RUHR UNIVERSITÄT BOCHUM RUB

Spontaneous symmetry breaking at all temperatures

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Asymptotic Safety Meets Particle Physics and Friends

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Collaborators

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SSB at all T: Why not?

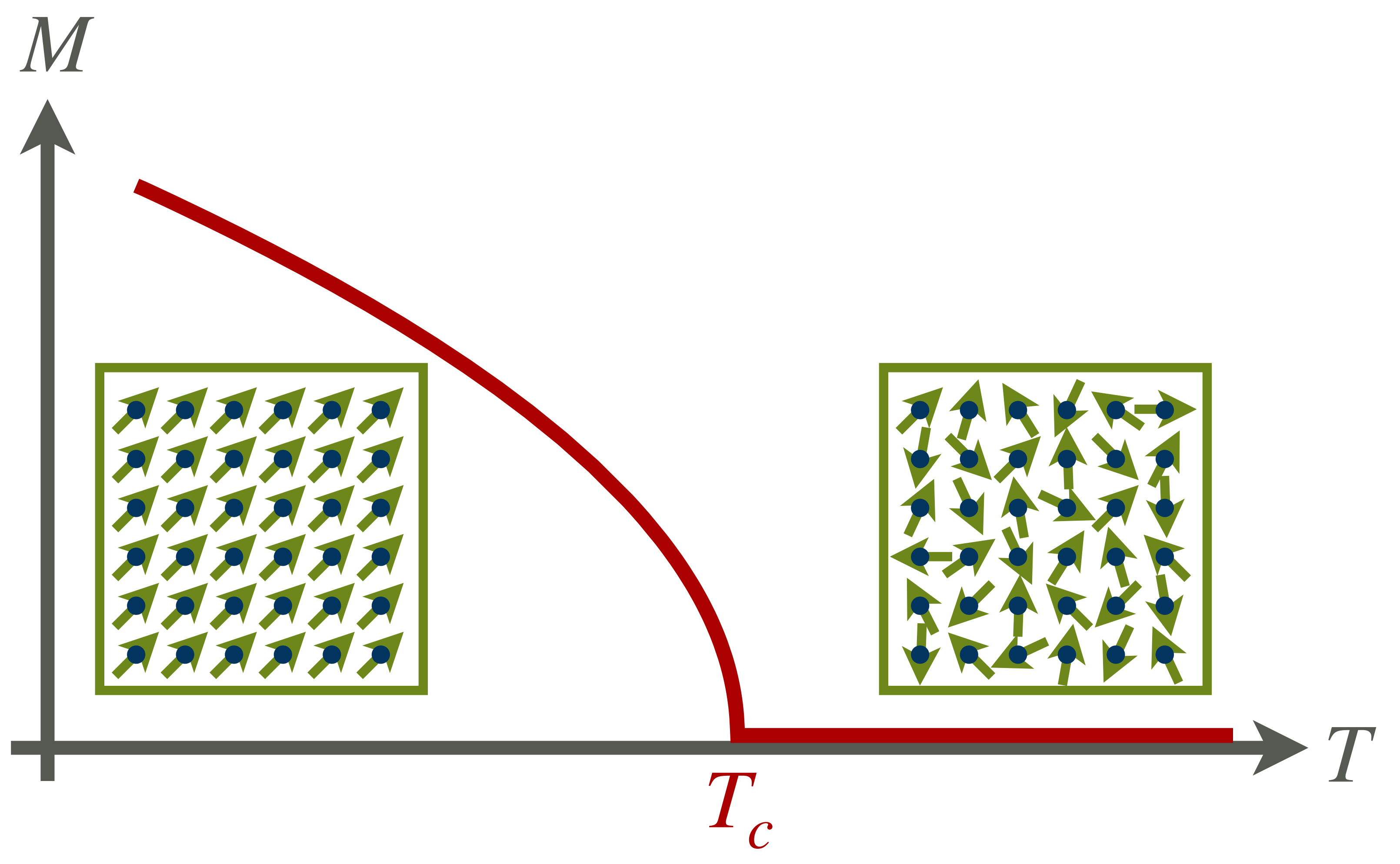
Spontaneous symmetry breaking **usually** only occurs at sufficiently **low temperatures**

Simple argument:

Free energy $F = E - TS$ **minimized** at **all temperatures**

➔ at **high** temperatures: **entropy** is **maximized**

➔ **usually**: high entropy $\longrightarrow$ **disorder** and **symmetry restoration**



Common examples: ferromagnetism, superconductivity, crystals, …

SSB at all T: Why not?

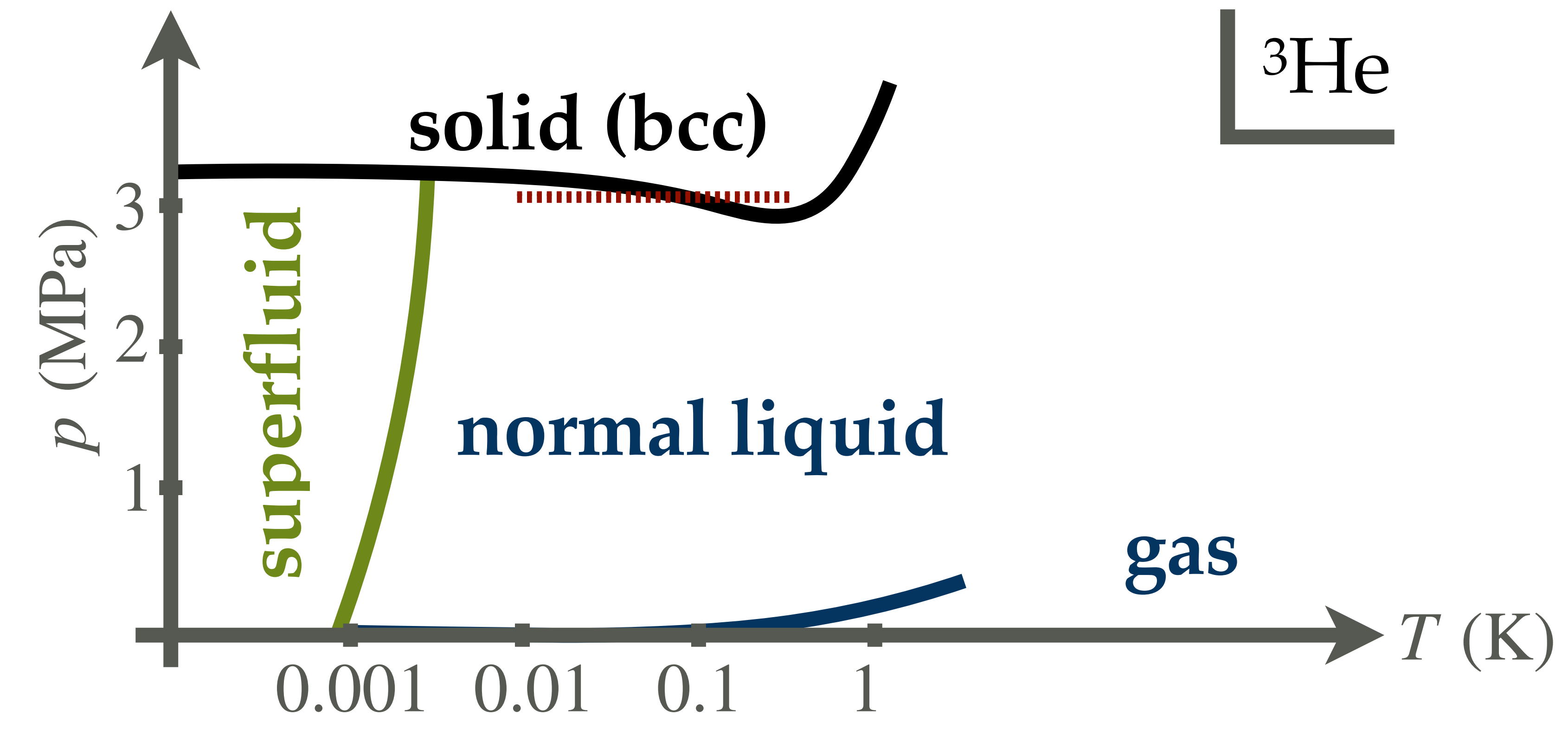
Spontaneous symmetry breaking **usually** only occurs at sufficiently **low temperatures**

But: symmetry can be broken when temperature is increased, e.g. **“Pomeranchuk effect”** in ^{3}**He**

For $T \lesssim 0.3\,\mathrm{K}:\ S_{\mathrm{liquid}} < S_{\mathrm{solid}}$ due to **disordered** nucleon spins

➜ **Additional** degree of freedom can lead to “inverted” transition

➜ *Can this also **persist** up to **highest temperatures**?*



I. Pomeranchuk, *Zh. Eksp. Teor. Fiz. Band 20, Nr. 919* (1950)

Can a symmetry be broken at all temperatures?

