

# Tri-unification

A separate  $SU(5)$  for each fermion family

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Based on work in collaboration with  
**Mario Fernández Navarro and Stephen F. King**

**2311.05683 & 2404.12442**

**New Physics Directions**  
Heidelberg

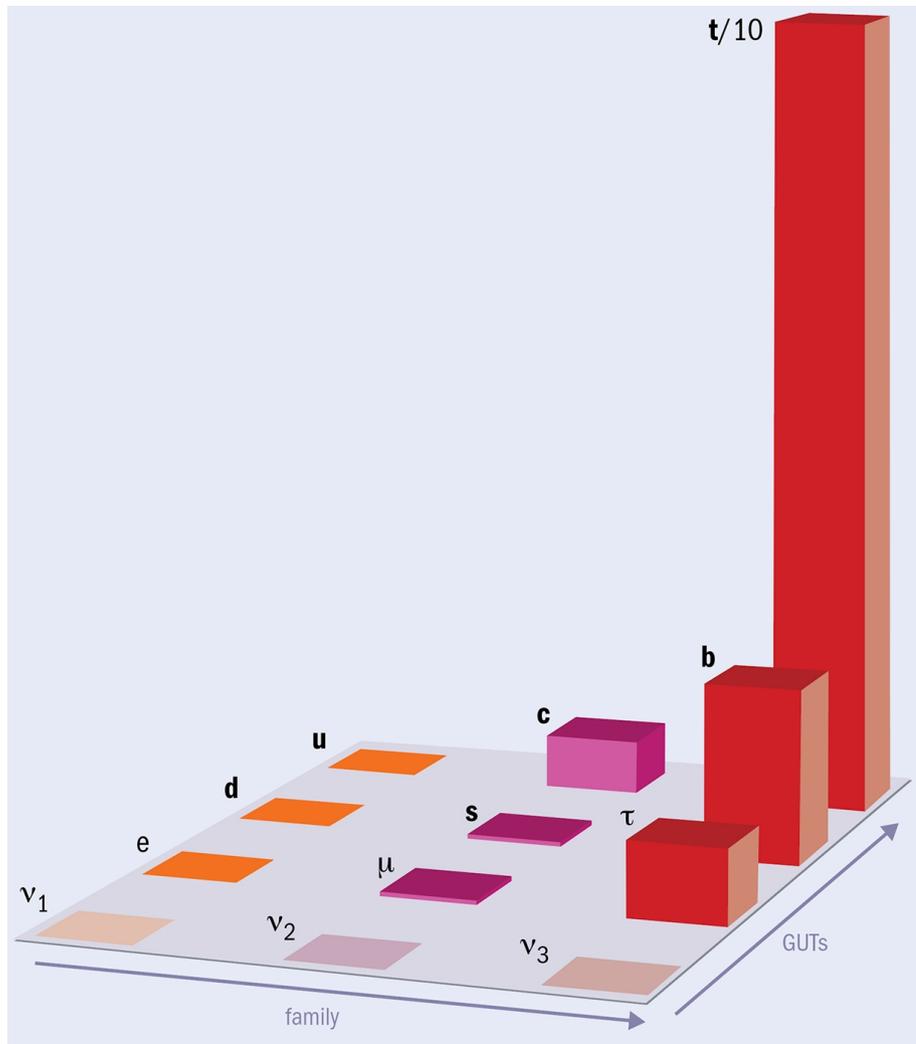


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# The flavor puzzle



★ Why are there 3 fermion families?

★ Why are fermion masses so hierarchical?

★ Why do **quarks** mix so little... and **leptons** so much?

Talks by Monika Blanke, Francesco Sannino, ...

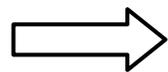
# Deconstructing flavor

## General idea:

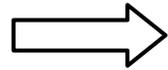
SM embedded in a larger gauge symmetry with **a separate factor for each family**

$$G = G_{\text{universal}} \times G_1 \times G_2 \times G_3$$

The SM Higgs is a **3<sup>rd</sup> family particle**: singlet of  $G_1$  and  $G_2$ , but not of  $G_3$



Only the 3<sup>rd</sup> family masses at renormalizable level



Explain the **SM flavor structure** with  $\mathcal{O}(1)$  Yukawa couplings

## Examples:

- Tri-hypercharge:  $SU(3)_c \times SU(2)_L \times U(1)_Y^3$  [ Fernández Navarro, King, AV ]
- $SU(3)_c \times SU(2)_L^3 \times U(1)_Y$  [ Li, Ma, Muller, Nandi, Chiang, Deshpande, He, Jiang... ]
- (Pati-Salam)<sup>3</sup> [ Bordone, Cornella, Fuentes-Martin, Isidori, Pagès, Stefanek... ]
- Grand unified models [ Rajpoot, Barbieri, Dvali, Strumia, Babu, Barr, Gogoladze... ]

**+ others**

# Tri-unification

$$SU(5)_1 \times SU(5)_2 \times SU(5)_3 \times \mathbb{Z}_3$$

★  $\mathbb{Z}_3$  cyclic symmetry

- Enforces a **common gauge coupling** (at unification scale)
- Invariance under cyclic permutations:  $(\mathbf{A}, \mathbf{B}, \mathbf{C}) + (\mathbf{B}, \mathbf{C}, \mathbf{A}) + (\mathbf{C}, \mathbf{A}, \mathbf{B})$

★ Symmetry breaking:

$$\begin{aligned} SU(5)^3 &\rightarrow SM_1 \times SM_2 \times SM_3 \\ &\rightarrow G_{\text{universal}} \times G_1 \times G_2 \times G_3 \\ &\rightarrow G_{\text{universal}} \times G_{1+2} \times G_3 \\ &\rightarrow SM_{1+2+3} \end{aligned}$$

# Minimal field content

In addition:

bi-representations, messenger fields, ...

Field	$SU(5)_1$	$SU(5)_2$	$SU(5)_3$
$F_1$	$\bar{5}$	$1$	$1$
$F_2$	$1$	$\bar{5}$	$1$
$F_3$	$1$	$1$	$\bar{5}$
$T_1$	$10$	$1$	$1$
$T_2$	$1$	$10$	$1$
$T_3$	$1$	$1$	$10$
$\Omega_1$	$24$	$1$	$1$
$\Omega_2$	$1$	$24$	$1$
$\Omega_3$	$1$	$1$	$24$
$H_1$	$5$	$1$	$1$
$H_2$	$1$	$5$	$1$
$H_3$	$1$	$1$	$5$

Note: These are only 4 irreps of the full symmetry group

# Minimal field content

In addition:

bi-representations, messenger fields, ...

