated)
    - `prc` (may contain user-specified `FORM` routines)
    - `results` (contains the results, populated by `MaRTIn`)

Workflow

- `MaRTIn` uses `qgraf` to generate diagrams (`FORM` / `.pdf`)
- Feynman rules provided via `FORM` model file
  - `MaRTIn` constructs amplitude, including sorting of fermion lines
- Dirac algebra – more later
- Tensor reduction / Passarino-Veltman
- IBP reduction to master integrals a la Davydychev-Tausk
- Insertion of master integrals



Momentum expansion vs. IRA

- Currently, only vacuum integrals. Two options:
- Taylor expansion in external momenta
- Infrared Rearrangement ("IRA")
  - Based on an exact decomposition of propagators [Chetyrkin et al. hep-ph/9711266]

$$\frac{1}{(p+q)^2 - M^2} = \frac{1}{p^2 - m_{\text{IRA}}^2} + \frac{M^2 - m_{\text{IRA}}^2 - 2\,p\cdot q - q^2}{p^2 - m_{\text{IRA}}^2}\frac{1}{(p+q)^2 - M^2}$$

  - p – loop momentum
  - q – external momentum
  - M – a generic mass (may be zero)
  - m_{IRA}^2 – an IR regulator mass

Dirac algebra

- Dirac algebra is implemented in d spacetime dimensions

- `MaRTIn` performs traces over closed fermion lines
  - Three options for $\gamma_5 = (i/4!)\epsilon_{\mu\nu\rho\sigma}\,\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma$ (*):
    - `define DSCHEME "NDR"`: use $\gamma_5\gamma^\mu = -\gamma^\mu\gamma_5$ if no γ_5 in trace
    - `define DSCHEME "HV"`: use $\gamma_5\gamma^\mu = -\gamma^\mu\gamma_5$ for $\mu = 0, 1, 2, 3$; $\gamma_5\gamma^\mu = \gamma^\mu\gamma_5$ otherwise
    - `define DSCHEME "LARIN"`: replace γ_5 using (*), treat Levi-Civita "projector" onto $\mu = 0, 1, 2, 3$
    - `define DSCHEME "sNDR"`: ask Tom S. about it

- `MaRTIn` sorts open fermion lines into a standard ordering. For instance,
  - `DIRAC(1,p1,mu1,mu2)*DIRAC(2,p2,mu1,mu2)` $\leftrightarrow \not{p}_1\gamma^{\mu_1\mu_2} \otimes \not{p}_2\gamma_{\mu_1\mu_2}$
  - `DIRAC(1,hat,mu1)*DIRAC(2,mu1,G5)` $\leftrightarrow \hat{\gamma}^{\mu_1} \otimes \gamma_{\mu_1}\gamma_5$

The `loop.dat` file

- Problem-specific information is contained in the problem file
- Contains several FORM folds
- Exception: the QGRAF fold specifies the diagrams via literal `qgraf` syntax:

```
*--#[ QGRAF :

  model = qmodel.prop.lag;
  model = qmodel.vrtx.lag;

  in  = field1[q1], field2[q2];
  out = field3[q3], field4[q4];

  loops = 2;
  loop_momentum = p;

  options = onepi;

*--#] QGRAF :
```

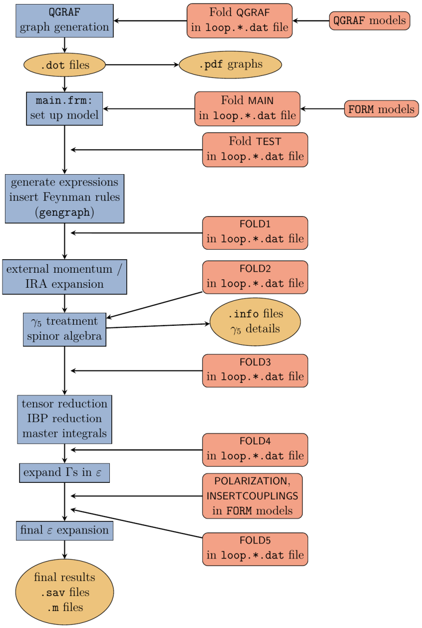
The `loop.dat` file

- The `MAIN` fold contains all other options, e.g.,:

```
*--#[ MAIN :

  #define NM "2"
  #define MODEL1 "modelA"
  #define MODEL2 "modelB"

  #define EXPDENO "1"
    -- or --
  #define IRA "1"

  #define FINALEPLIM "-1"

  #define DSCHEME "NDR/HV/LARIN/sNDR"

*--#] MAIN :
```

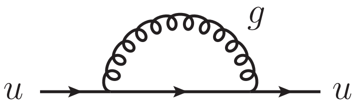
- There are many more options, see the manual for details
- Plus additional `FORM` folds to allow user interference at “default” places

Workflow



QGRAF graph generation
Fold QGRAF in loop.*.dat file
QGRAF models
.dot files
.pdf graphs
main.frm: set up model
Fold MAIN in loop.*.dat file
FORM models
Fold TEST in loop.*.dat file
generate expressions insert Feynman rules (gengraph)
FOLD1 in loop.*.dat file
external momentum / IRA expansion
FOLD2 in loop.*.dat file
γ5 treatment spinor algebra
.info files γ5 details
FOLD3 in loop.*.dat file
tensor reduction IBP reduction master integrals
FOLD4 in loop.*.dat file
expand Γs in ε
POLARIZATION, INSERTCOUPLINGS in FORM models
final ε expansion
FOLD5 in loop.*.dat file
final results .sav files .m files

