Asymptotic Safety within on-shell perturbation theory

Renata Ferrero

based on work in collaboration with Kevin Falls
2411.00938 [hep-th]

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Asymptotic Safety meets Particle Physics & Friends

DESY, Hamburg

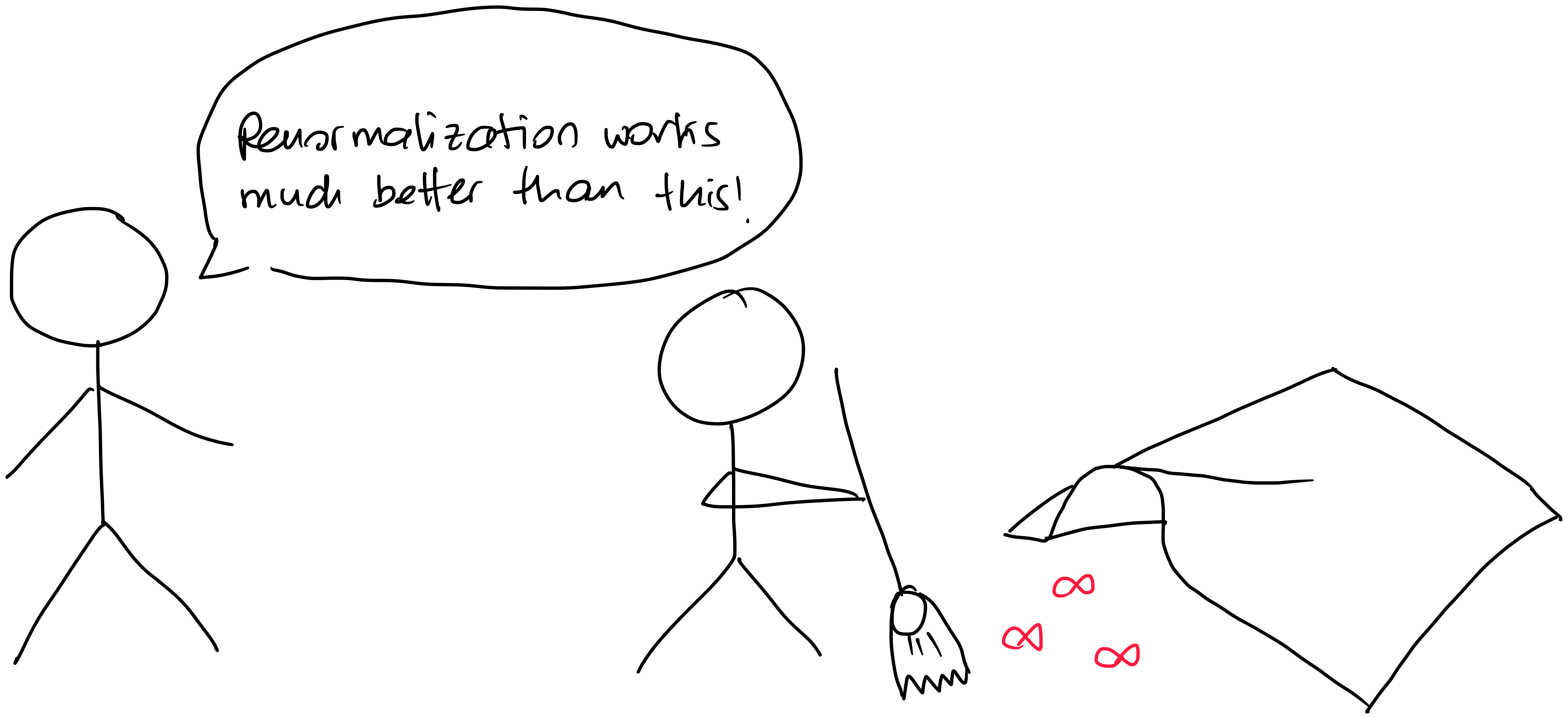
FRIEDRICH-ALEXANDER
UNIVERSITÄT
ERLANGEN-NÜRNBERG

Disclaimer

I will talk about Einstein gravity.
However, this approach can also be applied to different theories.

