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# Absence of SO(4) quantum criticality in Dirac semimetals at two-loop order

**based on MU, Herbut, Stamou, Scherer — arXiv:2308.12464 & PRB 108, 245130**

**Max Uetrecht**

Asymptotic Safety @ DESY

19 December, 2023

# Outline

- **Critical Phenomena in Dirac Materials**
  - Equivalent field-theoretical description of Criticality
    *[Liu, Huffmann, Chandrasekharan, Kaul '22, PRL] [Herbut, Scherer '22, PRB]*
  - Recent quantum Monte Carlo simulation → **Criticality found**
- **Describe QMC Criticality with a Gross—Neveu—Yukawa field theory**
  - RG Fixed-Point Analysis at **one-loop** order → **Criticality lost** → **two-loop?**
    *[Herbut, Scherer '22, PRB]*

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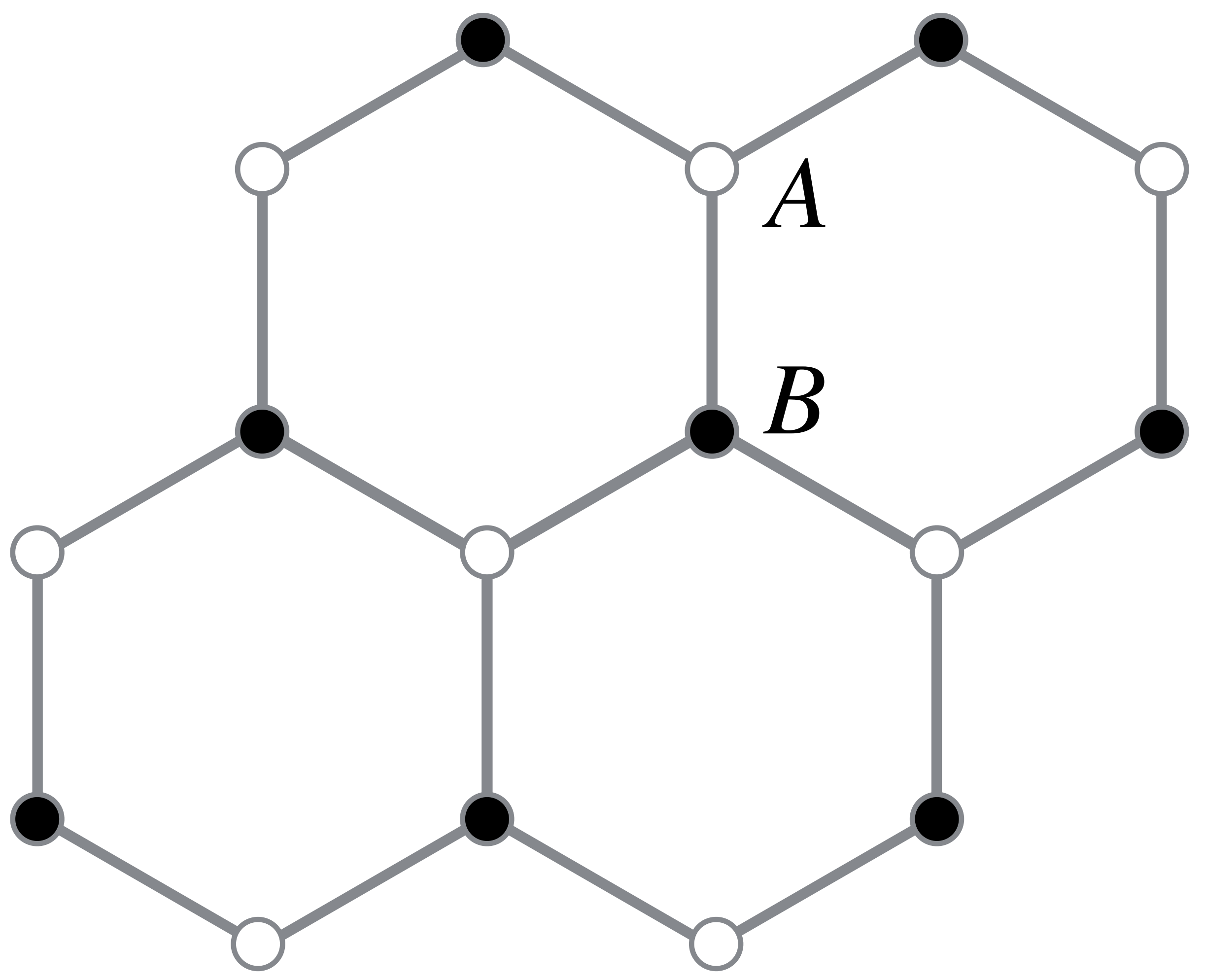
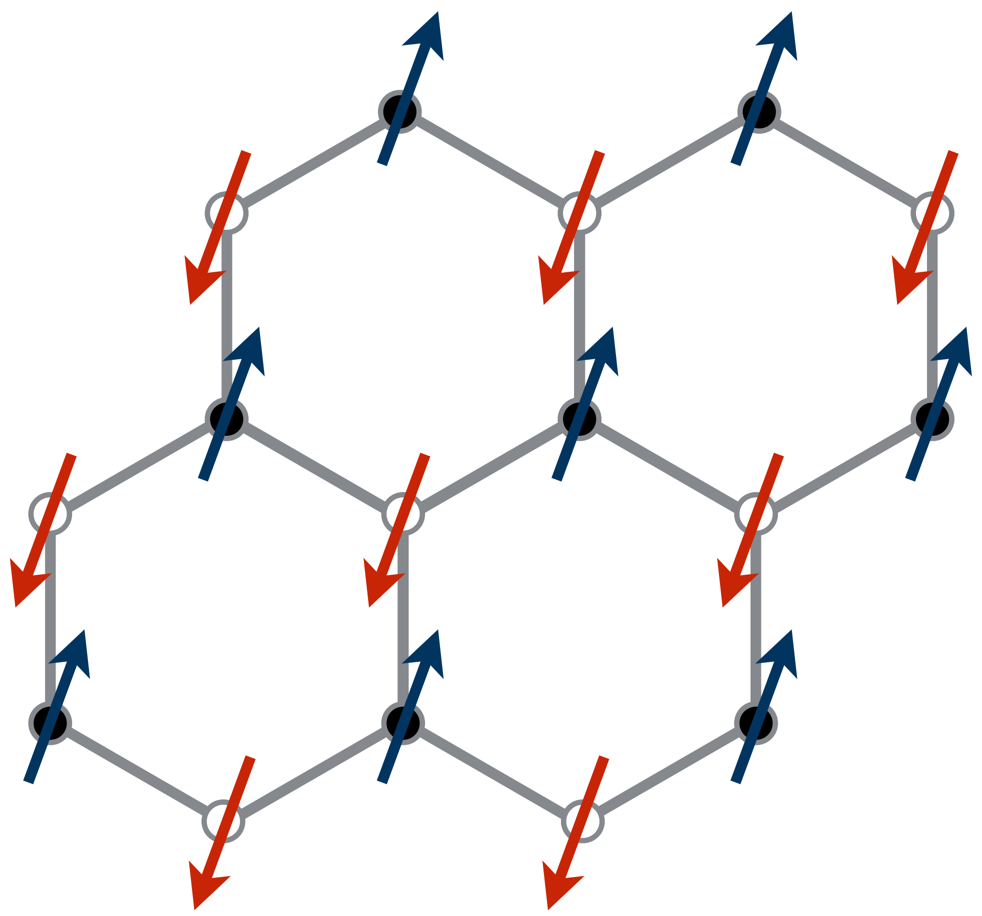
# Critical Phenomena in Dirac Materials

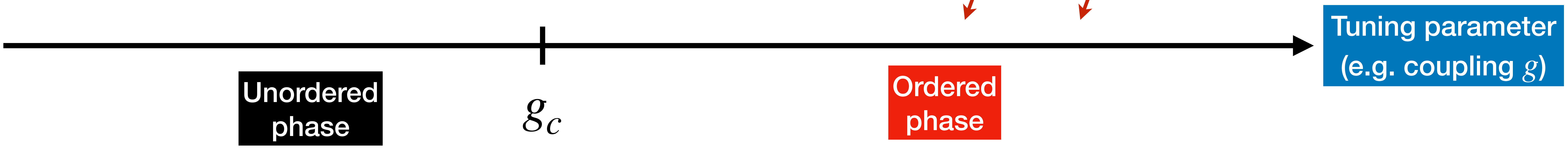



- Electrons in a solid effectively described by the **Dirac equation**
- Examples: graphene, topological insulators, …
- Strong electron-electron interaction ➡ massive fermion phase

e.g.:

