In Memoriam
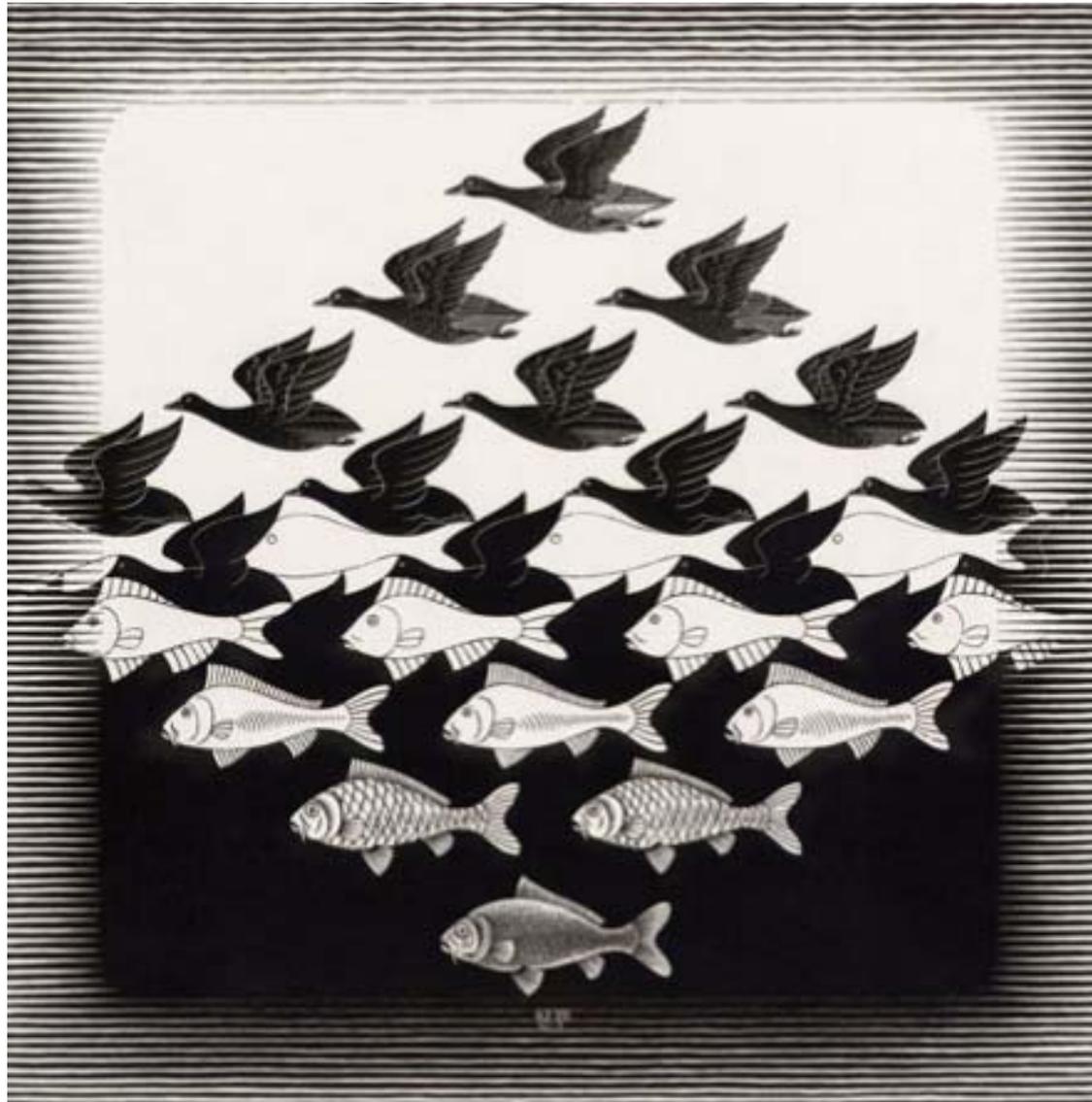

Durmuş Demir (1967-2024)
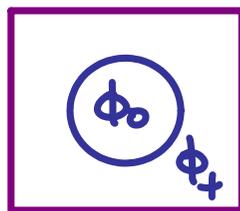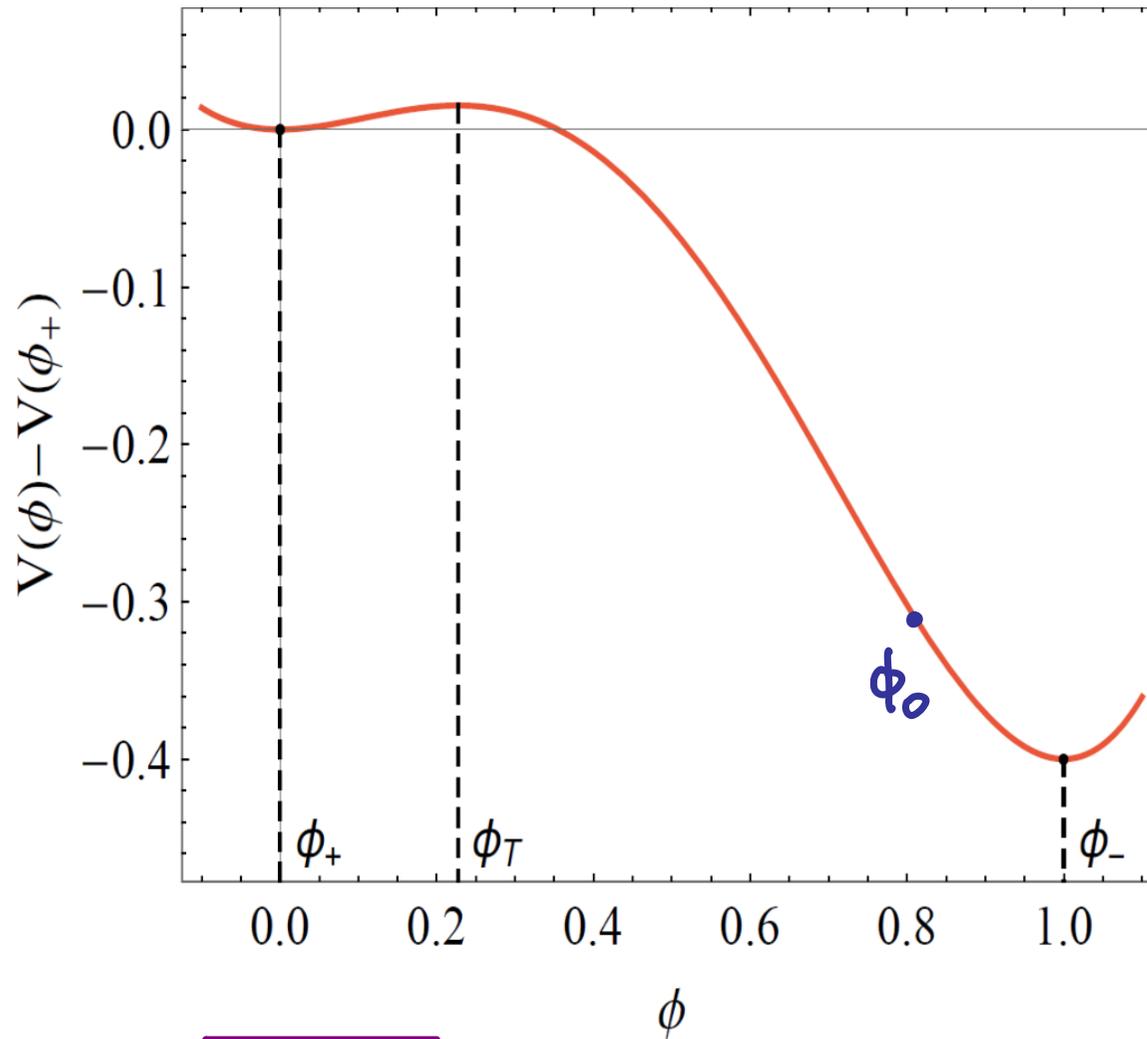
1. Tunneling Potential Formalism

WHY VACUUM DECAY ?

- BSM/SM in false vacuum

- Early universe phase transitions

- Vacuum decay triggered by inflation

- How the string vacuum landscape is populated

.
.

THE PROBLEM

$$\Gamma/V = A e^{-S}$$

calculate this

EUCLIDEAN FORMALISM

Coleman'77
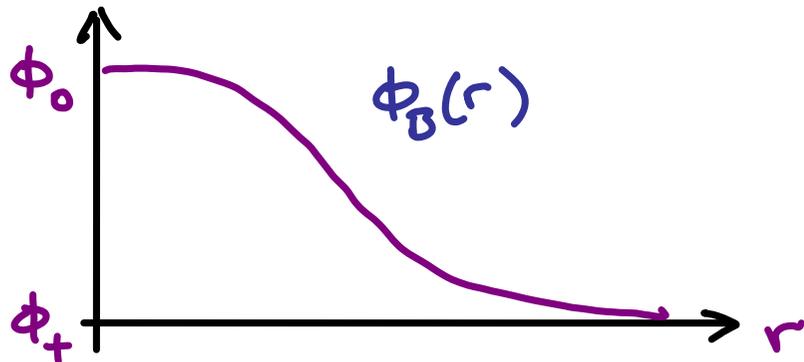
Euclidean bounce $\phi_B(r)$, $O(4)$-sym, extremizes

$$S_E = 2\pi^2 \int_0^\infty dr \, r^3 \left[\frac{1}{2} \dot\phi^2 + V(\phi) \right]$$