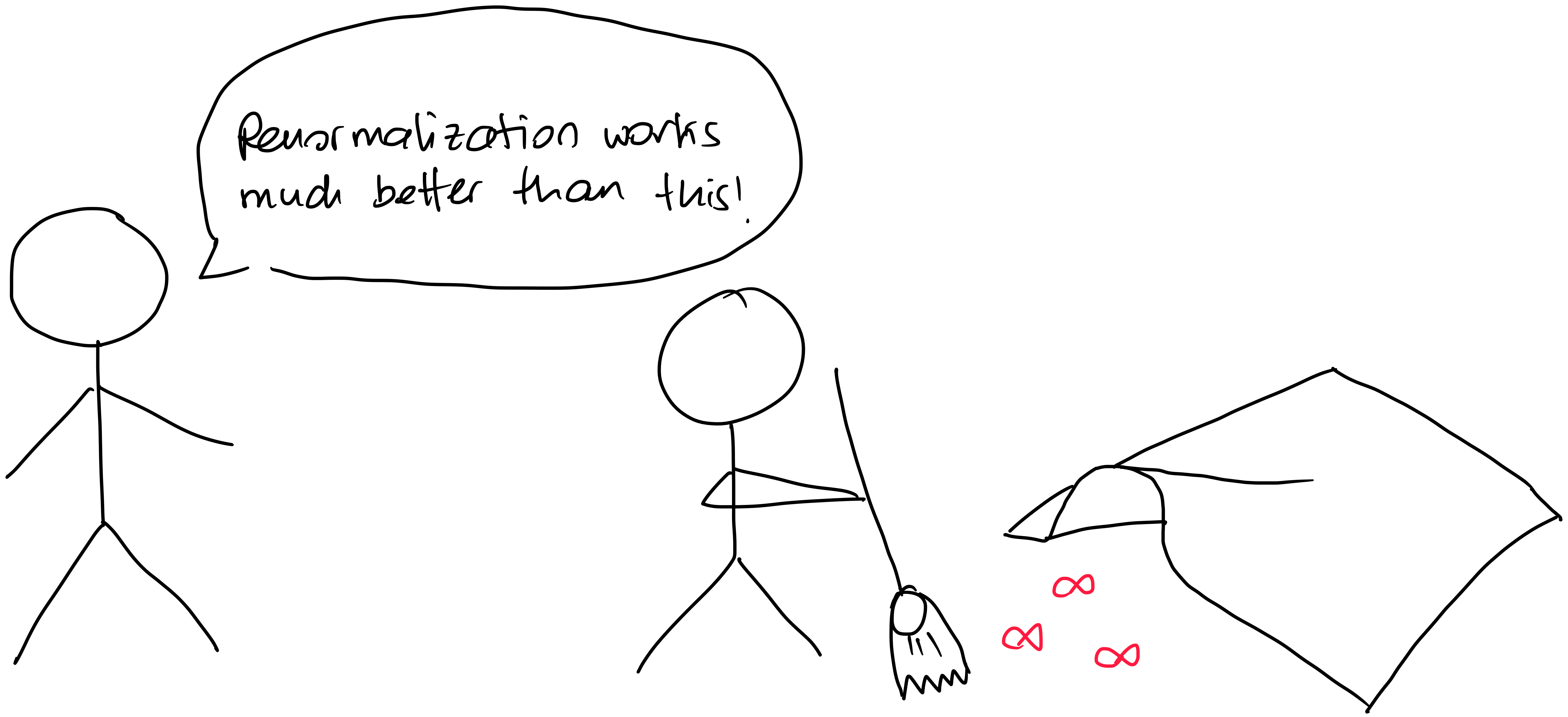
Gravity is perturbatively non-renormalizable at two-loops.

Asymptotic Safety



Asymptotic Safety

Weinberg's conjecture: There exists a nonperturbative dynamical mechanism which renders physical scattering amplitudes finite and computable at energy scales exceeding the Planck scale: a nontrivial UV fixed point.



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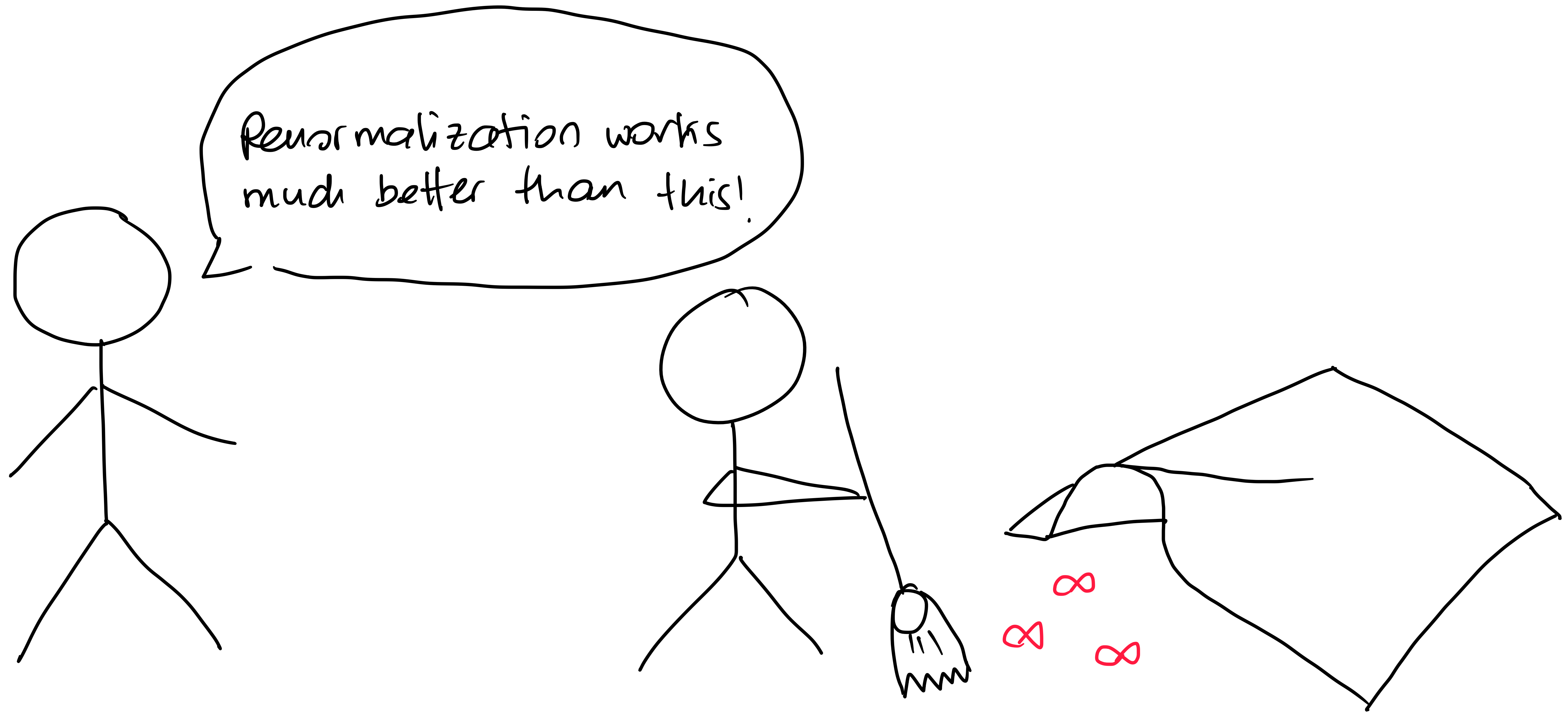
Usually technically investigated via the **Functional Renormalization Group**