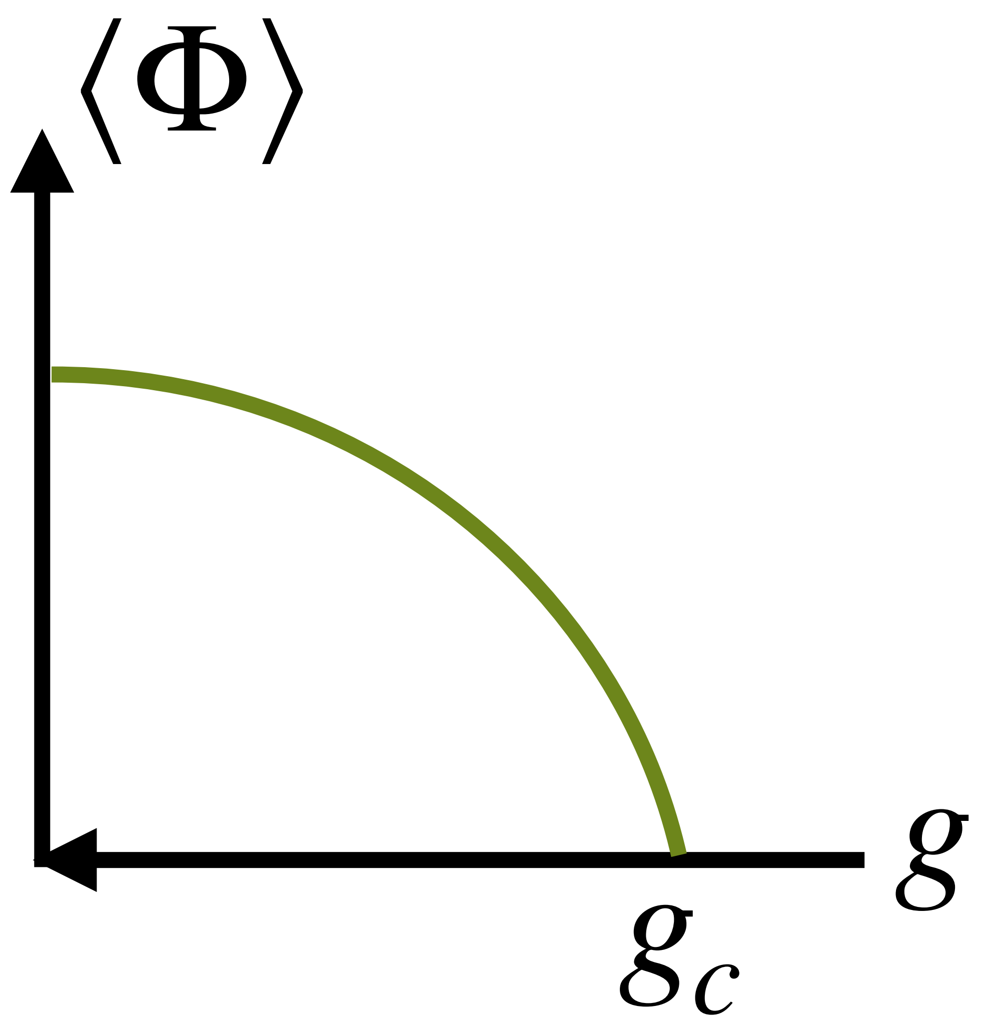


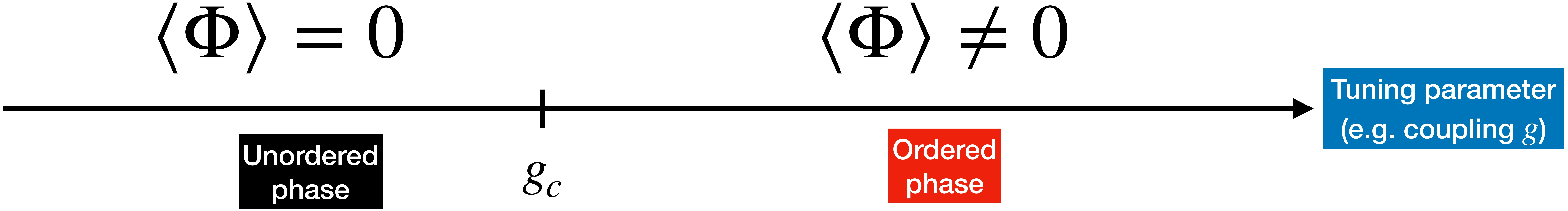
$m = 0$ $\quad$ $m > 0$



# Critical Phenomena in Dirac Materials

- Phases connected by continuous phase transition ➜ Critical Phenomena
- Mass generation described by **Spontaneous Symmetry Breaking (SSB)**
- Governed by Order Parameter $\Phi$ associated to the symmetry



# Critical Phenomena in Dirac Materials

- At critical point $g = g_c$ : system is **scale invariant**, i.e. $\xi \to \infty$

- **Connection to field theory?**

Renormalize $\mathscr{L}_{\text{Dirac}}$ ➡ Beta functions $\beta(g) = \dfrac{dg}{d\log\mu}$

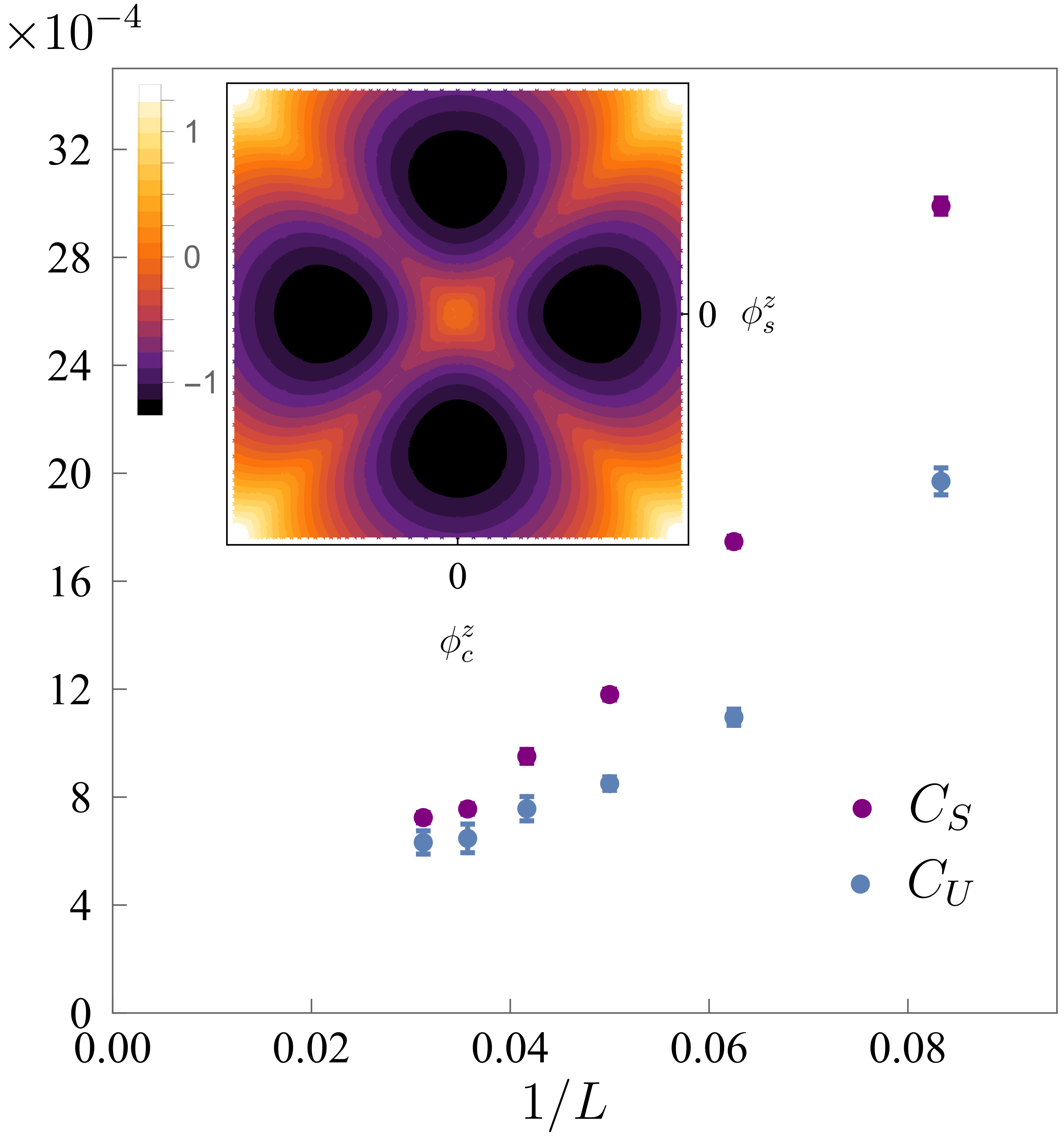
Roots of Beta function = Fixed Points (FPs) ➡ **scale invariance**

