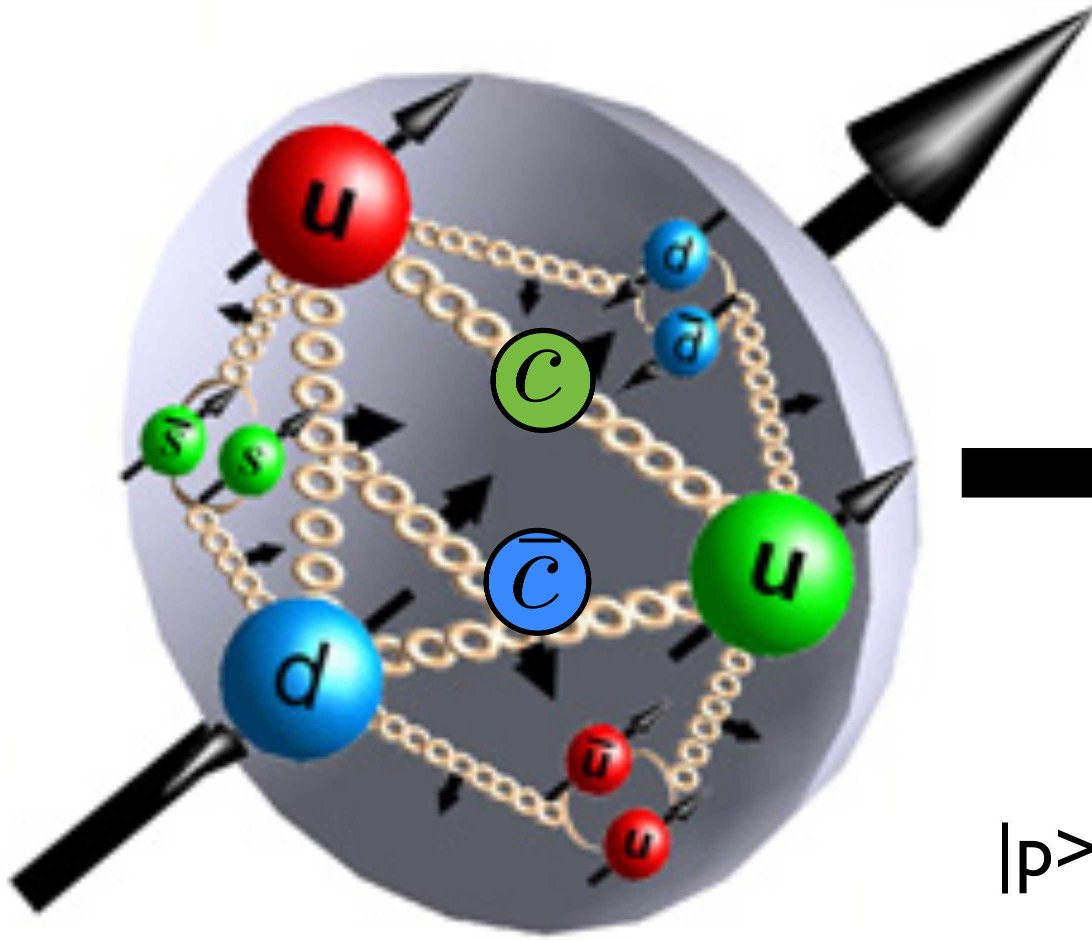
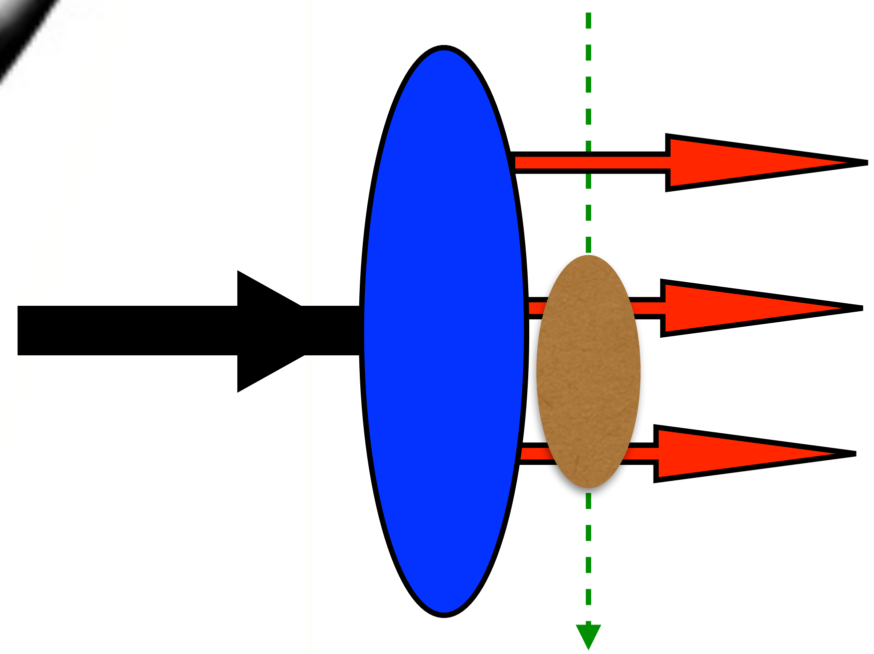
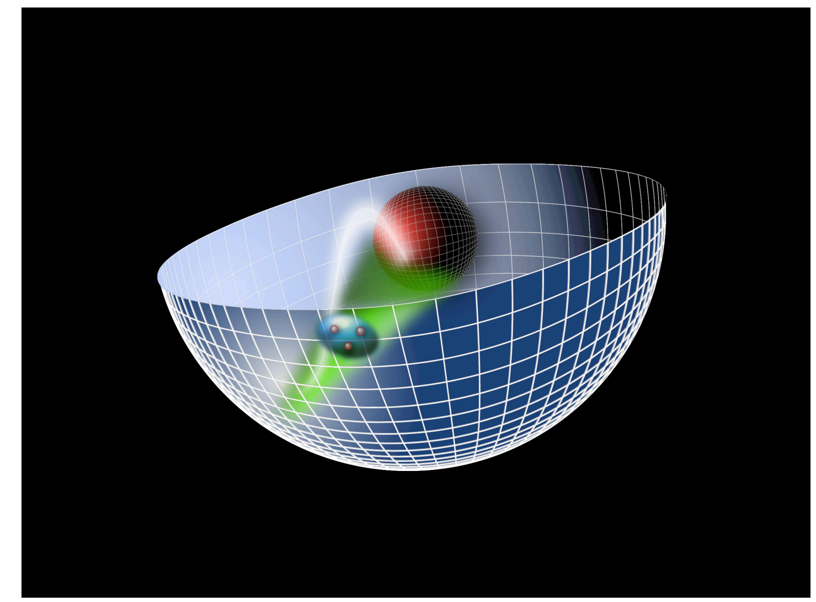
Holographic Light-Front QCD: A Novel Nonperturbative Approach to Color Confinement and Hadron Physics



Fixed $\tau = t + z/c$





|p>=|u[ud]> quark-diquark cluster

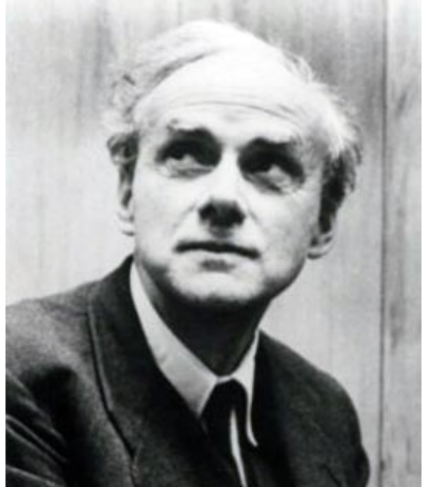
with Guy de Tèramond, Hans Günter Dosch, Cèdric Lorcè, Alexandre Deur, and Joshua Erlich

Fifty Years QCD
September 15, 2023
UCLA

Stan Brodsky

SLAC NATIONAL ACCELERATOR LABORATORY

Light-Front Quantization



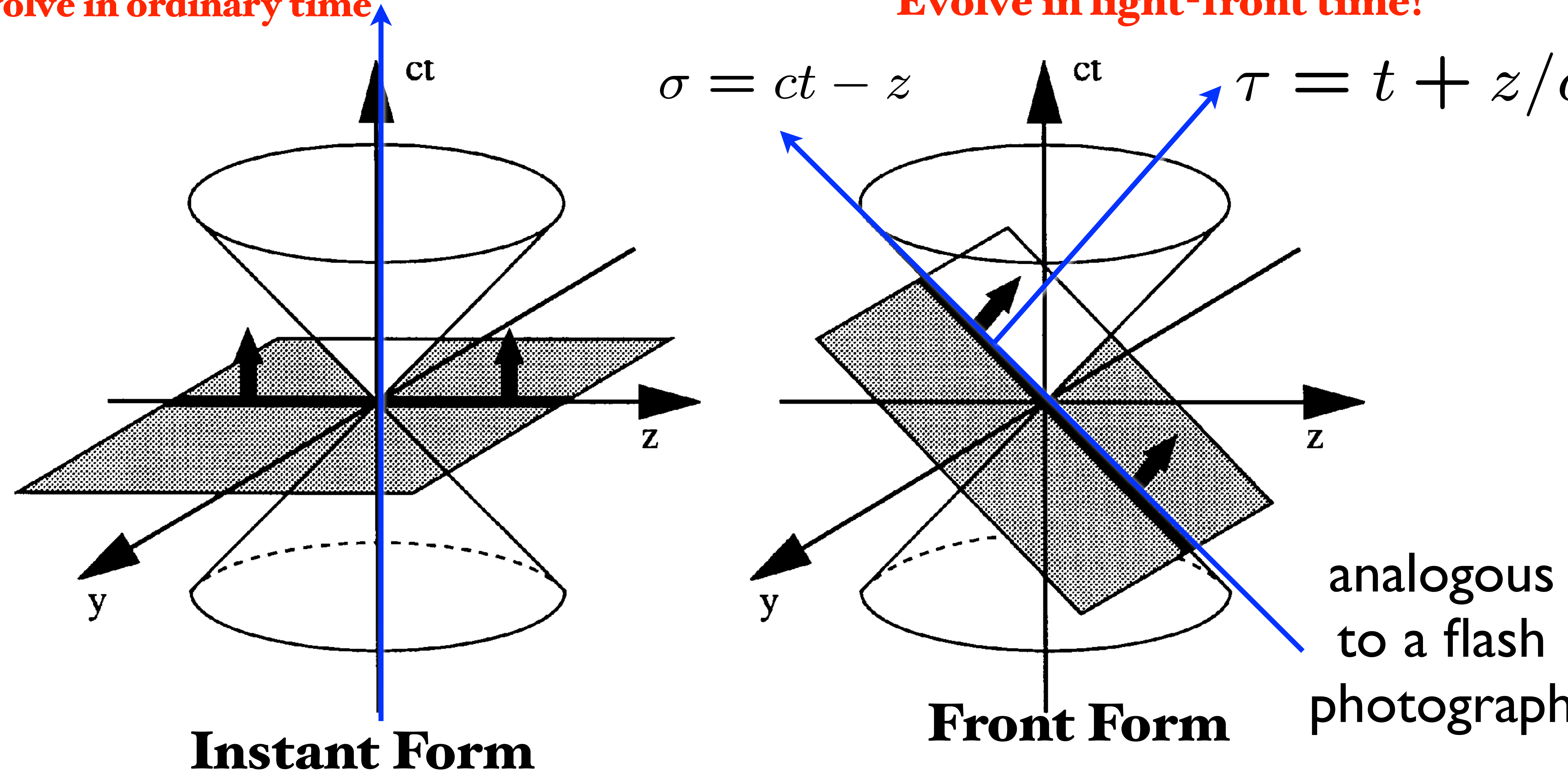
P.A.M Dirac, Rev. Mod. Phys. 21, 392 (1949)

Dirac's Amazing Idea: The "Front Form"

Evolve in ordinary time

Evolve in light-front time!

$$\sigma = ct - z \qquad \tau = t + z/c$$



Causal, Boost Invariant!

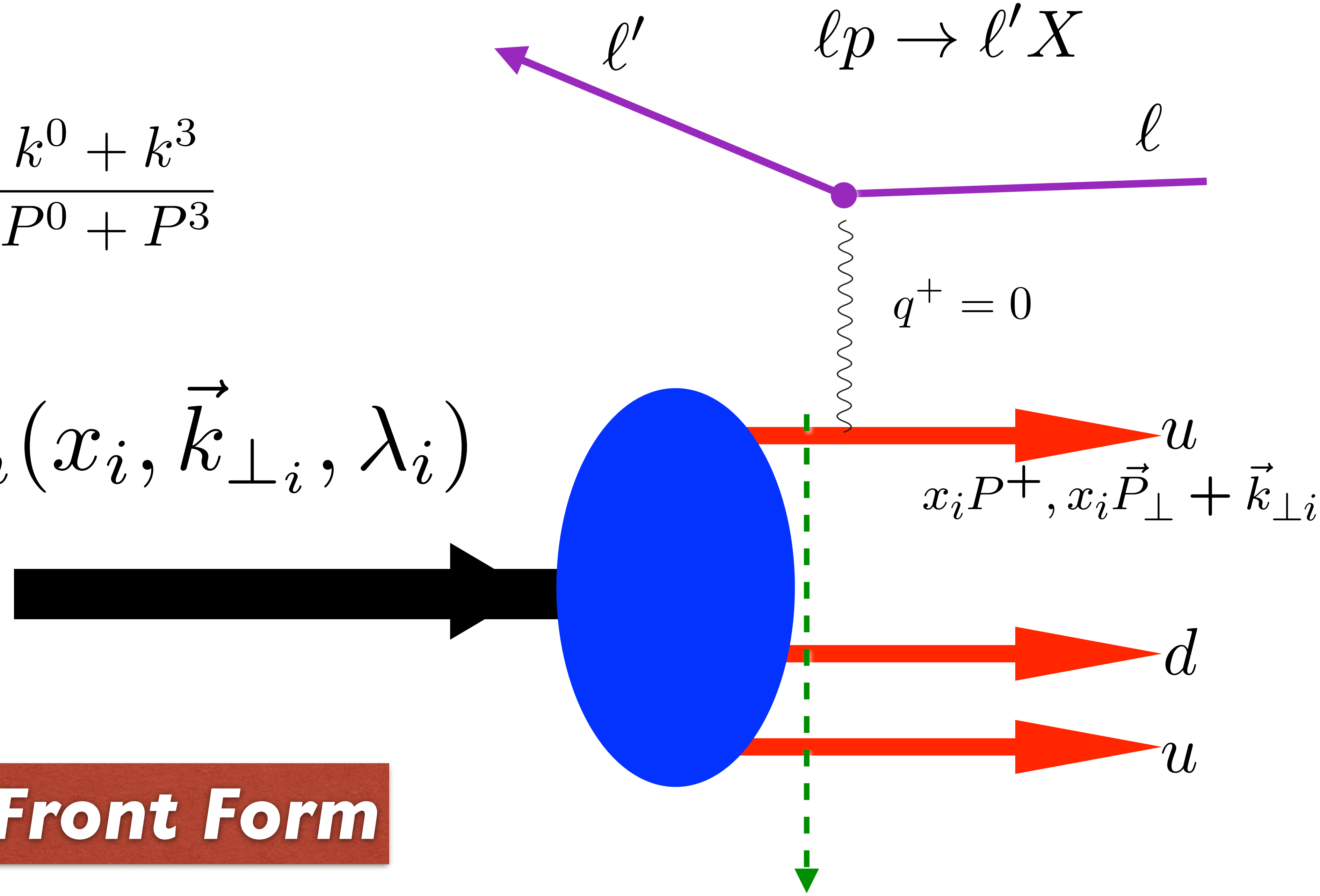
Comparing light-front quantization with instant-time quantization
Philip D. Mannheim(Connecticut U.),
Peter Lowdon(Ecole Polytechnique, CPHT),
Stanley J. Brodsky(SLAC)

- e-Print: 2005.00109 [hep-ph]

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$

$$\psi_n(x_i, \vec{k}_{\perp_i}, \lambda_i)$$

$P^+, \vec{P}_\perp$



Dirac's Front Form

Measurements of hadron LF wavefunction are at fixed LF time

Fixed $\tau = t + z/c$

Like a flash photograph

$$x_{bj} = x = \frac{k^+}{P^+}$$

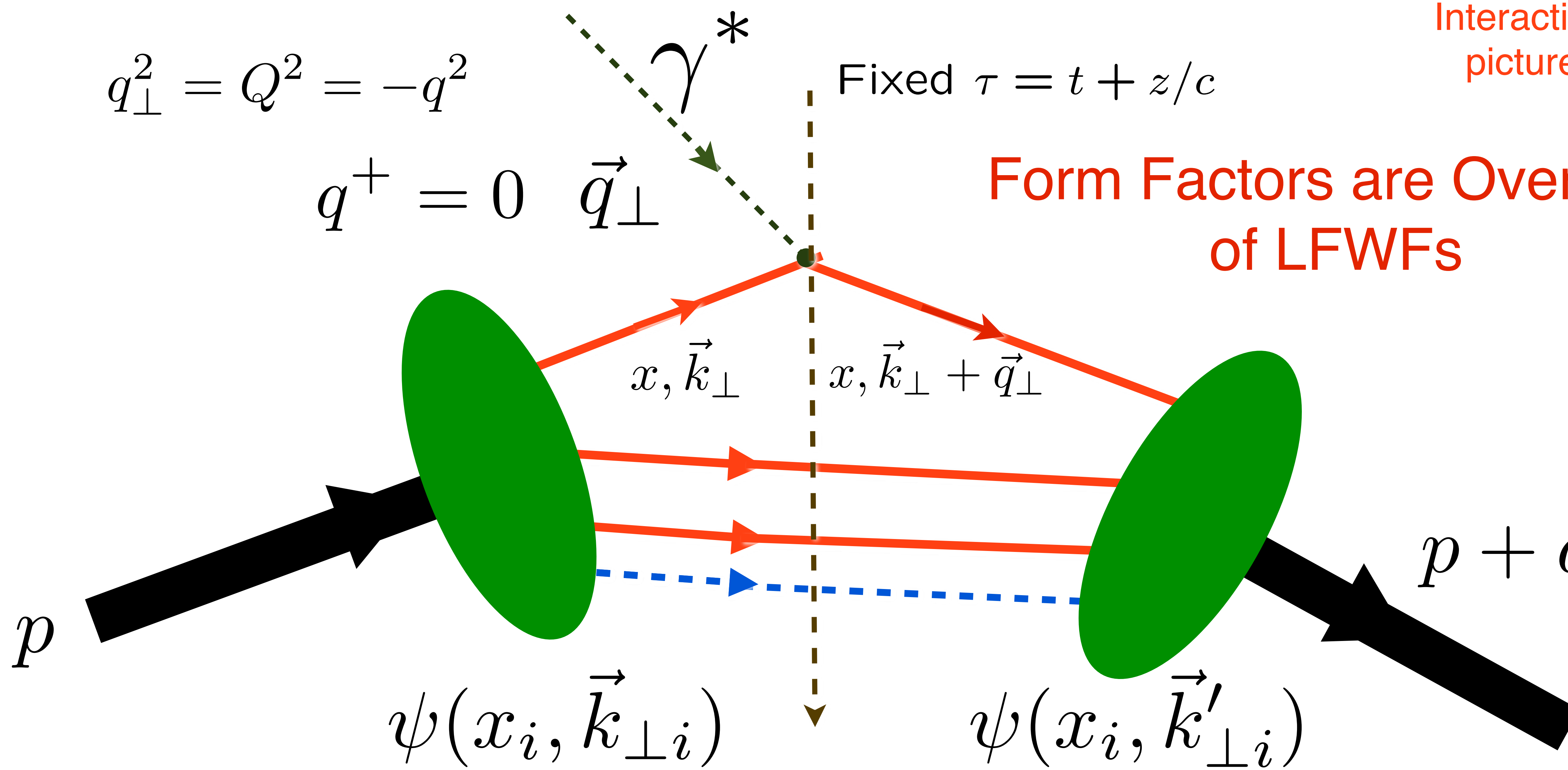
Invariant under boosts! Independent of P^μ

