Entanglement in flavored scalar scattering

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in collaboration with
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based on:
JHEP 07 (2024) 156 (arXiv:2404.13743)

National Science Centre Poland

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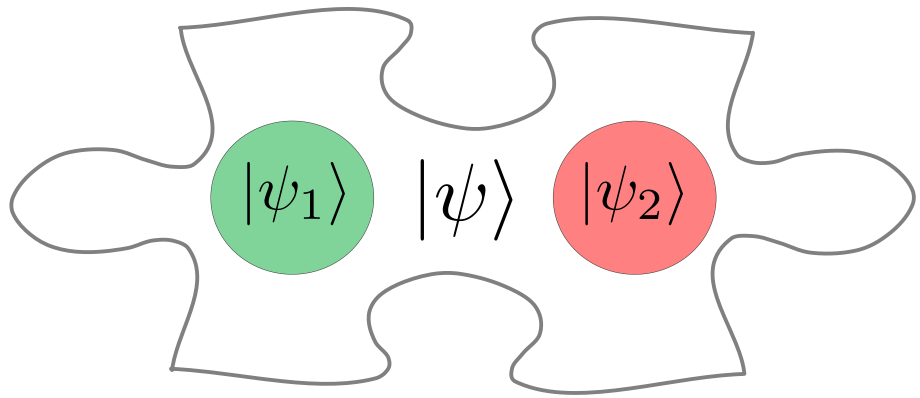
Asymptotic Safety meets Particle Physics (and friends)

NATIONAL CENTRE FOR NUCLEAR RESEARCH ŚWIERK

Outline

- Entanglement and symmetries in high-energy scattering
- From the S-matrix to the density matrix
- Measures of entanglement and partitions of the Hilbert space
- Application to the 2HDM and constraints
    - Entanglement generation
    - Entanglement transformation

Quantum entanglement



$$|\psi\rangle \neq |\psi_1\rangle \otimes |\psi_2\rangle$$

entanglement = non-separability

Example 1

Qubit space: $|1\rangle,\ |2\rangle \in \mathbb{C}^2 \to \mathbb{C}^4 = \mathbb{C}^2 \otimes \mathbb{C}^2$

A) $a_1 b_1|11\rangle + a_1 b_2|12\rangle + a_2 b_1|21\rangle + a_2 b_2|22\rangle = (a_1|1\rangle + a_2|2\rangle) \otimes (b_1|1\rangle + b_2|2\rangle)$ separable (not entangled)

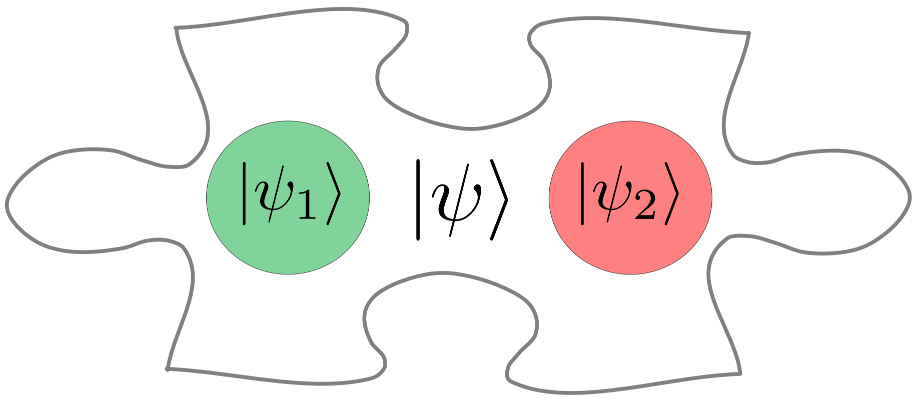
B)

$$\frac{1}{\sqrt{2}}|11\rangle + \frac{1}{\sqrt{2}}|22\rangle \qquad \frac{1}{\sqrt{2}}|11\rangle - \frac{1}{\sqrt{2}}|22\rangle$$

$$\frac{1}{\sqrt{2}}|12\rangle + \frac{1}{\sqrt{2}}|21\rangle \qquad \frac{1}{\sqrt{2}}|12\rangle - \frac{1}{\sqrt{2}}|21\rangle$$

non separable (entangled)

Quantum entanglement



$$|\psi\rangle \neq |\psi_1\rangle \otimes |\psi_2\rangle$$

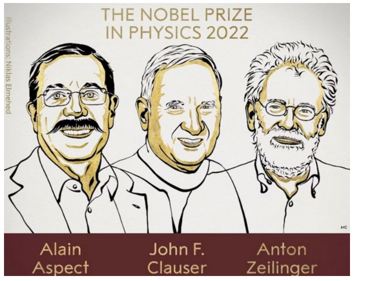
entanglement = non-separability

Example 2 $\mathcal{H}_{\mathrm{comp}} = L^2(\mathbb{R}) \otimes \mathbb{C}^2$ $\qquad |\psi\rangle_{\mathrm{comp}} = \,?$

A) $$\sum_{i=1}^{2} \int_{-\infty}^{\infty} dx\, \psi(x)\epsilon_i |x\rangle|i\rangle$$ separable

B) $$\sum_{i=1}^{2} \int_{-\infty}^{\infty} dx\, \psi_i(x) |x\rangle|i\rangle$$ non-separable

Entanglement is hot!



Measured experimentally (Bell inequalities violation)

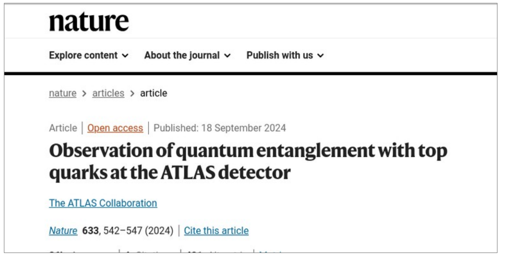
for linear polarization of low-energy photons

Clauser et al., '72; Aspect et al., '82; Zeilinger et al., '98;

… also at colliders

for spin of the top quarks

ATLAS '23; CMS '24



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Observation of quantum entanglement with top quarks at the ATLAS detector

The ATLAS Collaboration

Nature 633, 542–547 (2024) | Cite this article

Quantum information

dense coding (Bennett, Wiesner, '92), teleportation (Bennett et at., '93), key distrubution (Ekert, '91)

Emergence of space and time

Moreva *et al.* '13, Van Raamsdonk '10, Ryu and Takayanagi '06, Maldacena, Susskind '13

Applications in particle physics

- **quantum tomography** ex. Afik, Muñoz de Nova '20, Fabbrichesi *et al.* '21, many others after ATLAS, CMS results

  use entanglement (and other quantum observables) to enhance sensitivity of collider searches