$$\beta(g) = \frac{dg}{d\log\mu} = 0 \implies g = g_c$$

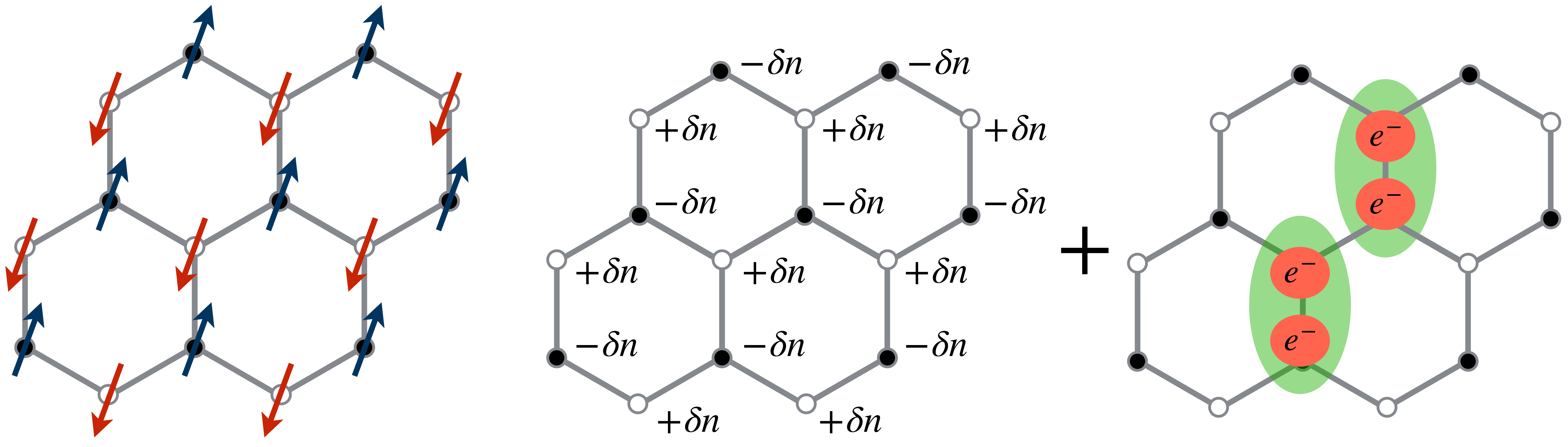
➡ **FPs of field theory correspond to critical points**

# Critical Phenomena in Dirac Materials: Recent QMC Analysis

*[Liu, Huffmann, Chandrasekharan, Kaul '22, PRL]*




- *H. Liu et. al. '22:* Quantum Monte Carlo (QMC) analysis of Dirac criticality with **two order parameters (OPs)**

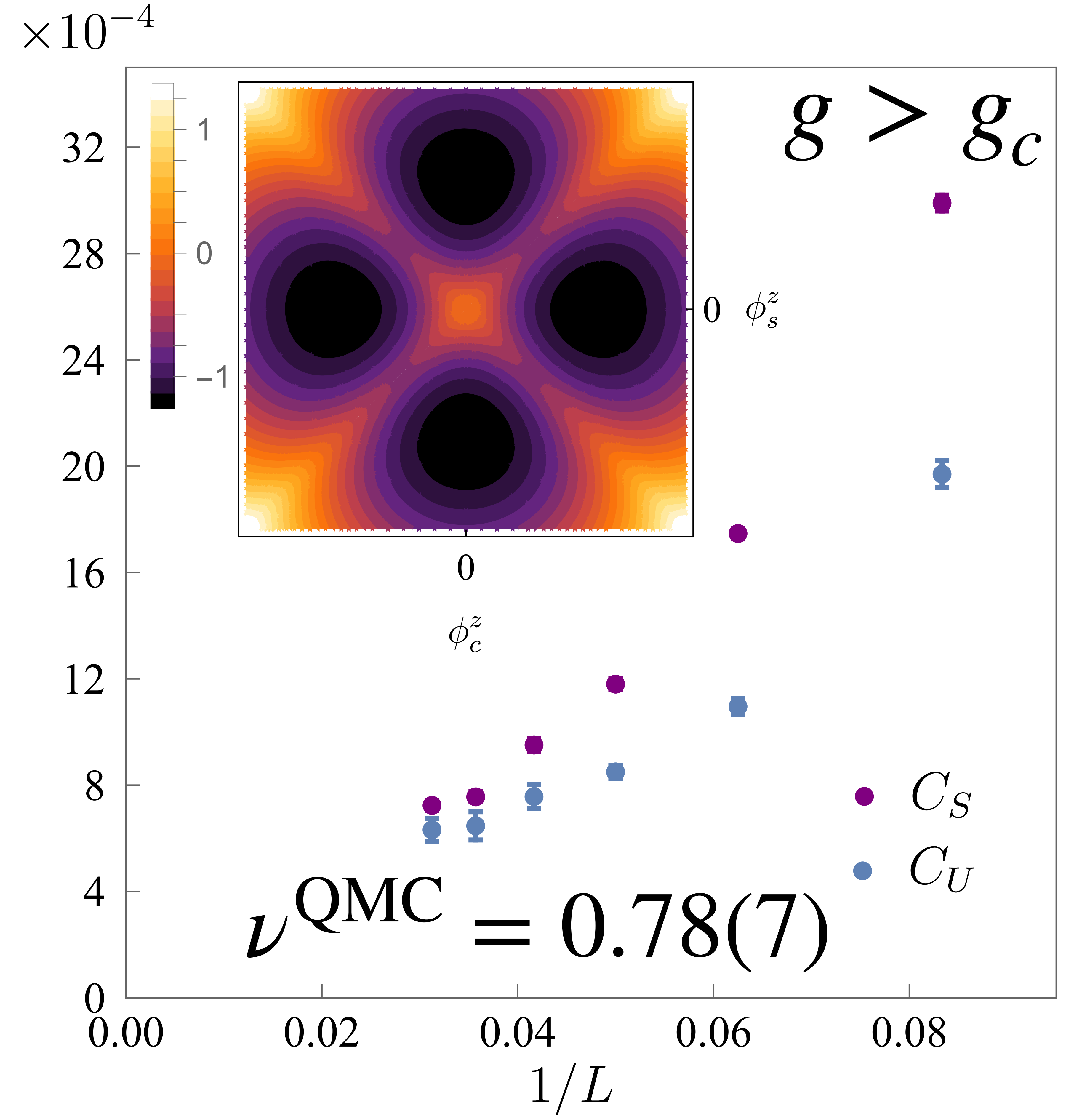



$C_S$ : Néel state

$C_U$ : superconductor-CDW-state

# Critical Phenomena in Dirac Materials: Recent QMC Analysis

*[Liu, Huffmann, Chandrasekharan, Kaul '22, PRL]*




- QMC data: **continuous transition** between massless Dirac phase and massive phase
- Both OPs **simultaneously** break their associated symmetry: $L \to 0 : C_{S/U} \neq 0$ **!**
- Divergence: $g \to g_c : \xi \propto |g - g_c|^{-\nu}$

➡ **Capture criticality with field theory**

$C_S, C_U = 0$ | $g_c$ | $C_S, C_U \neq 0$ → Tuning parameter (e.g. coupling $g$)

Unordered Dirac semimetal phase

Ordered phase

**Next:**

**Describe quantum Monte Carlo Criticality with a Field Theory**

# Gross—Neveu—Yukawa (GNY) Theory in $d = 2 + 1$

$$\mathscr{L} \simeq \mathscr{L}_{\text{free}} - g_a \overline{\Psi} \left( \vec{a}\vec{\sigma} \right) \Psi - g_b \overline{\Psi} \left( \vec{b}\vec{\sigma} \right) \Psi - \lambda_a \vec{a}^2 - \lambda_b \vec{b}^2 - \lambda_{ab} \vec{a}\vec{b}$$