$$< p+q|j^+(0)|p >= 2p^+F(q^2)$$

Front Form

Interaction picture

Form Factors are Overlaps of LFWFs



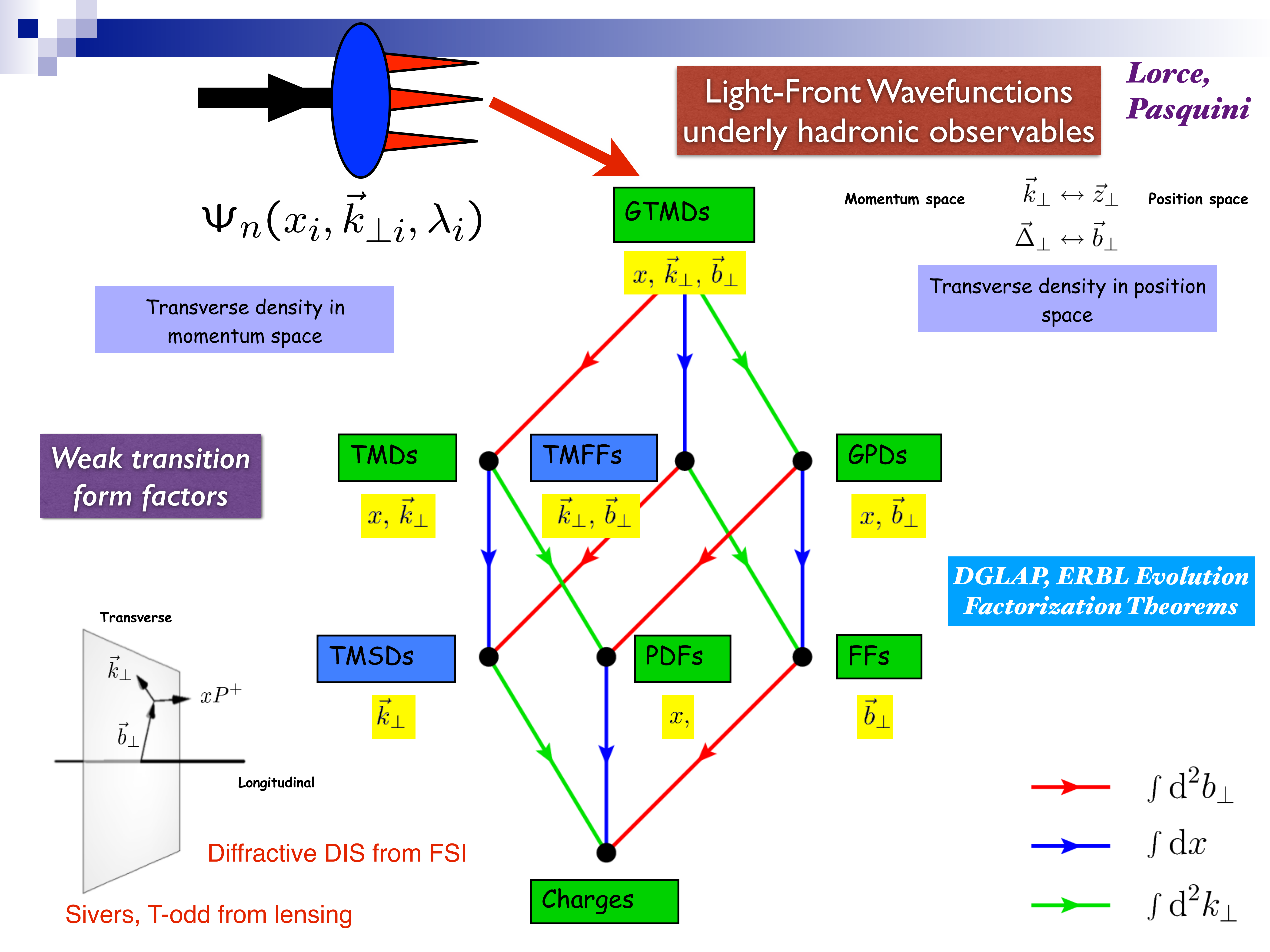
struck $\quad \vec{k}'_{\perp i} = \vec{k}_{\perp i} + (1 - x_i)\vec{q}_\perp$

spectators $\quad \vec{k}'_{\perp i} = \vec{k}_{\perp i} - x_i\vec{q}_\perp$

Drell &Yan, West
Exact LF formula!

Drell, sjb

Transverse size $\propto \frac{1}{Q}$



Light-Front Wavefunctions underly hadronic observables
Lorce, Pasquini
$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$
Momentum space
$\vec{k}_\perp \leftrightarrow \vec{z}_\perp$
$\vec{\Delta}_\perp \leftrightarrow \vec{b}_\perp$
Position space
GTMDs
$x, \vec{k}_\perp, \vec{b}_\perp$
Transverse density in momentum space
Transverse density in position space
Weak transition form factors
TMDs
$x, \vec{k}_\perp$
TMFFs
$\vec{k}_\perp, \vec{b}_\perp$
GPDs
$x, \vec{b}_\perp$
DGLAP, ERBL Evolution Factorization Theorems
Transverse
$\vec{k}_\perp$
xP^+
$\vec{b}_\perp$
Longitudinal
TMSDs
$\vec{k}_\perp$
PDFs
$x,$
FFs
$\vec{b}_\perp$
Diffractive DIS from FSI
Sivers, T-odd from lensing
Charges
$\int d^2 b_\perp$
$\int dx$
$\int d^2 k_\perp$

Fixed $\tau = t + z/c$

Light-Front QCD

Physical gauge: $A^+ = 0$

Exact frame-independent formulation of nonperturbative QCD!

$$L^{QCD} \to H_{LF}^{QCD}$$

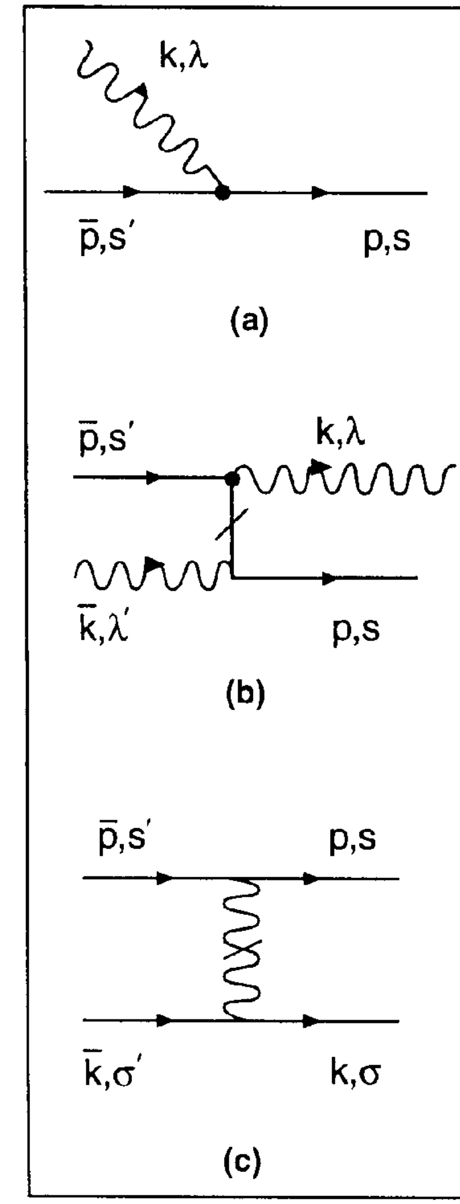
$$H_{LF}^{QCD} = \sum_i \left[\frac{m^2 + k_\perp^2}{x}\right]_i + H_{LF}^{int}$$

H_{LF}^{int}: Matrix in Fock Space

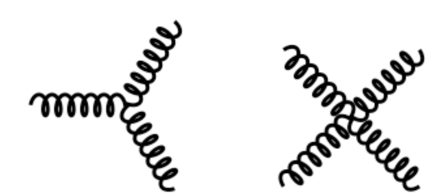
$$H_{LF}^{QCD} |\Psi_h> = \mathcal{M}_h^2 |\Psi_h>$$

$$|p, J_z> = \sum_{n=3} \psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) |n; x_i, \vec{k}_{\perp i}, \lambda_i>$$

Eigenvalues and Eigensolutions give Hadronic Spectrum and Light-Front wavefunctions



LFWFs: Off-shell in P- and invariant mass



H_{LF}^{int}

Solve nPQCD by matrix diagonalization: Hornbostel, Pauli, sjb

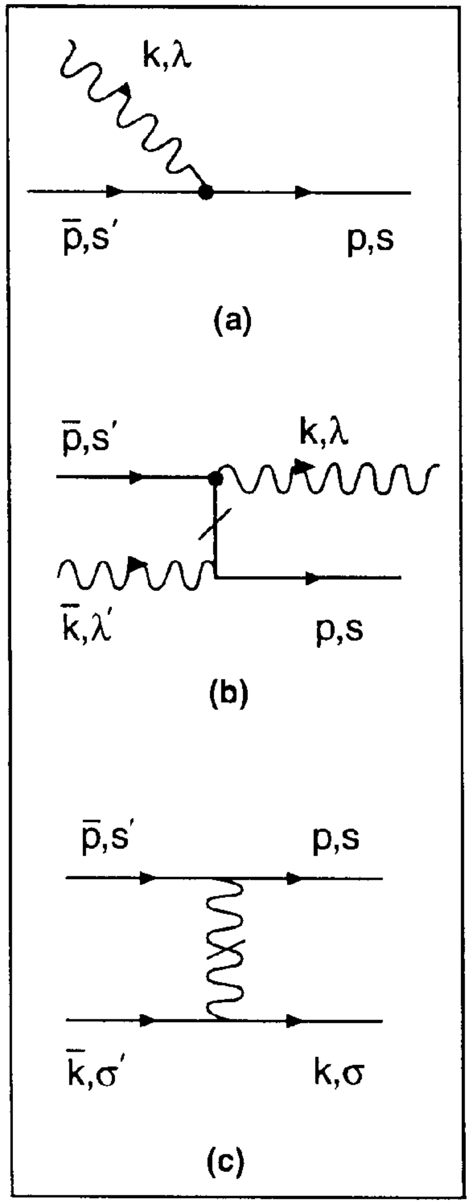
Light-Front QCD

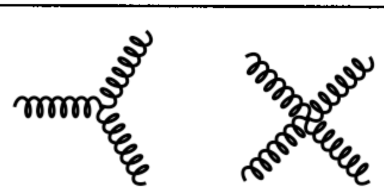
Heisenberg Equation

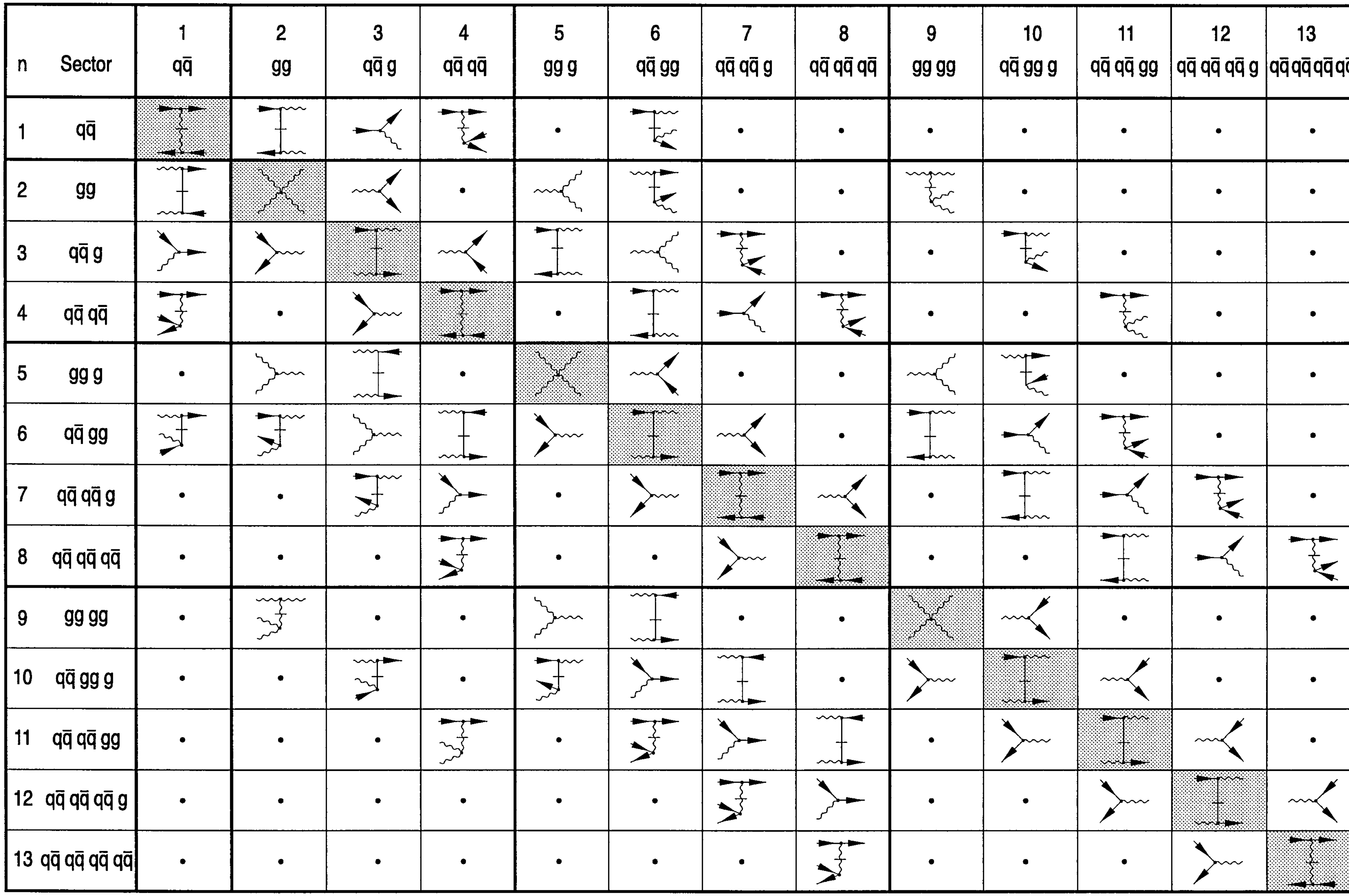
$$\boxed{H_{LC}^{QCD}|\Psi_h\rangle = \mathcal{M}_h^2\, |\Psi_h\rangle}$$

DLCQ: Solved QCD(1+1) for any quark mass and flavors

Hornbostel, Pauli, sjb







Minkowski space; frame-independent; no fermion doubling; no ghosts

Discretized LF Quantization

DLCQ: Diagonalize QCD Hamiltonian, periodic LF BC

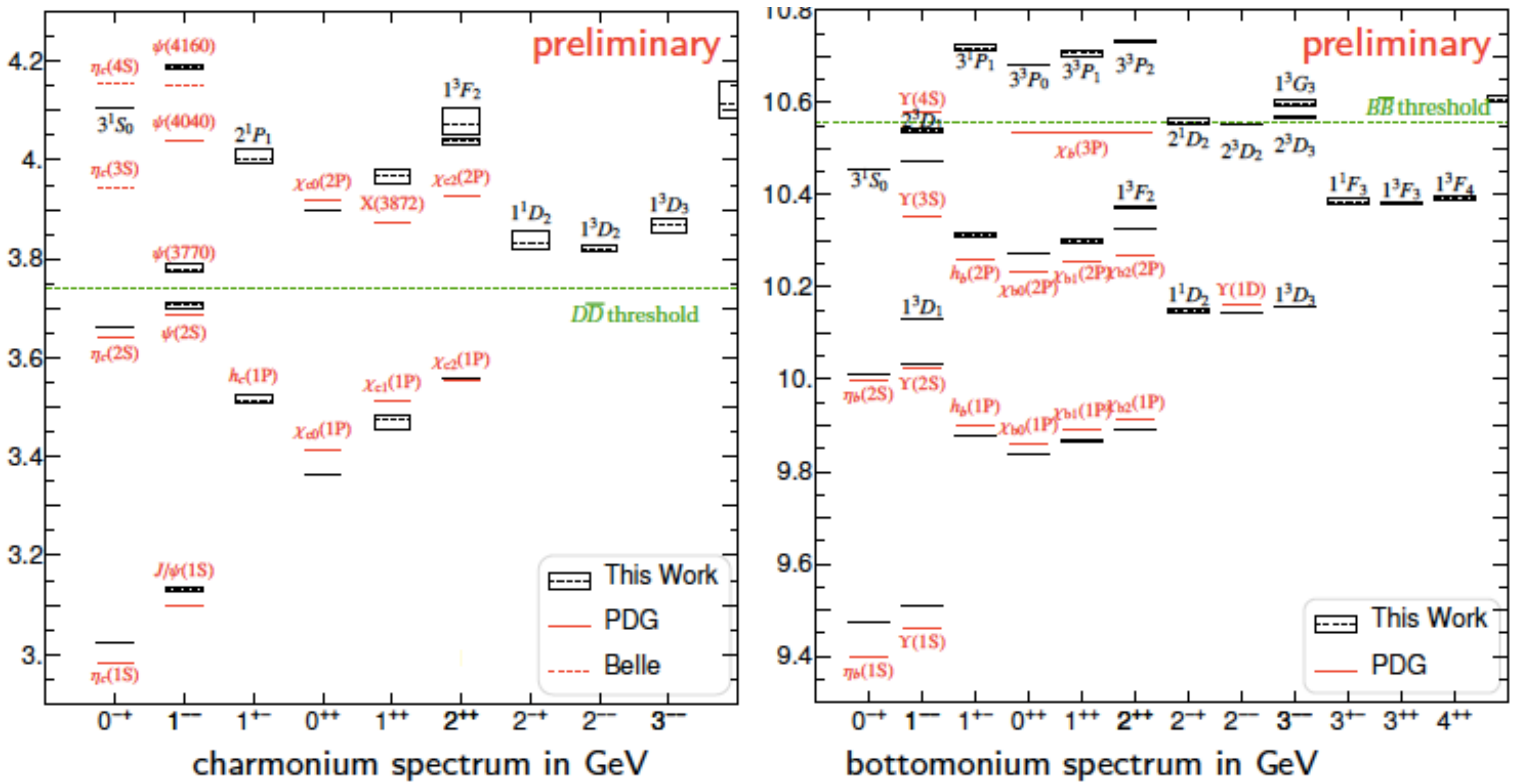
BLFQ (Vary et al)
Use LF Holographic Basis

Solve QCD by Matrix Diagonalization

Diagonalize the LF Hamiltonian on an Orthonormal Basis
Lorentz Frame-Independent,
Minkowski Causal LF Time
Compute Hadron masses, LF Wavefunctions
Successful applications to QCD(1+1)
Use advanced computer resources
Competitive with LGTh?

Heavy Quarkonium in a Light-Front Holographic Basis

BLFQ using AdS/QCD



Yang Li , Pieter Maris Xingbo Zhao James P. Vary ***PLB 758, 116 (2016)***

$$H_{\rm eff} = \underbrace{\frac{\vec{k}_\perp^2 + m_q^2}{x} + \frac{\vec{k}_\perp^2 + m_{\bar{q}}^2}{1-x}}_{\text{LF kinetic energy}} + \underbrace{\kappa^4 \zeta_\perp^2 - \frac{\kappa^4}{(m_q+m_{\bar{q}})^2}\partial_x\left[x(1-x)\partial_x\right]}_{\text{confinement}} - \underbrace{\frac{C_F 4\pi\alpha_s}{Q^2}\bar{u}_{s'}(k')\gamma_\mu u_s(k)\bar{v}_{\bar{s}}(\bar{k})\gamma^\mu v_{\bar{s}'}(\bar{k}')}_{\text{one-gluon exchange}}$$

PHYSICAL REVIEW D VOLUME 22, NUMBER 9 1 NOVEMBER 1980

Exclusive processes in perturbative quantum chromodynamics

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(Received 27 May 1980)

We present a systematic analysis in perturbative quantum chromodynamics (QCD) of large-momentum-transfer exclusive processes. Predictions are given for the scaling behavior, angular dependence, helicity structure, and normalization of elastic and inelastic form factors and large-angle exclusive scattering amplitudes for hadrons and photons. We prove that these reactions are dominated by quark and gluon subprocesses at short distances, and thus that the dimensional-counting rules for the power-law falloff of these amplitudes with momentum transfer are rigorous predictions of QCD, modulo calculable logarithmic corrections from the behavior of the hadronic wave functions at short distances. These anomalous-dimension corrections are determined by evolution equations for process-independent meson and baryon "distribution amplitudes" $\phi(x_i,Q)$ which control the valence-quark distributions in high-momentum-transfer exclusive reactions. The analysis can be carried out systematically in powers of $\alpha_s(Q^2)$, the QCD running coupling constant. Although the calculations are most conveniently carried out using light-cone perturbation theory and the light-cone gauge, we also present a gauge-independent analysis and relate the distribution amplitude to a gauge-invariant Bethe-Salpeter amplitude.