On a **lattice:**

density matrix $\rho \sim e^{-\beta H} \xrightarrow{\text{high } T} \mathbb{1}$

➜ all sites **disentangle** and system is **disordered**

Kliesch et al., *Phys. Rev. X* 4.3: 031019 *(2014)*

As a **QFT:** **evidence** for the existence of such theories, but only for

(a) non-unitary theories (i.e. in $d = 4 - \epsilon$, non-zero chemical potential, …)

(b) unitary but non-local theories in $d = 2 + 1$

(c) UV incomplete theories

(d) infinitely many fields

S. Weinberg, *Phys. Rev. D 9 3357 (1974)*
S.-I. Hong et al., *Phys. Rev. D 63, 085014 (2001)*
N. Chai et al., *Phys. Rev. Lett. 125, 131603 (2020)*
N. Chai et al., *Phys. Rev. D 102, 065014 (2020)*
Chaudhuri et al., *JHEP 2021.8: 1-38 (2021)*
B. Bajc et al., *Phys. Rev. D 103, 096014 (2021)*

In **low dimensions**: Coleman-Hohenberg-Mermin-Wagner theorem

➜ **forbids** SSB of **continuous symmetries** at finite temperature

Towards a theory with persistent SSB

If theory has non-trivial fixed-point at zero temperature: **UV completion** at **fixed-point**

➔ at $T > 0$, the time domain compactifies $\rightarrow$ **CFT** at **finite temperature**

A CFT at finite temperature is a CFT on $\mathbb{R}^{d-1} \times S^1_\beta$

- **One-point functions** now enter CFT data, e.g. for scalar operator $\mathcal{O}$, $\langle \mathcal{O} \rangle_\beta = \dfrac{b_\mathcal{O}}{\beta^{\Delta_\mathcal{O}}}$
- If $b_\mathcal{O} \neq 0$, the vev is **non-vanishing at all temperatures**

L. Iliesiu et al., *JHEP* 2018.10: 1-71 (2018)

Main question: does a non-trivial unitary and local CFT at finite T exist, that features SSB?

Conjecture: $\mathrm{O}(N) \times \mathbb{Z}_2$ symmetric models in $d = 2+1$ at biconical fixed-point

N. Chai et al., *Phys. Rev. Lett. 125, 131603 (2020)*

Towards a theory with persistent SSB

$\mathrm{O}(N) \times \mathbb{Z}_2$ symmetric theory with $\mathrm{O}(N)$ vector ϕ and Ising field χ

$$S[\phi,\chi] = \int d^dx \left[\frac{1}{2}(\partial\phi)^2 + \frac{1}{2}(\partial\chi)^2 + \frac{1}{2}m_\phi^2\phi^2 + \frac{1}{2}m_\chi^2\chi^2 + \frac{\lambda_\phi}{8}(\phi^2)^2 + \frac{\lambda_\chi}{8}\chi^4 + \frac{\lambda_{\phi\chi}}{4}\phi^2\chi^2\right]$$

Evidence for persistent SSB of $\mathrm{O}(N) \times \mathbb{Z}_2 \to \mathrm{O}(N)$ collected

- in $d = 4 - \epsilon$, i.e. unitarity is violated for $\epsilon > 0$
- in $d = 2 + 1$ for the non-local (or long-range) model or infinite N

N. Chai et al., *Phys. Rev. Lett. 125, 131603 (2020)*
N. Chai et al., *Phys. Rev. D 102, 065014 (2020)*
Chaudhuri et al., *JHEP 2021.8: 1-38 (2021)*

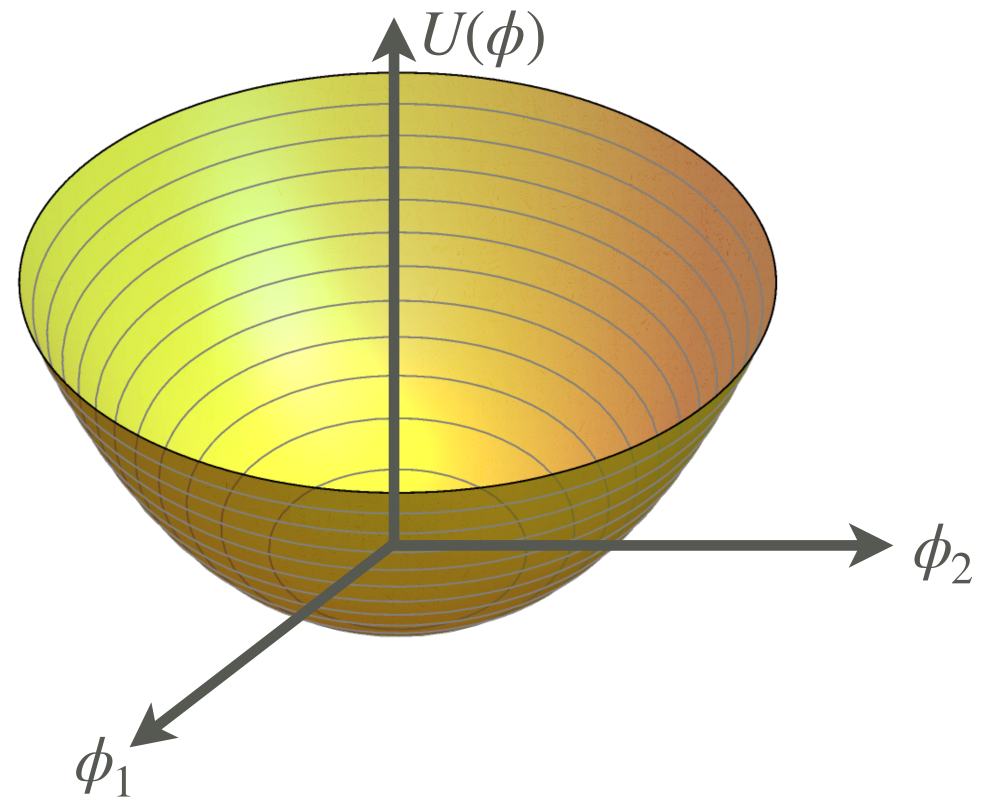
Here, we show by using **functional RG** in $d = 2 + 1$, that

- **Mermin-Wagner theorem** is **fulfilled** wherever applicable
- the theory **has** a **UV completion** at the **biconical fixed-point**
- **SSB** of $\mathbb{Z}_2$ **persist** to all temperatures if N is **sufficiently large**

LPA of O(N) model

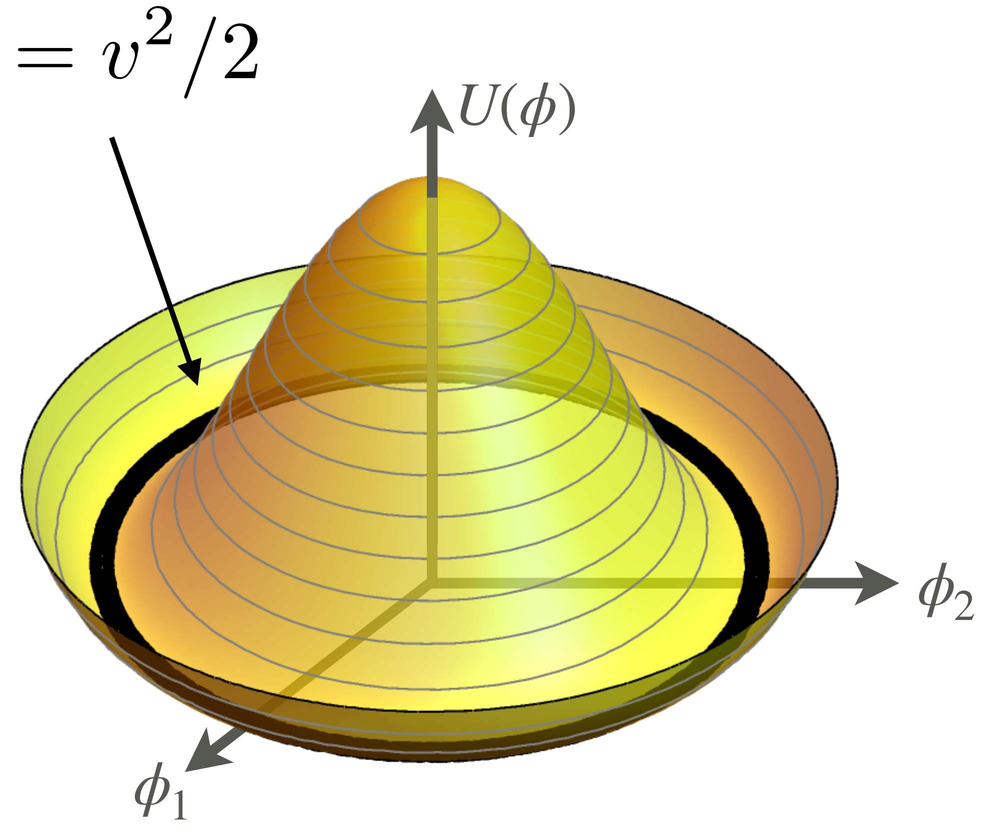
Extended Local Potential Approximation (LPA'):

$$\Gamma_k[\phi] = \int d^d x \left(\frac{Z_k}{2} (\partial\phi)^2 + U_k[\phi] \right)$$