- Represent two OPs as **massive scalar fields** $\vec{a}, \vec{b}$: vectors of $\mathrm{SO}(3) \simeq \mathrm{SU}(2)$
- Combine OPs into **global** $\mathrm{SO}(4) \simeq \mathrm{SU}(2)_\mathrm{A} \times \mathrm{SU}(2)_\mathrm{B}$ **symmetry; same as QMC lattice**
- Couple OPs to $\mathrm{SU}(2)-$bidoublet fermion ➡ **fermionic "mass terms"**
- Include fluctuations of the OPs ➡ **quartics**

| | $\mathrm{SU}(2)_\mathrm{A}$ | $\mathrm{SU}(2)_\mathrm{B}$ |
|---|---|---|
| $\Psi$ | $\mathbf{2}$ | $\mathbf{2}$ |
| $a_i$ | $\mathbf{3}$ | – |
| $b_j$ | – | $\mathbf{3}$ |

*[Liu, Huffmann, Chandrasekharan, Kaul '22, PRL]*
*[Herbut, Scherer '22, PRB]*

# Gross—Neveu—Yukawa (GNY) Theory in $d = 4 - \epsilon$

$$\mathscr{L} \simeq \mathscr{L}_{\text{free}} - g_a \overline{\Psi} \left(\vec{a}\vec{\sigma}\right) \Psi - g_b \overline{\Psi} \left(\vec{b}\vec{\sigma}\right) \Psi - \lambda_a \vec{a}^2 - \lambda_b \vec{b}^2 - \lambda_{ab} \vec{a}\vec{b}$$

- $d = 2 + 1$ : couplings are dimensionful ➡ **no direct perturbative expansion**
- Continue $\mathscr{L}$ analytically to $d = 4 - \epsilon$ and expand for **small** $\epsilon$
- Obtain predictions in the limit $\epsilon \to 1$

**RGEs**

➡ **Beta functions & anomalous dimensions at two-loops in** $d = 4 - \epsilon$

# Gross—Neveu—Yukawa (GNY) Theory in $d = 4 - \epsilon$

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**Checks of our $\beta$ functions?**

- 1-loop results ✓ *[Herbut, Scherer '22, PRB], [Liu, Huffmann, Chandrasekharan, Kaul '22, PRL]*
- **(Chiral) Heisenberg** subsectors @ 2-loop ✓ *[Zerf, Mihaila, Marquard, Herbut, Scherer '17, PRD]*
- Independent 2-loop results using **ARGES** ✓ *[Litim, Steudtner '21, Comp. Phys. Comm.]*

# Finally:
# Fixed-Point Analysis @ 2-loops for QMC model

# Fixed-Point Analysis @ 2-loop

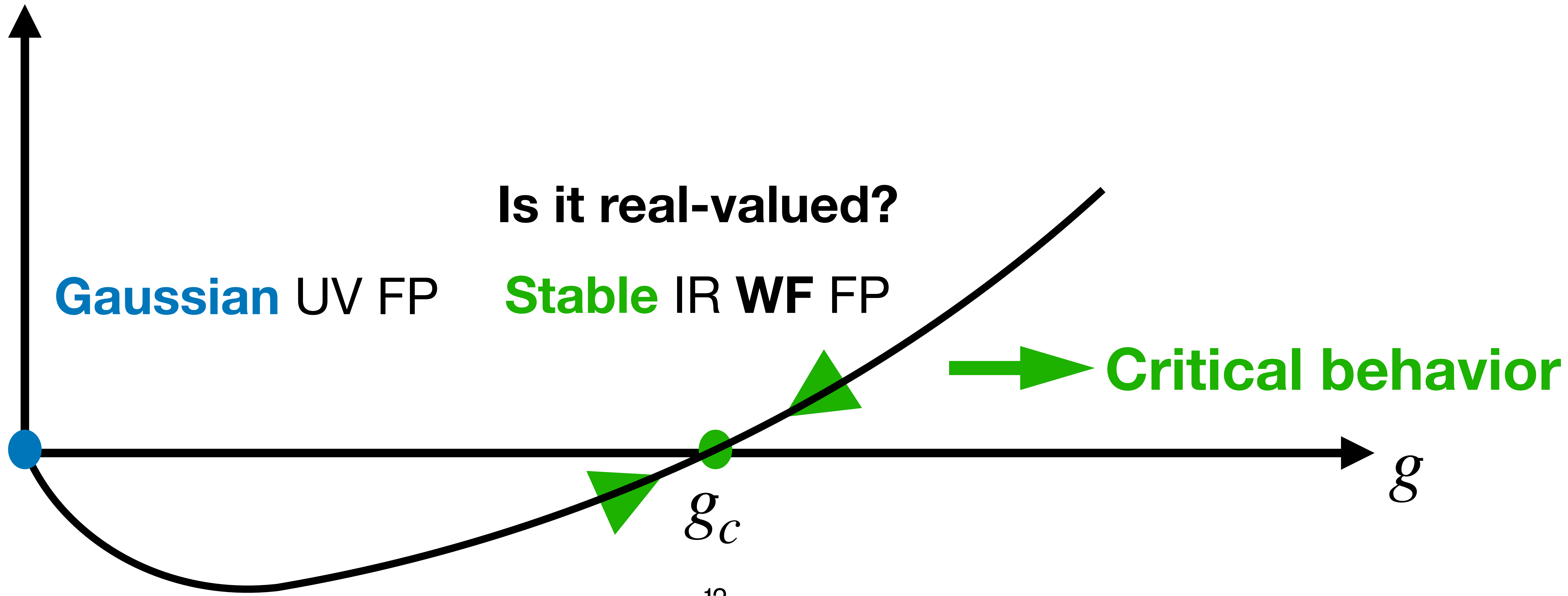
- $d = 4 - \epsilon$ theory has **Wilson—Fisher** FPs in the **IR** — **UV**: trivial Gaussian FP

- Linearize $\beta$ functions around FP $g_c$ ⟶ **stability matrix S**

$$\beta(g) = S(g - g_c) + \mathcal{O}((g - g_c)^2)$$

- **Eigenvalues** determine stability

$$\beta(g) = \frac{dg}{d\log\mu}$$



# Fixed-Point Analysis @ 2-loop

- Study model corresponding to **quantum Monte Carlo:** $g_a = g_b, \lambda_a = \lambda_b, N_f = 2$

$$\beta_{g^2} = -\epsilon g^2 + \frac{1}{16\pi^2} g^4 (5 + N_f) + \frac{1}{(16\pi^2)^2} g^2 \left[ g^4 \left( -6N_f - \frac{49}{8} \right) - g^2 \left( 5\lambda + 9\lambda_c \right) + \frac{5}{2}\lambda^2 + \frac{3}{2}\lambda_c^2 \right]$$

$$\beta_\lambda = -\epsilon\lambda + \frac{1}{16\pi^2} \left[ -N_f g^4 + 2N_f g^2 \lambda + 11\lambda^2 + 3\lambda_c^2 \right]$$

$$+ \frac{1}{(16\pi^2)^2} \left[ 8N_f g^6 + N_f g^4 \left( 3\lambda_c - \frac{9}{2}\lambda \right) - N_f g^2 \left( 11\lambda^2 + 3\lambda_c^2 \right) - 3\left( 23\lambda^3 + 5\lambda\lambda_c^2 + 4\lambda_c^3 \right) \right]$$

$$\beta_{\lambda_c} = -\epsilon\lambda_c + \frac{1}{16\pi^2} \left[ -3N_f g^4 + 2N_f g^2 \lambda_c + 2\lambda_c (5\lambda + 2\lambda_c) \right]$$

$$+ \frac{1}{(16\pi^2)^2} \left[ 16N_f g^6 + N_f g^4 \left( 5\lambda - \frac{5}{2}\lambda_c \right) - 2N_f g^2 \lambda_c (5\lambda + 2\lambda_c) - \lambda_c (5\lambda + \lambda_c)(5\lambda + 11\lambda_c) \right]$$

