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NEW PHYSICS DIRECTIONS IN THE LHC ERA AND BEYOND, 25/04/2024

# Heavy Dark Matter EFT for any Spin

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in collaboration with Fady Bishara, Joachim Brod, Emmanuel Stamou and Jure Zupan

ongoing work

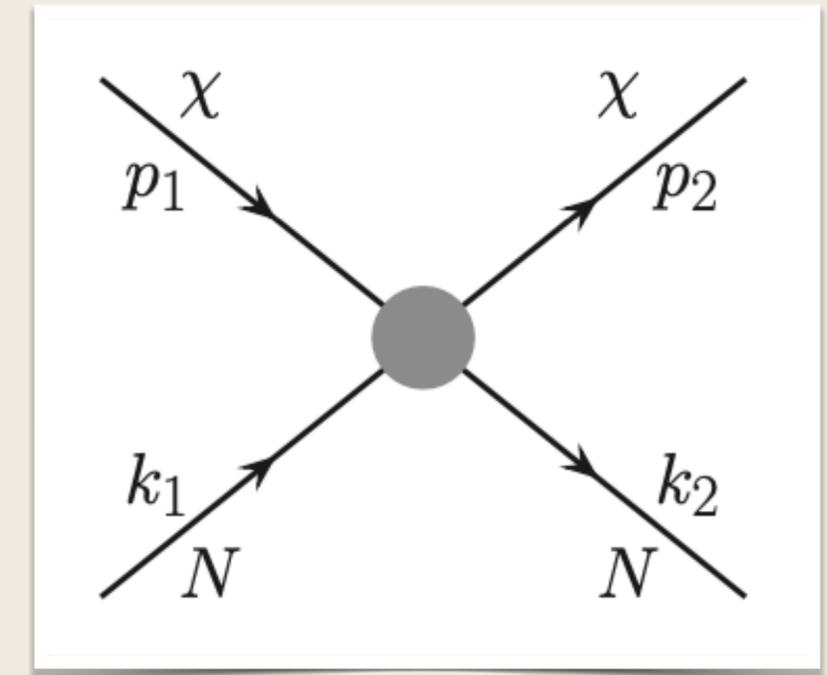


# Introduction

- Goal: model independent description of direct detection experimental results.

- Consider:

- non-relativistic DM;
- scattering processes;
- small momentum exchanges, i.e.  $p_1 - p_2 = q \ll m_\chi$ ;
- heavy mediators;
- arbitrary spin.



- E.g. DM bound states, composite DM.

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# Existing approaches

EFT for relativistic DM:  $\mathcal{L}_\chi = \sum_{a,d} \hat{\mathcal{C}}_a^d \mathcal{O}_a^d, \quad \hat{\mathcal{C}}_a^d = \frac{\mathcal{C}_a^d}{\Lambda^{d-4}}$

- $\mathcal{O}_a^d$ :  $(\bar{\chi}\chi)(\bar{q}q), (\bar{\chi}\gamma^\mu\chi)(\bar{q}\gamma_\mu q), \dots$  up to dim 7 for spin 0, 1/2, 1;
- EFT above EW scale - relate to indirect exp., LHC;
- No higher spins.

[Bishara et al, JCAP 02 (2017) 009]

[Goodman et al, Phys. Rev. D 82 (2010) 116010]

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EFT for non-relativistic DM interacting with non-relativistic nucleons:  $\mathcal{L}_{NR} = \sum_{i,N} c_i^N(q^2) \mathcal{O}_i^N$

- $\mathcal{O}_i^N$ :  $\mathbf{1}_\chi \mathbf{1}_N, \vec{S}_\chi \cdot \vec{S}_N, \dots$ , for any spin;
- Requires  $\vec{q} \ll m_\pi$ .

[Fitzpatrick et al, JCAP 02 (2013) 004]

[Jenkins, Manohar, Phys. Lett. B 255 (1991) 558-562]

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=> Combine the best of both: HDMEFT - NR DM for any spin with Lorentz symmetry and couplings to SM fields.

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# Heavy DM EFT - Little Group

- Little group is a subgroup of Lorentz transformations that is isomorphic to  $SO(3)$  group of rotations and leaves time-like vector  $v$  invariant. [\[Heinonen et al, Phys. Rev. D 86 \(2012\) 094020\]](#)
- Lorentz trans. encoded in generators  $\mathcal{J}^{\alpha\beta}$  which can be split into rotations  $J^{\alpha\beta}$  and boosts  $K^{\alpha\beta}$ .