Example: one-loop up-quark QCD self energy



Example: one-loop up-quark QCD self energy

```
*--#[ QGRAF :
*
model = 'sm.prop.lag' ;
model = 'sm.vrtx.lag' ;

in =  fu[q1];
out = fu[q1];

loops = 1 ;
loop_momentum = p;

options =;
true = iprop[g, 1, 1] ;
*
*--#] QGRAF :

*--#[ MAIN :
*
#define FINALEPLIM "-1"

#define NM "1"
#define MODEL1 "SM"

#define GAUGEG "gaugeg"
#define IRA "1"
#define DSCHEME "NDR"
*
*--#] MAIN :
```

Example: one-loop up-quark QCD self energy

Running

```
martin problems/SM/loop.1_uu.dat
```

gives

```
Computing xxx/user_template/results/SM/form.1_uu/dia1.sav ...
FORM 4.2.1 (Feb  6 2019, v4.2.1-3-g558b01f) 64-bits  Run: <date and time>
    #-

   dia1 =

    + ep^-1 * (
       - 3/4*UbarSp(fu,su3col,j1,mom,q1)*DIRAC(1,one)*USp(fu,su3col,j1,mom,
      q1)*i_*pi^-1*alphas*Mup*CF
       - 1/4*UbarSp(fu,su3col,j1,mom,q1)*DIRAC(1,one)*USp(fu,su3col,j1,mom,
      q1)*i_*pi^-1*xiqg*alphas*Mup*CF
       + 1/4*UbarSp(fu,su3col,j1,mom,q1)*DIRAC(1,q1)*USp(fu,su3col,j1,mom,
      q1)*i_*pi^-1*xiqg*alphas*CF
       );

  0.10 sec out of 0.10 sec
Done computing xxx/user_template/results/SM/form.1_uu/dia1.sav.

MaRTIn finished.
```

$$\texttt{dia1} = -\frac{i\alpha_s}{4\pi} C_F \frac{1}{\varepsilon} \bar{u}(\boldsymbol{q}_1, j_1) \Big[\xi_G \not{q}_1 + m_u (3 + \xi_G) \Big] u(\boldsymbol{q}_1, j_1) \,.$$

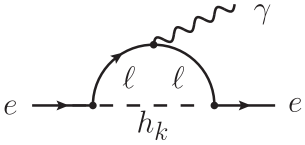
Implementing a new model

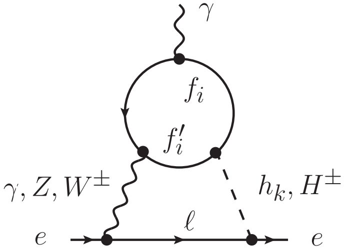
- Two necessary ingredients:
  - `qgraf` model file (propagators and vertices)
  - `FORM` model file
- Model implementation is somewhat of a bottleneck for the use
- Need to follow standard notation for propagators and vertices
- Most vertices are already implemented in generic form
  - E.g. vector-scalar-scalar $\propto (q_2 - q_3)^\mu$
- Any group structure needs to be implemented by hand
  - QCD color algebra is already implemented for many cases
- Advice: Read the manual, start with modifying `model_SM`, contact the authors if in doubt

Application: electron EDM in the 2HDM

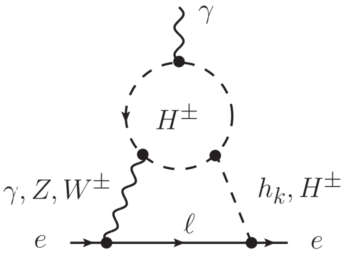
- The electron EDM is a sensitive of non-CKM CP violation
  - $\frac{d_e}{4e}\,\bar{e}\sigma_{\mu\nu}F^{\mu\nu}\gamma_5 e \quad \rightarrow \quad \frac{d_e}{e}\,\boldsymbol{E}\cdot\boldsymbol{s}$
- The experimental bound is $d_e < 4.1\times 10^{-30} e\,\mathrm{cm}$ @ 90% CL
  [Roussy et al. 2212.11841]
- Electron EDM in the most general 2HDM [Altmannshofer et al. 2410.17313]
  - Extend SM by second Higgs doublet
  - Extended scalar sector: three neutral and one charged Higgs boson
  - Generically, expect new CP phases in Yukawa couplings and Higgs potential
  - Popular “toy” model for electroweak baryogenesis
- In our context: non-trivial check of `MaRTIn`

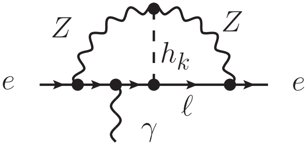
Barr-Zee diagrams



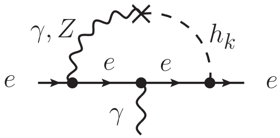


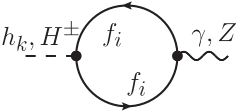
non-Barr-Zee diagrams





Technical scope of calculation





- We evaluated all closed fermion loops in the 't Hooft-Veltman (HV) scheme
    - Counterterm diagrams are evanescent (but non-zero) in HV
    - Divergent counterterm insertions lead to additional finite contributions
    - Final results agree with naive evaluation in NDR scheme
- Whole calculation performed in generalized R_ξ gauge
    - Background field gauge for external states
    - Gauge propagators and would-be Goldstone masses are ξ dependent
    - All physical results are manifestly ξ independent

Conclusion

- `MaRTIn` is a versatile all-in-one tool for multiloop calculations
  - Currently, up to two-loop vacuum integrals with any masses
- Not (yet well) optimized for speed
- Implementation of new models possible (if somewhat cumbersome)
- Active work on extension up to four-loop