- **theoretical implications** ⟹ **emergent symmetries**

  investigate connections between entanglement and fundamental properties of quantum field theories

Emergent symmetries

Symmetries arising from **entanglement extremization**
in scattering processes

- non-relativistic baryon-baryon scattering → SU(4) and SU(16) global symmetries from **minimizing** entanglement among spins

  Beane, Kaplan, Klco, Savage,
  PRL 122 (2019) 102001

- quantum electrodynamics → U(1) gauge symmetry from **maximizing** entanglement among helicities

  Cervera-Lierta et al., SciPost Phys 3 (2017) 036
  Fedida, Serafini, PRD 107 (2023) 116007

- Two-Higgs-doublet model → indications of SO(8) global symmetry from **minimizing** entanglement among Higgs boson “flavors”

  Carena, Low, Wagner, Xiao
  PRD 109 (2024) L051901

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  - What happens if more than one scattering channel is considered
  - Implications of preserving unitarity in the scattering process

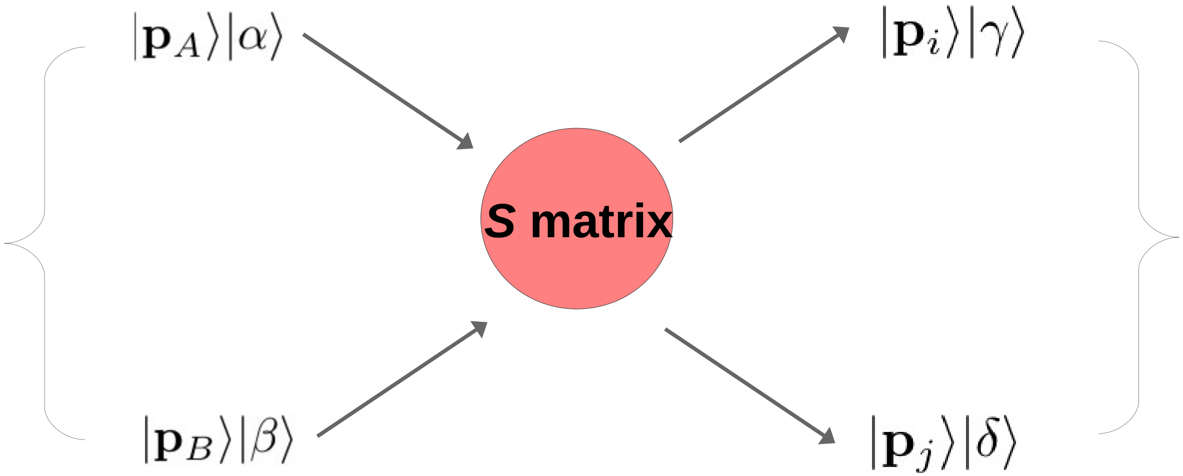
Kowalska, EMS, 2404.13743; Chang, Jacobo, 2409.13030; Low, Yin, 2405.08056, 2410.22414

Entanglement in scattering

2 → 2 scattering particles *A*, *B* with internal "qubit" quantum number: $|\mathbf{p}_A\rangle|\alpha\rangle,\ |\mathbf{p}_B\rangle|\beta\rangle$

$$|\text{in}\rangle = \frac{1}{\sqrt{V}}\,|\mathbf{p}_A\mathbf{p}_B\rangle$$

$$\otimes \sum_{\alpha,\beta=1,2} a_{\alpha\beta}|\alpha\beta\rangle$$



$$|\text{out}\rangle = S\,|\text{in}\rangle$$

Hilbert space: **momentum** + **flavor** (qubit)

$$\mathcal{H}_{\text{tot}} = L^2(\mathbb{R}^3 \otimes \mathbb{R}^3) \otimes \mathbb{C}^4$$

In perturbation theory:

$$S^{ijab}_{\gamma\delta\alpha\beta} = (\mathcal{I} + iT)^{ijab}_{\gamma\delta\alpha\beta}$$

$$= (2\pi)^6\,4\,E_a E_b\,\delta^{ijab}_{\gamma\delta\alpha\beta} + (2\pi)^4\delta^4(p_a + p_b - p_i - p_j)\,i\mathcal{M}_{\gamma\delta,\alpha\beta}(p_a, p_b \to p_i, p_j)$$

The final-state density matrix:

$$\rho = |\text{out}\rangle\langle\text{out}|$$

encodes all the properties of a quantum system **(entanglement)**

Perturbative density matrix

$$\rho = |\mathrm{out}\rangle\langle\mathrm{out}|$$

Shao et al., '08; Seki et al., '15; Peschanski, Seki, '16, '19; Carney et al., '16; Rätzel et al., '17; Fan et al., '17; Araujo et al., '19; Fan et al., '21; Fonseca et al., '22; Shivashankara '23; Aoude et al., '24

Properties:

$$1)\ \mathrm{Tr}(\rho) = 1$$

⟹ unitarity of the *S*-matrix

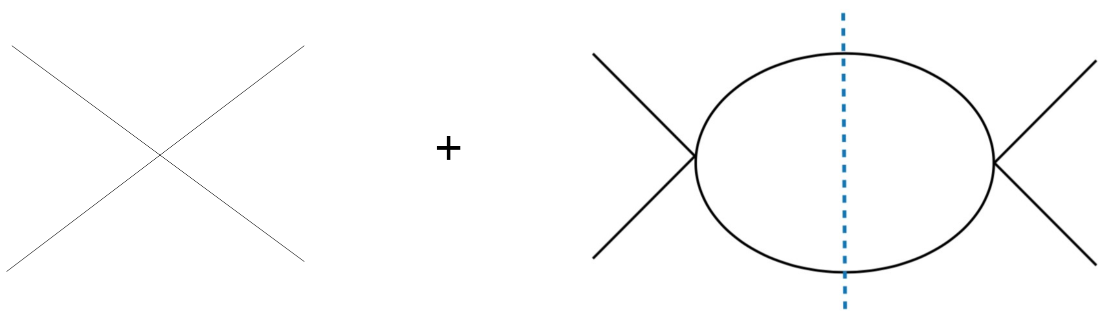
optical theorem

$$\langle\mathrm{out}|\mathrm{out}\rangle = 1 + \Delta\left(i \sum_{\alpha\beta,\gamma\delta} a^*_{\alpha\beta}\mathcal{M}_{\alpha\beta,\gamma\delta}(p_A,p_B \to p_A,p_B)a_{\gamma\delta} + \mathrm{c.c.}\right)$$
$$+\ \Delta \int\int \frac{d^3p_i}{(2\pi)^3}\frac{1}{2E_i}\frac{d^3p_j}{(2\pi)^3}\frac{1}{2E_j}(2\pi)^4\delta^4(p_A+p_B-p_i-p_j)$$
$$\times \sum_{\alpha\beta,\rho\epsilon,\sigma\tau} \mathcal{M}_{\alpha\beta,\rho\epsilon}(p_A,p_B\to p_i,p_j)a_{\rho\epsilon}\mathcal{M}^*_{\alpha\beta,\sigma\tau}(p_A,p_B\to p_i,p_j)a^*_{\sigma\tau}$$

$$\Delta = \frac{(2\pi)^4\delta^4(p_A+p_B-p_A-p_B)}{4E_AE_B\left[(2\pi)^3\,\delta^3(0)\right]^2}$$

