Wormholes in the Axiverse

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New Physics Directions in the LHC era and beyond 4/2024, MPI
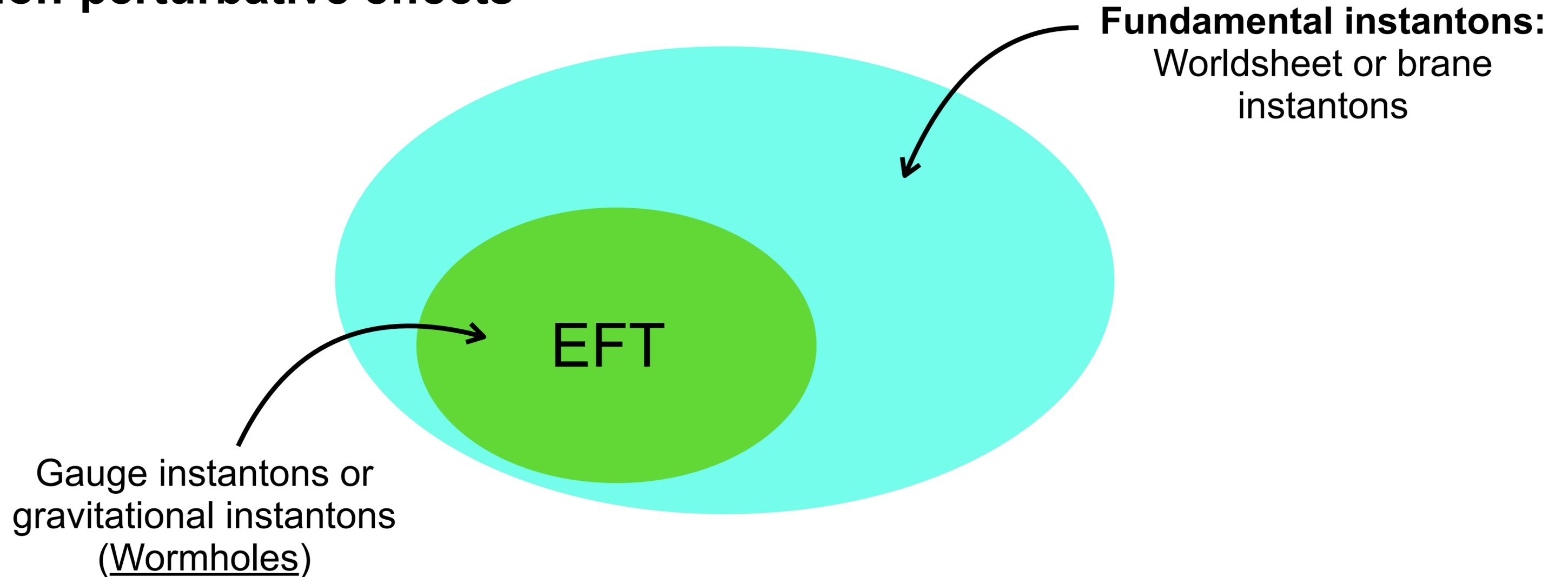
In QFT

— QCD axion is a pseudo-Nambu-Goldstone boson from an approximate (PQ) symmetry

— Approximate symmetry ➔ axion potential of order $V(a) = \epsilon f_a^4$

— Solution of strong CP iff $V(a) = \epsilon f_a^4 \ll V_{\text{QCD}}$

— Large f_a ➔ sensitivity to UV cutoff, <u>even if Planck-suppressed</u>! (quality problem)

Quantum gravity? String theory…

In string theory: $\underline{N \gg 1 \text{ periodic } (a^i \simeq a^i + 1) \text{ fundamental axions}}$ (Axiverse) carrying approximate shift-symmetries $(a^i \to a^i + c^i)$ $\underline{\text{broken only non-perturbatively}}$

Non-perturbative effects

Fundamental instantons:
Worldsheet or brane
instantons

EFT

Gauge instantons or
gravitational instantons
(Wormholes)

An EFT for the $\mathcal{N} = 1$ Axiverse

(Approximate) Shift symmetry & SUSY

Kinetic function $\mathscr{G}_{ij} \equiv \dfrac{1}{2}\dfrac{\partial^2 K}{\partial s^i \partial s^j}$

$$\mathcal{L}_{\text{EFT}} = \frac{1}{2}M_{\text{P}}^2 \int d^4x \sqrt{-g}\left[R - \mathscr{G}_{ij}(s)\left(\partial_\mu a^i \partial^\mu a^j + \partial_\mu s^i \partial^\mu s^j\right) + \mathcal{O}\left(\frac{\partial^4}{M_{\text{UV}}^2}\right)\right]$$

Quantum Gravity constraints

non-perturbative effects dominated by fund. BPS instantons: <u>s must be large</u>

Instanton charge (integers)

$t^i = a^i + is^i$

$$e^{2\pi i q_i t^i} = e^{2\pi i q_i a^i} e^{-2\pi q_i s^i}$$

$$\implies \quad s > 1/\alpha$$

Kinetic term controlled by a <u>homogeneous function</u>

$$K(s) = -\ln P(s)$$

with $P(\lambda s) = \lambda^n P(s)$

and $n = 1,2,3,4,5,6,7$

According to $\hbar$ counting, $[s] = [\hbar]$ ➜ "secret loop-coupling parameter"

$$\boxed{\frac{\hbar}{2\pi s} \ll 1}$$

➜ s must be large.

➜ At a given order, $P(s)$ must be a homogeneous function.

