50 Years of QCD, UCLA 9/11-15/2023

From QCD to QGP: Strong interaction in extremis

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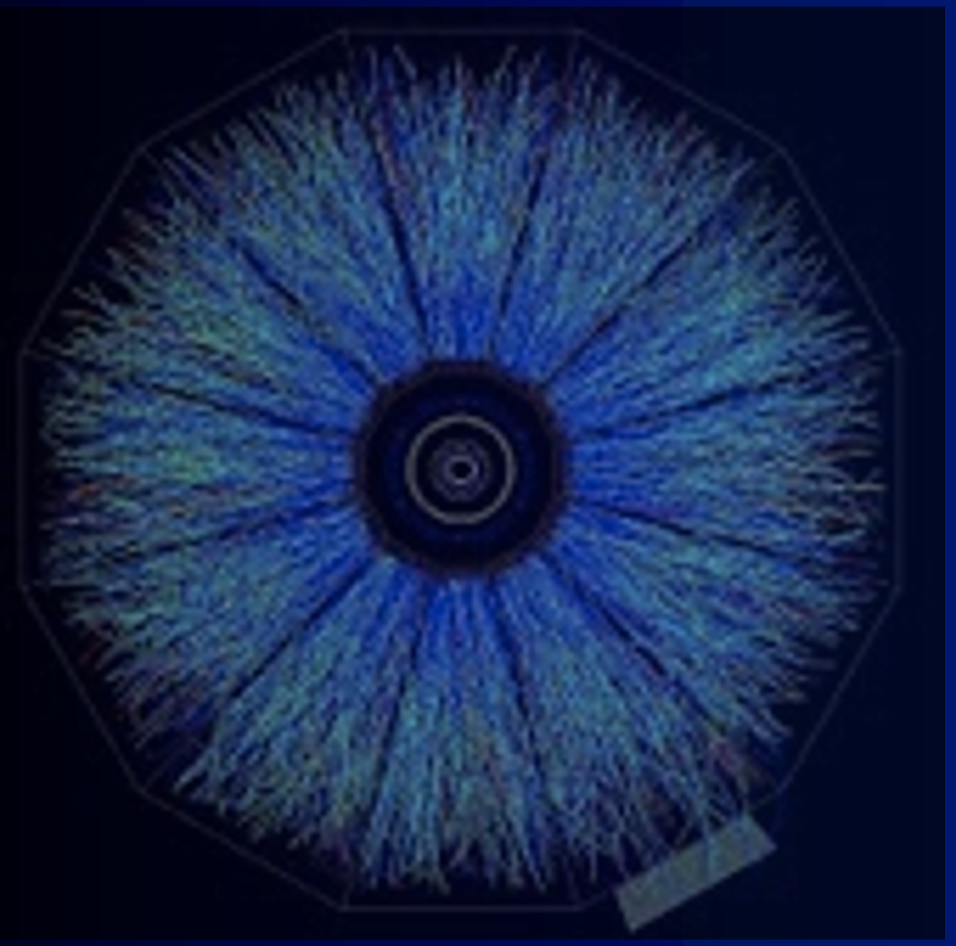
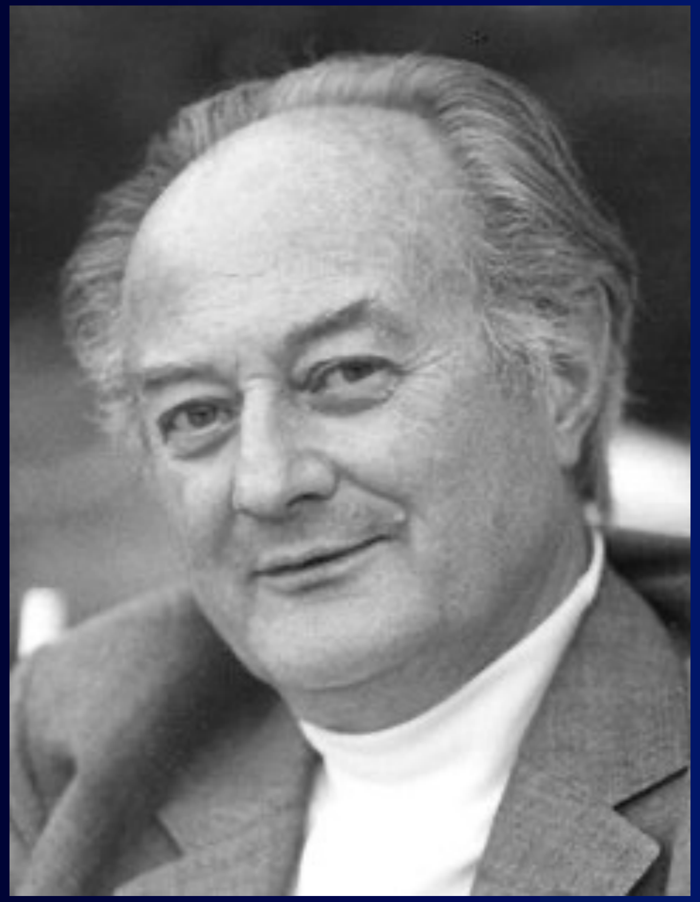


Hagedorn limiting temperature

Increasing number of hadron production (and decays) in high-energy collisions



Hagedorn statistic boost trap model (1968):

$$\rho(m,V_0)=\delta(m-m_0)+\sum_N \frac{1}{N!}\left[\frac{V_0}{(2\pi)^3}\right]^N \int \prod_{i=1}^{N}\left[dm_i\rho(m_i)d^3p_i\right]\delta^4(\sum_i p_i-p)$$

With the solution: $\rho(m,V_0)=\mathrm{const}.m^{-3}e^{m/T_H}$

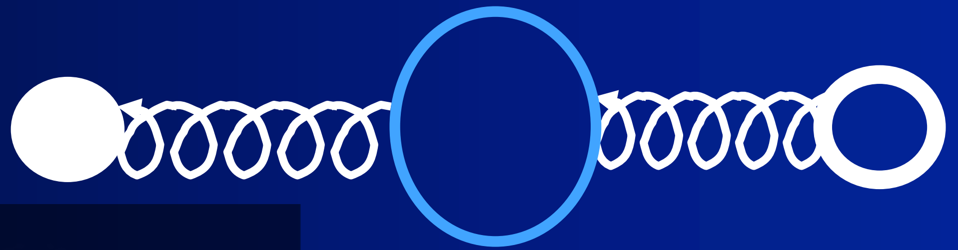
Partition function of the Hagedorn (hadron) resonance gas (HRG) model:

$$\ln\mathcal{Z}(T,V)=\frac{VT}{2\pi^2}\int dm m^2\rho(m)K_2(m/T)\approx V\left[\frac{T}{2\pi}\right]^{3/2}\int dm m^{-3/2}e^{-m\left[\frac{1}{T}-\frac{1}{T_H}\right]}\rightarrow\infty \quad \text{when} \;\; T>T_H$$

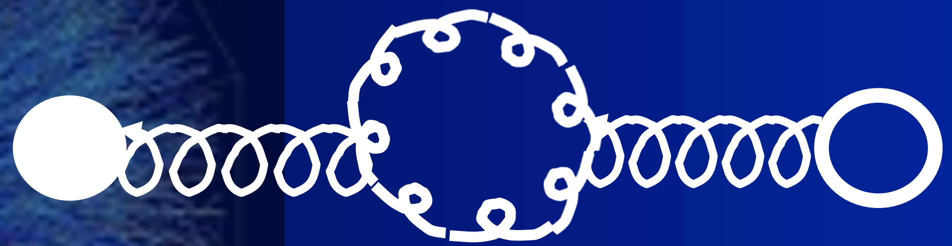
Asymptotic freedom & confinement in QCD

Gross & Wilczek; Politzer (1973)

$$\alpha_s(Q^2) = \frac{4\pi/(11-2n_f/3)}{\ln(Q^2/\Lambda^2_{\rm QCD})}$$

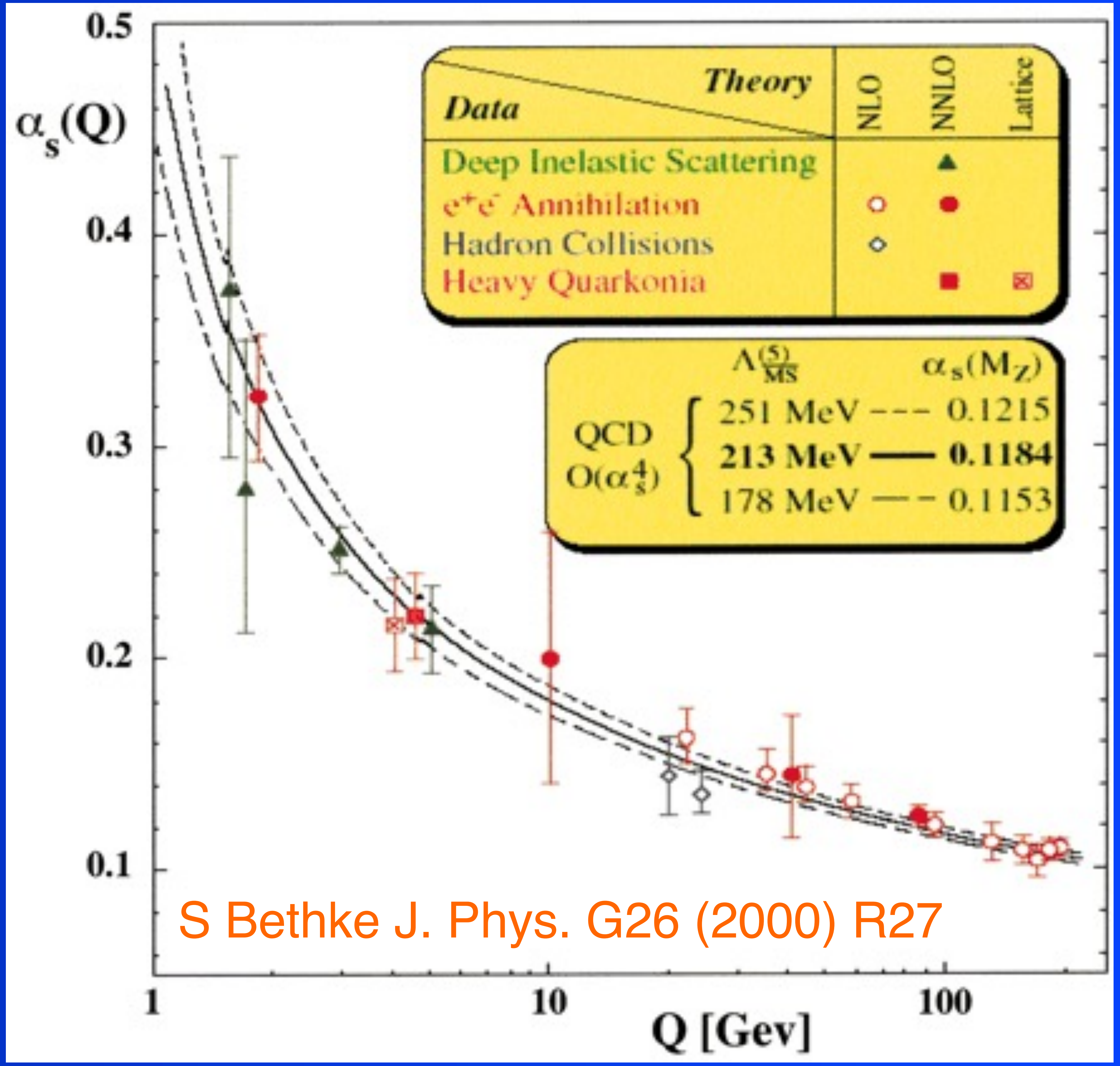


screening



anti-screening

← Confinement



S Bethke J. Phys. G26 (2000) R27

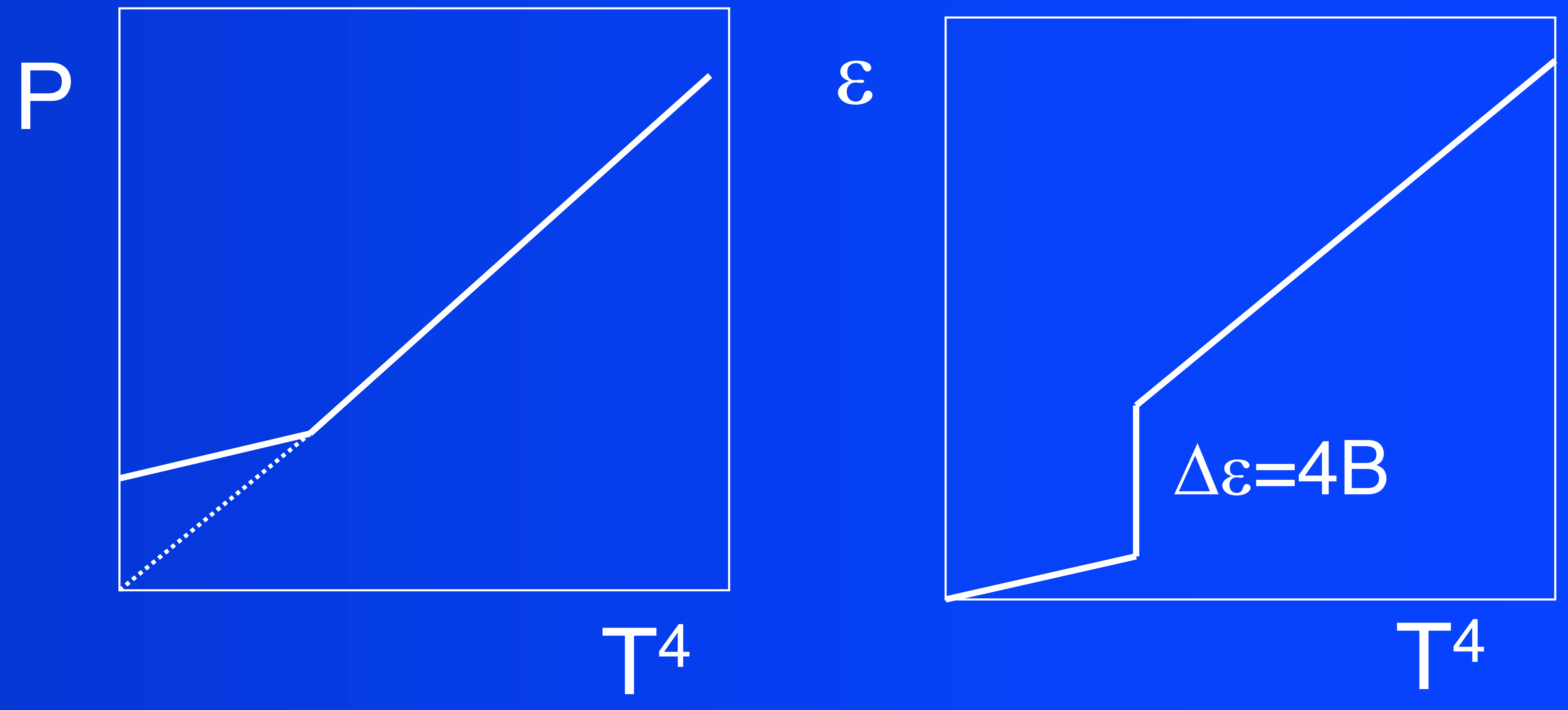
Asymptotically free →

Quark-gluon plasma in a MIT bag model

J Collins and M. Perry (1975) G. Baym and S Chin (1976), E. Shuryak (1978)

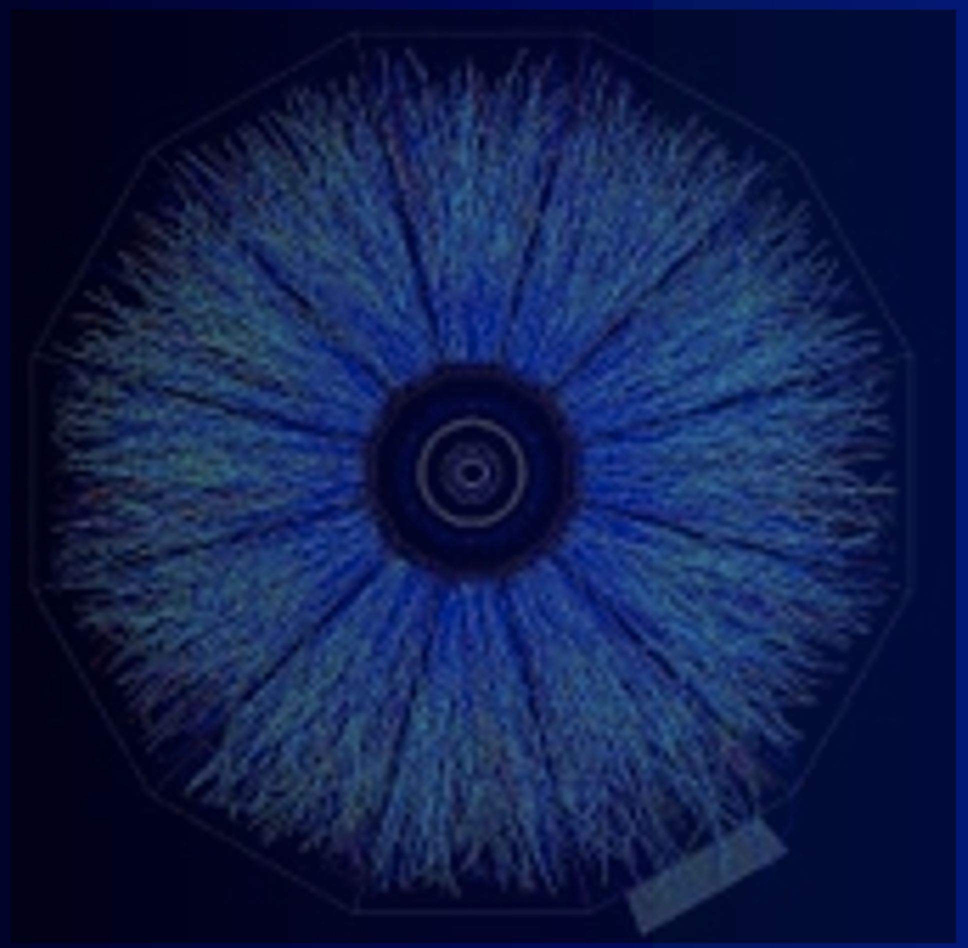
Ideal QGP: $\epsilon_{q,g} = 6n_f \frac{7\pi^2}{120} T^4 + 16 \frac{\pi^2}{30} T^4$

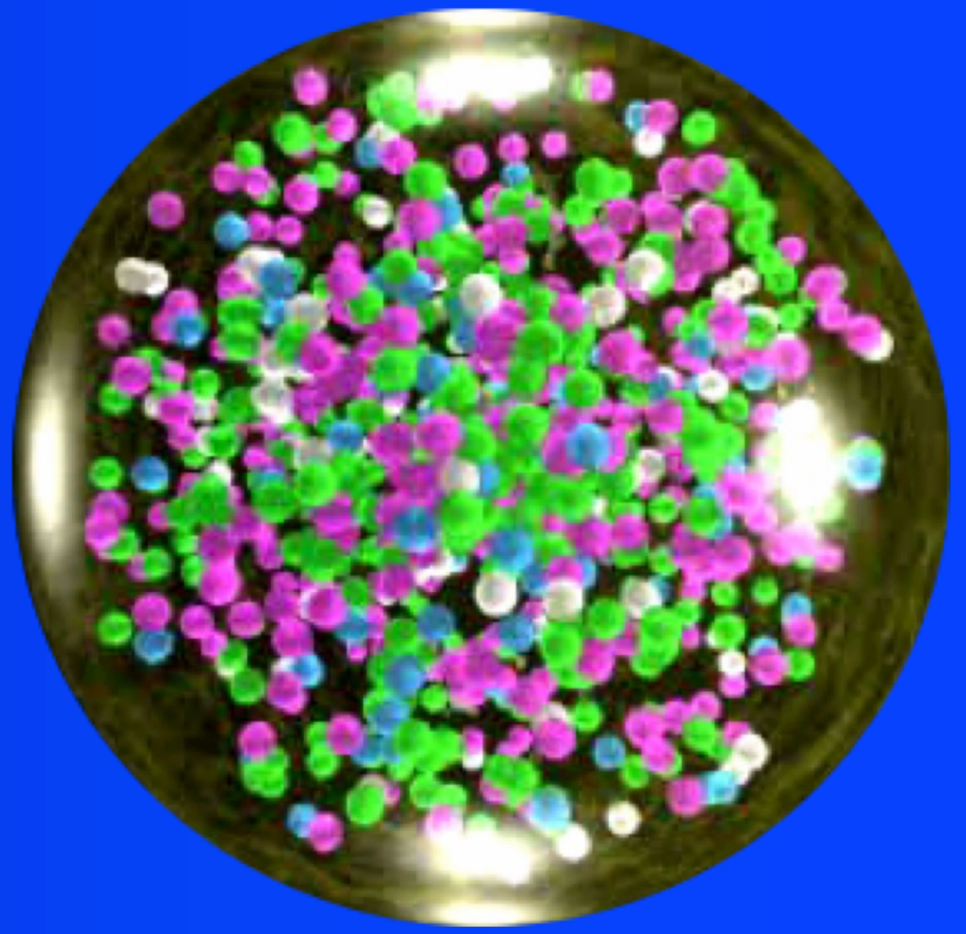
$$\epsilon = \epsilon_{q,g} + B \qquad P = \frac{1}{3}\epsilon_{q,g} - B$$



Massless π gas: $\epsilon_\pi = 3\frac{\pi^2}{30} T^4 \qquad P_\pi = \frac{1}{3}\epsilon_\pi$

$$P_\pi(T_c) = P_{q+g}(T_c) \longrightarrow T_c \approx 0.72 B^{1/4}$$

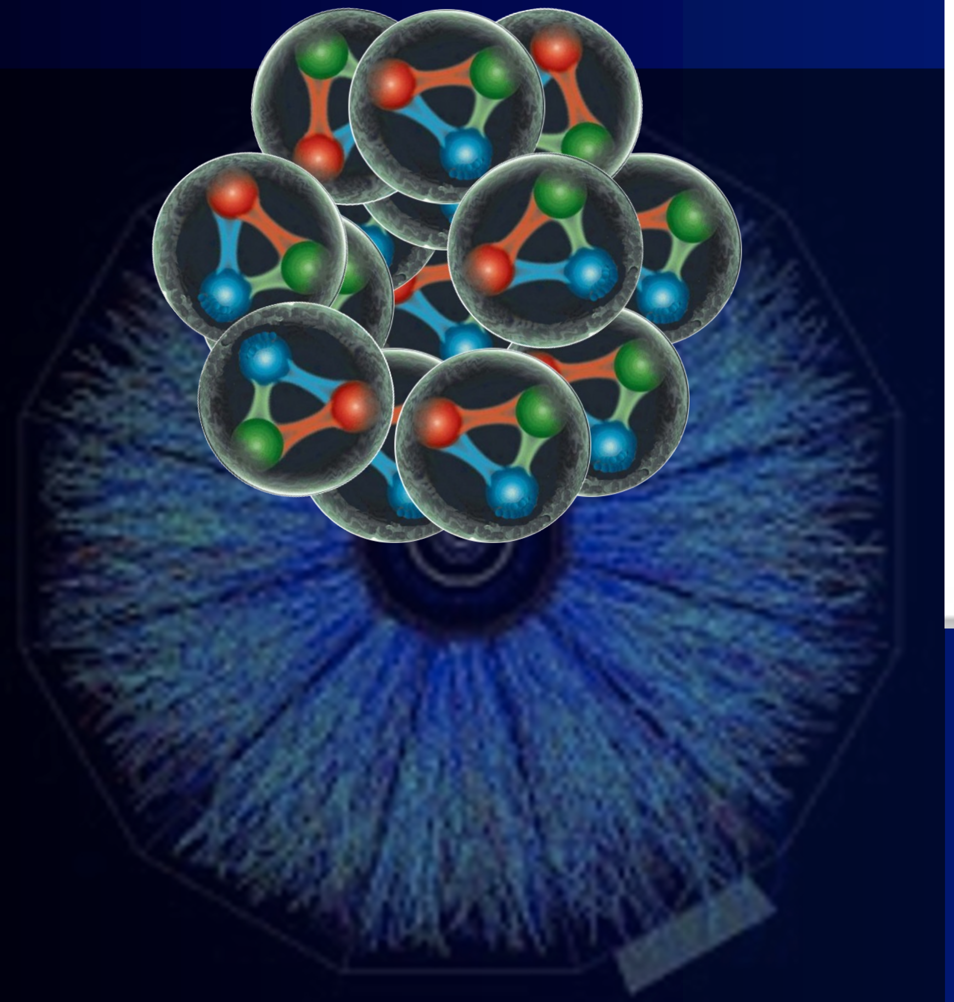


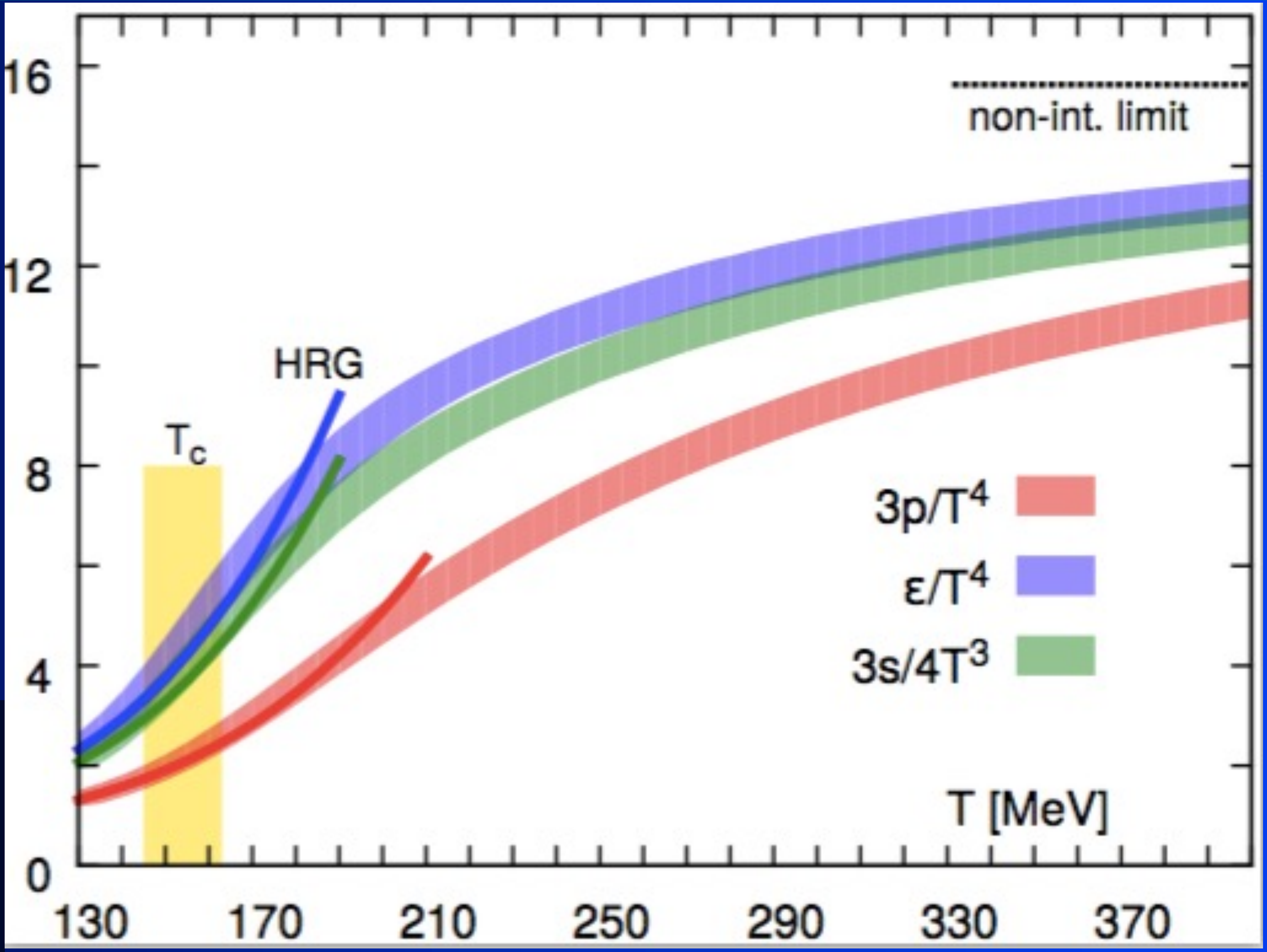


First-order phase transition

Phase transition in QCD

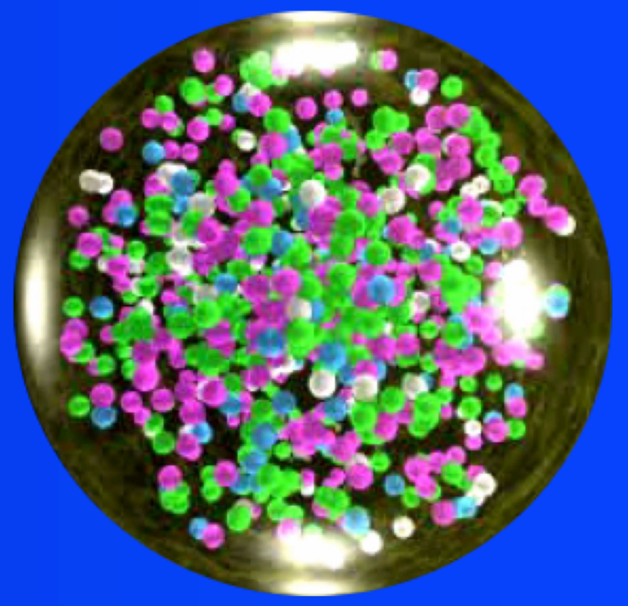
Normal
nuclear matter





F. Karch et al., 2014

$$\epsilon_{SB} = \left[6n_f \frac{7\pi^2}{120} + 16\frac{\pi^2}{30}\right] T^4$$

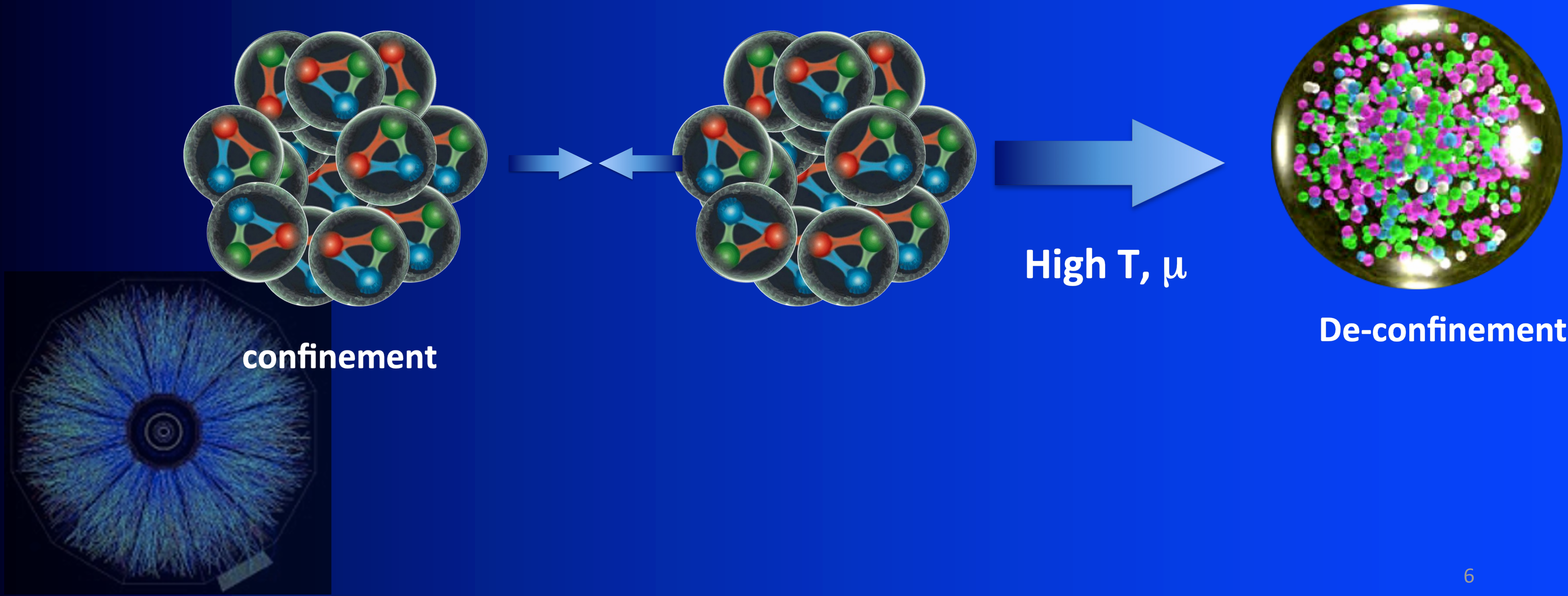


Quark gluon plasma

$$\langle \bar{\psi}\psi \rangle \to 0$$

$$\langle \alpha_s F^2 \rangle \to 0$$

New state of matter: quark-gluon plasma (QGP)



nucleus + nucleus

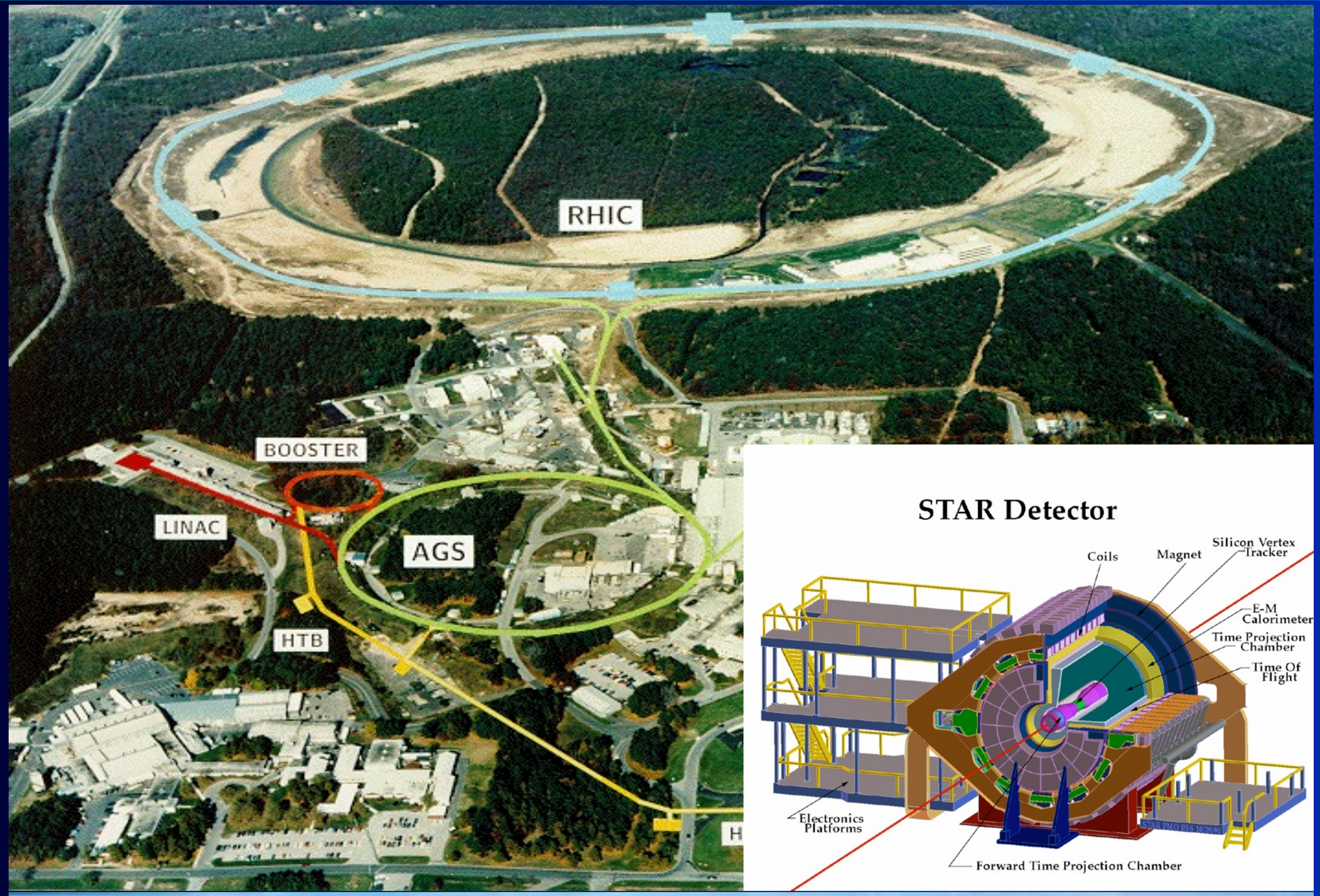
quark-gluon plasma (QGP)