(indeterminate normalization)

We work at 1-loop order Carena *et al.* '23 tree level



Perturbative density matrix

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Properties:

$$1)\ \text{Tr}(\rho) = 1$$

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optical theorem

$$\langle\text{out}|\text{out}\rangle = 1 + \Delta\left(i\sum_{\alpha\beta,\gamma\delta} a^*_{\alpha\beta}\mathcal{M}_{\alpha\beta,\gamma\delta}(p_A,p_B\to p_A,p_B)a_{\gamma\delta} + \text{c.c.}\right)$$
$$+\ \Delta\int\!\!\int \frac{d^3p_i}{(2\pi)^3}\frac{1}{2E_i}\frac{d^3p_j}{(2\pi)^3}\frac{1}{2E_j}(2\pi)^4\delta^4(p_A+p_B-p_i-p_j)$$
$$\times\sum_{\alpha\beta,\rho\epsilon,\sigma\tau}\mathcal{M}_{\alpha\beta,\rho\epsilon}(p_A,p_B\to p_i,p_j)a_{\rho\epsilon}\mathcal{M}^*_{\alpha\beta,\sigma\tau}(p_A,p_B\to p_i,p_j)a^*_{\sigma\tau}$$

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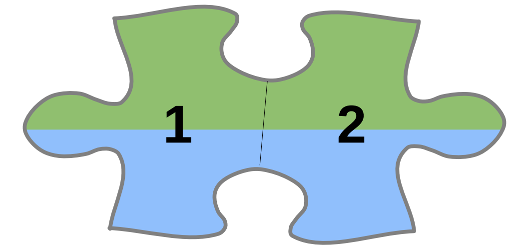
(indeterminate normalization)

$$2)\ \text{Tr}(\rho^2)\begin{cases} = 1 & \text{pure state} \\ < 1 & \text{mixed state}\end{cases}$$

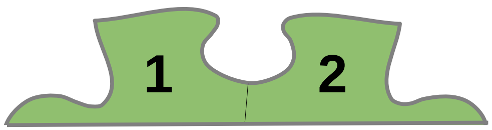
⇒ **Leads to different entanglement measures**

Entanglement of the final state

$\rho = |\text{out}\rangle\langle\text{out}|$ is <u>pure</u>

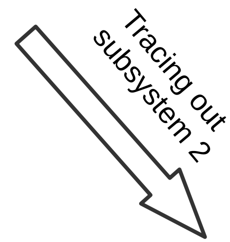


$\mathcal{H}_{\text{tot}} = L^2(\mathbb{R}^3) \otimes L^2(\mathbb{R}^3) \otimes \mathbb{C}^2 \otimes \mathbb{C}^2$



$\mathcal{H}_{\text{red}} = \mathbb{C}^2 \otimes \mathbb{C}^2$

bipartitions:

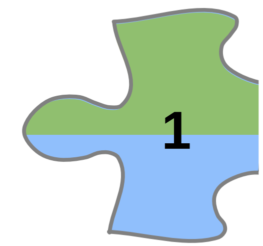


$$\tilde{\rho} = \text{Tr}_{\mathbf{p}}(\rho)$$

basis : $|\alpha\beta\rangle\langle\gamma\delta|$

$$\tilde{\rho} = \text{Tr}_2(\rho)$$

basis : $|\mathbf{p}_i\alpha\rangle\langle\mathbf{p_j}\gamma|$



$\mathcal{H}_{\text{red}} = L^2(\mathbb{R}^3) \otimes \mathbb{C}^2$

- entanglement between bipartite states:

  von Neumann entropy

$$S_N(\tilde{\rho}) = -\text{Tr}(\tilde{\rho}\,\log_2 \tilde{\rho})$$

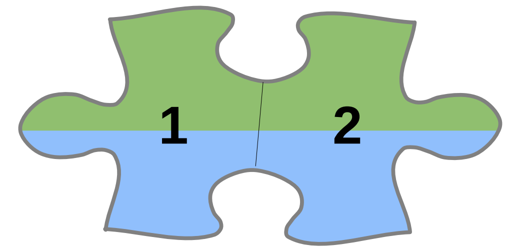
Entanglement monotone:

$S_N = 0$ no entanglement

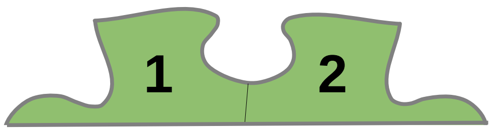
$S_N = 1$ maximal entanglement

Entanglement of the final state

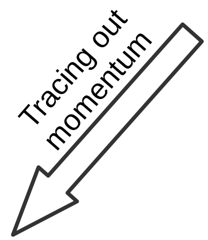
$$\rho = |\text{out}\rangle\langle\text{out}|$$ is pure



$$\mathcal{H}_{\text{tot}} = L^2(\mathbb{R}^3) \otimes L^2(\mathbb{R}^3) \otimes \mathbb{C}^2 \otimes \mathbb{C}^2$$



$$\mathcal{H}_{\text{red}} = \mathbb{C}^2 \otimes \mathbb{C}^2$$

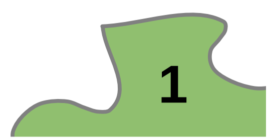


$$\tilde{\rho} = \text{Tr}_{\mathbf{p}}(\rho)$$

$\tilde{\rho}_{\alpha\beta,\gamma\delta}$ in general is mixed

basis : $|\alpha\beta\rangle\langle\gamma\delta|$

- entanglement between 2 qubits:



Concurrence

$$C(\tilde{\rho}) = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}$$

λ: eigenvalues of $\tilde{\rho}(\sigma_y \otimes \sigma_y)\tilde{\rho}^*(\sigma_y \otimes \sigma_y)$

Entanglement monotone:

$C = 0$ no entanglement

$C = 1$ maximal entanglement

Perturbative density matrix

$$2)\ \mathrm{Tr}(\rho^2) \begin{cases} = 1 & \text{pure state} \\ < 1 & \text{mixed state} \end{cases}$$

$$\begin{aligned} \mathrm{Tr}(\tilde\rho^2) = \sum_{\alpha\beta,\gamma\delta} \tilde\rho_{\alpha\beta,\gamma\delta}\,\tilde\rho_{\gamma\delta,\alpha\beta} = 1 + 2\,\Delta \Bigg[& i \sum_{\alpha\beta,\epsilon\rho} \mathcal{M}_{\alpha\beta,\epsilon\rho}(p_A,p_B \to p_A,p_B)\, a^*_{\alpha\beta} a_{\epsilon\rho} + \text{c.c.} \\ & + \int\!\!\int \frac{d^3p_i}{(2\pi)^3}\frac{1}{2E_i}\frac{d^3p_j}{(2\pi)^3}\frac{1}{2E_j}(2\pi)^4\delta^4(p_A+p_B-p_i-p_j) \\ & \times \sum_{\epsilon\rho,\gamma\delta,\tau\sigma,\alpha\beta} \mathcal{M}_{\alpha\beta,\epsilon\rho}(p_A,p_B\to p_i,p_j)\mathcal{M}^*_{\gamma\delta,\tau\sigma}(p_A,p_B\to p_i,p_j)\, a_{\gamma\delta}\, a^*_{\alpha\beta}\, a_{\epsilon\rho}\, a^*_{\tau\sigma} \Bigg] \\ - \Delta^2 \Bigg[& \sum_{\alpha\beta,\gamma\delta,\epsilon\rho,\tau\sigma} \mathcal{M}_{\alpha\beta,\epsilon\rho}(p_A,p_B\to p_A,p_B)\mathcal{M}_{\gamma\delta,\tau\sigma}(p_A,p_B\to p_A,p_B)\, a^*_{\alpha\beta}\, a_{\epsilon\rho}\, a^*_{\gamma\delta}\, a_{\tau\sigma} + \text{c.c.} \\ & - 2 \sum_{\alpha\beta,\epsilon\rho,\tau\sigma} \mathcal{M}_{\alpha\beta,\epsilon\rho}(p_A,p_B\to p_A,p_B)\,\mathcal{M}^*_{\alpha\beta,\tau\sigma}(p_A,p_B\to p_A,p_B) a^*_{\tau\sigma} a_{\epsilon\rho} \Bigg]. \end{aligned} \quad (2.18)$$

$$\Delta \leq \frac{1}{16\pi}$$

wave packet normalization:

$$|\text{in}\rangle = \left(\prod_{i=1,2}\int \frac{d^3p_i}{(2\pi)^3}\frac{1}{\sqrt{2E_i}}\right)\phi_A(\mathbf{p}_1)\phi_B(\mathbf{p}_2)\,|\mathbf{p}_1\mathbf{p}_2\rangle \sum_{\alpha,\beta=1,2} a_{\alpha\beta}|\alpha\beta\rangle \quad\longrightarrow\quad \int \frac{|\mathbf{p}|^2 dp\, d\Omega}{(2\pi)^3}|\phi_{A,B}(\mathbf{p})|^2 = 1 \ \text{finite}$$

$$\approx \frac{1}{\sqrt{V}} \sum_{\alpha,\beta=1,2} a_{\alpha\beta}|\mathbf{p}_A\mathbf{p}_B\rangle|\alpha\beta\rangle\,, \quad\longrightarrow\quad \phi_{A,B}(\mathbf{p}) = \sqrt{\frac{(2\pi)^3}{\delta^3(0)}}\,\delta^3(\mathbf{p}-\mathbf{p}_{A,B})$$

Indeterminate: it does not belong to the Hilbert space

Entanglement is perturbatively suppressed

Final state with measured momenta

Consider a final state with "measured" momentum

$$|f\rangle = \left(\prod_{i=1,2}\int \frac{d^3p_i}{(2\pi)^3}\frac{1}{\sqrt{2E_i}}\right)\phi_C(\mathbf{p}_1)\phi_D(\mathbf{p}_2)\,|\mathbf{p}_1\mathbf{p}_2\rangle \approx \frac{1}{\sqrt{V}}\,|\mathbf{p}_C\mathbf{p}_D\rangle$$

Project the "out" state along "*f* "

$$\begin{aligned}|\mathrm{proj}\rangle \equiv |f\rangle\langle f|\mathrm{out}\rangle &= |f\rangle\sum_{\gamma\delta}\int\int\frac{d^3p_i}{(2\pi)^3}\frac{1}{2E_i}\frac{d^3p_j}{(2\pi)^3}\frac{1}{2E_j}\,\psi_{\gamma\delta}(\mathbf{p}_i,\mathbf{p}_j)\langle f|\mathbf{p}_i\mathbf{p}_j\rangle|\gamma\delta\rangle\\ &= \frac{1}{V}\sum_{\gamma\delta}\psi_{\gamma\delta}(\mathbf{p}_C,\mathbf{p}_D)|\mathbf{p}_C\mathbf{p}_D\rangle|\gamma\delta\rangle\,.\end{aligned}$$

Density matrix:

$$\tilde{\rho}_p = \frac{|\mathrm{proj}\rangle\langle\mathrm{proj}|}{\langle\mathrm{proj}|\mathrm{proj}\rangle} = \sum_{\alpha\beta,\gamma\delta}(\tilde{\rho}_p)_{\alpha\beta,\gamma\delta}|\alpha\beta\rangle\langle\gamma\delta|$$

No dependence on Δ

$$(\tilde{\rho}_p)_{\alpha\beta,\gamma\delta} = \frac{\sum_{\epsilon\rho,\tau\sigma}\mathcal{M}_{\alpha\beta,\epsilon\rho}(p_A,p_B\to p_C,p_D)\mathcal{M}^*_{\gamma\delta,\tau\sigma}(p_A,p_B\to p_C,p_D)\,a_{\epsilon\rho}\,a^*_{\tau\sigma}}{\sum_{\gamma\delta,\epsilon\rho,\tau\sigma}\mathcal{M}_{\gamma\delta,\epsilon\rho}(p_A,p_B\to p_C,p_D)\mathcal{M}^*_{\gamma\delta,\tau\sigma}(p_A,p_B\to p_C,p_D)\,a_{\epsilon\rho}\,a^*_{\tau\sigma}}$$