Dynamical Triangulations



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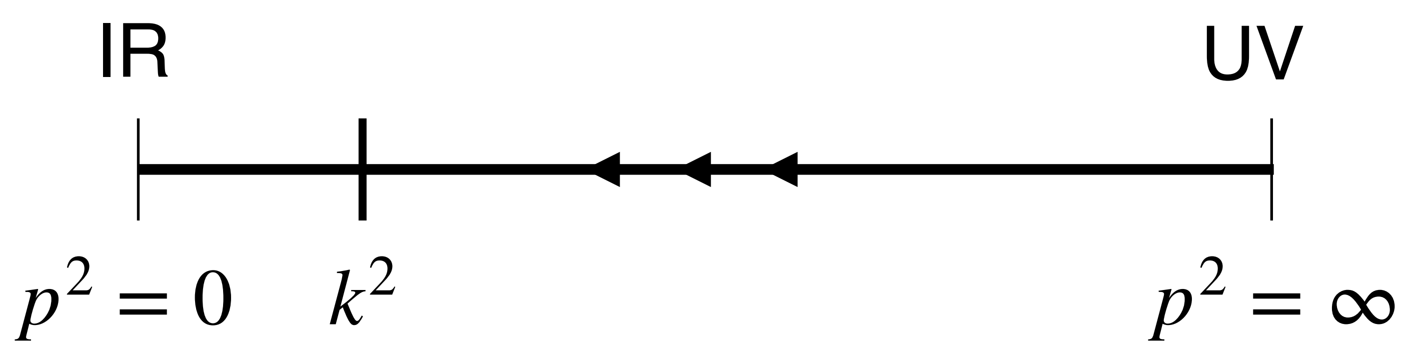
It represents the scale-dependent version of the standard effective action.



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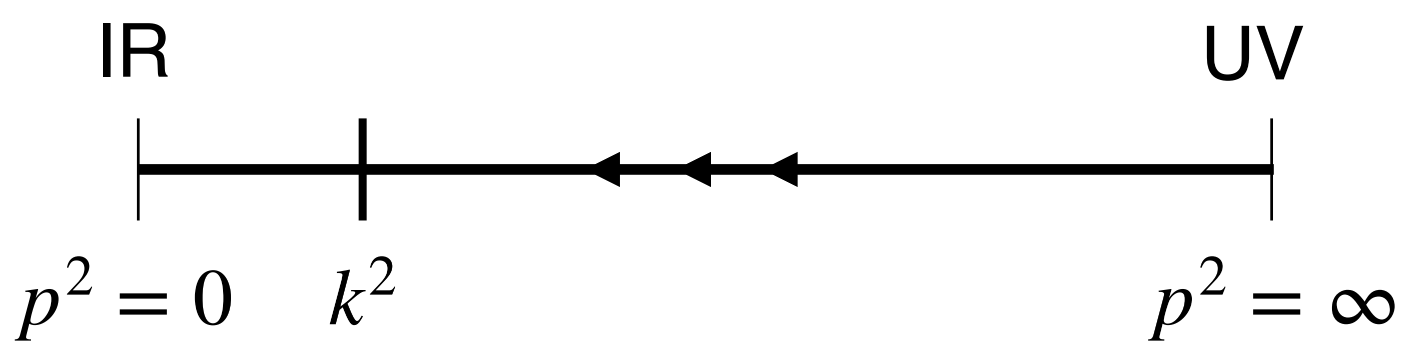

$$\lim_{k \to 0} \Gamma_k \to \Gamma \qquad \lim_{k \to \infty} \Gamma_k \to S$$

It satisfies the Functional Renormalization Group Equation:

$$k\partial_k \Gamma_k[\phi] = \frac{1}{2} \mathrm{Tr} \left[(\Gamma_k^{(2)} + \mathcal{R}_k)^{-1} \, k\partial_k \mathcal{R}_k \right]$$

- UV- and IR finite
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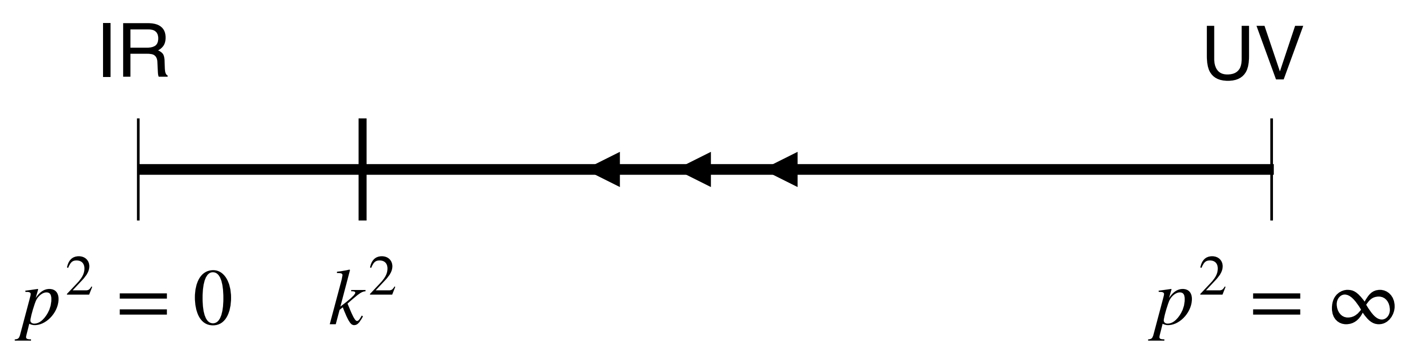