$$U_k = \frac{1}{2} m^2 \phi^2 + \frac{\lambda}{8} (\phi^2)^2$$

$\lambda, m^2 > 0$



$$U_k = \frac{\lambda}{2} \left(\frac{\phi^2}{2} - \kappa \right)^2$$

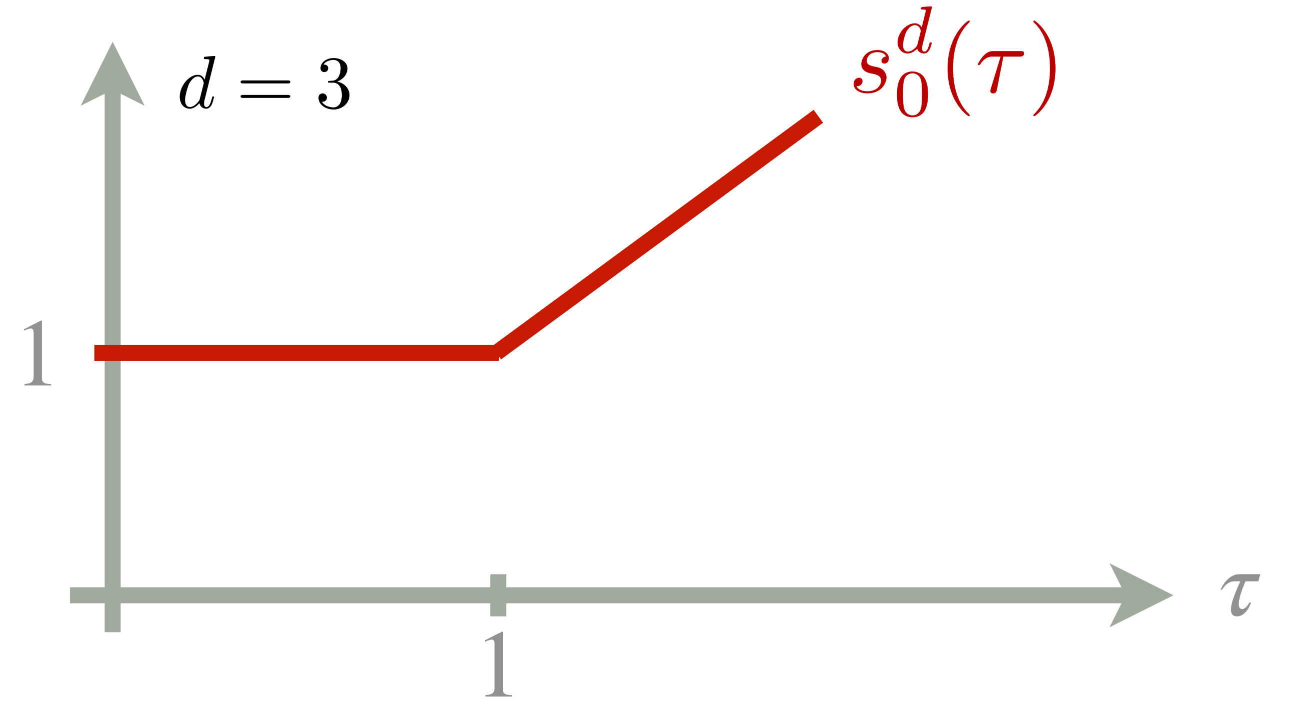
$\lambda, \kappa > 0$

Continuation to finite temperature

$$q_0 \to i\omega_n, \qquad \int \frac{d^D q}{(2\pi)^D} \to T \sum_{n\in\mathbb{Z}} \int \frac{d^d q}{(2\pi)^d}$$

Flow of dimensionless effective potential $u = k^{-d} U_k$

as a function of $\rho = \phi^2/2$ and $\tau = 2\pi T/k$



$$\partial_t u = -du + (d-2)\rho u' + \frac{4v_d}{d} s_0^d(\tau) \frac{1}{1 + u' + 2\rho u''} + \frac{4v_d}{d} s_0^d(\tau) \frac{N-1}{1+u'}$$

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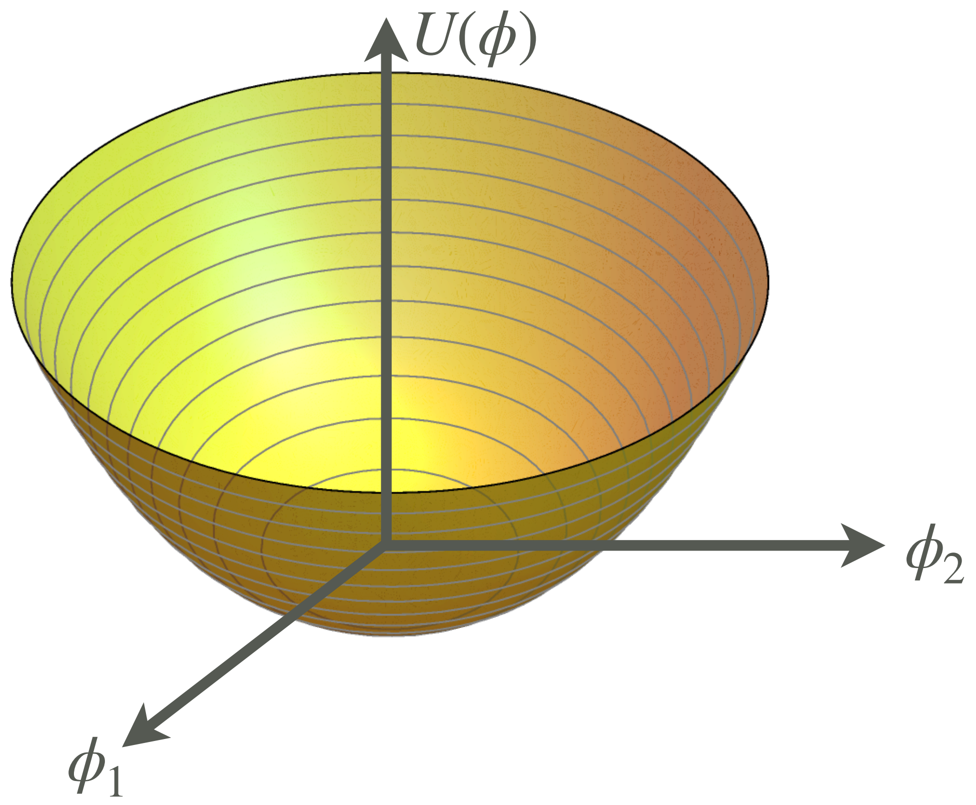
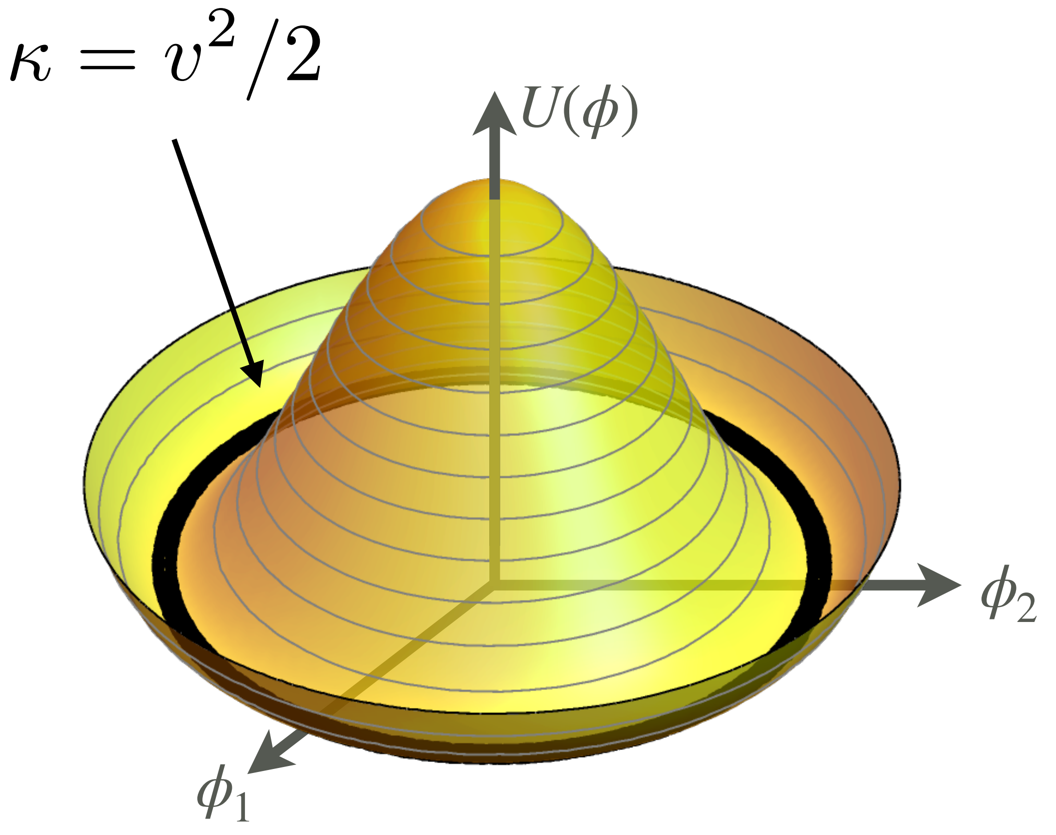