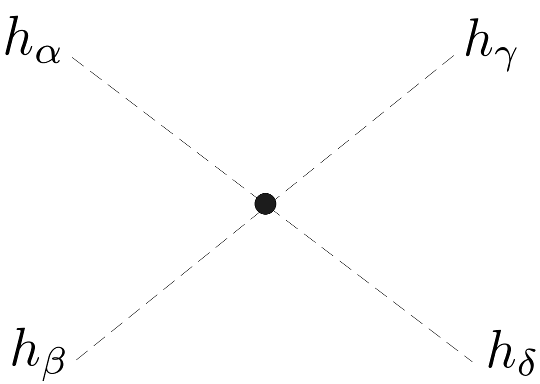
- Can use to quantify/measure final-state entanglement → *e.g.* **quantum tomography**
- Potentially phase space-point dependent

2HDM in a nutshell

SU(2) doublets: $H_\alpha = \begin{pmatrix} h_\alpha^+ \\ h_\alpha^0 \end{pmatrix}_{Y=\frac{1}{2}}$ $\quad \alpha = 1,2 \rightarrow |1\rangle, |2\rangle$ **two flavors**

scalar potential:
$$V(H_1,H_2) = \mu_1^2\, H_1^\dagger H_1 + \mu_2^2\, H_2^\dagger H_2 + \left(\mu_3^2\, H_1^\dagger H_2 + \text{H.c.}\right)$$
$$+ \lambda_1\, (H_1^\dagger H_1)^2 + \lambda_2\, (H_2^\dagger H_2)^2 + \lambda_3\, (H_1^\dagger H_1)(H_2^\dagger H_2) + \lambda_4\, (H_1^\dagger H_2)(H_2^\dagger H_1)$$
$$+ \left(\lambda_5\, (H_1^\dagger H_2)^2 + \lambda_6\, (H_1^\dagger H_1)(H_1^\dagger H_2) + \lambda_7\, (H_2^\dagger H_2)(H_1^\dagger H_2) + \text{H.c.}\right)$$

contact interactions (we assume *p^2 >> μ^2*)



$$i\,\mathcal{M}^{(0+1)}_{\gamma\delta,\alpha\beta}$$

$$i\mathcal{M}^{(0)}(h^+h^0 \to h^+h^0) = -i \begin{pmatrix} 2\lambda_1 & \lambda_6 & \lambda_6 & 2\lambda_5 \\ \lambda_6 & \lambda_3 & \lambda_4 & \lambda_7 \\ \lambda_6 & \lambda_4 & \lambda_3 & \lambda_7 \\ 2\lambda_5 & \lambda_7 & \lambda_7 & 2\lambda_2 \end{pmatrix}$$

Carena *et al.* '23

$$i\mathcal{M}^{(0)}(h^+h^{0*} \to h^+h^{0*}) = -i \begin{pmatrix} 2\lambda_1 & \lambda_6 & \lambda_6 & \lambda_4 \\ \lambda_6 & \lambda_3 & 2\lambda_5 & \lambda_7 \\ \lambda_6 & 2\lambda_5 & \lambda_3 & \lambda_7 \\ \lambda_4 & \lambda_7 & \lambda_7 & 2\lambda_2 \end{pmatrix}$$

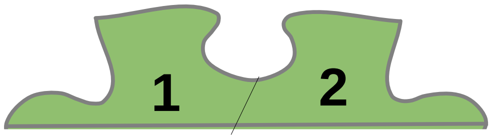
$$i\mathcal{M}^{(0)}(h^0h^0 \to h^0h^0) = -i \begin{pmatrix} 4\lambda_1 & 2\lambda_6 & 2\lambda_6 & 4\lambda_5 \\ 2\lambda_6 & \lambda_3+\lambda_4 & \lambda_3+\lambda_4 & 2\lambda_7 \\ 2\lambda_6 & \lambda_3+\lambda_4 & \lambda_3+\lambda_4 & 2\lambda_7 \\ 4\lambda_5 & 2\lambda_7 & 2\lambda_7 & 4\lambda_2 \end{pmatrix}$$

$$i\mathcal{M}^{(0)}(h^0h^{0*} \to h^+h^-) = i\mathcal{M}^{(0)}(h^+h^- \to h^0h^{0*}) = -i \begin{pmatrix} 2\lambda_1 & \lambda_6 & \lambda_6 & \lambda_3 \\ \lambda_6 & \lambda_4 & 2\lambda_5 & \lambda_7 \\ \lambda_6 & 2\lambda_5 & \lambda_4 & \lambda_7 \\ \lambda_3 & \lambda_7 & \lambda_7 & 2\lambda_2 \end{pmatrix}$$

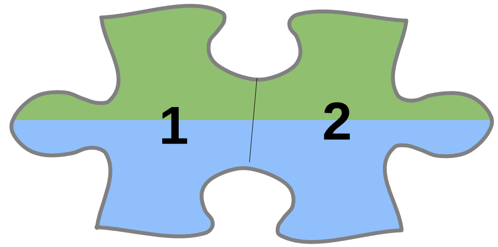
$$i\mathcal{M}^{(0)}(h^+h^- \to h^+h^-) = i\mathcal{M}^{(0)}(h^0h^{0*} \to h^0h^{0*}) = -i \begin{pmatrix} 4\lambda_1 & 2\lambda_6 & 2\lambda_6 & \lambda_3+\lambda_4 \\ 2\lambda_6 & \lambda_3+\lambda_4 & 4\lambda_5 & 2\lambda_7 \\ 2\lambda_6 & 4\lambda_5 & \lambda_3+\lambda_4 & 2\lambda_7 \\ \lambda_3+\lambda_4 & 2\lambda_7 & 2\lambda_7 & 4\lambda_2 \end{pmatrix}$$

Q: any constraints on λ from entanglement extremization?

Entanglement generation



$\subset$



$$\mathcal{H}_{\rm red} = \mathbb{C}^2 \otimes \mathbb{C}^2$$

$$\mathcal{H}_{\rm tot} = L^2(\mathbb{R}^3) \otimes L^2(\mathbb{R}^3) \otimes \mathbb{C}^2 \otimes \mathbb{C}^2$$

no initial entanglement: $$|\text{in}\rangle = \frac{1}{\sqrt{V}} |\mathbf{p}_A \mathbf{p}_B\rangle |11\rangle$$

$$|\text{out}\rangle = S\,|\text{in}\rangle$$

Trace out momentum $$\tilde{\rho} = \mathrm{Tr}_{\mathbf{p}}(\rho)$$

$$\tilde{\rho}(h^0 h^0 \to h^0 h^0) = \quad \tilde{\rho}_{11,11} = 1 - \Delta \left(\frac{\lambda_5^2}{\pi} + \frac{\lambda_6^2}{2\pi} \right),$$

$$\tilde{\rho}_{11,12} = \tilde{\rho}_{11,21} = \tilde{\rho}^*_{12,11} = \tilde{\rho}^*_{21,11} = \Delta \left(2\,i\,\lambda_6 + \frac{2\lambda_1\lambda_6 - \lambda_3\lambda_6 - \lambda_4\lambda_6 - 2\lambda_5\lambda_7}{8\pi} \right),$$

$$\tilde{\rho}_{11,22} = \tilde{\rho}^*_{22,11} = \Delta \left(4\,i\,\lambda_5 + \frac{2\lambda_1\lambda_5 - 2\lambda_2\lambda_5 - \lambda_6\lambda_7}{4\pi} \right),$$

$$\tilde{\rho}_{12,12} = \tilde{\rho}_{12,21} = \tilde{\rho}^*_{21,12} = \tilde{\rho}^*_{21,21} = \Delta\, \frac{\lambda_6^2}{4\pi},$$