Rigorous QCD analysis of exclusive reactions
Hadron Distribution amplitudes
ERBL Evolution

Also: Efremov and Radyshkin

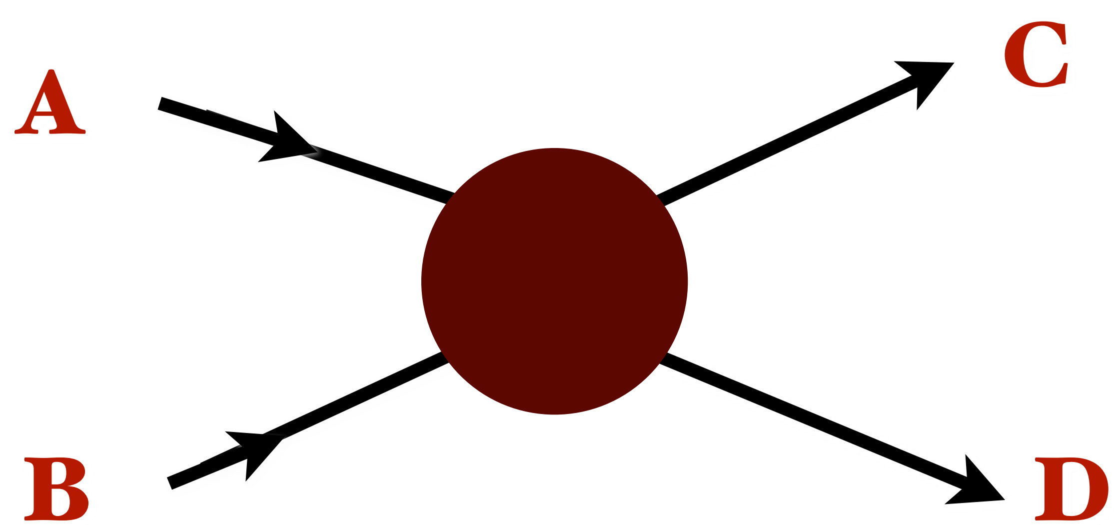
Simple properties of Hard Exclusive Processes

1973: Farrar and sjb

Scaling Laws at Large Transverse Momentum

$$\frac{d\sigma}{dt}(A + B \to C + D) = \frac{F(t/s)}{s^{n_{tot}-2}}$$

$$n_{tot} = n_A + n_B + n_C + n_D$$



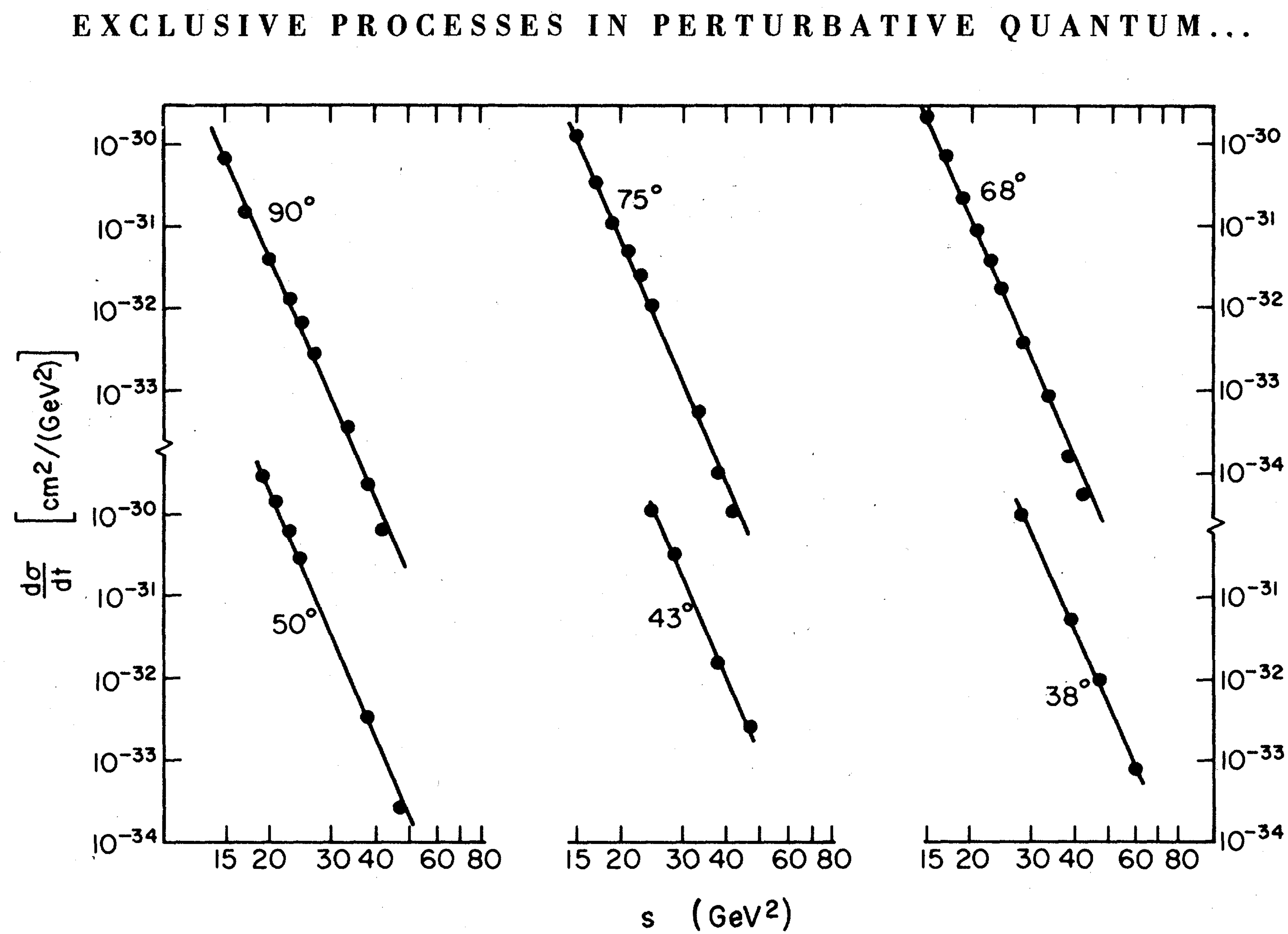
Counting rules
n = twist =
dimension-spin

$$\text{e.g. } n_{tot} - 2 = n_A + n_B + n_C + n_D - 2 = 10 \text{ for } pp \to pp$$

Predict: $$\frac{d\sigma}{dt}(p + p \to p + p) = \frac{F(\theta_{CM})}{s^{10}}$$

Manifestation of
Asymptotic Freedom

Scaling of Hard Exclusive reactions: Fixed t/s



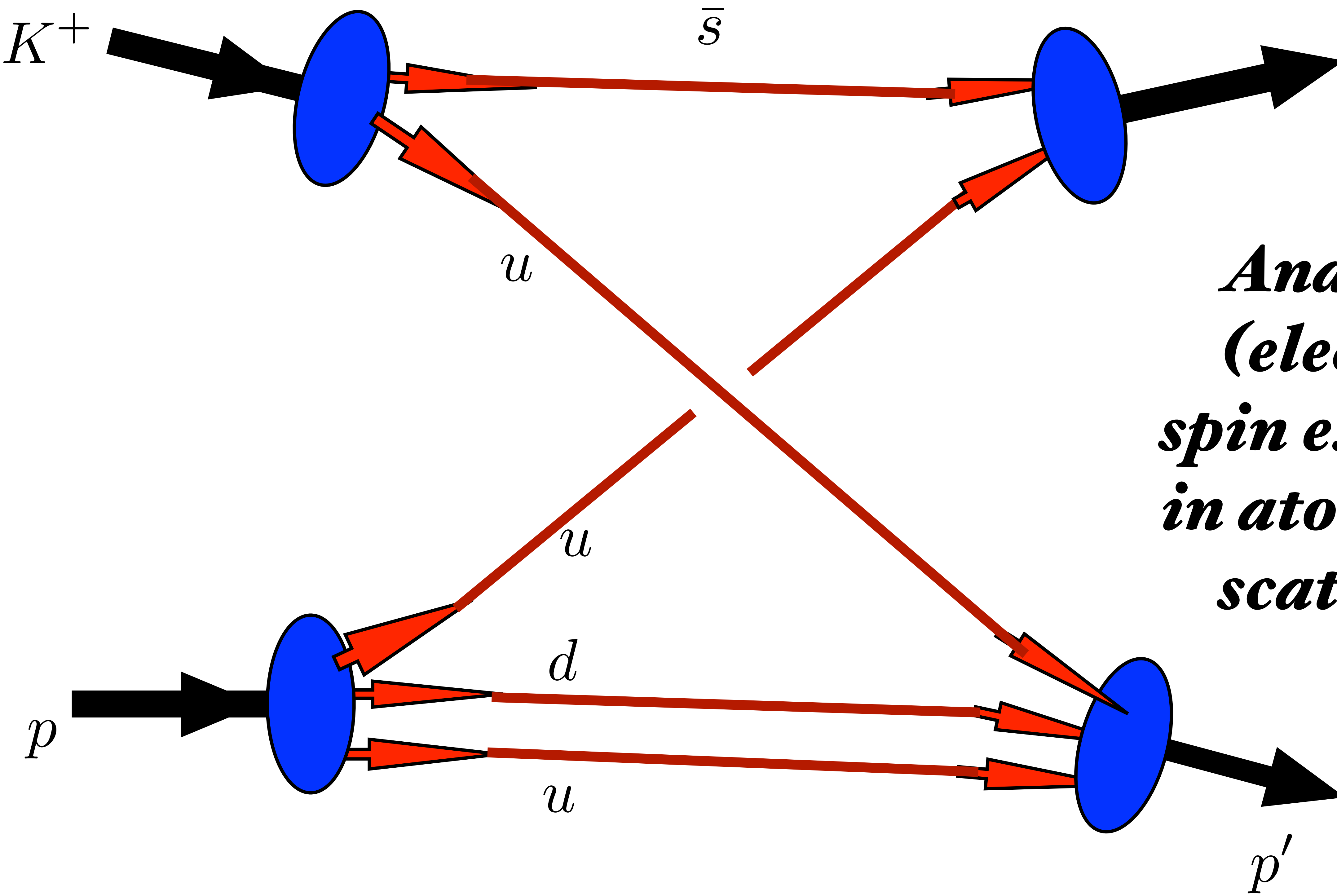
Cross sections for $pp \to pp$ at wide angles

The straight lines correspond to a falloff of $1/s^{10}$.

$$\frac{d\sigma}{dt}(p+p \to p+p) = \frac{F(\theta_{CM})}{s^{10}}$$

Manifestation of Asymptotic Freedom

$K^+ p \to K^+ p$

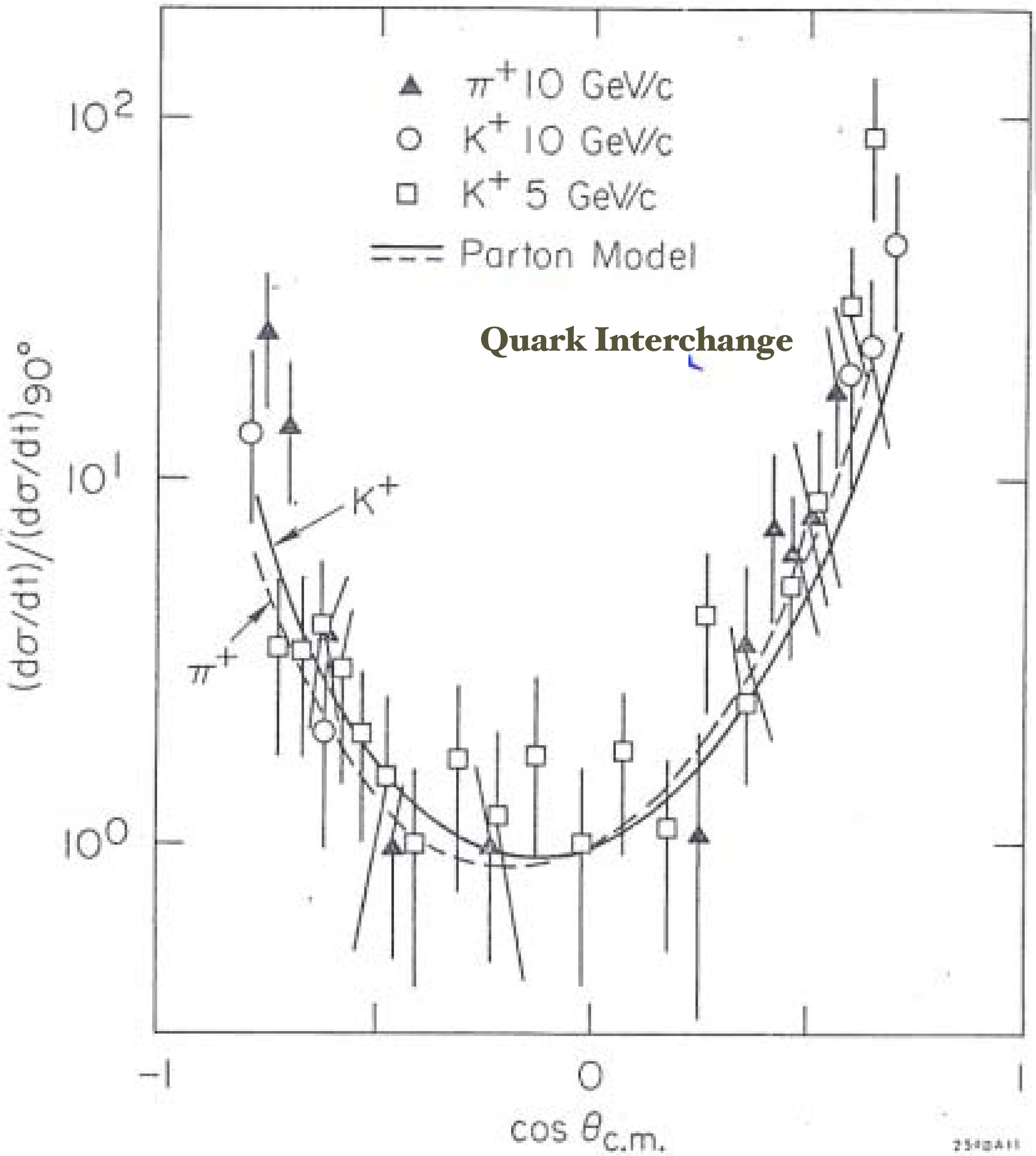


Analog of (electron) spin exchange in atom-atom scattering

Quark Interchange

Blankenbecler, Gunion, sjb

Interactions between exchanged quarks suppressed at high momentum transfer



Quark Interchange
Blankenbecler, Gunion, sjb

$$M(t,u)_{\text{interchange}} \propto \frac{1}{ut^2}$$

$$\frac{d\sigma}{dt}(K^+p \to K^+p) = \frac{F(t/s)}{s^8}$$

Non-linear Regge behavior:

$$\alpha_R(t) \to -1$$

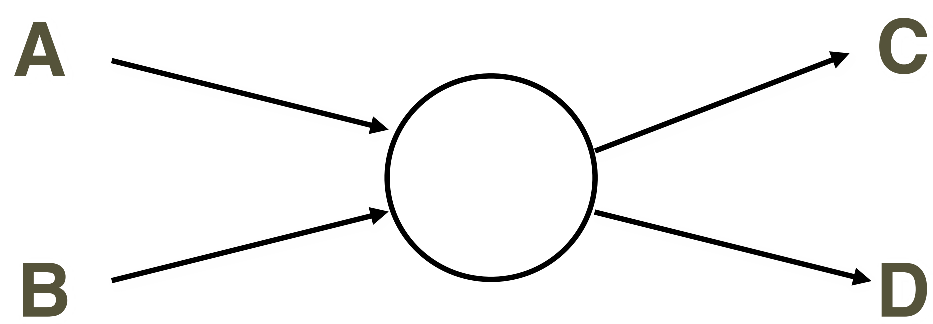
$$\frac{d\sigma}{dt} = \frac{f(t/s)}{s^{N-2}}$$

N-2 =# fundamental constituents -2 = 2+3+2 +3- 2=8

"Counting Rules" *Farrar and sjb; Muradyan, Matveev, Tavkelidze*

Scaling: manifestation of asymptotically free hadronic interactions

From dimensional arguments at high energies in binary reactions:



CONSTITUENT COUNTING RULE

Brodsky and Farrar, Phys. Rev. Lett. 31 (1973) 1153
Matveev et al., Lett. Nuovo Cimento, 7 (1973) 719

helicity conservation

Counting Rules:

$$q(x) \sim (1-x)^{2n_{spect}-1} \text{ for } x \to 1$$

$$F(Q^2) \sim (\frac{1}{Q^2})^{(n-1)}$$

$$\frac{d\sigma}{dt}(AB \to CD) \sim \frac{F(t/s)}{s^{(n_{participants}-2)}}$$

$$n_{participants} = n_A + n_B + n_C + n_D$$

$$\frac{d\sigma}{d^3p/E}(AB \to CX) \sim F(\hat{t}/\hat{s}) \times \frac{(1-x_R)^{(2n_{spectators}-1)}}{(p_T^2)^{(n_{participants}-2)}}$$

Farrar, Jackson; Lepage, sjb; Burkardt, Schmidt, Sjb

1979: G.P Lepage and sjb

Exclusive Processes in Perturbative Quantum Chromodynamics:

Distribution Amplitudes, ERBL Evolution Equations

Richard Feynman

Cal Tech (1979)— First QCD Conference

R.F. to sjb:
What you said today
was wrong!

R.F. You are right!

.

Challenge: Compute Hadron Structure, Spectroscopy, and Dynamics from QCD!

- *Color Confinement*
- *Origin of the QCD Mass Scale*
- *Meson and Baryon Spectroscopy*
- *Exotic States: Tetraquarks, Pentaquarks, Gluonium,*
- *Universal Regge Slopes: n, L, Mesons and Baryons*
- *Almost Massless Pion: GMOR Chiral Symmetry Breaking*
  $$M_\pi^2 f_\pi^2 = -\frac{1}{2}(m_u + m_d)\langle \bar{u}u + \bar{d}d \rangle + \mathcal{O}\big((m_u + m_d)^2\big)$$
- *QCD Coupling at all Scales* $\alpha_s(Q^2)$
- *Eliminate Scale Uncertainties and Scheme Dependence: BLM/PMC (Principle of Maximum Conformality)*

BLM Renormalization Scale Setting

On the elimination of scale ambiguities in perturbative quantum chromodynamics

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(Received 23 November 1982)

We present a new method for resolving the scheme-scale ambiguity that has plagued perturbative analyses in quantum chromodynamics (QCD) and other gauge theories. For Abelian theories the method reduces to the standard criterion that only vacuum-polarization insertions contribute to the effective coupling constant. Given a scheme, our procedure automatically determines the coupling-constant scale appropriate to a particular process. This leads to a new criterion for the convergence of perturbative expansions in QCD. We examine a number of well known reactions in QCD, and find that perturbation theory converges well for all processes other than the gluonic width of the Υ. Our analysis calls into question recent determinations of the QCD coupling constant based upon Υ decay.

All orders: PMC (Principle of Maximum Conformality)
Satisfies all principles of renormalization theory
Eliminates n! renormalons
Commensurate scale relations between observables
Abelian limit: Standard QED Scale-Setting

M. Mojaza, sjb
L. di Giustino, Xing-Gang Wu

Need a First Approximation to QCD

Comparable in simplicity to Schrödinger Theory in Atomic Physics

Relativistic, Frame-Independent, Color-Confining

Origin of hadronic mass scale if $m_q=0$

Semi-Classical Approximation to QCD

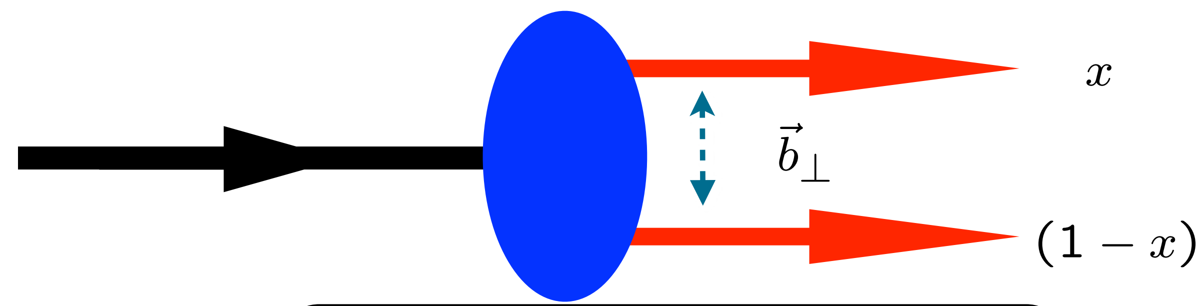
de Téramond, Dosch, Lorcé, sjb

AdS/QCD
Light-Front Holography

Light-Front QCD

Fixed $\tau = t + z/c$

$$\mathcal{L}_{QCD} \longrightarrow \boxed{H^{LF}_{QCD}}$$



$$\boxed{\zeta^2 = x(1-x)b^2_\perp}$$

$$(H^0_{LF} + H^I_{LF})|\Psi> = M^2|\Psi>$$

Coupled Fock states

Eliminate higher Fock states
and retarded interactions

$$\left[\frac{\vec{k}^2_\perp + m^2}{x(1-x)} + V^{LF}_{\rm eff}\right] \psi_{LF}(x, \vec{k}_\perp) = M^2 \ \psi_{LF}(x, \vec{k}_\perp)$$

Effective two-particle equation

Azimuthal Basis ζ, ϕ

$$\left[-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right]\psi(\zeta) = \mathcal{M}^2\psi(\zeta)$$

Single variable Equation

$$m_q = 0$$

AdS/QCD: LF Holography

$$\boxed{U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)}$$

Confining AdS/QCD potential!