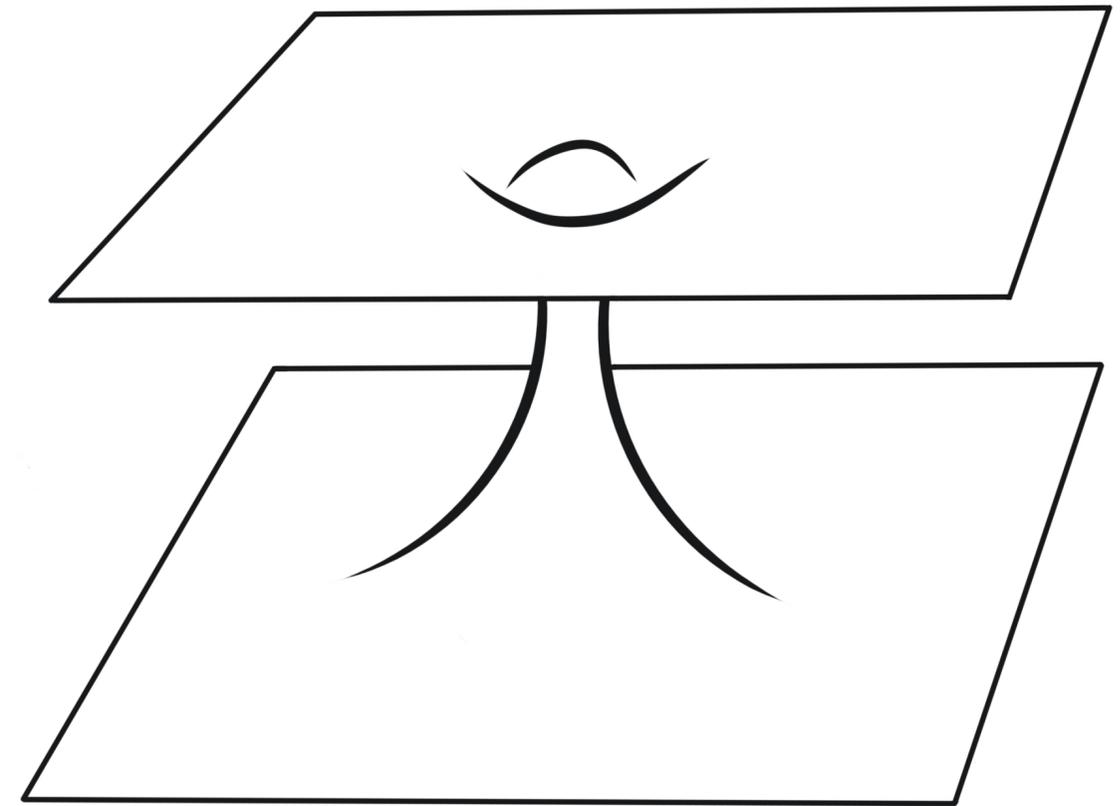"$s > 1/\alpha$" ➜ **Perturbative corrections down by $\alpha/2\pi \ll 1$**

UV non-perturbative corrections by $e^{-\frac{2\pi}{\alpha}} \ll 1$

Wormholes

Creation and annihilation of "baby universes"

Release charge

Withdraw charge

Hawking (1987)
Giddings-Strominger (1988)
Coleman (1988)
Coleman-Lee (1989)
Klebanov-Susskind-Banks (1989)
Preskill (1989)
Kallosh-Linde-Linde-Susskind (1994)
Arkani Hamed-Orgera-Polchinski (2007)
Almheiri-Hartman-Maldacena-Shaghoulian-Tajdini (2019)
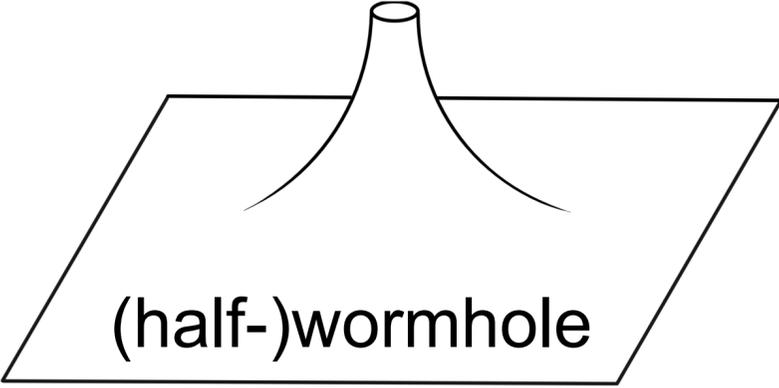Marolf-Maxfield (2020)
McNamara-Vafa (2020)

Wormhole mysteries

$e^{-S_{\mathsf{W}}}$

(half-)wormhole

$e^{-S_{\mathsf{hw}}} = e^{-S_{\mathsf{W}}/2}$

$$\exp\left\{-\int d^4x\sqrt{|g(x)|}\int d^4y\sqrt{|g(y)|}\,C^{IJ}\,\overline{\mathcal{O}}_I(x)\mathcal{O}_J(y)\right\} =$$

$$\int d\alpha\,d\bar{\alpha}\,e^{-\alpha^I C_{IJ}^{-1}\bar{\alpha}^J}\exp\left\{-\int d^4x\sqrt{|g|}\,\alpha^I\mathcal{O}_I(x) + \text{c.c.}\right\}$$

<u>Mysteries</u>:
— Loss of locality and coherence? Euclidean path integral of quantum gravity?
— Cluster decomposition generically violated (factorization) $\langle A(x)B(y)\rangle = \int d\alpha\,d\bar{\alpha}\,e^{-\alpha^I C_{IJ}^{-1}\bar{\alpha}^J}\langle A(x)B(y)\rangle_{\alpha,\bar{\alpha}}$
— Undetermined EFT couplings in conflict with string theory experience
— Charge is conserved (globally): how can we induce an axion potential?

Hebecker–Mikhail–Soler (2018)

Wormhole mysteries

$$\exp\left\{-\int d^4x\sqrt{|g(x)|}\int d^4y\sqrt{|g(y)|}\; C^{IJ}\,\overline{\mathcal{O}}_I(x)\mathcal{O}_J(y)\right\} =$$

$$\int d\alpha\, d\bar{\alpha}\, e^{-\alpha^I C^{-1}_{IJ}\bar{\alpha}^J}\exp\left\{-\int d^4x\sqrt{|g|}\,\alpha^I\mathcal{O}_I(x)+\text{c.c.}\right\}$$

Avoided if Coleman's α-parameters are uniquely determined

Mysteries:
— Loss of locality and coherence? Euclidean path integral of quantum gravity?
— Cluster decomposition generically violated (factorization)
— Undetermined EFT couplings in conflict with string theory experience
— Charge is conserved (globally): how can we induce an axion potential?