$\Rightarrow$

$$\ddot\phi + \frac{3}{r}\dot\phi = \frac{\partial V}{\partial \phi}$$

with $\quad \phi(0)=\phi_0 \quad \dot\phi(0)=0 \quad \phi(\infty)=\phi_+$

$\phi_B(r)$

$$S = \Delta S_E \equiv S_E[\phi_B] - S_E[\phi_+]$$

TUNNELING POTENTIAL FORMALISM

$$S[V_t] = 54\pi^2 \int_{\phi_+}^{\phi_e} \frac{(V - V_t)^2}{(-V_t')^3} \, d\phi$$

$$S = \underset{V_t}{\text{Min}} \, S[V_t]$$

$$\phi_e = \phi_o = \phi(0), \text{Euclidean}$$

DE-EUCLIDEANIZE

⇨ Get rid of Euclidean quantities in terms of V & V_t ones

LINK : $\qquad V_t = V - \frac{1}{2}\dot{\phi}^2$

⇨ $\dot{\phi} = -\sqrt{2(V-V_t)}$ $\qquad \ddot{\phi} = \frac{d}{d\phi}\left(-\sqrt{2(V-V_t)}\right)\dot{\phi} = V' - V_t'$

EoM : $\qquad \ddot{\phi} + \frac{3}{r}\dot{\phi} = V'$ $\qquad$ ⇨ $\qquad r = \dfrac{3\sqrt{2(V-V_t)}}{-V_t'}$

$\frac{d}{dr}(\ldots)$ ⇨ $(4V_t' - 3V')V_t' + 6(V-V_t)V_t'' = 0$

Action gives this EoM for V_t

PROPERTIES & APPS OF V_t-FORM.

JRE '1805

★ Field can be a distraction.

V_t on same footing as V

★ V_t monotonic ➪ Easy to approximate
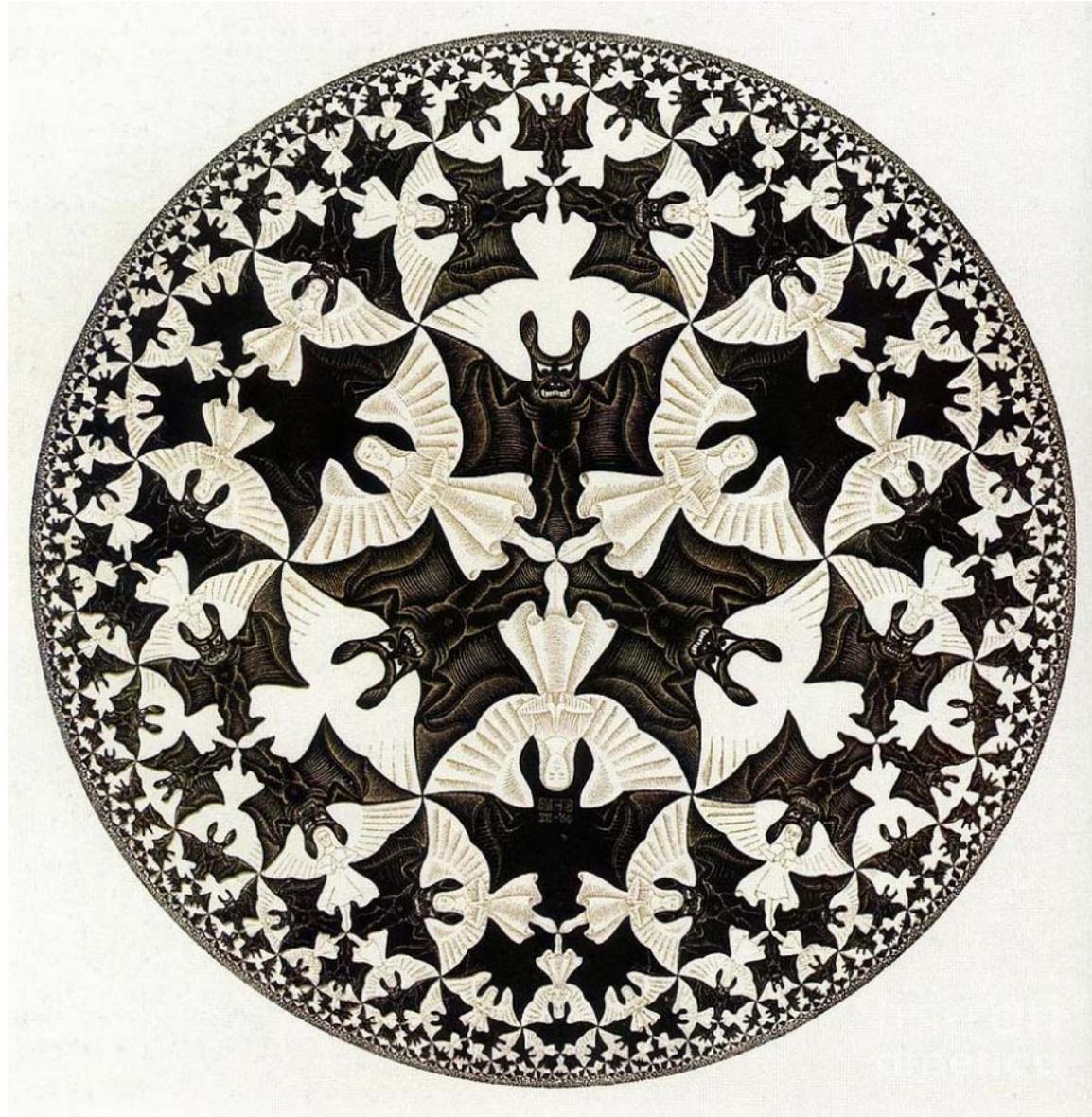
★ $S[V_t]$ minimized ($\phi_B(r)$ saddle-point)

➪ Good for numerics

➪ Useful for multi-field case   JRE, Konstandin '1811

★ Generalizes to any dimension

$d = 3$ applicable to finite T phase transitions

2. GRAVITY CORRECTIONS

JRE '1808
JRE, Huertas,
 Fortin '2106
JRE, Fortin '2211

WHY ?

Gravity relevant if tunneling involves

$$\Delta \phi \lesssim m_P \qquad \text{or} \qquad \Delta V \lesssim m_P^4 \qquad \text{or} \qquad |V_+| \lesssim m_P^4$$

Note $\Delta \phi \lesssim m_P$ already in SM...

When relevant :

Need to include reaction of the metric.

Gravity can cause qualitative changes

EUCLIDEAN FORMALISM W/GRAVITY

Assuming $O(4)$

Euclidean bounce $\phi_B(\xi)$ and metric function $\rho(\xi)$

$$ds^2 = d\xi^2 + \rho^2(\xi)\, d\Omega_3^2$$

↳ line-element S^3

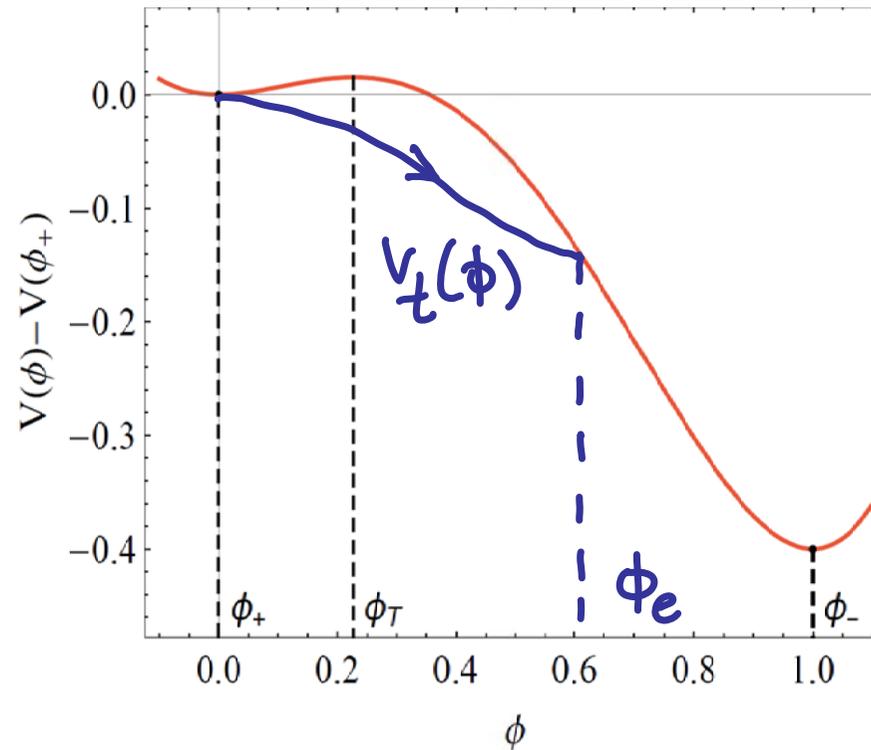$$\Rightarrow \quad \ddot{\phi} + \frac{3\dot{\rho}}{\rho}\dot{\phi} = \frac{\partial V}{\partial \phi} \qquad\qquad \dot{\rho}^2 = 1 + \frac{\kappa \rho^2}{3}\left(\frac{1}{2}\dot{\phi}^2 - V\right)$$

$$\kappa \equiv 1/m_P^2$$

$$S = \Delta S_E = S_E[\phi_B, \rho_B] - S_E[\phi_+, \rho_+]$$

V_t FORMALISM w/ GRAVITY

$$S[V_t] = \frac{6\pi^2}{\kappa^2} \int_{\phi_+}^{\phi_e} \frac{(D+V_t')^2}{DV_t^2} \, d\phi \qquad = \Delta S_E$$