Semiclassical first approximation to QCD

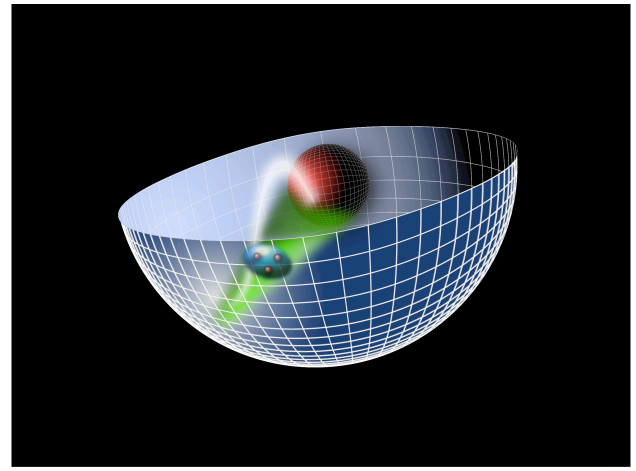
Sums an infinite # diagrams

de Téramond, Dosch, Lorcé, sjb

de Tèramond, Dosch, sjb

AdS/QCD
Soft-Wall Model

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$



$$\boxed{\zeta^2 = x(1-x)\mathbf{b}_\perp^2.}$$

Light-Front Holography

$$\left[-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right]\psi(\zeta) = M^2\psi(\zeta)$$

Light-Front Schrödinger Equation

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

Single variable ζ

Unique Confinement Potential!

Conformal Symmetry of the AdS action

Confinement scale: $\kappa \simeq 0.5\ GeV$

- **de Alfaro, Fubini, Furlan:**
- **Fubini, Rabinovici:**

Scale can appear in Hamiltonian and EQM without affecting conformal invariance of AdS action!

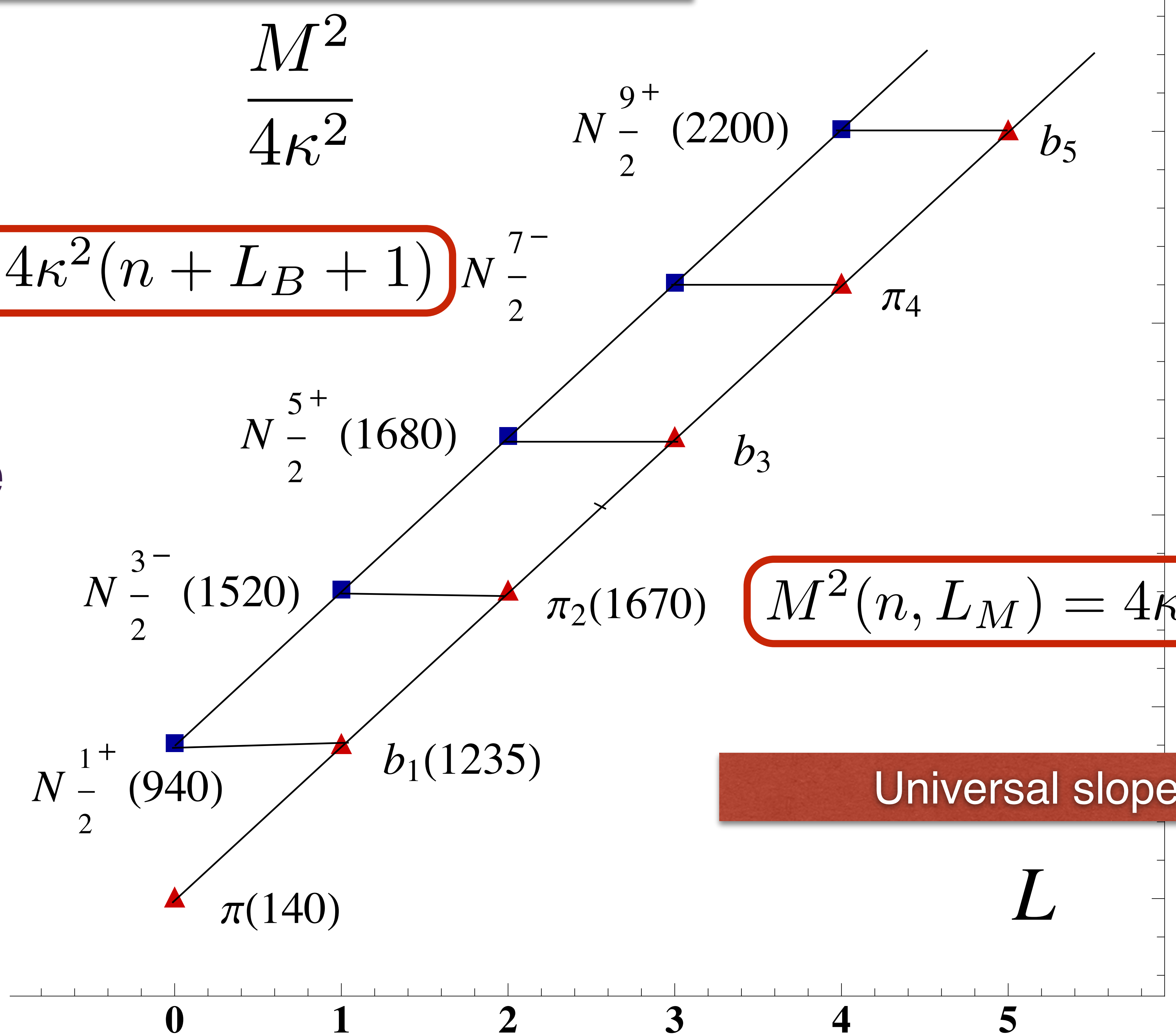
GeV units external to QCD: Ratios of Masses Determined

Superconformal Quantum Mechanics Light-Front Holography

de Téramond, Dosch, Lorcé, sjb

$$M^2(n, L_B) = 4\kappa^2(n + L_B + 1)$$

Same slope

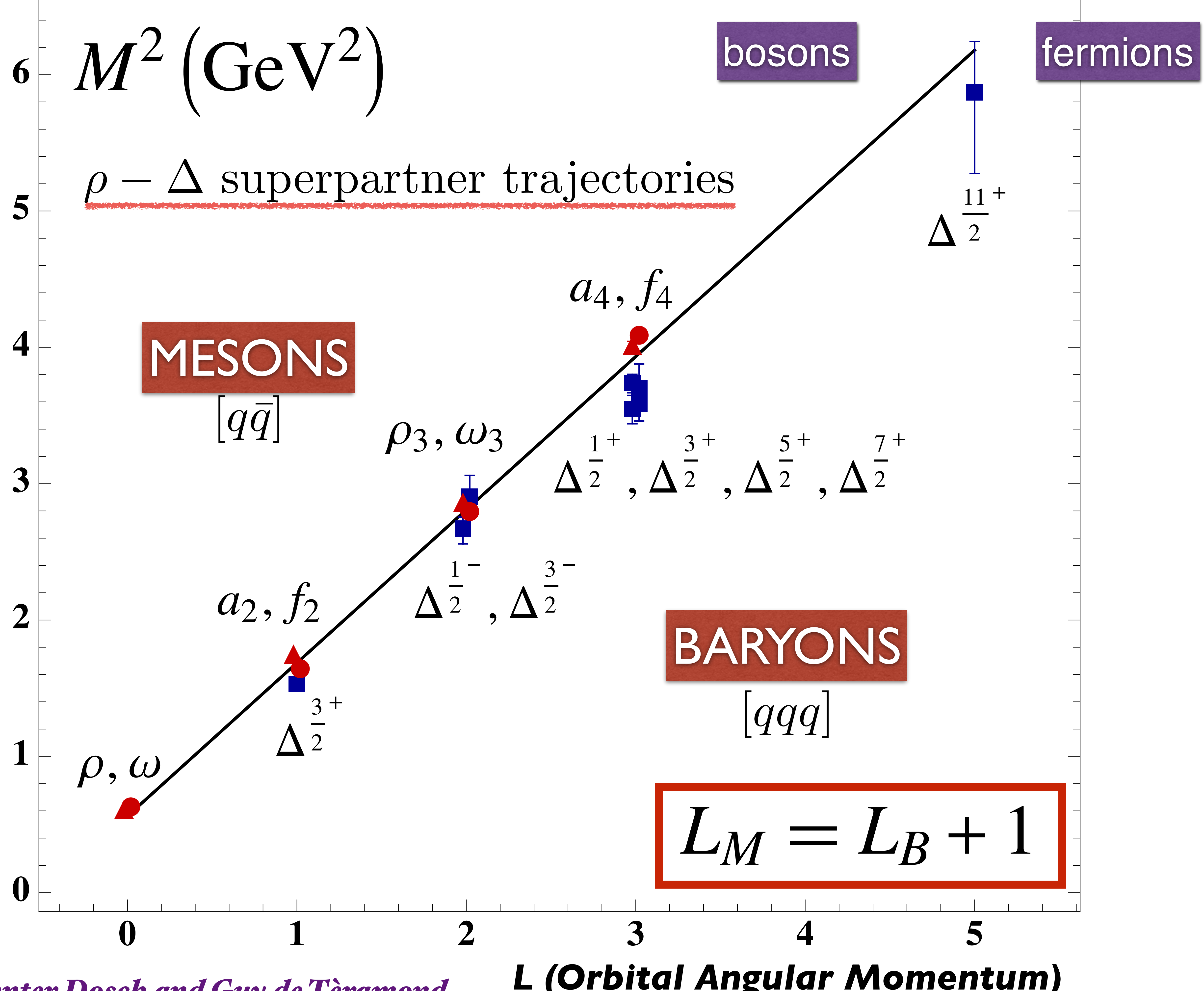


$$M^2(n, L_M) = 4\kappa^2(n + L_M)$$

Universal slopes in n, L

$$\frac{M^2_{meson}}{M^2_{nucleon}} = \frac{n + L_M}{n + L_B + 1}$$

Meson-Baryon Mass Degeneracy for $L_M = L_B + 1$

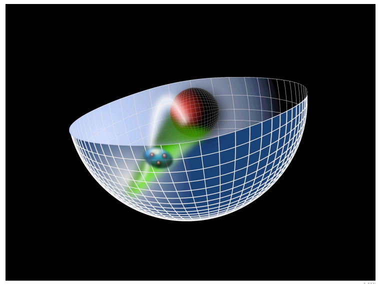


with Guenter Dosch and Guy de Tèramond

Light-Front Holography

Dilaton-Modified AdS

$$ds^2 = e^{\varphi(z)} \frac{R^2}{z^2}(\eta_{\mu\nu} x^\mu x^\nu - dz^2)$$



- ***Soft-wall dilaton profile breaks***

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

- ***Color Confinement in z***
- ***Introduces confinement scale κ***
- ***Uses AdS_5 as template for conformal theory***

AdS/CFT

D. Gross: duality of QCD with string theory

Introduce "Dilaton" to simulate confinement analytically

- Nonconformal metric dual to a confining gauge theory

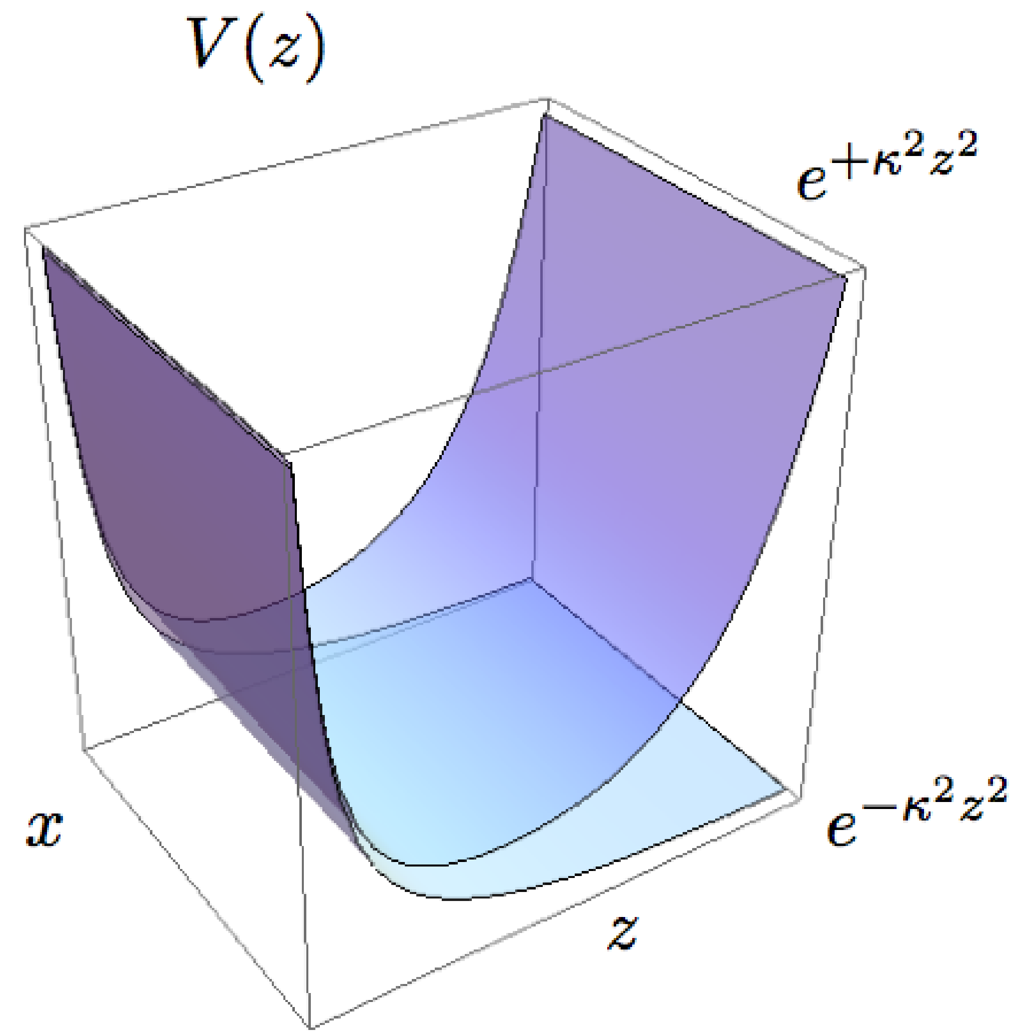
$$ds^2 = \frac{R^2}{z^2}\boxed{e^{\varphi(z)}}\left(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2\right)$$

where $\varphi(z) \to 0$ at small z for geometries which are asymptotically AdS$_5$

- Gravitational potential energy for object of mass m

$$V = mc^2\sqrt{g_{00}} = mc^2 R\,\frac{e^{\varphi(z)/2}}{z}$$

- Consider warp factor $\exp(\pm\kappa^2 z^2)$
- Plus solution: $V(z)$ increases exponentially confining any object in modified AdS metrics to distances $\langle z\rangle \sim 1/\kappa$



Klebanov and Maldacena

$$\boxed{e^{\varphi(z)} = e^{+\kappa^2 z^2}}$$

Positive-sign dilaton

- de Teramond, sjb

Bosonic Solutions: Hard Wall Model

- Conformal metric: $ds^2 = g_{\ell m} dx^\ell dx^m$. $x^\ell = (x^\mu, z)$, $g_{\ell m} \to \left(R^2/z^2\right) \eta_{\ell m}$.

- Action for massive scalar modes on AdS$_{d+1}$:

$$S[\Phi] = \frac{1}{2} \int d^{d+1} x \sqrt{g} \, \tfrac{1}{2} \left[g^{\ell m} \partial_\ell \Phi \partial_m \Phi - \mu^2 \Phi^2 \right], \quad \sqrt{g} \to (R/z)^{d+1}.$$

- Equation of motion

$$\frac{1}{\sqrt{g}} \frac{\partial}{\partial x^\ell} \left(\sqrt{g} \, g^{\ell m} \frac{\partial}{\partial x^m} \Phi \right) + \mu^2 \Phi = 0.$$

- Factor out dependence along x^μ-coordinates , $\Phi_P(x,z) = e^{-iP\cdot x} \, \Phi(z)$, $P_\mu P^\mu = \mathcal{M}^2$:

$$\boxed{\left[z^2 \partial_z^2 - (d-1) z \, \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2 \right] \Phi(z) = 0.}$$

- Solution: $\Phi(z) \to z^\Delta$ as $z \to 0$,

$$\Phi(z) = C z^{d/2} J_{\Delta - d/2}(z\mathcal{M}) \qquad \Delta = \tfrac{1}{2}\left(d + \sqrt{d^2 + 4\mu^2 R^2}\,\right).$$

$$\Delta = 2 + L \qquad d = 4 \qquad (\mu R)^2 = L^2 - 4$$

$$\boxed{e^{\varphi(z)} = e^{+\kappa^2 z^2}}$$

Positive-sign dilaton

• de Teramond, sjb

AdS Soft-Wall Schrödinger Equation for bound state of two scalar constituents:

$$\left[-\frac{d^2}{dz^2} - \frac{1-4L^2}{4z^2} + U(z)\right]\Phi(z) = \mathcal{M}^2\Phi(z)$$

$$U(z) = \kappa^4 z^2 + 2\kappa^2(L+S-1)$$

Derived from variation of Action for Dilaton-Modified AdS$_5$

Identical to Single-Variable Light-Front Bound State Equation in ζ!

$$z \longleftrightarrow \zeta = \sqrt{x(1-x)\vec{b}_\perp^2}$$

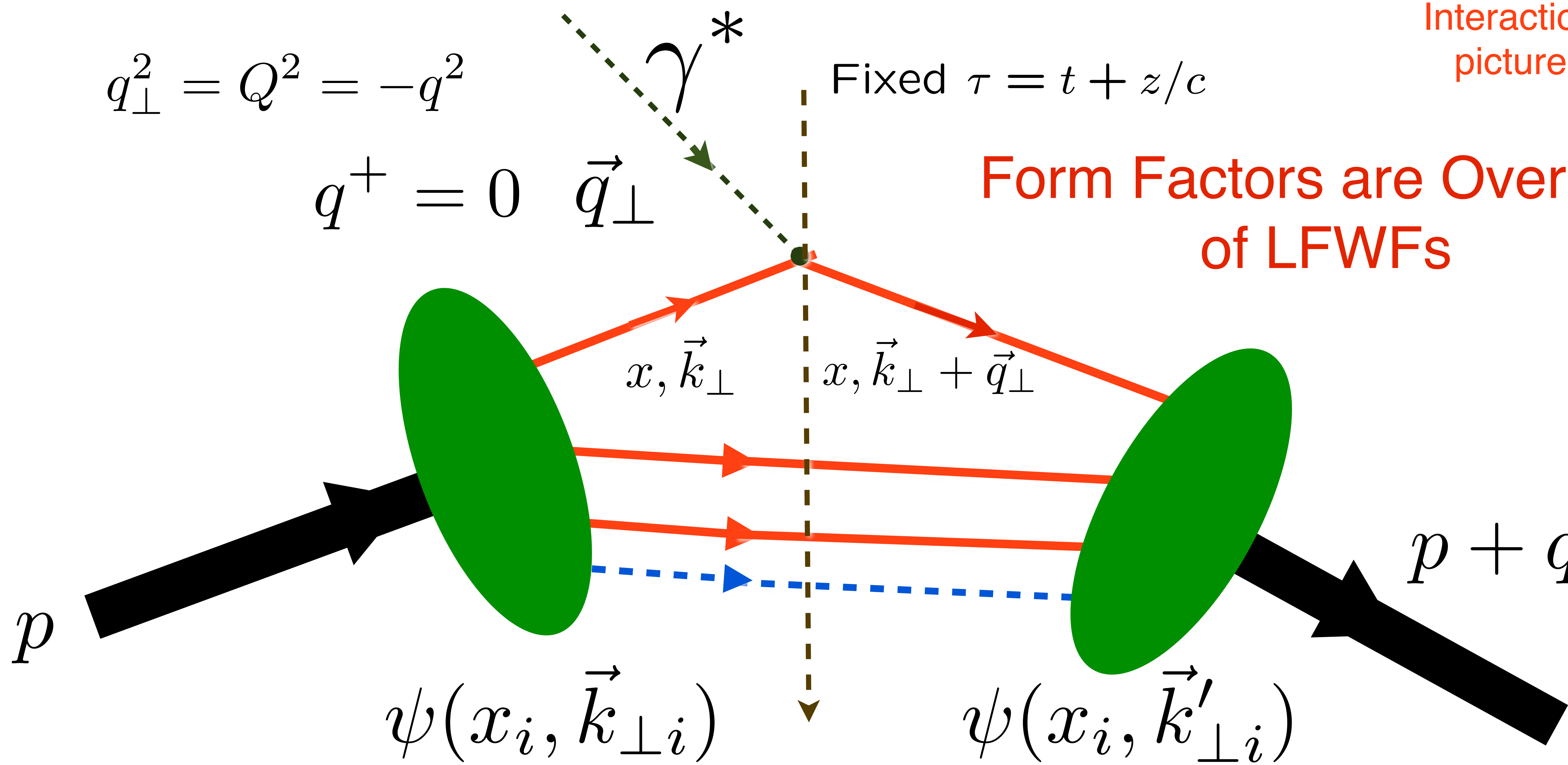
Light-Front Holography

$$< p+q|j^+(0)|p >= 2p^+F(q^2)$$

Front Form

Interaction picture

Form Factors are Overlaps of LFWFs



struck $\quad \vec{k}'_{\perp i} = \vec{k}_{\perp i} + (1 - x_i)\vec{q}_\perp$

spectators $\quad \vec{k}'_{\perp i} = \vec{k}_{\perp i} - x_i\vec{q}_\perp$

Drell &Yan, West
Exact LF formula!