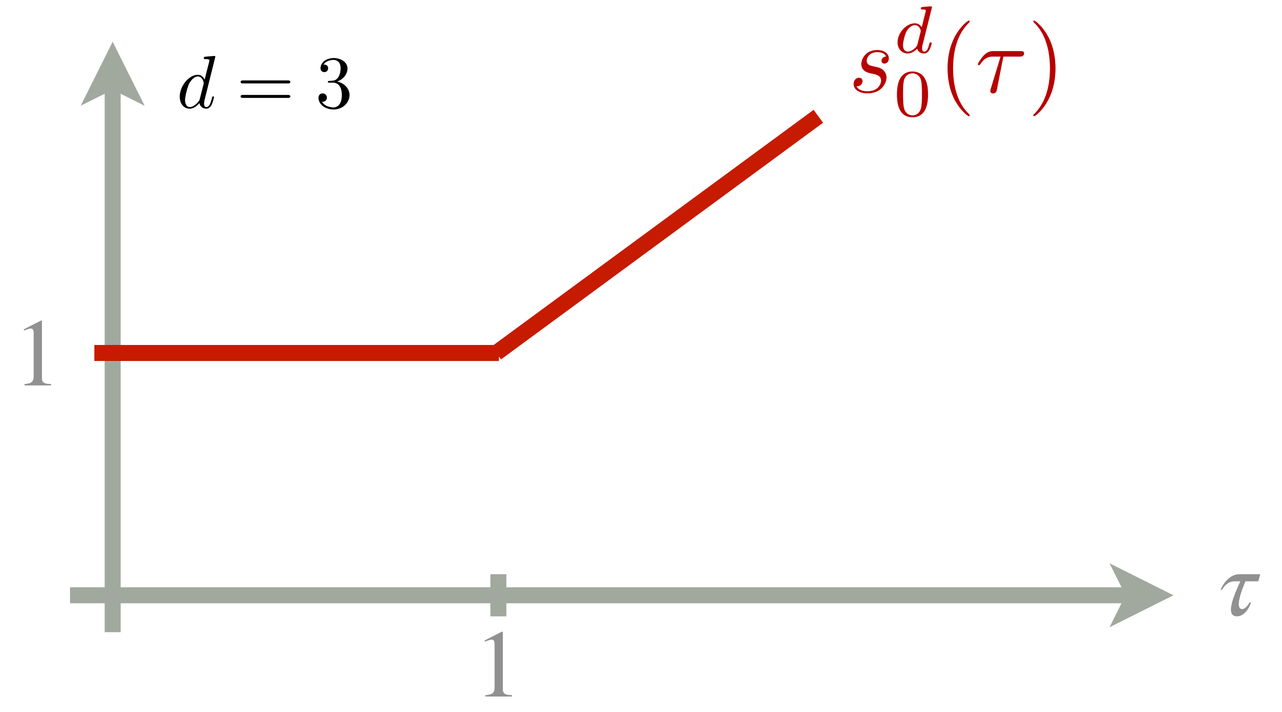
$$U_k = \frac{1}{2} m^2 \phi^2 + \frac{\lambda}{8} (\phi^2)^2$$
$$\lambda, m^2 > 0$$



$$U_k = \frac{\lambda}{2} \left(\frac{\phi^2}{2} - \kappa \right)^2$$
$$\lambda, \kappa > 0$$

Assume $\kappa > 0$ at small scales $k < 2\pi T$

$$k\partial_k \kappa = \frac{4v_{d-1}}{d-1} T(N-1) k^{d-3} + \mathcal{O}(k^{d+1})$$



➡ $d = 1+1: \quad \partial_k \kappa \sim T(N-1)/k$

$d = 2+1: \quad \partial_k \kappa \sim T(N-1)/k^2$

➡ **not possible** to maintain $\lim_{k\to 0} \kappa > 0$ if $d \leq 3$ and $N > 1$

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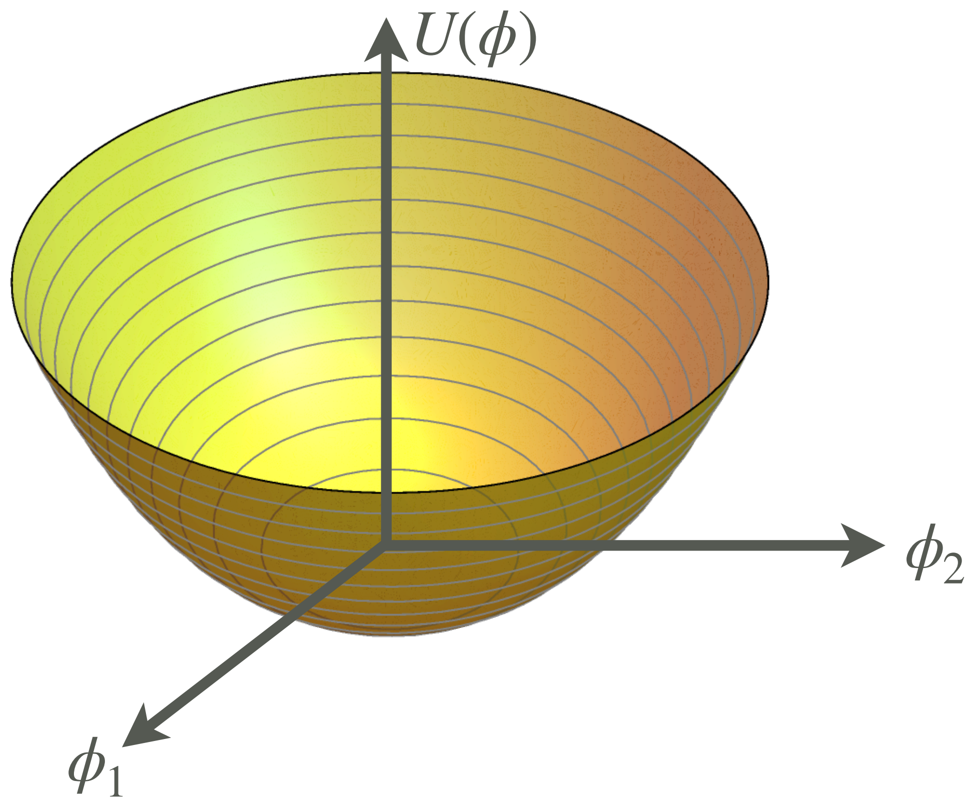

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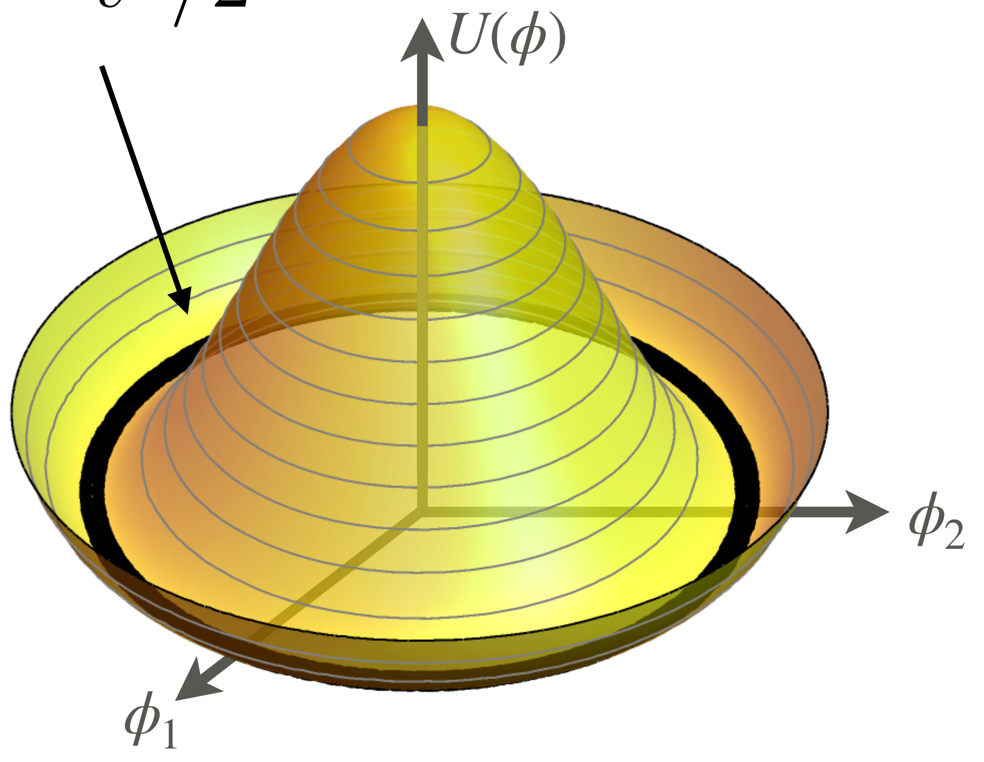
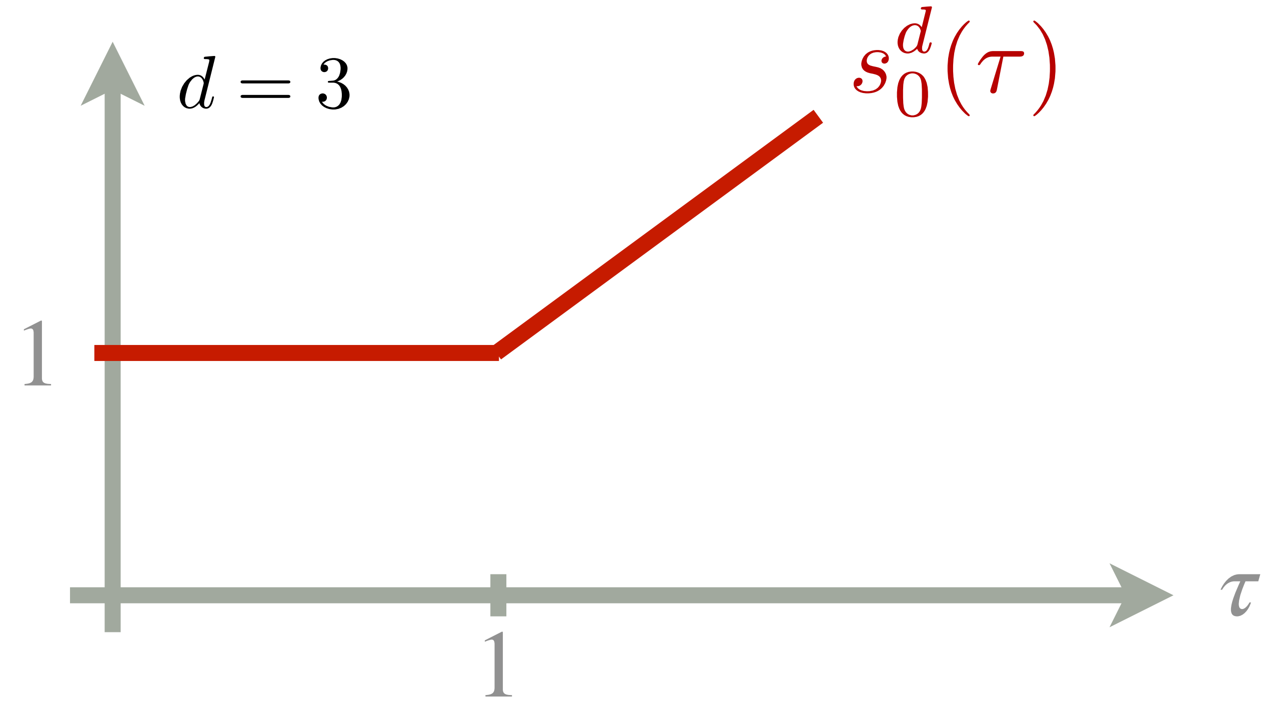


$$U_k = \frac{\lambda}{2} \left(\frac{\phi^2}{2} - \kappa \right)^2$$

$$\lambda, \kappa > 0$$

Assume $m^2 > 0$ at some RG scale $k > 0$

$$k\partial_k m^2 = -\frac{4v_d}{d} s_0^d(\tau)(N+2) \frac{\lambda}{(k^2+m^2)^2} < 0$$