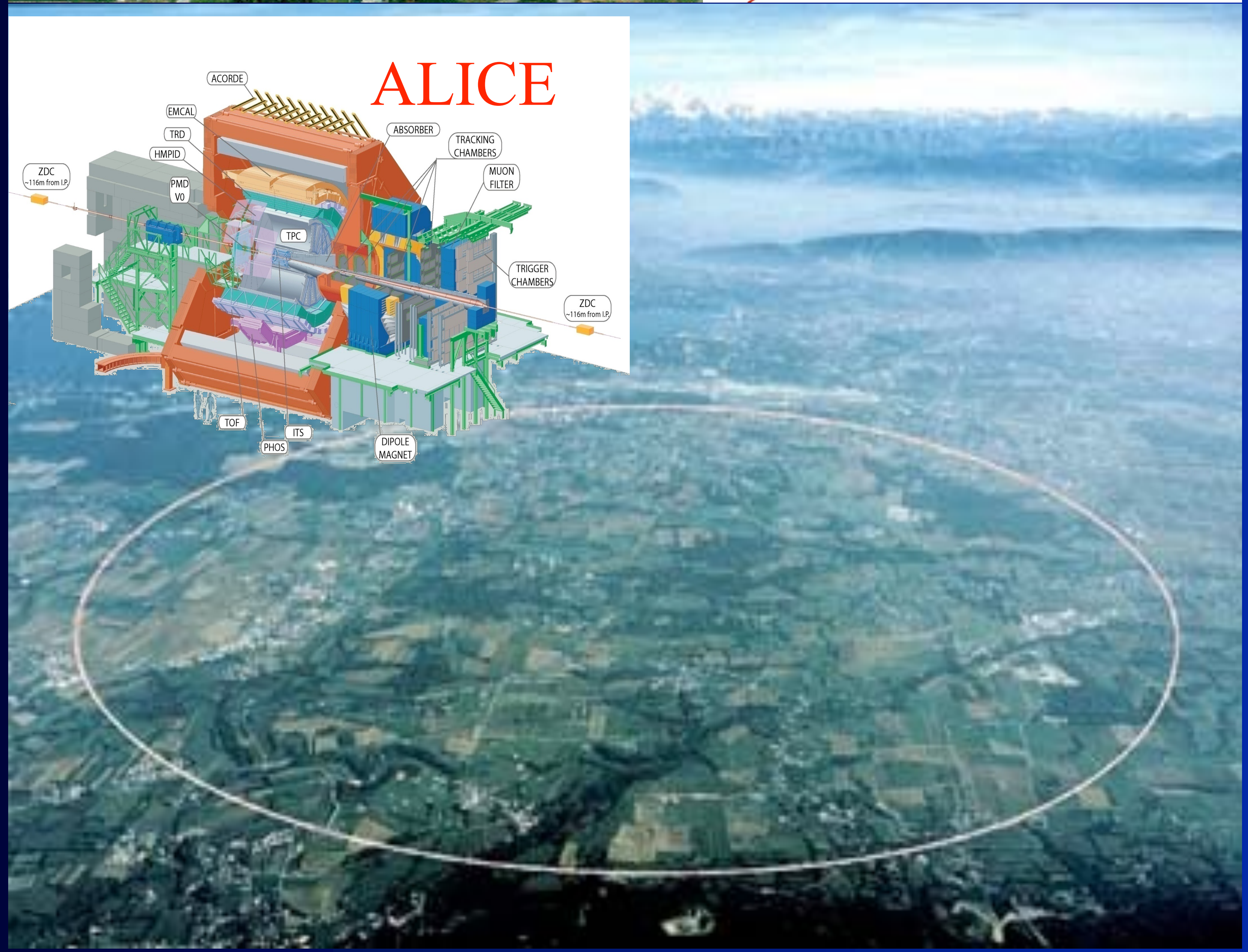
High T, μ

confinement

De-confinement

Relativistic Heavy-ion Collider/Large Hadron Collider





RHIC@BNL:

Proposed ~ 1980

Approved ~ 1990

Operation ~ 2000

Proposed ~ 1990
First results ~ 2010

Quark Matter Conferences

AND THE QUEST CONTINUES....

2023 · HOUSTON (USA)
2022 · CRACOW (POLAND)
2019 · WUHAN (CHINA)
2018 · VENICE (ITALY)
2017 · CHICAGO (USA)
2015 · KOBE (JAPAN)
2014 · DARMSTADT (GERMANY)
2012 · WASHINGTON (USA)
2011 · ANNECY (FRANCE)
2009 · KNOXVILLE (USA)
2008 · JAIPUR (INDIA)
2006 · SHANGHAI (CHINA)
2005 · BUDAPEST (HUNGARY)
2004 · OAKLAND (USA)
2002 · NANTES (FRANCE)
2001 · STONY BROOK (USA)
1999 · TORINO (ITALY)
1997 · TSUKUBA (JAPAN)
1996 · HEIDELBERG (GERMANY)
1995 · MONTEREY (USA)
1993 · BORLANGE (SWEDEN)
1991 · GATLINBURG (USA)
1990 · MENTON (FRANCE)
1988 · LENOX (USA)
1987 · NORDKIRCHEN (GERMANY)
1986 · PACIFIC GROVE (USA)
1984 · HELSINKI (FINLAND)
1983 · UPTON (USA)
1982 · BIELEFELD (GERMANY)
1980 · DARMSTADT (GERMANY)

Properties of QGP in A+A Collisions

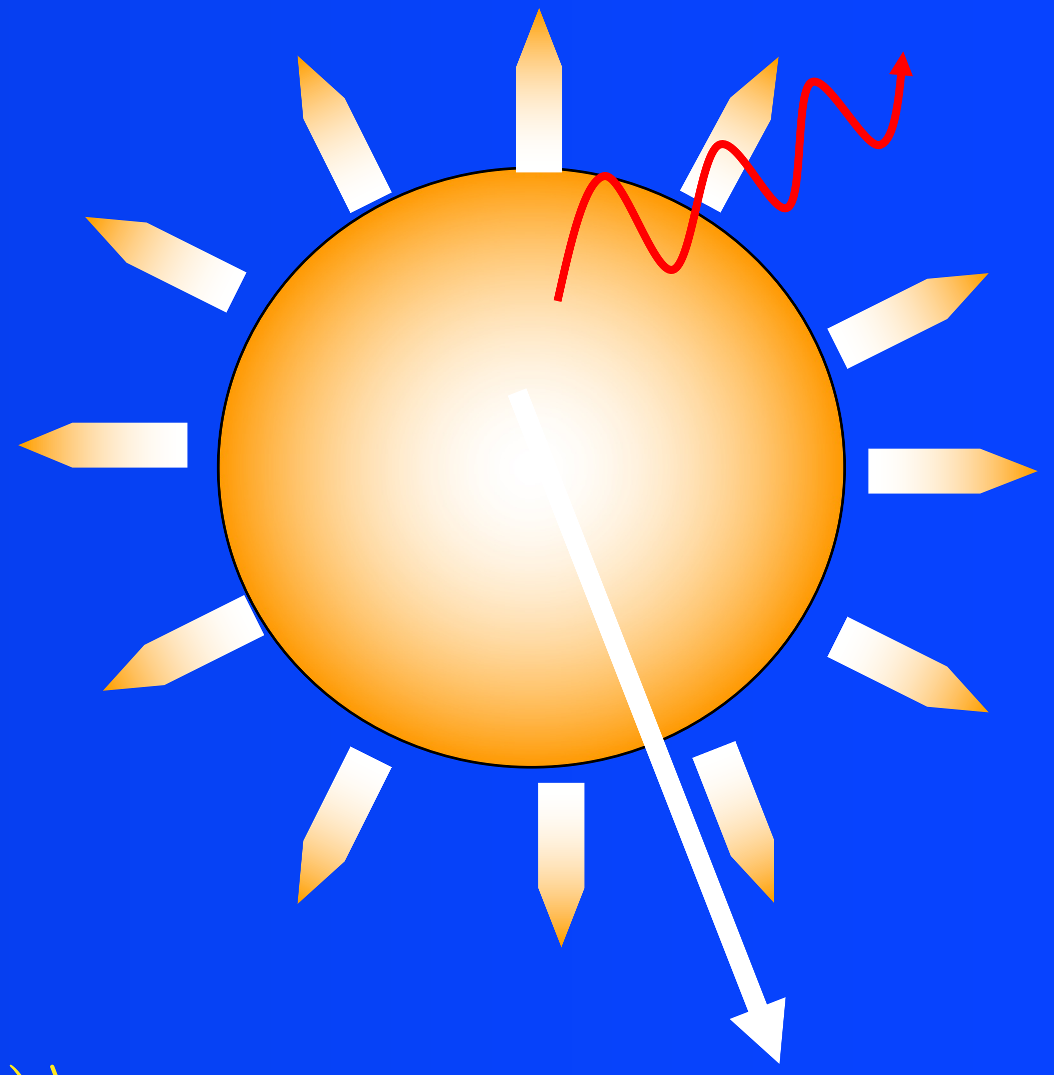
Multi-messenger study of dynamics and properties of QGP

- Soft probes: collective flow - bulk properties, EoS, transport properties, initial conditions

$$T_{\mu\nu}(x) : T(x), u(x)$$

$$T_{\mu\nu} \Longleftrightarrow \epsilon, P, s, c_s^2 = \partial p/\partial\epsilon$$

$$\eta = \lim_{\omega\to 0}\frac{1}{2\omega}\int dtdx e^{i\omega t}\langle[T_{xy}(0), T_{xy}(x)]\rangle$$



- EM Probes: EM emission – Temperature, EM response, medium modification of resonances

$$W_{\mu\nu}(q) = \int \frac{d^4x}{4\pi} e^{iq\cdot x}\langle j_\mu(0) j_\nu(x)\rangle$$

- Hard probes: Jet quenching, heavy quarks– Jet transport coefficients, diffusion constant

$$\hat{q} = \frac{4\pi^2\alpha_s C_R}{N_c^2 - 1}\int \frac{dy^-}{\pi}\langle F^{\sigma+}(0) F_\sigma^{+}(y)\rangle$$

Collective flow of QGP

- Hydrodynamics:

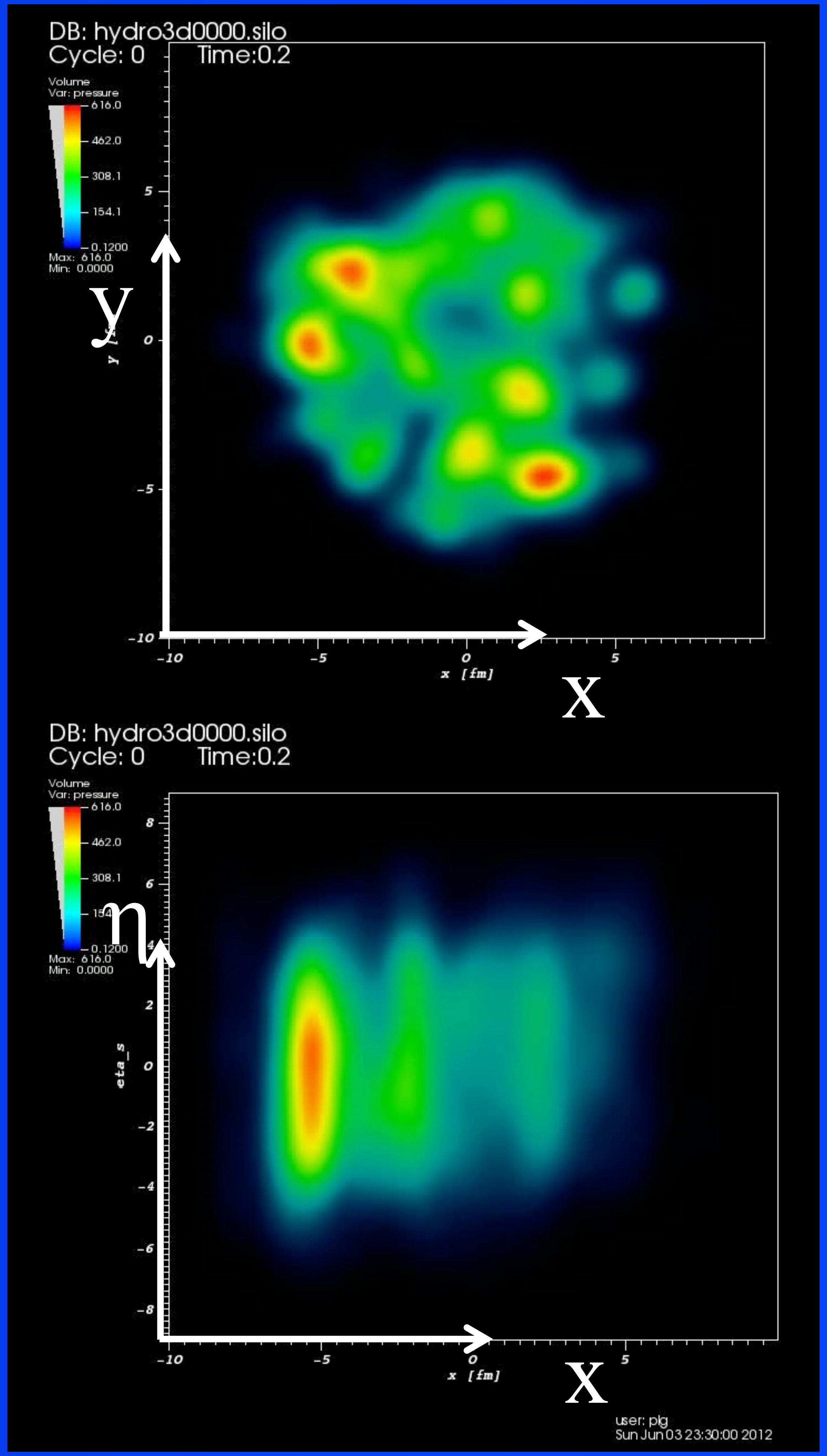
$$\partial_\mu T^{\mu\nu} = 0$$

$$T^{\mu\nu} = (\epsilon + P)u^\mu u^\nu - P g^{\mu\nu} + \Delta T^{\mu\nu}$$

$$\Delta T^{\mu\nu} = \eta(\Delta^\mu u^\nu + \Delta^\nu u^\mu) + (\frac{2}{3}\eta - \zeta)H^{\mu\nu}\partial_\rho u^\rho$$

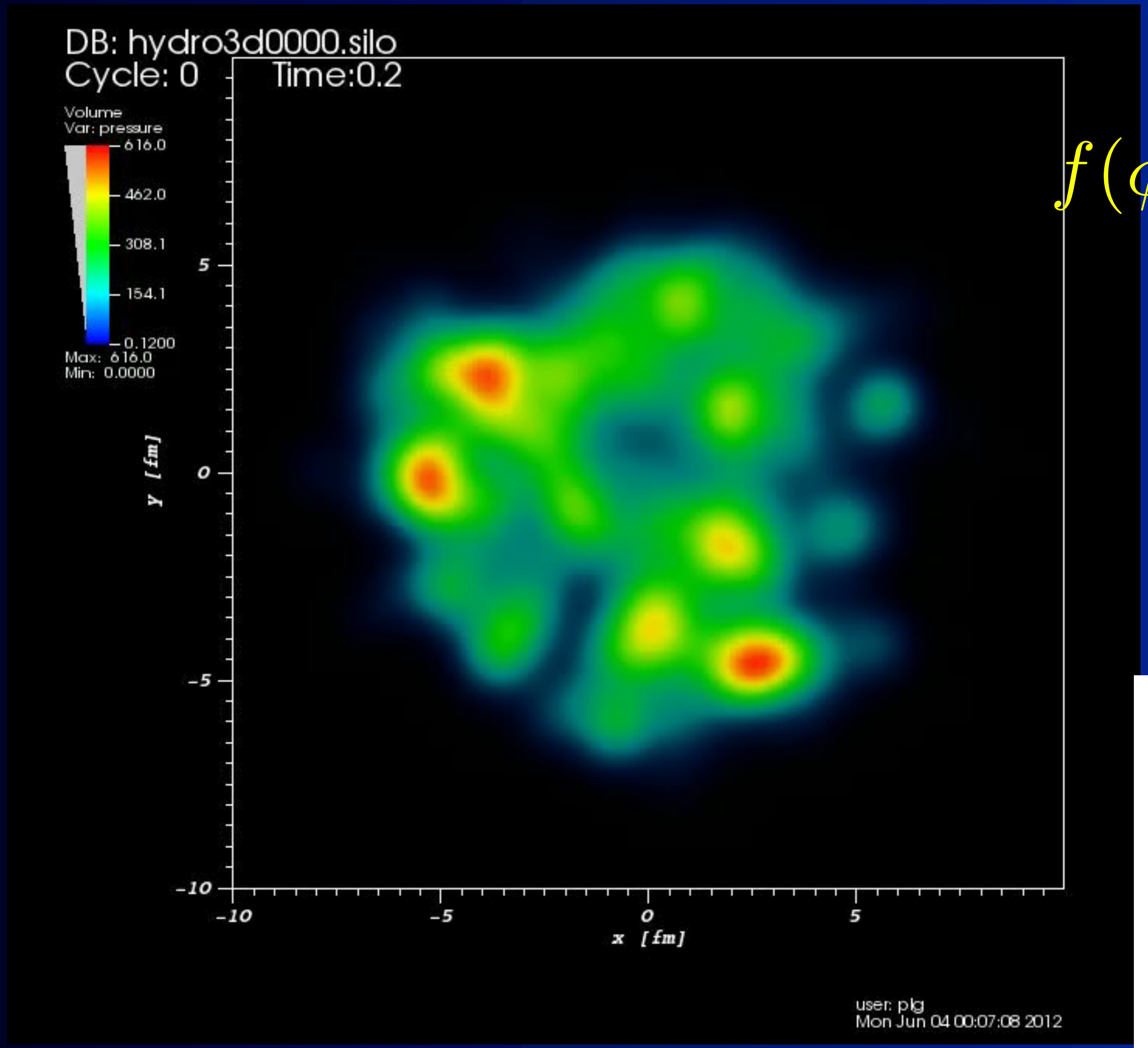
  – a low-momentum effective theory
  – Inputs from first principle QCD (lattice QCD)
  EoS p(ε), transport coefficients ξ(T), ζ(T) (??)
  – Initial condition: parton prod. & thermalization

Initial thermalization: hydrodynamic attractors, hydrodynamization, anisotropic hydrodynamics, kinetic theory, etc



(3+1)D viscous hydro (CLVisc) with AMPT initial condition

“CMB” of the little bang: Anisotropic flow of QGP



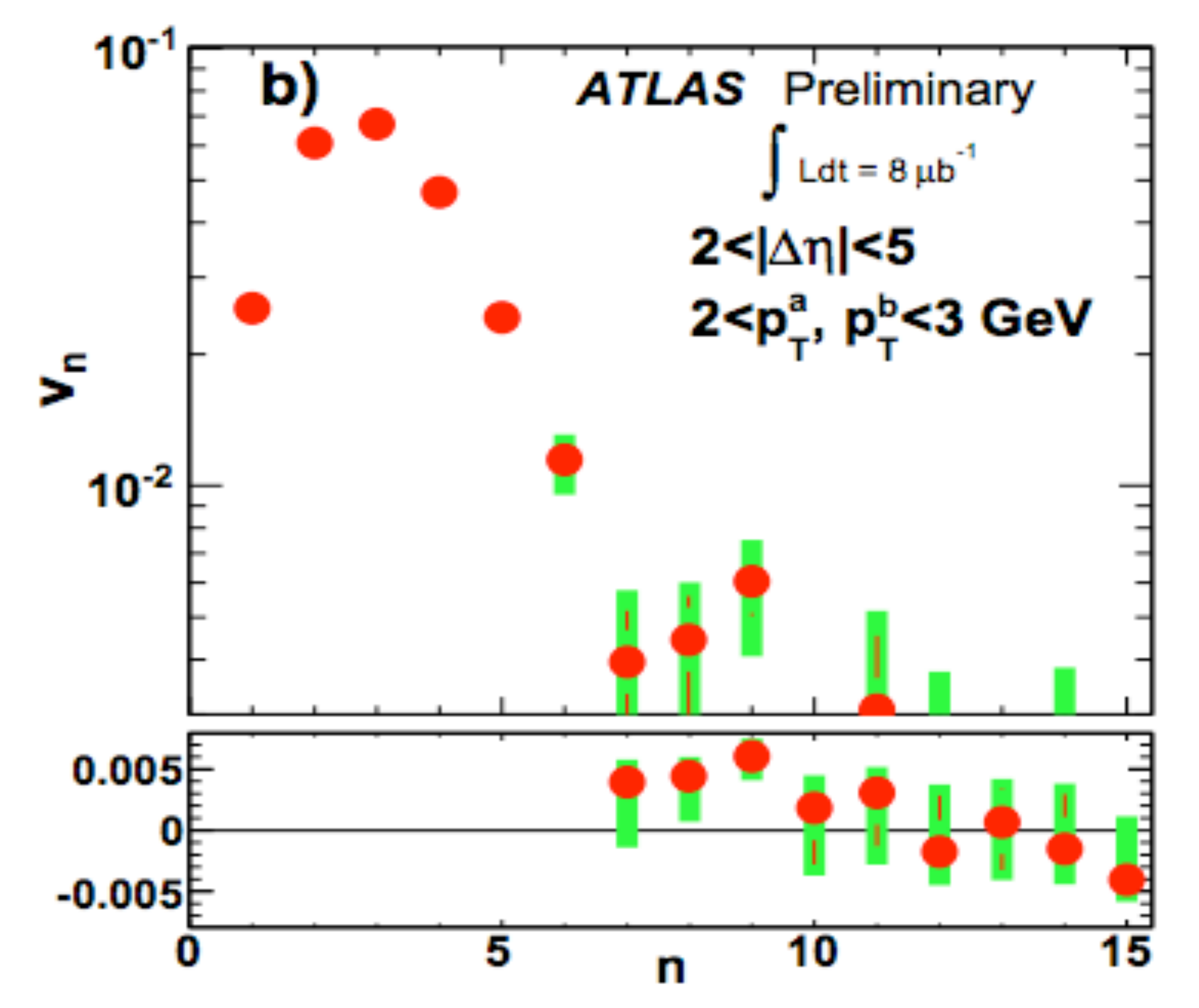
$$f(\phi) = f_0 \left[1 + 2 \sum_{n=1} v_n \cos n(\phi - \Psi_n) \right]$$

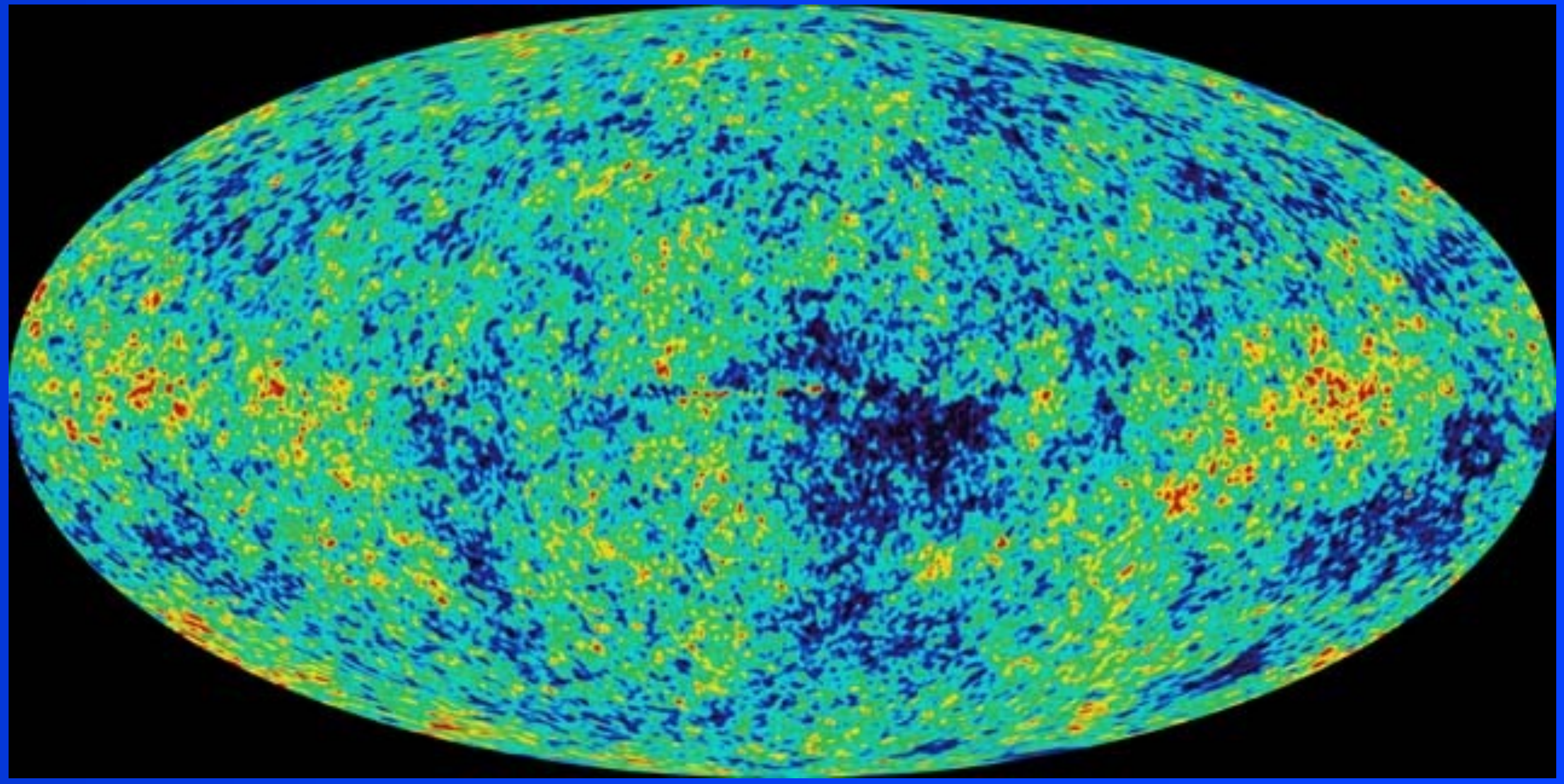
$$\Psi_n = \frac{1}{n} \arctan \frac{\langle p_T \sin(n\phi) \rangle}{\langle p_T \cos(n\phi) \rangle}$$

$V_n \sim \kappa \, \varepsilon_n$

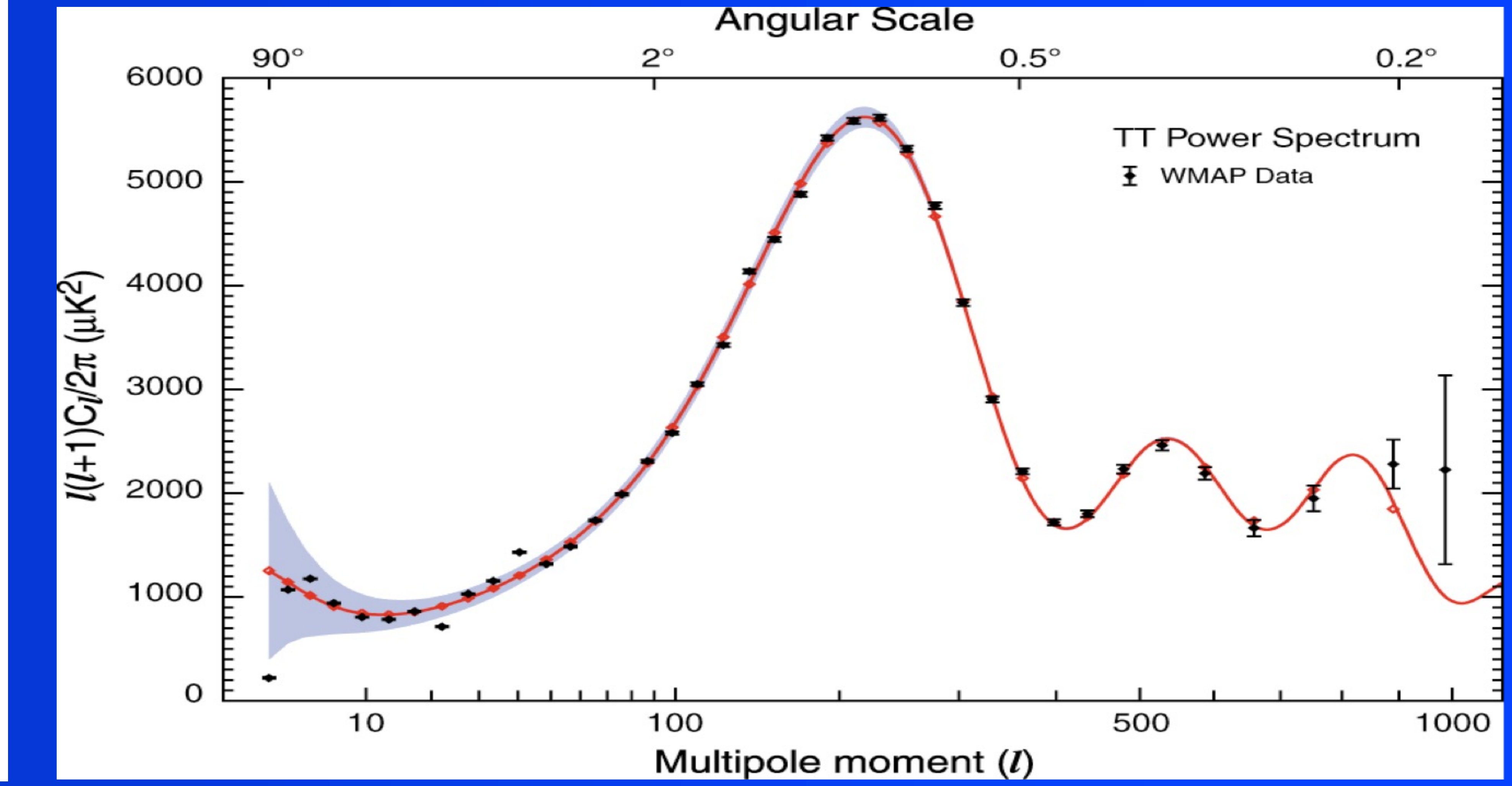
ε_n: Initial geometrical anisotropy

κ: Encodes transport coefficients





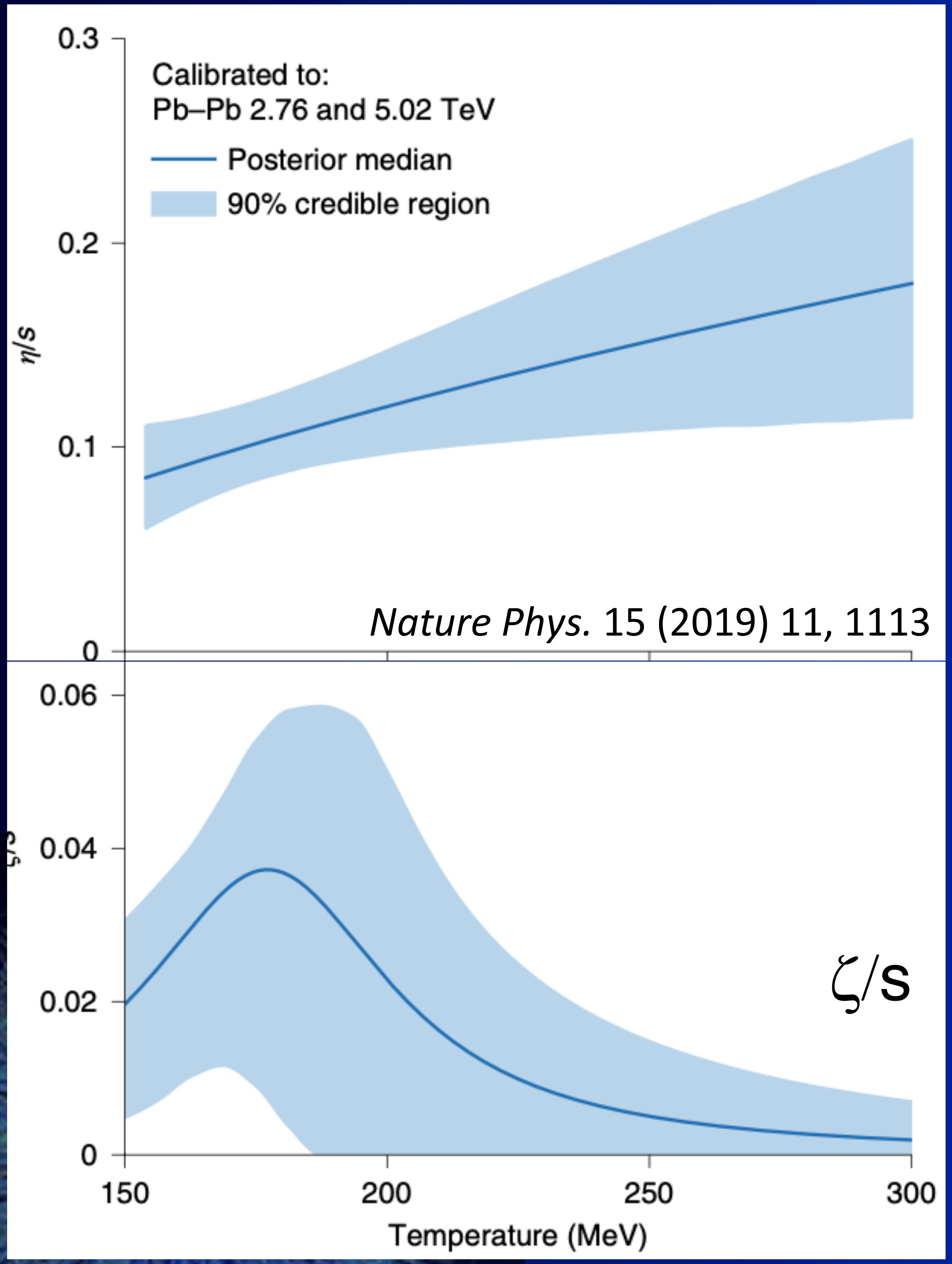
Anisotropy in CMB



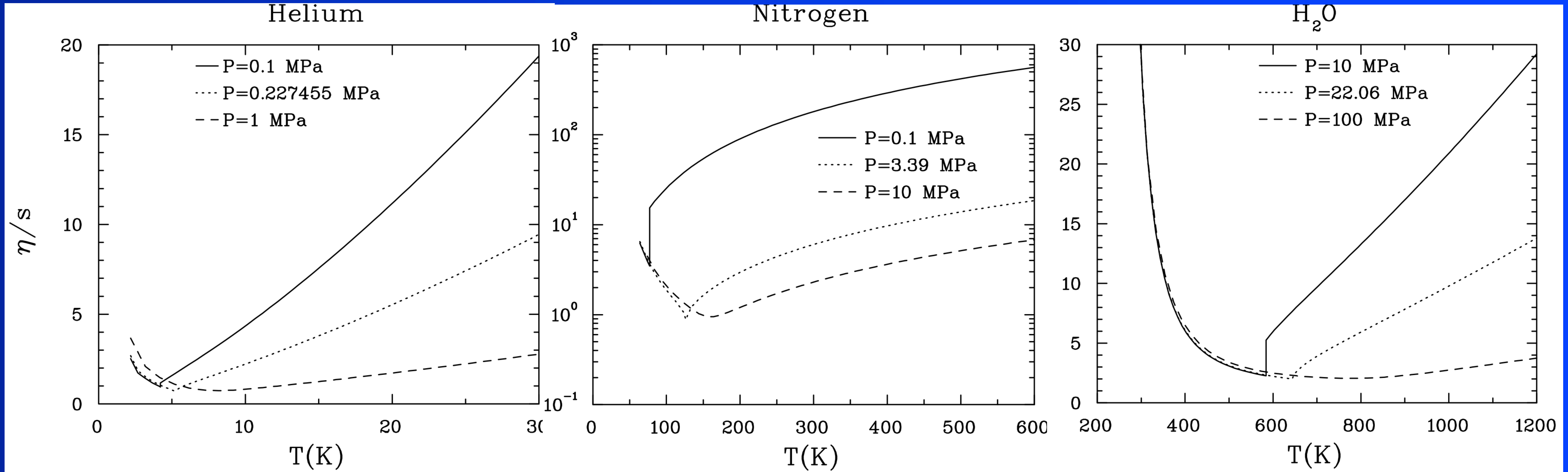
QGP: the most perfect fluid

η/s Bayesian inference:

$$\mathcal{P}^{(i)}(\boldsymbol{x}|\mathbf{y}_{\rm exp}) = \frac{\mathcal{P}^{(i)}(\mathbf{y}_{\rm exp}|\boldsymbol{x})\mathcal{P}(\boldsymbol{x})}{\mathcal{P}^{(i)}(\mathbf{y}_{\rm exp})}$$