# Heavy DM EFT - Little Group

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- Lorentz trans. encoded in generators  $\mathcal{J}^{\alpha\beta}$  which can be split into rotations  $J^{\alpha\beta}$  and boosts  $K^{\alpha\beta}$ .
- Build Lorentz inv. HDMEFT by
  - embedding DM fields into little group rep. and defining trans. under Lorentz group
  - requiring rotational invariance:  $R_\nu^\mu v^\nu = v^\mu$  (sufficient for LO in  $1/m_\chi$ );
  - requiring inv. under small boosts:  $\chi(x) \rightarrow \exp(i\vec{\eta} \cdot \vec{K}) \chi(x') = e^{-iq \cdot x} (1 + \mathcal{O}(1/m_\chi^2)) \chi_\nu(x')$ , where  $\vec{\eta} = -\vec{q}/M \Rightarrow$  **RPI relates operators of different dimensions.**
- Choose  $v$  such that  $v \rightarrow v + \frac{q}{M}$ , e.g.  $v^\mu = (1, \vec{0})$ .
- Define spin operator:  $S_\mu = -\frac{1}{2} \epsilon_{\mu\alpha\beta\gamma} J^{\alpha\beta} v^\gamma$ , e.g. spin - 1/2:  $S_\mu = \gamma_\mu \gamma_5 / 2$ .

# Reparametrization invariance

To find RPI relations perform a Lorentz boost that shifts  $v^\mu$

- Choose a boost transformation  $\mathcal{B}(q) = \Lambda(v - q/M, v)$

$$\mathcal{B}(q)^\mu{}_\nu = \delta^\mu_\nu + \frac{1}{m_\chi} [v^\mu q_\nu - q^\mu v_\nu - (v \cdot q) v^\mu v_\nu] + \mathcal{O}(q^2), \quad \text{Vector rep.}$$

$$\mathcal{B}_{\frac{1}{2}}(q) = \mathbb{1} - \frac{\not{q}\not{v}}{2m_\chi} + \mathcal{O}(q^2) \quad \text{Dirac rep.}$$

- SM currents ( $\bar{f}\gamma^\mu f$ ) transform as vectors
- Heavy DM as

$$\chi_v \xrightarrow{\mathcal{B}(q)} e^{iq \cdot x} \left[ 1 + \frac{iq \cdot \partial_\perp}{2m_\chi^2} - \frac{\epsilon_{\mu\nu\rho\sigma} v^\mu S_\perp^\nu q^\rho \partial_\perp^\sigma}{2m_\chi^2} \right] \chi_v + \mathcal{O}(1/m_\chi^4)$$

Here  $(v \cdot \dots)$  longitudinal and  $\dots_\perp$  perpendicular components

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Here  $(v \cdot \dots)$  longitudinal and  $\dots_\perp$  perpendicular components

- Derivatives and field strength tensors as

$$v \cdot D \longrightarrow v \cdot D + \frac{1}{m_\chi} q \cdot D_\perp + \mathcal{O}(q^2),$$

$$D_\perp^\mu \longrightarrow D_\perp^\mu - \frac{1}{m_\chi} q^\mu (v \cdot D) + \mathcal{O}(q^2),$$

$$(v \cdot F)^\mu \longrightarrow (v \cdot F)^\mu + \frac{1}{m_\chi} (q \cdot F_\perp)^\mu + \mathcal{O}(q^2),$$

$$F_\perp^{\mu\nu} \longrightarrow F_\perp^{\mu\nu} - \frac{1}{m_\chi} \left[ q^\mu (v \cdot F)^\nu - (v \cdot F)^\mu q^\nu \right] + \mathcal{O}(q^2)$$

- EOM used to remove higher order terms

$$\left[ i(v \cdot \partial) + \frac{1}{2m_\chi} (i\partial_\perp)^2 + \dots \right] \chi_v = 0,$$

- Bianchi identity  $\partial^\mu \tilde{F}_{\mu\nu} = 0$

# Reparametrization invariance

$$\mathcal{L}_{\text{int}} = \sum_{a,d} \frac{w_a^{(d)}}{m_\chi^{d-4}} \mathcal{P}_a^{(d)}$$