with

$$D = \sqrt{(V_t')^2 + 6\kappa(V-V_t)V_t}$$

$$\kappa \equiv 1/m_P^2$$

DE-EUCLIDEANIZE

⇨ Get rid of Eucl. quant. in terms of V & V_t
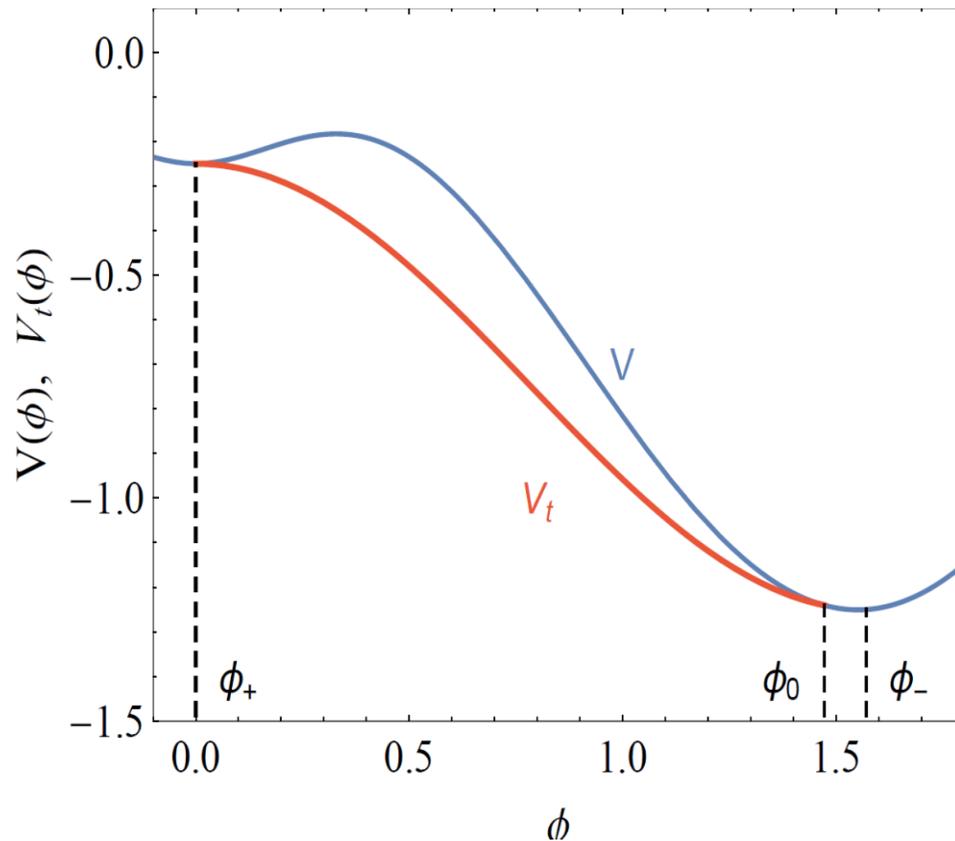
$$V_t = V - \frac{1}{2}\dot{\phi}^2$$

As before. Now

$$r = \frac{3\sqrt{2(v-v_t)}}{-v_t'} \longrightarrow \rho = \frac{3\sqrt{2(v-v_t)}}{D}$$

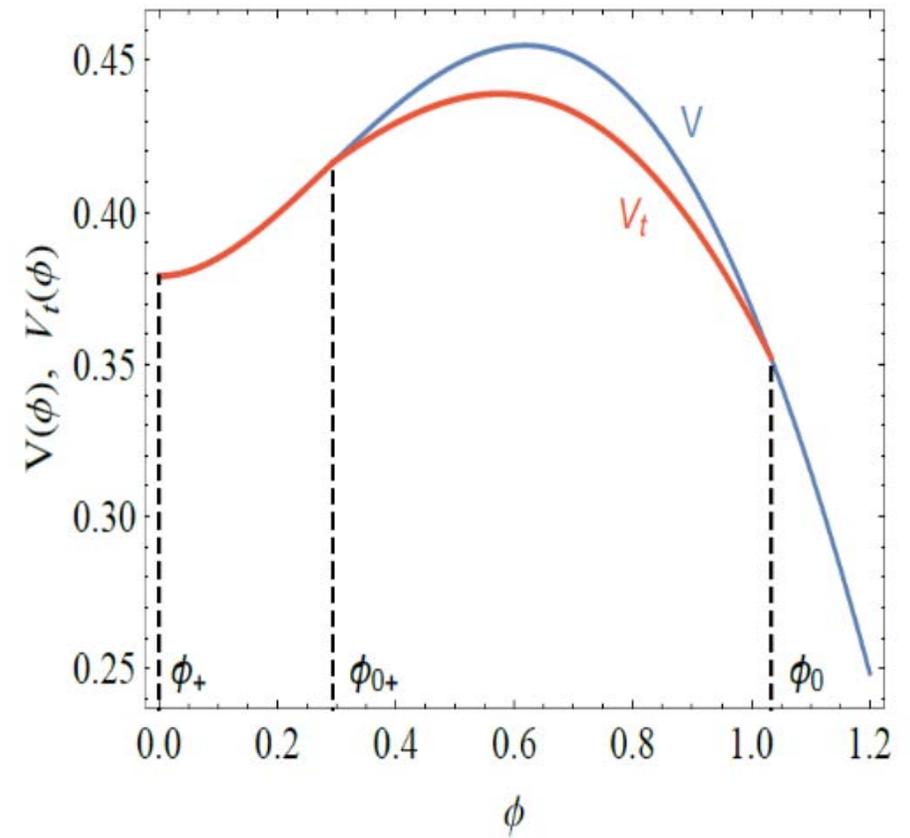$$\frac{d}{d\xi}(\ldots) \Rightarrow 0 = (4v_t' - 3v')v_t' + 6(v-v_t)\left[v_t'' + \kappa(3v - 2v_t)\right]$$

Action gives this EoM for V_t

PROPERTIES & APPS OF V_t-FORM.

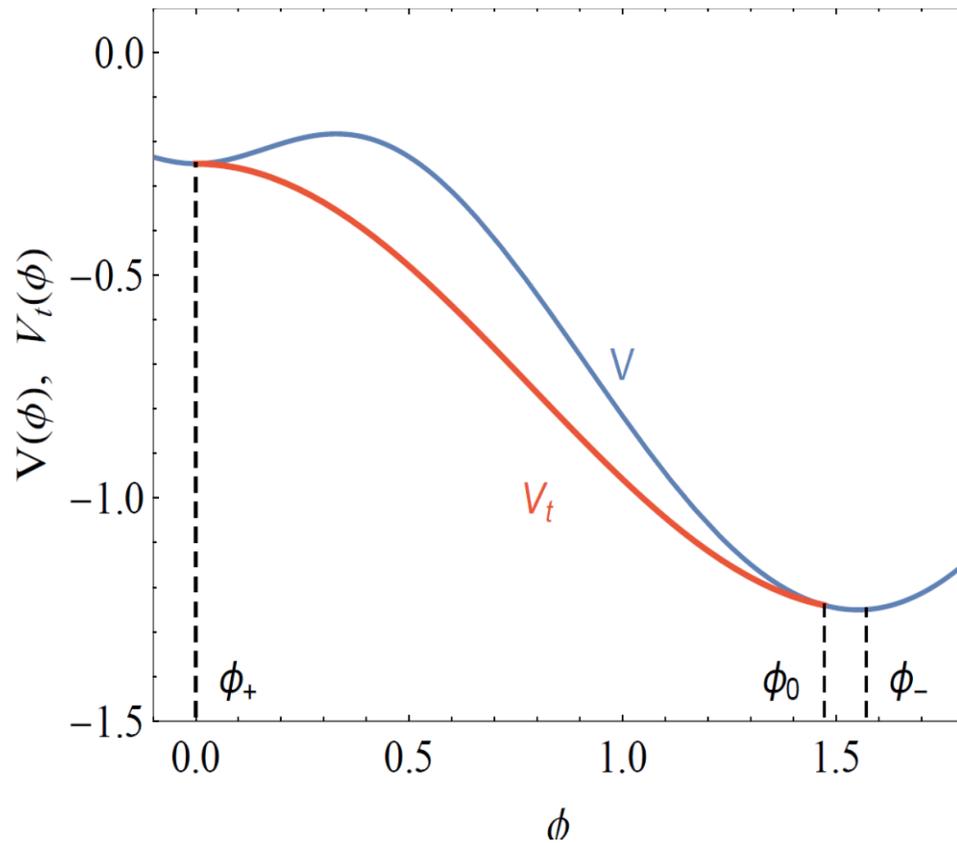Universal formula, valid for AdS, Minkowski or dS
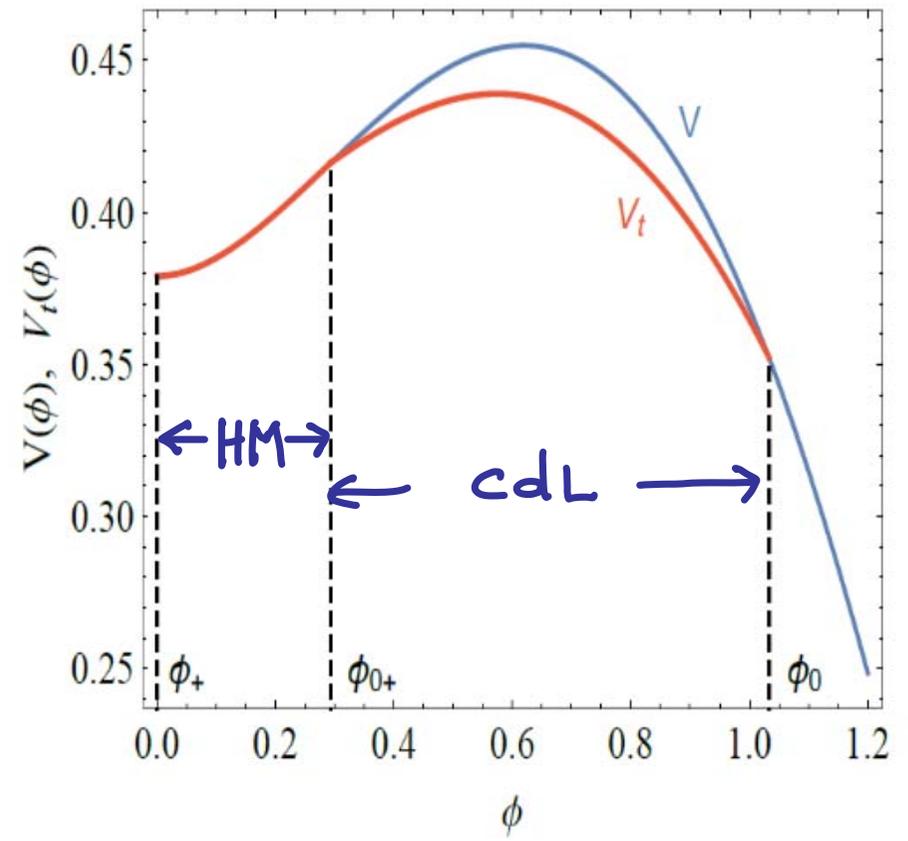

Minkowski or AdS

dS

PROPERTIES & APPS OF V_t-FORM.