Nature Phys. 15 (2019) 11, 1113



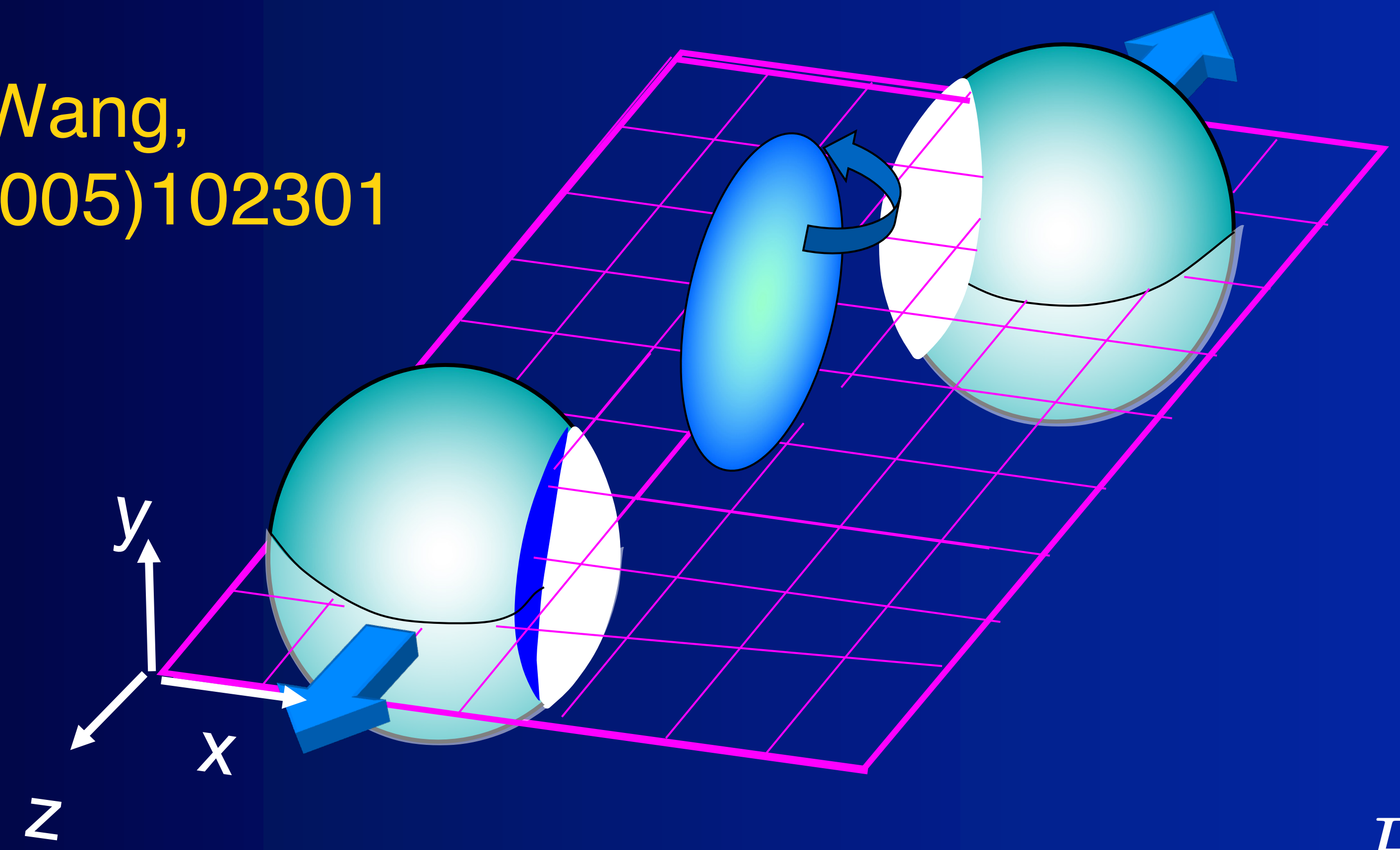
Csernai, Kapusta and McLerran, PRL 97, 152303 (2006)

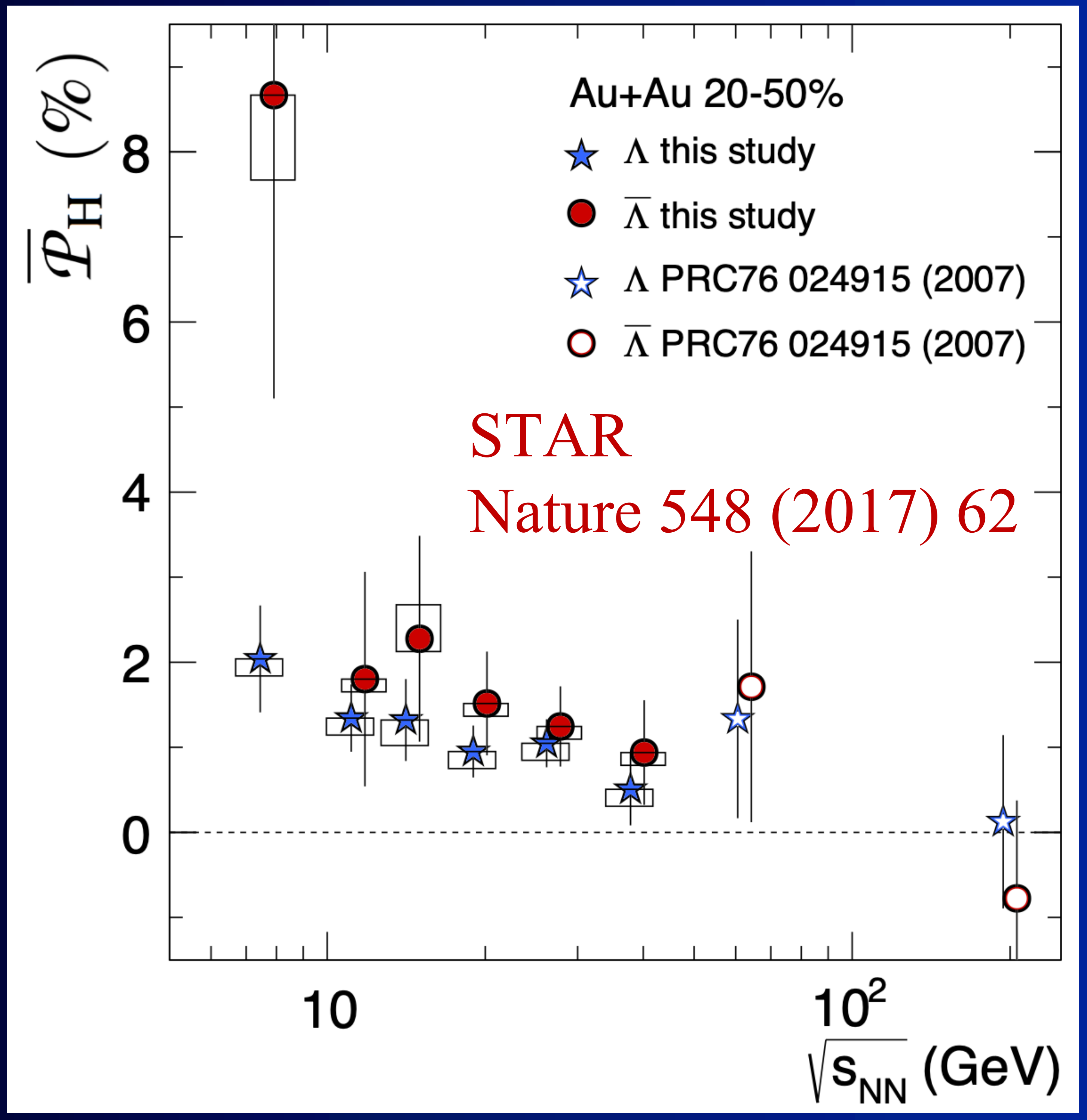
AdS/CFT limit: 1/4π ~ 0.08

Kovtun, Son and Starinets, PRL 94, 111601 (2005)

Spin dynamics in heavy-ion collisions

Liang & Wang,
PRL94(2005)102301





STAR
Nature 548 (2017) 62

Single scattering:

$$P_q \approx -\pi \frac{\mu p}{m_q^2} \sim \frac{\omega}{T}$$

In equilibrium:

$$P^{\mu}_{q(\bar{q})} \approx \frac{1}{4m_q}\epsilon^{\mu\nu\rho\sigma}\left[\omega_{\rho\sigma} \pm \frac{e_q}{(u\cdot p)T}F_{\rho\sigma}\right]p_{\nu}$$

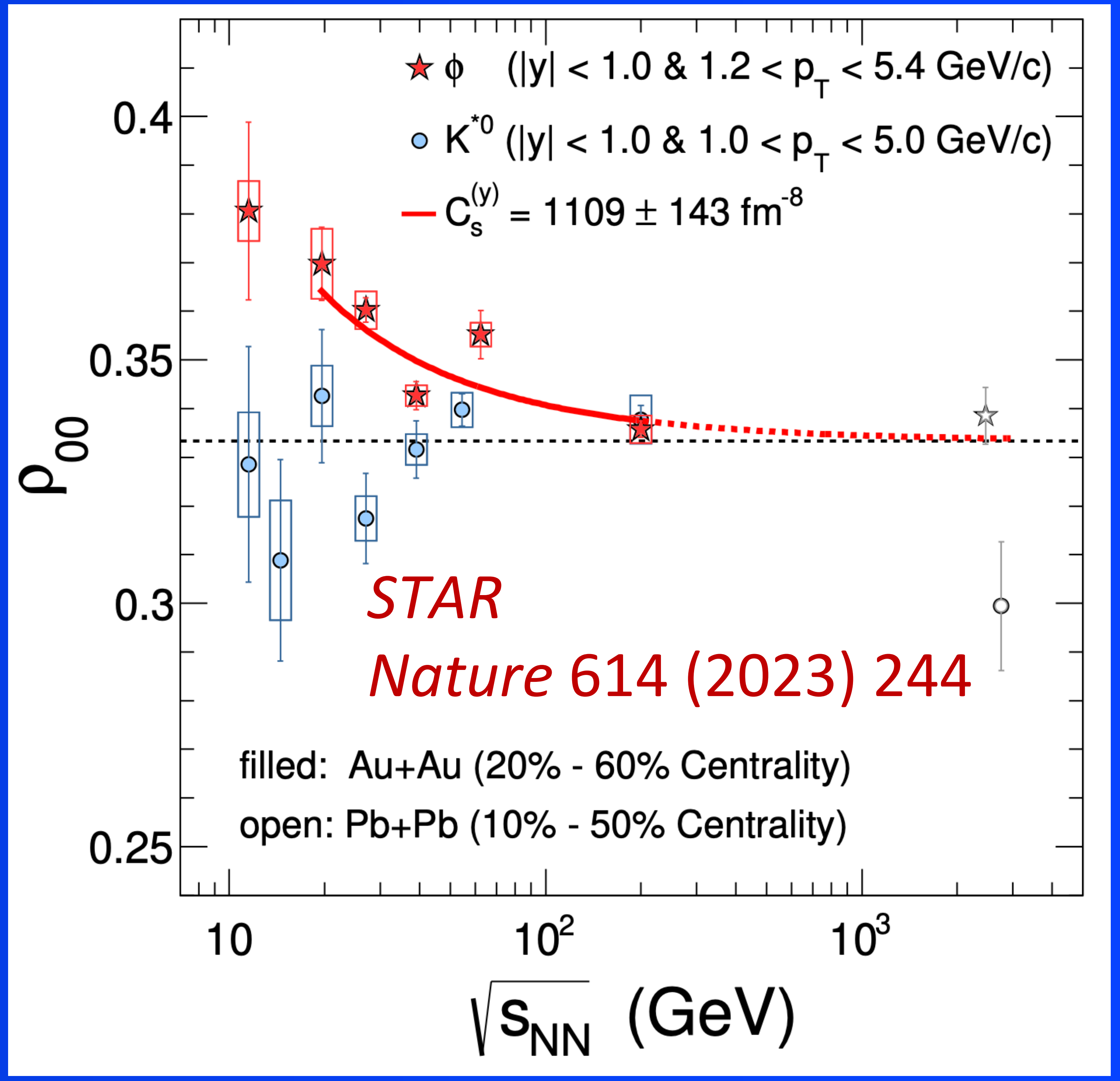
F. Becattini et al, Annsla Phys. 338, 32 (2013)

$P_{\Lambda}=P_s$ (quark model)

$\omega \sim 10^{19}$ s^{-1}

The most vortical fluid!

Vector meson spin alignment



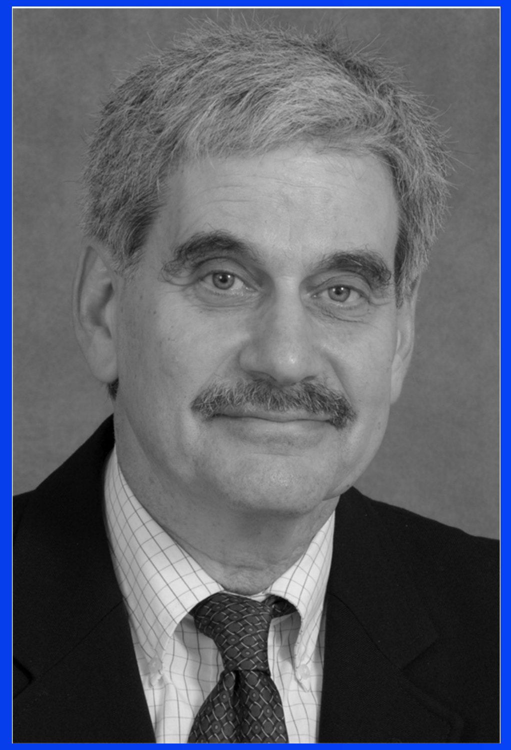
STAR
Nature 614 (2023) 244

$$\rho_{00} \approx \frac{1}{3} + \frac{g_{\phi}^2}{m_{\phi}^2 T_{\mathrm{eff}}^2}(C_1 B^2\phi + C_2 E_{\phi}^2)$$

Sheng, Oliva, Liang, Wang & XNW,
PRL. 131 (2023) 4, 042304

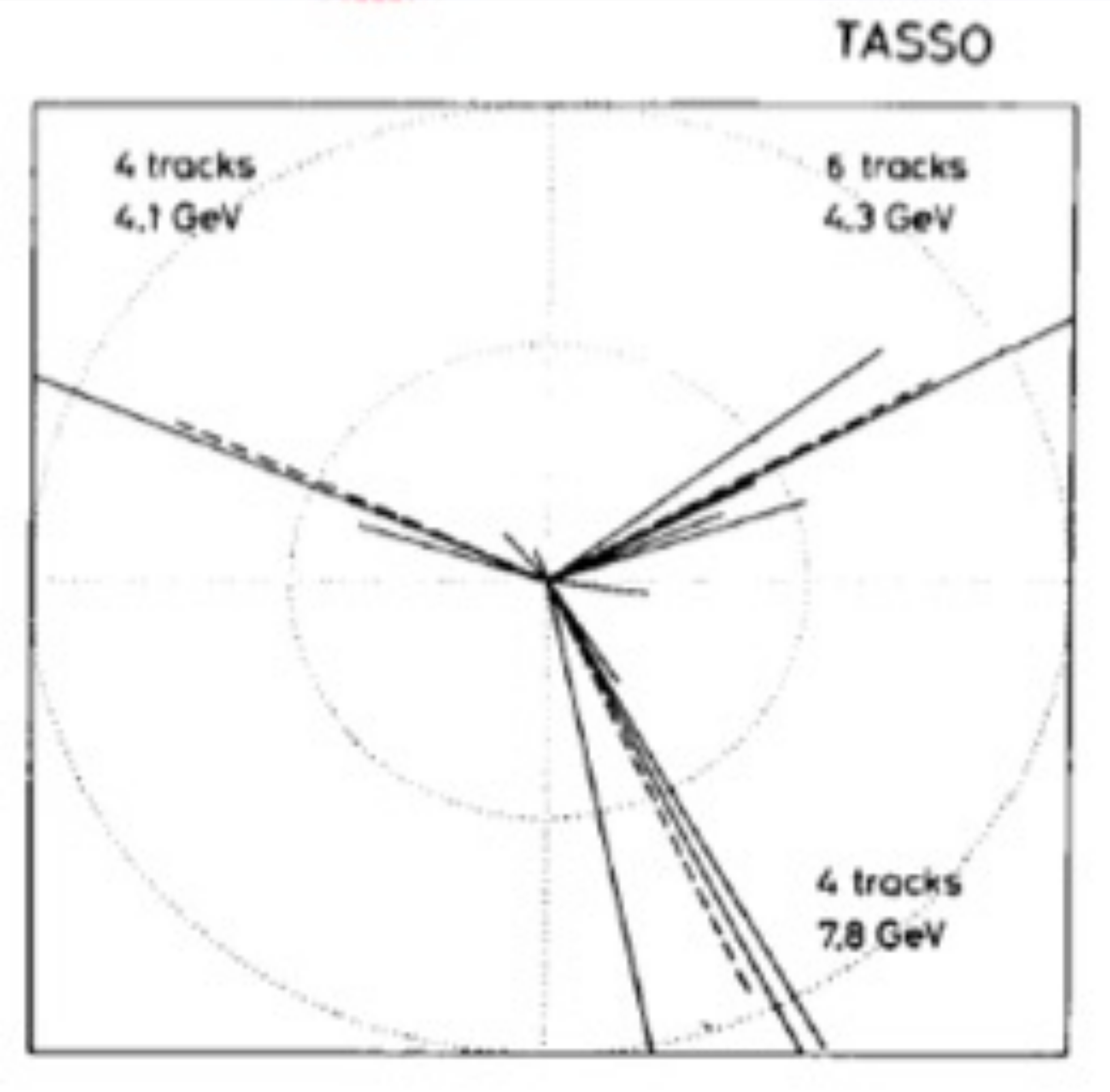
Jets in high-energy collisions

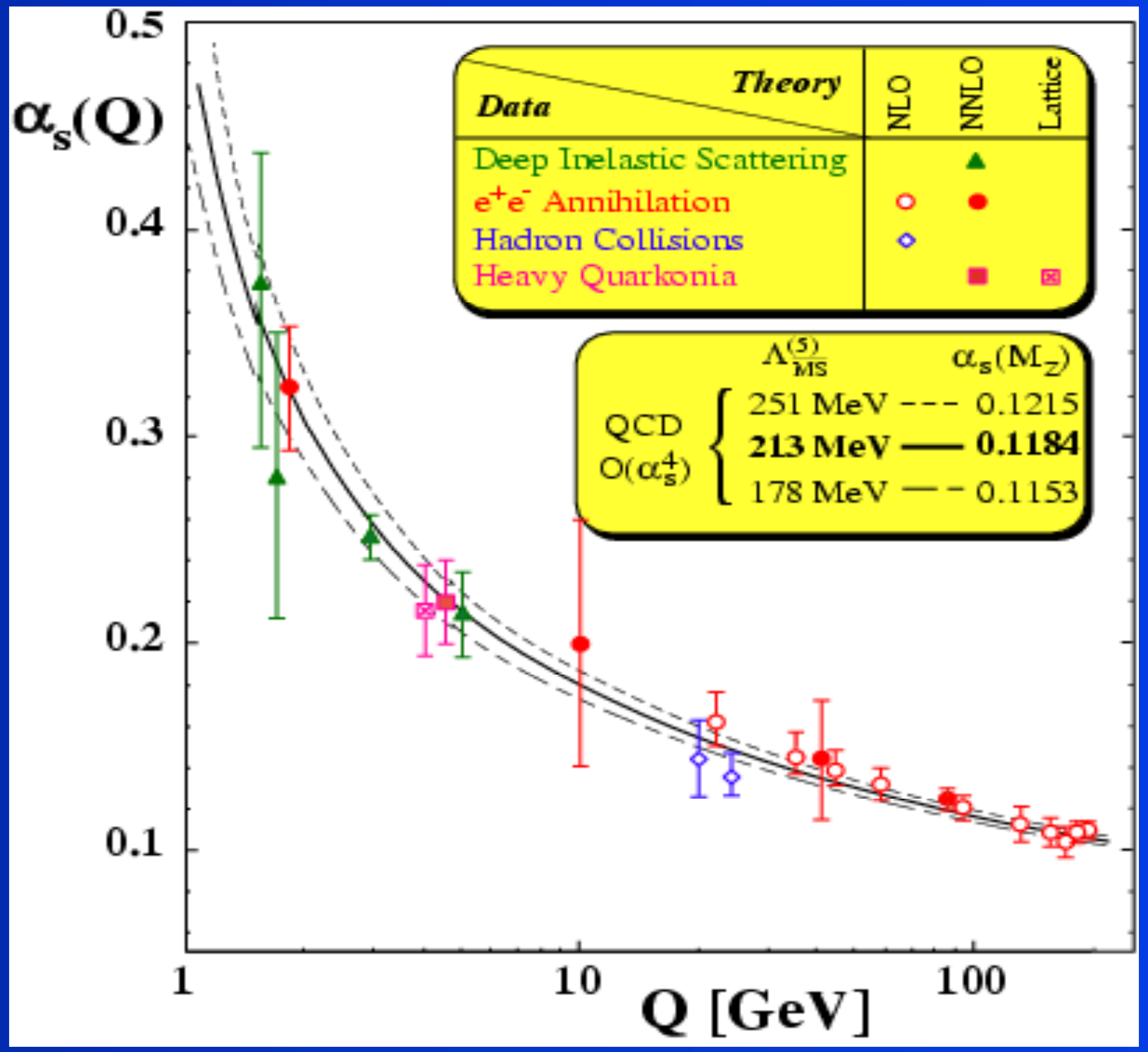
- Partons in QCD: Ellis, Gaillard & Ross (1976), Feymann & Fields, Georgi & Machacek (1977)
- Jets in QCD: Sterman & Weinberg (1977)



Sterman

Weinberg

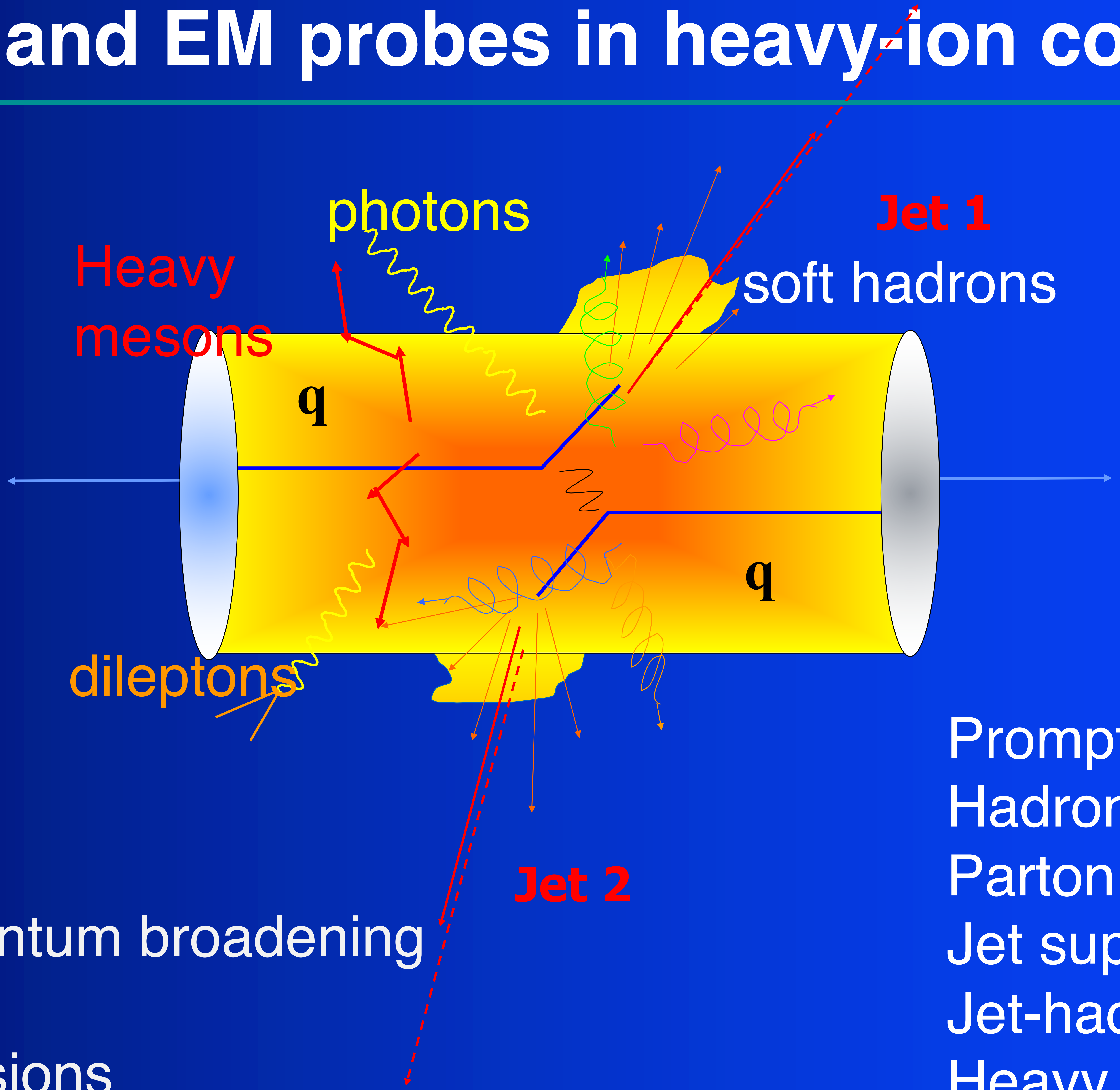




S Bethke J. Phys. G26 (2000) R27

Powerful tools for studying QGP in heavy-ion experoiments

Hard and EM probes in heavy-ion collisions

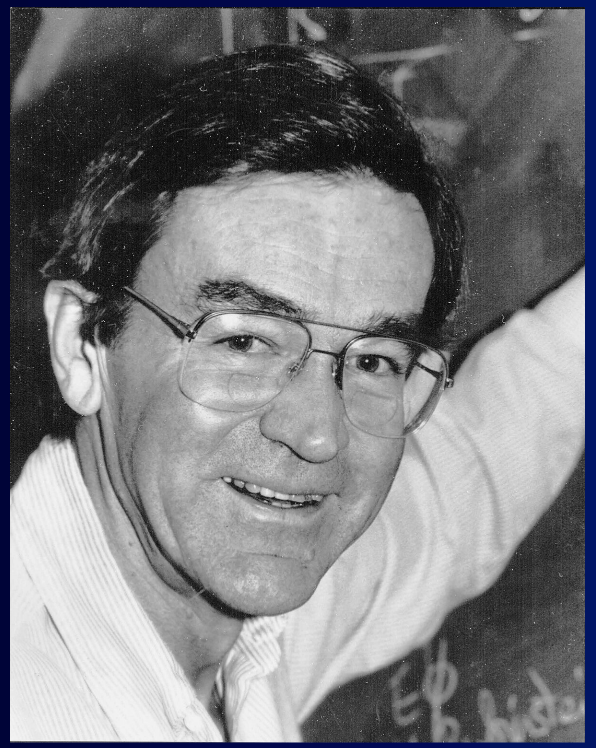


EM response
Multiple scattering
Transverse momentum broadening
Medium response
Heavy quark diffusions

Prompt γ emission
Hadron properties in medium
Parton energy loss
Jet suppression
Jet-hadron correlation
Heavy meson modification

Parton propagation in QCD medium

Elastic parton energy loss:

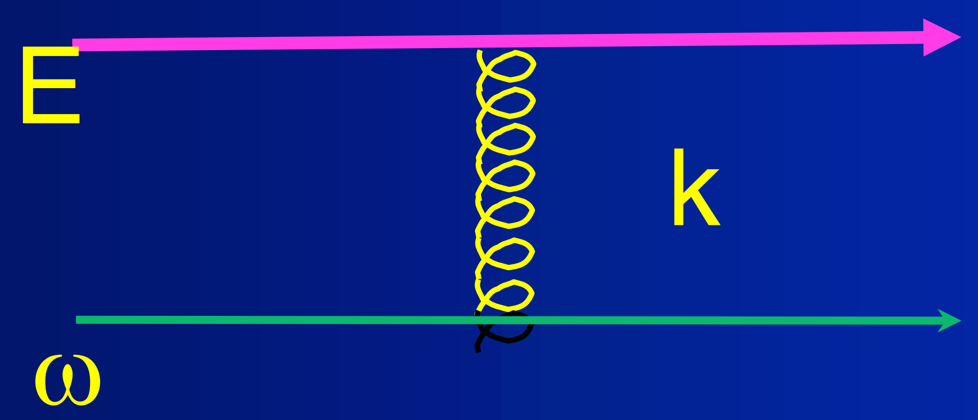


Bjorken (1982)

Thoma & Gyulassy (1990)

$$\frac{dE^a_{el}}{dx} = \sum_b \int d\omega f_b(\omega/T) \int dk_\perp^2 \frac{d\sigma_{ab}}{dk_\perp^2} k_0$$

$k_0 \approx k_\perp^2/2\omega$



$$\approx C_a \frac{3\pi}{2} \alpha_s^2 T^2 \log \frac{2.6ET}{4\mu_D^2}$$

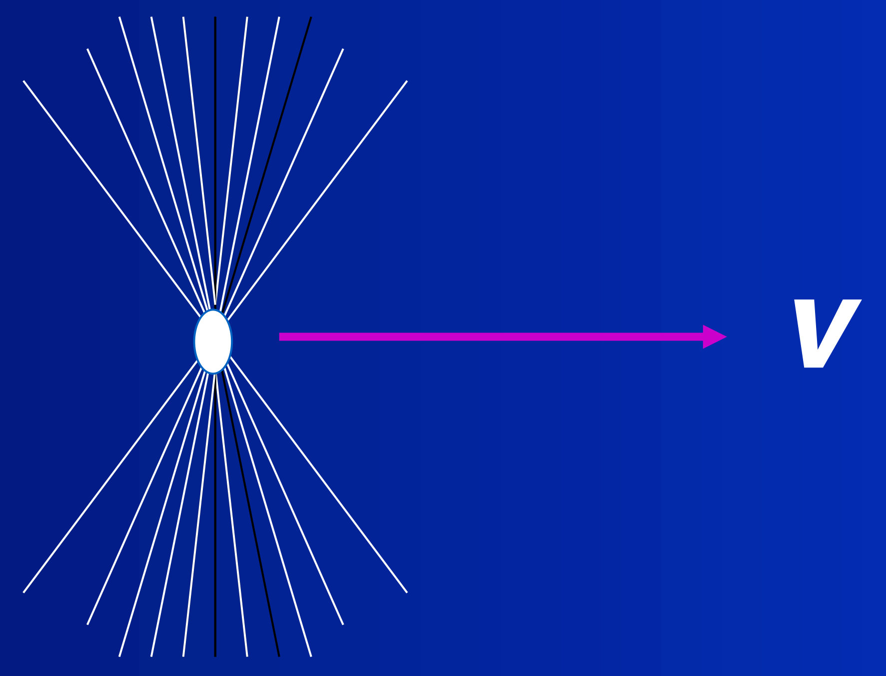
Inelastic parton energy loss:

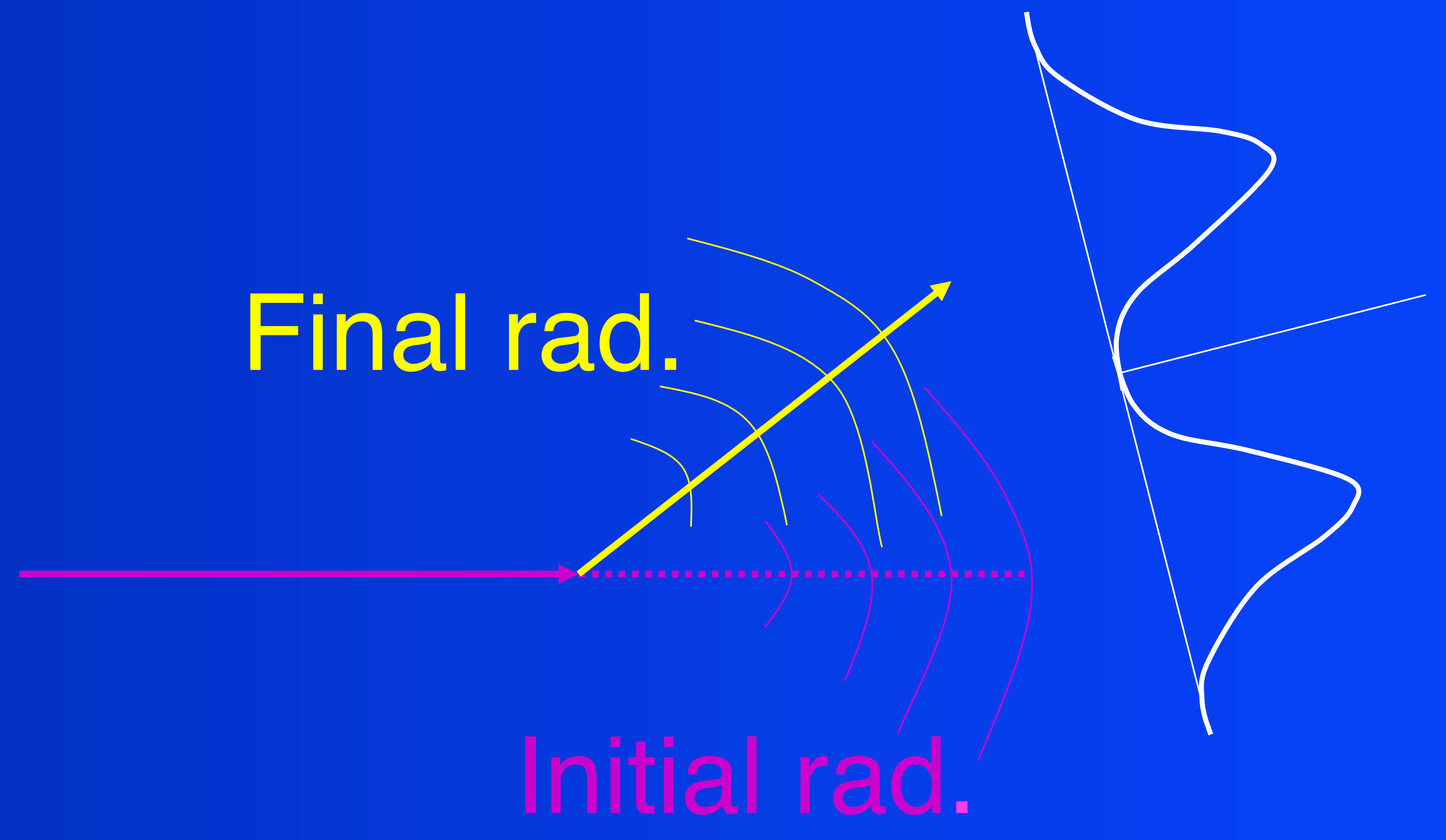
Gyulassy & XNW (1994),

BDMPS (1995), Zakharov (1996)

EM Radiation: Single scattering

EM field carried by a fast charge particle before and after scattering





EM Radiation by scattering: Interference between initial and final state radiation

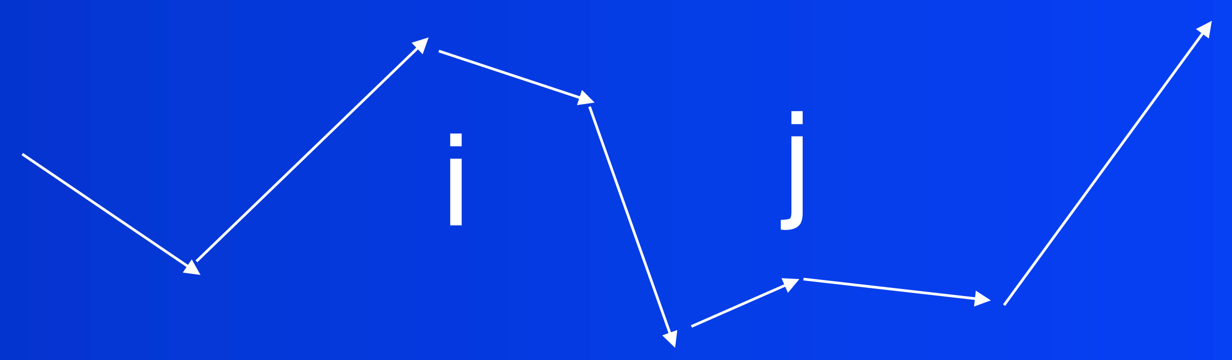
$$\omega \frac{d^2 I}{d\omega\, d\Omega} = \frac{e^2}{4\pi^2}\left|\frac{\vec{k}\times\vec{v}_i}{\vec{k}\cdot\vec{v}_i-\omega} - \frac{\vec{k}\times\vec{v}_f}{\vec{k}\cdot\vec{v}_f-\omega}\right|^2$$

Bethe Heitler

$$\omega\frac{dI}{d\omega}\approx\frac{2\alpha}{\pi}\left[\ln\frac{2E^2(1-\vec{v}_i\cdot\vec{v}_f)}{m^2}-1\right]$$

EM Radiation: multiple scattering

Classical radiation of a point charge **(Jackson, p671)**



$$\omega \frac{d^2 I}{d\omega\, d\Omega} = \frac{e^2}{4\pi^2} \left| \sum_i \left(\frac{\vec{k}\times\vec{v}_i}{\vec{k}\cdot\vec{v}_i - \omega} - \frac{\vec{k}\times\vec{v}_{i+1}}{\vec{k}\cdot\vec{v}_{i+1} - \omega} \right) e^{i(\omega t_i - \vec{k}\cdot\vec{r}_i)} \right|^2$$

Lorentz Invariant form:

$$\omega \frac{d^3 I}{d^3 k} = \frac{e^2}{2(2\pi)^3} \sum_\lambda \left| \varepsilon_\lambda(k) \cdot \sum_i J_i(k) e^{ik\cdot x_i} \right|^2$$

$$J_i^\mu(k) = \frac{p_{i-1}}{k\cdot p_{i-1}} - \frac{p_i}{k\cdot p_i}$$

EM current of a charged through a scattering

Two Limits: (In)coherent radiation

$$\exp[ik\cdot(x_i-x_j)]=\exp[i\Delta x_{ij}/\tau_f]$$

Photon formation time: $\tau_f=\frac{1}{\omega(1-\cos\theta)}\approx\frac{2}{\omega\theta^2}$

Coherent Limit: $\tau_f\gg\Delta x_{ij}$ single coherent scattering

$$J\mu(k)=\sum_i\left(\frac{p_{i-1}}{k\cdot p_{i-1}}-\frac{p_i}{k\cdot p_i}\right)e^{ik\cdot x_i}\approx\frac{p_1}{k\cdot p_1}-\frac{p_N}{k\cdot p_N}$$

Incoherent Bethe Heitler Limit: $\tau_f\ll\Delta x_{ij}$

$$\omega\frac{d^3I}{d^3k}=\frac{e^2}{4\pi^2}\left[\sum_{i,\lambda}|\varepsilon_\lambda\cdot J_i|^2+2Re\,\xcancel{\sum_{i,\lambda}\sum_{j>i,\lambda'}(\varepsilon_\lambda\cdot J_i)(\varepsilon_{\lambda'}\cdot J_j)}e^{ik\cdot(x_i-x_j)}\right]$$

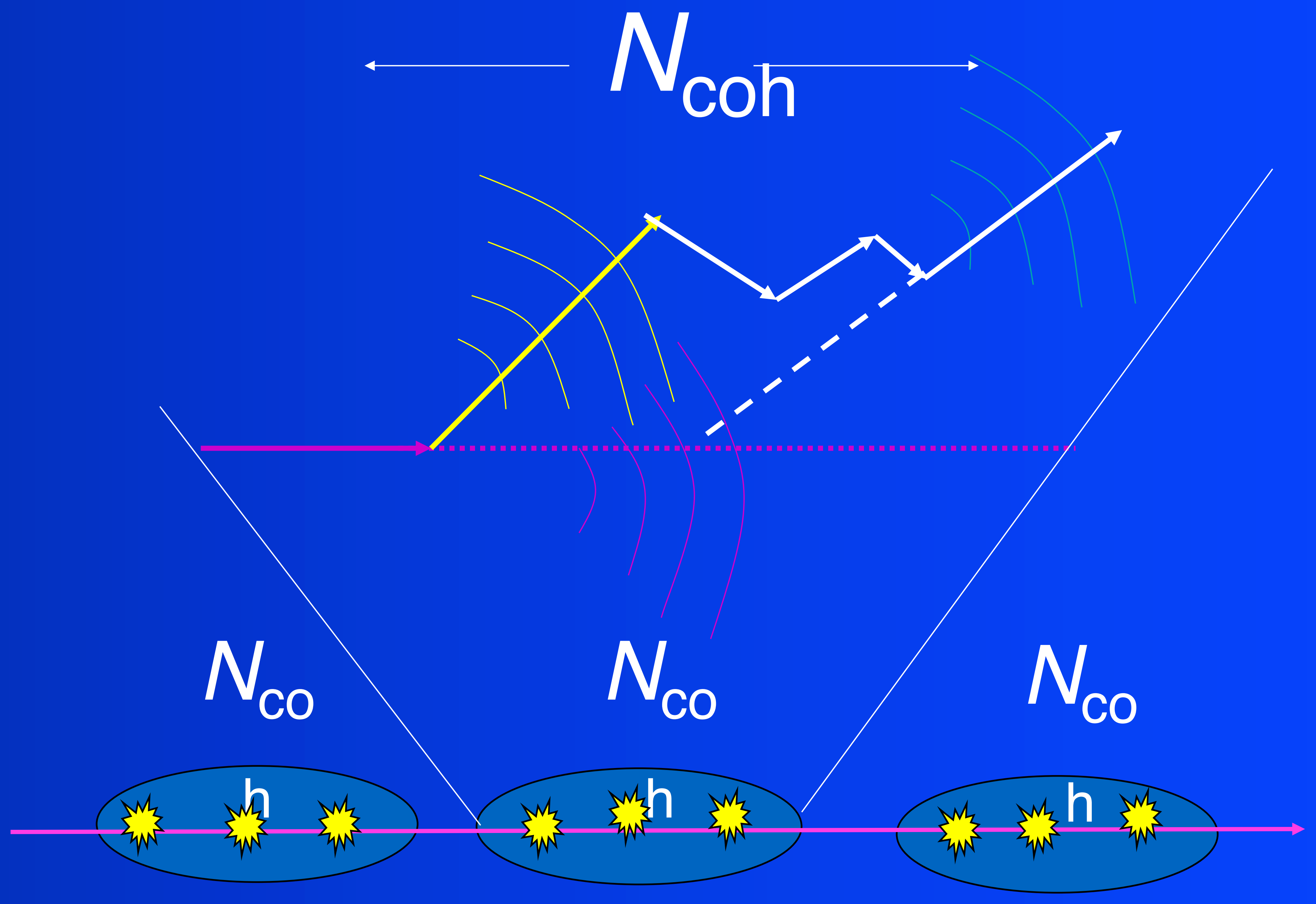
$$\omega\frac{dI}{d\omega}=\frac{L}{\lambda_{mfp}}\left(\omega\frac{dI}{d\omega}\right)_{\mathrm{BH}}\propto N\frac{2\alpha}{\pi}$$

LPM Interference

$$\tau_f = \frac{2}{\omega\theta^2} \qquad \theta^2 = N_{\mathrm{coh}}\frac{q_\perp^2}{E^2}$$

$$N_{\mathrm{coh}}\lambda \approx \tau_f$$

$$\rightarrow N_{\mathrm{coh}} = \frac{2E}{\sqrt{\omega\langle q_\perp^2\rangle\lambda}}$$

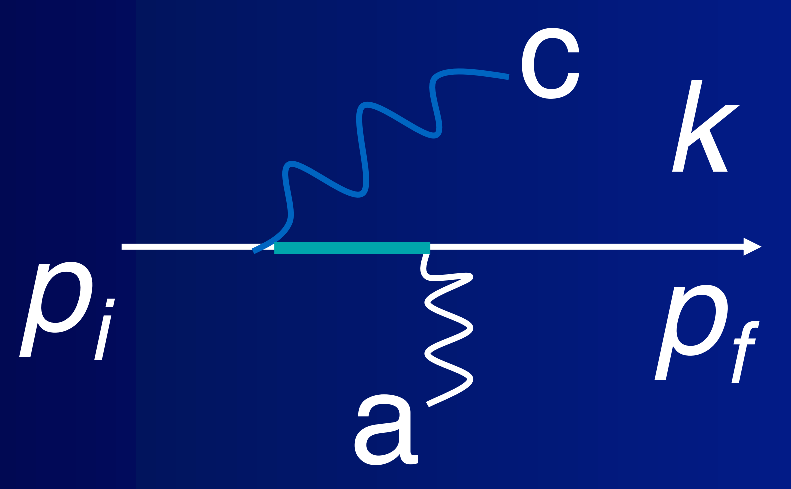


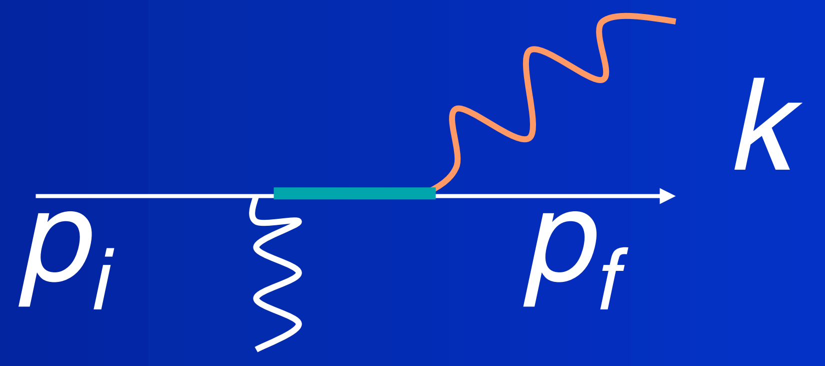
N_{coh} # of scattering for a coherent radiation

Effective spectra

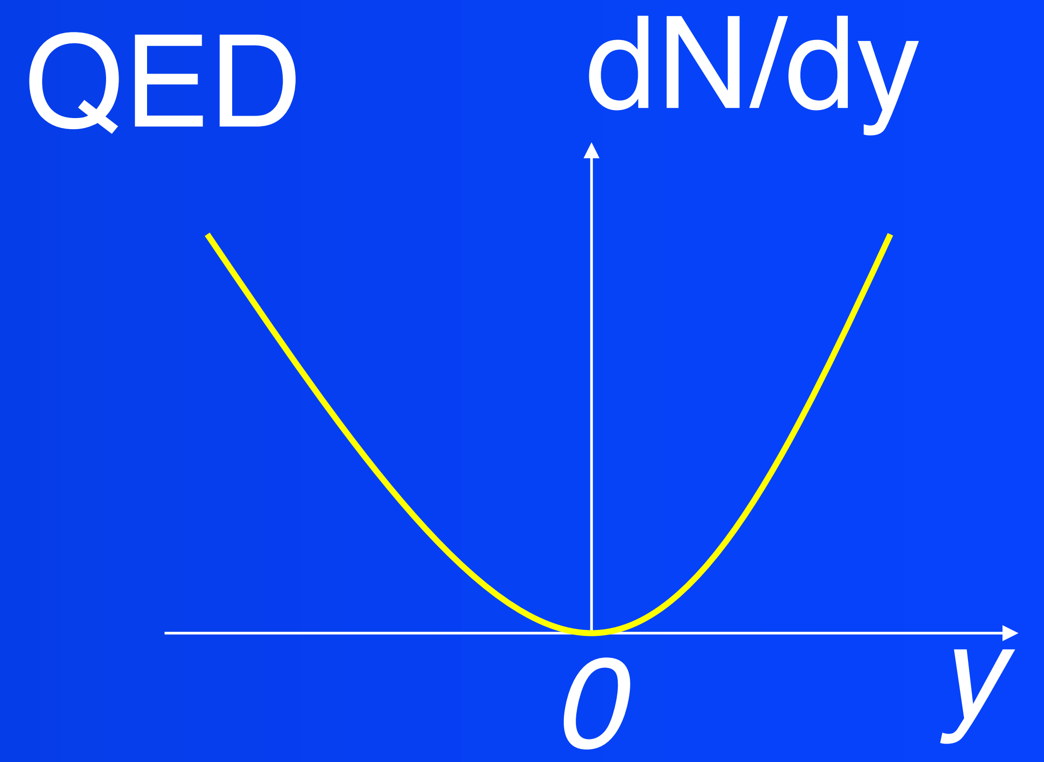
$$\omega\frac{dI}{d\omega} = \frac{L}{\lambda}\left(\omega\frac{dI}{d\omega}\right)_{\mathrm{BH}}\frac{1}{N_{\mathrm{coh}}} \propto N\frac{\alpha}{\pi}\sqrt{\frac{\langle q_\perp^2\rangle}{E^2}\lambda\omega}$$

Radiation in QCD: Colors Makes the Difference

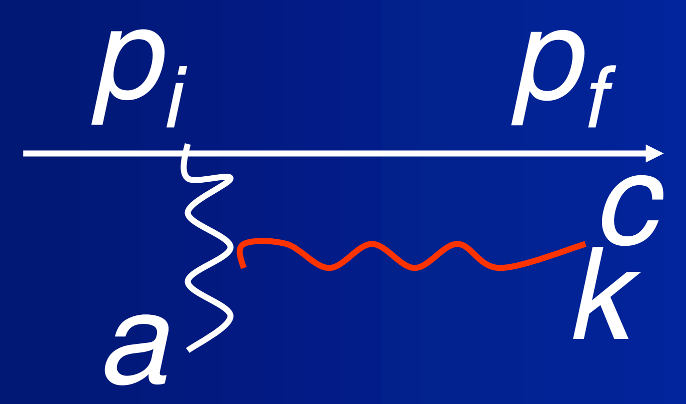




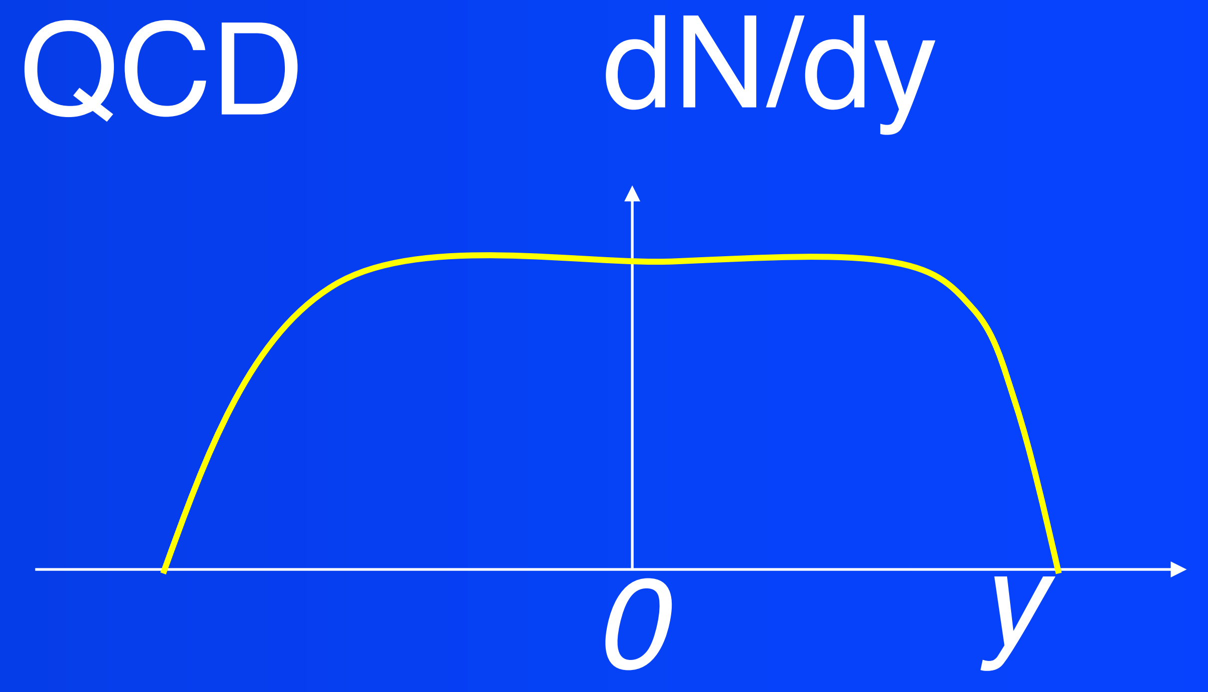
$$R_S^{(1)} \approx ig\frac{2\vec{\epsilon}_\perp \cdot \vec{k}_\perp}{k_\perp^2}[T_aT_c - T_cT_a]$$



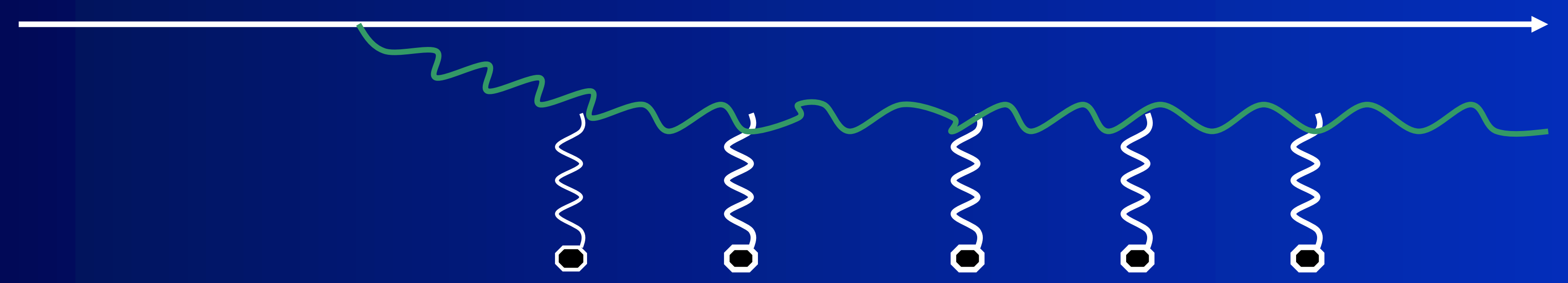
QCD: gluons carry **color**: interference incomplete



$$R_S^{(2)} \approx ig\frac{2\vec{\epsilon}_\perp \cdot (\vec{q}_\perp - \vec{k}_\perp)}{(\vec{q}_\perp - \vec{k}_\perp)^2}[T_a, T_c]$$

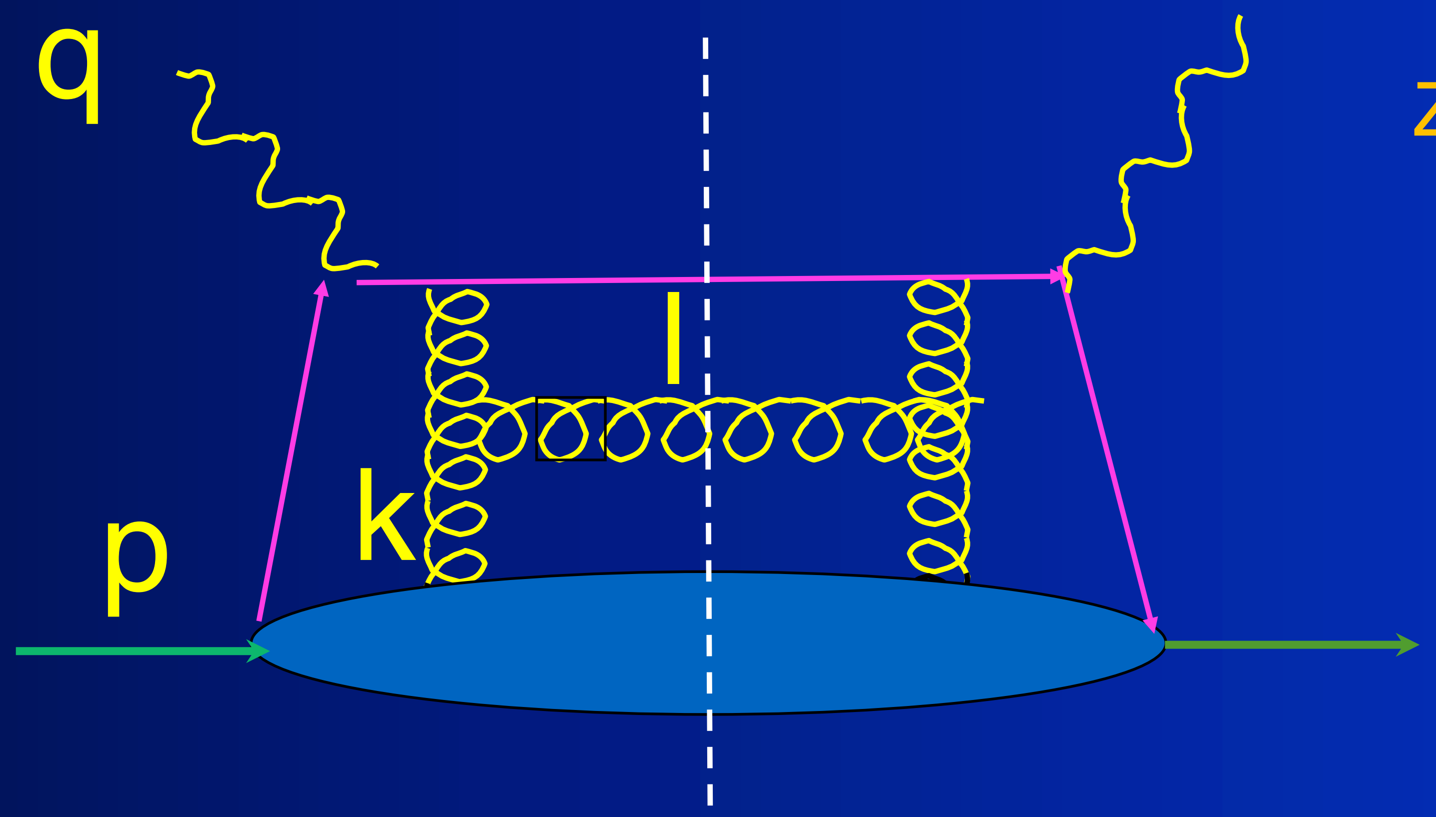


Gluon multiple scattering (BDMP'96, Zakharov'96)



$$\Delta E = \approx \frac{\alpha_s N_c}{4}\frac{\langle q_\perp^2 \rangle}{\lambda}L^2$$

Parton propagation in QCD medium



Zhang, Qin and XNW arXiv:1905.12699

medium TMD gluon density

$$\frac{dN_g}{dl_\perp^2\,dz} = \int_{y^-}^{\infty} dy_1^- \left[\rho_A(y_1^-,\vec{y}_\perp)\frac{2\pi\alpha_s}{N_c}\pi\int\frac{dk_\perp^2}{(2\pi)^2}\frac{\phi_N(0,\vec{k}_\perp)}{k_\perp^2}\right]\pi\frac{\alpha_s}{2\pi}P_{qg}(z)\frac{C_A}{l_\perp^2}\mathcal{N}_g(\vec{l}_\perp,\vec{k}_\perp)$$

$$\mathcal{N}_g^{static+soft} = \int\frac{d\varphi}{2\pi}\frac{2\vec{k}_\perp\cdot\vec{l}_\perp}{(\vec{l}_\perp-\vec{k}_\perp)^2}\left(1-\cos\Big[\frac{(\vec{l}_\perp-\vec{k}_\perp)^2}{2q^- z(1-z)}y_1^-\Big]\right)$$

$$\downarrow$$

Formation time of the gluon emission $\tau_f \longleftarrow y_1^-/\tau_f$

Parton energy loss and jet transport

$$\frac{dE_{rad}}{dx} \approx E\frac{2C_A\alpha_s}{\pi}\hat{q}(x)\int dz\frac{d\ell_\perp^2}{\ell_\perp^4}zP(z)\sin^2\frac{\ell_\perp^2(x-x_0)}{4z(1-z)E}$$

Inelastic energy loss $\Delta E_{inel} \propto \alpha_s \hat{q} L^2$

$$\frac{dE_{el}}{dx} = \int\frac{d^3k}{(2\pi)^3}dq_\perp^2 f(k)\frac{q_\perp^2}{2k}\frac{d\sigma}{dq_\perp^2} \approx \langle\frac{1}{2\omega}\rangle\hat{q}$$

Elastic energy loss $\Delta E_{el} \propto \hat{q}L/T$

Jet transport coefficient:

$$\hat{q}(y) = \frac{4\pi^2\alpha_s C_R}{N_c^2-1}\rho(y)xG(x)|_{x\approx 0} = \frac{\langle q_\perp^2\rangle}{\lambda}$$

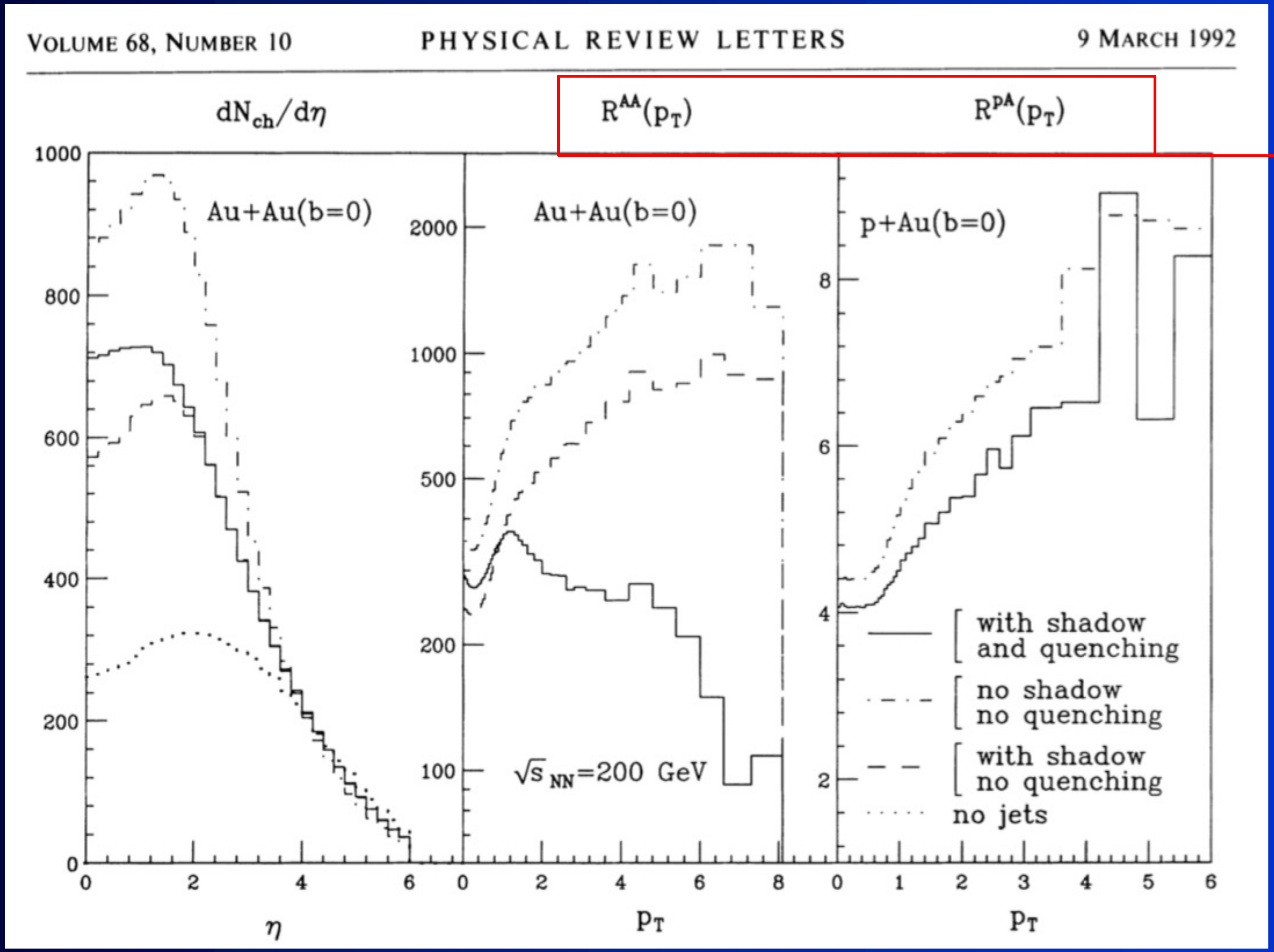
pQCD (BDMPS'96)
AdS/CFT (Liu,Rajagopal &Wideman'06)
lattice QCD (Majumder'12)

Extract jet transport coefficient from parton energy loss

Jet quenching in heavy-ion collisions

Gyulassy, XNW: Suppression of leading hadrons due to jet quenching

- *Phys. Rev. Lett.* 68 (1992) 1480-1483


VOLUME 68, NUMBER 10 PHYSICAL REVIEW LETTERS 9 MARCH 1992

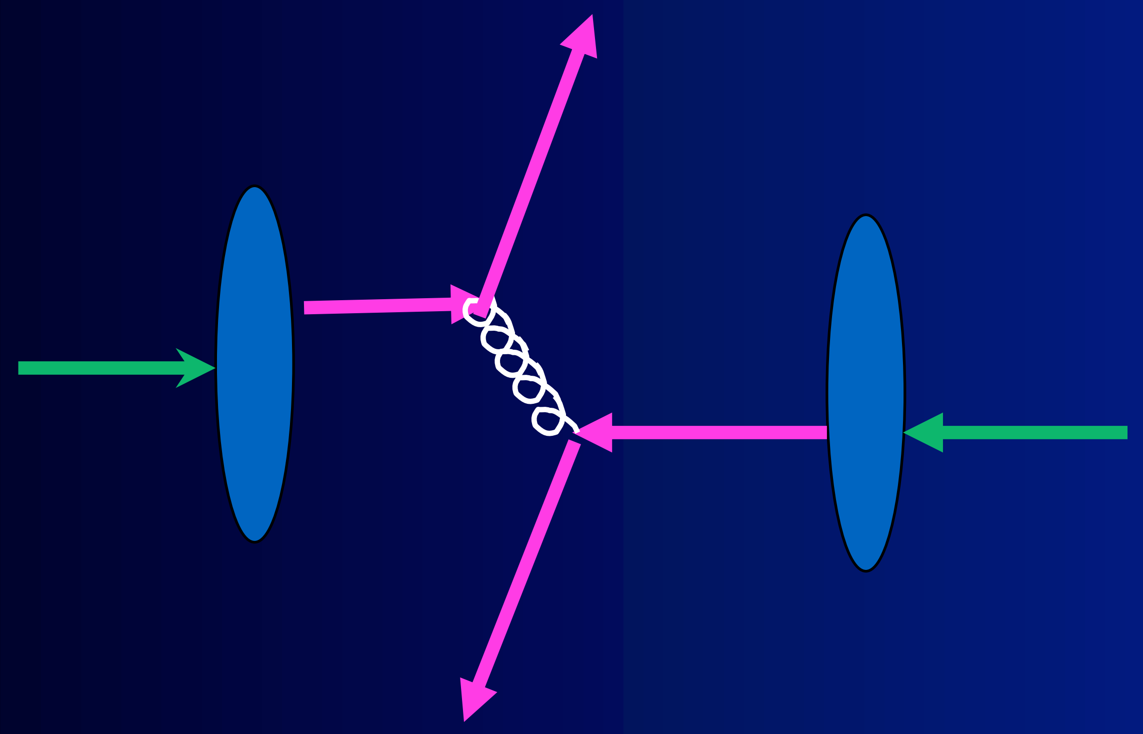
$$\frac{(dN/d\eta dp_T)_{AA}}{(dN/d\eta dp_T)_{NN}}$$