- All RPI's up to  $\mathcal{O}(q)$
- Dim 5 up to  $\mathcal{O}(1/m_\chi^2)$
- Dim 6 up to  $\mathcal{O}(1/m_\chi)$
- Dim 7 up to  $\mathcal{O}(1/m_\chi^0)$

$$\begin{aligned} \mathcal{P}_{1\gamma}^{(5)} &\rightarrow \mathcal{P}_{1\gamma}^{(5)} + \frac{e}{16\pi^2} \frac{1}{m_\chi} (\chi_v^\dagger S_\perp^\mu \chi_v) \tilde{F}_{\perp q\mu} \\ &\quad + \frac{e}{16\pi^2} \frac{1}{2m_\chi^2} \left[ (\chi_v^\dagger S_\perp^\mu i \hat{\partial}_\perp^q \chi_v) (v \cdot \tilde{F})_\mu \right. \\ &\quad \quad - \frac{1}{2} (\chi_v^\dagger S_\perp^q i \hat{\partial}_\perp^\mu \chi_v) (v \cdot \tilde{F})_\mu + \frac{1}{2} (\chi_v^\dagger S_\perp^\mu i \hat{\partial}_\perp^\mu \chi_v) (v \cdot \tilde{F})_q \\ &\quad \quad - \partial_\perp^\mu (\chi_v^\dagger \overline{S_\perp^{\mu\nu}} \chi_v) F_{\perp q\nu} \\ &\quad \quad \left. - (\chi_v^\dagger \overline{S_\perp^{q\mu}} \chi_v) \underbrace{\partial^\nu F_{\nu\mu}}_{e \sum_f Q_f \bar{f} \gamma_{\mu f}} - \frac{j(j+1)}{3} (\chi_v^\dagger \chi_v) \underbrace{\partial^\mu F_{\mu\nu}}_{e \sum_f Q_f \bar{f} \gamma_{\nu f}} q^\nu \right], \\ \mathcal{P}_{1f}^{(6)} &\rightarrow \mathcal{P}_{1f}^{(6)} + \frac{q_\mu}{m_\chi} (\chi_v^\dagger \chi_v) (\bar{f} \gamma_\perp^\mu f), \\ \mathcal{P}_{1\gamma}^{(6)} &\rightarrow \mathcal{P}_{1\gamma}^{(6)} + \frac{e}{16\pi^2} \frac{1}{m_\chi} \partial_{\perp\mu} (\chi_v^\dagger \overline{S_\perp^{\mu\nu}} \chi_v) F_{\perp q\nu}. \end{aligned}$$

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## Minimal basis

$$\begin{aligned} \mathcal{P}_{1\gamma}^{(5)} &\rightarrow \mathcal{P}_{1\gamma}^{(5)} + \frac{e}{16\pi^2} \frac{1}{m_\chi} (\chi_v^\dagger S_\perp^\mu \chi_v) \tilde{F}_{\perp q\mu} \\ &+ \frac{e}{16\pi^2} \frac{1}{2m_\chi^2} \left[ (\chi_v^\dagger S_\perp^\mu i \hat{\partial}_\perp^q \chi_v) (v \cdot \tilde{F})_\mu \right. \\ &\quad - \frac{1}{2} (\chi_v^\dagger S_\perp^q i \hat{\partial}_\perp^\mu \chi_v) (v \cdot \tilde{F})_\mu + \frac{1}{2} (\chi_v^\dagger S_\perp^\mu i \hat{\partial}_\perp^\mu \chi_v) (v \cdot \tilde{F})_q \\ &\quad - \partial_\perp^\mu (\chi_v^\dagger \overline{S_\perp^{\mu\nu}} \chi_v) F_{\perp q\nu} \\ &\quad \left. - (\chi_v^\dagger \overline{S_\perp^{q\mu}} \chi_v) \underbrace{\partial^\nu F_{\nu\mu}}_{e \sum_f Q_f \bar{f} \gamma_{\mu f}} - \frac{j(j+1)}{3} (\chi_v^\dagger \chi_v) \underbrace{\partial^\mu F_{\mu\nu}}_{e \sum_f Q_f \bar{f} \gamma_{\nu f}} q^\nu \right], \\ \mathcal{P}_{1f}^{(6)} &\rightarrow \mathcal{P}_{1f}^{(6)} + \frac{q_\mu}{m_\chi} (\chi_v^\dagger \chi_v) (\bar{f} \gamma_\perp^\mu f), \\ \mathcal{P}_{1\gamma}^{(6)} &\rightarrow \mathcal{P}_{1\gamma}^{(6)} + \frac{e}{16\pi^2} \frac{1}{m_\chi} \partial_\perp^\mu (\chi_v^\dagger \overline{S_\perp^{\mu\nu}} \chi_v) F_{\perp q\nu}. \end{aligned}$$