**SM fermions**



$$\bar{\mathbf{5}} = (\bar{\mathbf{3}}, \mathbf{1})_{\frac{1}{3}} \oplus (\mathbf{1}, \mathbf{2})_{-\frac{1}{2}}$$

$$\mathbf{10} = (\mathbf{3}, \mathbf{2})_{\frac{1}{6}} \oplus (\bar{\mathbf{3}}, \mathbf{1})_{-\frac{2}{3}} \oplus (\mathbf{1}, \mathbf{1})_1$$

Field	$SU(5)_1$	$SU(5)_2$	$SU(5)_3$
$F_1$	$\bar{\mathbf{5}}$	$\mathbf{1}$	$\mathbf{1}$
$F_2$	$\mathbf{1}$	$\bar{\mathbf{5}}$	$\mathbf{1}$
$F_3$	$\mathbf{1}$	$\mathbf{1}$	$\bar{\mathbf{5}}$
$T_1$	$\mathbf{10}$	$\mathbf{1}$	$\mathbf{1}$
$T_2$	$\mathbf{1}$	$\mathbf{10}$	$\mathbf{1}$
$T_3$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{10}$
$\Omega_1$	$\mathbf{24}$	$\mathbf{1}$	$\mathbf{1}$
$\Omega_2$	$\mathbf{1}$	$\mathbf{24}$	$\mathbf{1}$
$\Omega_3$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{24}$
$H_1$	$\mathbf{5}$	$\mathbf{1}$	$\mathbf{1}$
$H_2$	$\mathbf{1}$	$\mathbf{5}$	$\mathbf{1}$
$H_3$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{5}$

Note: These are only 4 irreps of the full symmetry group

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$$\bar{\mathbf{5}} = (\bar{\mathbf{3}}, \mathbf{1})_{\frac{1}{3}} \oplus (\mathbf{1}, \mathbf{2})_{-\frac{1}{2}}$$

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**Scalars in the adjoint** →

Breaking of  $SU(5)^3$  and  $SM^3$

Field	$SU(5)_1$	$SU(5)_2$	$SU(5)_3$
$F_1$	$\bar{\mathbf{5}}$	$\mathbf{1}$	$\mathbf{1}$
$F_2$	$\mathbf{1}$	$\bar{\mathbf{5}}$	$\mathbf{1}$
$F_3$	$\mathbf{1}$	$\mathbf{1}$	$\bar{\mathbf{5}}$
$T_1$	$\mathbf{10}$	$\mathbf{1}$	$\mathbf{1}$
$T_2$	$\mathbf{1}$	$\mathbf{10}$	$\mathbf{1}$
$T_3$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{10}$
$\Omega_1$	$\mathbf{24}$	$\mathbf{1}$	$\mathbf{1}$
$\Omega_2$	$\mathbf{1}$	$\mathbf{24}$	$\mathbf{1}$
$\Omega_3$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{24}$
$H_1$	$\mathbf{5}$	$\mathbf{1}$	$\mathbf{1}$
$H_2$	$\mathbf{1}$	$\mathbf{5}$	$\mathbf{1}$
$H_3$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{5}$

Note: These are only 4 irreps of the full symmetry group

# Minimal field content

In addition:

bi-representations, messenger fields, ...

**SM fermions** →

$$\bar{\mathbf{5}} = (\bar{\mathbf{3}}, \mathbf{1})_{\frac{1}{3}} \oplus (\mathbf{1}, \mathbf{2})_{-\frac{1}{2}}$$

$$\mathbf{10} = (\mathbf{3}, \mathbf{2})_{\frac{1}{6}} \oplus (\bar{\mathbf{3}}, \mathbf{1})_{-\frac{2}{3}} \oplus (\mathbf{1}, \mathbf{1})_1$$

**Scalars in the adjoint** →

Breaking of  $SU(5)^3$  and  $SM^3$

**Higgs doublets** →

Only  $H_3$  in most non-universal theories

$\mathbb{Z}_3$  breaking to get a light  $H_3$

Field	$SU(5)_1$	$SU(5)_2$	$SU(5)_3$
$F_1$	$\bar{\mathbf{5}}$	$\mathbf{1}$	$\mathbf{1}$
$F_2$	$\mathbf{1}$	$\bar{\mathbf{5}}$	$\mathbf{1}$
$F_3$	$\mathbf{1}$	$\mathbf{1}$	$\bar{\mathbf{5}}$
$T_1$	$\mathbf{10}$	$\mathbf{1}$	$\mathbf{1}$
$T_2$	$\mathbf{1}$	$\mathbf{10}$	$\mathbf{1}$
$T_3$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{10}$
$\Omega_1$	$\mathbf{24}$	$\mathbf{1}$	$\mathbf{1}$
$\Omega_2$	$\mathbf{1}$	$\mathbf{24}$	$\mathbf{1}$
$\Omega_3$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{24}$
$H_1$	$\mathbf{5}$	$\mathbf{1}$	$\mathbf{1}$
$H_2$	$\mathbf{1}$	$\mathbf{5}$	$\mathbf{1}$
$H_3$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{5}$

Note: These are only 4 irreps of the full symmetry group

# A complete model

$$G_{\text{universal}} = SU(3)_c \times SU(2)_L$$

$$G_1 \times G_2 \times G_3 = U(1)_{Y_1} \times U(1)_{Y_2} \times U(1)_{Y_3}$$

Tri-hypercharge

★ Symmetry breaking:

$$SU(5)^3 \xrightarrow{v_{\text{GUT}}} SM_1 \times SM_2 \times SM_3$$

$$\xrightarrow{v_{\text{SM}^3}} SU(3)_{1+2+3} \times SU(2)_{1+2+3} \times U(1)_1 \times U(1)_2 \times U(1)_3$$

$$\xrightarrow{v_{12}} SU(3)_{1+2+3} \times SU(2)_{1+2+3} \times U(1)_{1+2} \times U(1)_3$$

$$\xrightarrow{v_{23}} SU(3)_{1+2+3} \times SU(2)_{1+2+3} \times U(1)_{1+2+3}$$

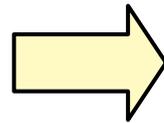
# A complete model

★ Higgs fields:

	Field	$SU(5)_1$	$SU(5)_2$	$SU(5)_3$
Up Higgses	$H_1^u$	5	1	1
	$H_2^u$	1	5	1
	$H_3^u$	1	1	5
'Down' Higgses	$H_1^{\bar{5}}$	$\bar{5}$	1	1
	$H_2^{\bar{5}}$	1	$\bar{5}$	1
	$H_3^{\bar{5}}$	1	1	$\bar{5}$
d-quarks and charged leptons splitting via <b>Georgi-Jarlskog factor</b>	$H_1^{45}$	45	1	1
	$H_2^{45}$	1	45	1
	$H_3^{45}$	1	1	45

Three pairs of doublets below  $M_{\text{GUT}}$

$$H_1^{u,d} \quad H_2^{u,d} \quad H_3^{u,d}$$



$$M_{H_3^{u,d}} \ll M_{H_2^{u,d}} \ll M_{H_1^{u,d}}$$

$$\tan \beta = \frac{v_3^u}{v_3^d} \approx 20$$

# A complete model

★ Bi-representations:

Field	$SU(5)_1$	$SU(5)_2$	$SU(5)_3$
$\Phi_{12}^F$	$\mathbf{5}$	$\bar{\mathbf{5}}$	$\mathbf{1}$
$\Phi_{13}^F$	$\bar{\mathbf{5}}$	$\mathbf{1}$	$\mathbf{5}$
$\Phi_{23}^F$	$\mathbf{1}$	$\mathbf{5}$	$\bar{\mathbf{5}}$
$\Phi_{12}^T$	$\overline{\mathbf{10}}$	$\mathbf{10}$	$\mathbf{1}$
$\Phi_{13}^T$	$\mathbf{10}$	$\mathbf{1}$	$\overline{\mathbf{10}}$
$\Phi_{23}^T$	$\mathbf{1}$	$\overline{\mathbf{10}}$	$\mathbf{10}$

“Hyperons”

$$\phi_{q12} \sim (\mathbf{1}, \mathbf{1})_{(-1/6, 1/6, 0)} \quad \phi_{q23} \sim (\mathbf{1}, \mathbf{1})_{(0, -1/6, 1/6)} \quad \phi_{\ell 23} \sim (\mathbf{1}, \mathbf{1})_{(0, 1/2, -1/2)}$$

$$\phi_{q13} \sim (\mathbf{1}, \mathbf{1})_{(-1/6, 0, 1/6)} \quad \phi_{\ell 13} \sim (\mathbf{1}, \mathbf{1})_{(1/2, 0, -1/2)}$$

- Spontaneous breaking of individual hypercharges
- Generate the **SM flavor structure**

**Embedding**

For instance:

$$\phi_{q12} \sim (\mathbf{1}, \mathbf{1})_{(-1/6, 1/6, 0)} \subset \Phi_{12}^T \sim (\overline{\mathbf{10}}, \mathbf{10}, \mathbf{1})$$

# A complete model

★ Messenger fields:

Field	$SU(5)_1$	$SU(5)_2$	$SU(5)_3$
$\chi_1$	<b>10</b>	<b>1</b>	<b>1</b>
$\chi_2$	<b>1</b>	<b>10</b>	<b>1</b>
$\chi_3$	<b>1</b>	<b>1</b>	<b>10</b>

(Vector-like fermions)

- Provide **UV-complete theory** by mediating the effective operators behind the SM flavor structure
- The heavy Higgses also act as messengers

**Embedding**

For instance:

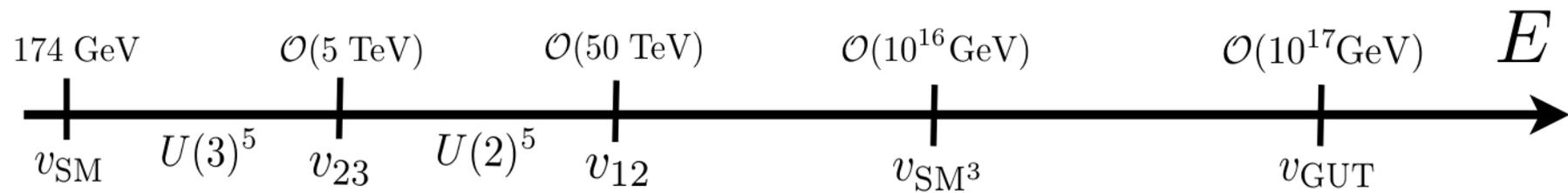
$$Q_1 \sim (\mathbf{3}, \mathbf{2})_{(1/6, 0, 0)} \subset \chi_1 \sim (\mathbf{10}, \mathbf{1}, \mathbf{1})$$

# GUTs checklist

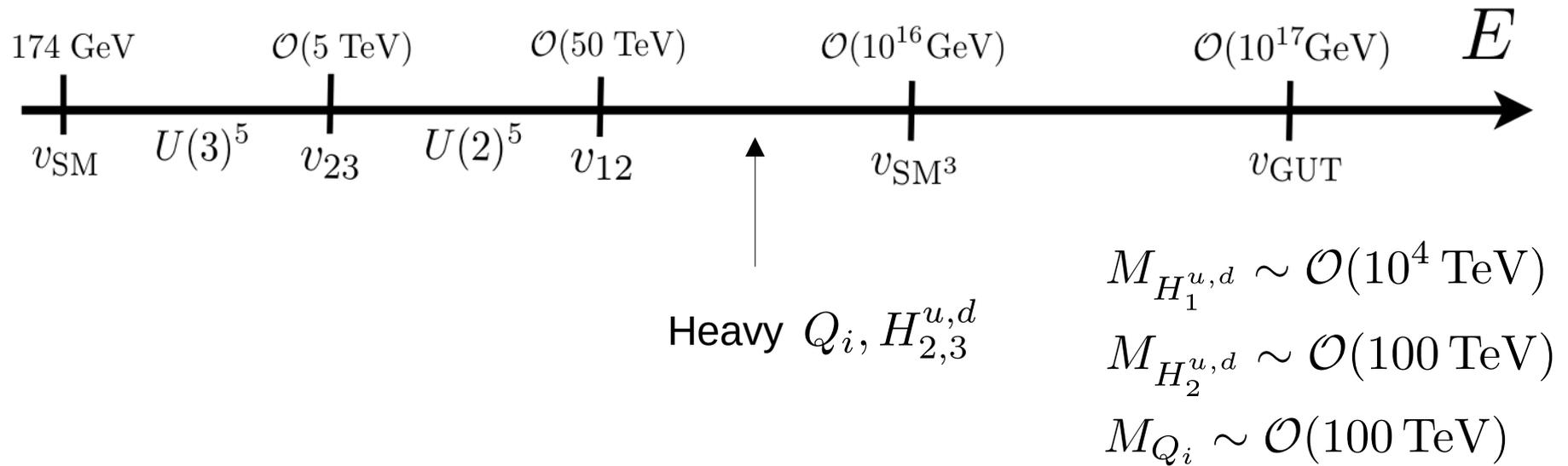
- Low-energy theory
- Gauge coupling unification
- Proton decay