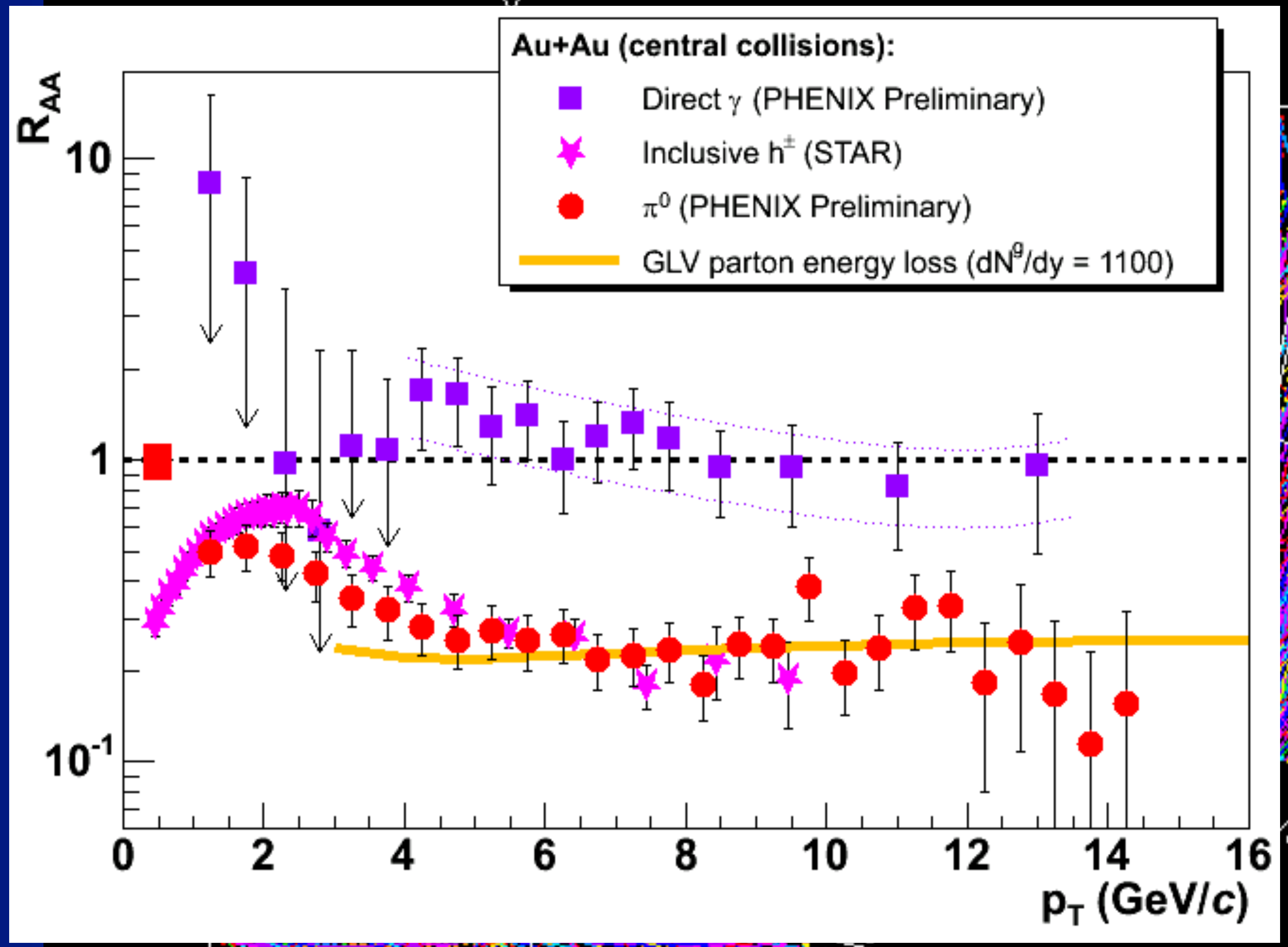
$$R_{AB}(p_T) = \frac{d\sigma_{AB}/dyd^2p_T}{\underline{\langle N_{\rm binary}\rangle} d\sigma_{NN}/dyd^2p_T}$$

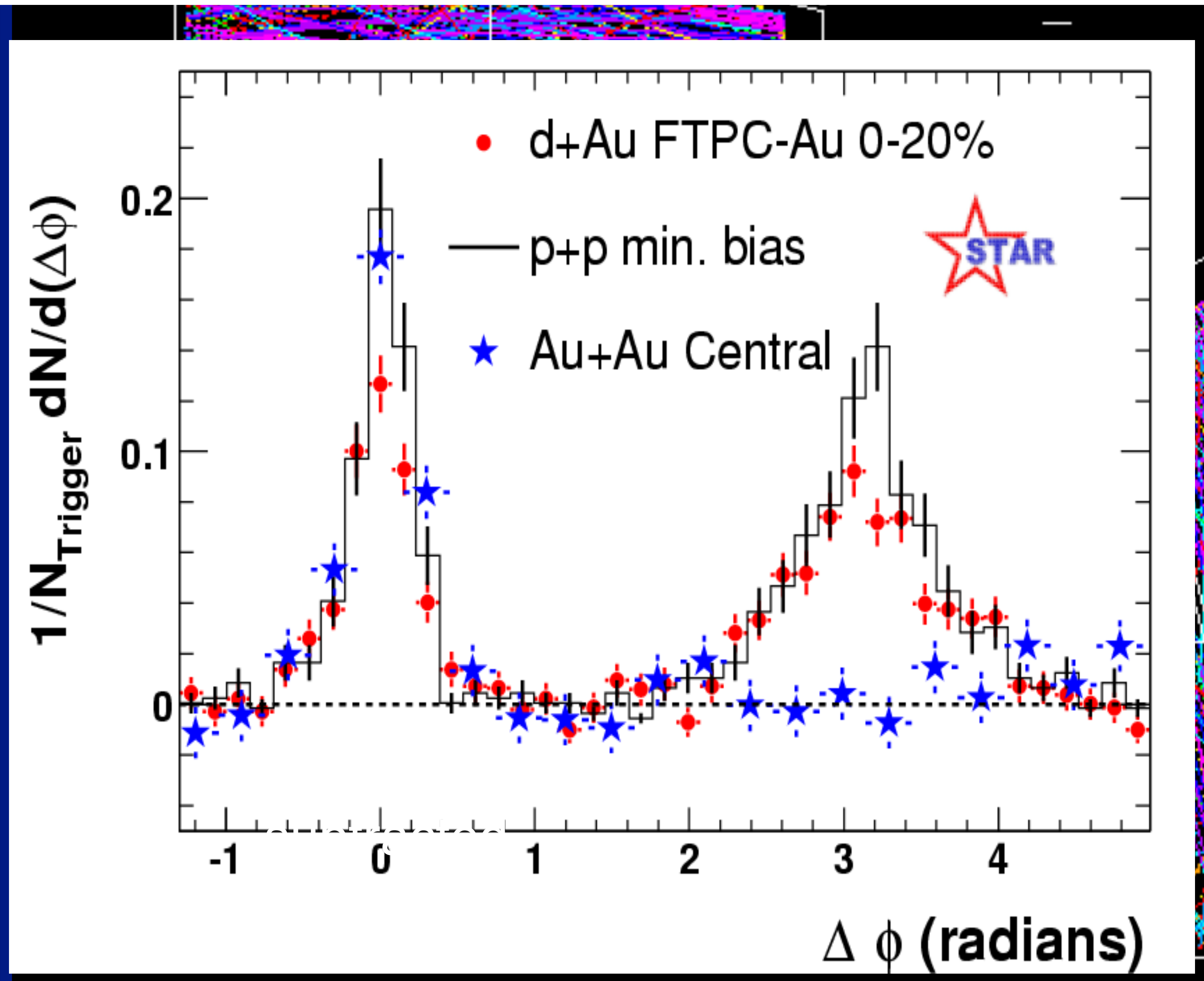
E Wang & XNW, PRC 64 (2001) 034901

Jet Quenching at RHIC

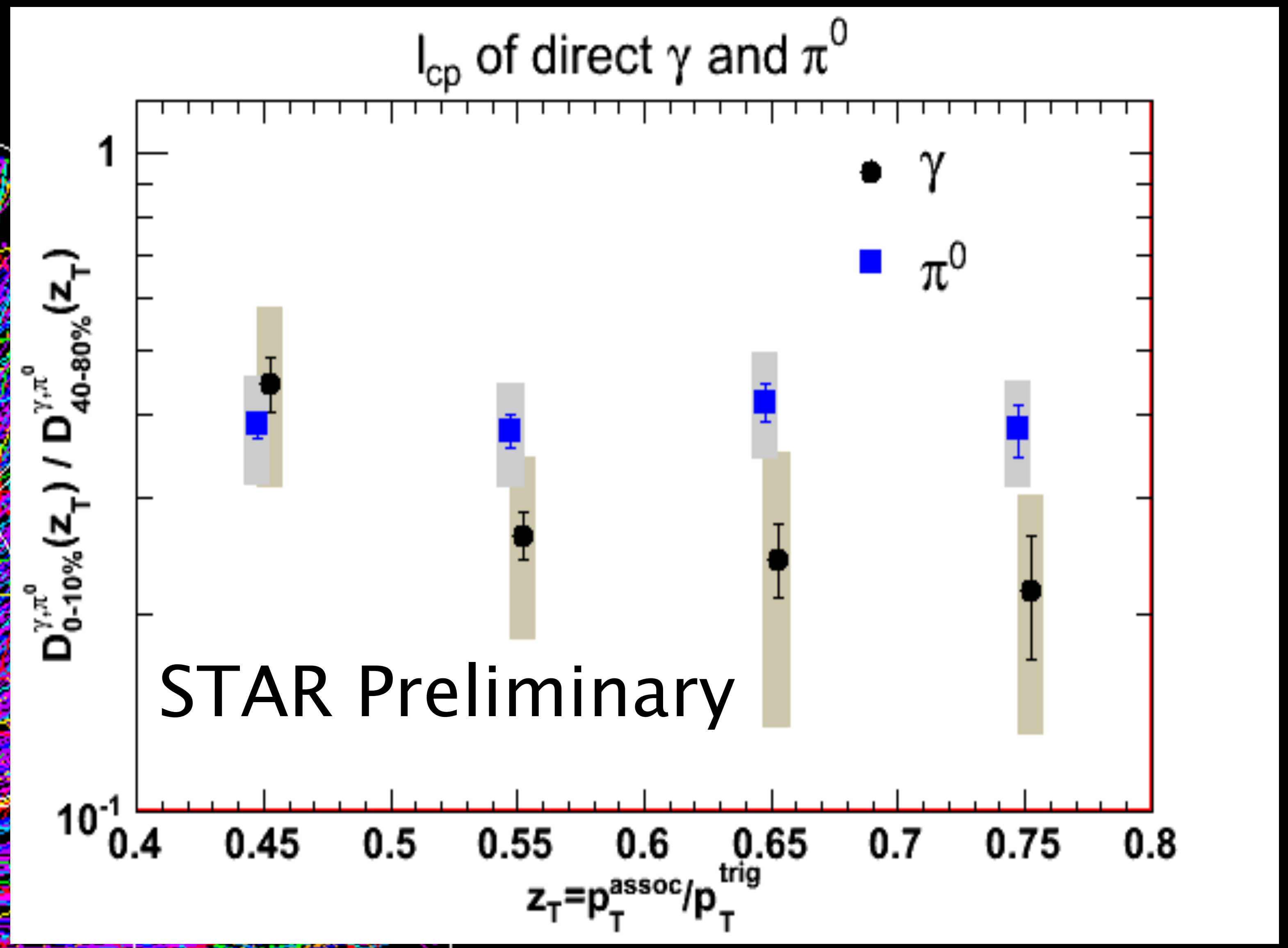
di-jet

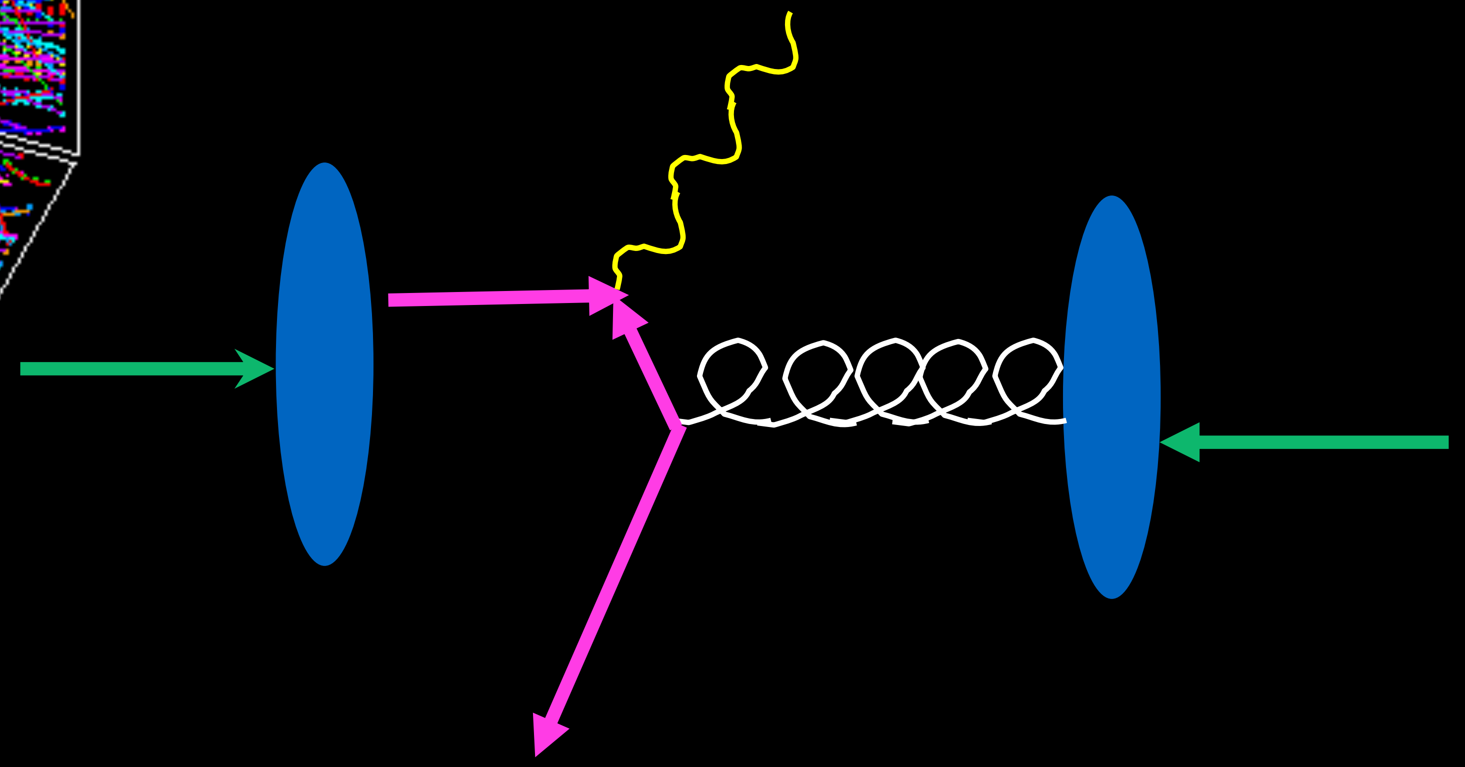




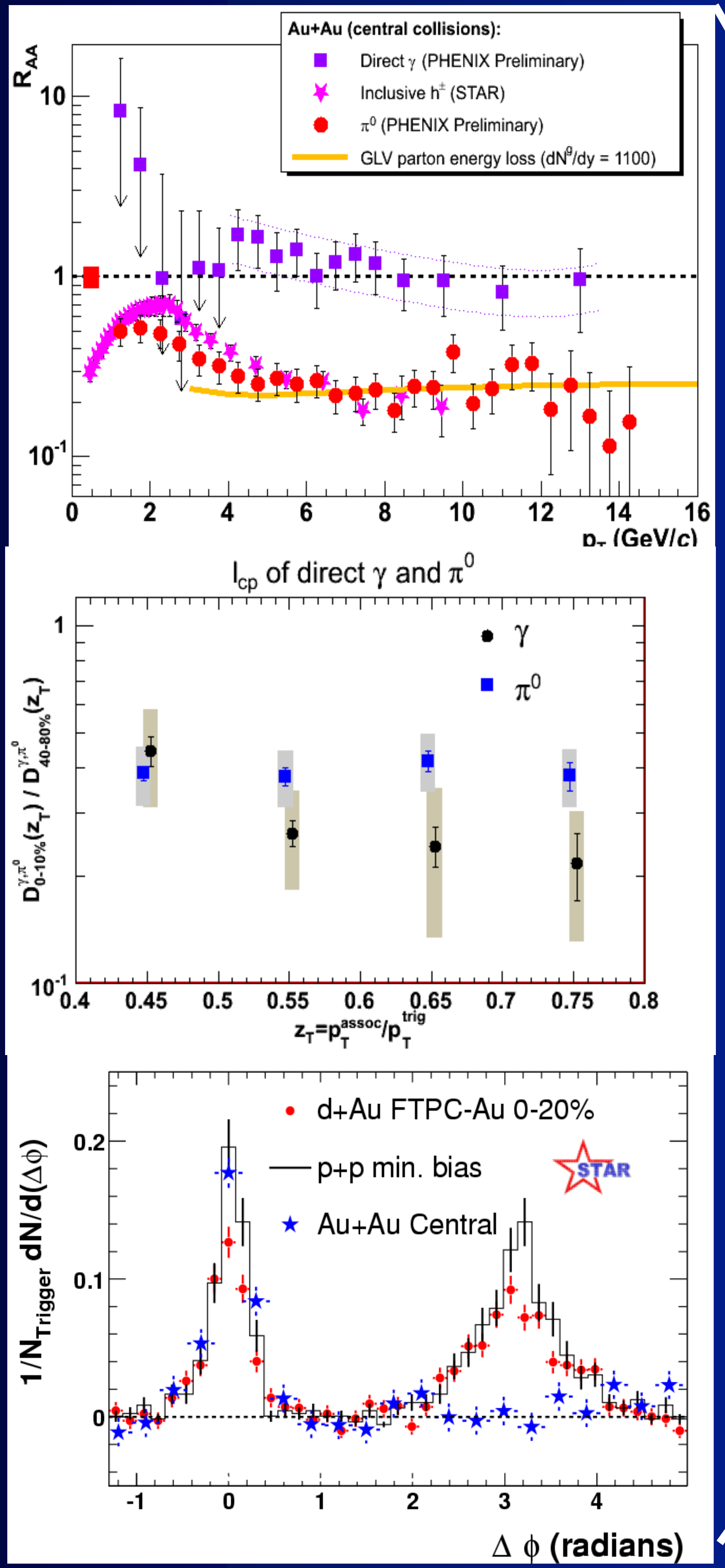


photon-jet





Bayesian inference of jet transport coefficient

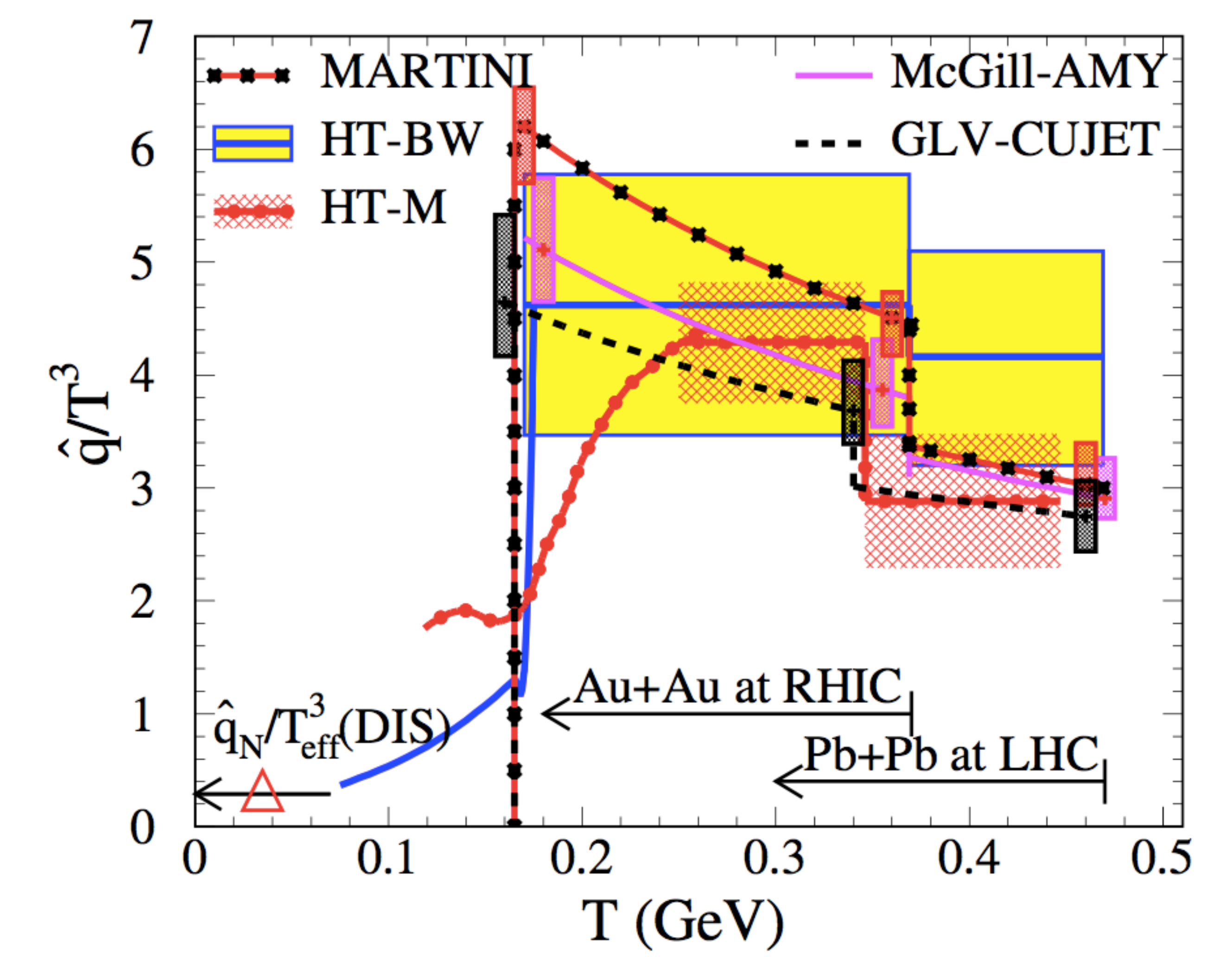


LIDO	e-Print: 2010.13680
JETSCAPE	e-Print: 2102.11337
QLBT:	e-Print: 2206.01340
IF Bayesian	e-Print: 2107.11713

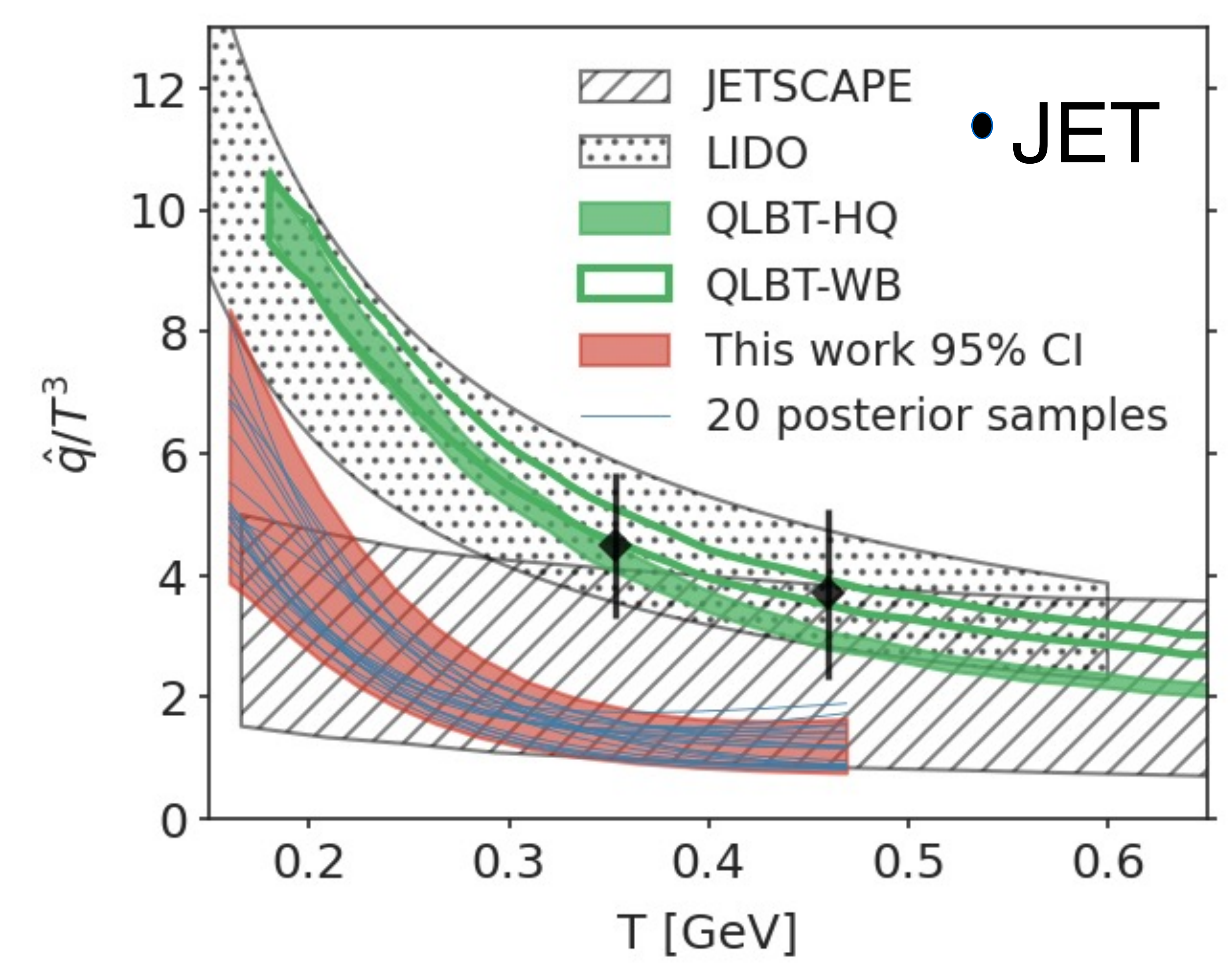
Strong T-dependence
Weak E-dependence

Information-Field approach to priors is free of long-range correlation

JET Collaboration: **e-Print: 1312.5003**

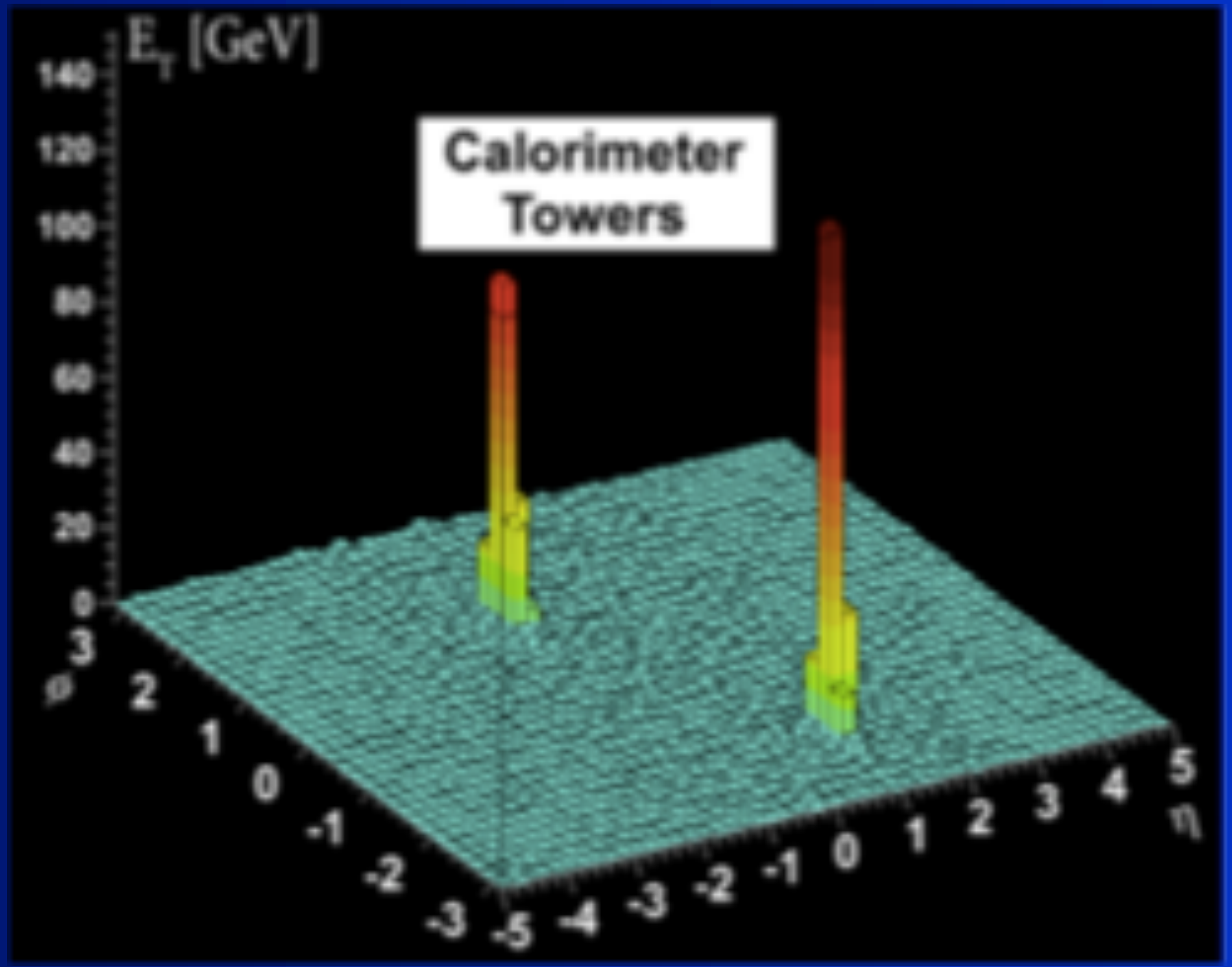


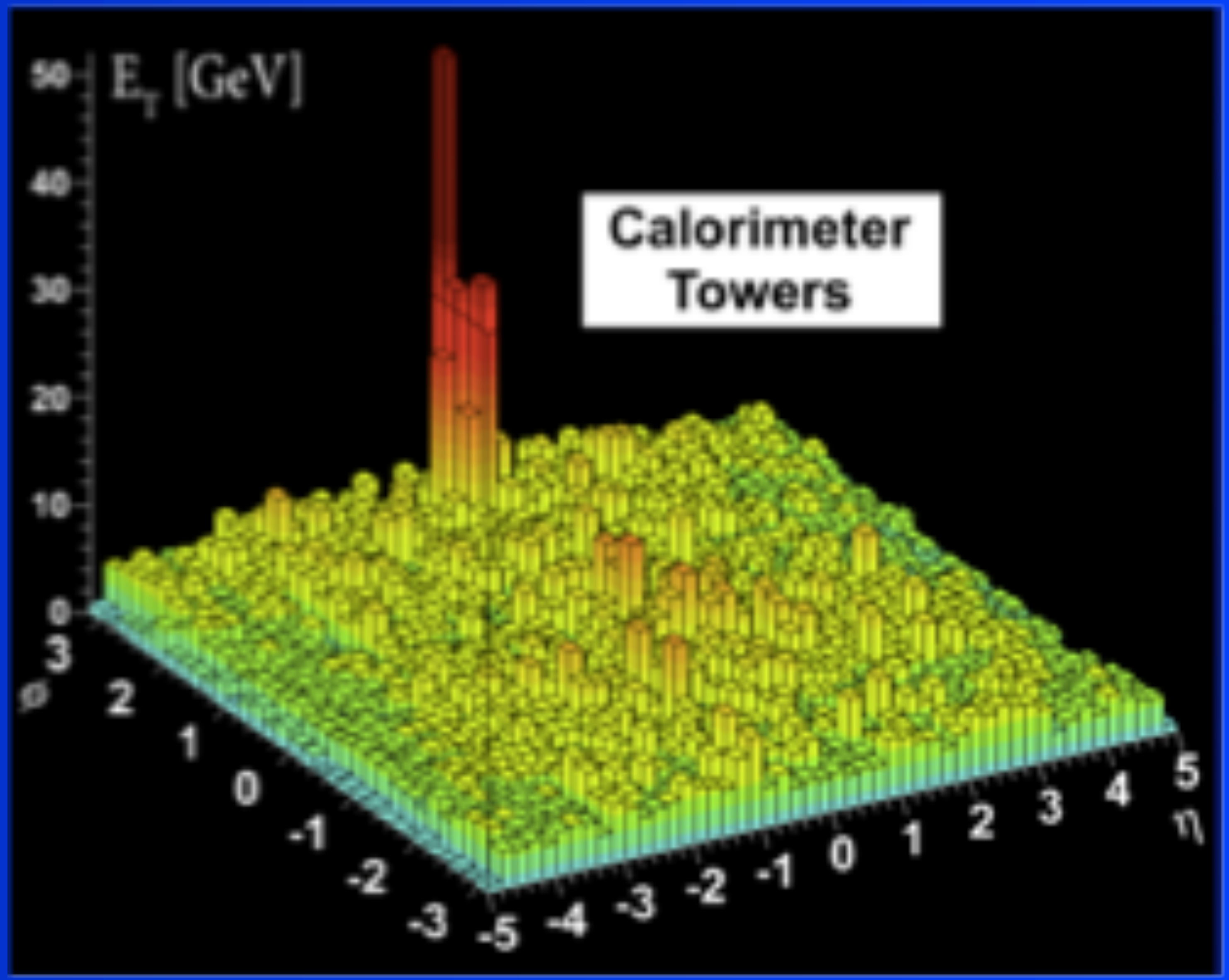
$$\hat{q} \approx \begin{cases} 1.2 \pm 0.3 \\ 1.9 \pm 0.7 \end{cases} \mathrm{GeV}^2/\mathrm{fm} \ \ \text{at} \ \ \begin{matrix} \mathrm{T}{=}370\ \mathrm{MeV}, \\ \mathrm{T}{=}470\ \mathrm{MeV}, \end{matrix}$$



Xie, Ke, Zhang & XNW (2022)

Jet energy, medium response and background





Jet energy as defined in the jet reconstruction algorithm with a jet cone R
Uncorrelated background should be subtracted
Jet-induced medium response is correlated with jet: not background
Some of the energy lost by leading partons remain inside jet-cone