Drell, sjb

Transverse size $\propto \frac{1}{Q}$

Holographic Mapping of AdS Modes to QCD LFWFs

Drell-Yan-West: Form Factors are Convolution of LFWFs

- Integrate Soper formula over angles:

$$F(q^2) = 2\pi \int_0^1 dx \, \frac{(1-x)}{x} \int \zeta d\zeta J_0\left(\zeta q \sqrt{\frac{1-x}{x}}\right) \tilde{\rho}(x, \zeta),$$

with $\widetilde{\rho}(x, \zeta)$ QCD effective transverse charge density.

- Transversality variable

$$\boxed{\zeta = \sqrt{x(1-x)\vec{b}_\perp^2}}$$

- Compare AdS and QCD expressions of FFs for arbitrary Q using identity:

$$\int_0^1 dx J_0\left(\zeta Q \sqrt{\frac{1-x}{x}}\right) = \zeta Q K_1(\zeta Q),$$

the solution for $J(Q, \zeta) = \zeta Q K_1(\zeta Q)$!

de T`eramond, sjb

Identical to Polchinski-Strassler Convolution of AdS Amplitudes

LF(3+1) ⟷ *AdS_5*

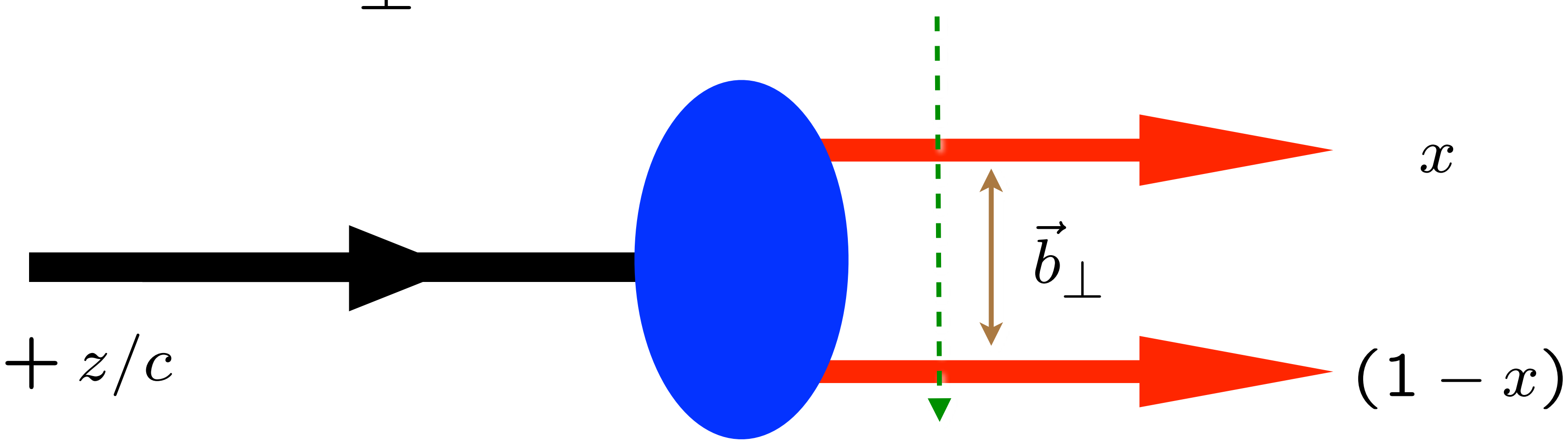
de Teramond, sjb

LF(3+1) $\longleftrightarrow$ AdS_5

Light-Front Holographic Dictionary

$$\psi(x, \vec{b}_\perp) \longleftrightarrow \phi(z)$$

$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2} \longleftrightarrow z$$

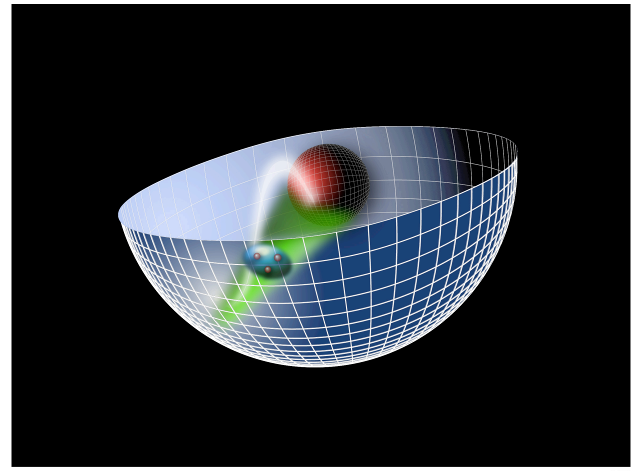


Fixed $\tau = t + z/c$

$$\psi(x, \zeta) = \sqrt{x(1-x)}\zeta^{-1/2}\phi(\zeta)$$

$$(\mu R)^2 = L^2 - (J-2)^2$$

Light-Front Holography: *Unique mapping derived from equality of LF and AdS formula for EM and gravitational current matrix elements and identical equations of motion* *de Tèramond, Dosch, sjb*



AdS/QCD
Soft-Wall Model

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

$$\boxed{\zeta^2 = x(1-x)\mathrm{b}_\perp^2.}$$

Light-Front Holography

$$\left[-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right]\psi(\zeta) = M^2\psi(\zeta)$$

Light-Front Schrödinger Equation

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

Single variable ζ

Unique Confinement Potential!

Conformal Symmetry of the action

Confinement scale: $\kappa \simeq 0.5\ GeV$

- **de Alfaro, Fubini, Furlan:**
- **Fubini, Rabinovici:**

Scale can appear in Hamiltonian and EQM without affecting conformal invariance of action!

GeV units external to QCD: Ratios of Masses Determined de Téramond, Dosch, Lorcé, sjb

LF Holography

Baryon Equation

Superconformal Quantum Mechanics

$$\left(-\partial_\zeta^2+\kappa^4\zeta^2+2\kappa^2(L_B+1)+\frac{4L_B^2-1}{4\zeta^2}\right)\psi_J^+=M^2\psi_J^+$$

$$\left(-\partial_\zeta^2+\kappa^4\zeta^2+2\kappa^2 L_B+\frac{4(L_B+1)^2-1}{4\zeta^2}\right)\psi_J^-=M^2\psi_J^-$$

$$M^2(n,L_B)=4\kappa^2(n+L_B+1)$$

S=1/2, P=+

Meson Equation

$$\lambda=\kappa^2$$

$$\left(-\partial_\zeta^2+\kappa^4\zeta^2+2\kappa^2(J-1)+\frac{4L_M^2-1}{4\zeta^2}\right)\phi_J=M^2\phi_J$$

S=0, P=+

$$M^2(n,L_M)=4\kappa^2(n+L_M)$$

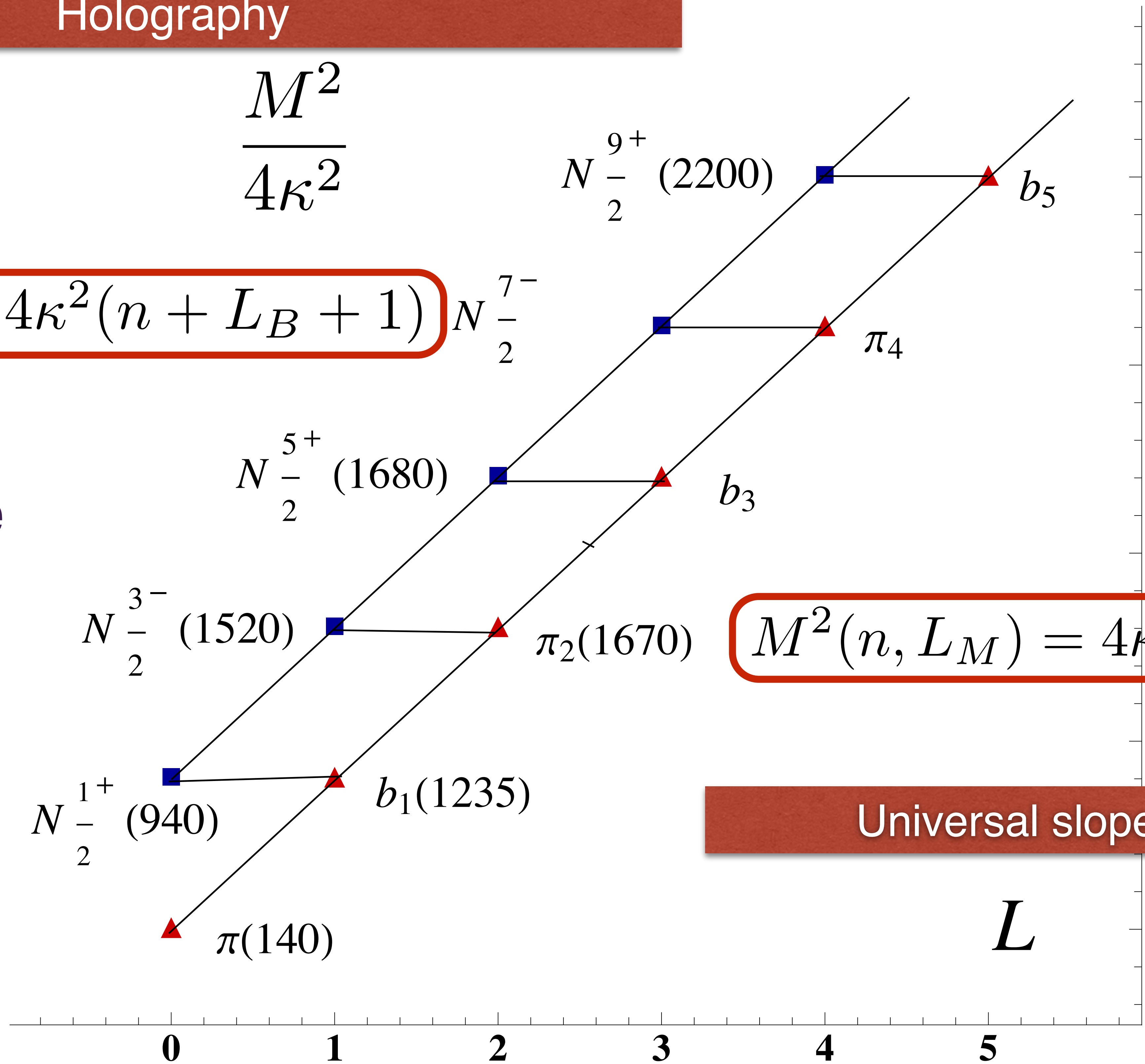
Same κ!

S=0, I=1 Meson is superpartner of S=1/2, I=1 Baryon

Meson-Baryon Degeneracy for $L_M=L_B+1$

Superconformal Quantum Mechanics Light-Front Holography

de Téramond, Dosch, Lorcé, sjb



$$M^2(n, L_B) = 4\kappa^2(n + L_B + 1)$$

Same slope

$$M^2(n, L_M) = 4\kappa^2(n + L_M)$$

Universal slopes in n, L

$$\frac{M^2_{meson}}{M^2_{nucleon}} = \frac{n + L_M}{n + L_B + 1}$$

Meson-Baryon Mass Degeneracy for $L_M = L_B + 1$

Meson Spectrum in Soft Wall Model

Massless pion!

$$\boxed{m_\pi = 0 \text{ if } m_q = 0}$$

Pion: Negative term for J=0 cancels positive terms from LFKE and potential

- Effective potential: $U(\zeta^2) = \kappa^4\zeta^2 + 2\kappa^2(J-1)$
- LF WE

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + \kappa^4\zeta^2 + 2\kappa^2(J-1)\right)\phi_J(\zeta) = M^2\phi_J(\zeta)$$

- Normalized eigenfunctions $\langle\phi|\phi\rangle = \int d\zeta\, \phi^2(z)^2 = 1$

$$\phi_{n,L}(\zeta) = \kappa^{1+L}\sqrt{\frac{2n!}{(n+L)!}}\,\zeta^{1/2+L}e^{-\kappa^2\zeta^2/2}L_n^L(\kappa^2\zeta^2)$$

- Eigenvalues

$$\boxed{\mathcal{M}^2_{n,J,L} = 4\kappa^2\left(n + \frac{J+L}{2}\right)}$$

$$\vec{\zeta}^2 = \vec{b}^2_\perp x(1-x)$$

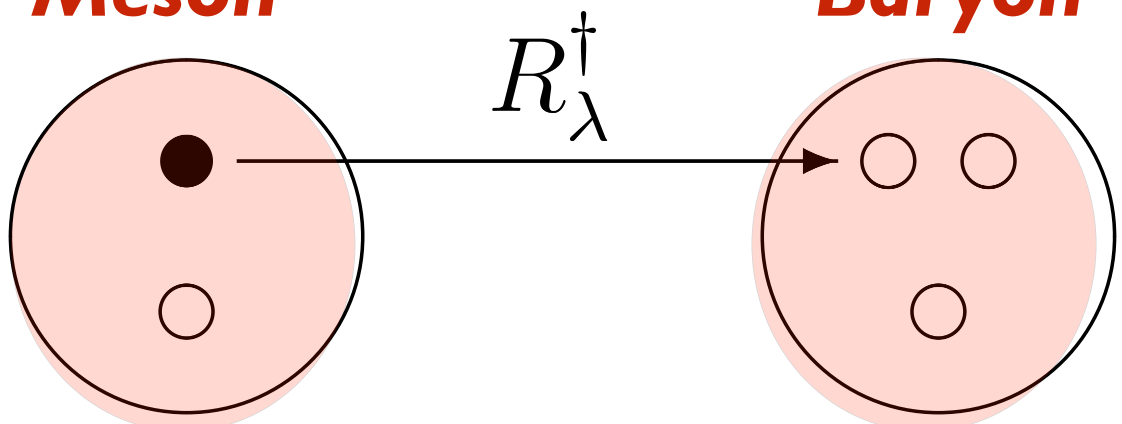
G. de Teramond, H. G. Dosch, sjb

Superconformal Algebra

Four-Plet Representations

Bosons, Fermions with Equal Mass!

Meson **Baryon**



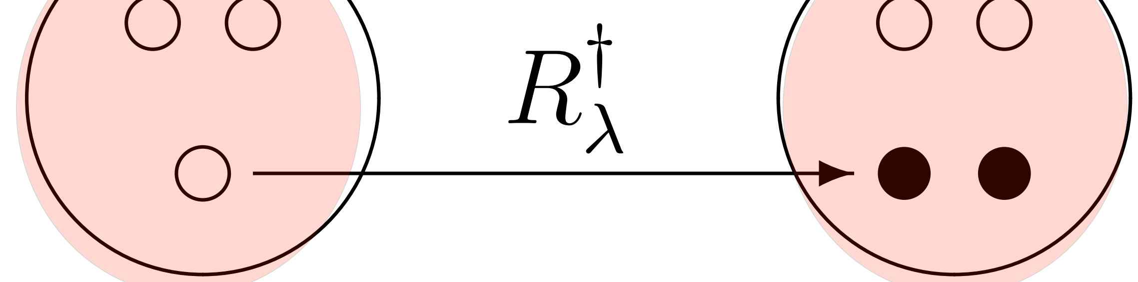
$\phi_M, \; L_B + 1$ $\quad$ $\psi_{B+}, \; L_B$

$$R_\lambda^\dagger \; \bar{q} \to [qq]$$
$$\bar{3}_C \to \bar{3}_C$$

Baryon

Tetraquark: diquark + antidiquark

$$R_\lambda^\dagger \; q \to [\bar{q}\bar{q}]$$
$$3_C \to 3_C$$



$\psi_{B-}, \; L_B + 1$ $\quad$ $\phi_T, \; L_B$

Proton: |u[ud]> Quark + Scalar Diquark
Equal Weight: L=0, L=1

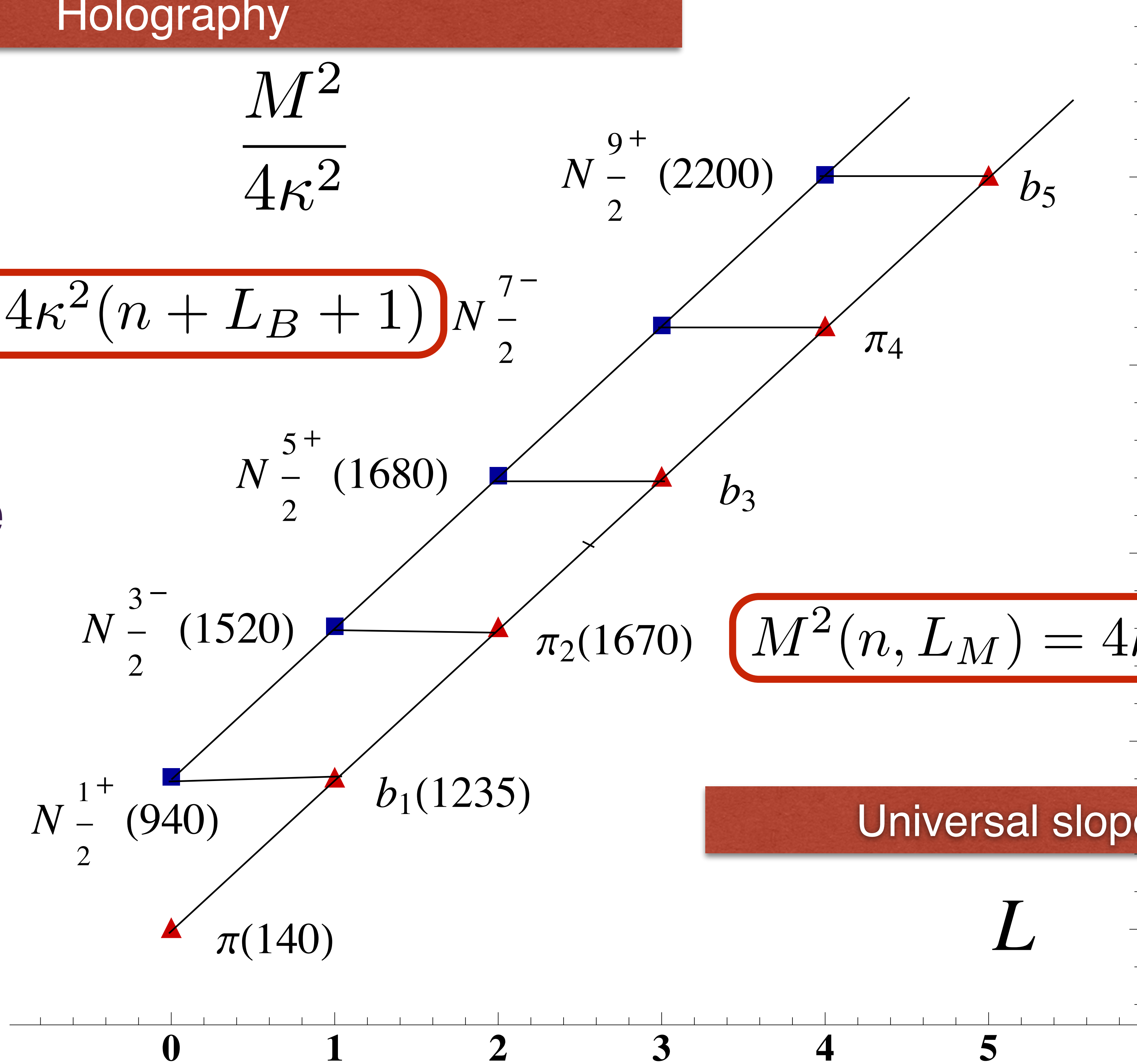
Superconformal Quantum Mechanics Light-Front Holography

