UCLA College | Physical Sciences Physics & Astronomy

Partonic distribution functions and amplitudes using tensor network methods

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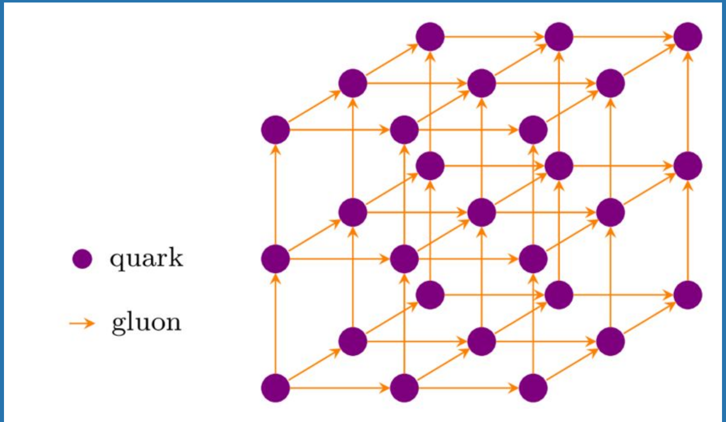
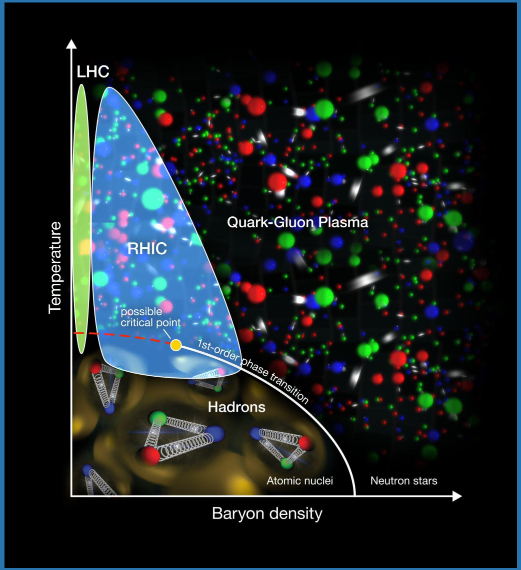
Why Lattice Theory?

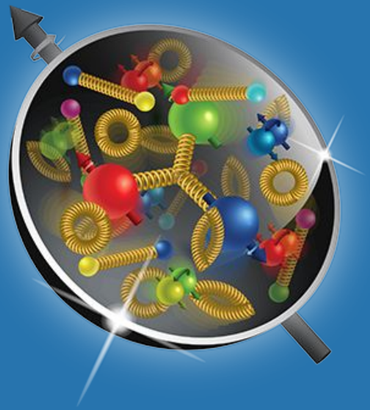
- Study of non-perturbative regime of QCD at lower energies
- QCD phenomena (e.g. confinement, phase transitions)
- First-principles calculations of quantities like PDFs



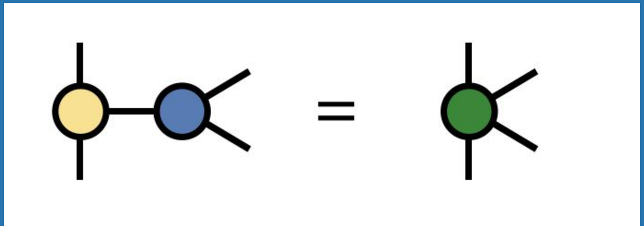


Parton Distribution Functions and Amplitudes

- Parton Distribution Function (PDF)
- Light-cone Distribution Amplitude (LCDA)
- Provide non-perturbative inputs to calculate deep inelastic scattering cross sections
- Essential to understanding of hadron internal structure, collider physics



Why Tensor Networks?



- TN have recently become an active research area in lattice field theory and hep
- TNS are efficient ansatz schemes for quantum many body problems free from the sign problem
- MPS can efficiently handle many common sources of error/complexity like high dimensionality and entanglement!