*[MU, Herbut, Stamou, Scherer ’23, PRB]*

# Fixed-Point Analysis @ 2-loop

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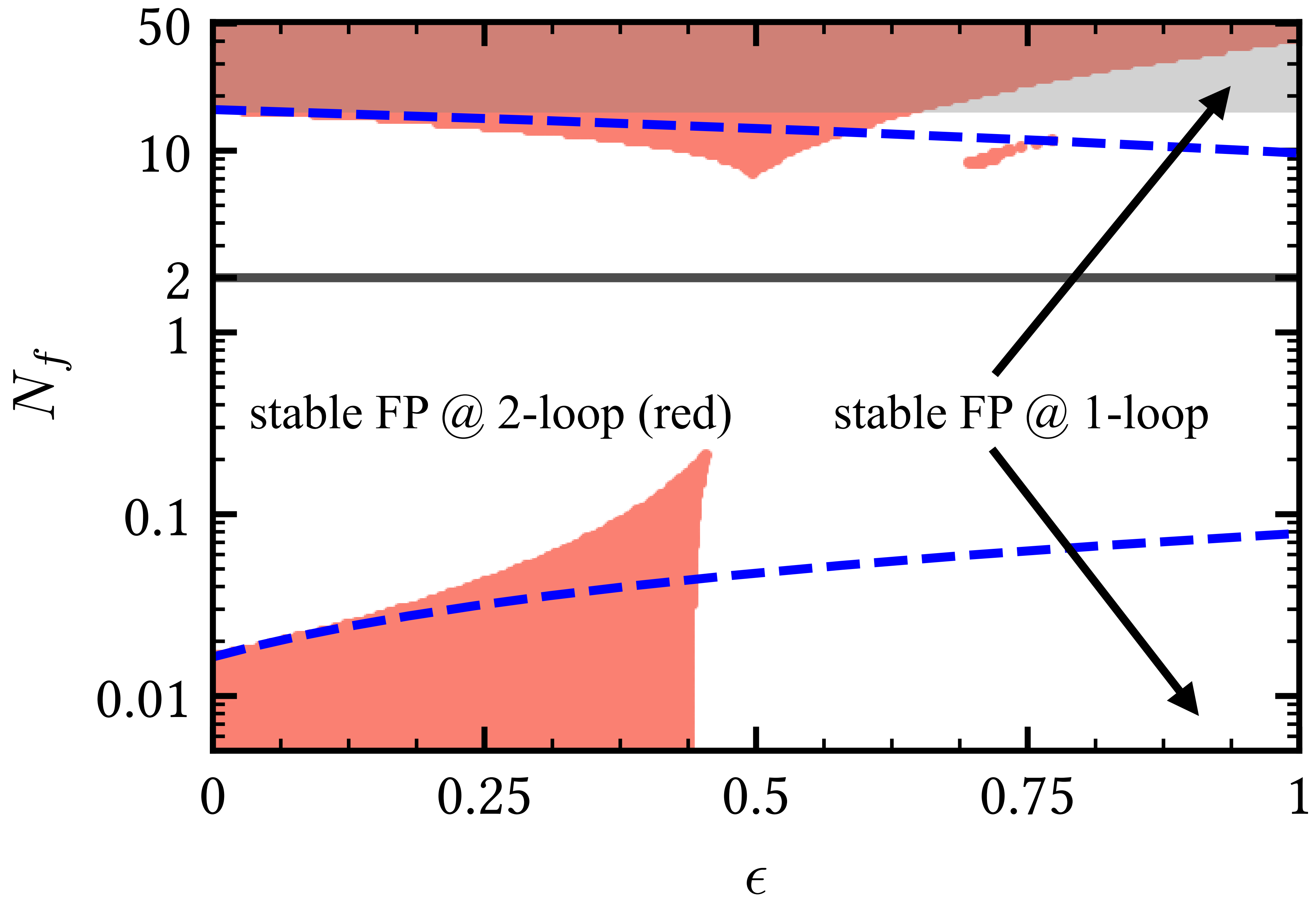
*[MU, Herbut, Stamou, Scherer '23, PRB]*




**Numerically search for FPs** @ FP: $m_{\text{OPs}} = 0$

- **1-loop**: no stable FP — FP Annihilation at

$$N_c^{>} \approx 16.83$$

# Fixed-Point Analysis @ 2-loop

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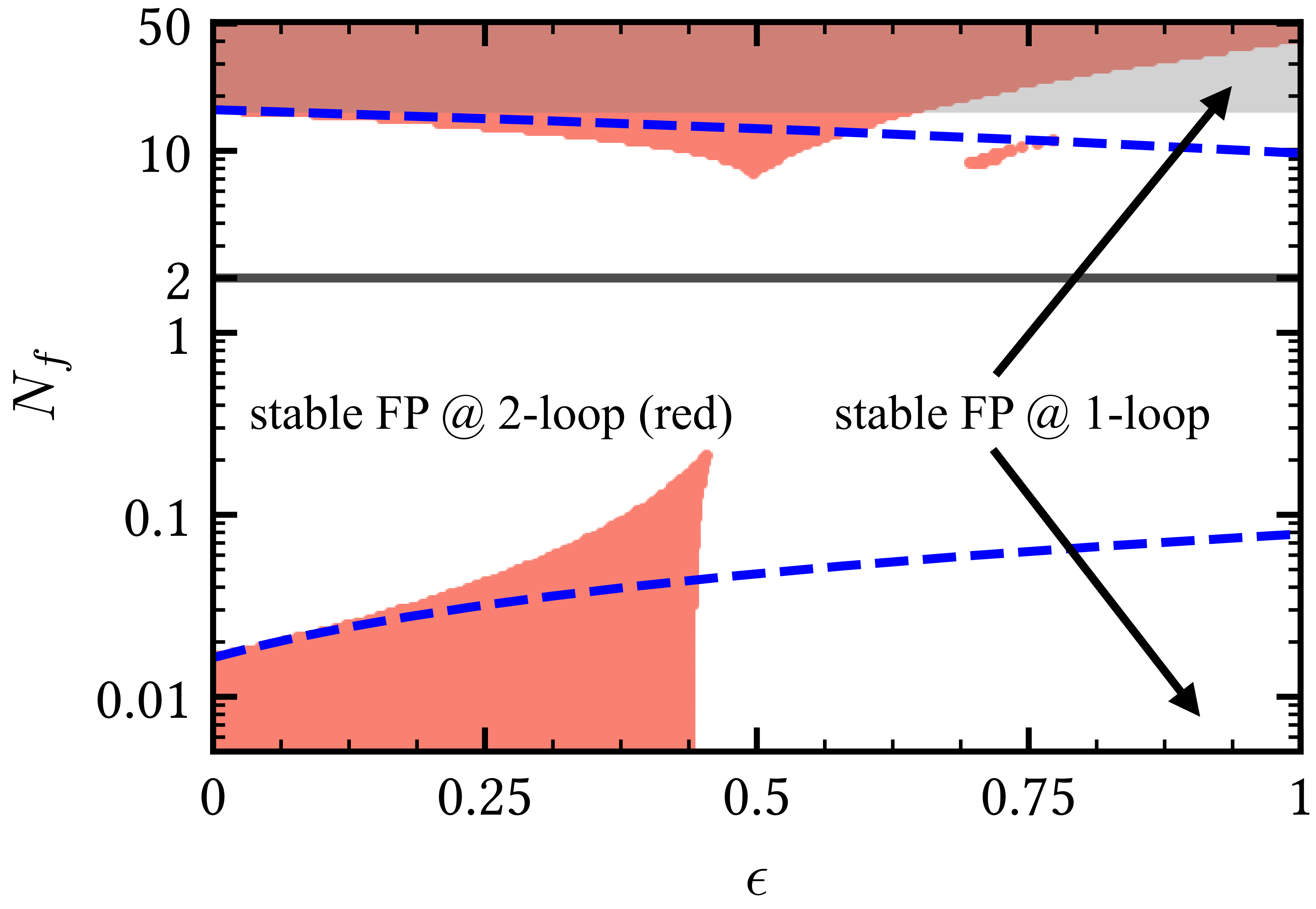
*[MU, Herbut, Stamou, Scherer '23, PRB]*