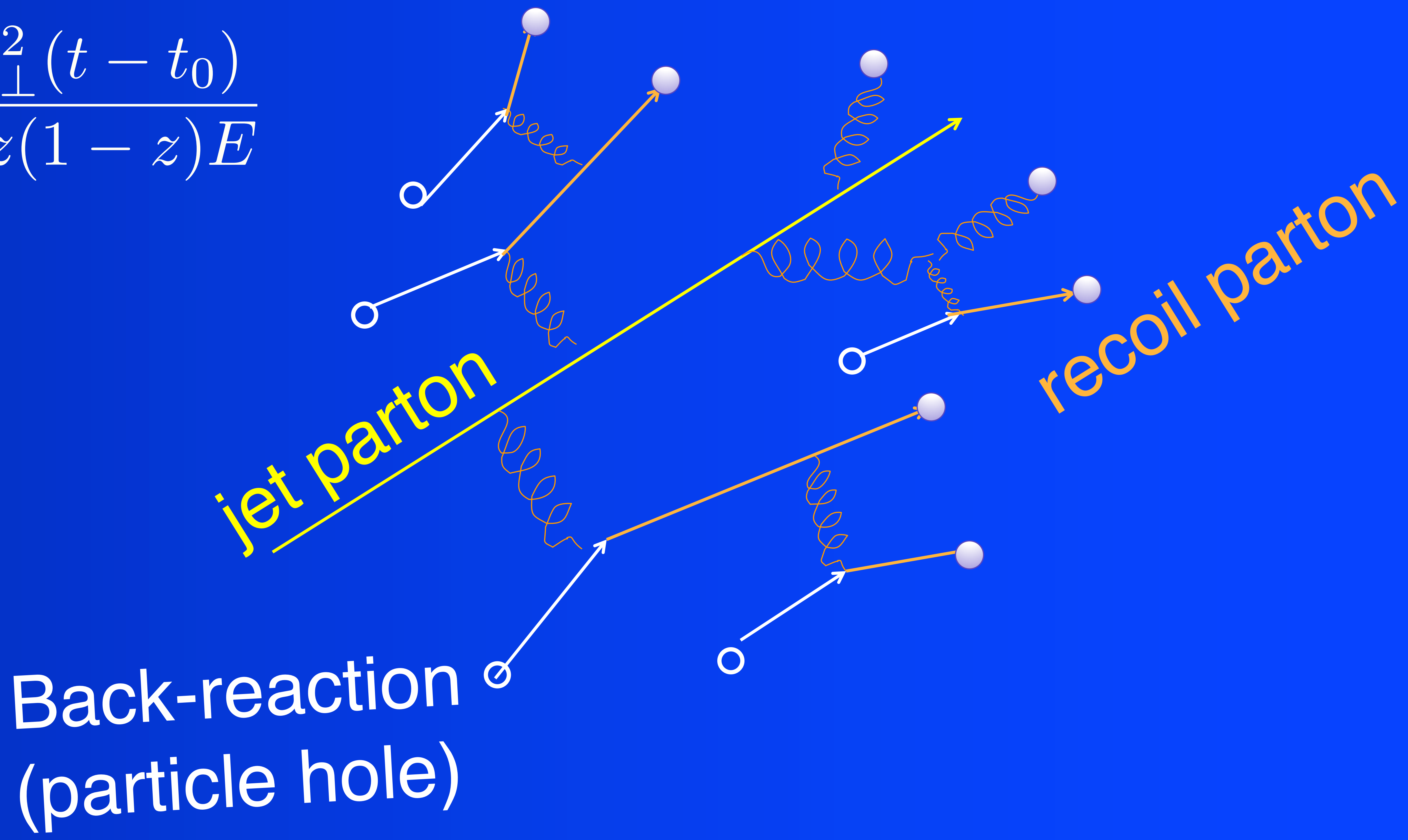
Jet Boltzmann Transport

$$p_1 \cdot \partial f_1 = -\int dp_2 dp_3 dp_4 (f_1 f_2 - f_3 f_4) |M_{12\to 34}|^2 (2\pi)^4 \delta^4(\sum_i p_i) + \text{inelastic}$$

Inelastic processes:

$$\frac{dN_g}{dz d^2k_\perp dt} \approx \frac{2C_A\alpha_s}{\pi k_\perp^4} P(z)\hat{q}(\hat{p}\cdot u)\sin^2\frac{k_\perp^2(t-t_0)}{4z(1-z)E}$$

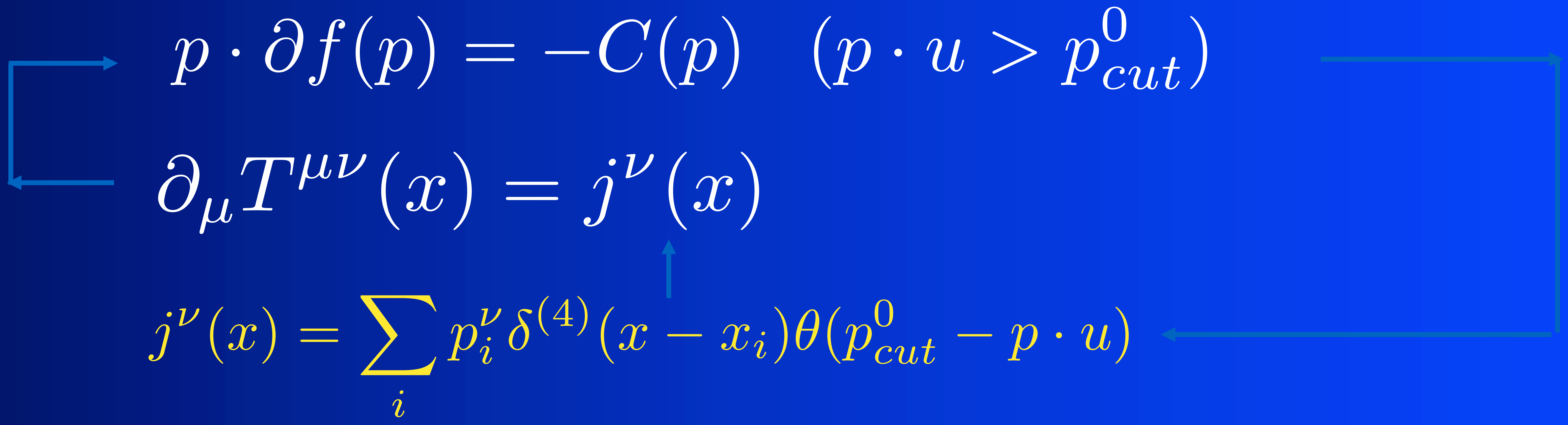
- pQCD elastic and radiative processes (high-twist)
- Transport of medium recoil partons (and back-reaction)
- CLVisc 3+1D hydro bulk evolution



He, Luo, Zhu & XNW, *PRC* 91 (2015) 054908

Jet hydro coupling

Concurrent and coupled evolution of bulk medium and jet showers



$$p \cdot \partial f(p) = -C(p) \quad (p \cdot u > p^0_{cut})$$

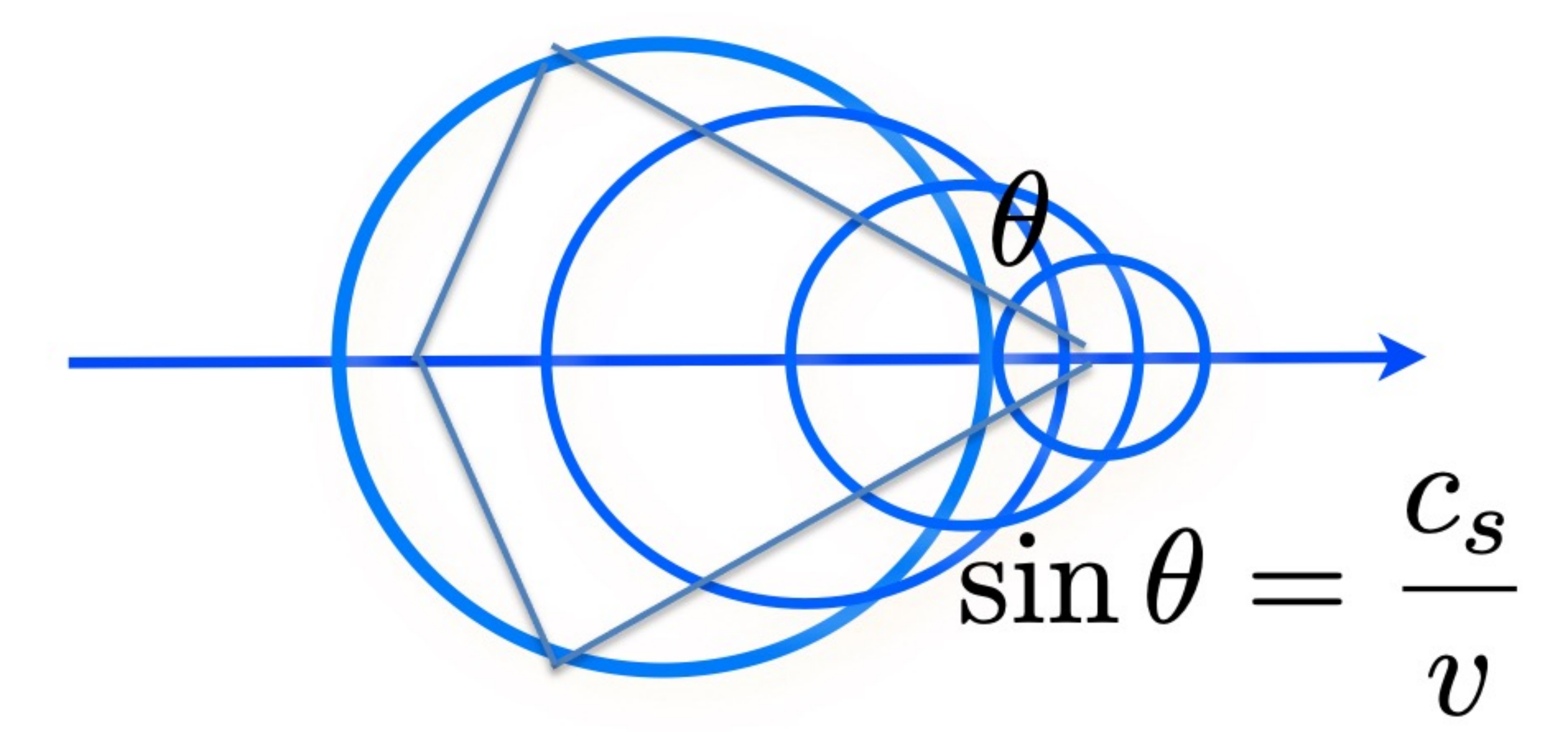
$$\partial_\mu T^{\mu\nu}(x) = j^\nu(x)$$

$$j^\nu(x) = \sum_i p_i^\nu \delta^{(4)}(x - x_i)\theta(p^0_{cut} - p \cdot u)$$

- LBT for energetic partons (jet shower and recoil)
- Hydrodynamic model for bulk and soft partons: CLVisc
- Parton coalescence (thermal-shower)+ jet fragmentation
- Hadron cascade using UrQMD

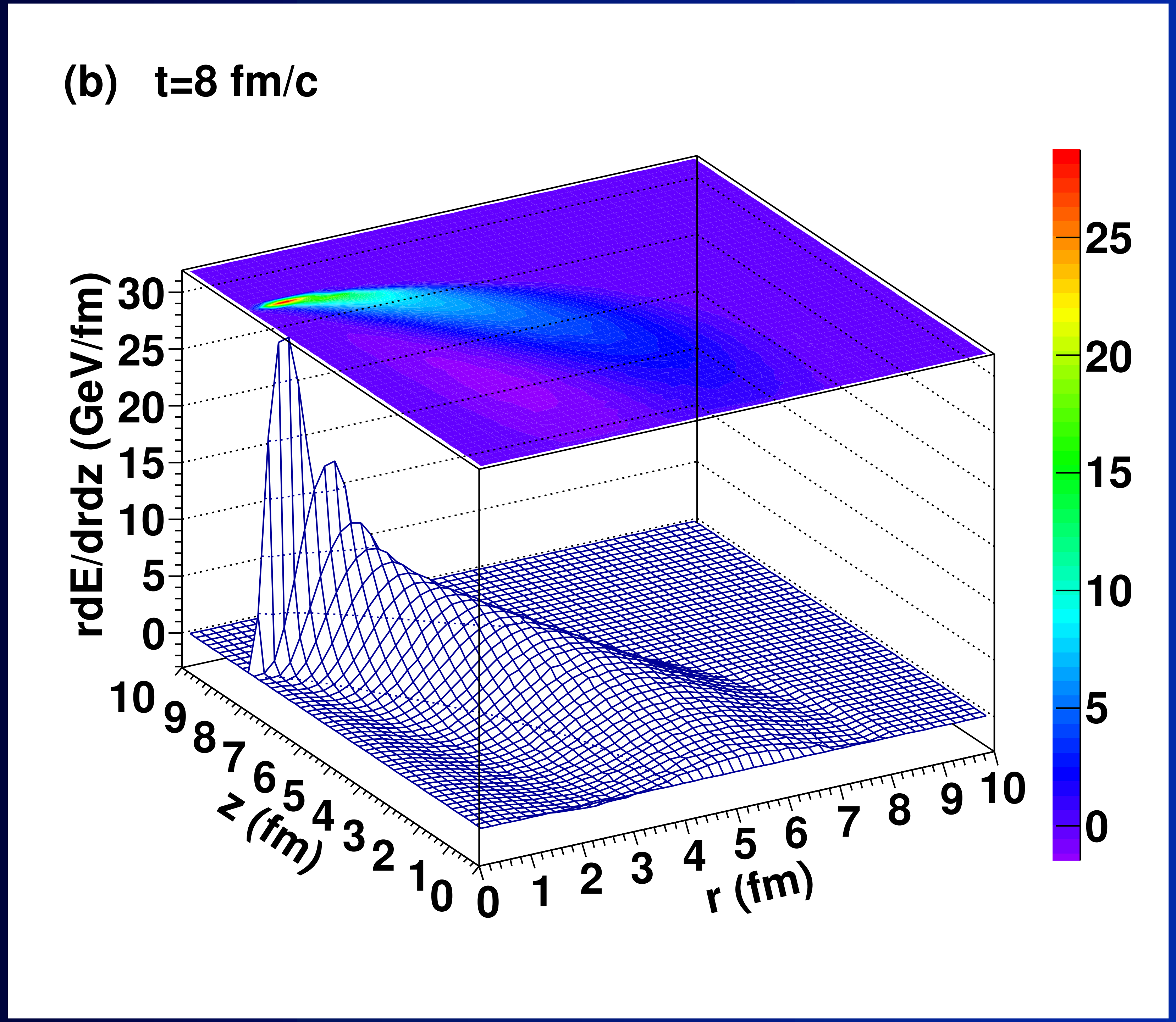
Chen, Cao，Luo, Pang & XNW, PLB777(2018)86,
Zhao, Ke, Chen, Luo & XNW, PRL 128(2022) 022302.

Mach cones and diffusion wakes

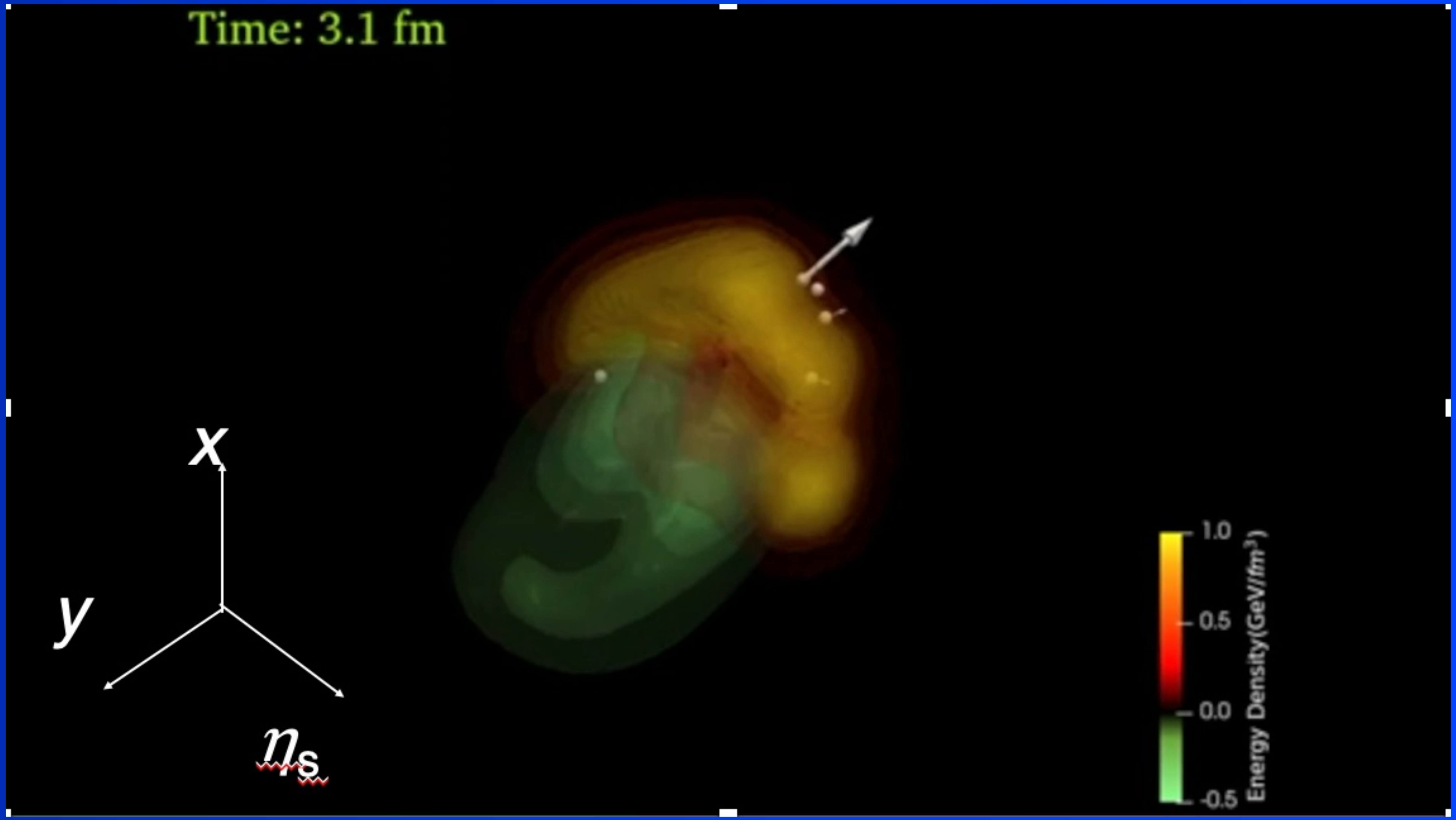


$$\sin\theta = \frac{c_s}{v}$$

LBT: Jet-induced medium response

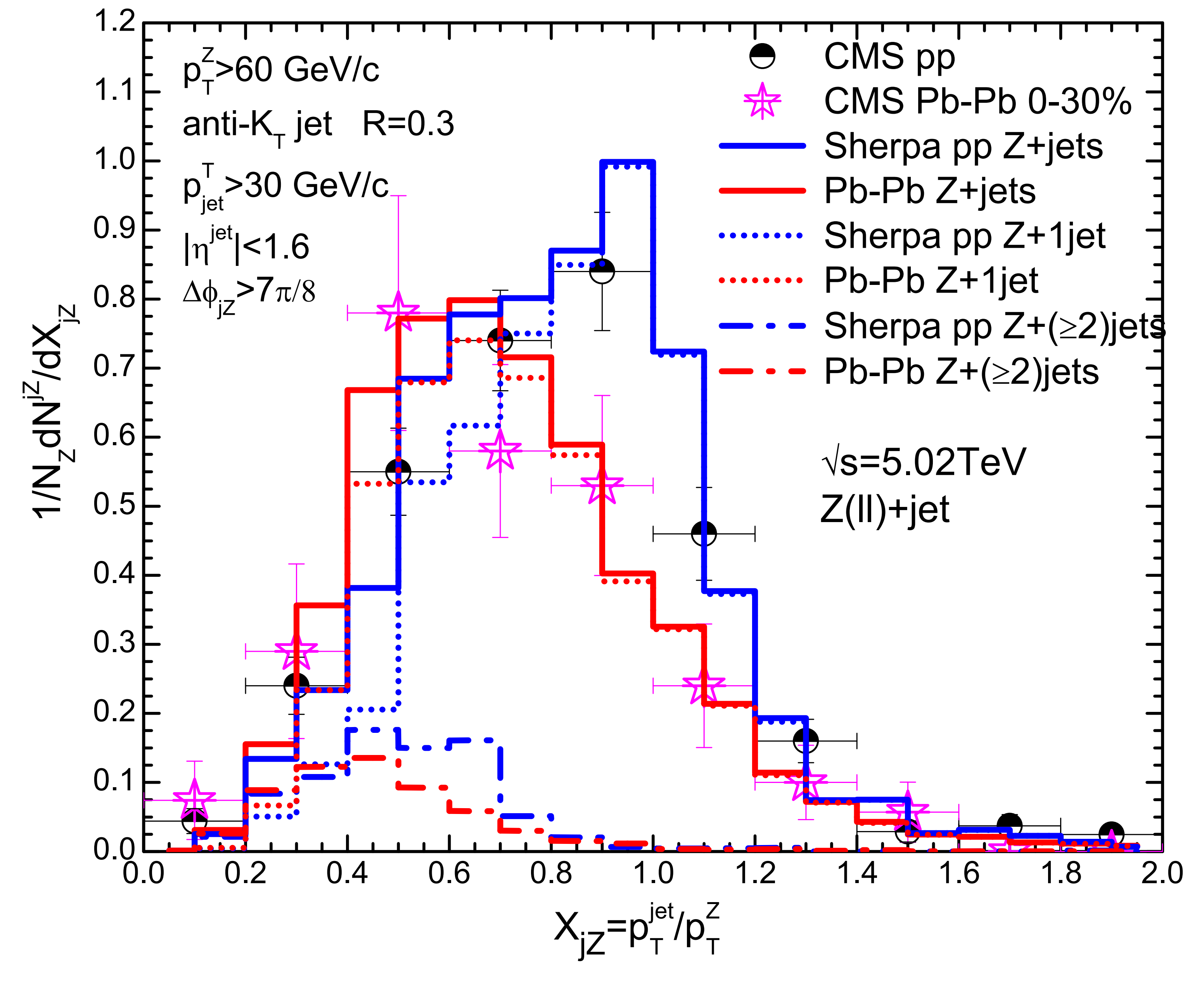


Energy transverse distribution of medium response in a static medium

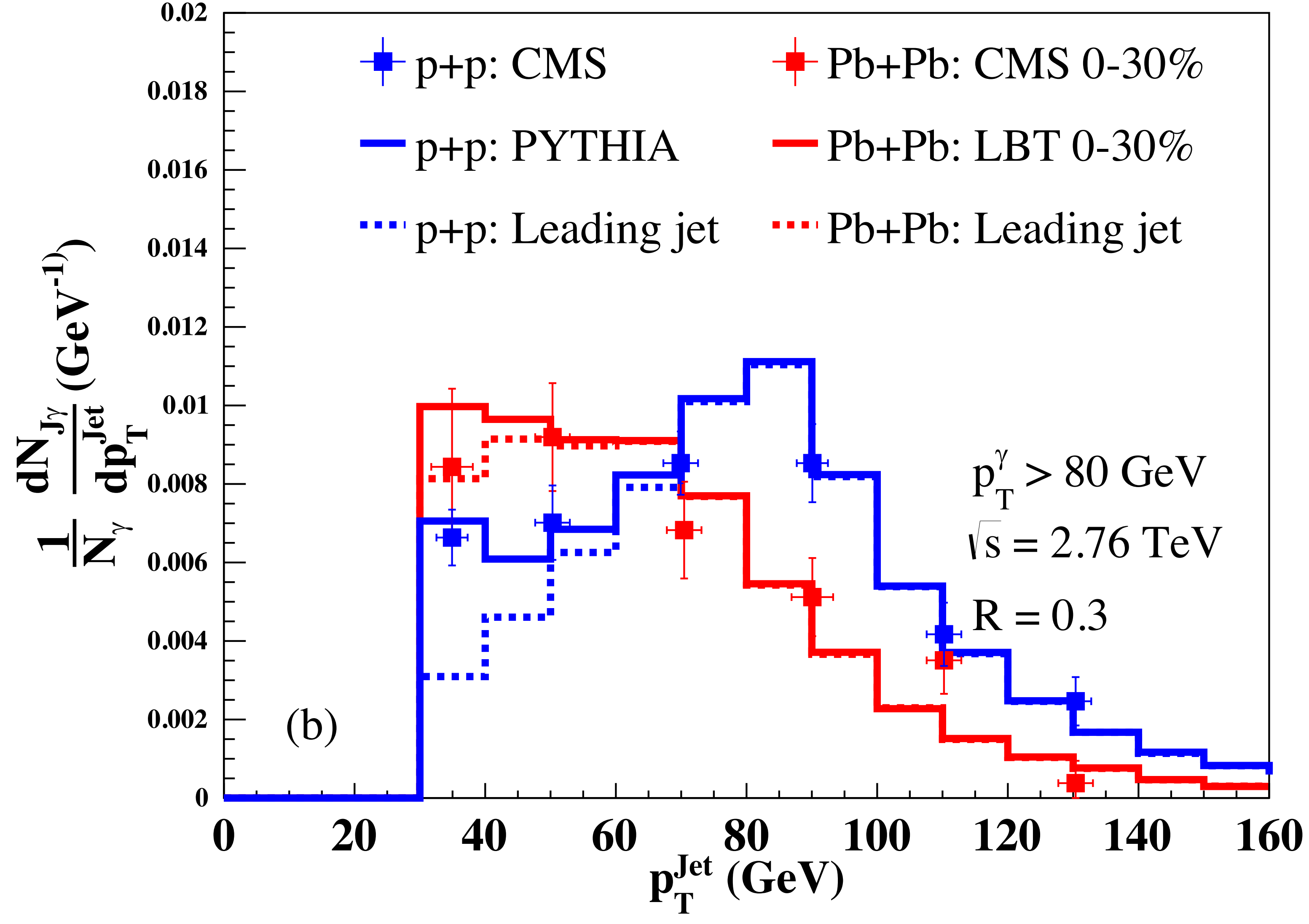


3D energy density distribution of the medium response induced by a γ-jet in a 0-10% Pb+Pb event

Jet suppression and medium response at LHC

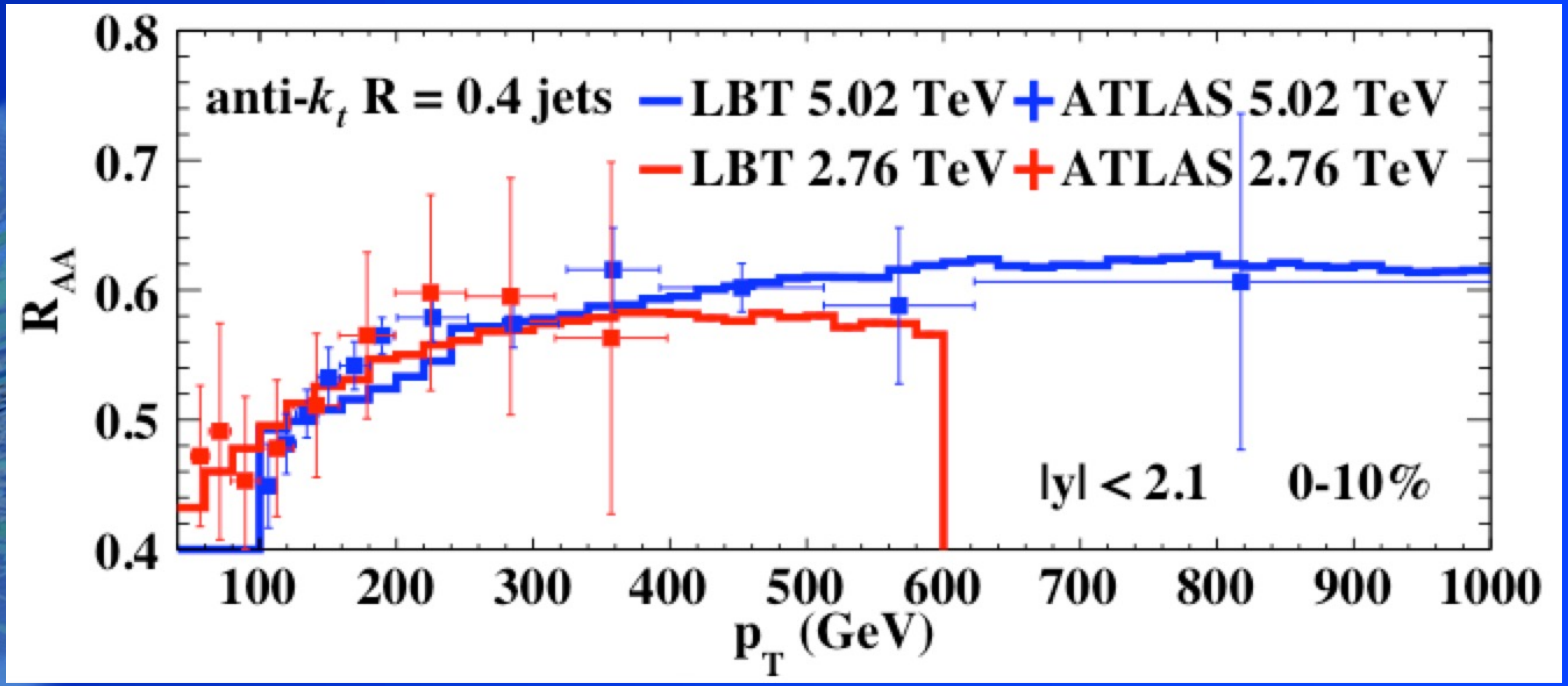


Z-jet



γ-jet

Single inclusive jets



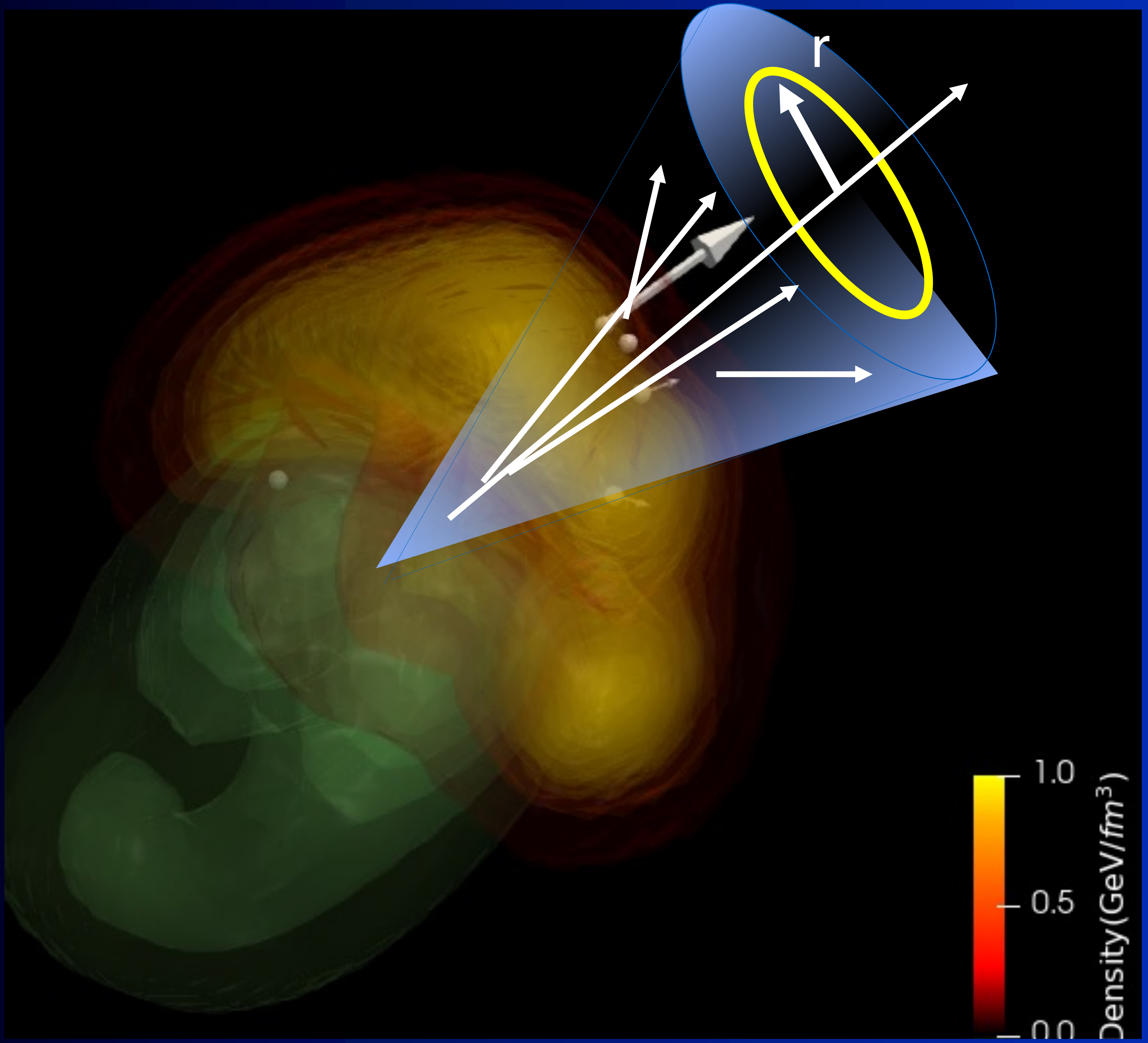
He, Cao, Chen, Luo, Pang & XNW 1809.02525