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$$\tilde{\rho}_{22,22} = \Delta\, \frac{\lambda_5^2}{\pi}.$$

<u>von Neumann entropy</u>

$$S_N(\tilde{\rho}) = -\mathrm{Tr}(\tilde{\rho}\, \log_2 \tilde{\rho})$$

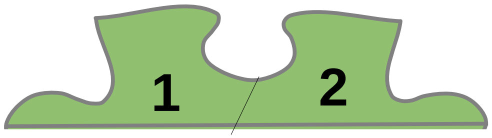
$h^0 h^0 \to h^0 h^0$

$$\theta_1 = 1 - \Delta \left(\frac{\lambda_5^2}{\pi} + \frac{\lambda_6^2}{2\pi} \right) + 16\,\Delta^2 \left(\lambda_5^2 + \frac{\lambda_6^2}{2} \right),$$

$$\theta_2 = \Delta \left(\frac{\lambda_5^2}{\pi} + \frac{\lambda_6^2}{2\pi} \right) - 16\,\Delta^2 \left(\lambda_5^2 + \frac{\lambda_6^2}{2} \right),$$

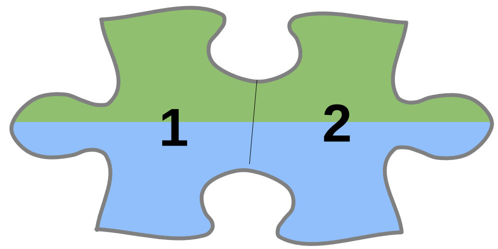
λ_5, λ_6 generate entanglement between **flavor and momentum**

Entanglement generation



$$\mathcal{H}_{\rm red} = \mathbb{C}^2 \otimes \mathbb{C}^2$$

$\subset$



$$\mathcal{H}_{\rm tot} = L^2(\mathbb{R}^3) \otimes L^2(\mathbb{R}^3) \otimes \mathbb{C}^2 \otimes \mathbb{C}^2$$

no initial entanglement: $$|\mathrm{in}\rangle = \frac{1}{\sqrt{V}} |\mathbf{p}_A \mathbf{p}_B\rangle |11\rangle$$

$$|\mathrm{out}\rangle = S\,|\mathrm{in}\rangle$$

Trace out momentum $$\tilde{\rho} = \mathrm{Tr}_{\mathbf{p}}(\rho)$$

$$\tilde{\rho}(h^0h^0 \to h^0h^0) = \quad \tilde{\rho}_{11,11} = 1 - \Delta\left(\frac{\lambda_5^2}{\pi} + \frac{\lambda_6^2}{2\pi}\right),$$

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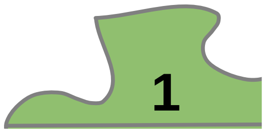
$$\theta_2 = \Delta\left(\frac{\lambda_5^2}{\pi} + \frac{\lambda_6^2}{2\pi}\right) - 16\,\Delta^2\left(\lambda_5^2 + \frac{\lambda_6^2}{2}\right),$$

$$\Delta \leq \frac{1}{16\pi}$$

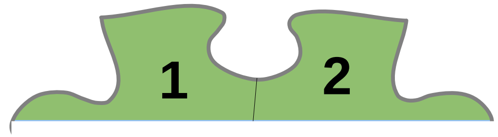
physicality!

entanglement is perturbatively small in λ, Δ

Entanglement generation



$\subset$



$$\mathcal{H}_{\text{red}} = \mathbb{C}^2 \otimes \mathbb{C}^2$$

qubit space

no initial entanglement:
$$|\text{in}\rangle = \frac{1}{\sqrt{V}} |\mathbf{p}_A \mathbf{p}_B\rangle |11\rangle$$
$$|\text{out}\rangle = S\,|\text{in}\rangle$$

Trace out momentum
$$\tilde{\rho} = \text{Tr}_{\mathbf{p}}(\rho)$$

$$\tilde{\rho}(h^0h^0 \to h^0h^0) = \quad \tilde{\rho}_{11,11} = 1 - \Delta\left(\frac{\lambda_5^2}{\pi} + \frac{\lambda_6^2}{2\pi}\right),$$
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$$\tilde{\rho}_{12,22} = \tilde{\rho}_{21,22} = \tilde{\rho}^*_{22,12} = \tilde{\rho}^*_{22,21} = \Delta\,\frac{\lambda_5\lambda_6}{2\pi},$$
$$\tilde{\rho}_{22,22} = \Delta\,\frac{\lambda_5^2}{\pi}.$$

<u>Concurrence:</u>

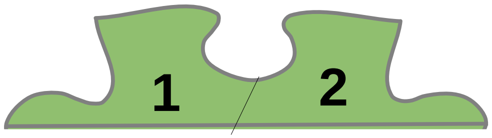
$$C(\tilde{\rho}) = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}$$

$h^0h^0 \to h^0h^0$

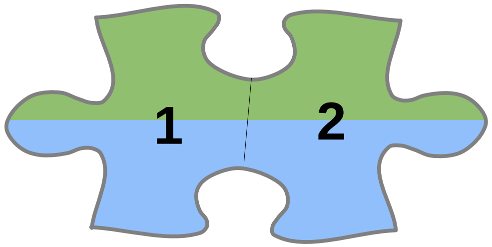
$$C(\tilde{\rho}) = \sqrt{\frac{2\,\Delta\,\lambda_5^2}{\pi} + 32\,\Delta^2\lambda_5^2} \approx \sqrt{\frac{2\Delta}{\pi}}|\lambda_5|$$

λ_5 generates entanglement **of 2 flavor qubits**

Entanglement generation



$\mathcal{H}_{\text{red}} = \mathbb{C}^2 \otimes \mathbb{C}^2$ $\subset$



$$\mathcal{H}_{\text{tot}} = L^2(\mathbb{R}^3) \otimes L^2(\mathbb{R}^3) \otimes \mathbb{C}^2 \otimes \mathbb{C}^2$$