de Téramond, Dosch, Lorcé, sjb

$$\boxed{M^2(n, L_B) = 4\kappa^2(n + L_B + 1)}$$

Same slope

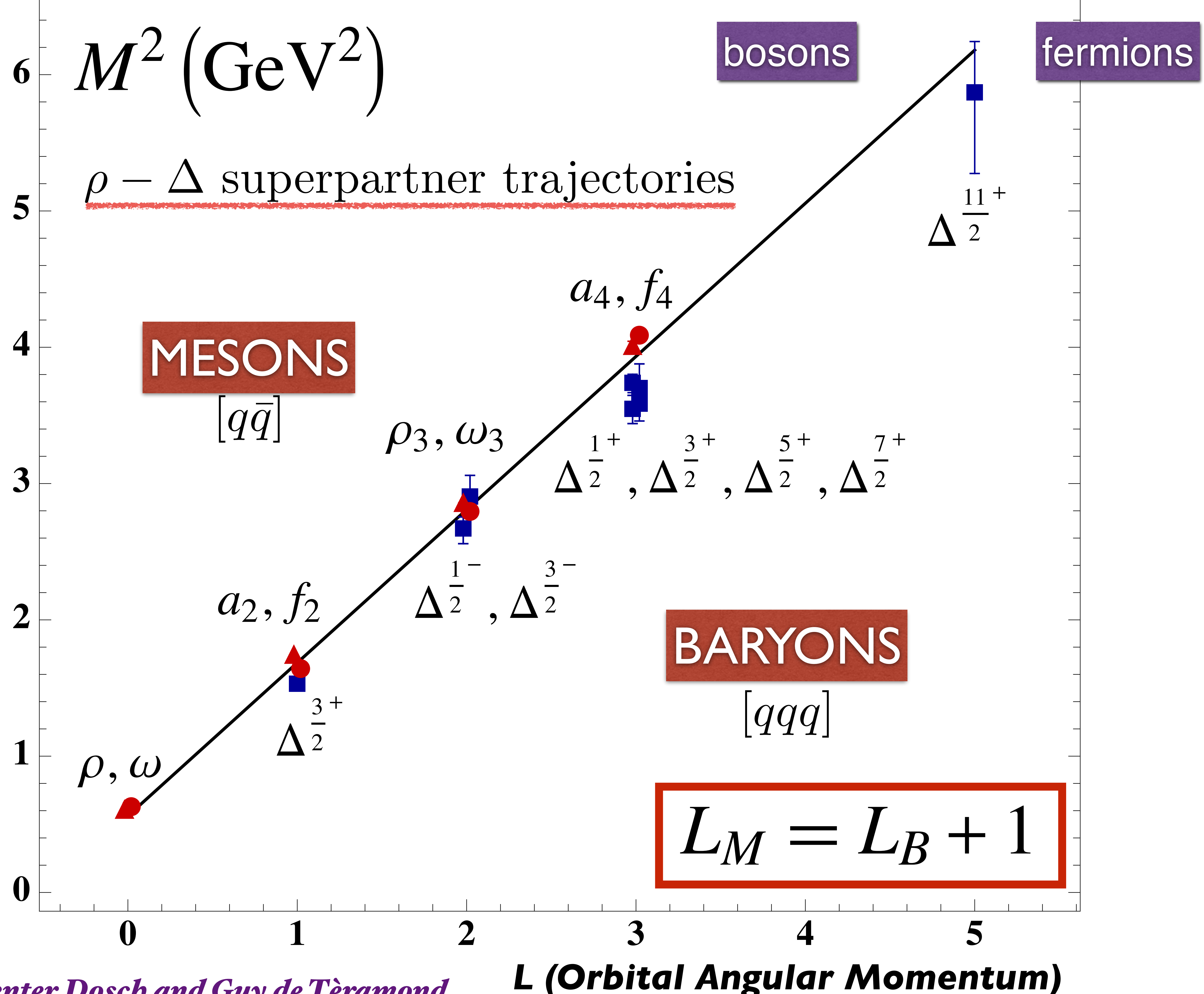
$$\boxed{M^2(n, L_M) = 4\kappa^2(n + L_M)}$$



Universal slopes in n, L

$$\frac{M^2_{meson}}{M^2_{nucleon}} = \frac{n + L_M}{n + L_B + 1}$$

Meson-Baryon Mass Degeneracy for L_M=L_B+1



with Guenter Dosch and Guy de Tèramond

Meson Spectrum in Soft Wall Model

Massless pion!

$$m_\pi = 0 \text{ if } m_q = 0$$

Pion: Negative term for J=0 cancels positive terms from LFKE and potential

- Effective potential: $U(\zeta^2) = \kappa^4\zeta^2 + 2\kappa^2(J-1)$
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$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + \kappa^4\zeta^2 + 2\kappa^2(J-1)\right)\phi_J(\zeta) = M^2\phi_J(\zeta)$$

- Normalized eigenfunctions $\langle\phi|\phi\rangle = \int d\zeta\, \phi^2(z)^2 = 1$

$$\phi_{n,L}(\zeta) = \kappa^{1+L}\sqrt{\frac{2n!}{(n+L)!}}\,\zeta^{1/2+L}e^{-\kappa^2\zeta^2/2}L_n^L(\kappa^2\zeta^2)$$

- Eigenvalues

$$\mathcal{M}^2_{n,J,L} = 4\kappa^2\left(n + \frac{J+L}{2}\right)$$

$$\vec{\zeta}^2 = \vec{b}^2_\perp x(1-x)$$

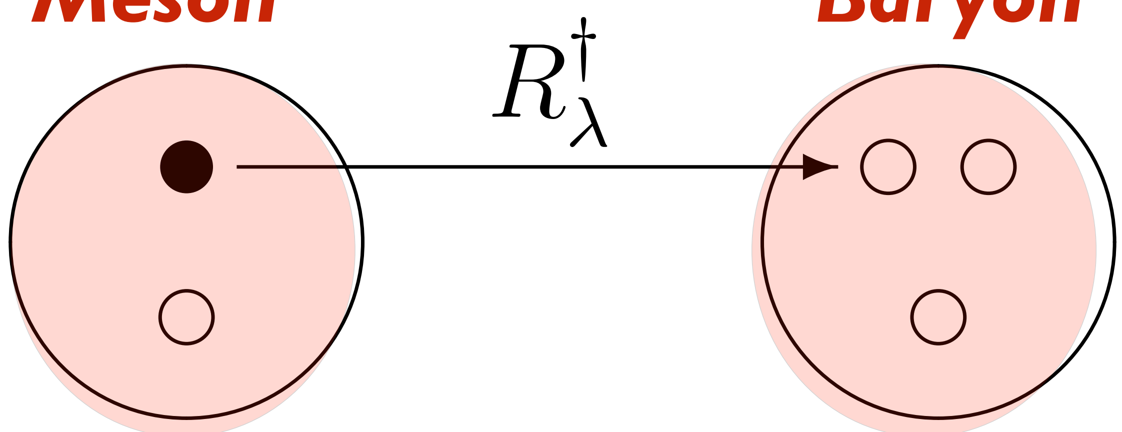
G. de Teramond, H. G. Dosch, sjb

Superconformal Algebra

Four-Plet Representations

Bosons, Fermions with Equal Mass!

Meson **Baryon**



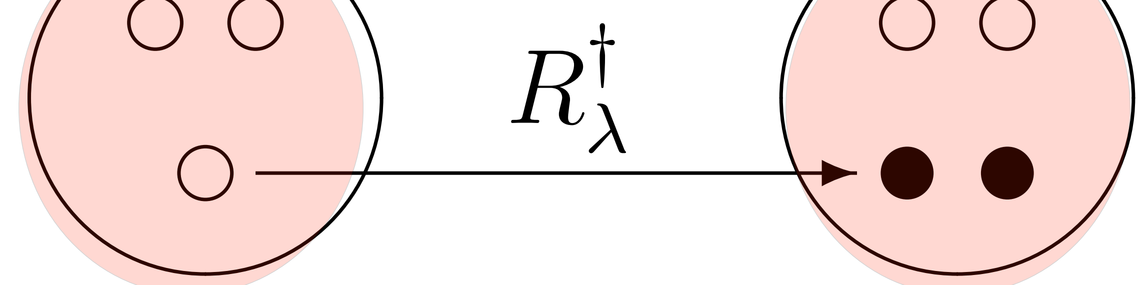
$\phi_M, \; L_B + 1$ $\psi_{B+}, \; L_B$

$$R^\dagger_\lambda \; \bar{q} \to [qq]$$

$$\bar{3}_C \to \bar{3}_C$$

Baryon

Tetraquark: diquark + antidiquark



$$R^\dagger_\lambda \; q \to [\bar{q}\bar{q}]$$

$$3_C \to 3_C$$

$\psi_{B-}, \; L_B + 1$ $\phi_T, \; L_B$

Proton: |u[ud]> Quark + Scalar Diquark
Equal Weight: L=0, L=1

Universal Hadronic Decomposition

$$\frac{\mathcal{M}_H^2}{\kappa^2} = (1 + 2n + L) + (1 + 2n + L) + (2L + 4S + 2B - 2)$$

- ***Universal quark light-front kinetic energy***

Equal: Virial Theorem

$$\Delta\mathcal{M}^2_{LFKE} = \kappa^2(1 + 2n + L)$$

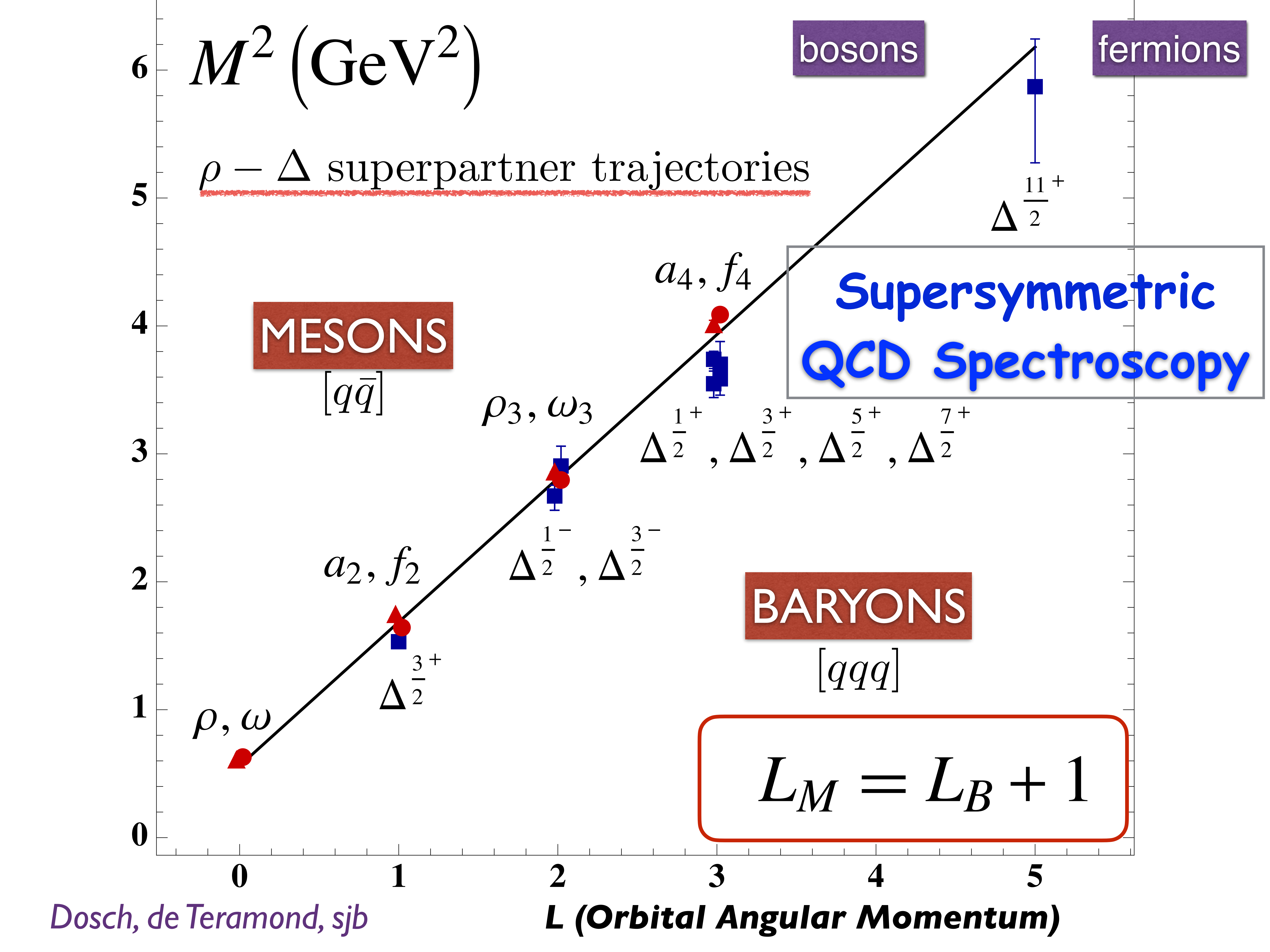
- ***Universal quark light-front potential energy***

$$\Delta\mathcal{M}^2_{LFPE} = \kappa^2(1 + 2n + L)$$

- ***Universal Constant Contribution from AdS and Superconformal Quantum Mechanics***

$$\Delta\mathcal{M}^2_{spin} = 2\kappa^2(L + 2S + B - 1)$$

hyperfine spin-spin



$M^2\,(\mathrm{GeV}^2)$
bosons
fermions
$\rho - \Delta$ superpartner trajectories
$\Delta^{\frac{11}{2}^+}$
a_4, f_4
Supersymmetric
QCD Spectroscopy
MESONS
$[q\bar{q}]$
ρ_3, ω_3
$\Delta^{\frac{1}{2}^+}, \Delta^{\frac{3}{2}^+}, \Delta^{\frac{5}{2}^+}, \Delta^{\frac{7}{2}^+}$
$\Delta^{\frac{1}{2}^-}, \Delta^{\frac{3}{2}^-}$
a_2, f_2
BARYONS
$[qqq]$
$\Delta^{\frac{3}{2}^+}$
ρ, ω
$L_M = L_B + 1$
0
1
2
3
4
5
6
L (Orbital Angular Momentum)
Dosch, de Teramond, sjb

Supersymmetry in QCD

- A hidden symmetry of Color $SU(3)_C$ in hadron physics:
- Relates meson and baryon spectroscopy
- QCD: No squarks or gluinos!
- Emerges from Light-Front Holography and Super-Conformal Algebra
- Color Confinement

de Téramond, Dosch, Lorcé, sjb

Input: one fundamental mass scale

$$\kappa = \sqrt{\lambda} = 0.523 \pm 0.024 \text{ GeV}$$

Remarkable Features of
Light-Front Schrödinger Equation

Dynamics + Spectroscopy!

- ***Relativistic, frame-independent***
- ***QCD scale appears - unique LF potential***
- ***Reproduces spectroscopy and dynamics of light-quark hadrons with one parameter***
- ***Zero-mass pion for zero mass quarks!***
- ***Regge slope same for n and L -- not usual HO***
- ***Splitting in L persists to high mass -- contradicts conventional wisdom based on breakdown of chiral symmetry***
- ***Phenomenology: LFWFs, Form factors, electroproduction***
- ***Extension to heavy quarks***

$$U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2 (L + S - 1)$$

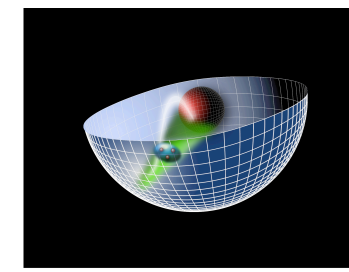
LFHQCD: Underlying Principles

- *Poincarè Invariance: Independent of the observer's Lorentz frame: Quantization at Fixed Light-Front Time* τ

- *Causality: Information within causal horizon: Light-Front*

- *Light-Front Holography: AdS₅ = LF (3+1)*

$$z \leftrightarrow \zeta \text{ where } \zeta^2 = b_\perp^2 x(1-x)$$



- *Introduce Mass Scale* κ *while retaining the Conformal Invariance of the AdS Action (dAFF)*

- *Unique Dilaton in AdS₅:* $e^{+\kappa^2 z^2}$

- *Unique color-confining LF Potential* $U(\zeta^2) = \kappa^4 \zeta^2$

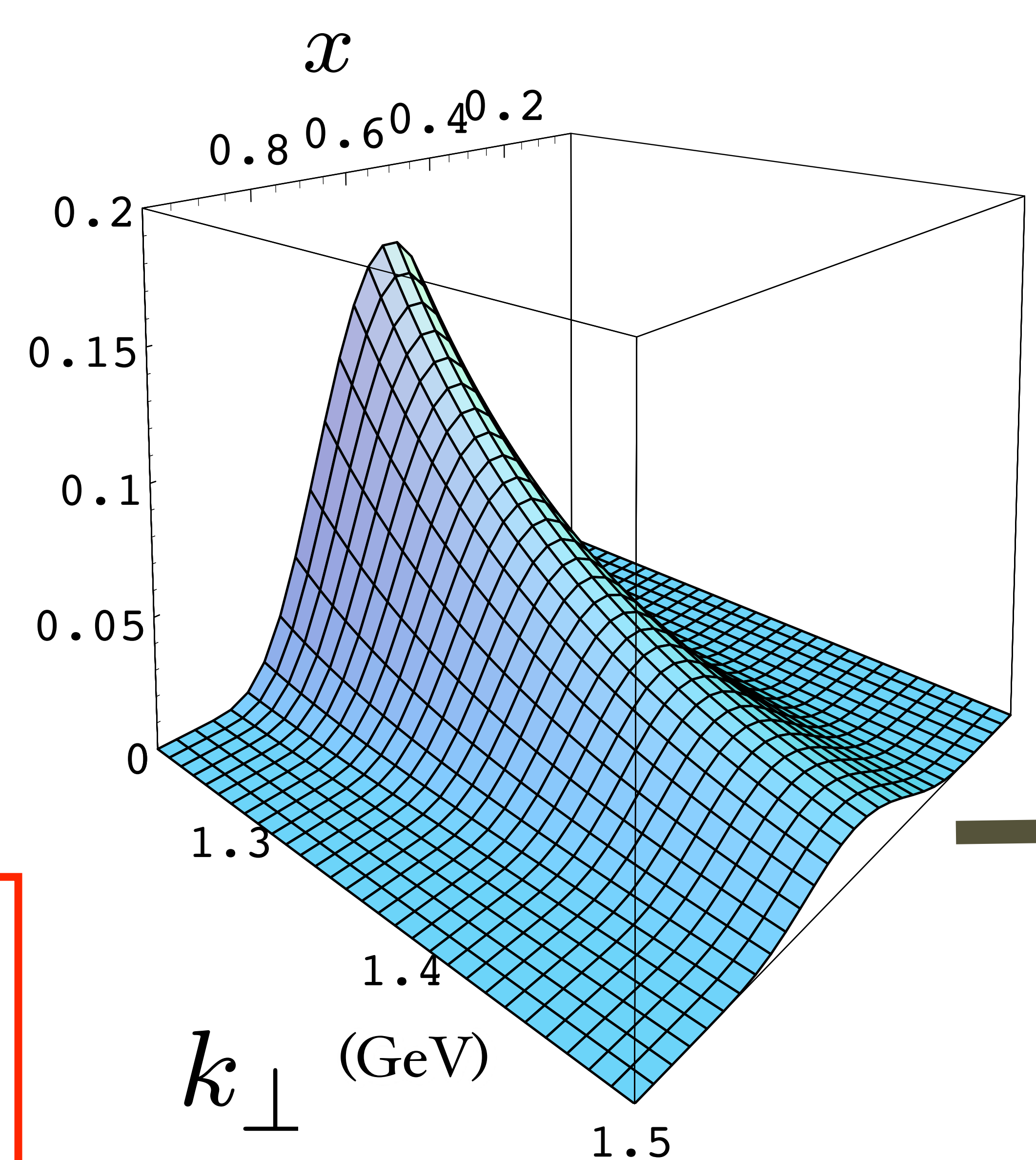
- *Superconformal Algebra: Mass Degenerate 4-Plet:*

$$\boxed{\text{Meson } q\bar{q} \leftrightarrow \text{Baryon } q[qq] \leftrightarrow \text{Tetraquark } [qq][\bar{q}\bar{q}]}$$