α's must be peaked away from 0 (Symmetry breaking)
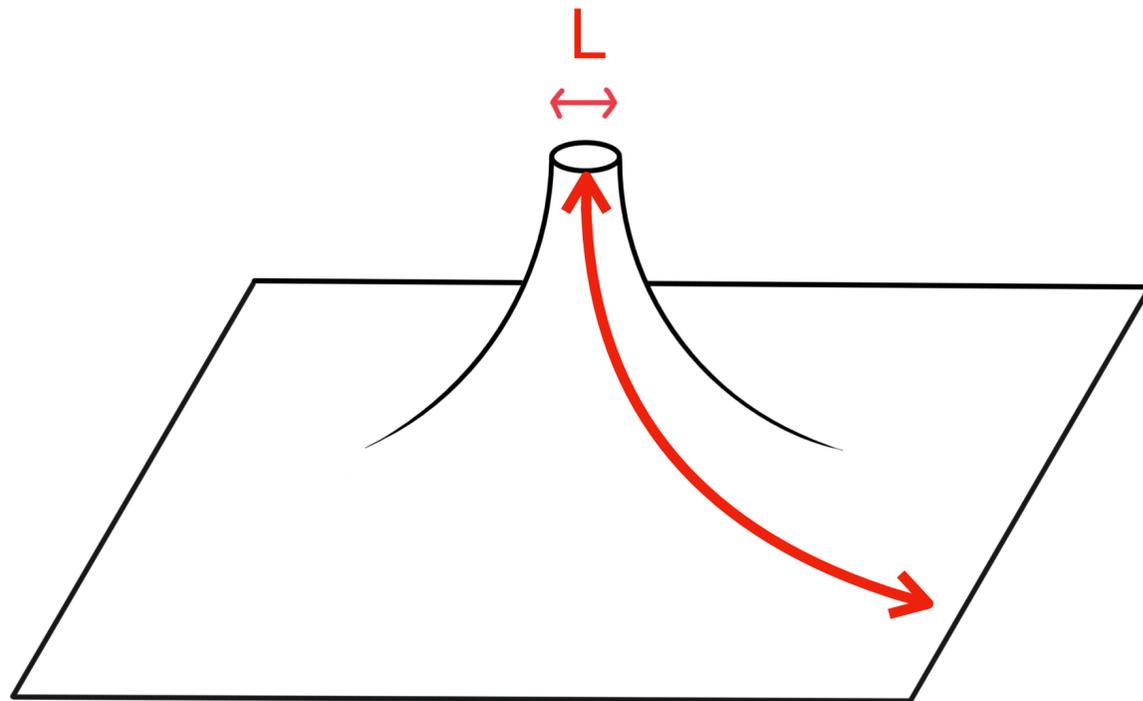
... in the $\mathcal{N} = 1$ Axiverse

General O(4)-invariant solutions carrying <u>quantized axionic charges q_i</u> ($J_i = \mathscr{G}_{ij} da^i$)
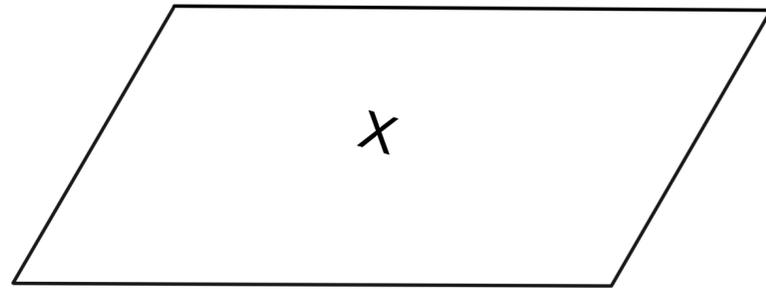
$$a^i = a^i(r)$$
$$s^i = s^i(r)$$

$$ds^2 = \frac{1}{1 - \frac{L^4}{r^4}} dr^2 + r^2 d\Omega_3^2$$

The problem is equivalent to the motion of a point particle in N dimensions with energy
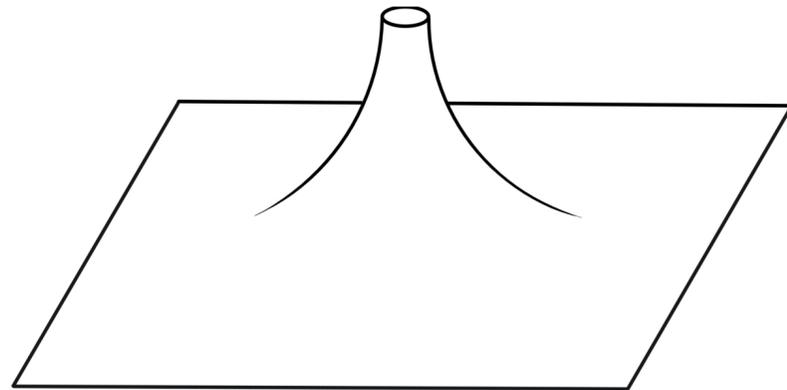
L

$$E \equiv \frac{1}{2} \mathscr{G}_{ij} \, \dot{s}^i \dot{s}^j - \frac{1}{2} \mathscr{G}^{ij} q_i q_j$$

$$L^4 \equiv \frac{|E|}{3\pi^2 M_{\mathrm{P}}^4}$$

Extremal wormholes (E=0)
(Singular scalars, flat geometry)

Non-extremal wormholes (E<0)
(Non-trivial regular geometry)

Extremal Wormholes (E=0)

$$\ell_i = -\frac{1}{2}\frac{\partial K}{\partial s^i}$$

Dual saxions

$$\ell_i(r) = \ell_i(\infty) + \frac{q_i}{2\pi M_{\mathrm{P}}^2 r^2}$$
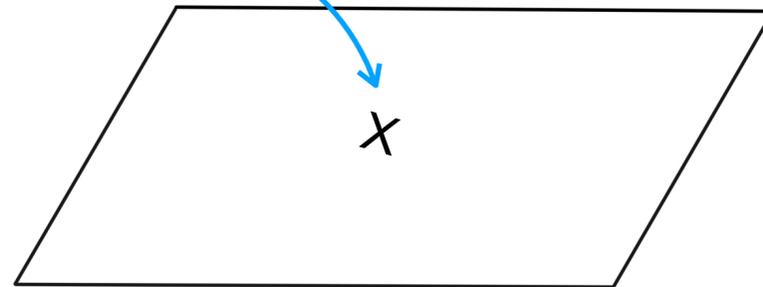
$$S|_{\mathrm{extr.}} = 2\pi q_i s_\infty^i - 2\pi q_i s_0^i$$
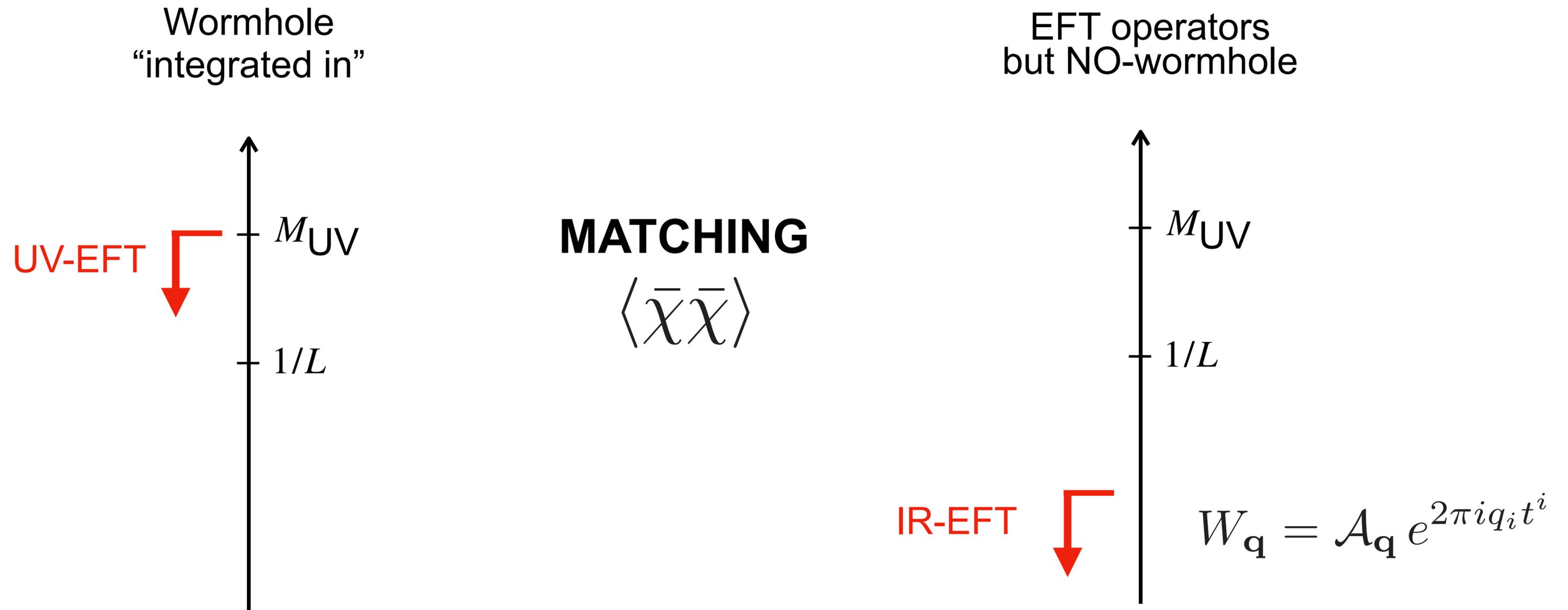
Quantum gravity input:

they describe the insertion of fundamental instantons at $r = 0$!

$$S|_{\mathrm{c.t.}} = 2\pi q_i s_0^i$$

$$S|_{\mathrm{extr.}} + S|_{\mathrm{c.t.}} = 2\pi q_i s_\infty^i$$

Let's integrate the wormhole out below 1/L…

Wormhole
"integrated in"

EFT operators
but NO-wormhole

UV-EFT

M_{UV}

$1/L$

MATCHING

$$\langle \bar{\chi}\bar{\chi} \rangle$$

M_{UV}

$1/L$

IR-EFT

$$W_{\mathbf{q}} = \mathcal{A}_{\mathbf{q}}\, e^{2\pi i q_i t^i}$$

**Extremal wormholes (aka fund. Instantons)
can induce effective (symmetry-breaking) superpotentials**

Non-Extremal Wormholes (E<0)

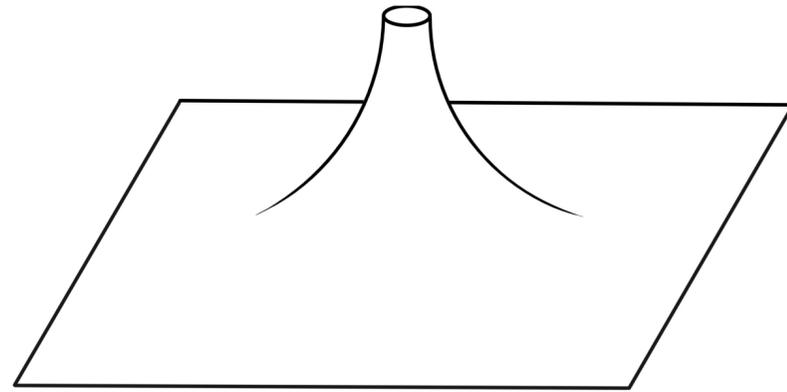Degree of homogeneity of P(s): $\begin{cases} n > 3 \text{ Fully regular solutions} \\ n = 3 \text{ Marginally-degenerate solutions} \\ n < 3 \text{ ~~Singular solutions~~} \end{cases}$

Non-Extremal, regular (n>3) Wormholes (E<0)

— We find analytic (homogeneous) solutions involving all N fields: $\ell_i(r) = q_i \ell_* \cos\left[\sqrt{\frac{3}{n}}\left(\frac{\pi}{2} - \arcsin\left(\frac{L^2}{r^2}\right)\right)\right]$

— Always "seed" of more general (numerical) configurations

MATCHING
(4 Goldstinos)

$$\langle \bar{\chi}\bar{\chi}\chi\chi \rangle$$

Non-Extremal regular wormholes can induce effective (symmetry-breaking) Kahler potential

$$K_{\mathbf{q}} \propto e^{-S|_{\mathrm{hw}}}$$

Non-Extremal, <u>marginally-degenerate</u> (n=3) Wormholes

These degenerate at $r = \infty$, have IR-divergent action…
Approximate analytic solution (not too large r)

$$\ell_i = \frac{q_i}{2\pi M_{\mathrm{P}}^2 r^2} + \ell_i(\infty) \left[1 + \mathcal{O}(L^4/r^4) \right]$$

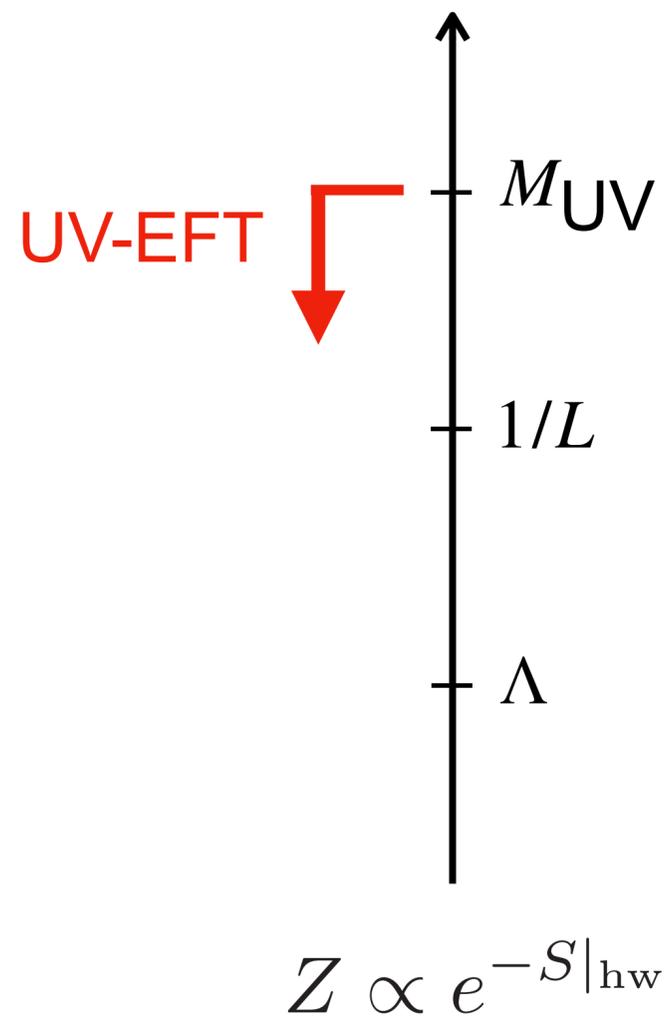Introducing an IR cutoff $r > 1/\Lambda$ the regularized action reads:

Look suspiciously similar to the extremal case…

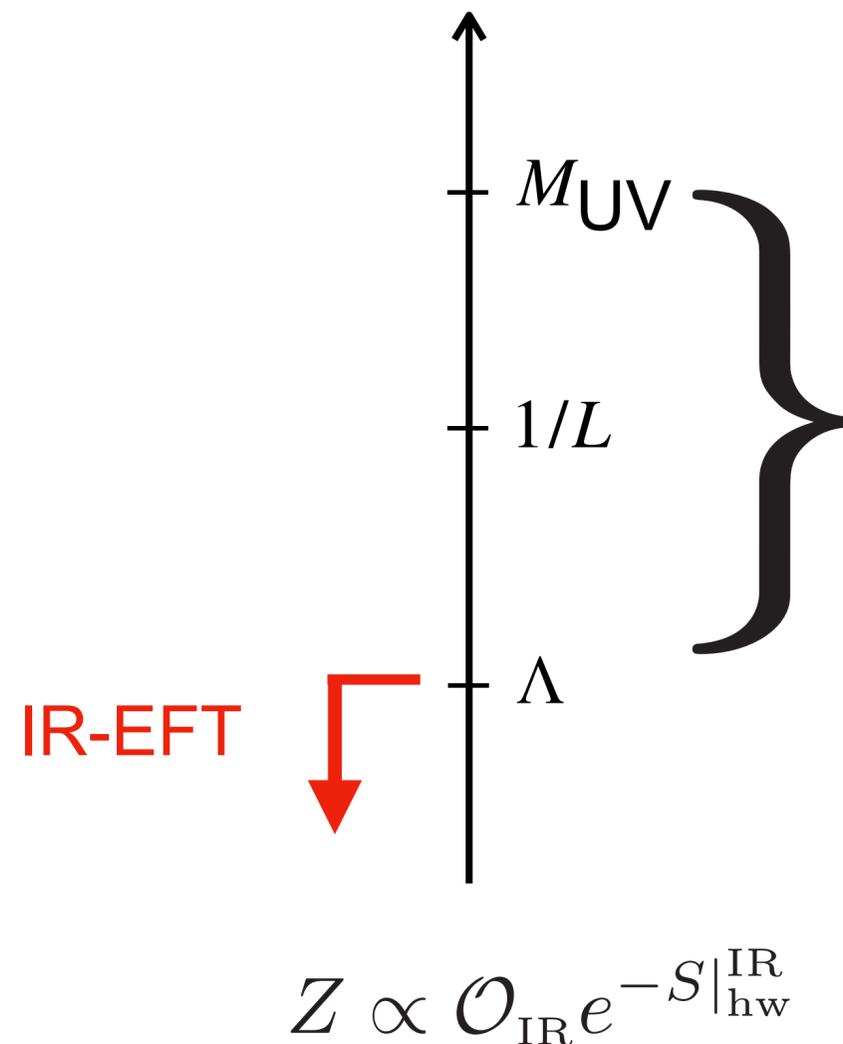$$S|_{\mathrm{hw}}^{\Lambda} = 2\pi q_i s_{\Lambda}^i \left[1 + \mathcal{O}(\Lambda^4 L^4) \right]$$

Let's integrate it out below 1/L…

Wormhole
"integrated in"

"Integrate out" the wormhole
below $\Lambda \ll 1/L$.

UV-EFT

M_{UV}

$1/L$

Λ

M_{UV}

$1/L$

IR-EFT

Λ

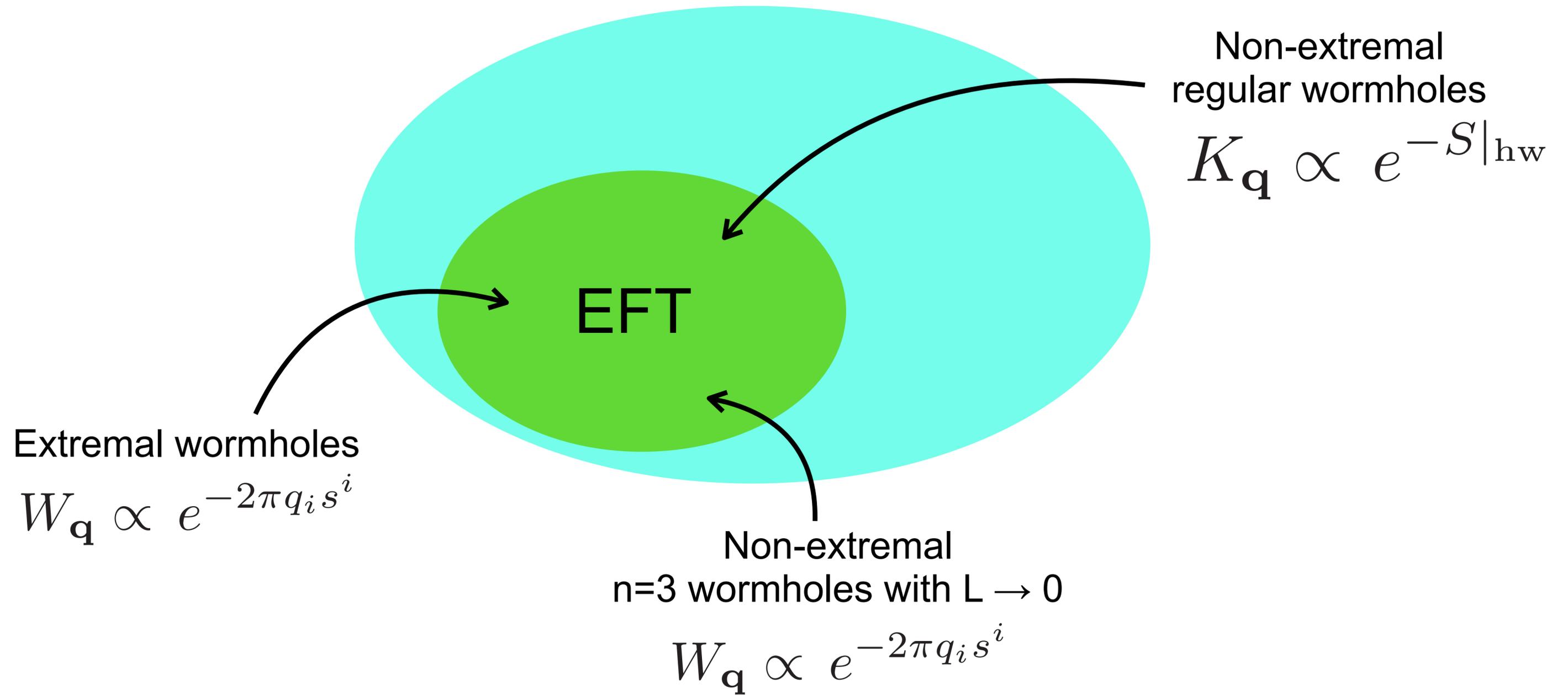Wilson's RG reproduces the counterterm that
describes fund. instantons (up to $\mathcal{O}(\Lambda^4 L^4)$)