- Repeating for |12>, |21>, |22> :

couplings that generate entanglement

$\|\text{in}\rangle_F$	momentum-flavor space	two-flavor space
$\|11\rangle$	$\lambda_3,\ \lambda_4,\ \lambda_5,\ \lambda_6$	$\lambda_3,\ \lambda_4,\ \lambda_5$
$\|12\rangle$	$\lambda_3,\ \lambda_4,\ \lambda_5,\ \lambda_6,\ \lambda_7$	$\lambda_3,\ \lambda_4,\ \lambda_5$
$\|21\rangle$	$\lambda_3,\ \lambda_4,\ \lambda_5,\ \lambda_6,\ \lambda_7$	$\lambda_3,\ \lambda_4,\ \lambda_5$
$\|22\rangle$	$\lambda_3,\ \lambda_4,\ \lambda_5,\ \lambda_7$	$\lambda_3,\ \lambda_4,\ \lambda_5$

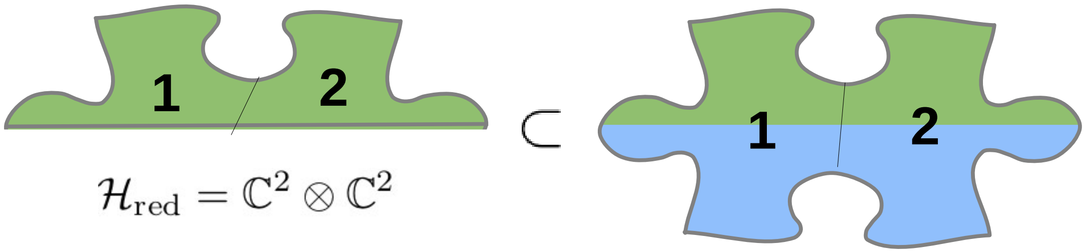
- Combination of basis vectors **minimal entanglement** →

$$\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = \lambda_5 = \lambda_6 = \lambda_7 = 0$$ momentum-flavor space

$$\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = \lambda_5 = 0\,, \quad \lambda_6 = \lambda_7$$ two-flavor space

- Computational basis: **no entanglement iff no flavor changing** $S\left(|\mathbf{p}_i\mathbf{p}_j\rangle|\alpha\alpha\rangle\right) = |\alpha\alpha\rangle S(\lambda_\alpha)|\mathbf{p}_i\mathbf{p}_j\rangle$ $\alpha = 1, 2$
- **There always exists some "in" state leading to entanglement creation**

Entanglement generation



$\mathcal{H}_{\rm red} = \mathbb{C}^2 \otimes \mathbb{C}^2 \subset$

$$\mathcal{H}_{\rm tot} = L^2(\mathbb{R}^3) \otimes L^2(\mathbb{R}^3) \otimes \mathbb{C}^2 \otimes \mathbb{C}^2$$

- Repeating for |12>, |21>, |22> :

couplings that generate entanglement

$\lvert \text{in} \rangle_F$	momentum-flavor space	two-flavor space
$\lvert 11\rangle$	$\lambda_3,\ \lambda_4,\ \lambda_5,\ \lambda_6$	$\lambda_3,\ \lambda_4,\ \lambda_5$
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$\lvert 21\rangle$	$\lambda_3,\ \lambda_4,\ \lambda_5,\ \lambda_6,\ \lambda_7$	$\lambda_3,\ \lambda_4,\ \lambda_5$
$\lvert 22\rangle$	$\lambda_3,\ \lambda_4,\ \lambda_5,\ \lambda_7$	$\lambda_3,\ \lambda_4,\ \lambda_5$

- Combination of basis vectors **minimal entanglement** →

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$$\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = \lambda_5 = 0\,, \quad \lambda_6 = \lambda_7$$ two-flavor space

- **λ6, λ7 are LOCC of C^4** (can't generate entangled out of separable)
- **No SO(8) Lagrangian symmetry emerges from MinEnt**

$$\neq \{2\lambda_1 = 2\lambda_2 = \lambda_3 \neq 0\}$$

cf. also Chang, Jacobo, 2409.13030

Entanglement transformation

maximal initial flavor entanglement: $|\text{in}\rangle = \frac{1}{\sqrt{V}} |\mathbf{p}_A \mathbf{p}_B\rangle \frac{1}{\sqrt{2}} (|11\rangle + |22\rangle)$

pre-scattering: $S_N(\tilde{\rho}_{\text{in}}) = 0 \qquad C(\tilde{\rho}_{\text{in}}) = 1$

post-scattering: $S_N(\tilde{\rho}_{\text{out}}) > 0 \qquad C(\tilde{\rho}_{\text{out}}) < 1$

<u>von Neuman entropy</u>

$$\theta_1 = 1 - \Delta\,(1 - 16\pi\Delta)\frac{(\lambda_1-\lambda_2)^2 + (\lambda_6+\lambda_7)^2}{4\pi}$$

$$\theta_2 = \Delta\,(1 - 16\pi\Delta)\frac{(\lambda_1-\lambda_2)^2 + (\lambda_6+\lambda_7)^2}{4\pi}$$

S>0, entanglement increases

<u>concurrence</u>

$$C(\rho^F) = \sqrt{1 - \Delta\,(1 - 16\pi\Delta)\frac{(\lambda_1-\lambda_2)^2 + (\lambda_6+\lambda_7)^2}{2\,\pi}}$$

C<1, entanglement is reduced

Entanglement "flows"
from
flavor Hilbert space to full Hilbert space

	Entanglement transformation flavor space $\to$ full Hilbert space
$\frac{1}{\sqrt{2}}(\|11\rangle + \|22\rangle)$	$\lambda_1 - \lambda_2,\ \lambda_6 + \lambda_7$
$\frac{1}{\sqrt{2}}(\|11\rangle - \|22\rangle)$	$\lambda_1 - \lambda_2,\ \lambda_6 - \lambda_7$
$\frac{1}{\sqrt{2}}(\|12\rangle + \|21\rangle)$	$\lambda_6 + \lambda_7$
$\frac{1}{\sqrt{2}}(\|12\rangle - \|21\rangle)$	none

<u>Unless:</u>

Coupling relations

e.g. $\lambda_1 = \lambda_2,\ \lambda_6 = -\lambda_7$

(discrete symmetry CP2)

cf. Ferreira, Grządkowski, Ogreid, Osland, 2306.02410

Entanglement transformation

maximal initial flavor entanglement: $|\text{in}\rangle = \frac{1}{\sqrt{V}} |\mathbf{p}_A \mathbf{p}_B\rangle \frac{1}{\sqrt{2}} (|11\rangle + |22\rangle)$