➔ **Restored symmetry** can **not** be broken again

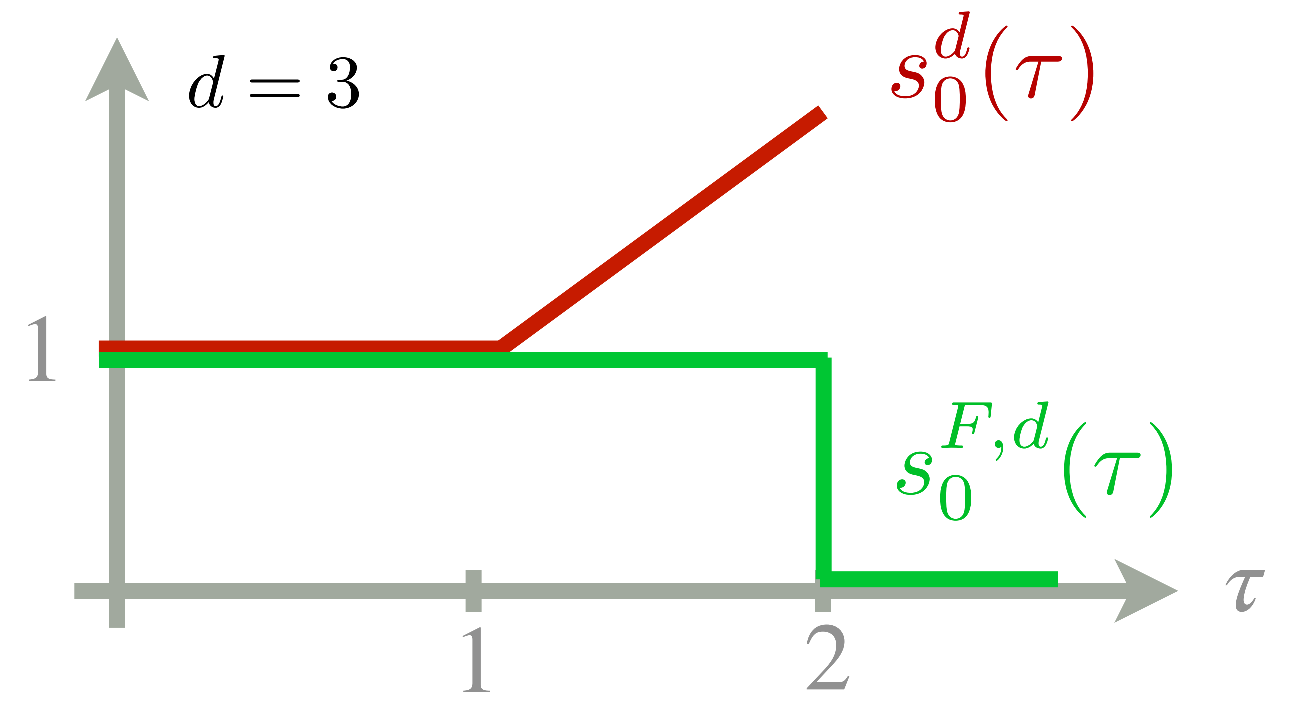
How can we drive high temperature SSB?

Couple fermions?

$$k\partial_k m^2 = -\frac{4v_d}{d}s_0^d(\tau)(N+2)\frac{\lambda}{(k^2+m^2)^2} + \frac{4v_d}{d}s_0^{F,d}(\tau)h^2$$

➜ at high temperatures, fermions **decouple**

→ recall talk by Mireia:



How can we drive high temperature SSB?

What about bosons?

Action of $\mathrm{O}(N) \times \mathbb{Z}_2$ theory at UV scale

$$S[\phi, \chi] = \int d^d x \left[\frac{1}{2}(\partial\phi)^2 + \frac{1}{2}(\partial\chi)^2 + \frac{1}{2}m_\phi^2 \phi^2 + \frac{1}{2}m_\chi^2 \chi^2 + \frac{\lambda_\phi}{8}(\phi^2)^2 + \frac{\lambda_\chi}{8}\chi^4 + \frac{\lambda_{\phi\chi}}{4}\phi^2\chi^2 \right]$$

Scalar potential **bounded** from below if $\lambda_\phi \lambda_\chi \geq \lambda_{\phi\chi}^2$ ➔ $\lambda_{\phi\chi} < 0$ possible!

LPA' flow equations

$$\begin{aligned} \partial_t u = & - Du + (D-2+\eta_\phi)\bar{\rho}_\phi u^{(1,0)} + (D-2+\eta_\chi)\bar{\rho}_\chi u^{(0,1)} && \text{canonical scaling} \\ & + I_R^D(\omega_\chi, \omega_\phi, \omega_{\phi\chi}) S_\phi(\tau) + (N-1) I_G^D(u^{(1,0)}) S_\phi(\tau) && \mathrm{O}(N) \text{ sector} \\ & + I_R^D(\omega_\phi, \omega_\chi, \omega_{\phi\chi}) S_\chi(\tau) && \mathbb{Z}_2 \text{ sector} \end{aligned}$$

with 'masses' $\omega_\phi = u_k^{(1,0)} + 2\bar{\rho}_\phi u_k^{(2,0)}$, $\omega_\chi = u_k^{(0,1)} + 2\bar{\rho}_\chi u_k^{(0,2)}$ and $\omega_{\phi\chi}^2 = 4\bar{\rho}_\phi \bar{\rho}_\chi (u_k^{(1,1)})^2$

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Scalar potential **bounded** from below if $\lambda_\phi\lambda_\chi \geq \lambda_{\phi\chi}^2$ ➡ $\lambda_{\phi\chi} < 0$ possible!

In $d = 2+1$, $\mathrm{O}(N)$ symmetry is **always restored** at a finite scale for $N > 1$.

Assume $\mathbb{Z}_2$ is **preserved** at that scale

$$k\partial_k m_\chi^2 = \frac{4v_3}{3}k^5 s_0^3(\tau)\left(-\frac{3\lambda_\chi}{(k^2+m_\chi^2)^2} - \frac{N\lambda_{\phi\chi}}{(k^2+m_\phi^2)^2}\right)$$

➡ $k\partial_k m_\chi^2 < 0$ for $\lambda_{\phi\chi} < 0$ and sufficiently large N!

… and SSB of $\mathbb{Z}_2$ **not excluded** by Mermin-Wagner!