Universal formula, valid for AdS, Minkowski or dS

Minkowski or AdS
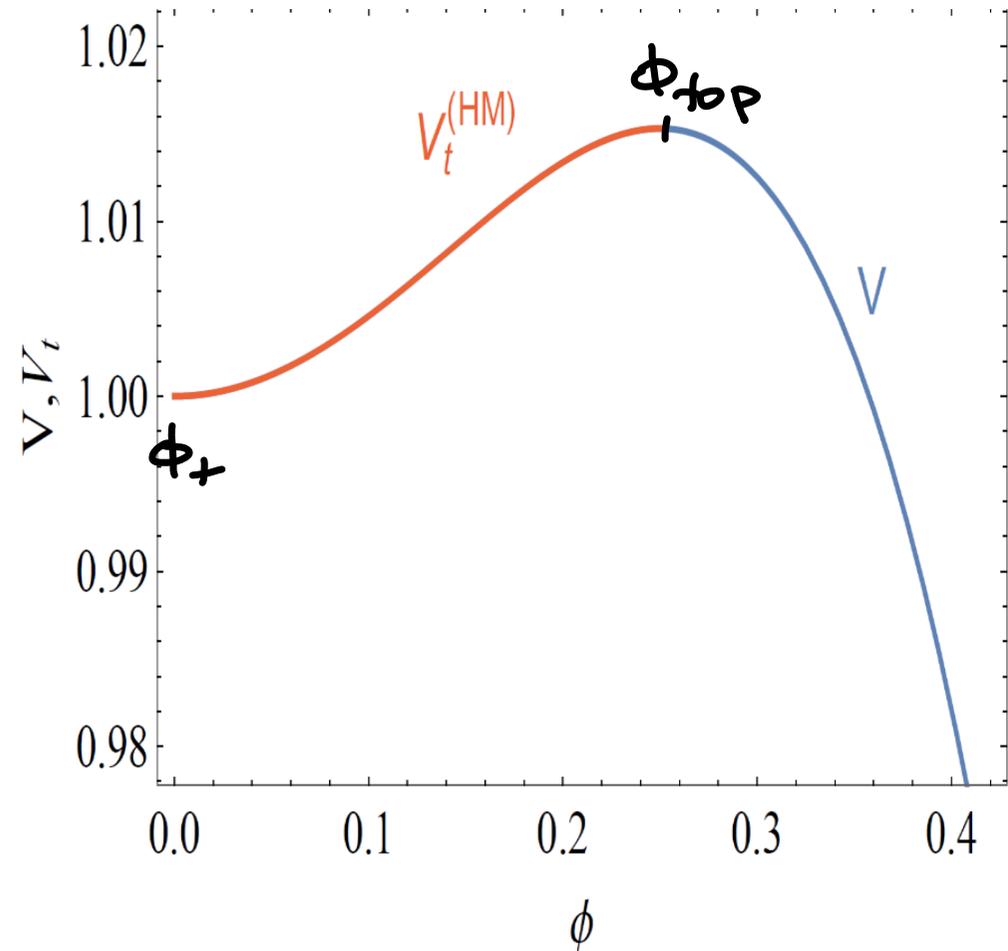
dS

(Th. int: Brown, Weinberg, 0706)

HAWKING-MOSS RATE

For dS, $V_t \uparrow \Rightarrow$ no CdL bounce

Hawking-Moss decay '82

with rate

$$S_{HM} = \frac{24\pi^2}{\kappa^2}\left(\frac{1}{V_+} - \frac{1}{V_{top}}\right)$$
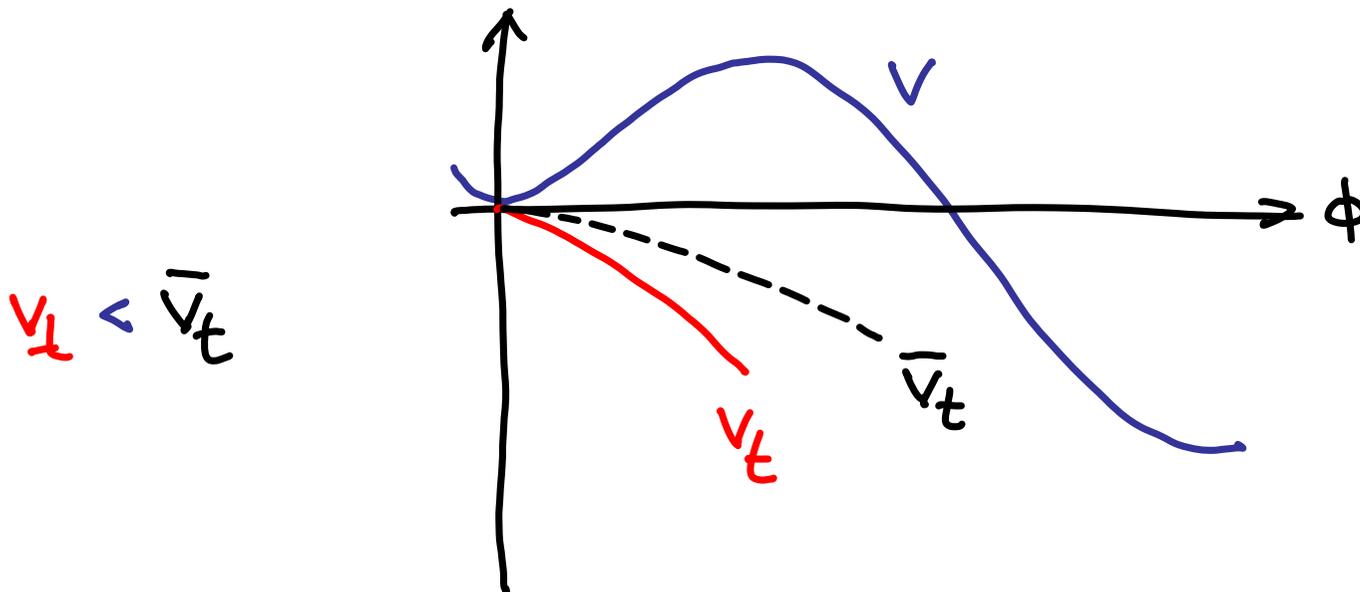
$S[V_t]$ reproduces this ✓

GRAVITATIONAL QUENCHING

For AdS, Minkowski $(V_t \leq 0)$

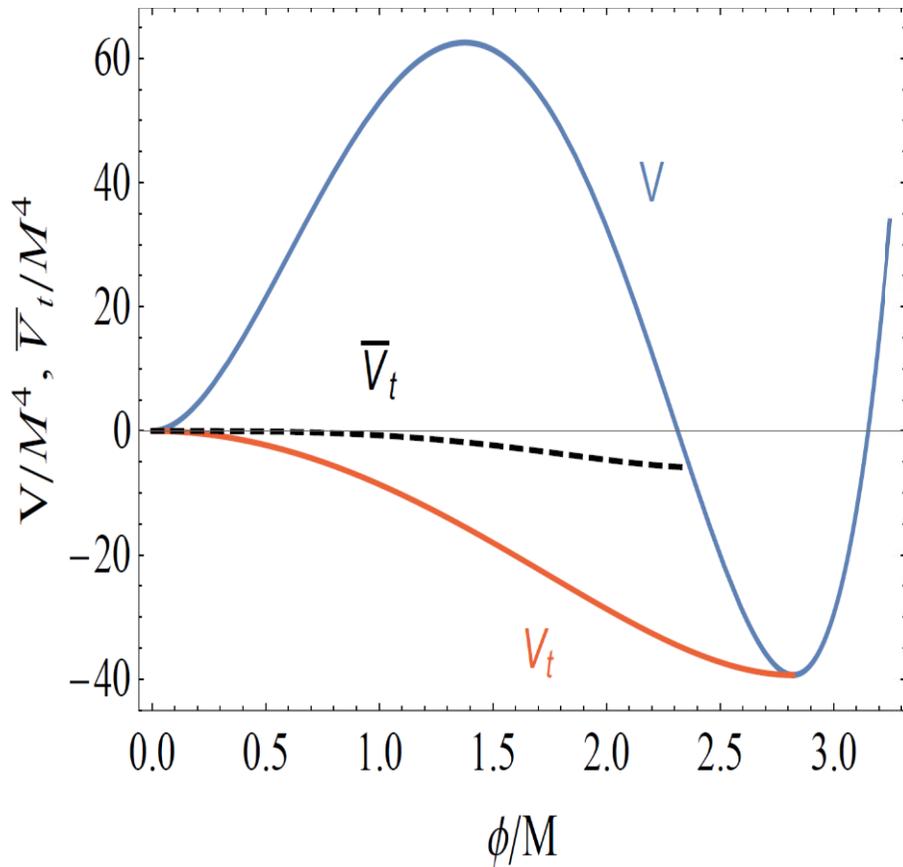gravitational quenching of decay possible