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$\frac{1}{\sqrt{2}}(\lvert 11\rangle - \lvert 22\rangle)$	$\lambda_1 - \lambda_2,\ \lambda_6 - \lambda_7$
$\frac{1}{\sqrt{2}}(\lvert 12\rangle + \lvert 21\rangle)$	$\lambda_6 + \lambda_7$
$\frac{1}{\sqrt{2}}(\lvert 12\rangle - \lvert 21\rangle)$	none

<u>Unless:</u>

$$\Delta_{\text{max}} = \frac{1}{16\pi}$$

spherical symmetry of
initial wave packet (s wave)

Entanglement transformation

maximal initial flavor entanglement: $|\text{in}\rangle = \frac{1}{\sqrt{V}} |\mathbf{p}_A \mathbf{p}_B\rangle \frac{1}{\sqrt{2}} (|11\rangle + |22\rangle)$

pre-scattering: $S_N(\tilde{\rho}_{\text{in}}) = 0 \qquad C(\tilde{\rho}_{\text{in}}) = 1$

post-scattering: $S_N(\tilde{\rho}_{\text{out}}) > 0 \qquad C(\tilde{\rho}_{\text{out}}) < 1$

<u>von Neuman entropy</u>

$$\theta_1 = 1 - \Delta(1 - 16\pi\Delta) \frac{(\lambda_1 - \lambda_2)^2 + (\lambda_6 + \lambda_7)^2}{4\pi}$$

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S>0, entanglement increases

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$$C(\rho^F) = \sqrt{1 - \Delta(1 - 16\pi\Delta) \frac{(\lambda_1 - \lambda_2)^2 + (\lambda_6 + \lambda_7)^2}{2\,\pi}}$$

C<1, entanglement is reduced

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	Entanglement transformation flavor space $\to$ full Hilbert space
$\frac{1}{\sqrt{2}}(\|11\rangle + \|22\rangle)$	$\lambda_1 - \lambda_2,\ \lambda_6 + \lambda_7$
$\frac{1}{\sqrt{2}}(\|11\rangle - \|22\rangle)$	$\lambda_1 - \lambda_2,\ \lambda_6 - \lambda_7$
$\frac{1}{\sqrt{2}}(\|12\rangle + \|21\rangle)$	$\lambda_6 + \lambda_7$
$\frac{1}{\sqrt{2}}(\|12\rangle - \|21\rangle)$	none

Q: Is conservation of entanglement related to symmetry?

Q: What relation Lagrangian / wave-packet?

work in progress...

To take home

- Post-scattering entanglement may provide a **complementary way of constraining** the interaction structure of BSM models.

- Scattering interaction **injects** entanglement in a separable system, but this is **perturbatively suppressed** in λ, Δ

- 2HDM: **all quartic couplings** can potentially create entanglement between momentum and “flavor” dof's

- 2HDM: entanglement can be **transformed** by some coupling combinations, may lead to symmetries

Backup

Final state with measured momenta

Project the "out" state along a choice of momentum:

$$|\text{proj}\rangle \equiv |f\rangle\langle f|\text{out}\rangle \qquad |f\rangle = \left(\prod_{i=1,2}\int \frac{d^3p_i}{(2\pi)^3}\frac{1}{\sqrt{2E_i}}\right)\phi_C(\mathbf{p}_1)\phi_D(\mathbf{p}_2)\,|\mathbf{p}_1\mathbf{p}_2\rangle \approx \frac{1}{\sqrt{V}}\,|\mathbf{p}_C\mathbf{p}_D\rangle$$

Density matrix: $$\tilde{\rho}_p = \frac{|\text{proj}\rangle\langle\text{proj}|}{\langle\text{proj}|\text{proj}\rangle} = \sum_{\alpha\beta,\gamma\delta}(\tilde{\rho}_p)_{\alpha\beta,\gamma\delta}|\alpha\beta\rangle\langle\gamma\delta|$$

$$(\tilde{\rho}_p)_{\alpha\beta,\gamma\delta} = \frac{\sum_{\epsilon\rho,\tau\sigma}\mathcal{M}_{\alpha\beta,\epsilon\rho}(p_A,p_B\to p_C,p_D)\mathcal{M}^*_{\gamma\delta,\tau\sigma}(p_A,p_B\to p_C,p_D)\,a_{\epsilon\rho}\,a^*_{\tau\sigma}}{\sum_{\gamma\delta,\epsilon\rho,\tau\sigma}\mathcal{M}_{\gamma\delta,\epsilon\rho}(p_A,p_B\to p_C,p_D)\mathcal{M}^*_{\gamma\delta,\tau\sigma}(p_A,p_B\to p_C,p_D)\,a_{\epsilon\rho}\,a^*_{\tau\sigma}}$$

No dependence on Δ

2HDM results

$\|\text{in}\rangle_F$	minimal entanglement	maximal entanglement
$\|11\rangle$	$2\lambda_1\lambda_3 = \lambda_6^2,\ \lambda_5 = \frac{1}{2}\lambda_3 = \frac{1}{2}\lambda_4$	$\lambda_1 = \lambda_3 = \lambda_4 = \lambda_5 = 0$ or $\lambda_6 = 0,\ \lambda_1 = \lambda_5 = \frac{1}{2}\lambda_3 = \frac{1}{2}\lambda_4$
$\|12\rangle,\ \|21\rangle$	$\lambda_6\lambda_7 = \lambda_3^2,\ \lambda_5 = \frac{1}{2}\lambda_3 = \frac{1}{2}\lambda_4$	$\lambda_6 = \lambda_7,\ \lambda_3 = \lambda_4 = \lambda_5 = 0$ or $\lambda_6 = \lambda_7 = 0,\ \lambda_3 = \lambda_4 = 2\lambda_5$
$\|22\rangle$	$2\lambda_2\lambda_3 = \lambda_7^2,\ \lambda_5 = \frac{1}{2}\lambda_3 = \frac{1}{2}\lambda_4$	$\lambda_2 = \lambda_3 = \lambda_4 = \lambda_5 = 0$ or $\lambda_7 = 0,\ \lambda_2 = \lambda_5 = \frac{1}{2}\lambda_3 = \frac{1}{2}\lambda_4$
Total	1) $\lambda_3 = \lambda_4 = \lambda_5 = \lambda_6 = \lambda_7 = 0$ 2) $\lambda_3^2 = \lambda_4^2 = 4\lambda_5^2 = \lambda_6\lambda_7 = 2\lambda_1\lambda_2$	1) $\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = \lambda_5 = 0,$ $\lambda_6 = \lambda_7$ 2) $\lambda_1 = \lambda_2 = \lambda_5 = \frac{1}{2}\lambda_3 = \frac{1}{2}\lambda_4,$ $\lambda_6 = \lambda_7 = 0$