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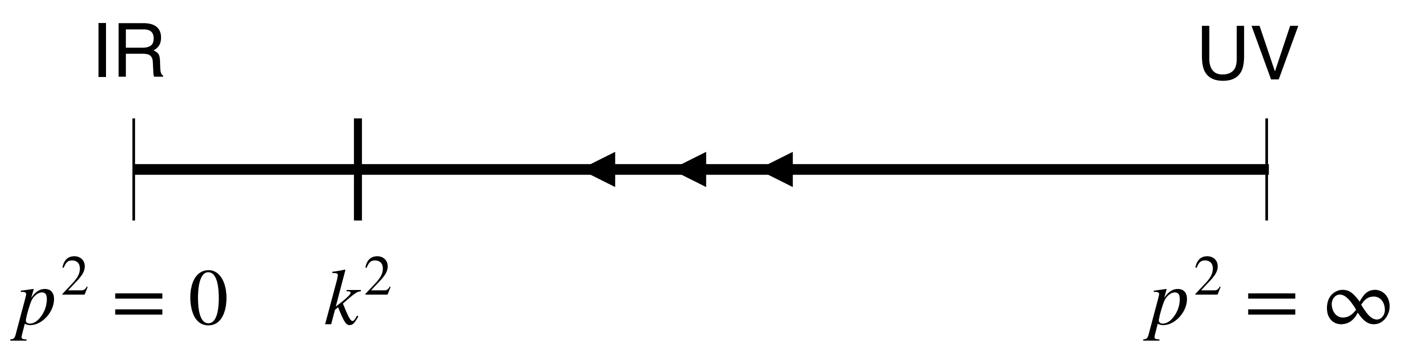

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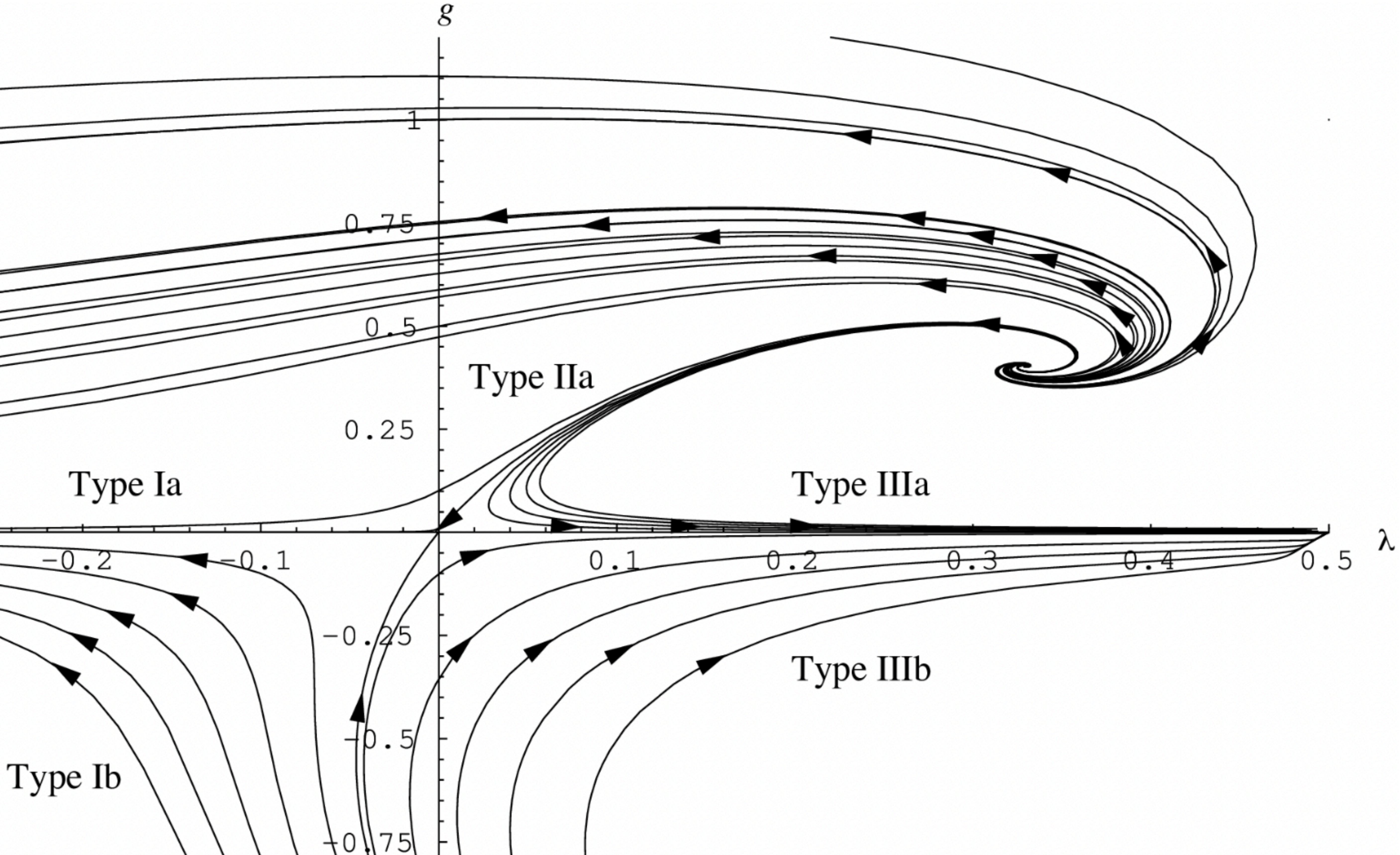
Predictive solutions do exist in theories that are otherwise perturbatively non-renormalizable.

Asymptotic Safety via FRG: A given trajectory has an acceptable UV limit, if and only if its endpoint in the UV is given by the nontrivial fixed point of the RG flow.