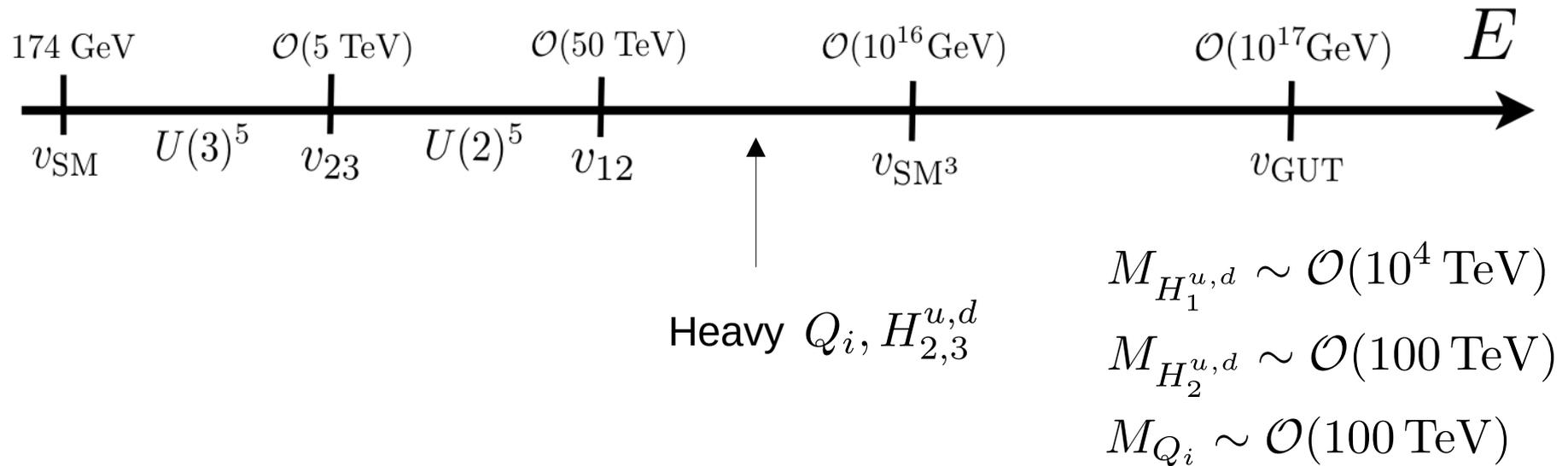
# Low-energy theory



# Low-energy theory



# Low-energy theory



Tri-hypercharge at low (intermediate) energies

[ Fernández Navarro, King, 2023 ]

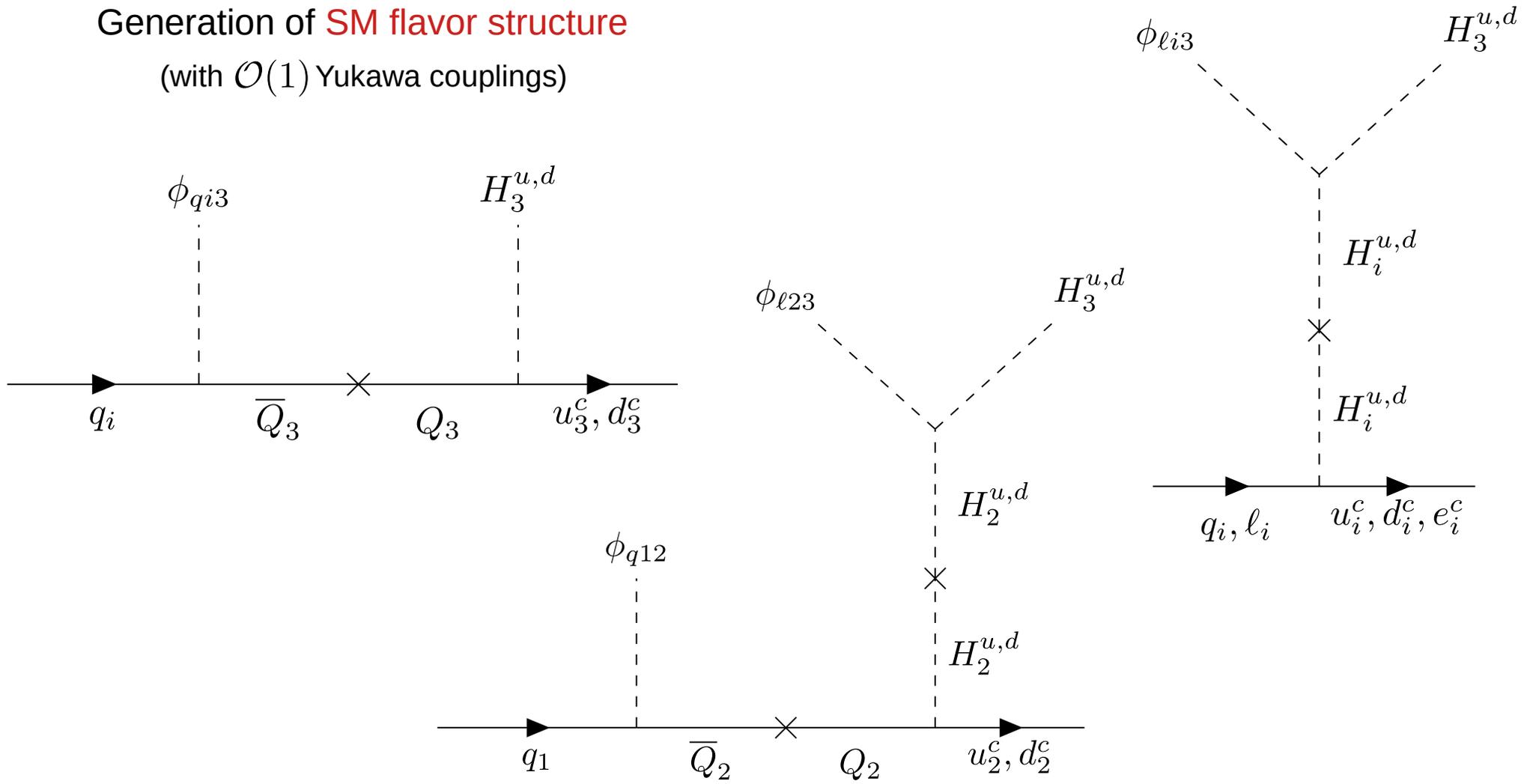
[ Fernández Navarro, King, AV, 2024 ]

$$SU(3)_c \times SU(2)_L \times U(1)_{Y_1} \times U(1)_{Y_2} \times U(1)_{Y_3}$$

# Low-energy theory

## Generation of SM flavor structure

(with  $\mathcal{O}(1)$  Yukawa couplings)



# Low-energy theory

Generation of **SM flavor structure**

(with  $\mathcal{O}(1)$  Yukawa couplings)

$\lambda \approx 0.224$   
Wolfenstein  
parameter

$$\frac{\langle \phi_{q23} \rangle}{M_{Q_i}} \approx \lambda^2 \quad \frac{\langle \phi_{q13} \rangle}{M_{Q_i}} \approx \lambda^3 \quad \frac{\langle \phi_{\ell 23} \rangle}{M_{H_2^{u,d}}} \approx \lambda^3 \quad \frac{\langle \phi_{q12} \rangle}{M_{Q_i}} \approx \lambda \quad \frac{\langle \phi_{\ell 23} \rangle}{M_{H_1^{u,d}}} \approx \lambda^6$$

$$\mathcal{M}_u = \begin{pmatrix} \lambda^6 & \lambda^4 & \lambda^3 \\ \lambda^7 & \lambda^3 & \lambda^2 \\ \lambda^9 & \lambda^5 & 1 \end{pmatrix} v_{\text{SM}} \quad \mathcal{M}_d = \begin{pmatrix} \lambda^6 & \lambda^4 & \lambda^3 \\ \lambda^7 & \lambda^3 & \lambda^2 \\ \lambda^9 & \lambda^5 & 1 \end{pmatrix} \lambda^2 v_{\text{SM}} \quad \mathcal{M}_e = \begin{pmatrix} \lambda^6 & 0 & 0 \\ 0 & \lambda^3 & 0 \\ 0 & 0 & 1 \end{pmatrix} \lambda^2 v_{\text{SM}}$$