$$\mathcal{O}_{\text{IR}} \propto e^{-(S|_{\text{hw}} - S|_{\text{hw}}^{\text{IR}})} = e^{-S|_{\text{hw}}^{\Lambda}}$$

**n=3 wormholes are
fundamental instantons
in disguise!**

$$Z \propto e^{-S|_{\text{hw}}}$$

$$Z \propto \mathcal{O}_{\text{IR}} e^{-S|_{\text{hw}}^{\text{IR}}}$$

Quality problem?

Non-perturbative effects

Non-extremal
regular wormholes

$$K_{\mathbf{q}} \propto e^{-S|_{\mathrm{hw}}}$$

EFT

Extremal wormholes

$$W_{\mathbf{q}} \propto e^{-2\pi q_i s^i}$$

Non-extremal
n=3 wormholes with L → 0

$$W_{\mathbf{q}} \propto e^{-2\pi q_i s^i}$$

Non-perturbative effects

Non-extremal
regular wormholes
$$K_{\mathbf{q}} \propto e^{-S|_{\mathrm{hw}}}$$

EFT

Extremal wormholes
$$W_{\mathbf{q}} \propto e^{-2\pi q_i s^i}$$

Non-extremal
n=3 wormholes with L → 0
$$W_{\mathbf{q}} \propto e^{-2\pi q_i s^i}$$

$$e^{-\mathcal{O}(1)\frac{2\pi}{\alpha}N}$$

For regular wormholes $\quad S|_{\mathrm{hw}} \geq 3\pi^3 M_{\mathrm{P}}^2 L^2$

Within the EFT $\quad L > 1/M_{\mathrm{UV}}$

Based on simple large N argument: $\quad M_{\mathrm{UV}} \lesssim \dfrac{2\pi M_P}{\sqrt{N}} \quad \Longrightarrow \quad S|_{\mathrm{hw}} = \mathcal{O}(N)$

Wormholes MUST be very
suppressed in the Axiverse

Easier to formalize for regular wormholes…
Detour: Gauss-Bonnet

Higher order terms are negligible for $p \ll M_{\mathsf{UV}}$, but (semi-)topological terms…

$$\int d^4x \sqrt{-g} \left[\frac{\tilde{C}_i s^i}{32\pi} \left(R_{abcd}R^{abcd} - 4R_{ab}R^{ab} + R^2 \right) + \frac{\tilde{C}_i a^i}{384\pi} \epsilon^{cdef} R^a_{\ bef} R^b_{\ acd} \right]$$