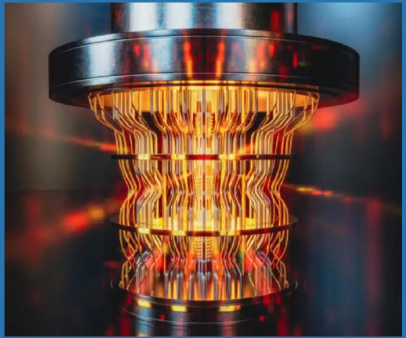
Tensor networks for High Energy Physics: contribution to Snowmass 2021

Tensor Networks for Lattice Gauge Theories beyond one dimension: a Roadmap

Giuseppe Magnifico,[1,2,3] Giovanni Cataldi,[3,4,5] Marco Rigobello,[3,4,5]
Peter Majcen ,[3,4,5] Daniel Jaschke,[3,4,5,6] Pietro Silvi,[3,4,5] and Simone Montangero[3,4,5]

Tensor Networks and Quantum Computing

- TNs easily simulate > 100 qubits
- Reasons for TNs
    a. Good testbed/alternative to quantum computing in the NISQ era
    b. Maximize what’s accomplishable by classical hardware



The (1+1) Dimension Lattice NJL Model

- NJL model is an effective field theory describing low-energy two-flavored QCD phenomena
- We study these phenomena on a lattice

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - m)\psi + g(\bar{\psi}\psi)^2$$

->

$$\mathcal{H} = \bar{\psi}(i\gamma_1\partial_1 + m)\psi - g(\bar{\psi}\psi)^2$$

Lagrangian

Hamiltonian

$$\psi(x = x_n) = \begin{pmatrix} \rho(x = x_n) \\ \eta(x = x_n) \end{pmatrix} = \frac{1}{\sqrt{a}} \begin{pmatrix} \chi_{2n} \\ \chi_{2n+1} \end{pmatrix}$$

Fermion field

The (1+1) Dimension Lattice NJL Model

- Apply Jordan-Wigner transformation to Hamiltonian

$$\chi_n = \sigma_n^- \prod_{i=0}^{n-1} (-i\sigma_i^z)$$

$$H = \frac{1}{2a}\left[\sum_{n=0}^{N-2}\left(-\sigma_n^+\sigma_n^z\sigma_{n+1}^- + \sigma_{n+1}^+\sigma_n^z\sigma_n^-\right)\right] + m\sum_{n=0}^{N-1}(-1)^n\sigma_n^+\sigma_n^- - \frac{g}{a}\sum_{n=0}^{N/2-1}\left(\sigma_{2n}^+\sigma_{2n}^- - \sigma_{2n+1}^+\sigma_{2n+1}^-\right)^2 .$$

Spin Hamiltonian

PDF and LCDA Computations

- Continuum limit PDF $$f_{q/h}(x) = \int_{-\infty}^{\infty} \frac{dt}{4\pi} e^{-itx\vec{n}M_h} \langle h| \bar{\psi}(t\vec{n}) W(t\vec{n} \leftarrow \vec{0}) \vec{n} \cdot \vec{\gamma} \psi(\vec{0}) |h\rangle$$

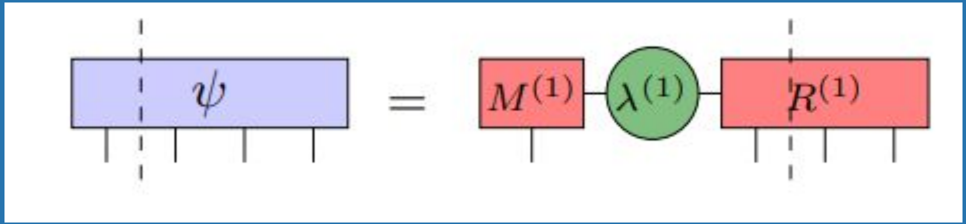
- Lattice PDF $$f_{q/h}(x) = \frac{\delta z}{4\pi} \sum_{z=\bar{N}_{\text{min}}}^{\bar{N}_{\text{max}}} e^{-ix\vec{n}M_h z} \langle h| \left(M_{00}(z) + M_{11}(z) - M_{01}(z) - M_{10}(z) \right) |h\rangle$$

- Lattice LCDA $$\phi_h(x) = \frac{\delta z}{f_h} \sum_{z=\bar{N}_{\text{min}}}^{\bar{N}_{\text{max}}} e^{-i(x-1)\vec{n}M_h z} \langle \Omega| \left(M_{00}(z) + M_{11}(z) - M_{01}(z) - M_{10}(z) \right) |h\rangle$$

- Matrix Elements $$M_{ij}(z) = e^{iHz} \chi^{\dagger}_{-2z+i+\bar{N}} e^{-iHz} \chi_{j+\bar{N}}$$