- Correct mass hierarchies
- Small quark mixing
- Lepton mixing from neutrinos (*in backup*)

# Gauge coupling unification

## Discontinuities due to matching conditions

For instance

$$\alpha_{Y_{12}}^{-1} + \alpha_{Y_3}^{-1} = \alpha_Y^{-1}$$

at  $v_{23} = 5 \text{ TeV}$

## Unification achieved with 'light' (~ 100 TeV) color octets

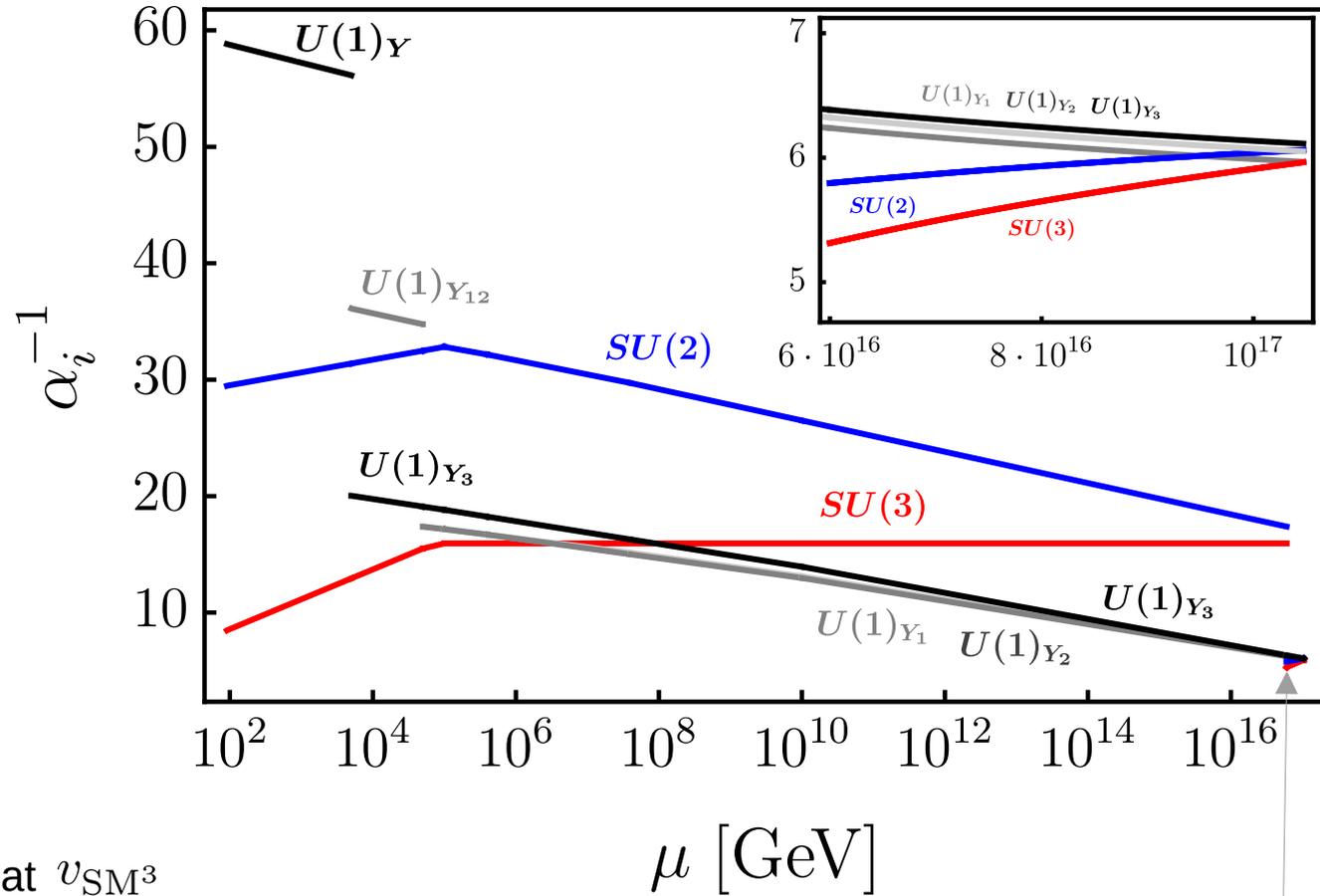
$$\Theta_i \sim (8, 1, 0)_i$$

(from cyclic 24)

## Approximate $\mathbb{Z}_3$ at low energies (exact at the unification scale)

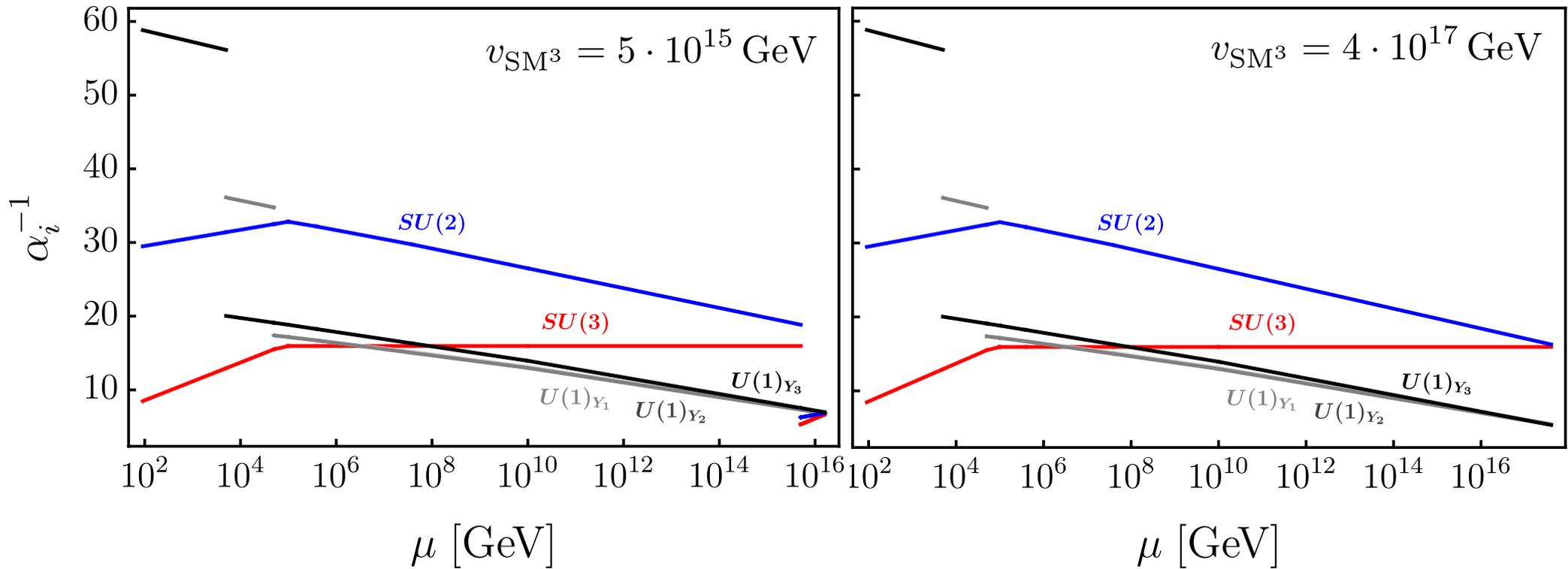
$$g_{s,1} \approx g_{s,2} \approx g_{s,3} \approx \sqrt{3}g_s \text{ at } v_{\text{SM}^3}$$