Zhang, Luo, XNW, Zhang, arXiv:1804.11041

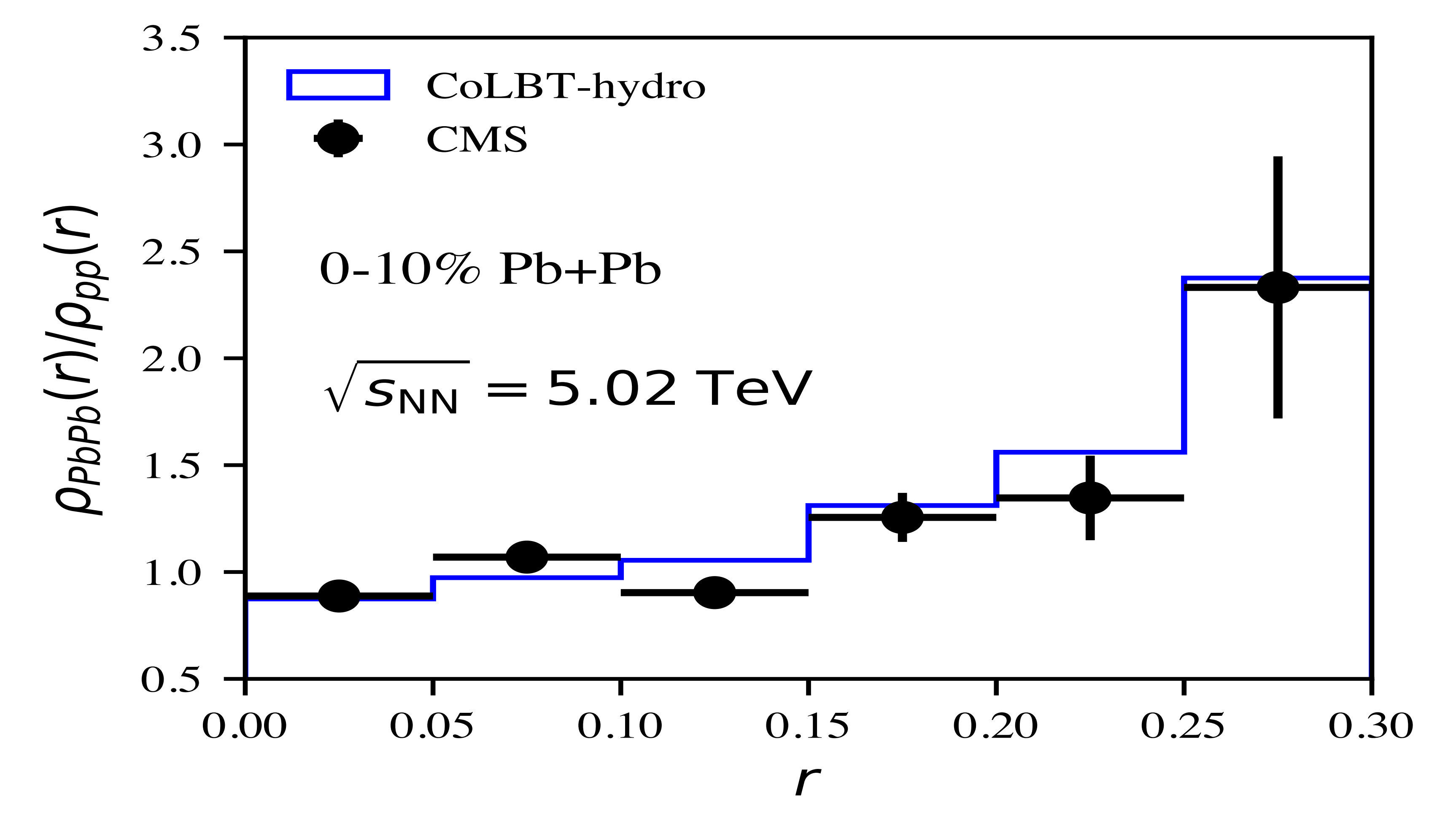
Luo, Cao, He & XNW, arXiv:1803.06785

Modification of jets and medium response

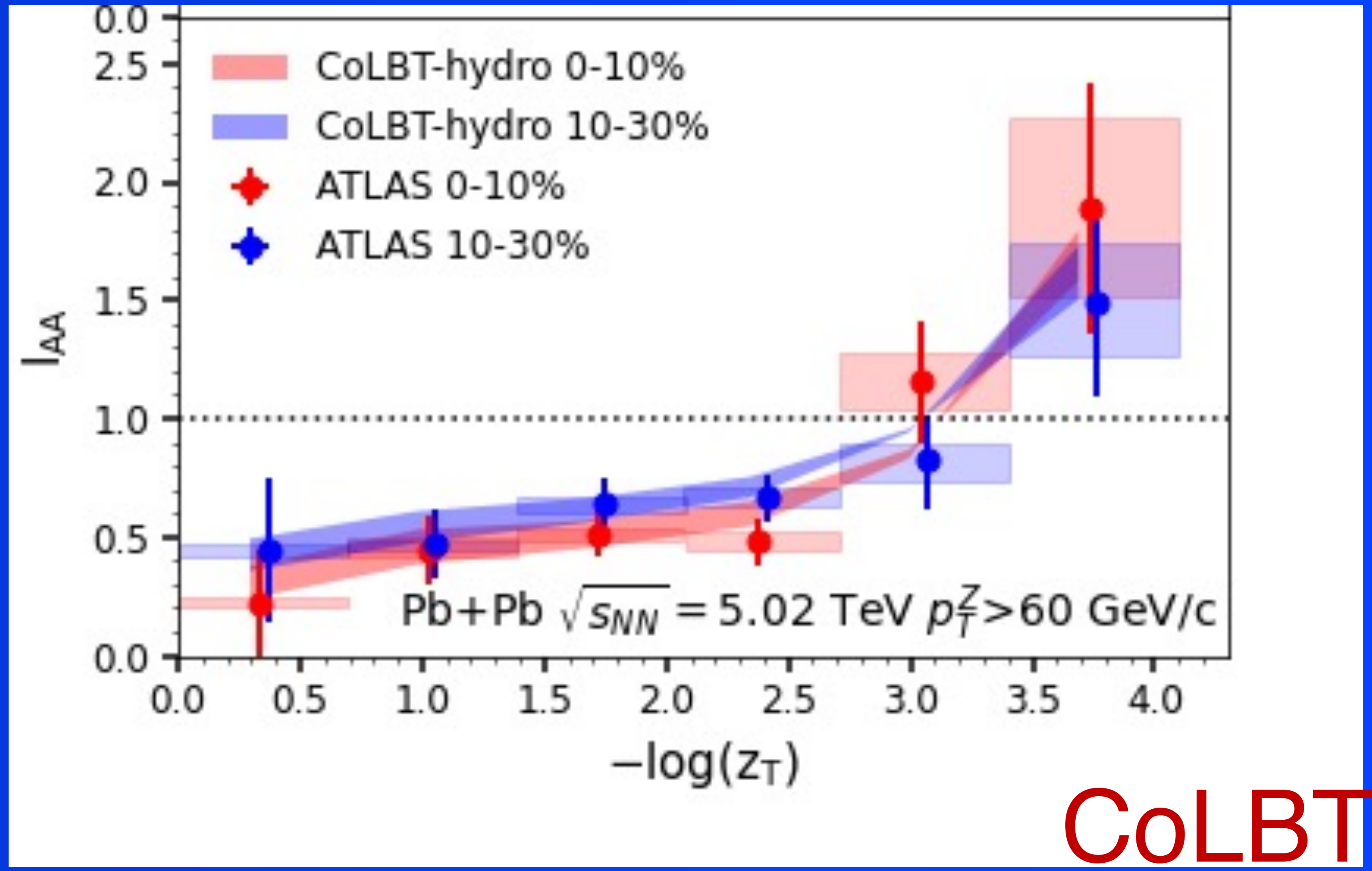
$$\rho(r) = \frac{1}{\Delta r}\frac{1}{N_{jet}}\sum_{jet}\frac{p_T^{jet}(r-\Delta r/2, r+\Delta r/2)}{p_T^{jet}(0,R)}$$



$$\frac{\rho_{AA}(r)}{\rho_{pp}(r)}$$



$$I_{AA} = \frac{D_{AA}(z_T)}{D_{pp}(z_T)}$$



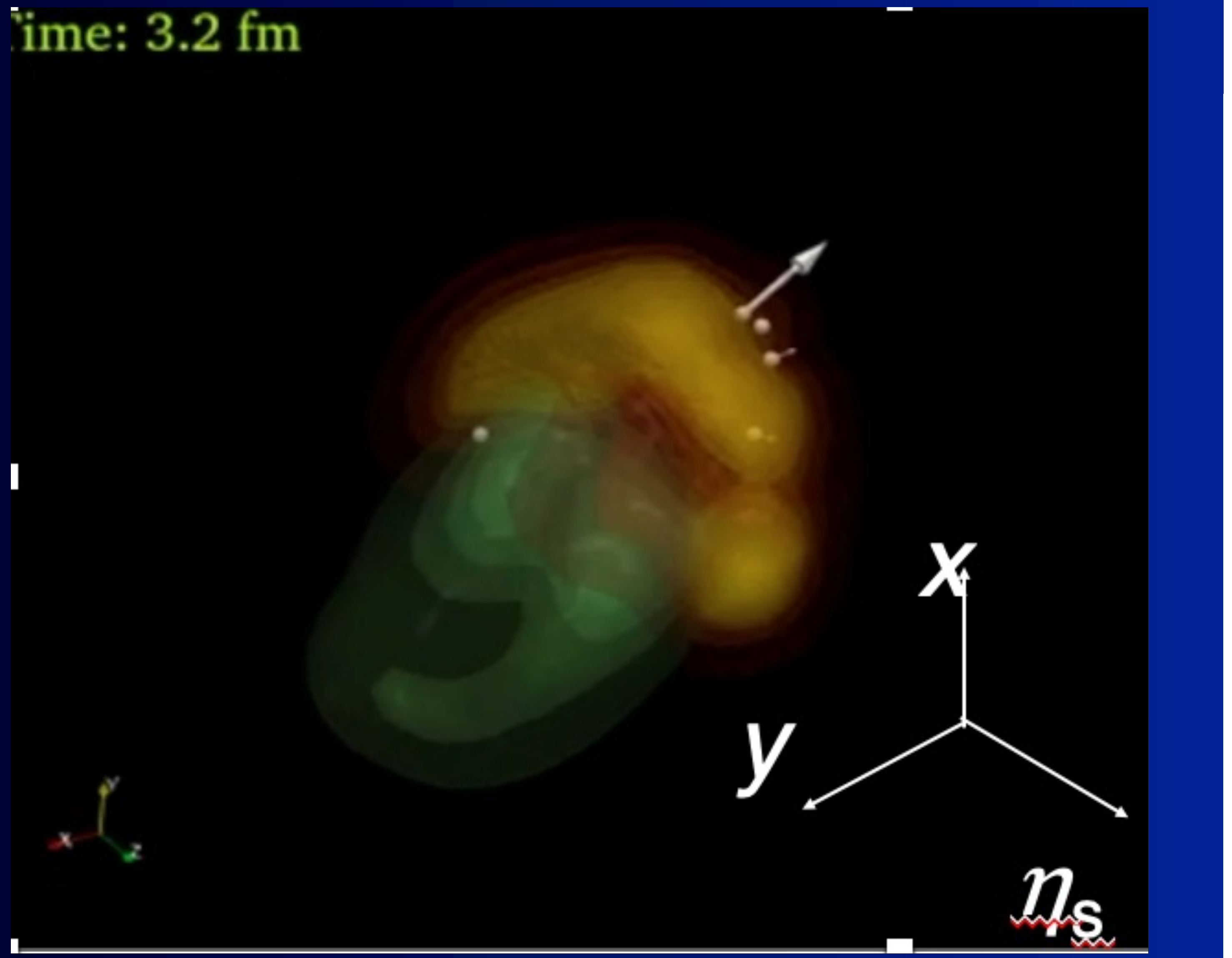
CoLBT

e-Print: 2101.05422

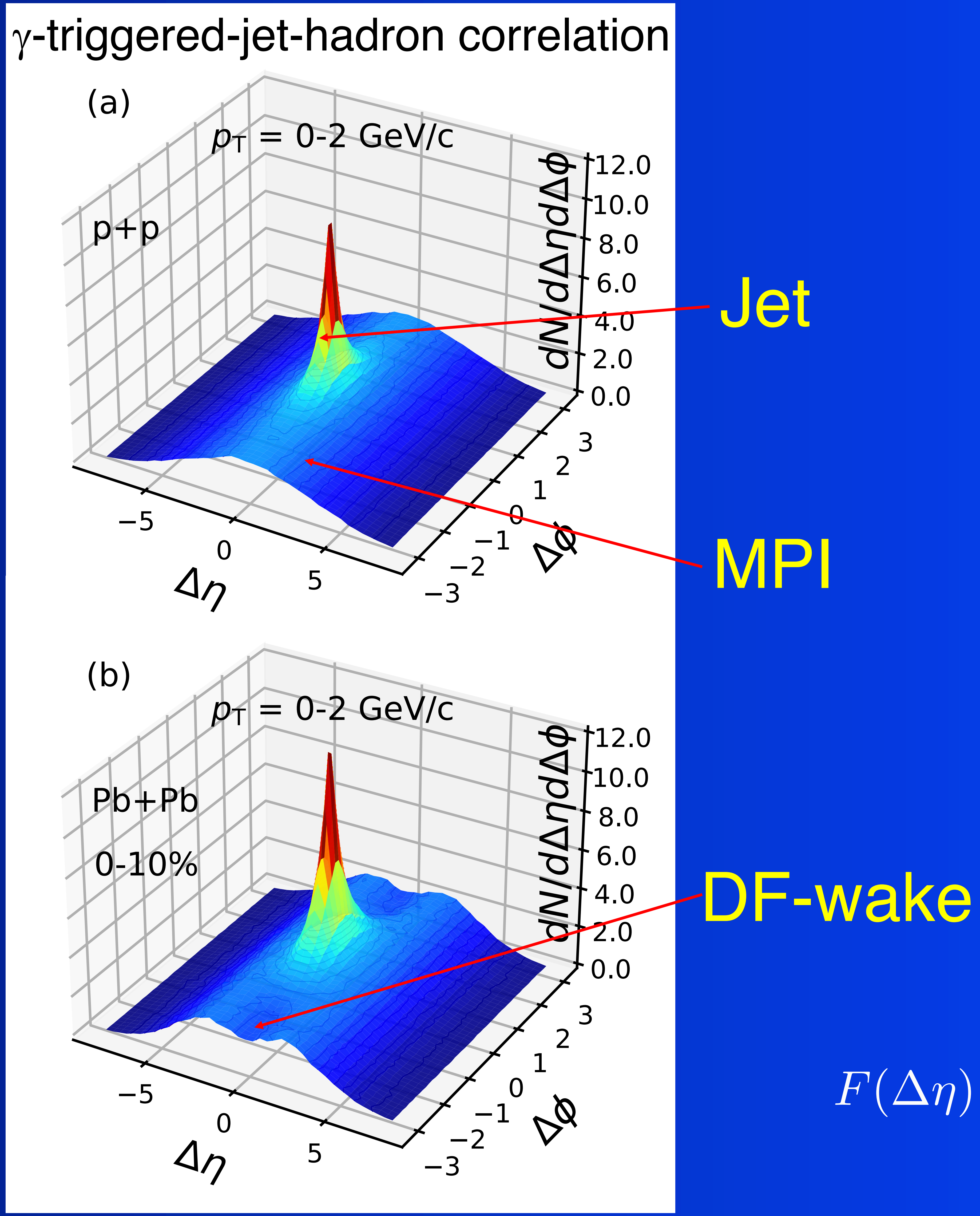
Search for jet-induced diffusion wake

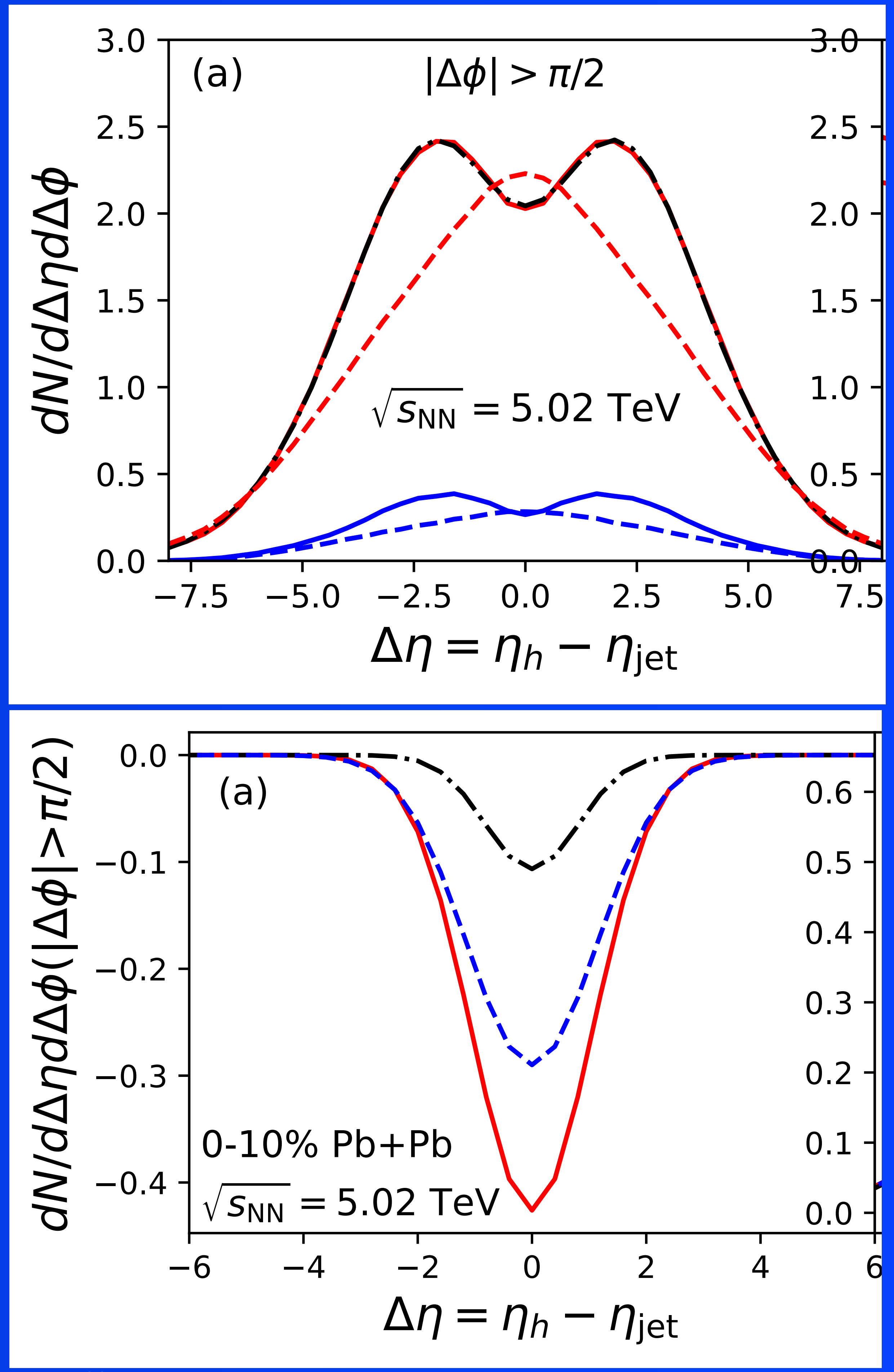
Diffusion (DF) wake leads to depletion of soft hadron yield in the back of jet direction

Yang, Tan, Chen,Pang & XNW, *PRL,* 130 (2023) , 052301



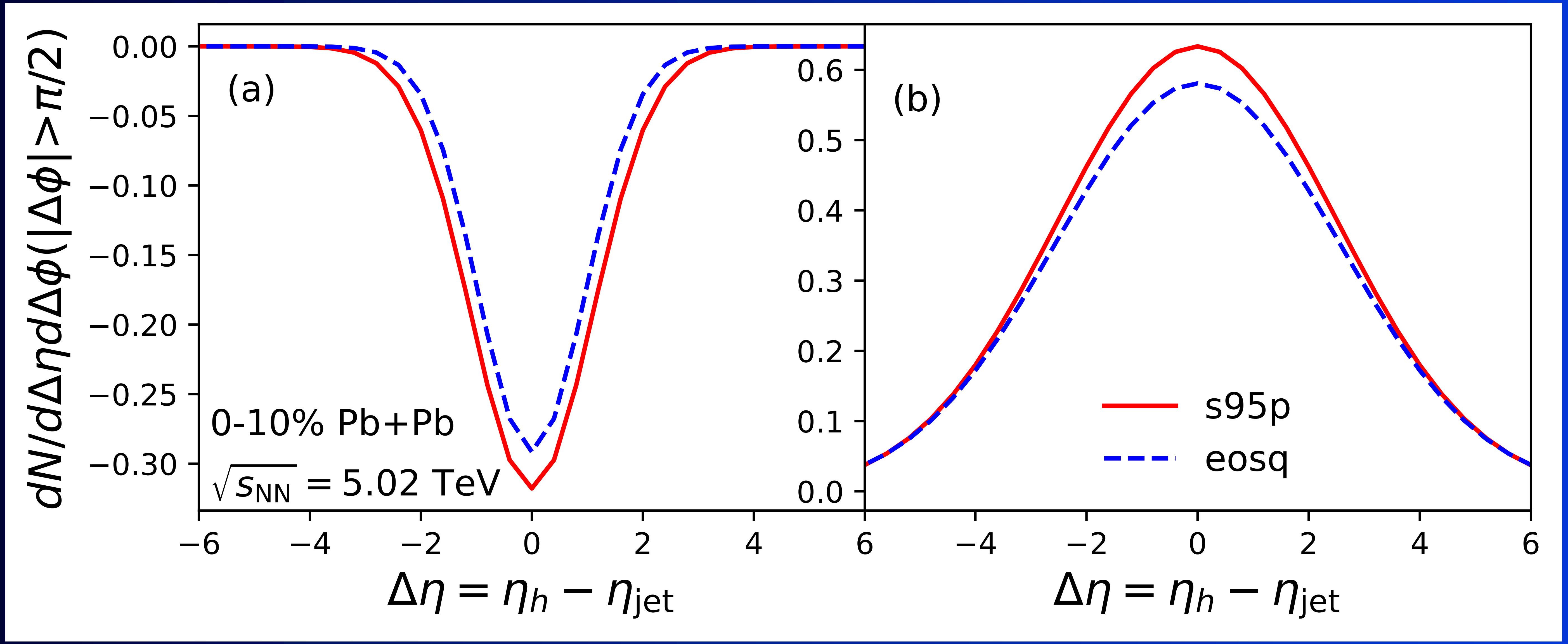
γ-triggered-jet-hadron correlation





$$F(\Delta\eta) = \int_{\eta_{j1}}^{\eta_{j2}} d\eta_j \underset{\substack{\uparrow\\ \text{Jet-distr}}}{F_3(\eta_j)}(\underset{\substack{\uparrow\\ \text{MPI}}}{F_2(\Delta\eta,\eta_j)} + \underset{\substack{\uparrow\\ \text{DF-wake}}}{F_1(\Delta\eta)}),$$

Sensitivity to EoS and shear viscosity

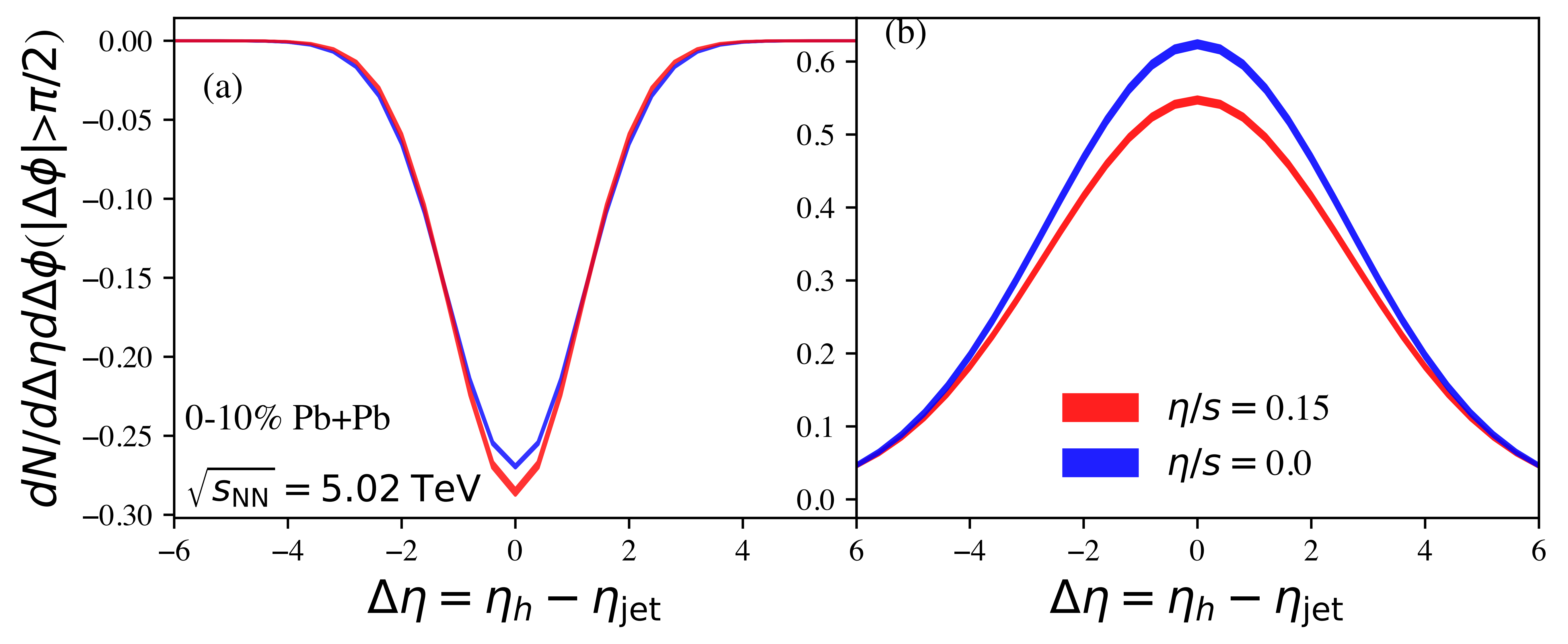


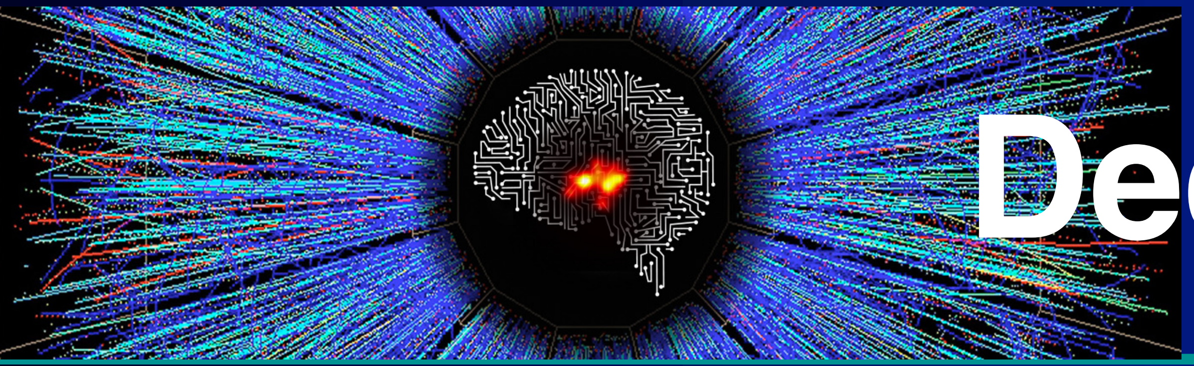
eosq: first order
s95p: rapid crossover from LQCD

Larger effective c_s in eosq → :
larger Mach cone angle → shallower DF valley
Stronger radial flow → smaller soft MPI

Competition of:

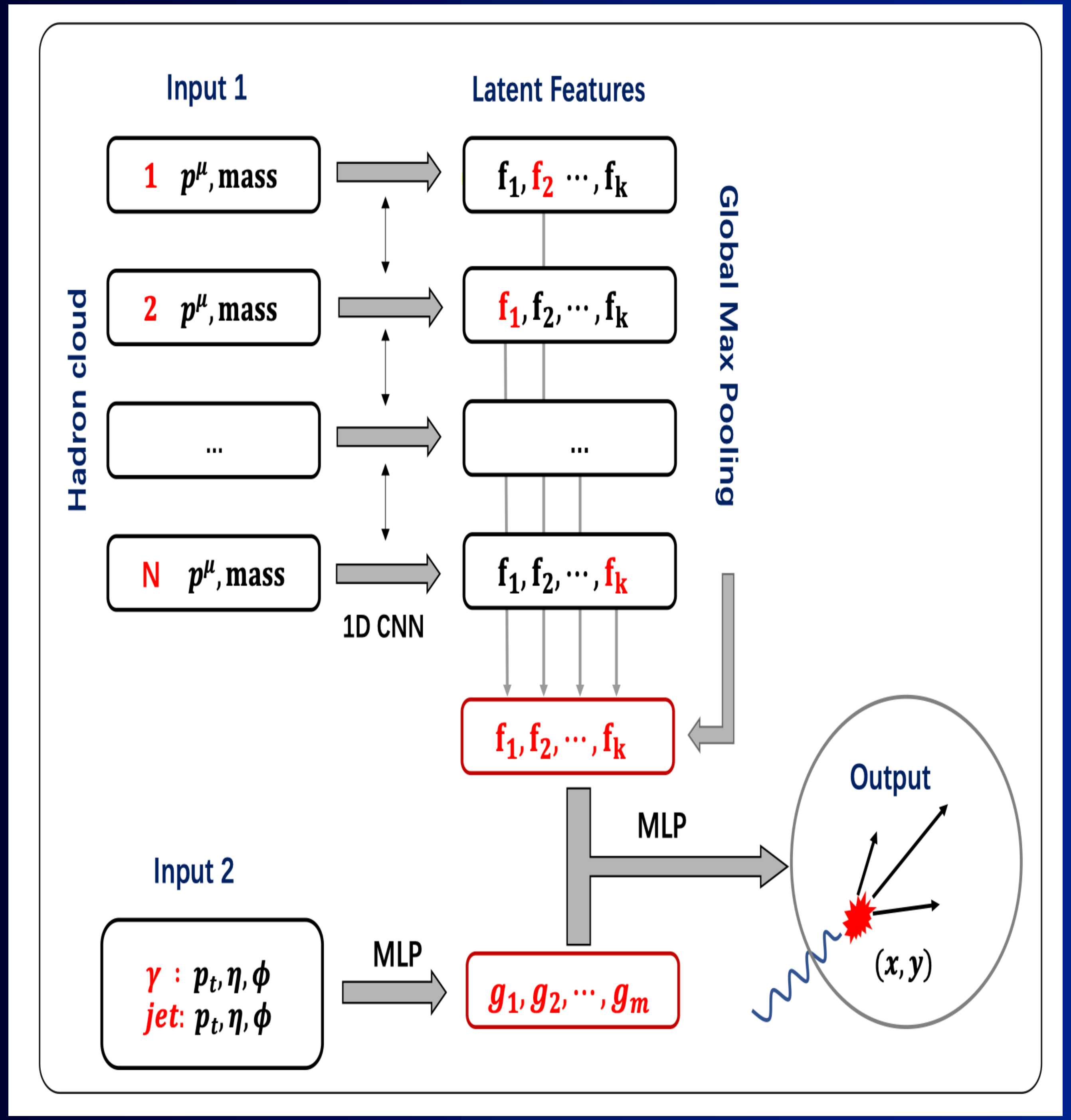
- η/s increase transverse flow → suppression of soft MPI and DF valley
- Negative shear correction of longitudinal pressure → impede longitudinal expansion → increase MPI and DF valley





Deep learning assisted jet tomography

PCN (point cloud network)



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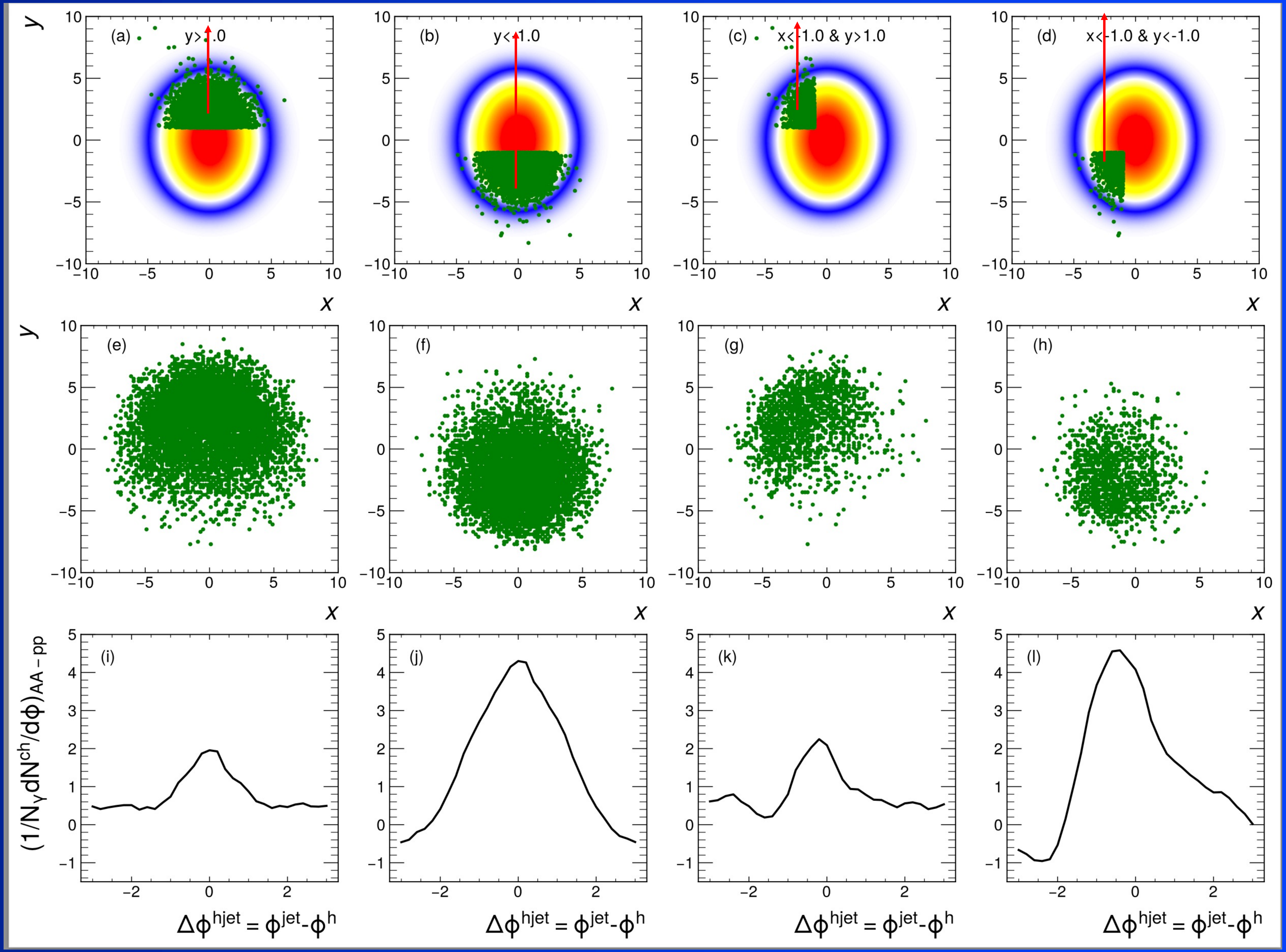
Yang, He，Chen，Ke，Pang & XNW