Tensor Network Methods and Algorithms

- We use ITensor and Julia for our code implementation
- Two challenges: state preparation and real-time evolution
- The MPS is efficient at simulating lattice theory in 1 spatial dimension
- entropy area laws force an upper bound on entanglement entropy



State Preparation: Density Matrix Renormalization Group

- Instantiate a charge-preserving lattice to stay in the Q = 0 sector
- Apply DMRG to obtain the ground (vacuum) state
- Apply DMRG with penalty to obtain the excited (hadron) state

$$H' = H - p|\psi_0\rangle\langle\psi_0| \qquad Q = \sum_{i=1}^{N}(\sigma_n^z + (-1)^n)/2$$

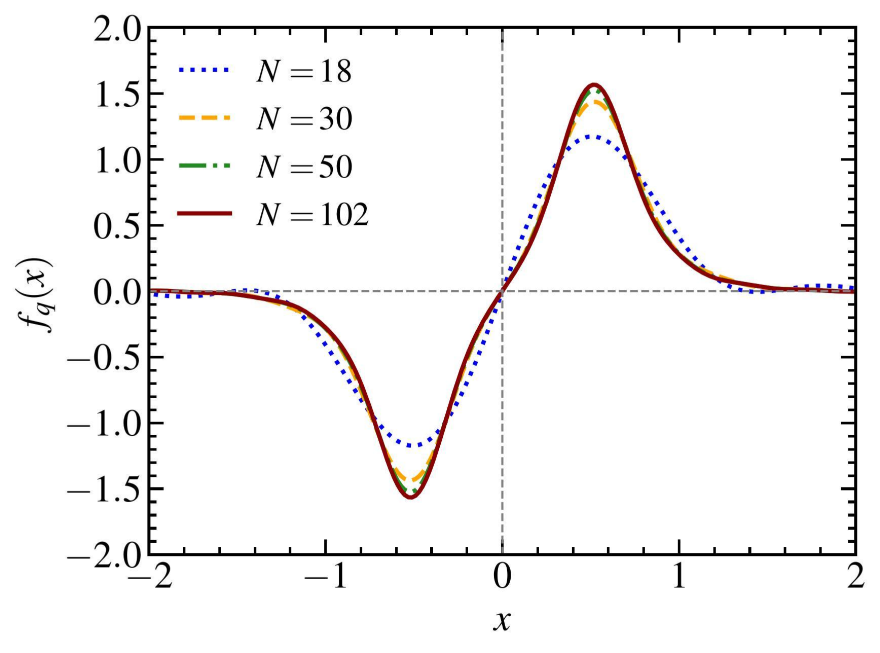
Real-time Evolution: Time-Dependent Variational Principle

- Apply TDVP to time evolve MPS
- We found TDVP more accurate and less computationally demanding than TEBD (trotterization)