Einstein-Hilbert

[Reuter 1996, Reuter-Saueressig and Percacci's book]

$$\Gamma_k[h;\bar{g}] = \frac{1}{16\pi\, G(k)} \int \mathrm{d}^4x\, \sqrt{-g}\, \Big(R(g) - 2\Lambda(k)\Big)\Big|_{g=\bar{g}+h} + \text{gauge fixing} + \text{ghosts}$$



$$g(k) = G(k)k^2 \qquad \lambda(k) = \frac{\Lambda(k)}{k^2}$$

$$\begin{cases} k\partial_k g_k = \beta_g(g,\lambda) \\ k\partial_k \lambda_k = \beta_\lambda(g,\lambda) \end{cases}$$

$$\begin{cases} \beta_g(g_*,\lambda_*) = 0 \\ \beta_\lambda(g_*,\lambda_*) = 0 \end{cases}$$

$g_* = \lambda_* = 0$

Gaussian fixed point

$g_* > 0, \quad \lambda_* > 0$

Non-Gaussian fixed point

ASYMPTOTIC SAFETY

Motivation

Perturbative renormalization	Non-perturbative renormalization
Dimensional regularization, proper time, …	Asymptotic Safety - Functional Renormalization Group

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The fixed point of order lies beyond
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no unphysical dependencies

On-shell effective action results tested in precision measurements in QCD.

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Perturbative renormalization	Non-perturbative renormalization
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Gravity is perturbative non-renormalizable	UV-completion via an interacting fixed point
E.g. dimensional regularization in $d = 2 + \epsilon$ The fixed point of order lies beyond perturbation theory. **On-shell effective action**: no unphysical dependencies	Effective action contains **unphysical information**: Field parametrization - gauge dependence 1. Effective action is **off-shell** 2. Regulator **breaks symmetry** (diffeomorpshism invariance)
On-shell effective action results tested in precision measurements in QCD.	Extract physical information from the flow of the effective action is a arduous task.

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Perturbative renormalization	Non-perturbative renormalization
Perturbative non-renormalizable (−)	Non-perturbative renormalizable (+)
Contains only physical information (+)	Affected by non-physical information (−)

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New subtraction scheme + essential renormalization group

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Use perturbative methods to investigate asymptotic safety

E.g. dimensional regularization with non-minimal subtraction scheme

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Do a fully functional approximation to keep invariants to all orders:
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RG improvement of one-loop effective action “looks like” non-perurbative RG

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Essential RG: RG scheme to keep unphysical dependencies under control

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1. One-loop effective action

For **Einstein gravity**:

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Faddeev-Popov ghost's term

DeWitt metric

Hessian

gauge fixing: e.g. background covariant harmonic gauge

$$S = \int \mathrm{d}^d x\, \sqrt{g}\left(\frac{\rho}{8\pi} - \frac{R}{16\pi G} + \vartheta \mathfrak{E}(g)\right)$$

Vacuum energy

Euler topological term in d = 4

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In this first analysis: investigation of the **parametrization dependence** [Gies, Knorr, Lippoldt 1507.08859]

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu} + \frac{1}{2}\left(\tau_1 h_{\mu\rho}h^{\rho}_{\nu} + \tau_2 h h_{\mu\nu} + \tau_3\bar{g}_{\mu\nu}h_{\rho\sigma}h^{\rho\sigma} + \tau_4\bar{g}_{\mu\nu}h^2\right) + O(h^2)$$

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Background field method:

$$S^{(2)\mu\nu,\rho\lambda}(x,y) = \frac{\delta S}{\delta h_{\mu\nu}(x)\delta h_{\rho\lambda}(y)}$$

Use the EoM for S: dependence on the parametrization should disappear.

2. Dimensional regularization

Revisit Weinberg's original conjecture of Asymptotic safety: gravity has a fixed point in $d = 2 + \epsilon$

[Weinberg, Niedermaier, Benedetti, Falls, Jack & Jones, Christensten & Duff, Kawai, Kluth 2409.09252, …]

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New subtraction scheme: keep (power law) divergences that appear also in $d_c = 0, 2, 4$

preserves gauge simmetry

Idea: in gravity we should keep track of **two dimensionalities**.
One gets regularized, one is dynamical $d = g^\mu_\mu$ (components of the field)

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Expand the trace up to second order in curvature, exploiting the EOM

$$\Gamma = \int \mathrm{d}^d x \, \sqrt{g} \left(\frac{a_0(d)}{d} \mu^d + \frac{a_2(d)}{d-2} \mu^{d-2} R + \frac{a_4(d)}{d-4} \mu^{d-4} G \rho R + \frac{a_4'(d)}{d-4} \mu^{d-4} \mathfrak{E} \right) + \text{finite terms}$$

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1. We do not need to introduce counter-terms outside the Einstein-Hilbert action.
2. The vacuum energy is only renormalised due to the singular term in $d = 0$ dimensions.

2. Dimensional regularization

Non-minimal subtraction scheme

Compute traces using proper time techniques and evaluate UV singular part $\int_0^\infty \mathrm{d}s\, s^{\frac{d_c-d}{2}-1} \sim -2\frac{\mu^{d-d_c}}{d-d_c}$

Keep $d = g^\mu_\mu$ distinct from d_c, in order not to identify the components of the metric with the regularization parameter.

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The HK coefficients then become

$$\begin{aligned}
\bar{a}_0(d) &= \frac{1}{2}(d-3)d \\
\bar{a}_2(d) &= \frac{1}{(4\pi)}\frac{1}{12}\left(d^2-3d-36\right) \\
\bar{a}_4(d) &= \frac{1}{(4\pi)^2}\frac{d^3+19d^2-566d+1200}{120(d-2)} \\
\bar{a}_4'(d) &= \frac{1}{(4\pi)^2}\frac{1}{360}\left(d^2-33d+540\right) ,
\end{aligned}$$

2. Dimensional regularization

Essential renormalization group

[Baldazzi, Falls, Zinati 2105.11482, 2107.00671]

Renormalization scheme which restricts the analysis to the running of the **essential couplings**

Couplings which contribute to the **scaling of physical observables** such as scattering cross sections (scaling exponents)

Inessential couplings associated with redundant operators
→ **fixed by renormalization conditions** achieved by a field reparameterisation along the RG flow.