## Related via RPI

$$\begin{aligned} \mathcal{P}_{3\gamma}^{(6)} &\rightarrow \mathcal{P}_{3\gamma}^{(6)} + \frac{e}{16\pi^2} 2 (\chi_v^\dagger S_\perp^\mu \chi_v) \tilde{F}_{\perp q\mu} \\ &\quad - \frac{e}{16\pi^2} \frac{1}{m_\chi} (\chi_v^\dagger S_\perp^q i \hat{\partial}_\perp^\mu \chi_v) (v \cdot \tilde{F})_\mu \\ &\quad + \frac{e}{16\pi^2} \frac{1}{m_\chi} (\chi_v^\dagger S_\perp^\mu i \hat{\partial}_\perp^q \chi_v) (v \cdot \tilde{F})_\mu \\ \mathcal{P}_{21f}^{(7)} &\rightarrow \mathcal{P}_{21f}^{(7)} - 2q_\mu (\chi_v^\dagger \chi_v) (\bar{f} \gamma_\perp^\mu f), \\ \mathcal{P}_{25f}^{(7)} &\rightarrow \mathcal{P}_{25f}^{(7)} - 2 (\chi_v^\dagger \overline{S_\perp^{q\mu}} \chi_v) (\bar{f} \gamma_{\perp\mu} f), \\ \mathcal{P}_{14\gamma}^{(7)} &\rightarrow \mathcal{P}_{14\gamma}^{(7)} - \frac{e}{16\pi^2} 2 (\chi_v^\dagger S_\perp^q i \hat{\partial}_\perp^\mu \chi_v) (v \cdot \tilde{F})_\mu \\ &\quad - \frac{e}{16\pi^2} 2 (\chi_v^\dagger S_\perp^\mu i \hat{\partial}_\perp^q \chi_v) (v \cdot \tilde{F})_q, \\ \mathcal{P}_{16\gamma}^{(7)} &\rightarrow \mathcal{P}_{16\gamma}^{(7)} - \frac{e}{16\pi^2} 2 \partial_\perp^\mu (\chi_v^\dagger \overline{S_\perp^{\mu\nu}} \chi_v) F_{\perp\nu q}. \end{aligned}$$

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Solving the system of equations gives:

$$\begin{aligned} w_{3\gamma}^{(6)} &= -\frac{1}{2} w_{1\gamma}^{(5)}, & w_{21f}^{(7)} &= \frac{1}{2} w_{1f}^{(6)} - \frac{j(j+1)}{12} \frac{e}{16\pi^2} e Q_f w_{1\gamma}^{(5)}, & w_{25f}^{(7)} &= -\frac{1}{4} \frac{e}{16\pi^2} e Q_f w_{1\gamma}^{(5)}, \\ w_{14\gamma}^{(7)} &= \frac{1}{8} w_{1\gamma}^{(5)}, & w_{16\gamma}^{(7)} &= \frac{1}{4} (w_{1\gamma}^{(5)} - 2w_{1\gamma}^{(6)}). \end{aligned}$$

# HDMEFT basis

$$\mathcal{L}_{\text{int}} = \sum_{a,d} \frac{w_a^{(d)}}{m_\chi^{d-4}} \mathcal{P}_a^{(d)}$$

Minimal basis with linearly independent WC's

- Dimension 5

$$\mathcal{P}_{1\gamma}^{(5)} = \frac{e}{16\pi^2} (\chi_v^\dagger S_\perp^\mu \chi_v) (v \cdot \tilde{F})_\mu, \quad \mathcal{P}_{2\gamma}^{(5)} = \frac{e}{16\pi^2} (\chi_v^\dagger S_\perp^\mu \chi_v) (v \cdot F)_\mu$$

$$j_\chi \geq 1/2$$

- Dimension 6

$$\begin{aligned} \mathcal{P}_{1f}^{(6)} &= (\chi_v^\dagger \chi_v) (\bar{f} \psi f), & \mathcal{P}_{2f}^{(6)} &= (\chi_v^\dagger \chi_v) (\bar{f} \psi \gamma_5 f), \\ \mathcal{P}_{3f}^{(6)} &= (\chi_v^\dagger S_\perp^\mu \chi_v) (\bar{f} \gamma_{\perp\mu} \gamma_5 f), & \mathcal{P}_{4f}^{(6)} &= (\chi_v^\dagger S_\perp^\mu \chi_v) (\bar{f} \gamma_{\perp\mu} f), \\ \mathcal{P}_{1\gamma}^{(6)} &= \frac{e}{16\pi^2} \partial_{\perp\mu} (\chi_v^\dagger \overline{S_\perp^{\mu\nu}} \chi_v) (v \cdot F)_\nu, & \mathcal{P}_{2\gamma}^{(6)} &= \frac{e}{16\pi^2} \partial_{\perp\mu} (\chi_v^\dagger \overline{S_\perp^{\mu\nu}} \chi_v) (v \cdot \tilde{F})_\nu \end{aligned}$$

$$j_\chi \geq 0$$

$$j_\chi \geq 1/2$$

$$j_\chi \geq 1$$

Notation:

$$\gamma_{\perp\mu} = \gamma_\mu - v_\mu \psi$$

$$\overline{S_\perp^{\mu\nu}} = S_\perp^{\{\mu} S_\perp^{\nu\}} - \frac{j_\chi(j_\chi + 1)}{3} \hat{g}^{\mu\nu}$$

$$\hat{g}_{\mu\nu} = g_{\mu\nu} - v_\mu v_\nu$$

# HDMEFT basis - dim 7

Fermions, photons and similar for gluons

$$\overline{S_{\perp}^{\mu\nu\rho}} = S_{\perp}^{\{\mu} S_{\perp}^{\nu} S_{\perp}^{\rho\}} - \frac{3j_{\chi}(j_{\chi} + 1) - 1}{5} \hat{g}^{\{\mu\nu} S_{\perp}^{\rho\}}$$

Operators that do not vanish for  $j_{\chi} \geq 0$ .

$$\begin{aligned} \mathcal{P}_{1f}^{(7)} &= m_f (\chi_v^{\dagger} \chi_v) (\bar{f} f), & \mathcal{P}_{2f}^{(7)} &= m_f (\chi_v^{\dagger} \chi_v) (\bar{f} i \gamma_5 f), \\ \mathcal{P}_{3f}^{(7)} &= (\chi_v^{\dagger} \chi_v) (\bar{f} \psi i (v \cdot \vec{D}) f), & \mathcal{P}_{4f}^{(7)} &= (\chi_v^{\dagger} \chi_v) (\bar{f} \psi i (v \cdot \vec{D}) \gamma_5 f), \end{aligned}$$

Operators that do not vanish for  $j_{\chi} \geq 1/2$ .

$$\begin{aligned} \mathcal{P}_{5f}^{(7)} &= m_f (\chi_v^{\dagger} S_{\perp}^{\mu} \chi_v) (\bar{f} (v \cdot \sigma)_{\mu} i \gamma_5 f), & \mathcal{P}_{6f}^{(7)} &= m_f (\chi_v^{\dagger} S_{\perp}^{\mu} \chi_v) (\bar{f} (v \cdot \sigma)_{\mu} f), \\ \mathcal{P}_{7f}^{(7)} &= \partial_{\perp}^{\mu} (\chi_v^{\dagger} S_{\perp}^{\nu} \chi_v) \epsilon_{\mu\nu\rho\sigma} v^{\rho} (\bar{f} \gamma_{\perp}^{\sigma} f), & \mathcal{P}_{8f}^{(7)} &= \partial_{\perp}^{\mu} (\chi_v^{\dagger} S_{\perp}^{\nu} \chi_v) \epsilon_{\mu\nu\rho\sigma} v^{\rho} (\bar{f} \gamma_{\perp}^{\sigma} \gamma_5 f), \\ \mathcal{P}_{9f}^{(7)} &= \partial_{\perp\mu} (\chi_v^{\dagger} S_{\perp}^{\mu} \chi_v) (\bar{f} \psi \gamma_5 f), & \mathcal{P}_{10f}^{(7)} &= \partial_{\perp\mu} (\chi_v^{\dagger} S_{\perp}^{\mu} \chi_v) (\bar{f} \psi f), \\ \mathcal{P}_{11f}^{(7)} &= (\chi_v^{\dagger} S_{\perp}^{\mu} \chi_v) (\bar{f} \psi \gamma_5 i \vec{D}_{\perp\mu} f), & \mathcal{P}_{12f}^{(7)} &= (\chi_v^{\dagger} S_{\perp}^{\mu} \chi_v) (\bar{f} \psi i \vec{D}_{\perp\mu} f), \\ \mathcal{P}_{13f}^{(7)} &= (\chi_v^{\dagger} S_{\perp}^{\mu} \chi_v) (\bar{f} \gamma_{\perp\mu} i (v \cdot \vec{D}) \gamma_5 f), & \mathcal{P}_{14f}^{(7)} &= (\chi_v^{\dagger} S_{\perp}^{\mu} \chi_v) (\bar{f} \gamma_{\perp\mu} i (v \cdot \vec{D}) f), \\ \mathcal{P}_{15f}^{(7)} &= (\chi_v^{\dagger} S_{\perp}^{\mu} \chi_v) \epsilon_{\mu\nu\rho\sigma} v^{\nu} (\bar{f} \gamma^{\rho} i \vec{D}_{\perp}^{\sigma} f), & \mathcal{P}_{16f}^{(7)} &= (\chi_v^{\dagger} S_{\perp}^{\mu} \chi_v) \epsilon_{\mu\nu\rho\sigma} v^{\nu} (\bar{f} \gamma^{\rho} \gamma_5 i \vec{D}_{\perp}^{\sigma} f), \end{aligned}$$