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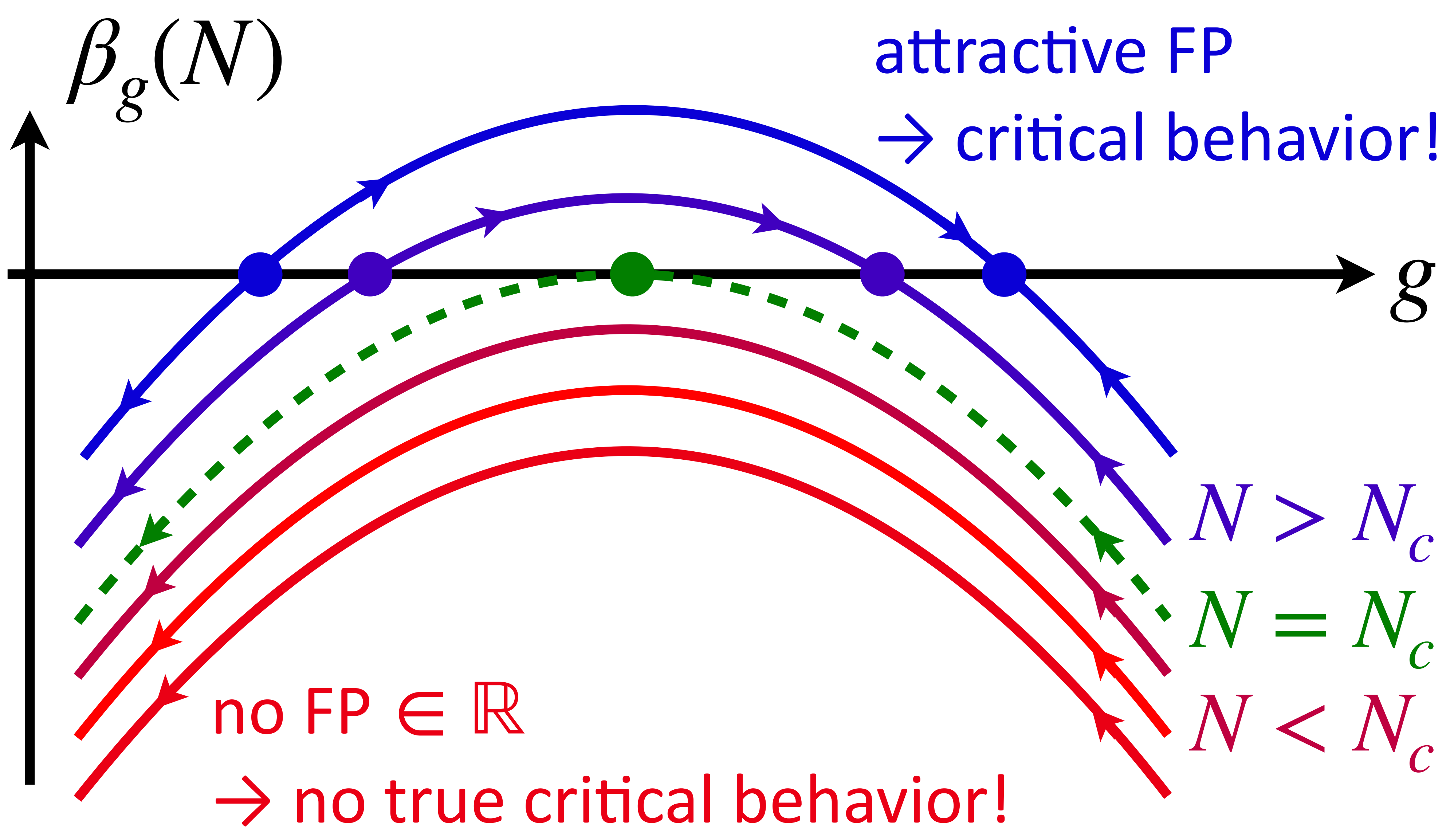




*Fig: [Song, Zhao, Janssen, Scherer, Meng '23]*

# Fixed-Point Analysis @ 2-loop

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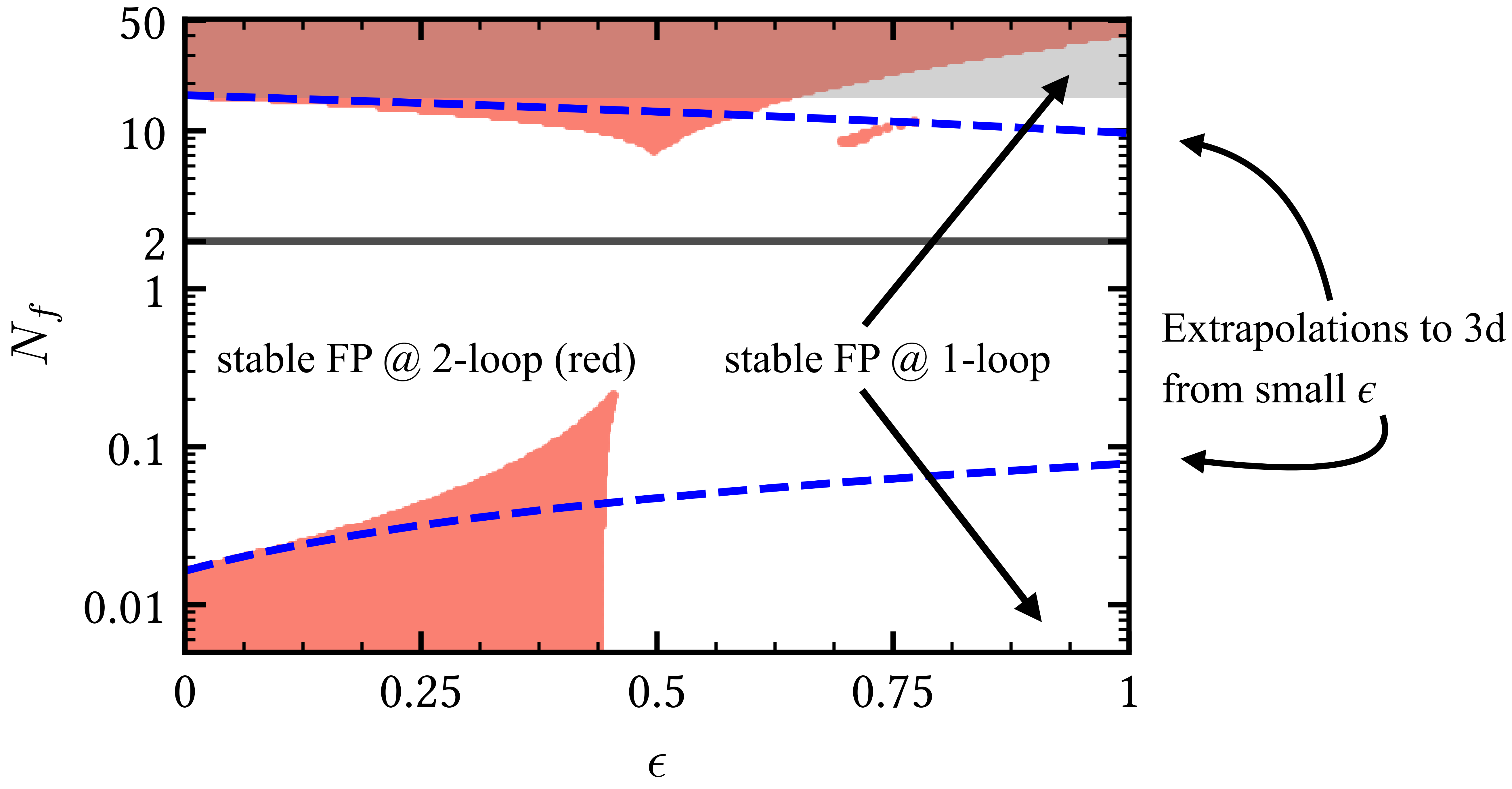
*[MU, Herbut, Stamou, Scherer '23, PRB]*