Prediction from AdS/QCD: Meson LFWF

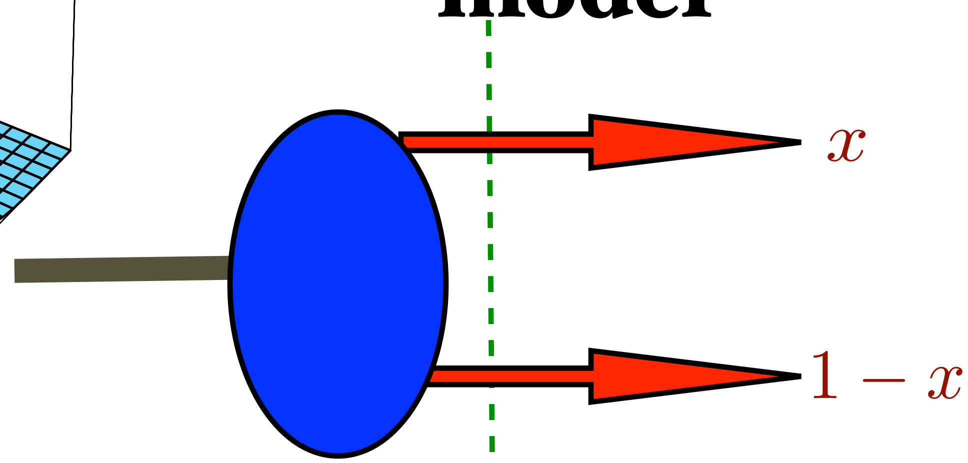
$$e^{\varphi(z)} = e^{+\kappa^2 z}$$

$$\psi_M(x, k_\perp^2)$$



de Teramond, Cao, sjb

"Soft Wall" model



massless quarks

Note coupling

$$k_\perp^2, x$$

$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa\sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}}$$

$$\boxed{\phi_\pi(x) = \frac{4}{\sqrt{3}\pi} f_\pi \sqrt{x(1-x)}}$$

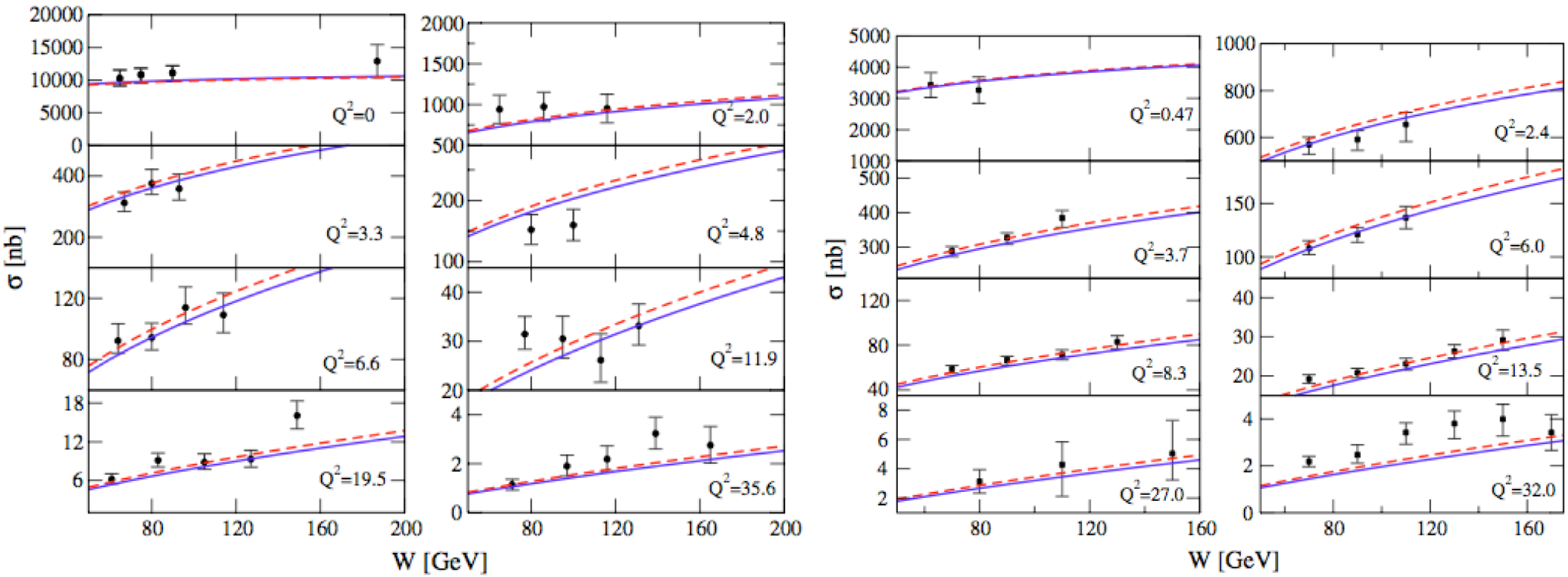
$$f_\pi = \sqrt{P_{q\bar{q}}} \frac{\sqrt{3}}{8} \kappa = 92.4 \text{ MeV}$$

Same as DSE! **C. D. Roberts et al.**

Provides Connection of Confinement to Hadron Structure

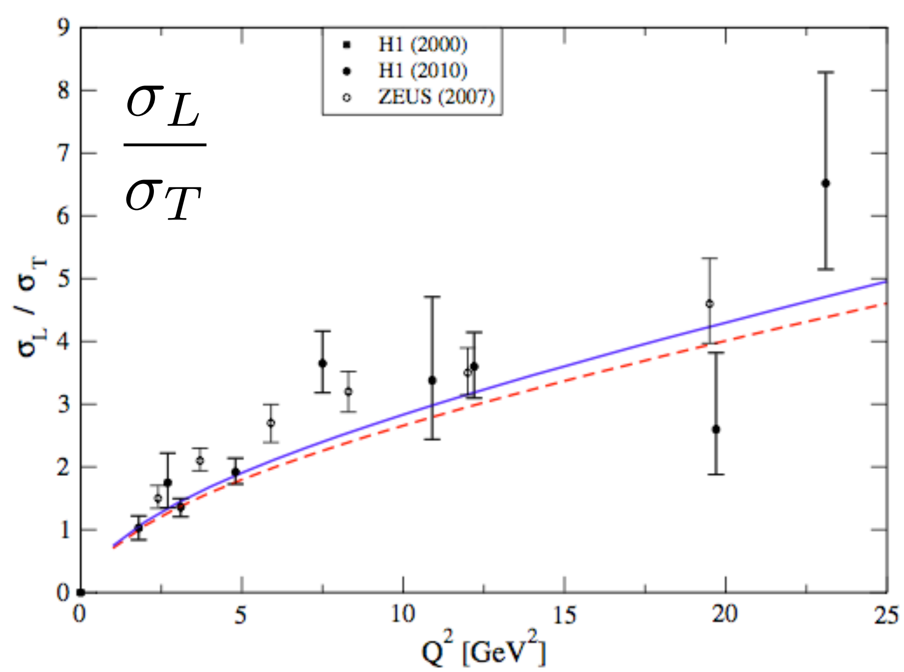
PRL 109, 081601 (2012) PHYSICAL REVIEW LETTERS week ending 24 AUGUST 2012

AdS/QCD Holographic Wave Function for the ρ Meson and Diffractive ρ Meson Electroproduction



(a) H1

(b) ZEUS



J. R. Forshaw,
R. Sandapen

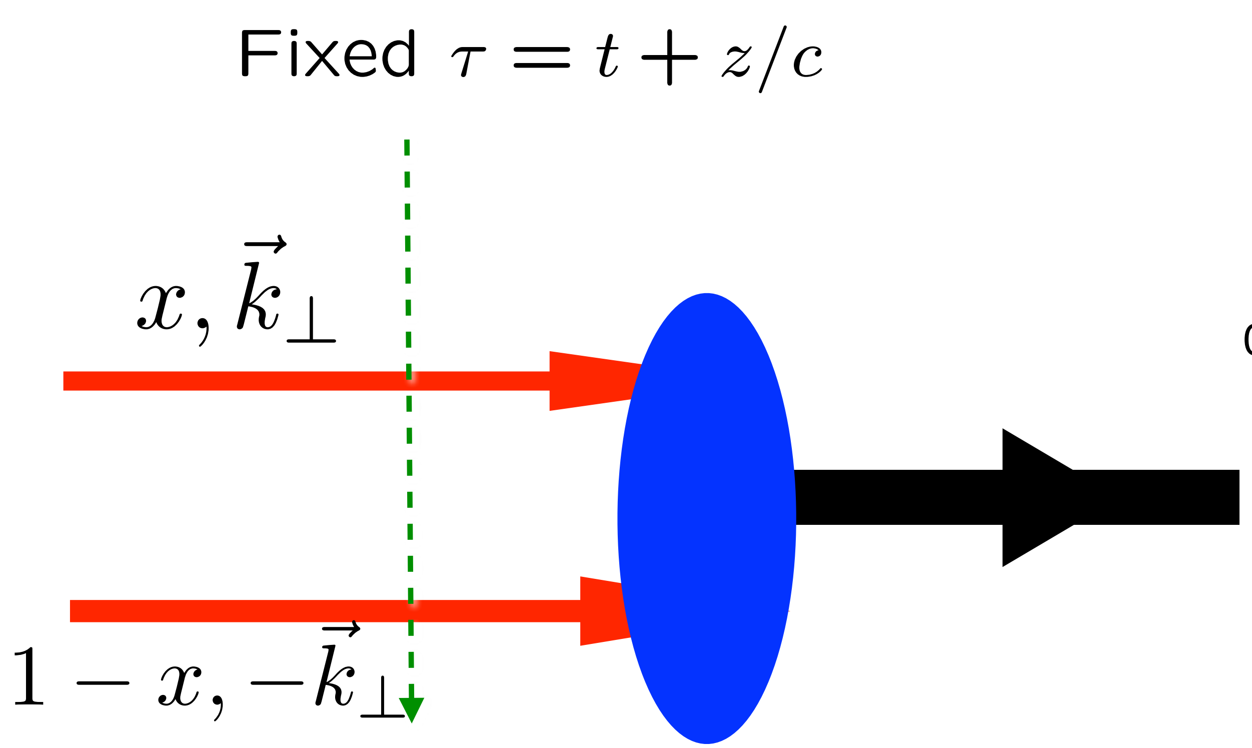
$$\gamma^* p \to \rho^0 p'$$

$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa\sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}}$$

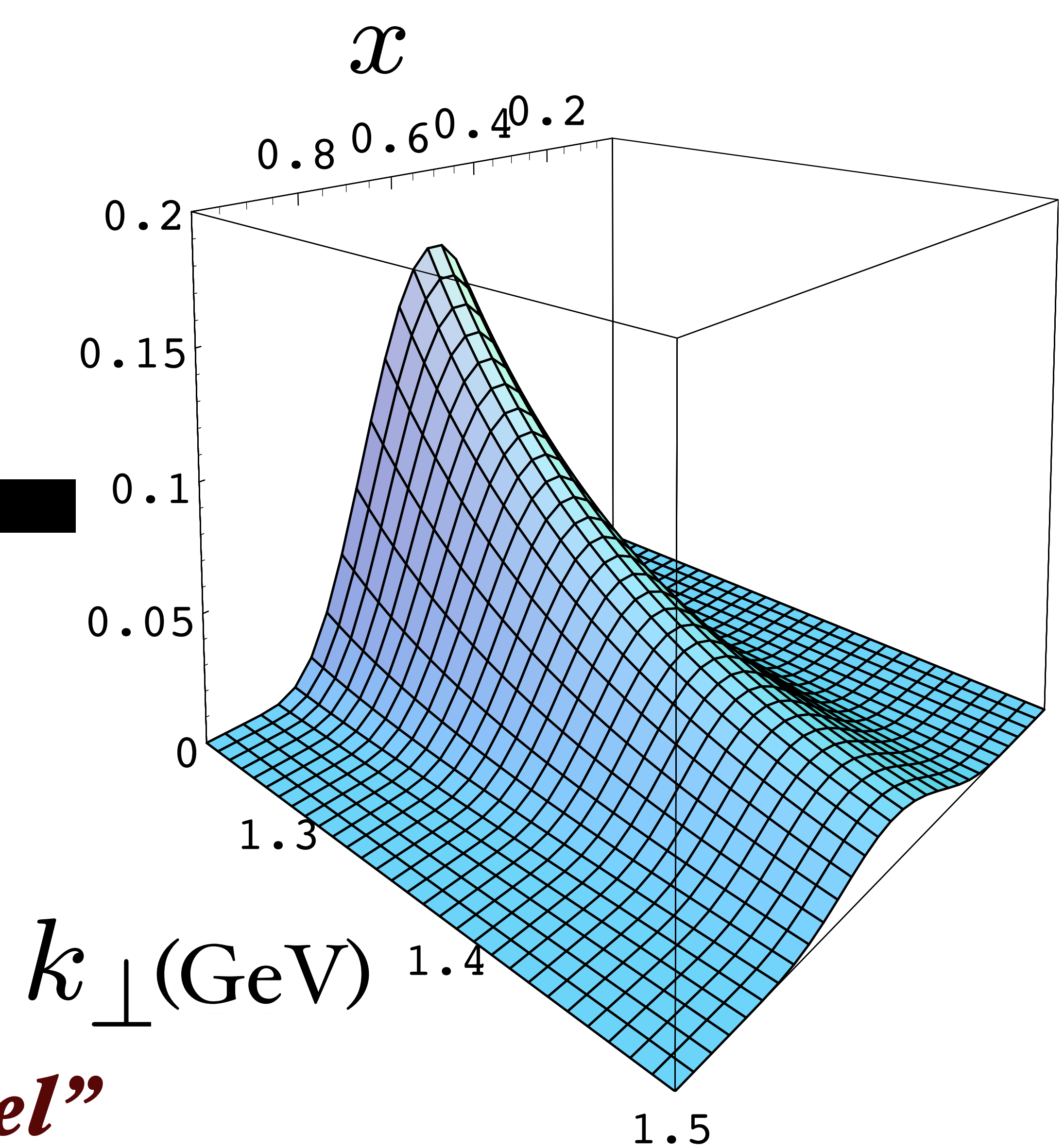
- *Light Front Wavefunctions:*

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

off-shell in P^- and invariant mass $\mathcal{M}^2_{q\bar{q}}$

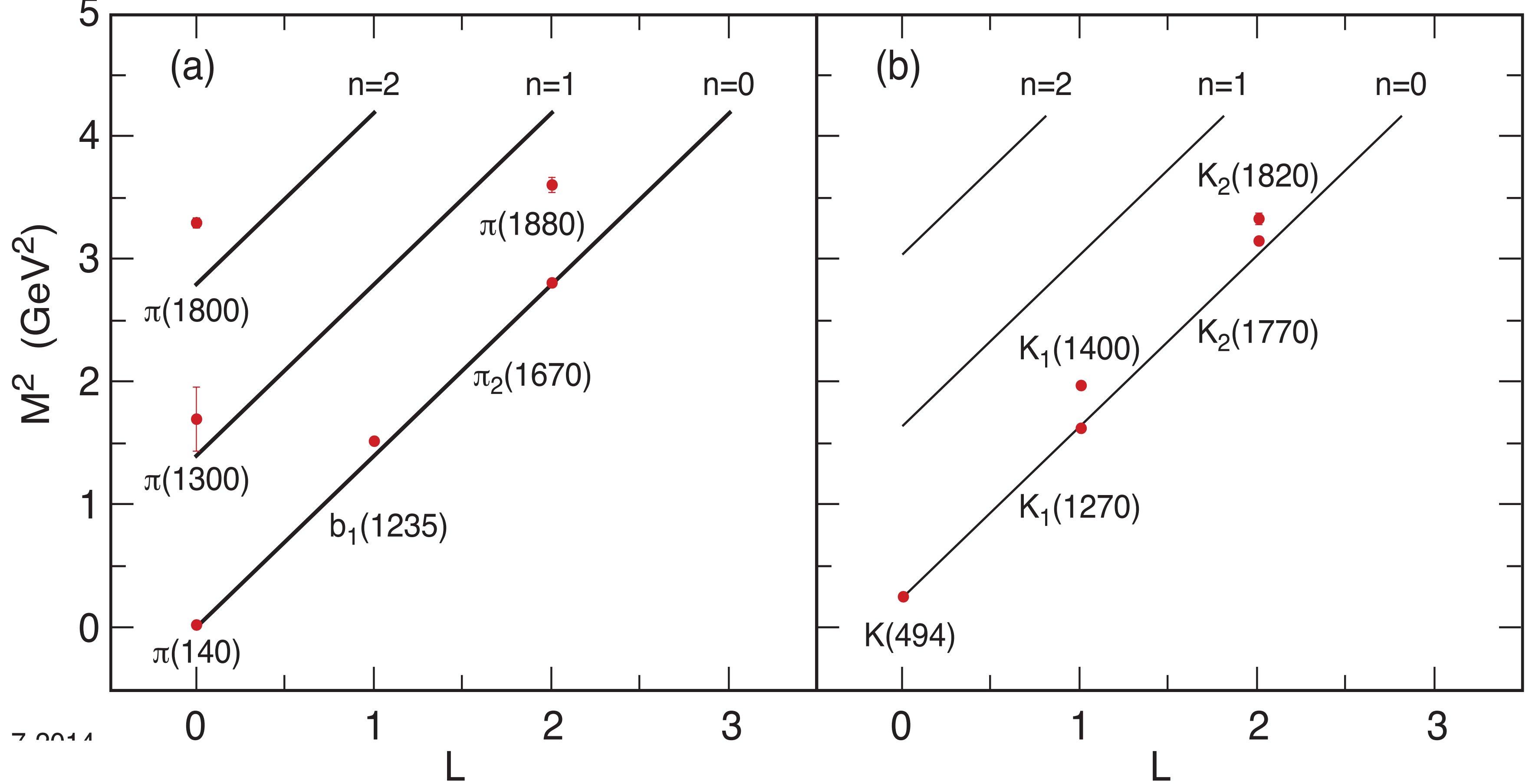


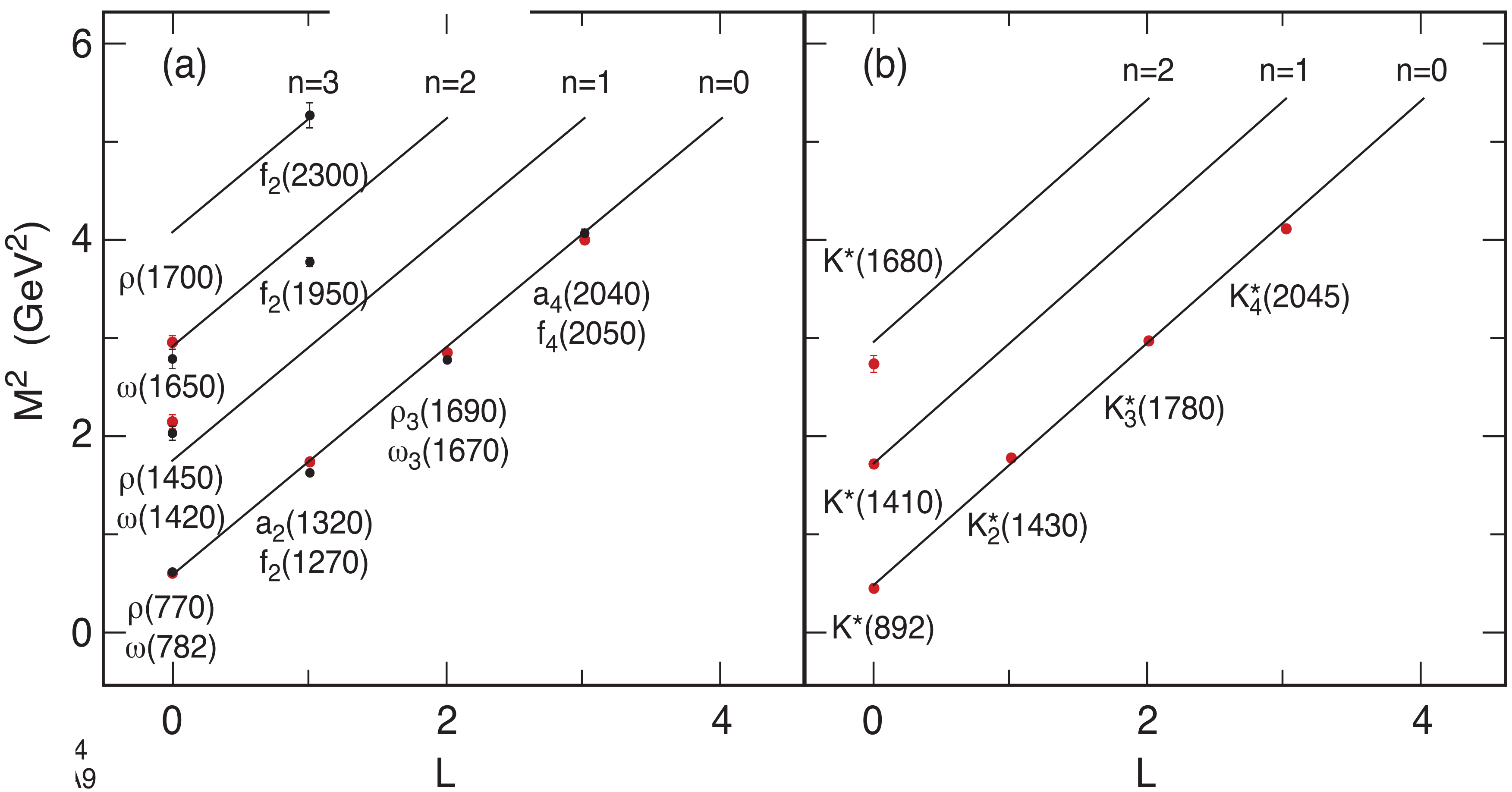
$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