Operators that do not vanish for  $j_{\chi} \geq 1$ .

$$\begin{aligned} \mathcal{P}_{17f}^{(7)} &= (\chi_v^{\dagger} \overline{S_{\perp}^{\mu\nu}} \chi_v) (\bar{f} \gamma_{\perp\mu} i \vec{D}_{\perp\nu} f), & \mathcal{P}_{18f}^{(7)} &= (\chi_v^{\dagger} \overline{S_{\perp}^{\mu\nu}} \chi_v) (\bar{f} \gamma_{\perp\mu} i \vec{D}_{\perp\nu} \gamma_5 f), \\ \mathcal{P}_{19f}^{(7)} &= \partial_{\perp\mu} (\chi_v^{\dagger} \overline{S_{\perp}^{\mu\nu}} \chi_v) (\bar{f} \gamma_{\perp\nu} f), & \mathcal{P}_{20f}^{(7)} &= \partial_{\perp\mu} (\chi_v^{\dagger} \overline{S_{\perp}^{\mu\nu}} \chi_v) (\bar{f} \gamma_{\perp\nu} \gamma_5 f). \end{aligned}$$

Operators that do not vanish for  $j_{\chi} \geq 0$ .

$$\begin{aligned} \mathcal{P}_{1\gamma}^{(7)} &= \frac{\alpha_{\text{em}}}{4\pi} (\chi_v^{\dagger} \chi_v) F_{\perp\mu\nu} F_{\perp}^{\mu\nu}, & \mathcal{P}_{2\gamma}^{(7)} &= \frac{\alpha_{\text{em}}}{4\pi} (\chi_v^{\dagger} \chi_v) F_{\perp\mu\nu} \tilde{F}_{\perp}^{\mu\nu}, \\ \mathcal{P}_{3\gamma}^{(7)} &= \frac{\alpha_{\text{em}}}{4\pi} (\chi_v^{\dagger} \chi_v) (v \cdot F)^{\mu} (v \cdot F)_{\mu}, \end{aligned}$$

Operators that do not vanish for  $j_{\chi} \geq 1/2$ .

$$\begin{aligned} \mathcal{P}_{4\gamma}^{(7)} &= \frac{\alpha_{\text{em}}}{4\pi} (\chi_v^{\dagger} S_{\perp}^{\mu} \chi_v) (v \cdot F)^{\nu} F_{\perp\mu\nu}, \\ \mathcal{P}_{5\gamma}^{(7)} &= \frac{e}{16\pi^2} \partial_{\perp}^2 (\chi_v^{\dagger} S_{\perp}^{\mu} \chi_v) (v \cdot \tilde{F})_{\mu}, & \mathcal{P}_{6\gamma}^{(7)} &= \frac{e}{16\pi^2} \partial_{\perp}^2 (\chi_v^{\dagger} S_{\perp}^{\mu} \chi_v) (v \cdot F)_{\mu}, \end{aligned}$$

Operators that do not vanish for  $j_{\chi} \geq 1$ .

$$\begin{aligned} \mathcal{P}_{7\gamma}^{(7)} &= \frac{\alpha_{\text{em}}}{4\pi} (\chi_v^{\dagger} \overline{S_{\perp}^{\mu\nu}} \chi_v) (v \cdot F)_{\mu} (v \cdot F)_{\nu}, & \mathcal{P}_{8\gamma}^{(7)} &= \frac{\alpha_{\text{em}}}{4\pi} (\chi_v^{\dagger} \overline{S_{\perp}^{\mu\nu}} \chi_v) (v \cdot F)_{\mu} (v \cdot \tilde{F})_{\nu}, \\ \mathcal{P}_{9\gamma}^{(7)} &= \frac{\alpha_{\text{em}}}{4\pi} (\chi_v^{\dagger} \overline{S_{\perp}^{\mu\nu}} \chi_v) (v \cdot \tilde{F})_{\mu} (v \cdot \tilde{F})_{\nu}, \end{aligned}$$

Operators that do not vanish for  $j_{\chi} \geq 3/2$ .

$$\mathcal{P}_{10\gamma}^{(7)} = \frac{e}{16\pi^2} \partial_{\perp\nu} \partial_{\perp\rho} (\chi_v^{\dagger} \overline{S_{\perp}^{\mu\nu\rho}} \chi_v) (v \cdot \tilde{F})_{\mu}, \quad \mathcal{P}_{11\gamma}^{(7)} = \frac{e}{16\pi^2} \partial_{\perp\nu} \partial_{\perp\rho} (\chi_v^{\dagger} \overline{S_{\perp}^{\mu\nu\rho}} \chi_v) (v \cdot F)_{\mu}.$$