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For the free theory (flat space, GFP)

$$S = \int \mathrm{d}^d x \, \sqrt{g} \left(\frac{\rho_k}{8\pi} - \frac{R}{16\pi G_k} + \vartheta \mathfrak{E}(g) \right)$$

$$S_{\mathrm{ct}} = - \int \mathrm{d}^d x \, \sqrt{g} \bar{a}_0(d) \frac{\mu^d}{d}$$

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$$\beta_{\tilde{\rho}} = -d\tilde{\rho} + 8\pi \bar{a}_0 \longrightarrow \frac{\tilde{\rho}}{8\pi} = \frac{\bar{a}_0}{d} = \frac{1}{2}(d-3)$$

Renormalization condition

2. Dimensional regularization
Essential renormalization group

Essential coupling (dimensionless - invariant under rescaling of metric)

$$\eta \equiv G \left(\frac{\rho}{4\pi(d-3)} \right)^{\frac{d-2}{d}} \quad \to \quad g$$

using ren. condition

$$\beta_\eta = (d-2)\eta + \frac{1}{3}((d-3)d - 36)\eta^2 + \frac{(d-3)(d(d(d+19)-566)+1200)\eta^3}{30(d-2)}$$

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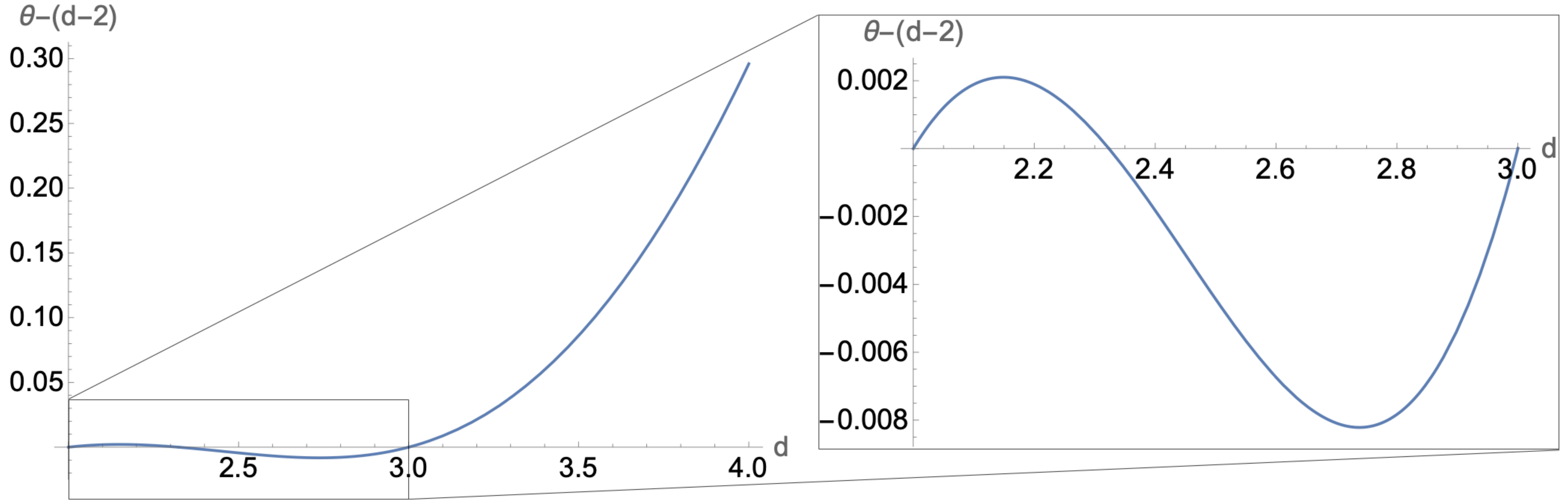
Fixed point and critical exponent (d = 4): $\eta_* = 0.16, \qquad \theta = -\left.\frac{\partial \beta_\eta}{\partial \eta}\right|_{\eta=\eta_*} = 2.296$



This was dim. reg.

Can we implement this into a flow equation?

3. Proper time regularization

Generalized proper time flow

One-loop effective action

$$\Gamma = S + \frac{\hbar}{2} \mathrm{Tr} \log K^{-1} S^{(2)}/M^2 + O(\hbar^2) \ .$$

arbitrary scale, take $M \to \infty$

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evaluate the trace via proper-time parametrization

IR and UV regulated effective action

$$\Gamma_{k,M} = S - \frac{\hbar}{2} \int_{M^{-2}}^{k^{-2}} ds \frac{1}{s} \mathrm{Tr} \left(\exp(-s K^{-1} S^{(2)}) - \exp(-s M^2) \right) + O(\hbar^2) \, .$$

IR-cutoff

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IR-cutoff

take k-derivative

Flow equation

$$k\partial_k\Gamma_k = \hbar\ \mathrm{Tr}\ \exp(-K^{-1}S_k^{(2)}k^{-2}) + O(\hbar^2)$$

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One-loop RG-improvement:

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Flow equation

$$k\partial_k\Gamma_k = \hbar\ \mathrm{Tr}\ \exp(-K^{-1}S_k^{(2)}k^{-2}) + O(\hbar^2)$$

↓ replace classical action and the metric by the effective action and an RG-improved DeWitt metric

One-loop RG-improvement:

$$k\partial_k\Gamma_k = \mathrm{Tr}\ \exp(-K_k^{-1}\Gamma_k^{(2)}[\phi]k^{-2})$$

It suffers from unphysical dependencies, it is off-shell

3. Proper time regularization

Essential renormalization group

Add an extra term: allow the field variable coupling to the source to flow with k

When deriving the effective action, do the Legendre transform:

$$e^{-\Gamma_k[\phi]} = \int d\hat{\chi} e^{-S[\hat{\chi}]+(\hat{\phi}_k[\hat{\chi}]-\phi)\cdot\frac{\delta\Gamma[\phi]}{\delta\phi}}$$

Introduce the RG kernel: $\Psi_k[\phi] := \langle k\partial_k\hat{\phi}_k[\hat{\chi}]\rangle$

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Generalized perturbative essential flow equation

$$k\partial_k\Gamma_k[\phi] = -\Psi_k[\phi]\cdot\frac{\delta}{\delta\phi}\Gamma_k[\phi] + \mathrm{Tr}\,\exp(-K_k^{-1}\Gamma_k^{(2)}[\phi]\;k^{-2})$$

absorb off-shell term into the kernel

proportional to the EoM

Only the essential couplings run.