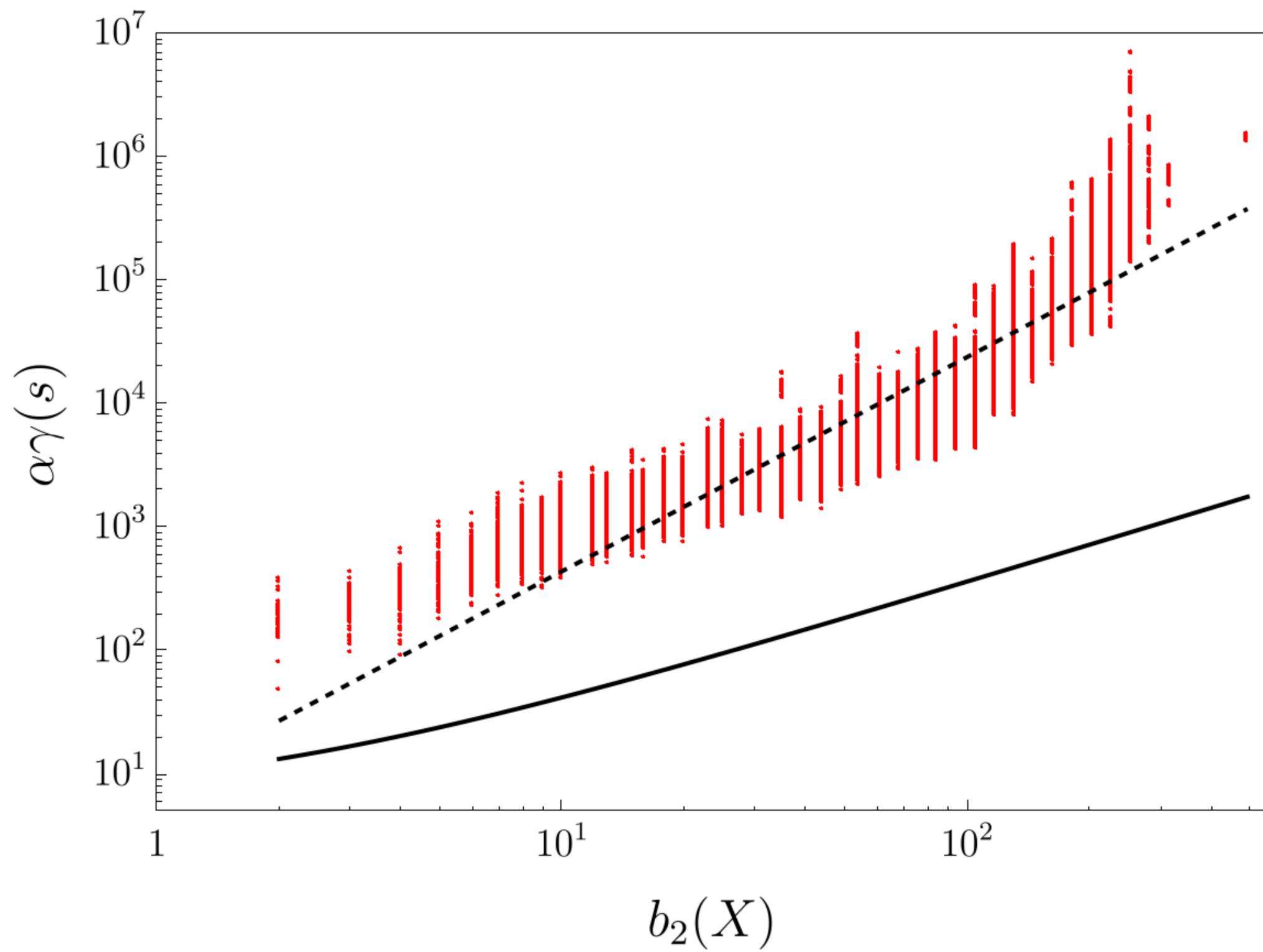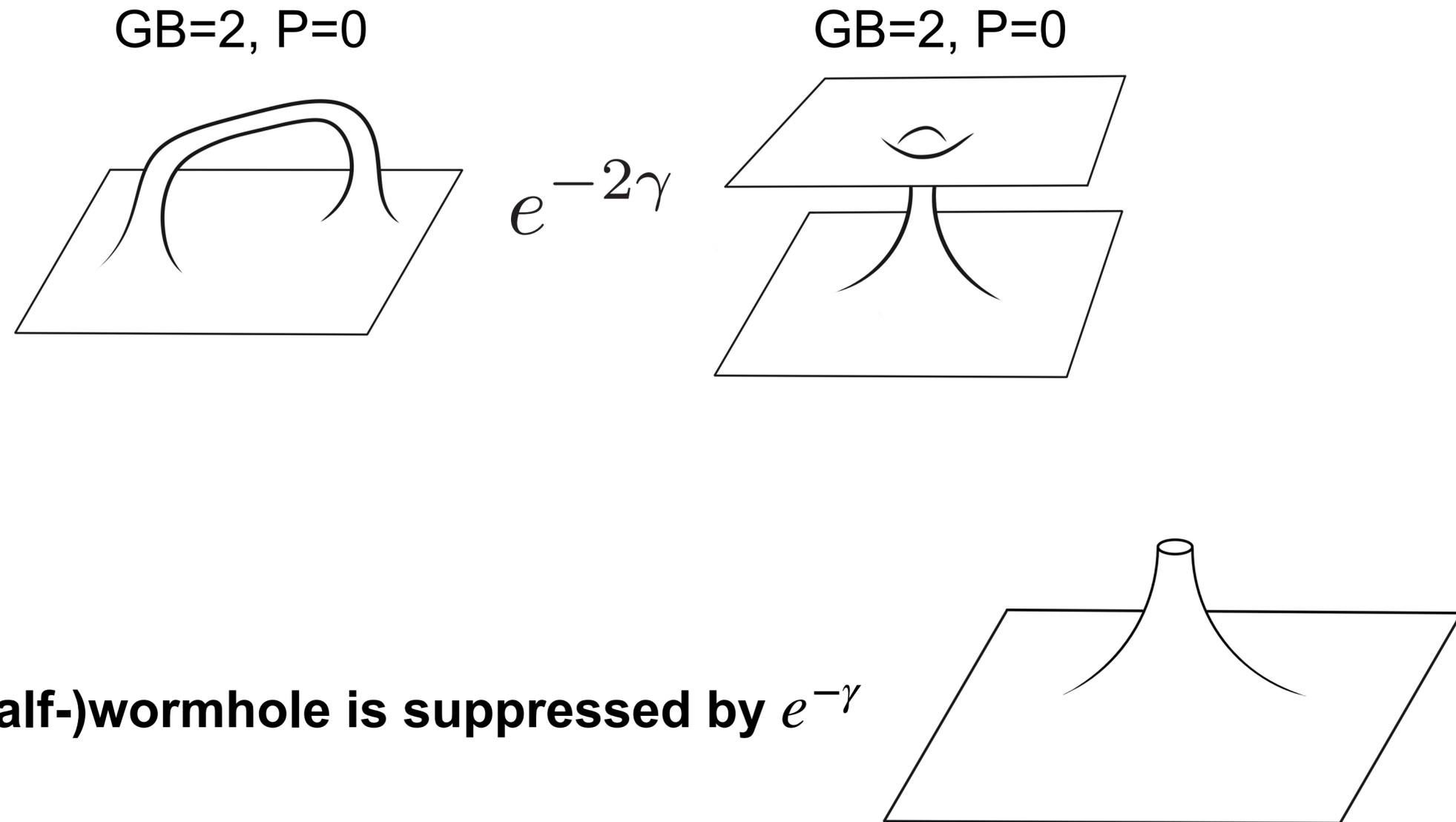Gauss-Bonnet

Pontryagin

$\tilde{C}_i \in \mathbb{Z}$ (axion periodicity)
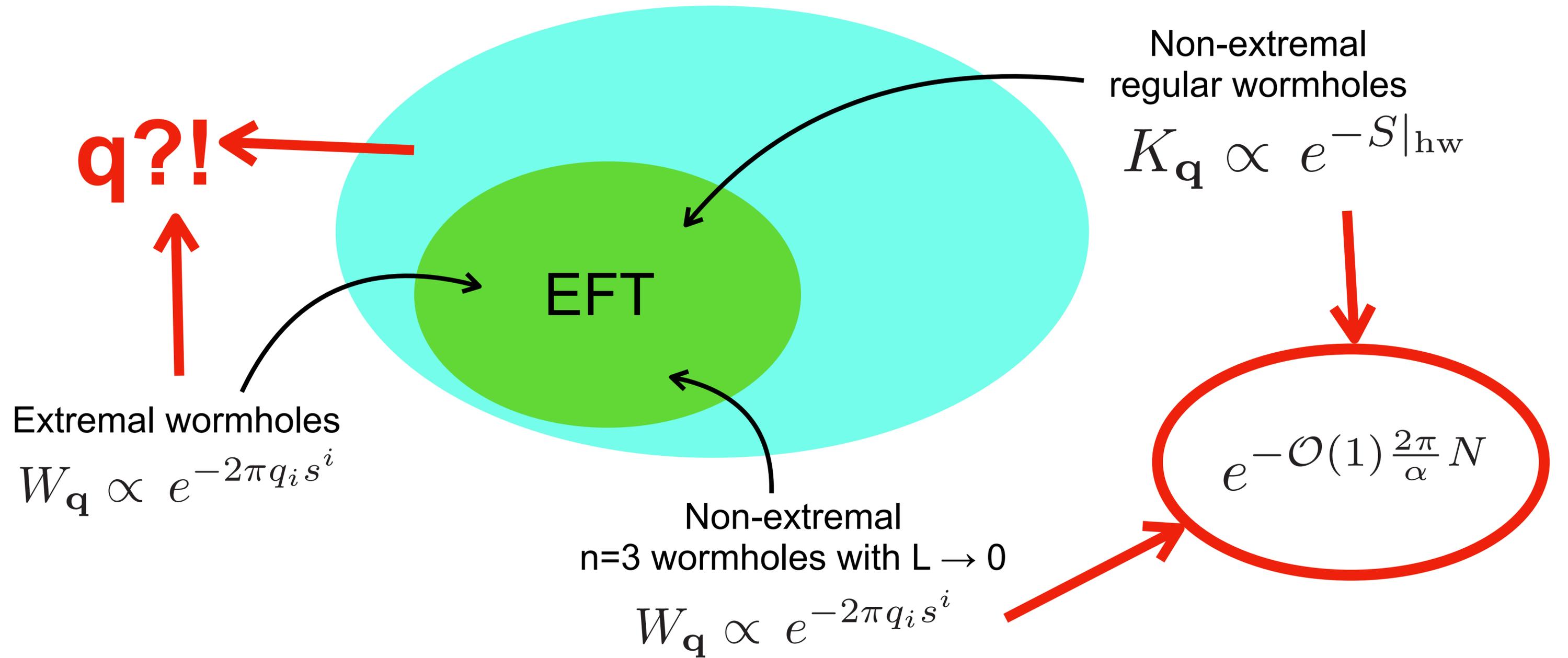
Another quantum gravity constraint on $\tilde{C}_i$ … Martucci-Risso-Weigand (2023)