↗ approx. if $3\lambda_\chi < N|\lambda_{\phi\chi}|$

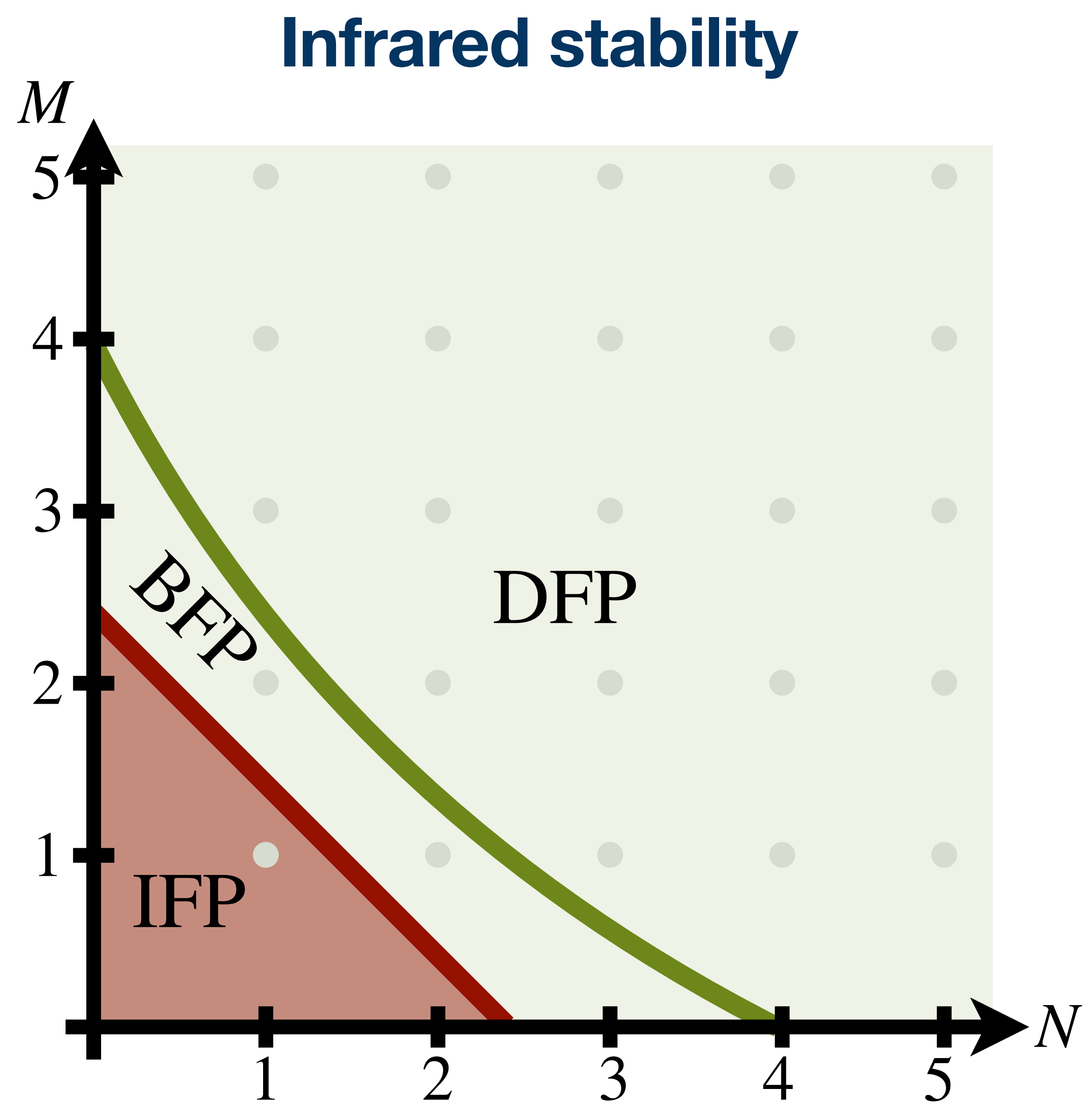
Quantum critical point

$\mathrm{O}(N) \times \mathrm{O}(M)$ theories feature several interacting fixed-points in $d = 2 + 1$

- **decoupled Wilson-Fisher** fixed point (DFP): $\lambda_\phi^*, \lambda_\chi^* > 0, \lambda_{\phi\chi}^* = 0$
- **isotropic** fixed point (IFP): $\lambda_\phi^* = \lambda_\chi^* = \lambda_{\phi\chi}^* > 0$
- **biconical** fixed point (BFP): $\lambda_\phi^*, \lambda_\chi^* > 0, \lambda_{\phi\chi}^* \neq 0$

➜ BFP has three relevant directions for $N \geq 3$ and is hence unstable

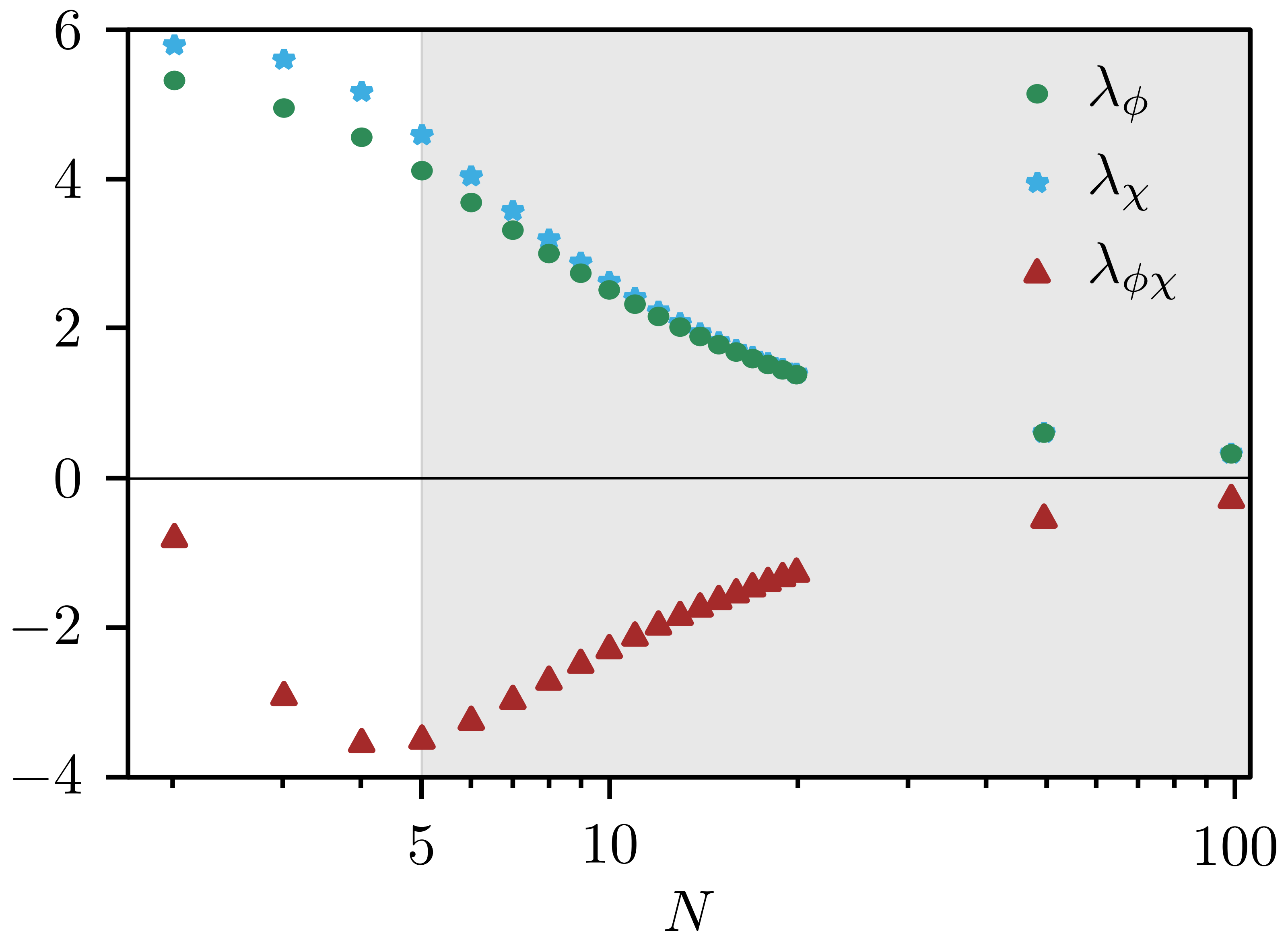
…but it features $\lambda_{\phi\chi}^* < 0$!



Calabrese et al., *Physical Review B 67 (5), 054505 (2003)*

Eichhorn et al., *Phys. Rev. E 88, 042141 (2013)*

$N = 10$	$\bar{\kappa}_\phi$	$\bar{\kappa}_\chi$	$\bar{\lambda}_\phi$	$\bar{\lambda}_\chi$	$\bar{\lambda}_{\phi\chi}$	θ_1	θ_2	θ_3
LPA6	0.25	0.10	2.62	2.54	-2.34	2.02	1.06	0.61
LPA8	0.24	0.09	2.59	2.84	-2.43	1.98	1.07	0.61
LPA′6	0.24	0.10	2.50	2.61	-2.30	1.99	1.09	0.56
LPA′8	0.24	0.09	2.47	2.81	-2.34	1.95	1.09	0.57



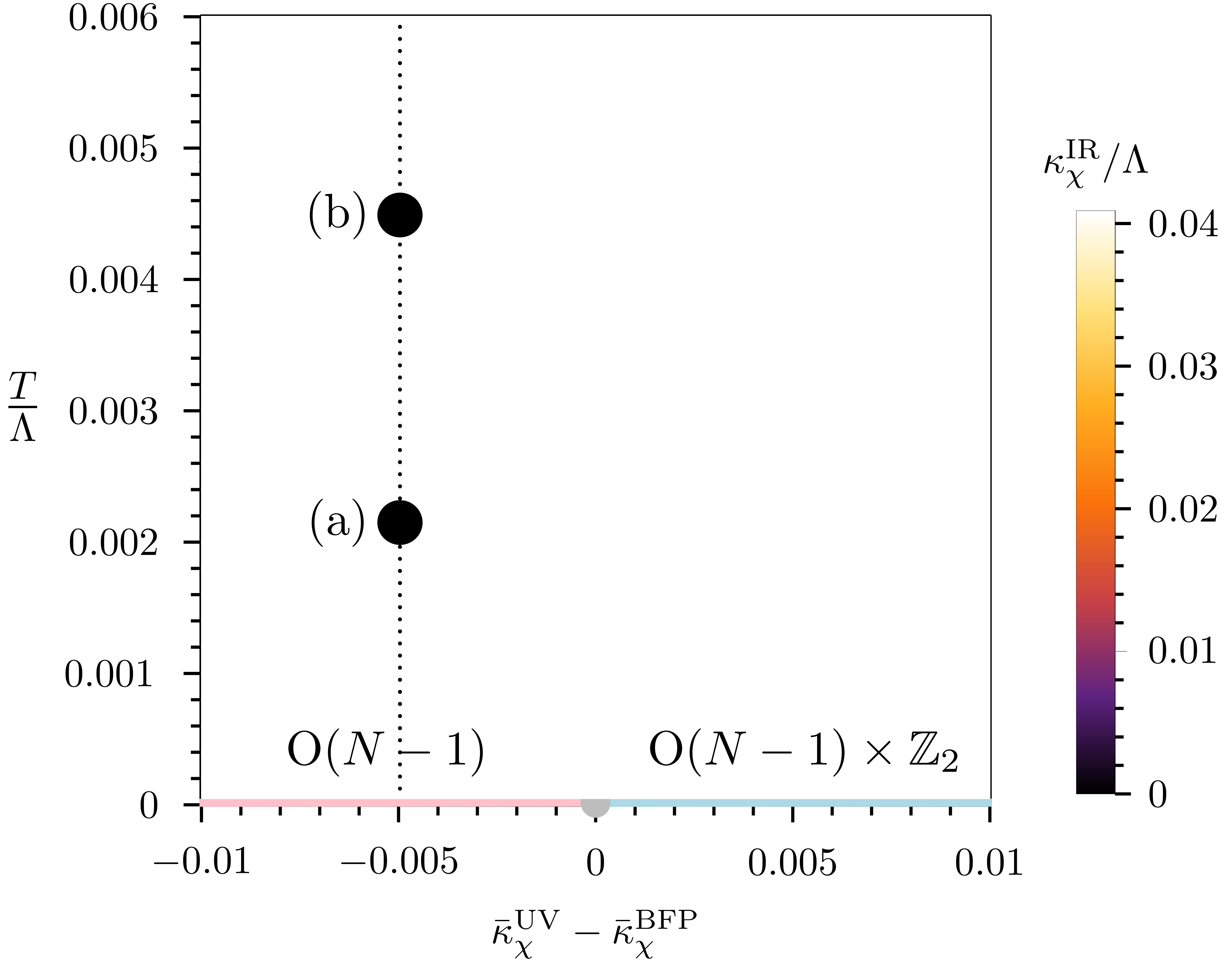
N=2	ν	η_ϕ	η_χ
5-loop	0.70(3)	0.037(5)	0.037(5)
FRG, LPA'6	0.68	0.040	0.040