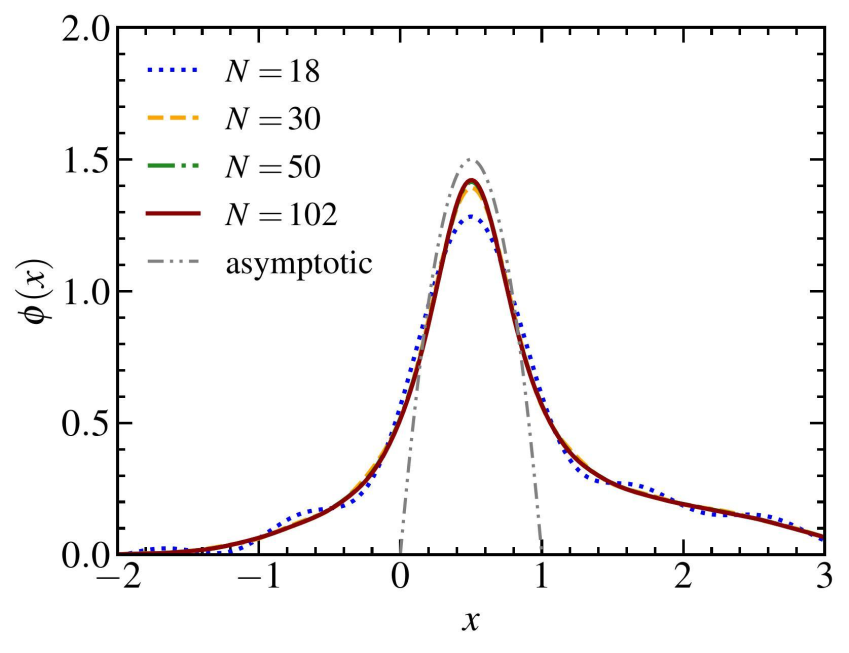
Results

- We fix g = 0.25, m = 0.7

PDF



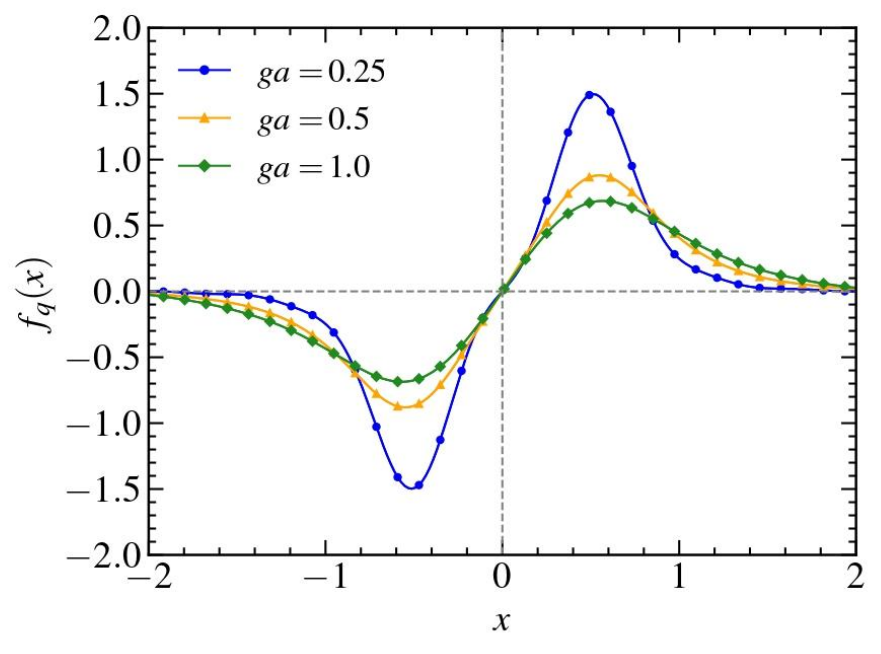
LCDA



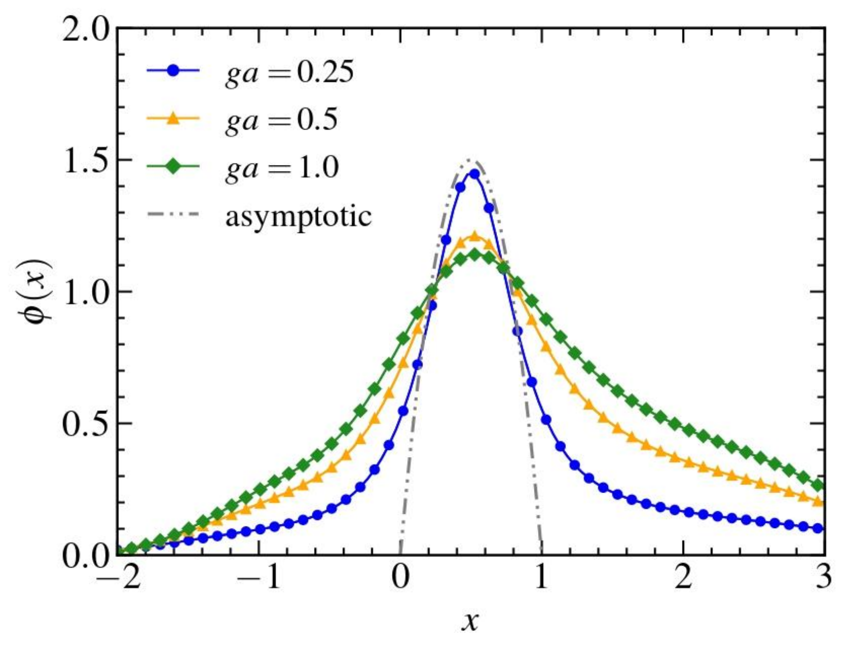
Results

- We fix N = 102, m = 0.7

PDF



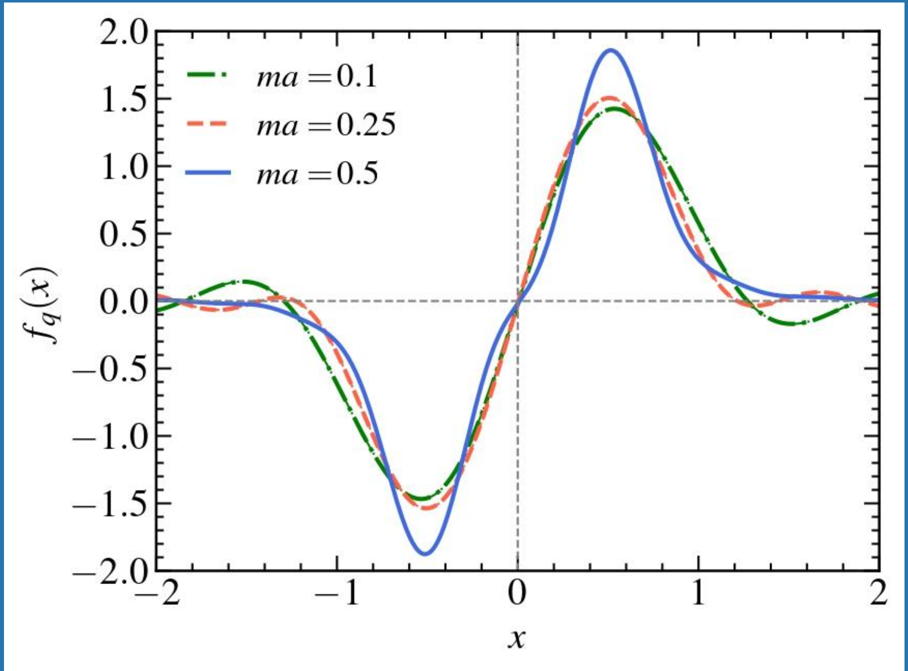
LCDA



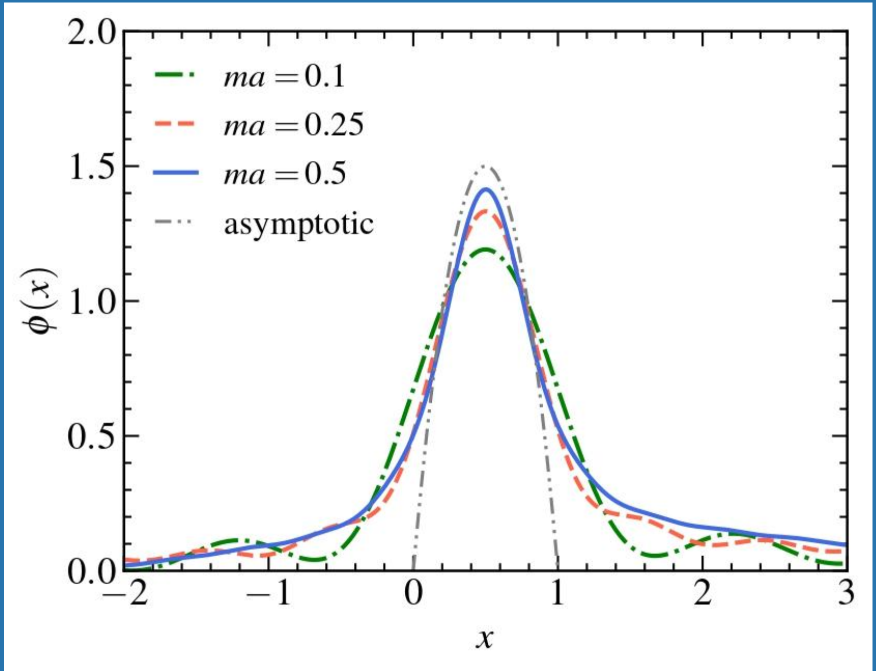
Results

- We fix N = 102, g = 0.25

PDF



LCDA



Conclusion

- Proposed a tensor network strategy for the (1+1)D NJL model
- Utilized a CP lattice and DMRG to prepare our hadron states
- Applied TDVP for real-time evolution of two-point correlators
- Simulated the PDF and LCDA in the continuum limit and observed agreement with pQCD and non-relativistic limits
- Demonstrated advantage of tensor networks in the NISQ era!

Outlook

- Include flavor degrees of freedom
- Study lattice gauge theories
- Extend to higher spatial dimensions (e.g. via PEPS)


Thank you!