DL network selection

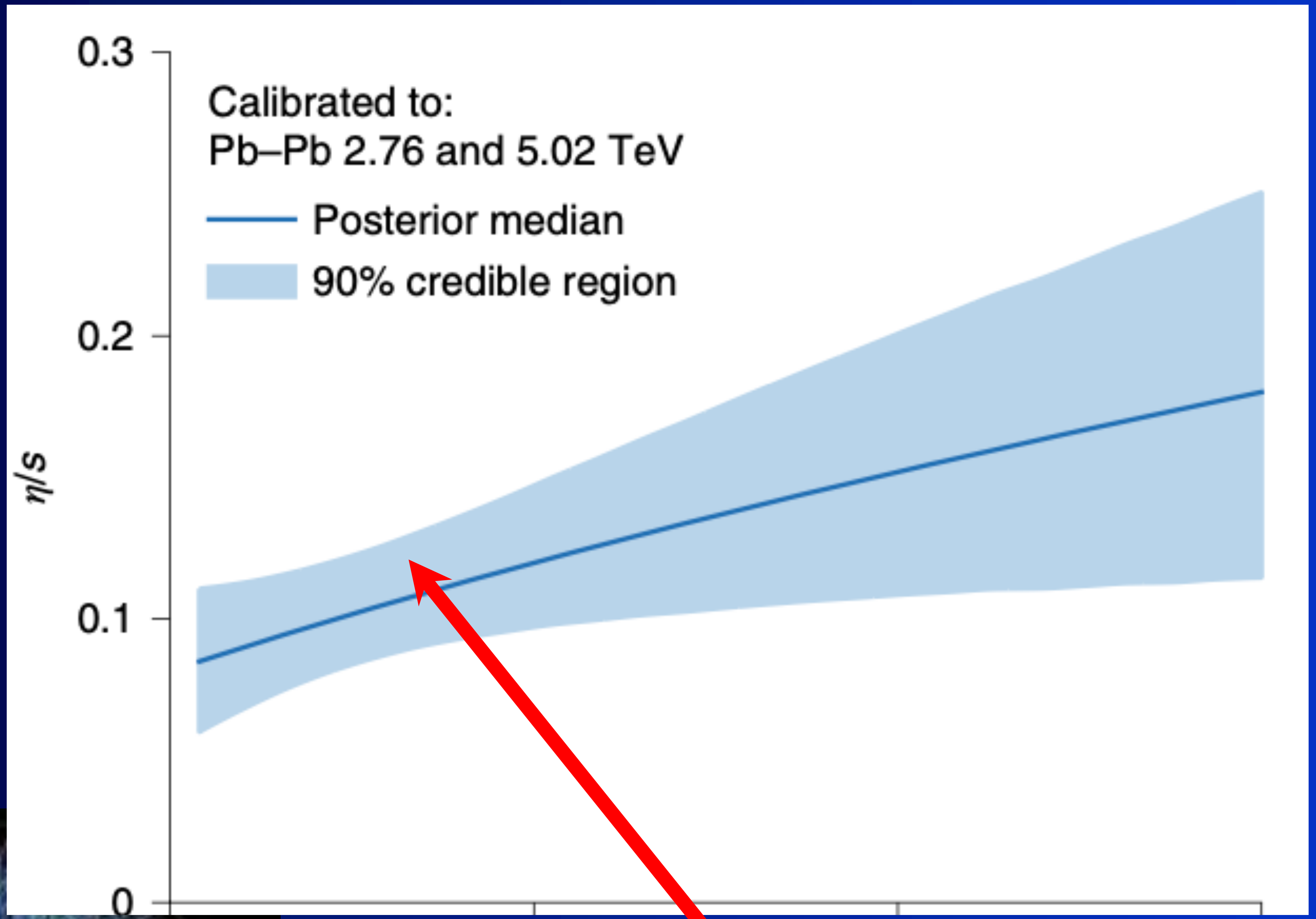
Actual distribution

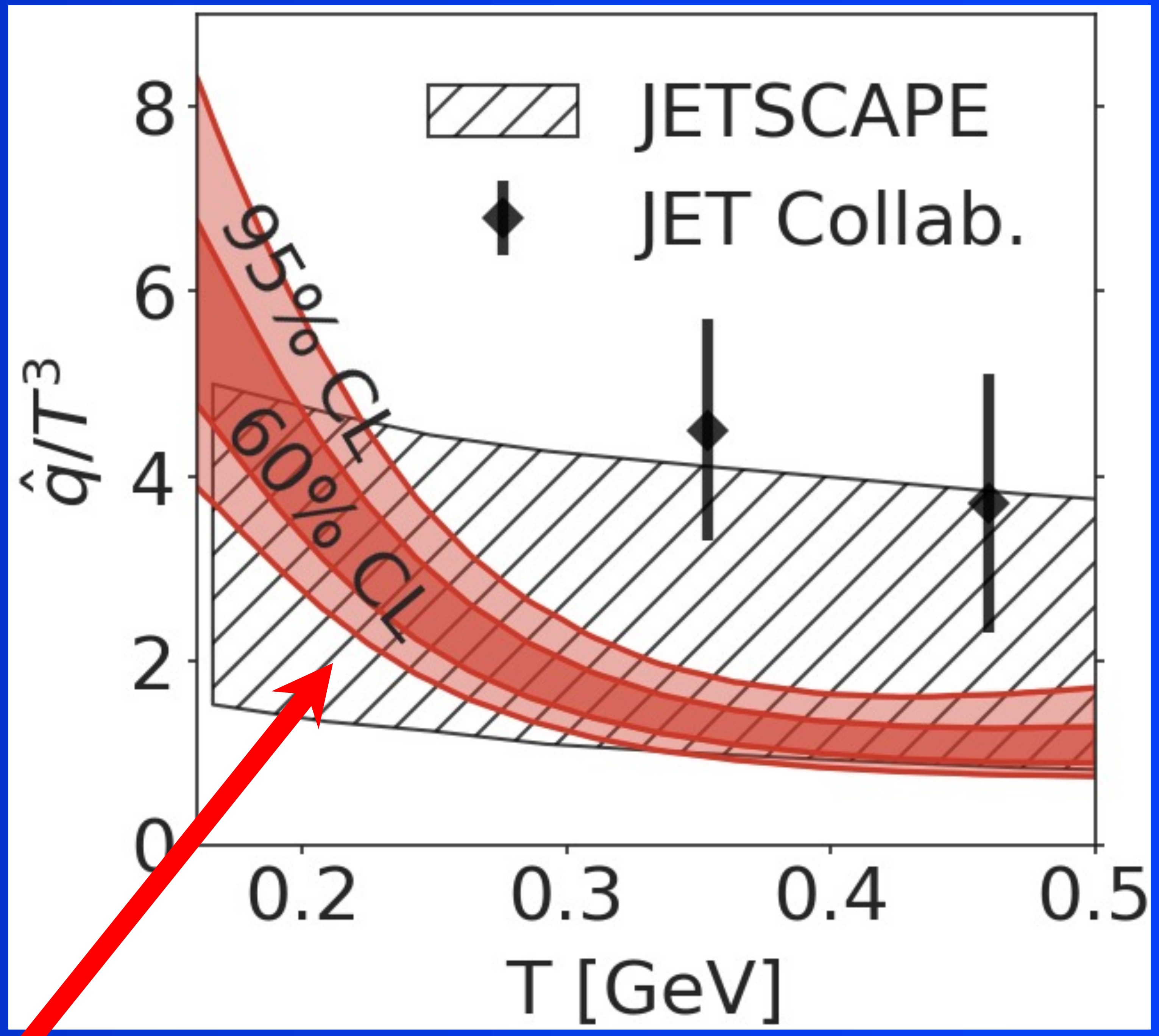
γ-soft hadron correlation

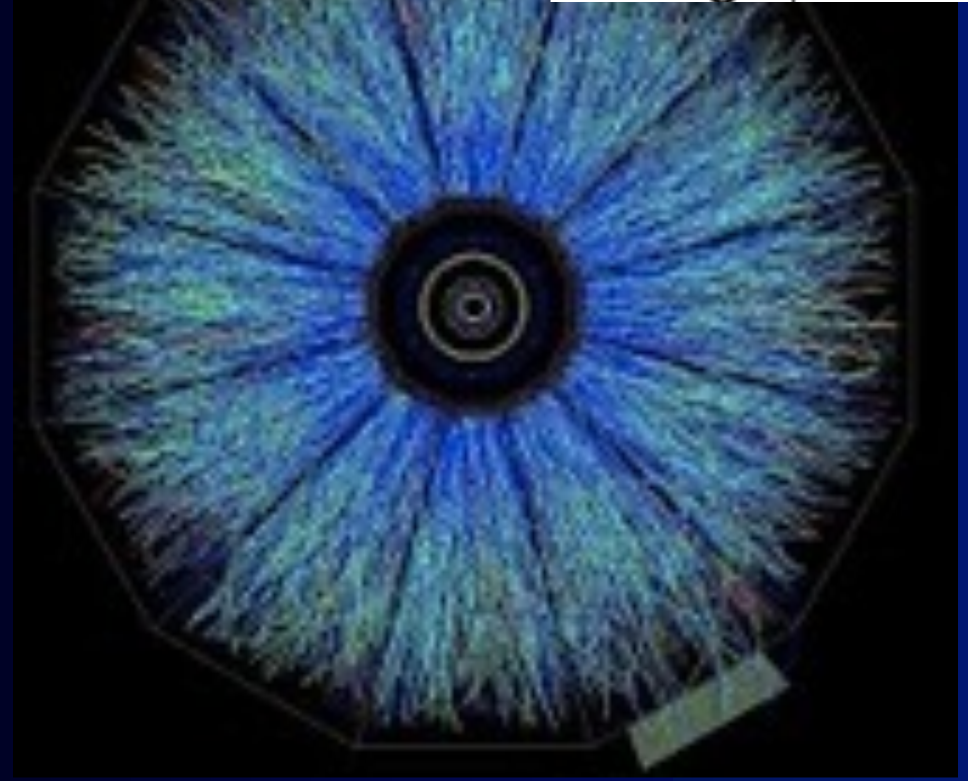
Jet and fluid transport property

Shear viscosity

jet transport





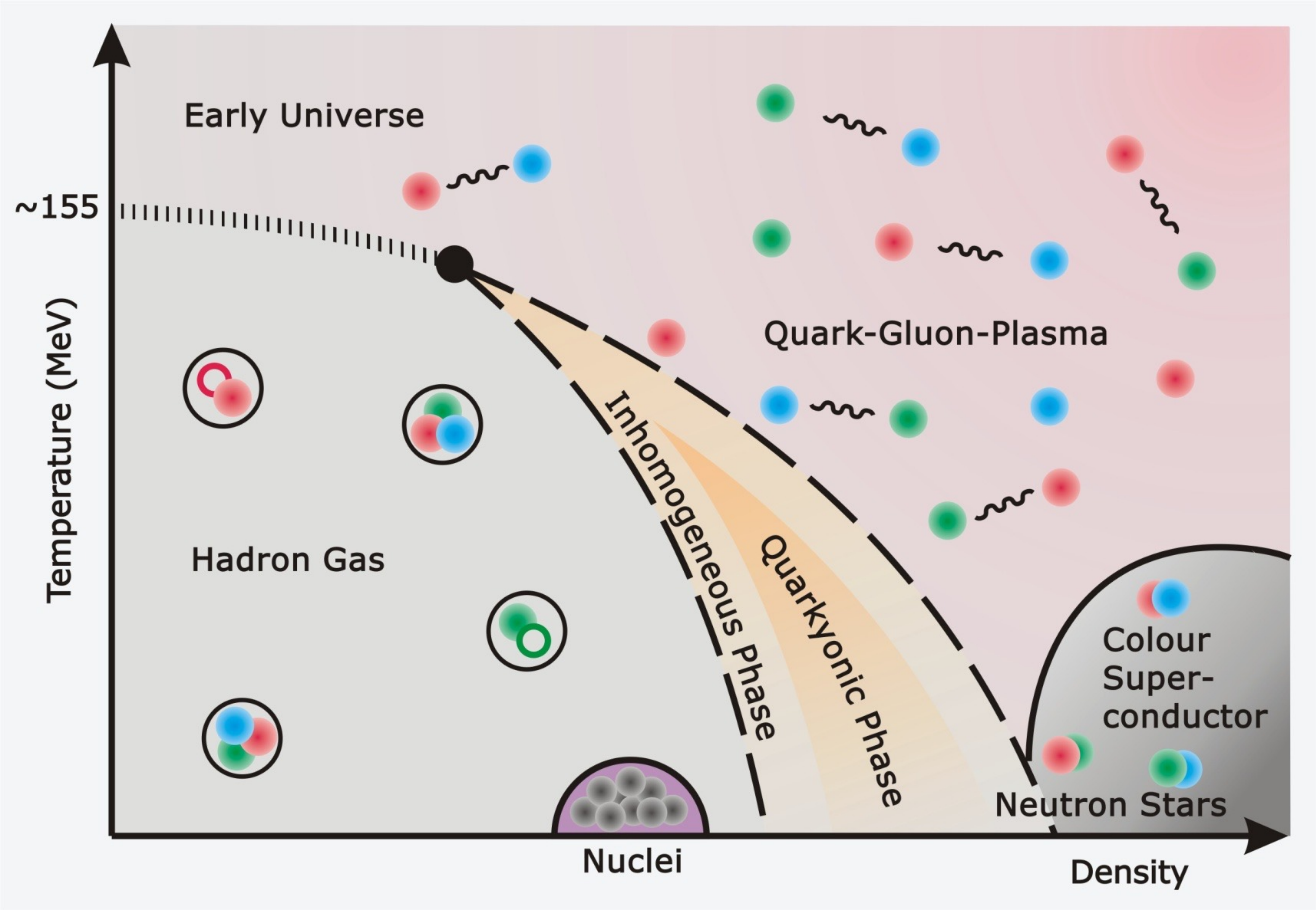


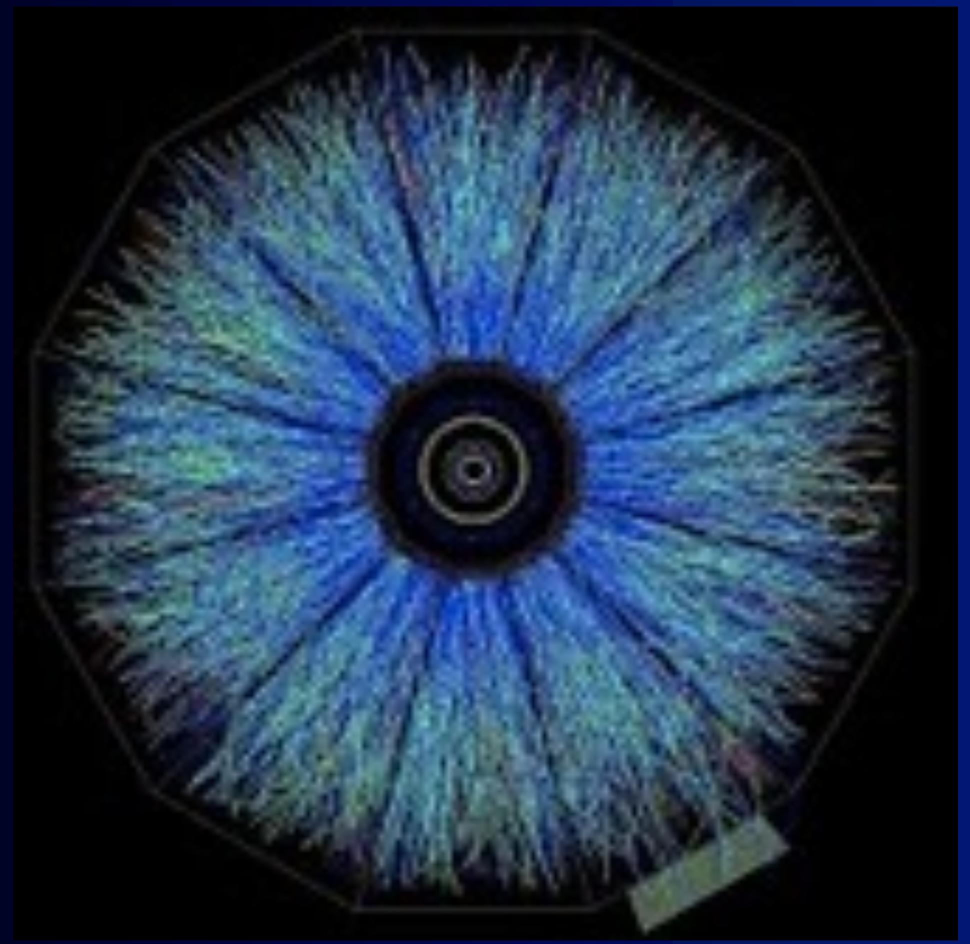
Connection? Majumder, Muller & XNW, PRL 99, 192301 (2007)

Perfect fluid? strongly coupled?

Mapping out the phase diagram of nuclear matter

QGP: The most perfect, most vortical and most opaque fluid

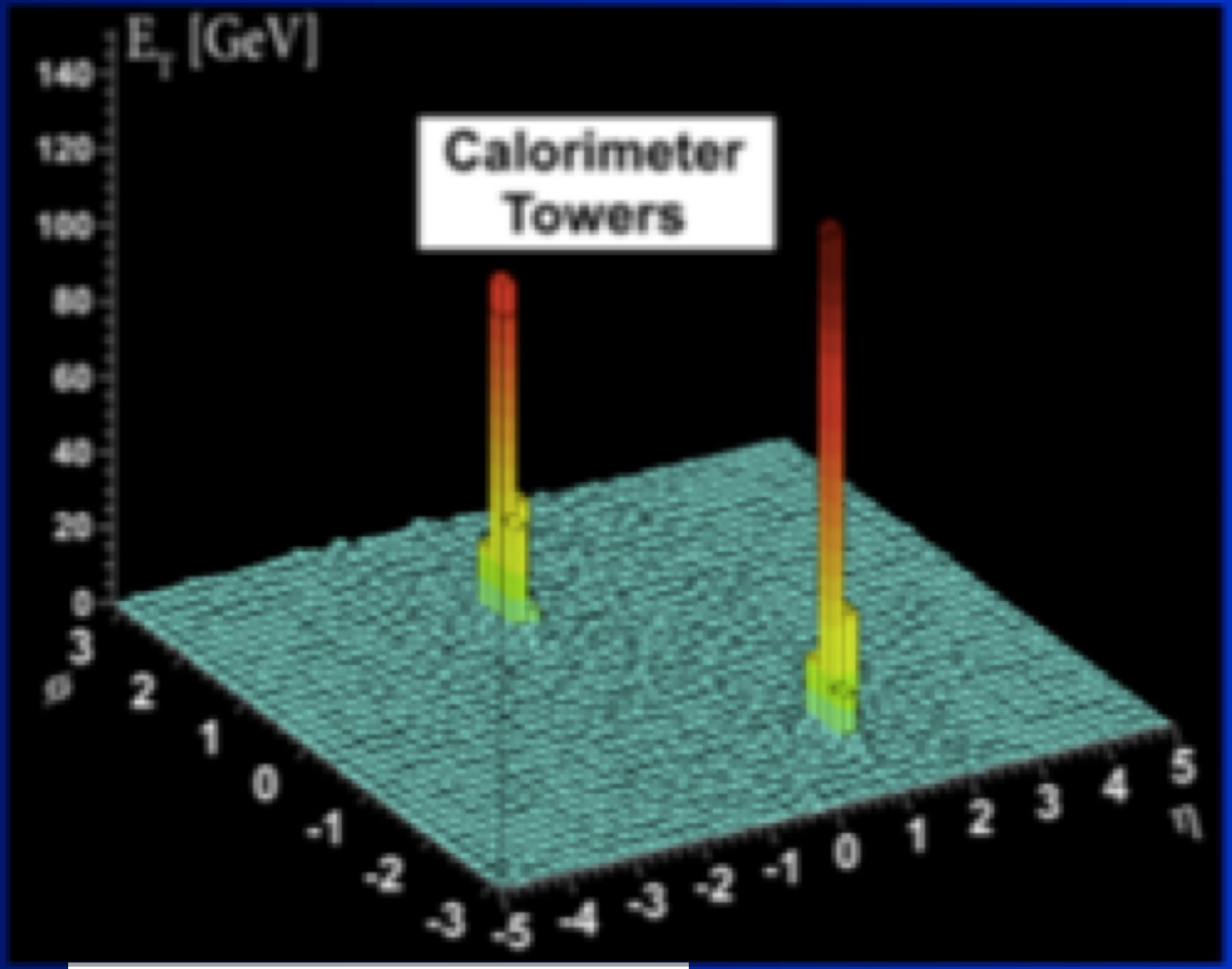


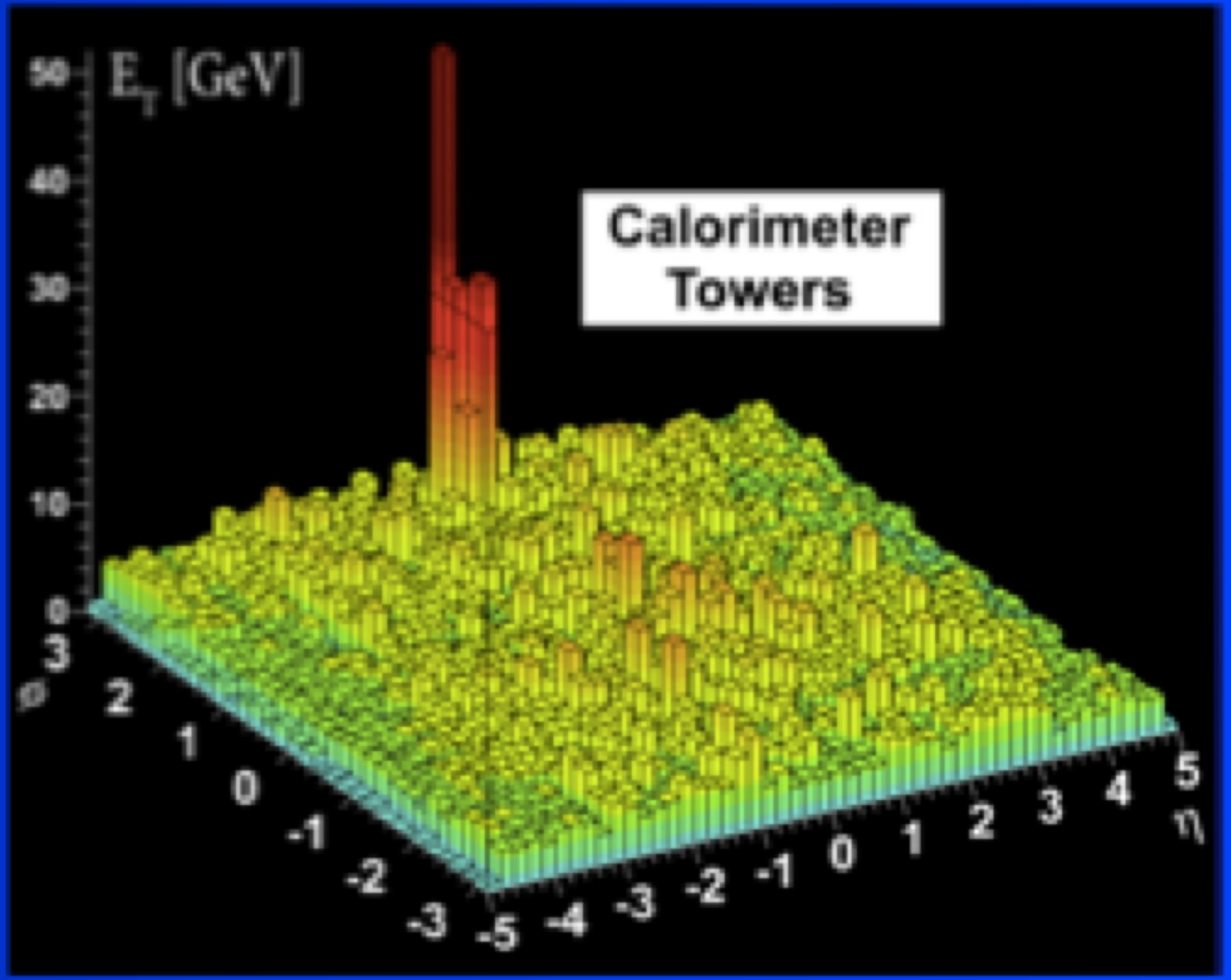


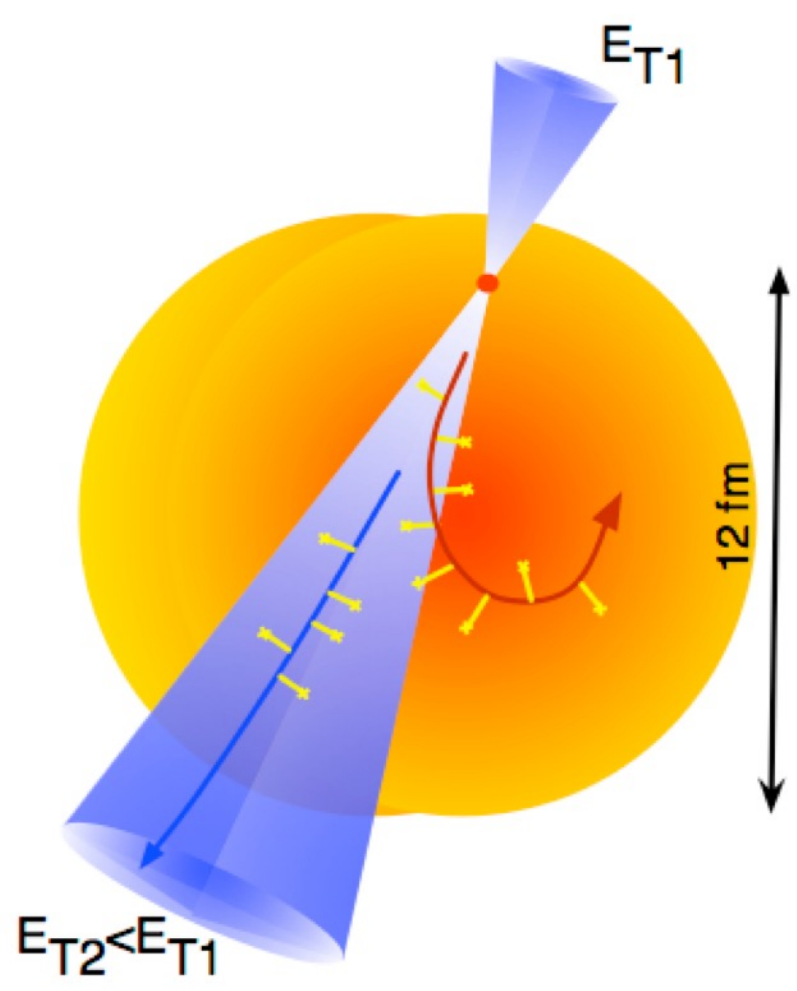
Happy 50th Birthday to QCD!

Many happy returns in the future…

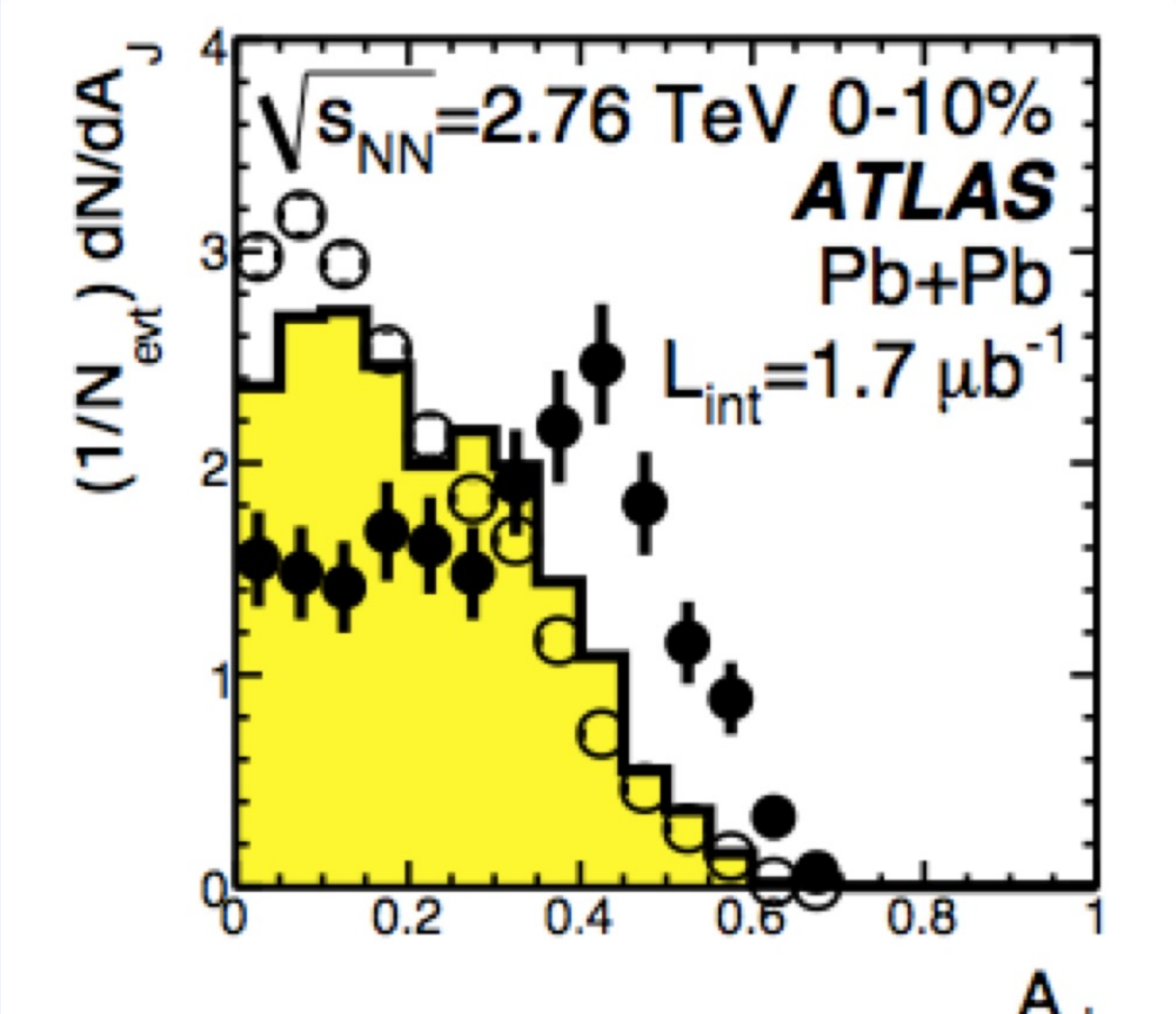
Jet quenching at LHC



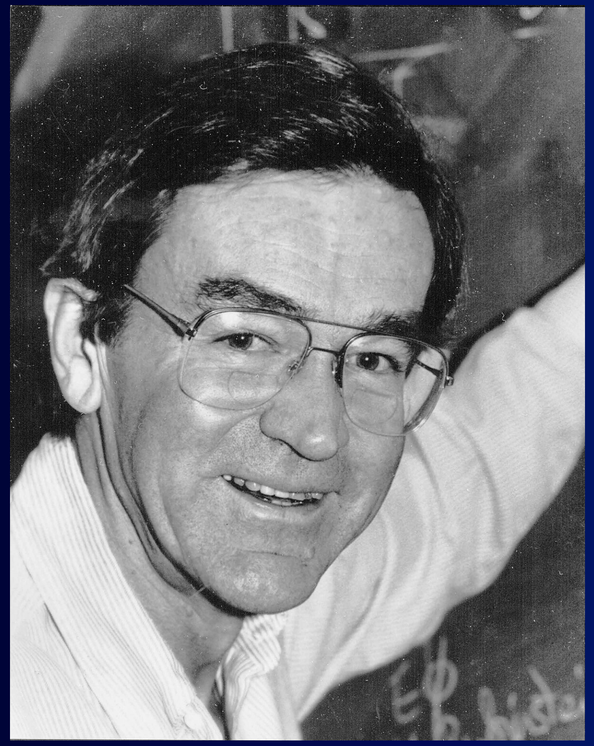




$$A_J = \frac{E_{T1} - E_{T2}}{E_{T1} + E_{T2}}$$



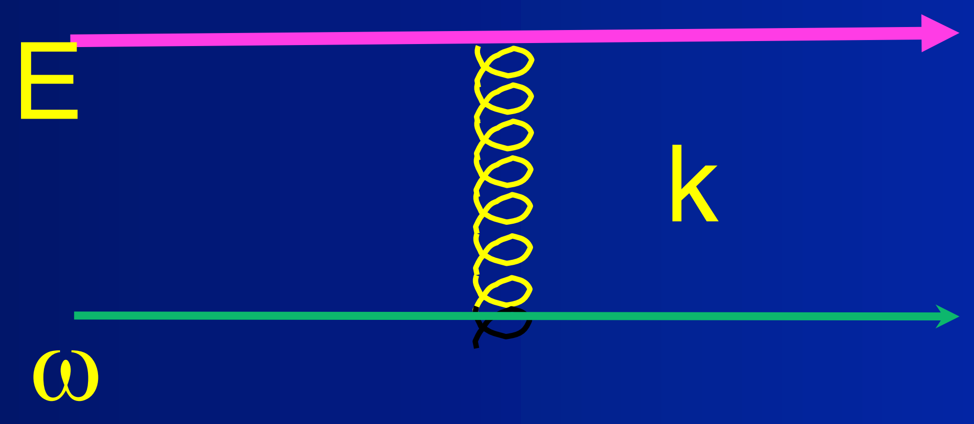
Parton propagation in QCD medium



Elastic parton energy loss:

Bjorken (1982)

Thoma & Gyulassy (1990)



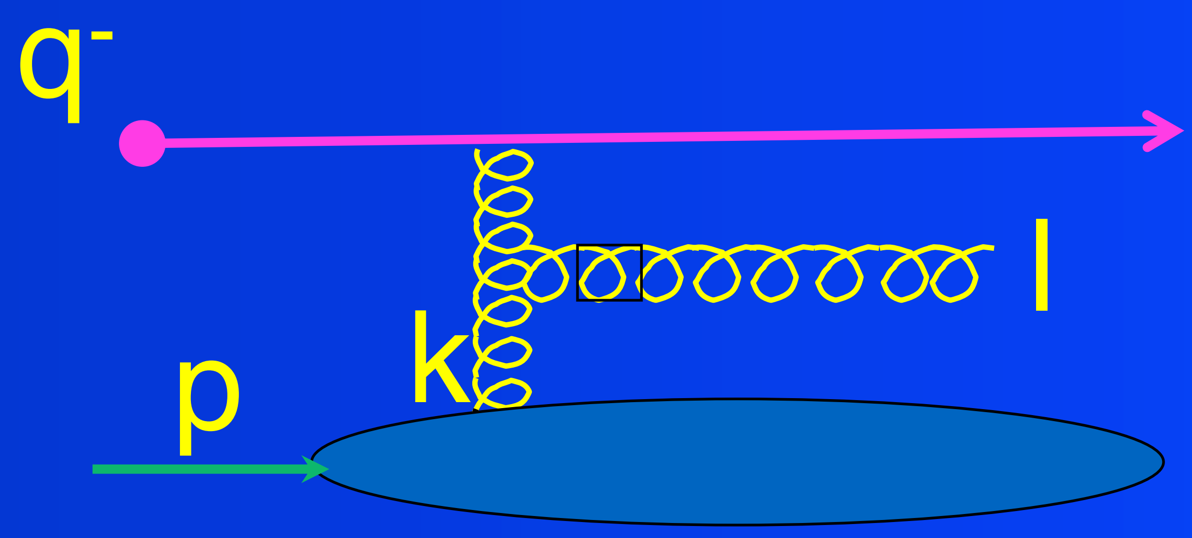
$$\frac{dE^a_{el}}{dx} = \sum_b \int d\omega f_b(\omega/T) \int dk_\perp^2 \frac{d\sigma_{ab}}{dk_\perp^2} k_0$$

$$k_0 \approx k_\perp^2/2\omega$$

$$\approx C_a \frac{3\pi}{2} \alpha_s^2 T^2 \log \frac{2.6ET}{4\mu_D^2}$$

Inelastic parton energy loss:

Gyulassy & XNW (1994), BDMPS (1995), Zakharov (1996)



Y Zhang & XNW
arXiv: 2104.04520

$$\frac{dN_g}{dl_\perp^2\, dz} = \int_{y^-}^{\infty} dy_1^- \left[\rho_A(y_1^-, \vec{y}_\perp) \frac{2\pi\alpha_s}{N_c} \pi \int \frac{dk_\perp^2}{(2\pi)^2} \frac{\phi_N(0, \vec{k}_\perp)}{k_\perp^2} \right] \times \pi \frac{\alpha_s}{2\pi} P_{qg}(z) \frac{C_A}{l_\perp^2} \mathcal{N}_g(\vec{l}_\perp, \vec{k}_\perp)$$

$$\mathcal{N}_g^{static+soft} = \int \frac{d\varphi}{2\pi} \frac{2\vec{k}_\perp \cdot \vec{l}_\perp}{(\vec{l}_\perp - \vec{k}_\perp)^2} \left(1 - \cos\left[\frac{(\vec{l}_\perp - \vec{k}_\perp)^2}{2q^- z(1-z)} y_1^- \right] \right)$$

τ_f Gluon formation time y_1^-/τ_f