**Numerically search for FPs** @ FP: $m_{\text{OPs}} = 0$

- **2-loop**: no stable FP — FP Annihilation at

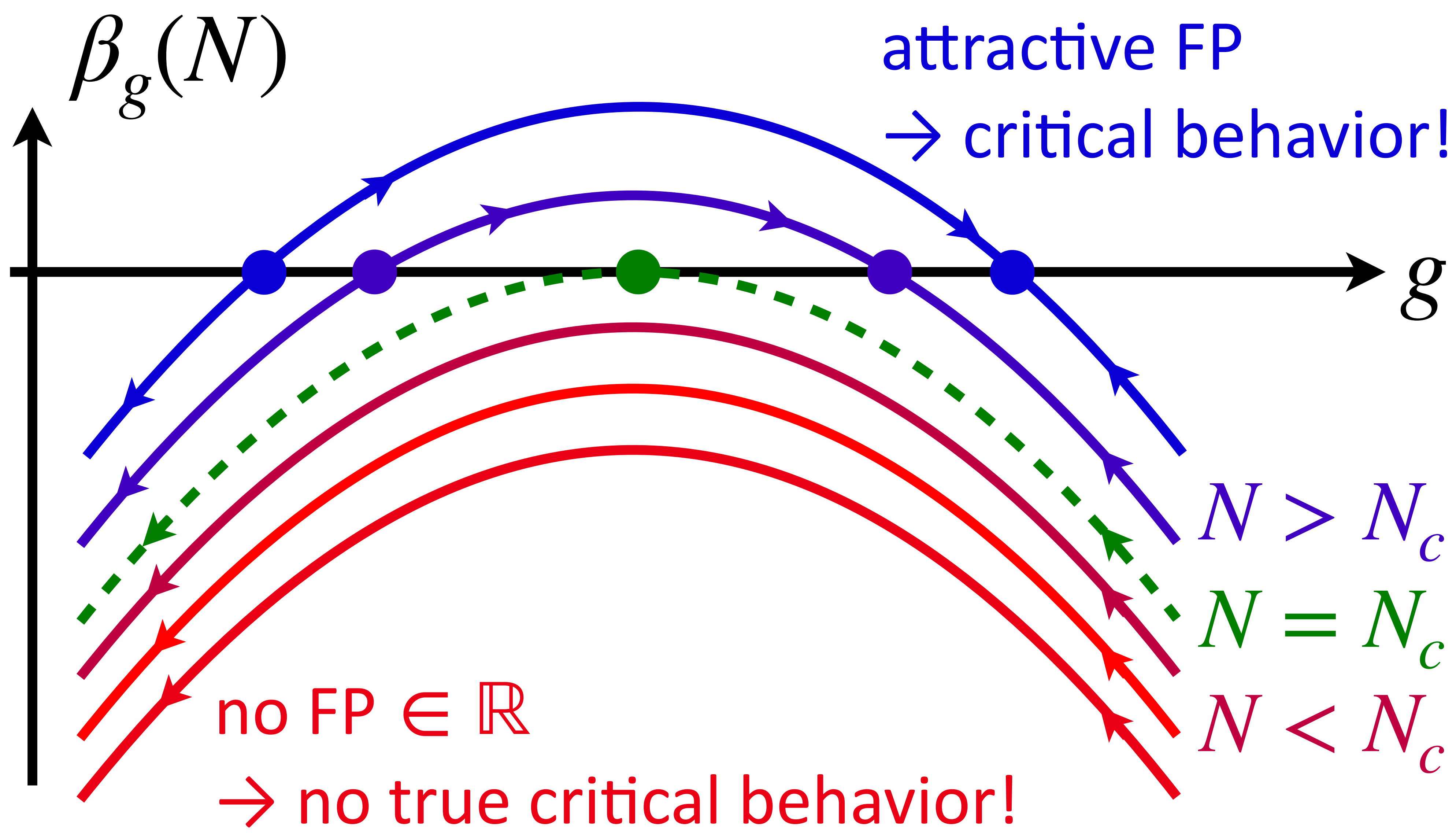
$$N_c^{>} \approx 16.83 - 7.14\epsilon$$

➡ **how to explain QMC data?**

# Fixed-Point Analysis @ 2-loop

- Study model corresponding to **quantum Monte Carlo:** $g_a = g_b, \lambda_a = \lambda_b, N_f = 2$

*Fig: [Song, Zhao, Janssen, Scherer, Meng '23]*




**How to possibly explain QMC data?**

- For $N_f < N_c$: complex FPs
- Close to $N_c$ : **slowly walking** RG flow
  - Corr. length $\xi$ large but finite

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- Study model corresponding to **quantum Monte Carlo:** $g_a = g_b, \lambda_a = \lambda_b, N_f = 2$

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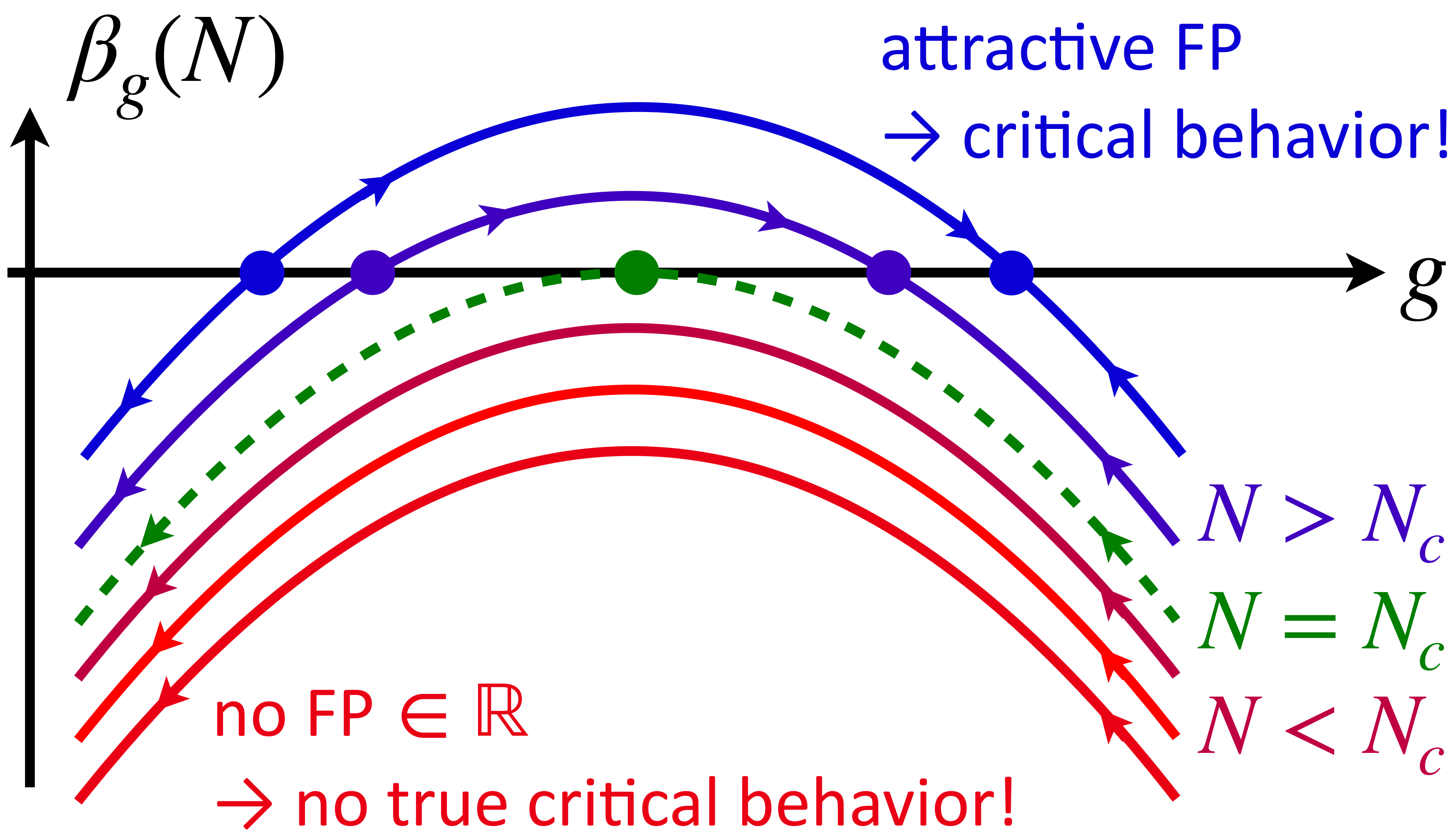