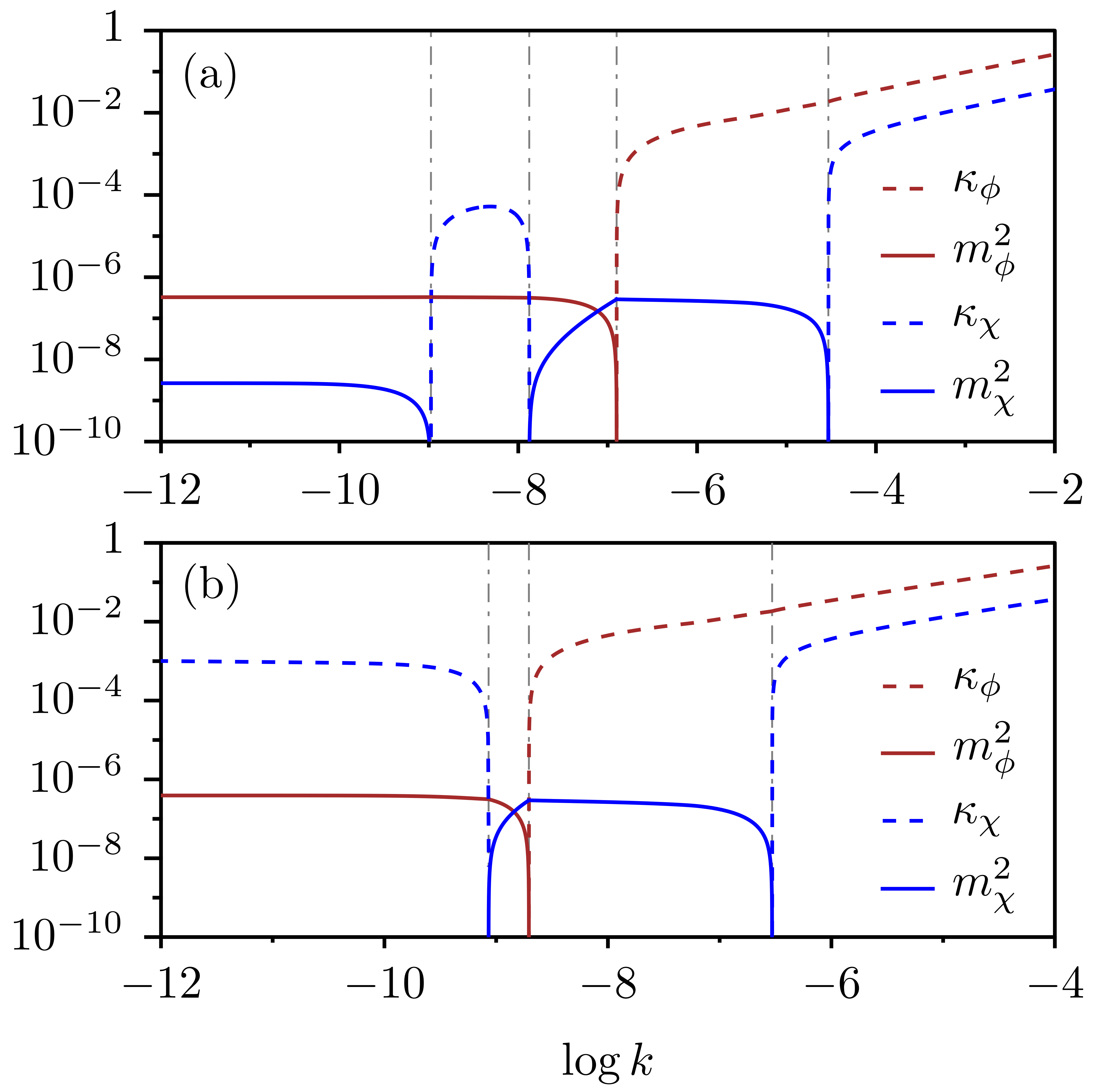
Calabrese et al., *Physical Review B 67 (5), 054505 (2003)*