Coleman, De Luccia '80

$$D = \sqrt{(V_t')^2 + 6\kappa(V - V_t)V_t} \quad \text{must be real}$$

$$\underset{\underset{0}{\vee}}{} \quad \underbrace{\qquad}_{\underset{0}{\wedge}}$$

$$|V_t'| > |\bar{V}_t'| = \sqrt{6\kappa(V - \bar{V}_t)(-\bar{V}_t)}$$
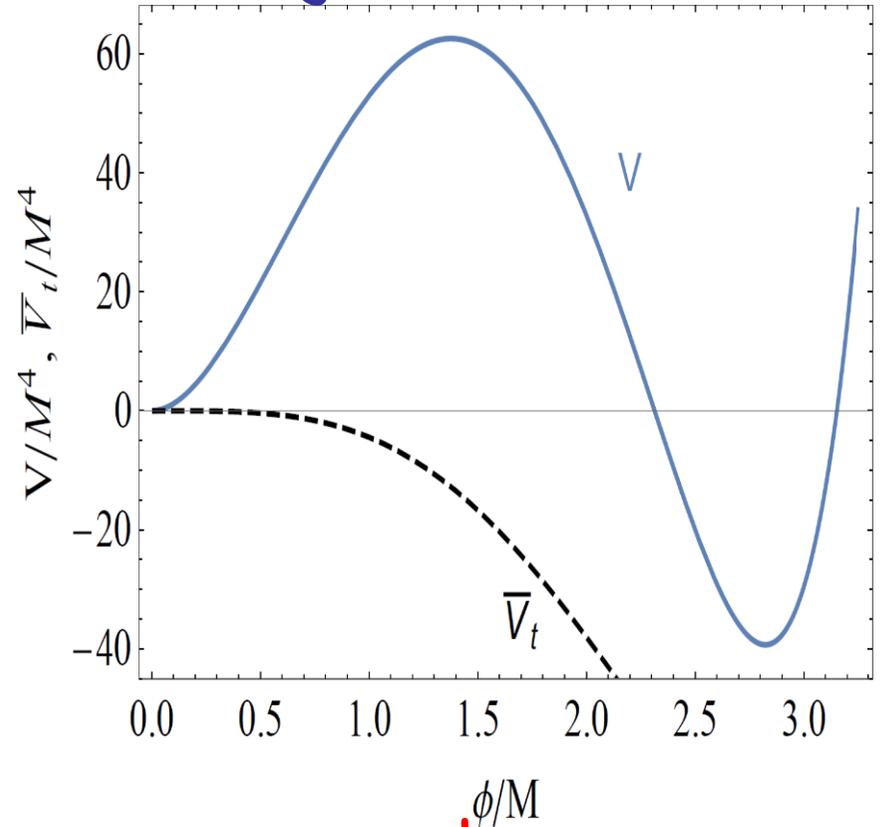

$V_t < \bar{V}_t$

GRAVITATIONAL QUENCHING

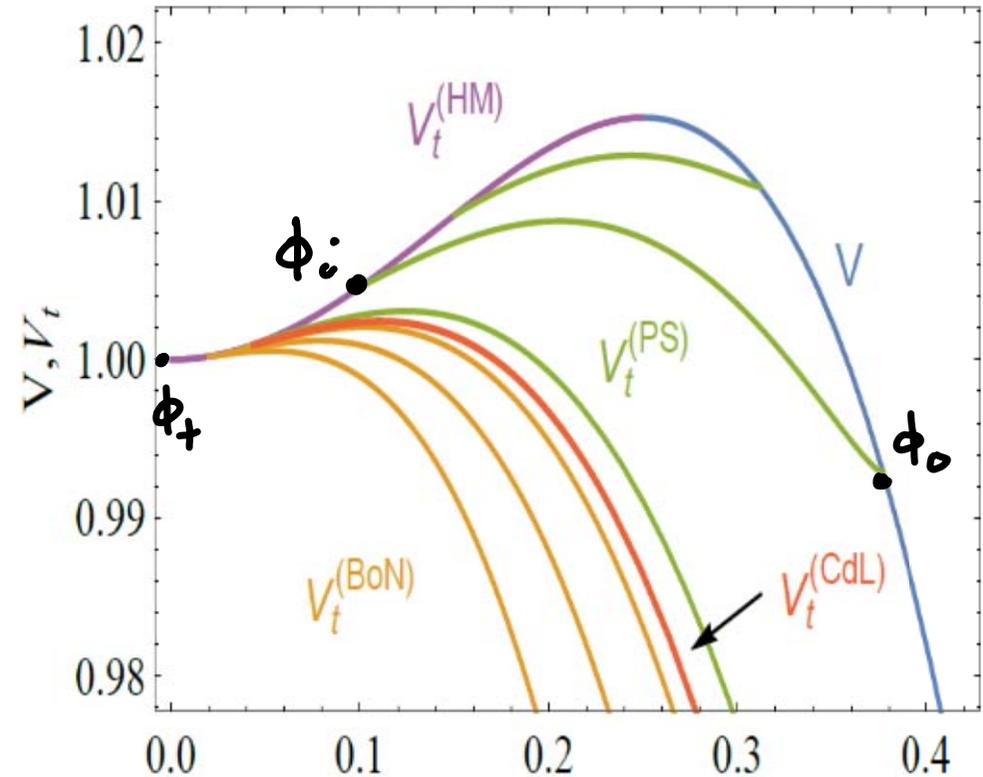**Weak grav. effects** $\longrightarrow$ **Strong grav. effects**
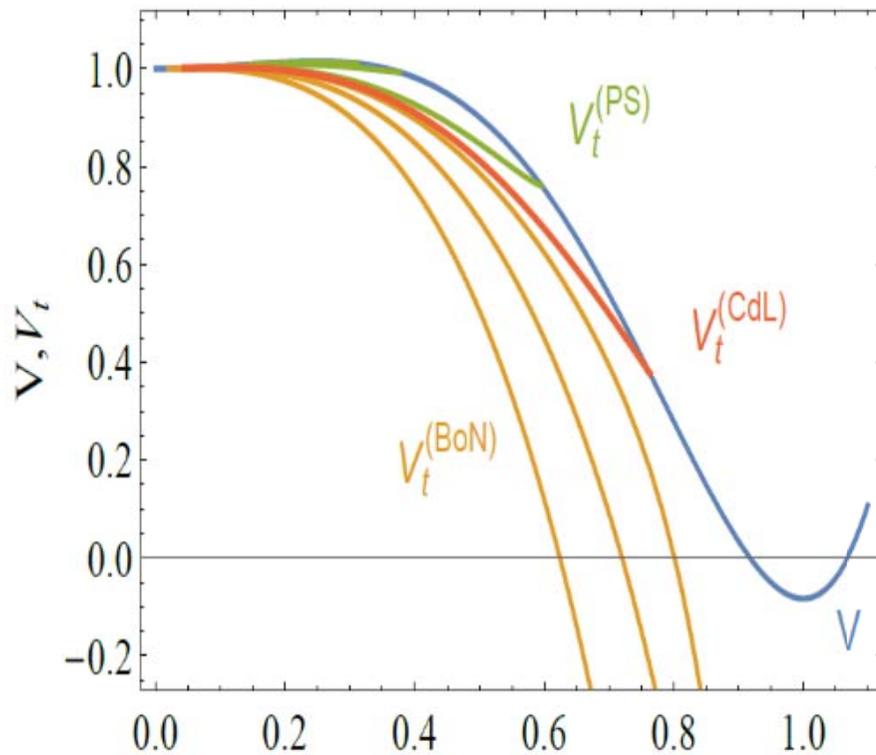

Decays

No decay

NO EXIT

Positive Energy Theorem

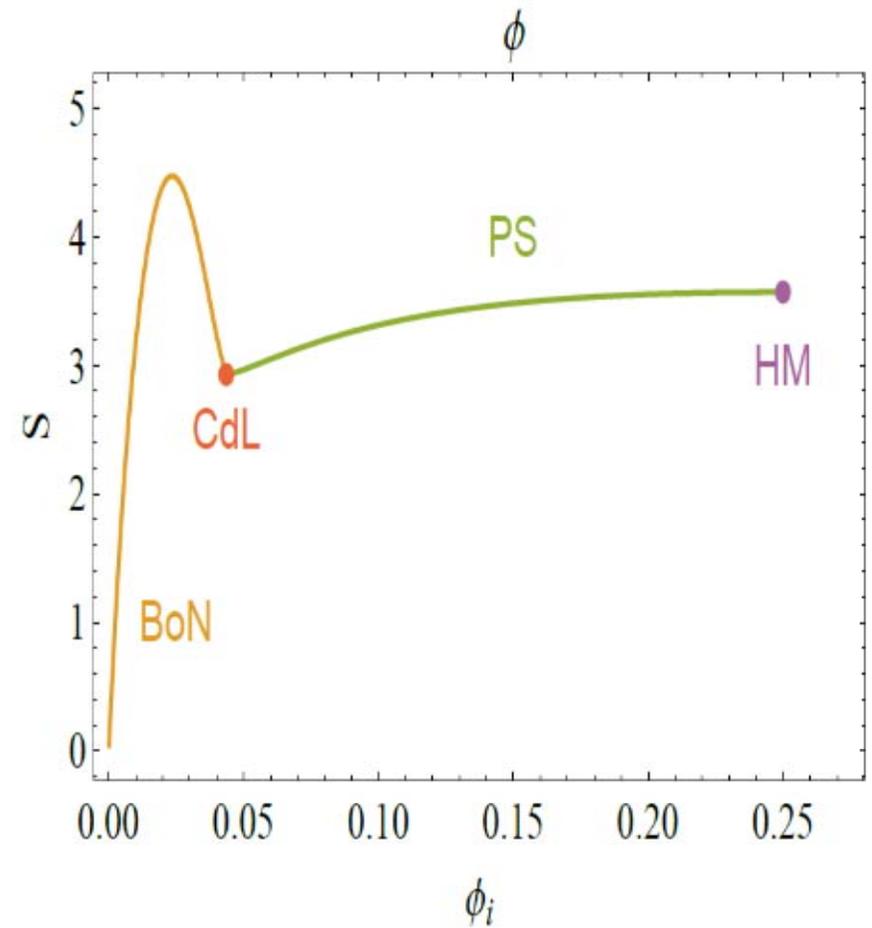JRE'2005

GLOBAL PICTURE (dS)