## Unification fully compatible with flavor



$$v_{\text{SM}^3} = 6 \cdot 10^{16} \text{ GeV}$$

# Gauge coupling unification



How low can we deconstruct  $SU(3)_c \times SU(2)_L$ ?

Very low unification scale: **proton decay!**

No intermediate **SM<sup>3</sup> scale**

Very high unification scale

# Proton decay

Proton decay mediated by **gauge leptoquarks** in  $SU(5)^3$

#LQs = 3 x #LQs in standard  $SU(5)$

However, 2<sup>nd</sup> and 3<sup>rd</sup> family LQs couple to 1<sup>st</sup> family fermions **only via mixing**

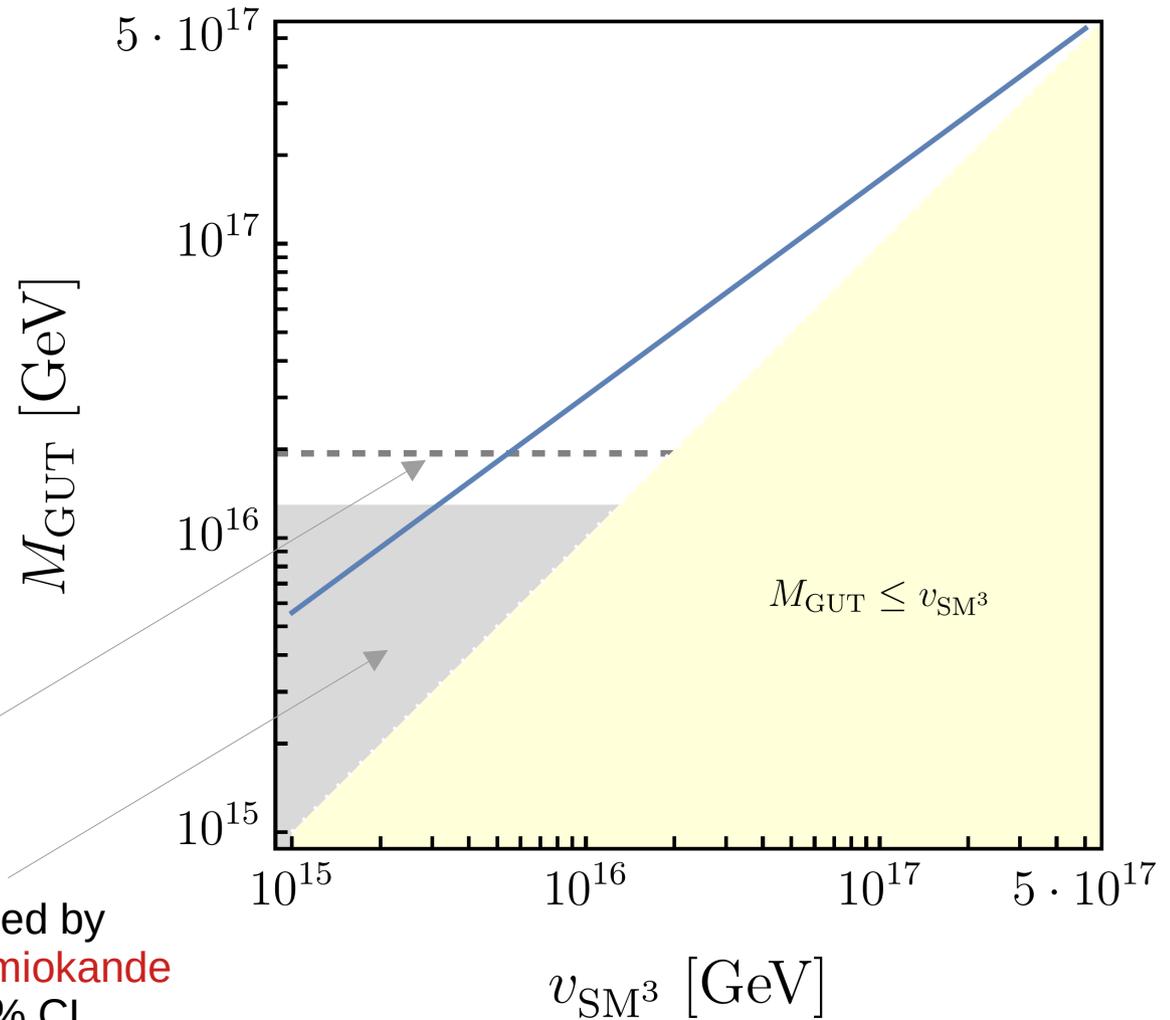
$$\tau_p \approx \tau_p^{\text{app}} = \frac{m_X^4}{\alpha_{\text{GUT}}^2 m_p^5}$$