Phase diagram

At $T = 0$: biconical fixed point as quantum critical point with SSB in the UV

➔ relevant coupling to tune in the UV is $\bar{\kappa}_\chi$





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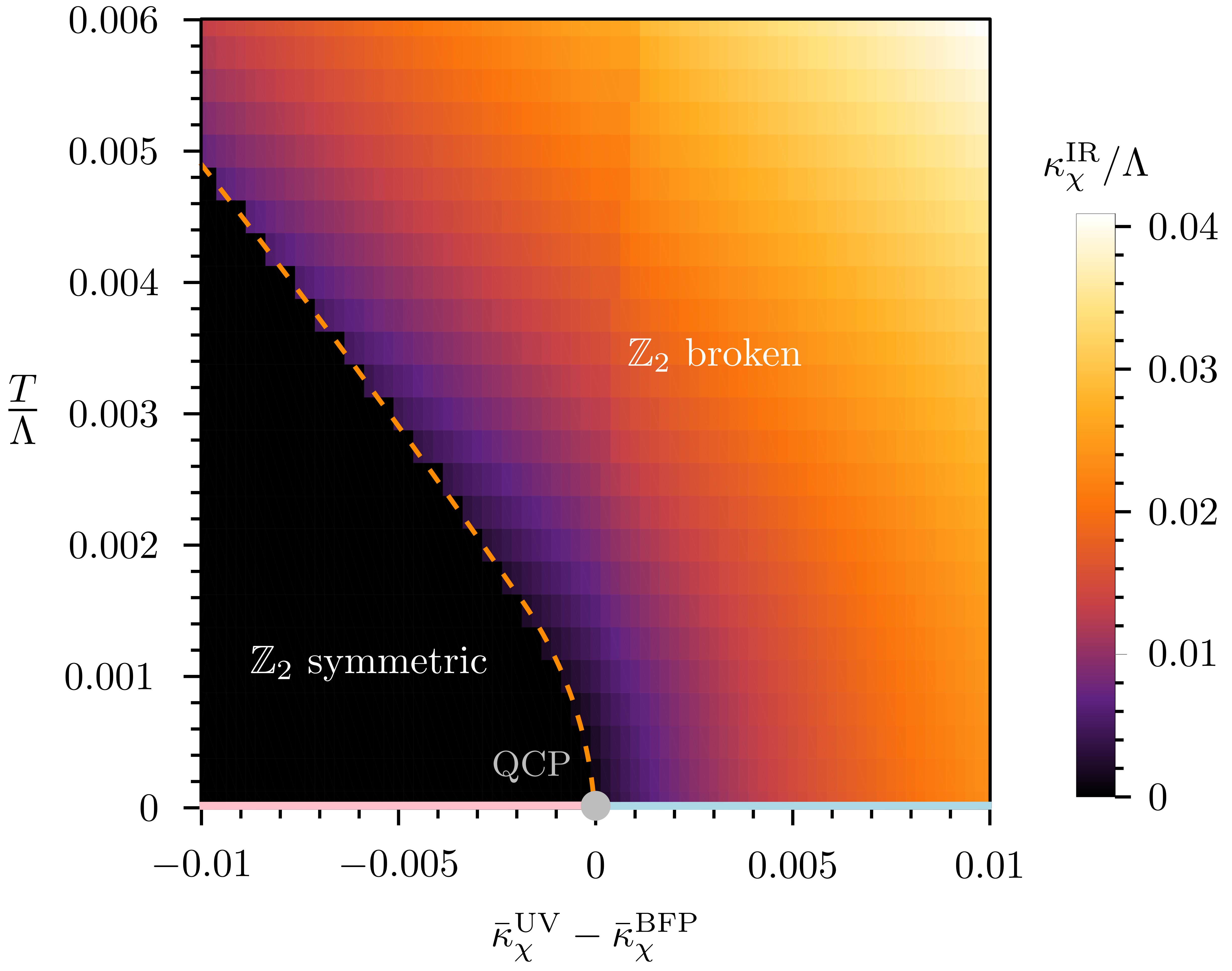
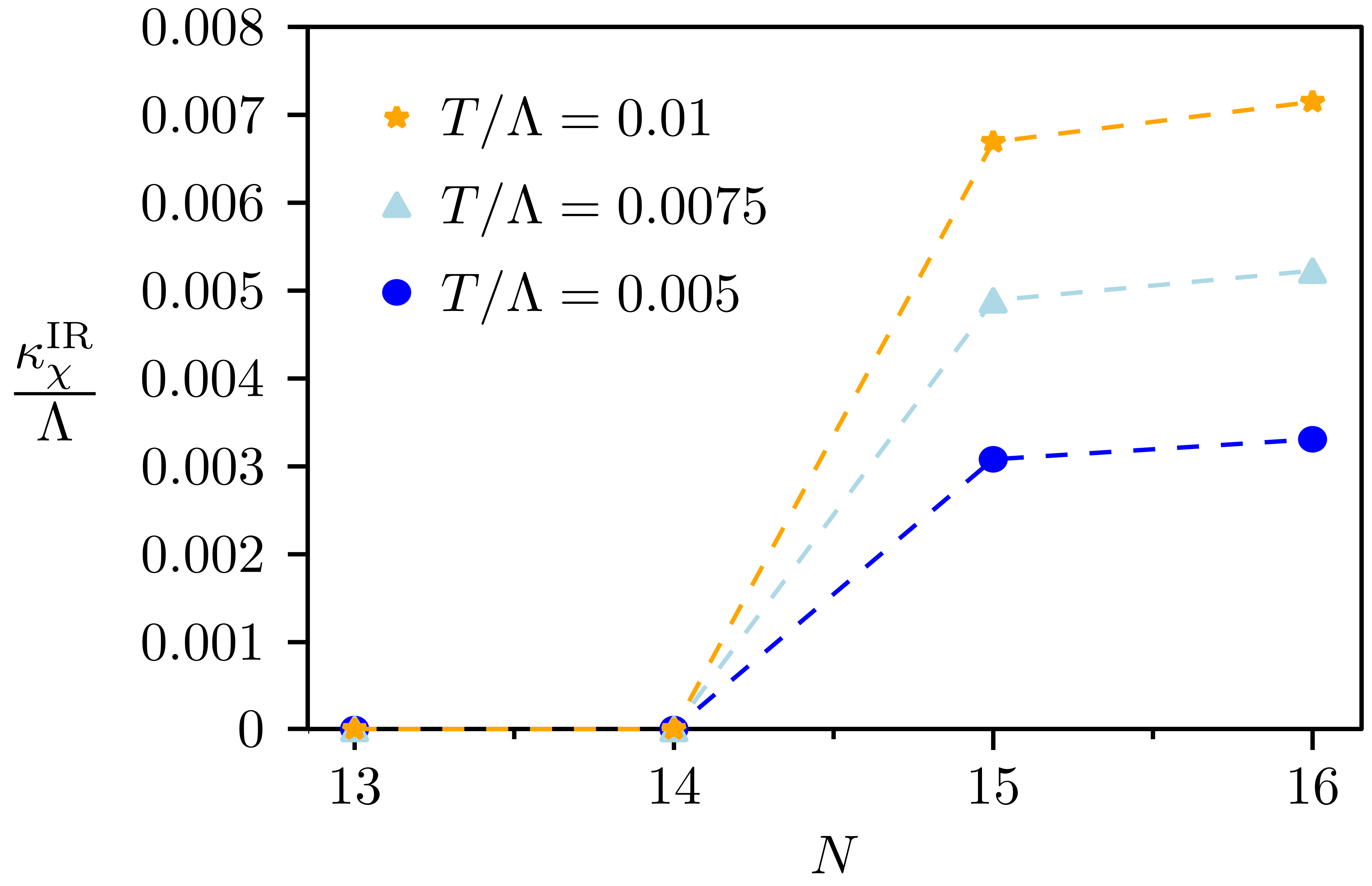




➔ persistent SSB for $N \geq N_c = 15$

Summary

High temperatures ⟶ **Entropy** is **maximized** in thermal equilibrium

⟶ **System** is expected to be in a **disordered** state

In nature: Systems contradicting this expectation, e.g., **Pomeranchuk effect** or **Rochelle salt**

We showed, that biconical $\mathrm{O}(N) \times \mathbb{Z}_2$ models in $d = 2 + 1$

- … posses a UV completion at the **biconical fixed-point**
- … spontaneously break $\mathbb{Z}_2$ symmetry at all temperatures for $N \geq N_c = 15$ above BFP

⟶ arXiv:2409.10606

Very recent (last Friday) follow-up:

Komargodski and Popov: shown for **large** but **finite** N

Komardgoski and Popov, *arXiv:2412.09459 (2024)*

Outlook

In $d = 2+1$: long-range order **forbidden** in $\mathrm{O}(N)$ at $T > 0$ for $N > 1$

➔ **bare** vev is always vanishing $\langle \phi_a \rangle_\beta = 0$

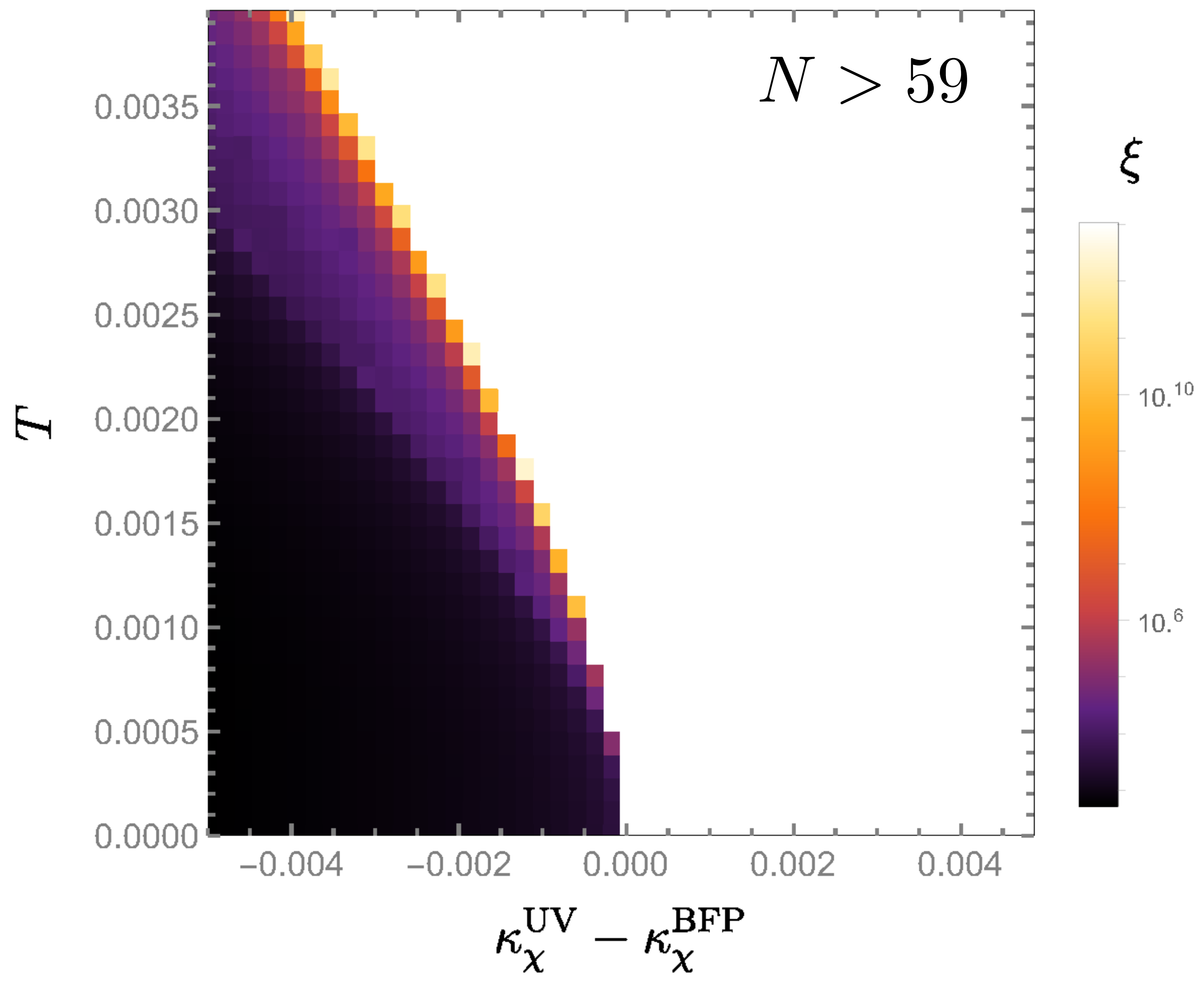
$$\langle \mathcal{O}(x)\mathcal{O}(0) \rangle \sim \frac{1}{|x|^\eta}$$

But: "renormalized" vev can still be finite if $Z_k \to \infty$ in the IR

➔ **Berezenskii-Kosterlitz-Thouless** phase and **quasi long-range order**

➔ line of fixed-points and **infinite corr. length**

➔ can we have a BKT phase at all temperatures?



To what extend is this included in the $\mathrm{O}(2)$ model?!

If it is… ➔ similar spirit as before in $\mathrm{O}(N) \times \mathrm{O}(2)$