**How to possibly explain QMC data?**

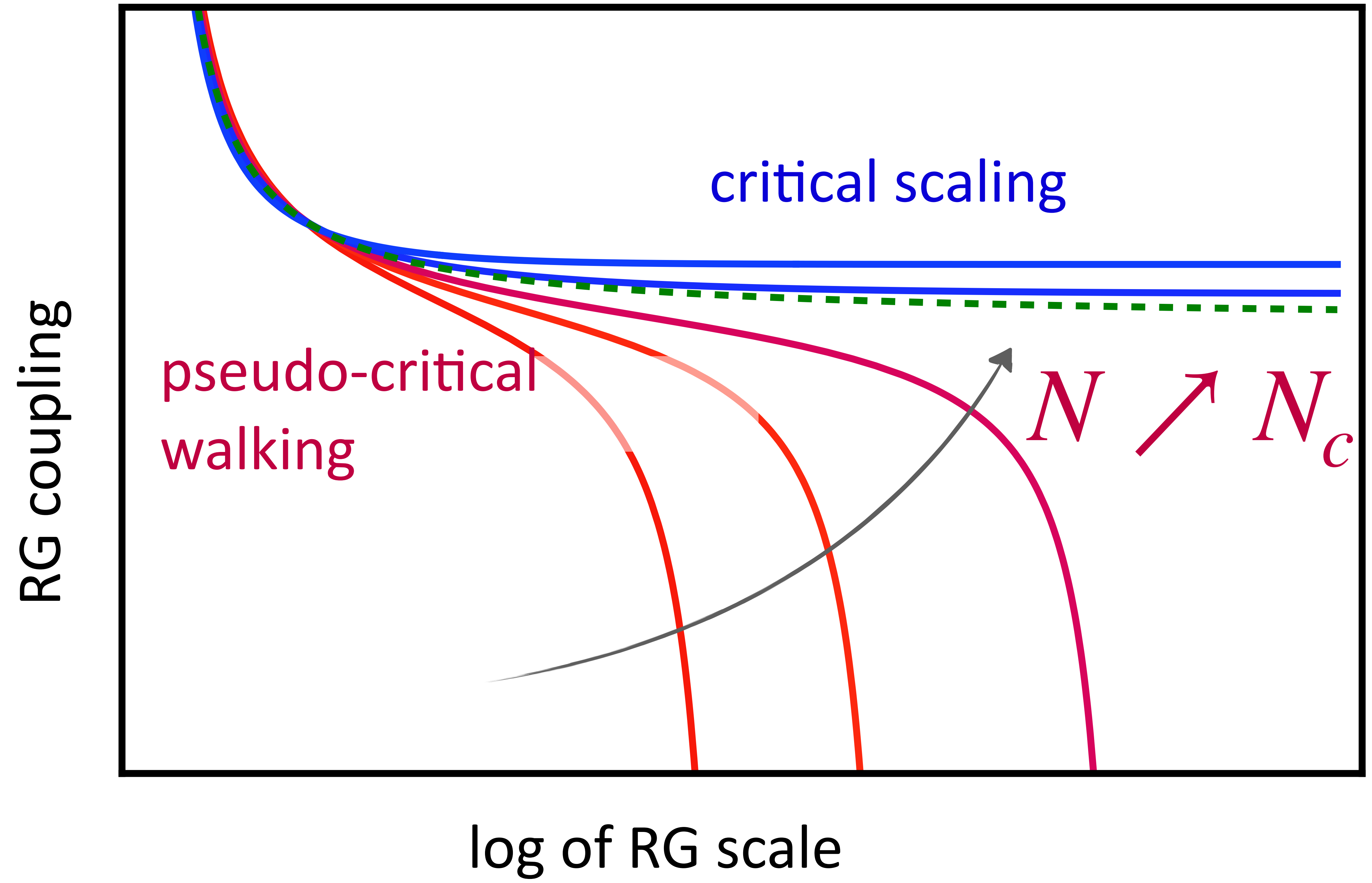
- For $N_f < N_c$: complex FPs
- Close to $N_c$ : **slowly walking** RG flow
  - Corr. length $\xi$ large but finite
  - **Higher orders suppress $N_c$ to $N_f = 2$**

→ **Stable FP restored**

# Fixed-Point Analysis @ 2-loop

- Study model corresponding to **quantum Monte Carlo:** $g_a = g_b, \lambda_a = \lambda_b, N_f = 2$

*Fig: [Song, Zhao, Janssen, Scherer, Meng '23]*



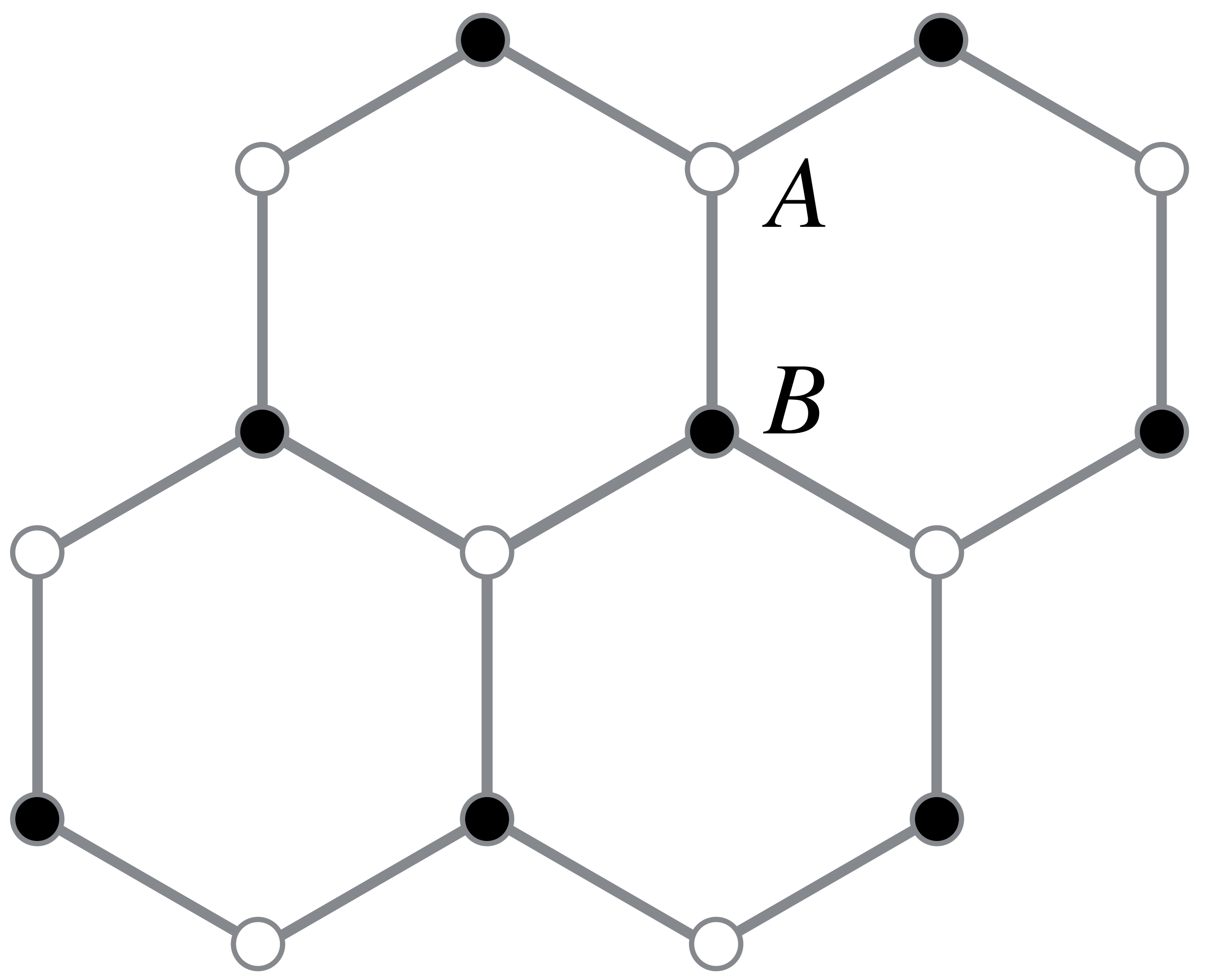

**How to possibly explain QMC data?**

- For $N_f < N_c$: complex FPs
- Close to $N_c$ : **slowly walking** RG flow
  - Corr. length $\xi$ large but finite
  - **Drifting critical exponents**

[Kaplan, Lee, Son, Stephanov '09, PRD]

**Challenging to confirm with QMC** → **Weak First-Order Transition**

# Conclusion & Outlook




- Dirac theory emerges in **Dirac Materials** like graphene
- **Criticality**: Dynamic fermion mass generation through **SSB**
- **Independent benchmark** of QMC / many-body methods *[Liu, Huffmann, Chandrasekharan, Kaul '22, PRL]*
- RG @ 2-loop in $d = 4 - \epsilon$: No stable FP ➡ **QMC-observed Criticality lost?**
  *[MU, Herbut, Stamou, Scherer '23, PRB]*
- FP restored at loop orders > 2 ?
- Pseudo-Critical Behavior/Walking ? *[Kaplan, Lee, Son '09, PRD]*