Excludes  $SM^3$  intermediate scale below

$$\sim 3 \cdot 10^{15} \text{ GeV}$$

Future  
**Hyper-Kamiokande**  
@ 90% CL

Excluded by  
**Super-Kamiokande**  
@ 90% CL



# Proton decay

Proton decay mediated by **gauge leptoquarks** in  $SU(5)^3$

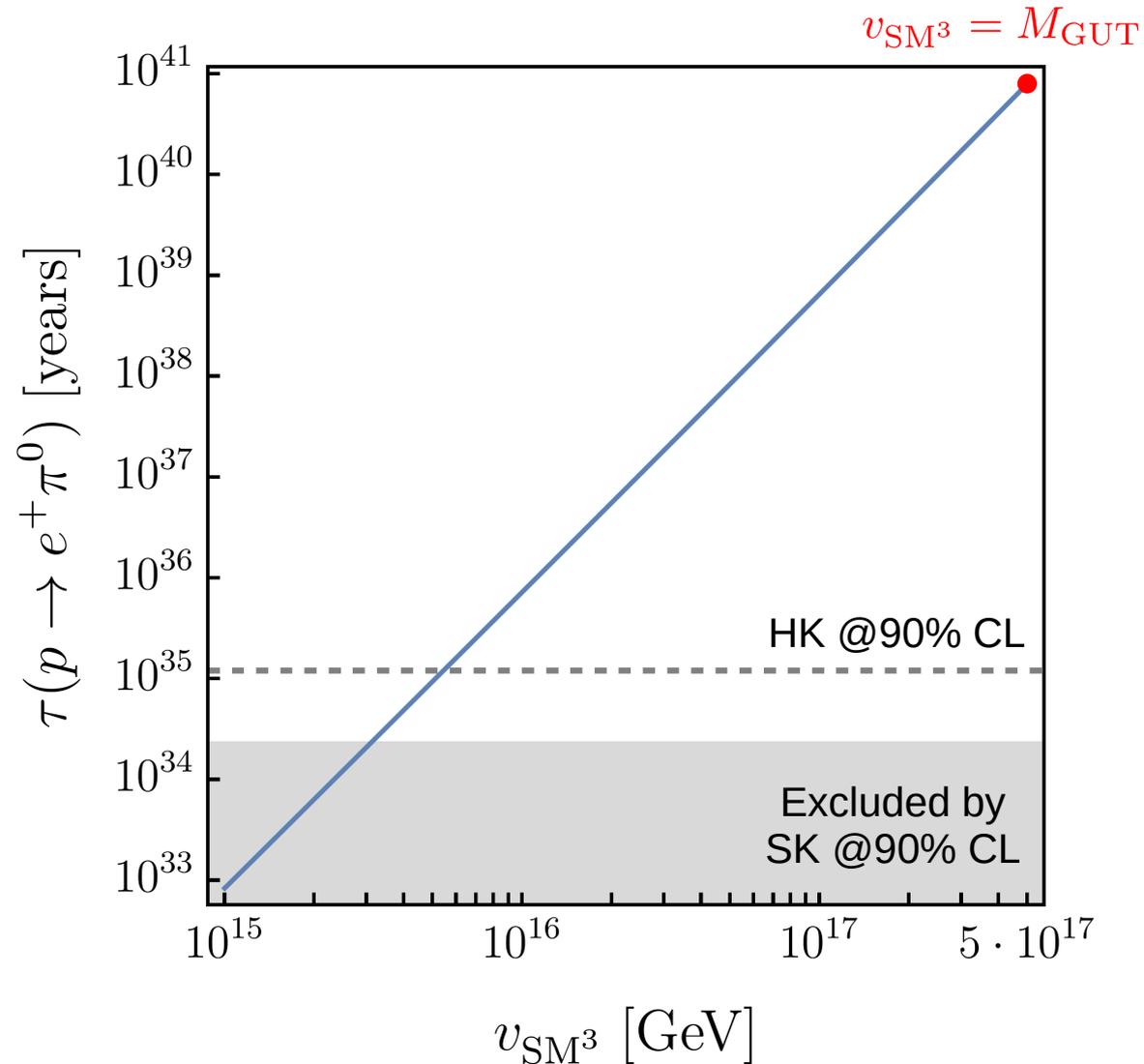
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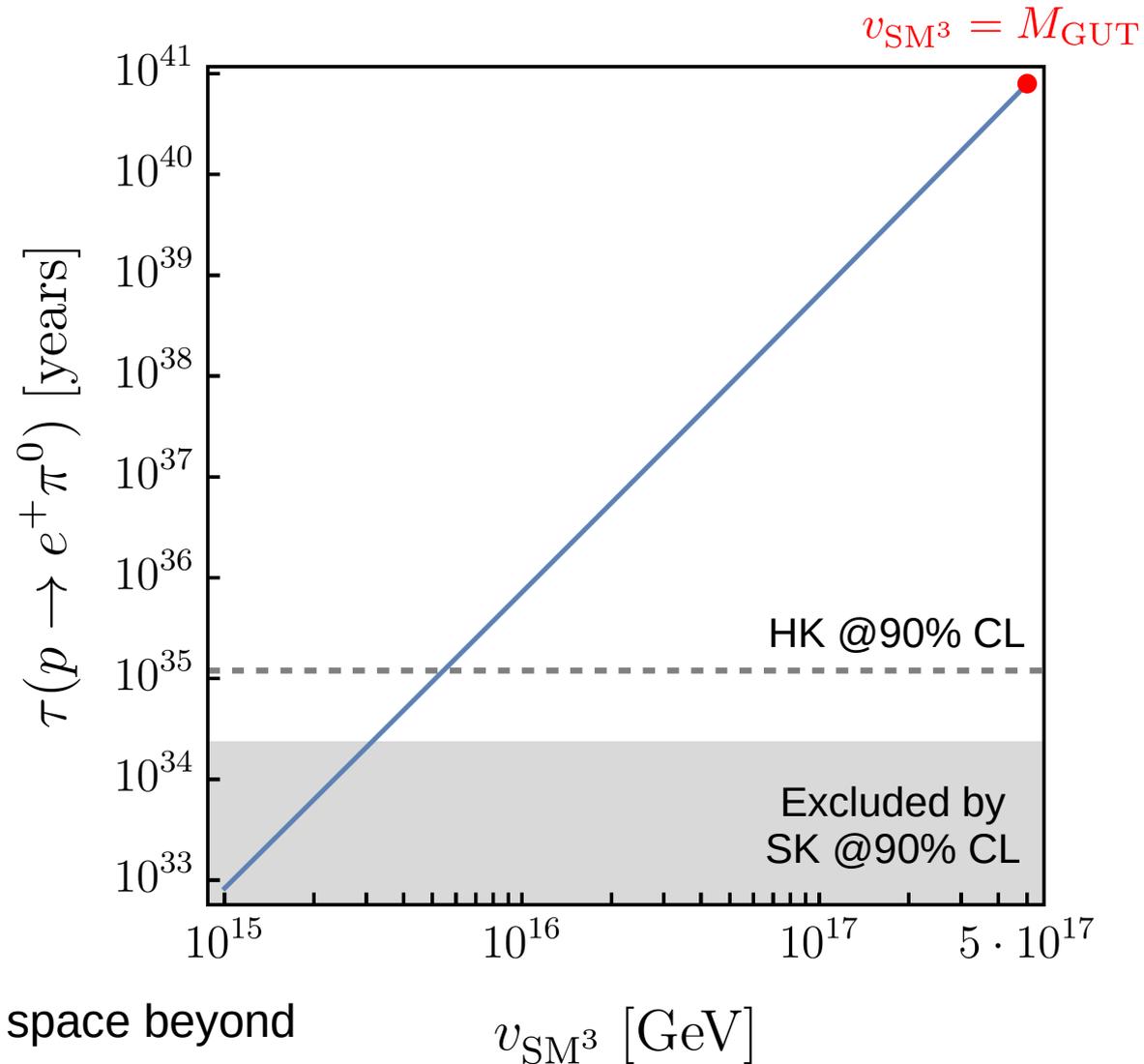
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Excludes SM<sup>3</sup> intermediate scale below

$$\sim 3 \cdot 10^{15} \text{ GeV}$$

**Model probed** in proton decay searches

Although large regions of the parameter space beyond any foreseen experiment



# Take-home messages

**Flavor deconstruction** is a successful way to generate the flavor structure of the SM

**Tri-unification** is a flavor deconstructed grand unified theory based on the symmetry

$$SU(5)_1 \times SU(5)_2 \times SU(5)_3 \times \mathbb{Z}_3$$

- ★ Explains **mass hierarchies and mixings**
- ★ **Unifies** the gauge couplings at high energies
- ★ Can be probed by **proton decay** experiments

An aerial photograph of a historic town, likely Regensburg, Germany. The town is built on a hillside, with a large stone castle (Regensburg Castle) at the top. The town is densely packed with buildings, many with red-tiled roofs. A large church with a tall spire is visible. A river flows through the town, and a stone bridge with multiple arches crosses it. The background is a lush green forested hillside.