3. Proper time regularization

MES @ order curvature squared

Consider again Einstein-Hilbert truncation $\longrightarrow$ remove curvature squared terms by field redefinitions

$$\left(k\partial_k + \int \mathrm{d}^d x \sqrt{g}\,\Psi_{\mu\nu}\frac{\delta}{\delta g_{\mu\nu}}\right)\Gamma = \mathrm{Tr}\, e^{-K^{-1}(\Gamma^{(2)}+S^{(2)}_{\mathrm{gf}})k^{-2}} - 2\mathrm{Tr}\, e^{-\mathcal{Q}_{\mathrm{FP}}[\phi]k^{-2}}$$

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RG kernel: linear combination of operators up to desired truncation order

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Our non-minimal variant of dimensional regularization coincides with proper time regularization, at least within the early time heat kernel expansion.

3. Proper time regularization

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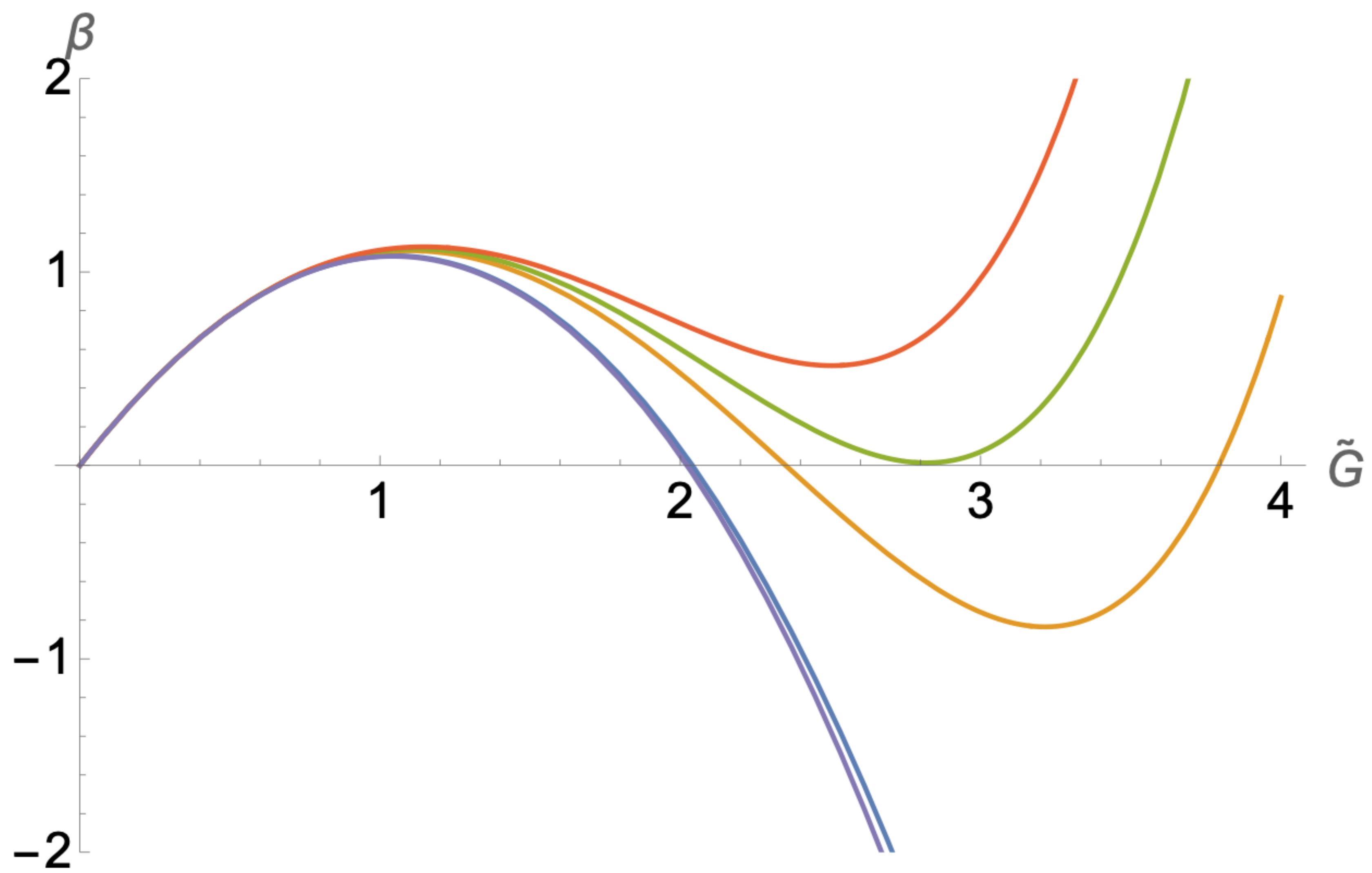
Let us use it as a tool to analyze parametrization dependence.