Family of solutions $V_t(\phi_i; \phi)$ of EoM for V_t

BCs $\qquad V_t(\phi_i) = V(\phi_i) \qquad V_t'(\phi_i) = \frac{3}{4} V'(\phi_i)$

PS = "pseudo-bounces"

GLOBAL PICTURE (dS)

DECAY MENU

Price

HM   Homogeneous jump $\phi_+ \rightarrow \phi_{top}$ S_{HM}

$\vee$

PS   Tunneling bubble with S_{PS}

central value ϕ_0 of choice

$\vee$

CdL   Cheapest tunneling bubble S_{CdL}

Extra

BoN   "Bubble of nothing" S_{BoN}

3. BUBBLE OF NOTHING DECAYS
w/ Blanco-Pillado, Huertas, Sousa

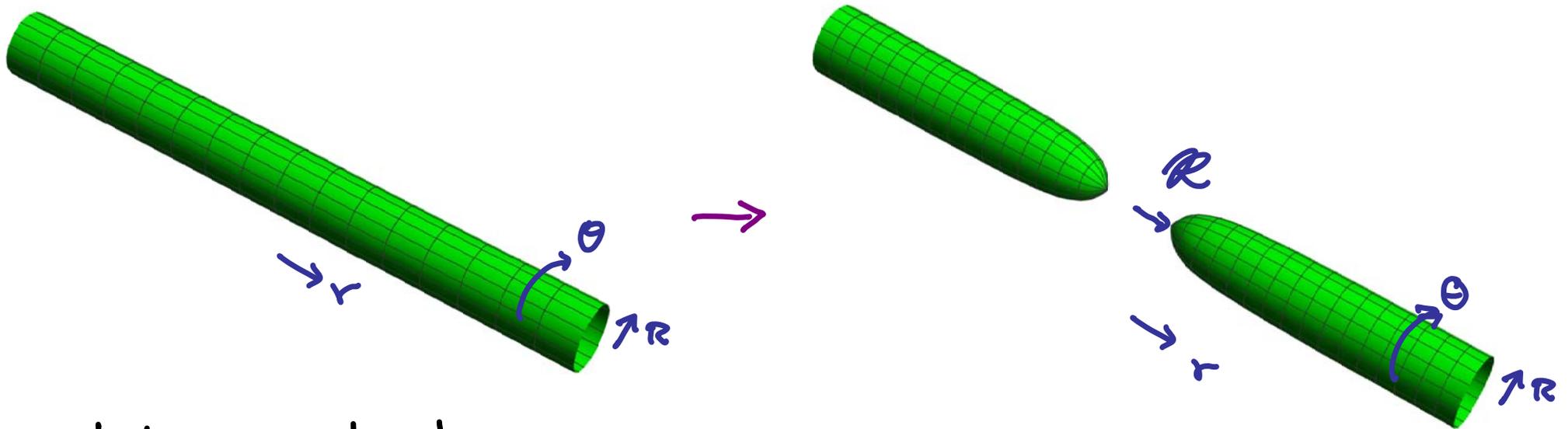# BUBBLE OF NOTHING DECAYS

Decays of spacetimes with compactified dim. like

5d KK $(M^4 \times S^1)$

Euclidean instanton:

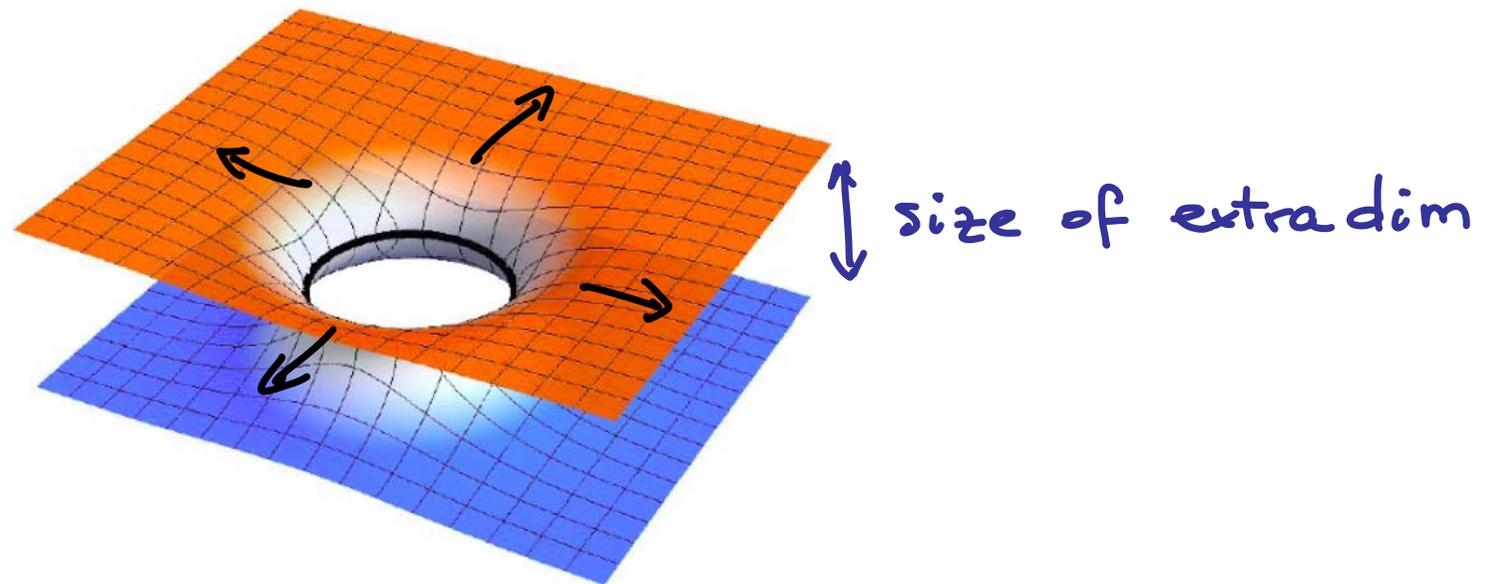$$ds^2 = \frac{dr^2}{1 - R^2/r^2} + r^2 d\Omega_3^2 + R^2 \left(1 - \frac{R^2}{r^2}\right) d\theta^2$$

with $S = (\pi R m_P)^2$

$R(r)^2 \to 0$ at $r \to \mathcal{R} = R$

BUBBLE OF NOTHING DECAYS

End-product of tunneling process is a hole
in space-time

size of extra dim

It's smooth, w/o singularities, curvature not large
But it grows, eating the whole space

WHY BONS ?

- Relevant for string vacua landscape

- BoNs thought to be ultimate cause of decay of non-SUSY vacua in string compactifications
  García-Etxebarría, Montero, Sousa, Valenzuela '2005

- Relevant for cobordism conjecture

4d VIEW

5d → 4d + Scalar ϕ (geometric modulus)

$$R_{S_1}^2 = \underbrace{R^2\left(1 - \frac{R^2}{r^2}\right)}_{5d} = \underbrace{R^2 e^{-2\sqrt{2\kappa/3}\,\phi}}_{4d}$$

$$\begin{cases} \text{BoN core}: \; r \to R & \phi \to \infty \\ \text{False vac}: \; r \to \infty & \phi \to 0 \end{cases}$$

and $ds_4^2 = d\xi^2 + \rho(\xi)^2 d\Omega_3^2$

with $\dfrac{d\xi}{dr} = \dfrac{1}{(1 - R^2/r^2)^{1/4}}$ (BoN core: $\xi \to 0$)