# HDM EFT basis - RPI

$$\mathcal{L}_{\text{int}} = \sum_{a,d} \frac{w_a^{(d)}}{m_\chi^{d-4}} \mathcal{P}_a^{(d)}$$

For the Lorentz invariant basis one also needs to include operators related via reparametrization invariance

Dimension-six

$$\mathcal{P}_{3\gamma}^{(6)} = \frac{e}{16\pi^2} (\chi_v^\dagger S_\perp^\mu i \overleftrightarrow{\partial}_\perp^\nu \chi_v) \tilde{F}_{\mu\nu},$$

$$\mathcal{P}_{4\gamma}^{(6)} = \frac{e}{16\pi^2} (\chi_v^\dagger S_\perp^\mu i \overleftrightarrow{\partial}_\perp^\nu \chi_v) F_{\mu\nu}.$$

Dimension-seven operators for fermions

$$\mathcal{P}_{21f}^{(7)} = (\chi_v^\dagger i \overleftrightarrow{\partial}_\perp^\mu \chi_v) (\bar{f} \gamma_\perp \mu f),$$

$$\mathcal{P}_{22f}^{(7)} = (\chi_v^\dagger i \overleftrightarrow{\partial}_\perp^\mu \chi_v) (\bar{f} \gamma_\perp \mu \gamma_5 f),$$

$$\mathcal{P}_{23f}^{(7)} = (\chi_v^\dagger i (S_\perp \cdot \overleftrightarrow{\partial}_\perp) \chi_v) (\bar{f} \psi \gamma_5 f),$$

$$\mathcal{P}_{24f}^{(7)} = (\chi_v^\dagger i (S_\perp \cdot \overleftrightarrow{\partial}_\perp) \chi_v) (\bar{f} \psi f),$$

$$\mathcal{P}_{25f}^{(7)} = (\chi_v^\dagger \overline{S_\perp^{\mu\nu}} i \overleftrightarrow{\partial}_{\perp\nu} \chi_v) (\bar{f} \gamma_\perp \mu f),$$

Dimension-seven operators for  $F$

$$\mathcal{P}_{12\gamma}^{(7)} = \frac{e}{16\pi^2} (\chi_v^\dagger S_{\perp\nu} i \overleftrightarrow{\partial}_\perp^\nu i \overleftrightarrow{\partial}_\perp^\mu \chi_v) (v \cdot \tilde{F})_\mu,$$

$$\mathcal{P}_{13\gamma}^{(7)} = \frac{e}{16\pi^2} (\chi_v^\dagger S_{\perp\nu} i \overleftrightarrow{\partial}_\perp^\nu i \overleftrightarrow{\partial}_\perp^\mu \chi_v) (v \cdot F)_\mu,$$

$$\mathcal{P}_{14\gamma}^{(7)} = \frac{e}{16\pi^2} \partial_{\perp\mu} (\chi_v^\dagger \overline{S_\perp^{\mu\nu}} i \overleftrightarrow{\partial}_\perp^\rho \chi_v) F_{\perp\nu\rho},$$

$$\mathcal{P}_{15\gamma}^{(7)} = \frac{e}{16\pi^2} \partial_{\perp\mu} (\chi_v^\dagger \overline{S_\perp^{\mu\nu}} i \overleftrightarrow{\partial}_\perp^\rho \chi_v) \tilde{F}_{\perp\nu\rho}.$$

RPI relations between Wilson Coefficients

$$w_{3\gamma}^{(6)} = -\frac{1}{2} w_{1\gamma}^{(5)},$$

$$w_{4\gamma}^{(6)} = -\frac{1}{2} w_{2\gamma}^{(5)},$$

$$w_{21f}^{(7)} = \frac{1}{2} w_{1f}^{(6)} - \frac{j(j+1)}{12} \frac{e}{16\pi^2} Q_f w_{1\gamma}^{(5)},$$

$$w_{22f}^{(7)} = \frac{1}{2} w_{2f}^{(6)},$$

$$w_{23f}^{(7)} = -\frac{1}{2} w_{3f}^{(6)},$$

$$w_{24f}^{(7)} = -\frac{1}{2} w_{4f}^{(6)},$$

$$w_{25f}^{(7)} = -\frac{1}{4} \frac{e}{16\pi^2} Q_f w_{1\gamma}^{(5)},$$

$$w_{12\gamma}^{(7)} = \frac{1}{8} w_{1\gamma}^{(5)},$$

$$w_{13\gamma}^{(7)} = \frac{1}{8} w_{2\gamma}^{(5)},$$

$$w_{14\gamma}^{(7)} = \frac{1}{4} (w_{1\gamma}^{(5)} - 2w_{1\gamma}^{(6)}),$$

$$w_{15\gamma}^{(7)} = -\frac{1}{4} (w_{2\gamma}^{(5)} + 2w_{2\gamma}^{(6)}).$$

Further operators removed by the RPI

# QM Basis and matching onto NR-nucleon EFT

1. Write the basis in the QM notation:

$$\chi_v^\dagger \chi_v \rightarrow 1_\chi, \quad \chi_v^\dagger S_\perp^\mu \chi_v \rightarrow (0, \vec{S}_\chi),$$