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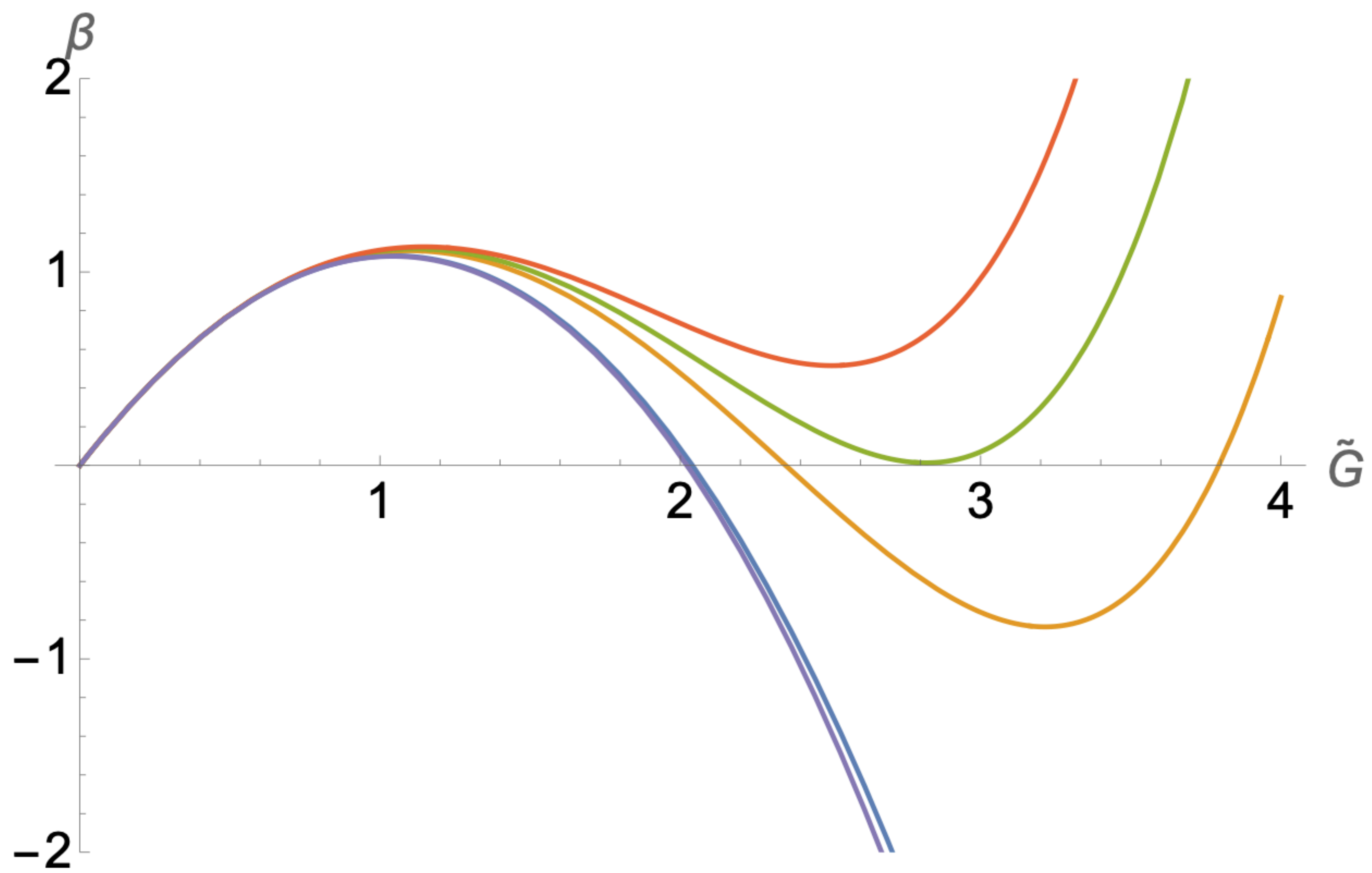

Expand the essential beta function:

$$\beta_{\tilde{G}} = (d-2)\tilde{G} + \frac{1}{3}(-36+(d-3)d)(4\pi)^{1-d/2}\tilde{G}^2 + \frac{(d-3)(1200+d(-566+d(d+19)))(4\pi)^{2-d}}{30(d-2)}\tilde{G}^3 + O(\tilde{G}^4)$$

No dependence on the τ's up to this order.

4. All curvature order on a max. symmetric background

Going to higher orders in curvature, one can lift the dependence on the parameterisation at higher order in $\tilde{G}$.

Expansion up to $O(R^2)$ $\longrightarrow$ No dependence on the parametrization up to $O(\tilde{G}^3)$

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Resolving terms $\int \mathrm{d}^d x \sqrt{g} R G_k^{N-1}$ requires going R^N

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Technical details:

- **Heat kernel expansion** on a d-sphere [Kluth, Litim 1910.00543]
- **Spectral sum** on a d-sphere or d-hyperboloid
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Fixed point and the critical exponent converges rapidly

$O(R^N)$	θ
R	2
R^2	2.296
R^3	2.312
R^4	2.312
R^5	2.311
R^∞	2.311

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The correlation length in units of the cutoff $\xi = \Lambda \ell_P$ $\longrightarrow$ $\lim_{\tilde{G}(\Lambda)\to\tilde{G}_\star} \xi \propto \frac{1}{|\tilde{G}_\star - \tilde{G}(\Lambda)|^{\frac{A}{d-2}}}$

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For different values of the bare coupling, one produces effective actions with an observed Planck length ℓ_P (minisuperspace).

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They work in Lorentzian signature: a and a_t distinguished but related

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$$\theta_{\mathrm{CDT}} = 4 \pm 1$$

[Ambjørn, Gizbert-Studnicki, Görlich, Németh 2408.07808, 2411.02330]

Reasons for such an incompatibility?

Conclusions & Outlook

Use perturbation theory to extract physical information (on-shell).

New subtraction scheme + essential renormalization group

1. Parametrization dependence disappears order by order in perturbation theory
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ToDo

- Analyze also gauge-dependence
- Test scheme in other theories - add matter
- Better understanding of comparison with lattice
- Go two loop in proper time?