BoN reduces to a CdL problem

$$\phi(0) = \infty \qquad \dot{\phi}(0) = -\infty \qquad \phi(\infty) = \phi_+ = 0$$
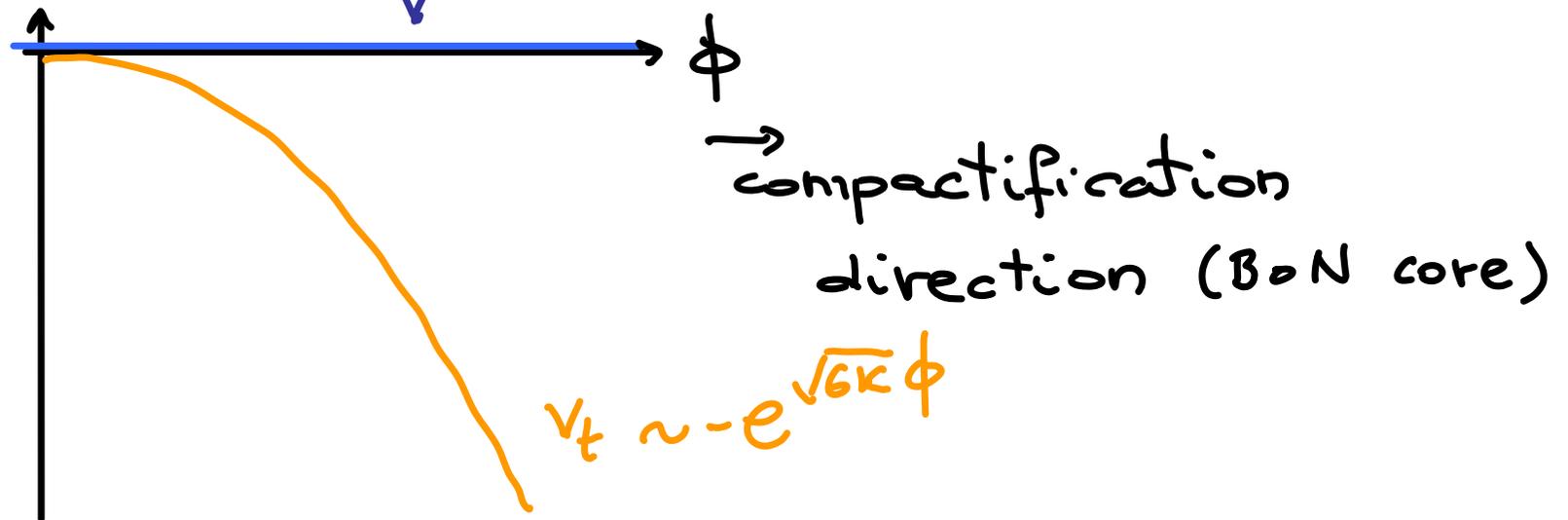
Action reproduced (paying attention to boundary terms)

V_t FOR BONS

Blanco-Pillado, JRE, Huertas, Sousa '2312

Witten's BoN

$$V_t = -6m_P^2 R^2 \sinh^3(\sqrt{2\kappa/3}\,\phi) , \qquad V = 0$$

V

ϕ

→ compactification direction (BoN core)

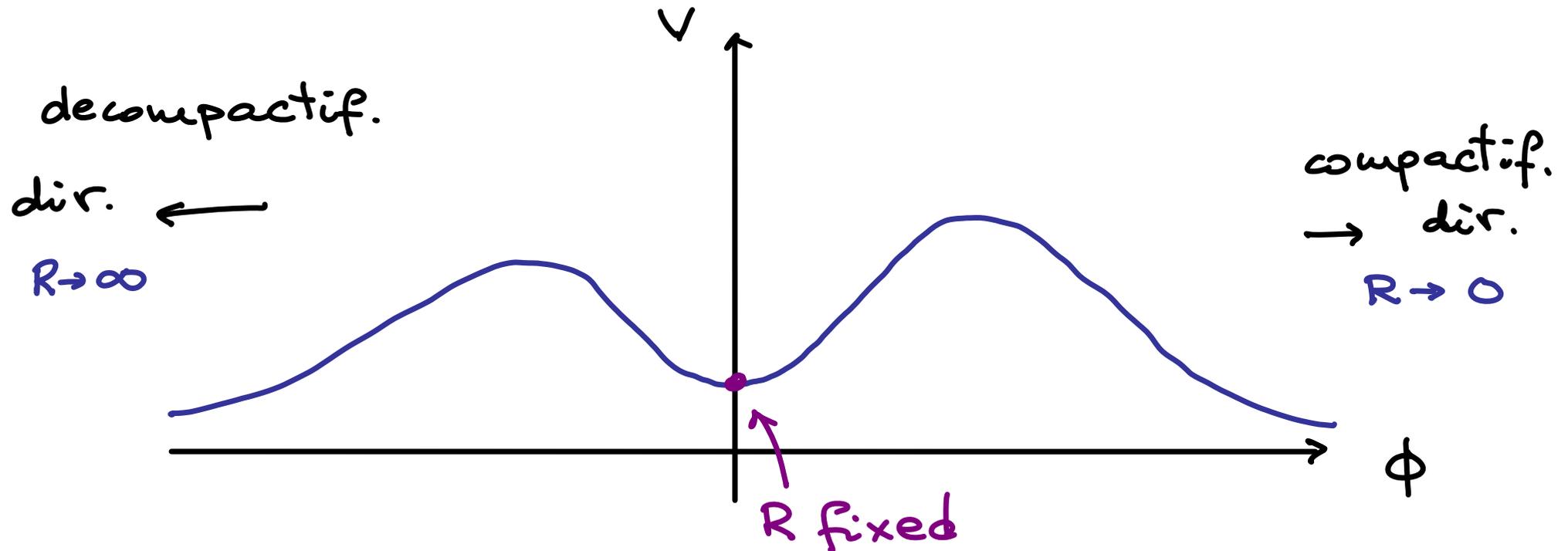$V_t \sim -e^{\sqrt{6\kappa}\,\phi}$

BoN tunneling to $-\infty$ AdS

★ $S[V_t]$ ✓ without additional boundary terms

Allows bottom-up approach to study which $V(\phi)$ admits BoN decays

Draper, García-García, Lillard '2105

BONS WITH $V(\phi)$?

decompactif. dir. ←

$R \to \infty$

compactif. → dir.

$R \to 0$

R fixed

★ Universal asymptotic V, V_t behaviours at $\phi \to \infty$ via

$$0 = (4V_t' - 3V')\,V_t' + 6(V - V_t)\left[V_t'' + \kappa\,(3V - 2V_t)\right]$$

→ which V, V_t can be obtained from extra-d theories?

→ Is BoN always the dominant decay?

$\phi \to \infty$ (BoN CoRE) ASYMPTOTICS

Example: $\mathbb{M}^4 \times S^1$, $V(\phi) \neq 0$

4d EFT:

$$V_t \simeq V_{tA}(A) \, e^{\sqrt{6\kappa}\,\phi} + \ldots \qquad D \simeq D_\infty(A) \, e^{\sqrt{8\kappa/3}\,\phi} + \ldots$$
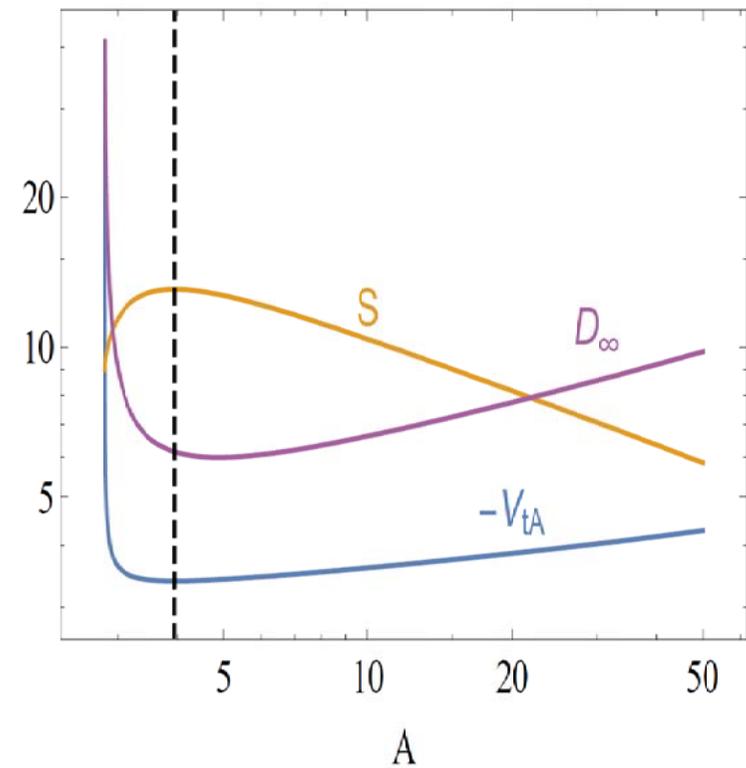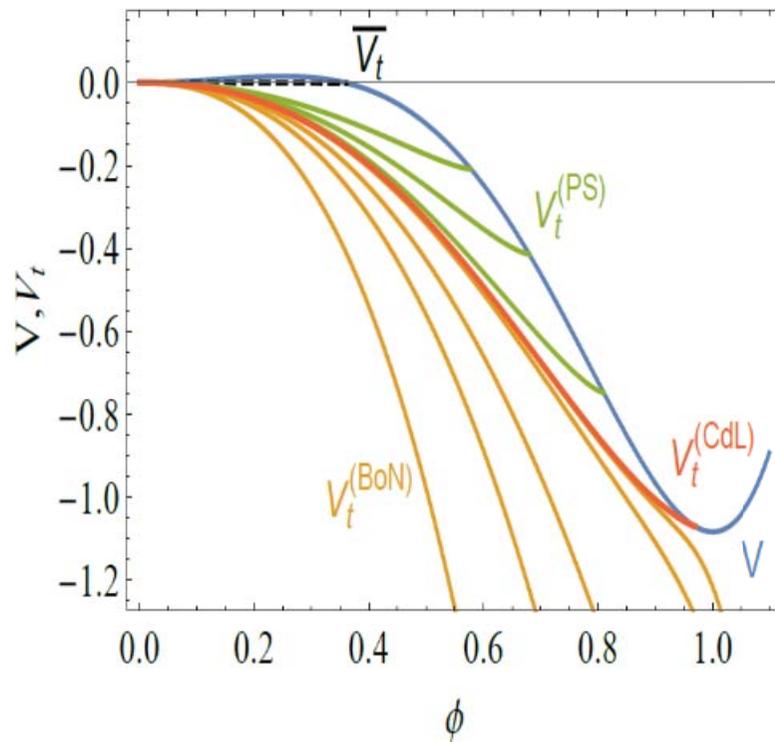
$\phi \rightarrow \infty$ (BoN CoRE) ASYMPTOTICS