$$\mathcal{P}_{1\gamma}^{(5)} \rightarrow \frac{e}{16\pi^2} \vec{S}_\chi \cdot \vec{B}, \quad \mathcal{P}_{2\gamma}^{(5)} \rightarrow \frac{e}{16\pi^2} \vec{S}_\chi \cdot \vec{E}.$$

$$\begin{aligned} \mathcal{P}_{1f}^{(6)} &\rightarrow 1_\chi (\bar{f} \gamma^0 f), & \mathcal{P}_{2f}^{(6)} &\rightarrow 1_\chi (\bar{f} \gamma^0 \gamma_5 f), \\ \mathcal{P}_{3f}^{(6)} &\rightarrow -\vec{S}_\chi \cdot (\bar{f} \vec{\gamma} \gamma_5 f), & \mathcal{P}_{4f}^{(6)} &\rightarrow -\vec{S}_\chi \cdot (\bar{f} \vec{\gamma} f), \\ \mathcal{P}_{1\gamma}^{(6)} &\rightarrow \frac{e}{16\pi^2} i q^i \overline{S_\chi^{ij}} E^j, & \mathcal{P}_{2\gamma}^{(6)} &\rightarrow \frac{e}{16\pi^2} i q^i \overline{S_\chi^{ij}} B^j. \end{aligned}$$

3. Obtain Wilson coefficients  $c_i^N(q^2)$  and compute cross-sections - direct detection experiments.

2. Hadronization and matching onto NR-nucleon EFT:

$$\mathcal{L}_{\text{NR}} = \sum_{i,N} c_i^N(q^2) \mathcal{O}_i^N.$$

$$\begin{aligned} \mathcal{O}_1^N &= 1_\chi 1_N, & \mathcal{O}_2^N &= (v_\perp)^2 1_\chi 1_N, \\ \mathcal{O}_3^N &= 1_\chi \vec{S}_N \cdot \left( \vec{v}_\perp \times \frac{i\vec{q}}{m_N} \right), & \mathcal{O}_4^N &= \vec{S}_\chi \cdot \vec{S}_N, \\ \mathcal{O}_5^N &= \vec{S}_\chi \cdot \left( \vec{v}_\perp \times \frac{i\vec{q}}{m_N} \right) 1_N, & \mathcal{O}_6^N &= \left( \vec{S}_\chi \cdot \frac{\vec{q}}{m_N} \right) \left( \vec{S}_N \cdot \frac{\vec{q}}{m_N} \right), \\ \mathcal{O}_7^N &= 1_\chi (\vec{S}_N \cdot \vec{v}_\perp), & \mathcal{O}_8^N &= (\vec{S}_\chi \cdot \vec{v}_\perp) 1_N, \\ \mathcal{O}_9^N &= \vec{S}_\chi \cdot \left( \frac{i\vec{q}}{m_N} \times \vec{S}_N \right), & \mathcal{O}_{10}^N &= -1_\chi \left( \vec{S}_N \cdot \frac{i\vec{q}}{m_N} \right), \\ \mathcal{O}_{11}^N &= -\left( \vec{S}_\chi \cdot \frac{i\vec{q}}{m_N} \right) 1_N, & \mathcal{O}_{12}^N &= \vec{S}_\chi \cdot \left( \vec{S}_N \times \vec{v}_\perp \right), \\ \mathcal{O}_{13}^N &= -\left( \vec{S}_\chi \cdot \vec{v}_\perp \right) \left( \vec{S}_N \cdot \frac{i\vec{q}}{m_N} \right), & \mathcal{O}_{14}^N &= -\left( \vec{S}_\chi \cdot \frac{i\vec{q}}{m_N} \right) \left( \vec{S}_N \cdot \vec{v}_\perp \right) \end{aligned}$$

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# Conclusions

- EFT for heavy Dark Matter of arbitrary spin at LO in  $1/m_\chi$ .
- Constraints on DM bound/composite states.
- DM fields embedded in little group rep. - rotational inv.
- Lorentz invariance is present through RPI relations.
- Matching onto NR-nucleon EFT.