**Thanks for your attention!**

**(time for questions & lunch)**

Backup slides

Field	$SU(5)_1$	$SU(5)_2$	$SU(5)_3$
$F_1$	$\bar{5}$	1	1
$F_2$	1	$\bar{5}$	1
$F_3$	1	1	$\bar{5}$
$T_1$	10	1	1
$T_2$	1	10	1
$T_3$	1	1	10
$\chi_1$	10	1	1
$\chi_2$	1	10	1
$\chi_3$	1	1	10
$\Xi_0$	1	1	1
$\bar{\Xi}_{12}$	5	$\bar{5}$	1
$\bar{\Xi}_{13}$	$\bar{5}$	1	5
$\bar{\Xi}_{23}$	1	5	$\bar{5}$
$\Sigma_{\text{atm}}$	1	10	$\bar{10}$
$\Sigma_{\text{sol}}$	10	1	$\bar{10}$
$\Sigma_{\text{cyclic}}$	10	$\bar{10}$	1
$\Omega_1$	24	1	1
$\Omega_2$	1	24	1
$\Omega_3$	1	1	24

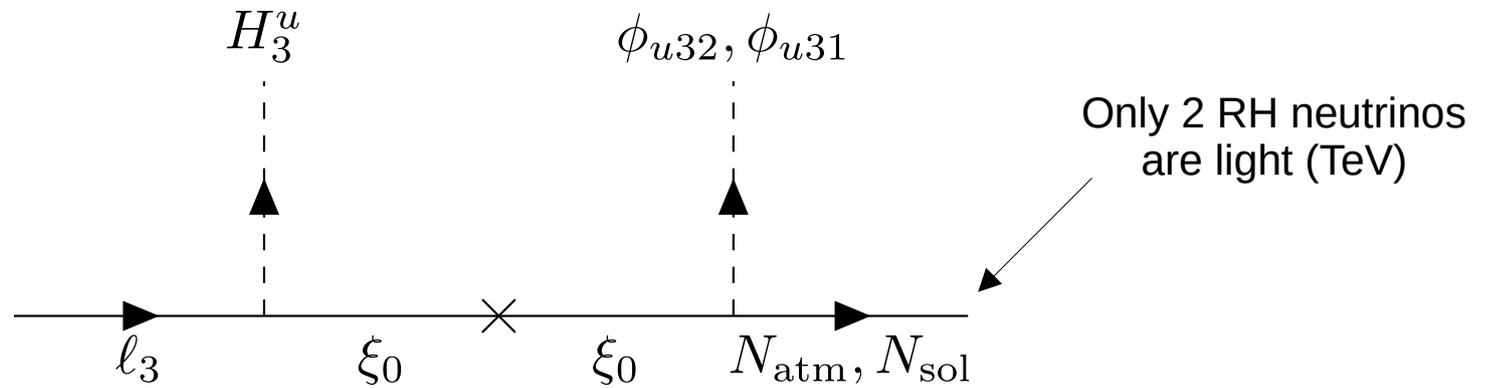
Field	$SU(5)_1$	$SU(5)_2$	$SU(5)_3$
$H_1^u$	5	1	1
$H_2^u$	1	5	1
$H_3^u$	1	1	5
$H_1^{\bar{5}}$	$\bar{5}$	1	1
$H_2^{\bar{5}}$	1	$\bar{5}$	1
$H_3^{\bar{5}}$	1	1	$\bar{5}$
$H_1^{45}$	45	1	1
$H_2^{45}$	1	45	1
$H_3^{45}$	1	1	45
$\Phi_{12}^F$	5	$\bar{5}$	1
$\Phi_{13}^F$	$\bar{5}$	1	5
$\Phi_{23}^F$	1	5	$\bar{5}$
$\Phi_{12}^T$	$\bar{10}$	10	1
$\Phi_{13}^T$	10	1	$\bar{10}$
$\Phi_{23}^T$	1	$\bar{10}$	10
$\Phi_{12}^{45}$	1	$\bar{45}$	45
$\Phi_{13}^{45}$	$\bar{45}$	1	45
$\Phi_{12}^{45}$	$\bar{45}$	45	1
$\Phi^{TFT}$	10	5	10
$\Phi^{FTT}$	5	10	10
$\Phi^{TTF}$	10	10	5

# Neutrino masses

	Field	$SU(5)_1$	$SU(5)_2$	$SU(5)_3$
New messenger fields	$\Xi_0$	1	1	1
	$\Xi_{12}$	5	$\bar{5}$	1
	$\Xi_{13}$	$\bar{5}$	1	5
	$\Xi_{23}$	1	5	$\bar{5}$
Right-handed neutrinos	$\Sigma_{\text{atm}}$	1	10	$\overline{10}$
	$\Sigma_{\text{sol}}$	10	1	$\overline{10}$
	$\Sigma_{\text{cyclic}}$	10	$\overline{10}$	1
New bi-representations (new hyperons)	$\Phi_{12}^{45}$	1	$\overline{45}$	45
	$\Phi_{13}^{45}$	$\overline{45}$	1	45
	$\Phi_{12}^{45}$	$\overline{45}$	45	1
	$\Phi^{TFT}$	10	5	10
	$\Phi^{FTT}$	5	10	10
	$\Phi^{TTF}$	10	10	5

More economical implementation in  
[ [Fernández Navarro, King, AV, 2024](#) ]

# Neutrino masses



$$m_\nu \simeq m_D M_N^{-1} m_D^T \approx \begin{pmatrix} 1 & 1 & \lambda \\ 1 & 1 & 1 \\ \lambda & 1 & 1 \end{pmatrix} v_{23} \frac{v_{\text{SM}}^2}{M_\xi^2}$$

Type-I seesaw mechanism

**Correct lepton mixing + a massless neutrino**

# Phenomenology

[ Fernández Navarro, King, AV, 2024 ]

All new particles are **too heavy** to be probed directly

**FCNCs** from heavy  $Z'$  bosons

VL fermions may induce large (but acceptable) contributions to **meson mixing** via boxes

General (**model-independent**) bounds:

- Dilepton tails at the LHC
- Modification of EW precision observables