In the domain "$s > 1/\alpha$" implies $\gamma(s) = \dfrac{\pi}{6} \tilde{C}_i s^i \geq \dfrac{N\pi}{6\alpha}$

GB=2, P=0   GB=2, P=0

$$e^{-2\gamma}$$

➔ **Each (half-)wormhole is suppressed by** $e^{-\gamma}$

Non-perturbative effects

q?!

Non-extremal
regular wormholes

$$K_{\mathbf{q}} \propto e^{-S|_{\mathrm{hw}}}$$

EFT

Extremal wormholes

$$W_{\mathbf{q}} \propto e^{-2\pi q_i s^i}$$

Non-extremal
n=3 wormholes with L → 0

$$W_{\mathbf{q}} \propto e^{-2\pi q_i s^i}$$

$$e^{-\mathcal{O}(1)\frac{2\pi}{\alpha}N}$$

Axion potential in the $\mathcal{N} = 1$ axiverse

$$V(a) \propto e^{-2S} + e^{-S} \left(\frac{M_{\mathrm{SSB}}}{M_{\mathrm{UV}}} \right)^k$$

Conclusions

* **A deep connection between fundamental and gravitational instantons**
  n=3 wormholes are fund. instantons in disguise ➔ α-parameters are fixed!
* **Non-extremal wormholes are suppressed by $e^{-\frac{2\pi}{\alpha}N}$.**
* **Bound on Gauss-Bonnet ➔ also QFT axions (in Axiverse) are $e^{-\frac{2\pi}{\alpha}N}$.**
* **Quality controlled by fund. instantons (model-dep.). How severe?***
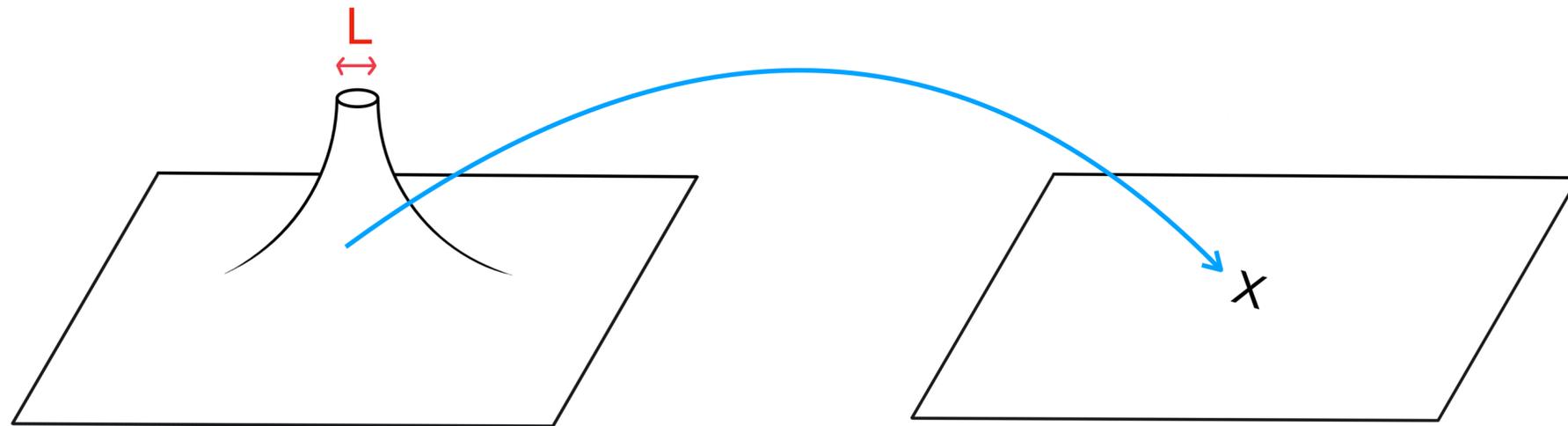
* For Type IIB, ask McAllister…

Additional material…

n=3 wormholes at low energy

In general…
— non-extremal n=3 wormholes have 4 zero-modes ➔ <u>no superpotential</u>
— 2 of the zero-modes are localized around L ➔ <u>cannot induce Kahler either!</u>

But… in the limit $L \to 0$ they are indistinguishable from fundamental instantons and must be able to induce effective superpotentials

First proposed in Park-Srednicki-Strominger (1990)

Gauss-bonnet and a bound on the species scale

$$\mathcal{L}|_{\Lambda < M_{\text{KK}}} = M_P^2 \sqrt{-g} \left\{ R + c_2 \frac{R^2}{M_{\text{UV}}^2} + c_3 \frac{R^3}{M_{\text{UV}}^4} + \cdots \right\}$$

Generic R^2 operators receive corrections from KK (if present)
Gauss-Bonnet does not ➜ suppressed by the highest (extra-dimensional) cutoff (species scale)

Our bound

EFT perturbativity

Compatible with "distance conjecture"

Large N expectation

$$\frac{N}{2\pi\alpha} < \frac{\gamma}{32\pi^2} < \frac{M_{\text{P}}^2}{M_{\text{UV}}^2} \qquad\qquad M_{\text{UV}}^2 < \min\left(M_{\text{KK}}^2, M_s^2\right) \leq \frac{\alpha}{2\pi} \frac{(2\pi M_P)^2}{N}$$

$$\implies \quad S|_{\text{hw}} > \mathcal{O}(1)\frac{2\pi}{\alpha} N$$