Example: $\mathbb{M}^4 \times S^1$, $V(\phi) \neq 0$

4d EFT:

$$V_t \simeq V_{tA}(A) \, e^{\sqrt{6k}\,\phi} + \dots \qquad D \simeq D_\infty(A) \, e^{\sqrt{8c/3}\,\phi} + \dots$$
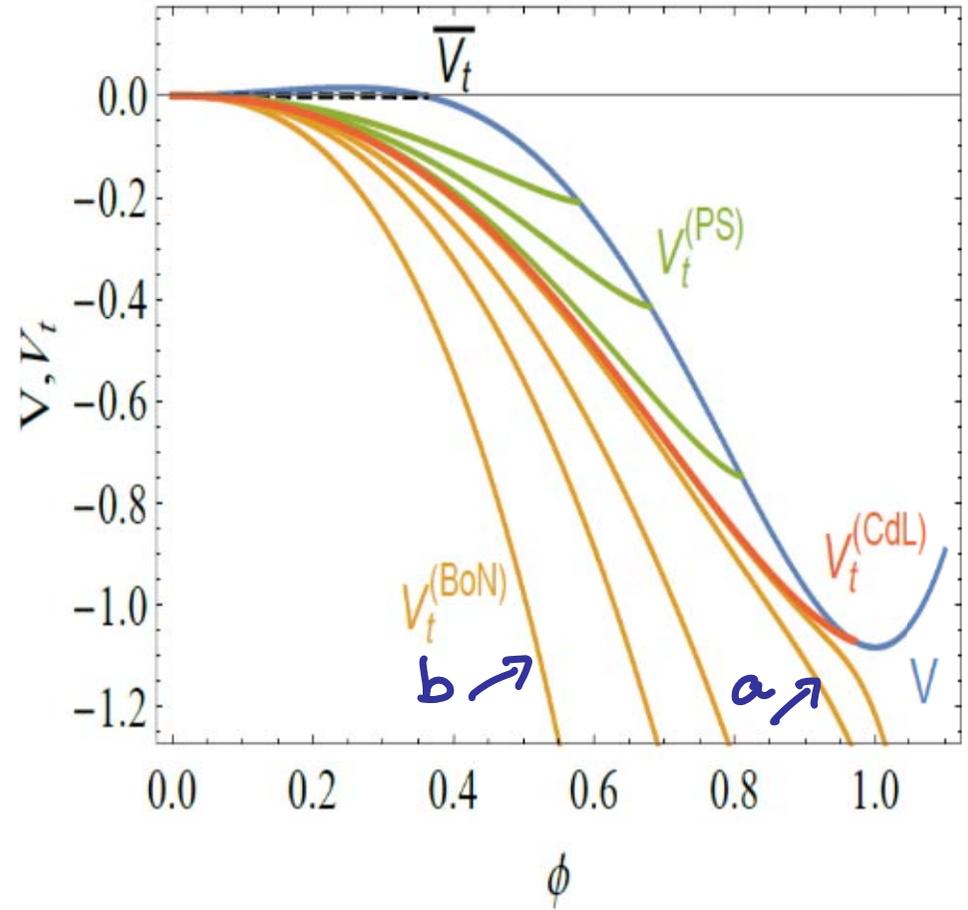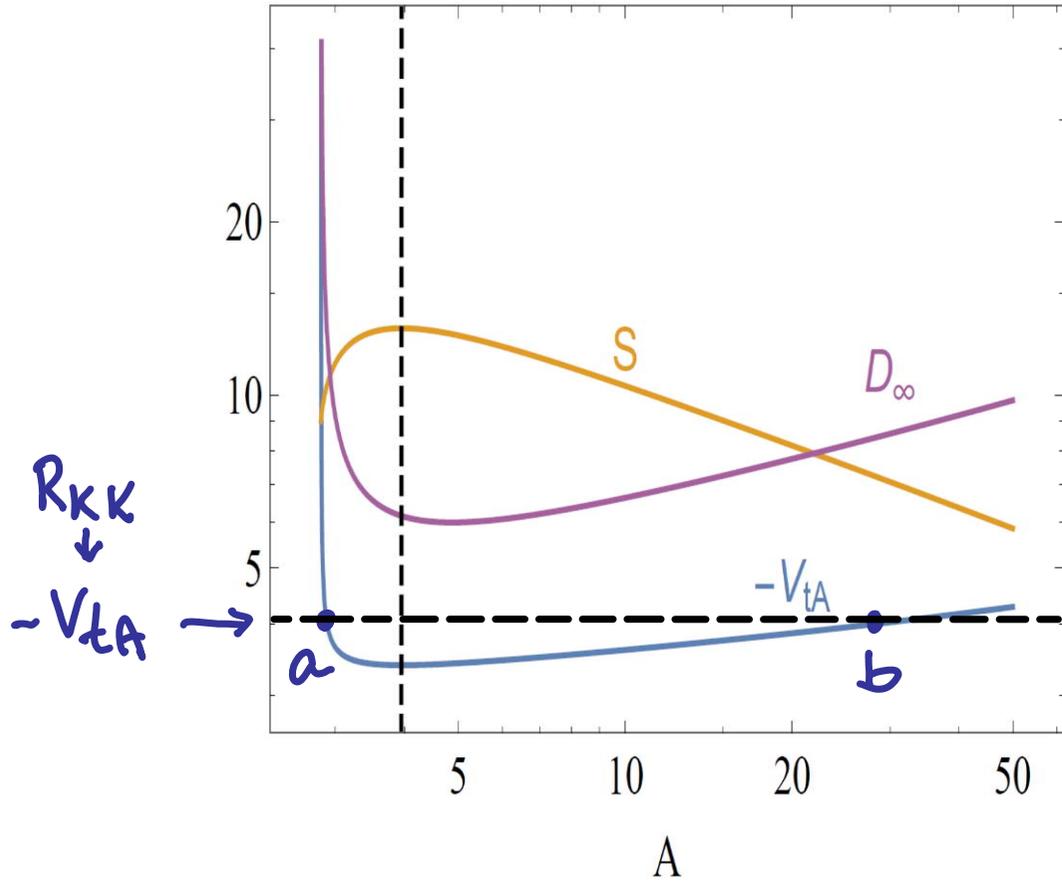
5d input:

$$V_{tA} = \frac{C}{k \, R_{KK}^2} \qquad\qquad D_\infty = \frac{C'}{\mathcal{R} \, R_{KK} \sqrt{k}}$$

BoN radius $\nearrow$ $\qquad$ $\nwarrow$ KK radius

For fixed R_{KK} $\Rightarrow$ fixed V_{tA}

WHICH V_t^{BoN} ?

Choose sol. with lowest S and D_∞ fixes $\mathcal{R}$

If R_{KK} gives $-V_{tA} < -V_{tA}^{min}$ $\Rightarrow$ No BoN decay

CONCLUSIONS

Tunneling potential formalism simple and useful in

- Vacuum decay

- Finite T transitions

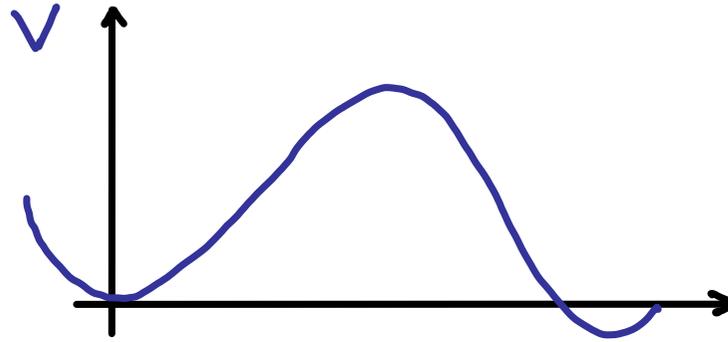- Gravity effects on vacuum decays

- Bubble-of-nothing decays

Further applications

- Q-balls   w/Heeck, Sokhashvili 2307

- Positive Energy theorems in AdS maxima   w/Jinno

- Flat/curved Domain Walls   w/C.Bachas, Z.Chen

More? Euclidean solutions $\Rightarrow$ V_t solutions
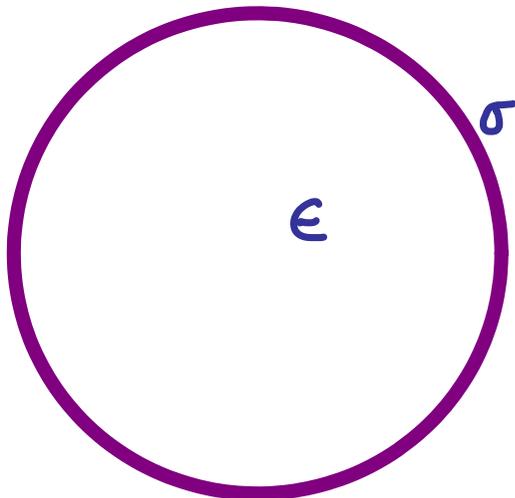
BACK UP SLIDES

GRAVITATIONAL QUENCHING

V

$V_- = -\epsilon$

No gravity

$$E \sim -\epsilon R^3 + \sigma R^2 = 0$$

σ

ϵ

large $R \sim \sigma/\epsilon$

With gravity

$$E \sim -\epsilon R^2 + \sigma R^2$$

$E = 0$ not guaranteed

GLOBAL PICTURE (Mink.)

Family of solutions $V_t(A; \phi)$