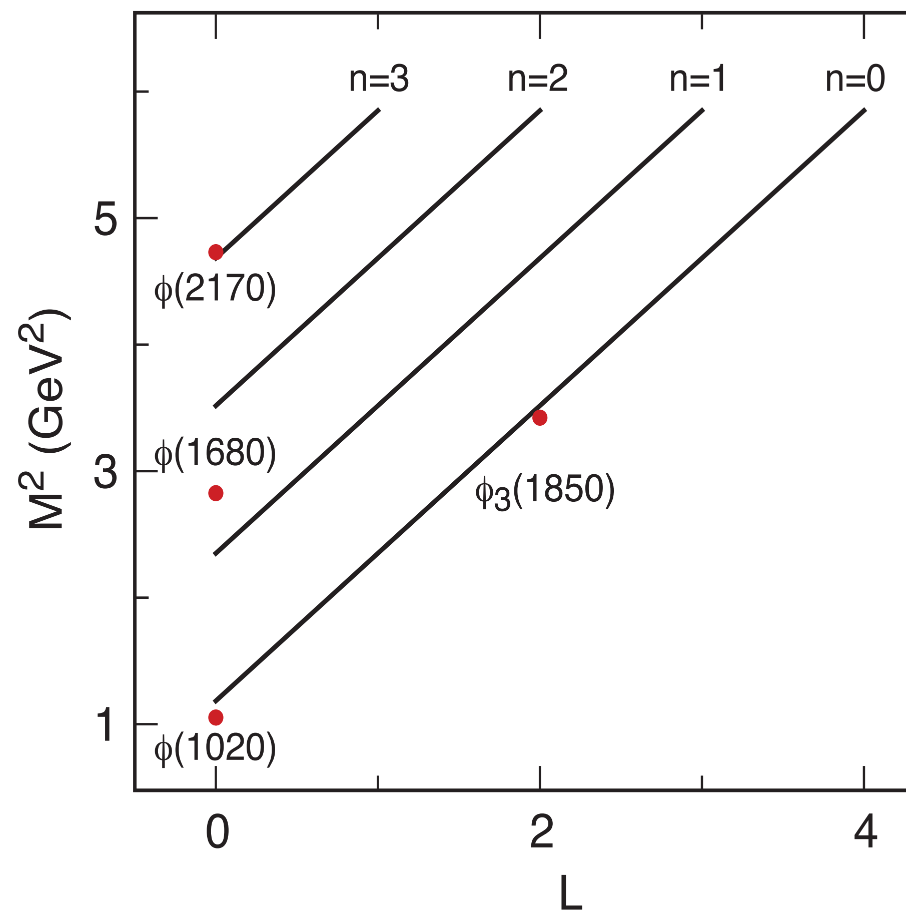


"Hadronization at the Amplitude Level"

Boost-invariant LFWF connects confined quarks and gluons to hadrons



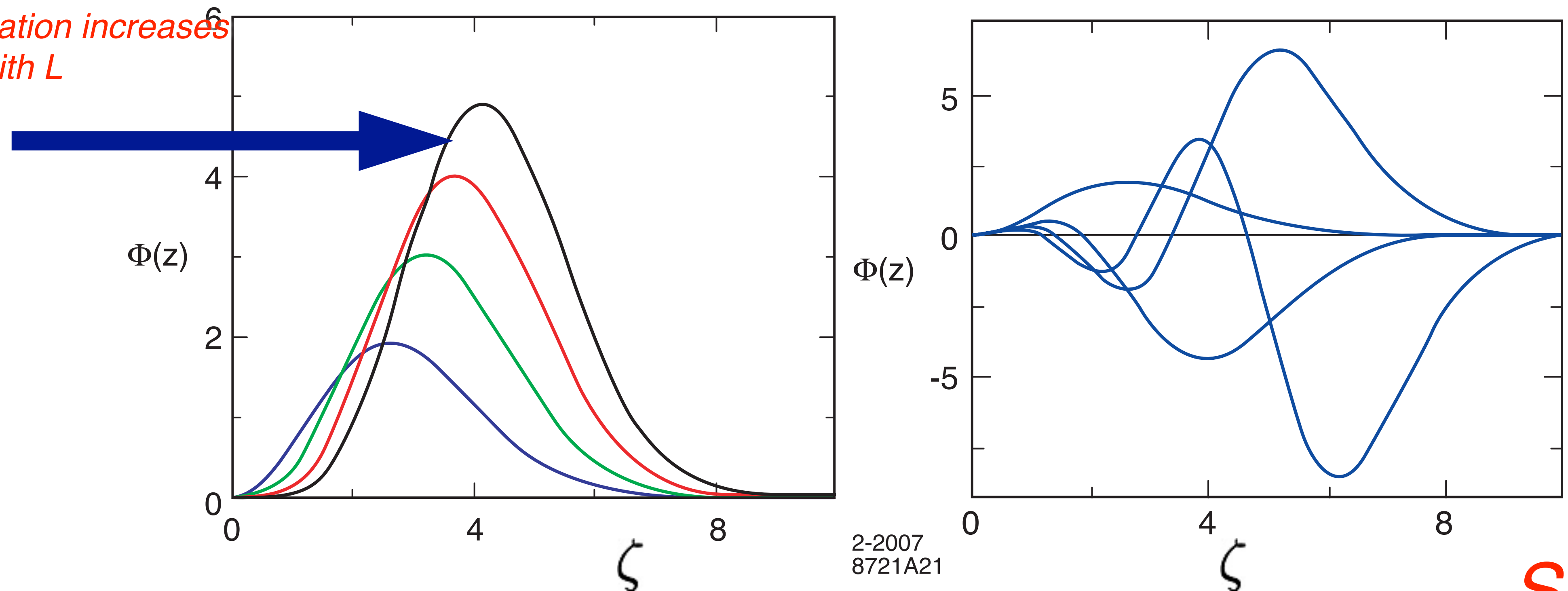




$$M^2(n, L, S) = 4\kappa^2(n + L + S/2)$$

Equal Slope in n and L

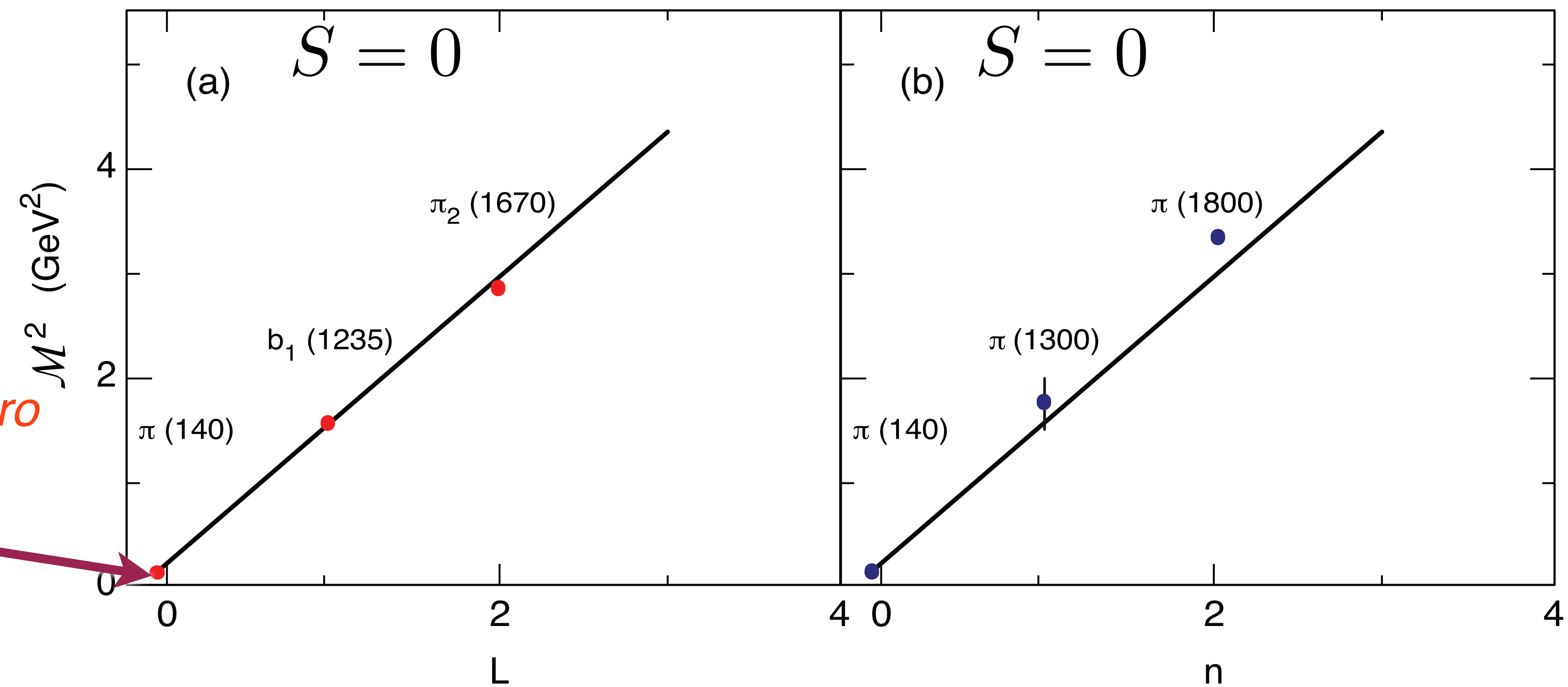
Quark separation increases with L



Soft Wall Model

Fig: Orbital and radial AdS modes in the soft wall model for κ = 0.6 GeV .

Same slope in n and L!

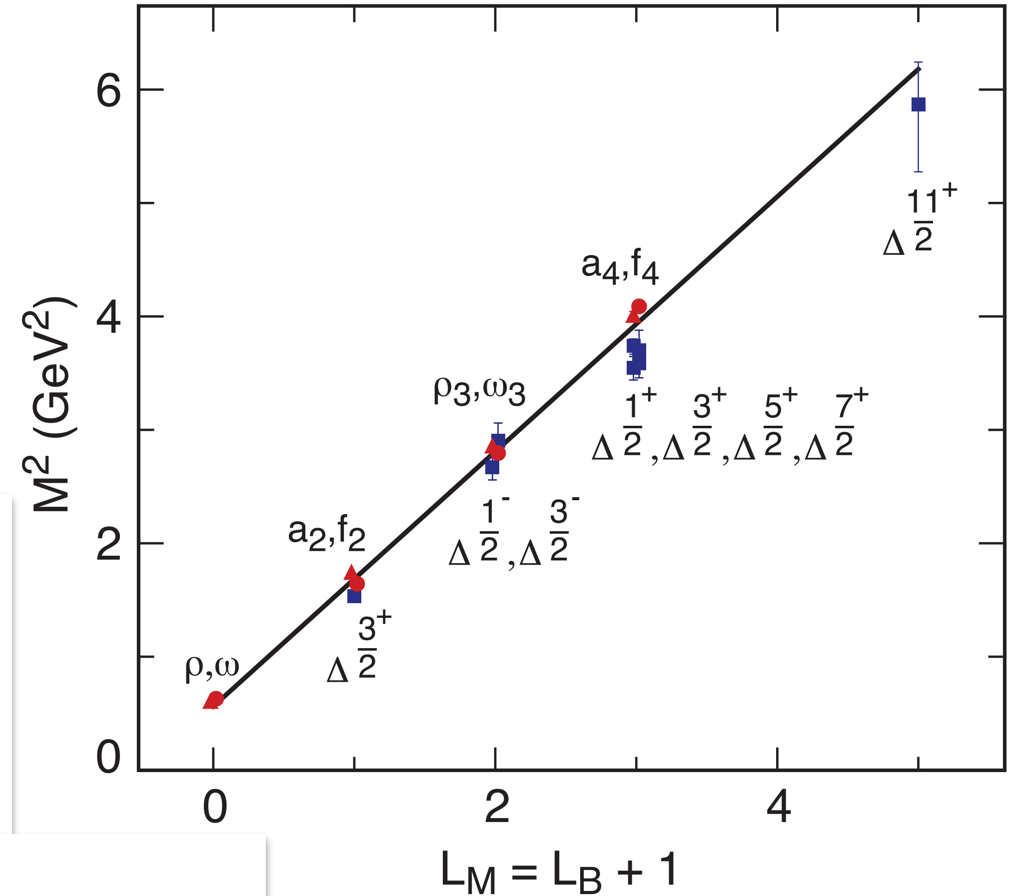


Pion has zero mass!

$$\boxed{m_q = 0}$$

Light meson orbital (a) and radial (b) spectrum for $\kappa = 0.6$ GeV.

Universal Regge Slope in L and n



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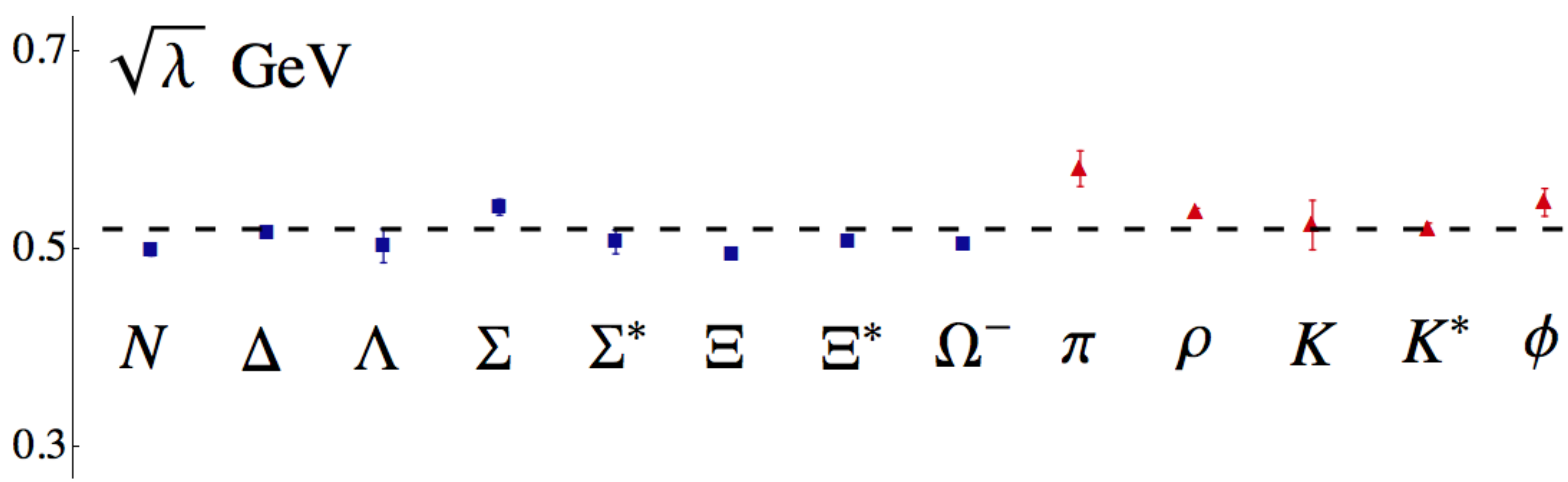
$$\kappa = \sqrt{\lambda} = 0.523 \pm 0.024$$

- How universal is the semiclassical approximation based on superconformal LFHQCD ?



Best fit for hadronic scale $\sqrt{\lambda}$ from different light hadron sectors including radial and orbital excitations

Light-Front Holography: First Approximation to QCD

- **Color Confinement, Analytic form of confinement potential**

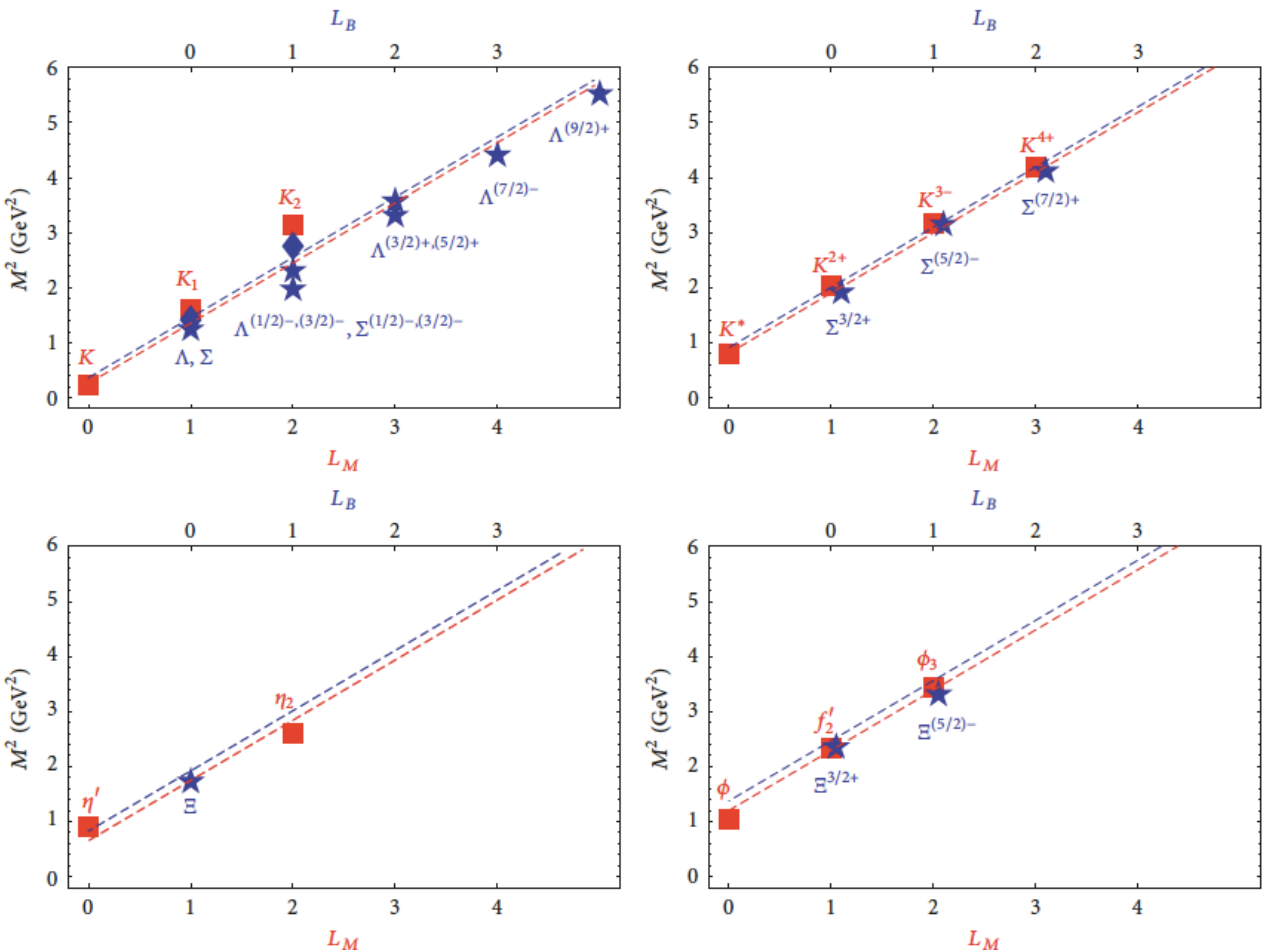
de Téramond, Dosch, Lorcé, sjb

- **Retains underlying conformal properties of QCD despite mass scale (DeAlfaro-Fubini-Furlan Principle)**
- **Massless quark-antiquark pion bound state in chiral limit, GMOR**
- **QCD coupling at all scales**
- **Connection of perturbative and nonperturbative mass scales**
- **Poincarè Invariant**
- **Hadron Spectroscopy-Regge Trajectories with universal slopes in n, L**
- **Supersymmetric 4-Plet: Meson-Baryon -Tetraquark Symmetry**
- **Light-Front Wavefunctions**
- **Form Factors, Structure Functions, Hadronic Observables**
- **OPE: Constituent Counting Rules**
- **Hadronization at the Amplitude Level: Many Phenomenological Tests**
- **Systematically improvable: Basis LF Quantization (BLFQ)**

Supersymmetric Features of Hadron Physics from Superconformal Algebra and Light-Front Holography

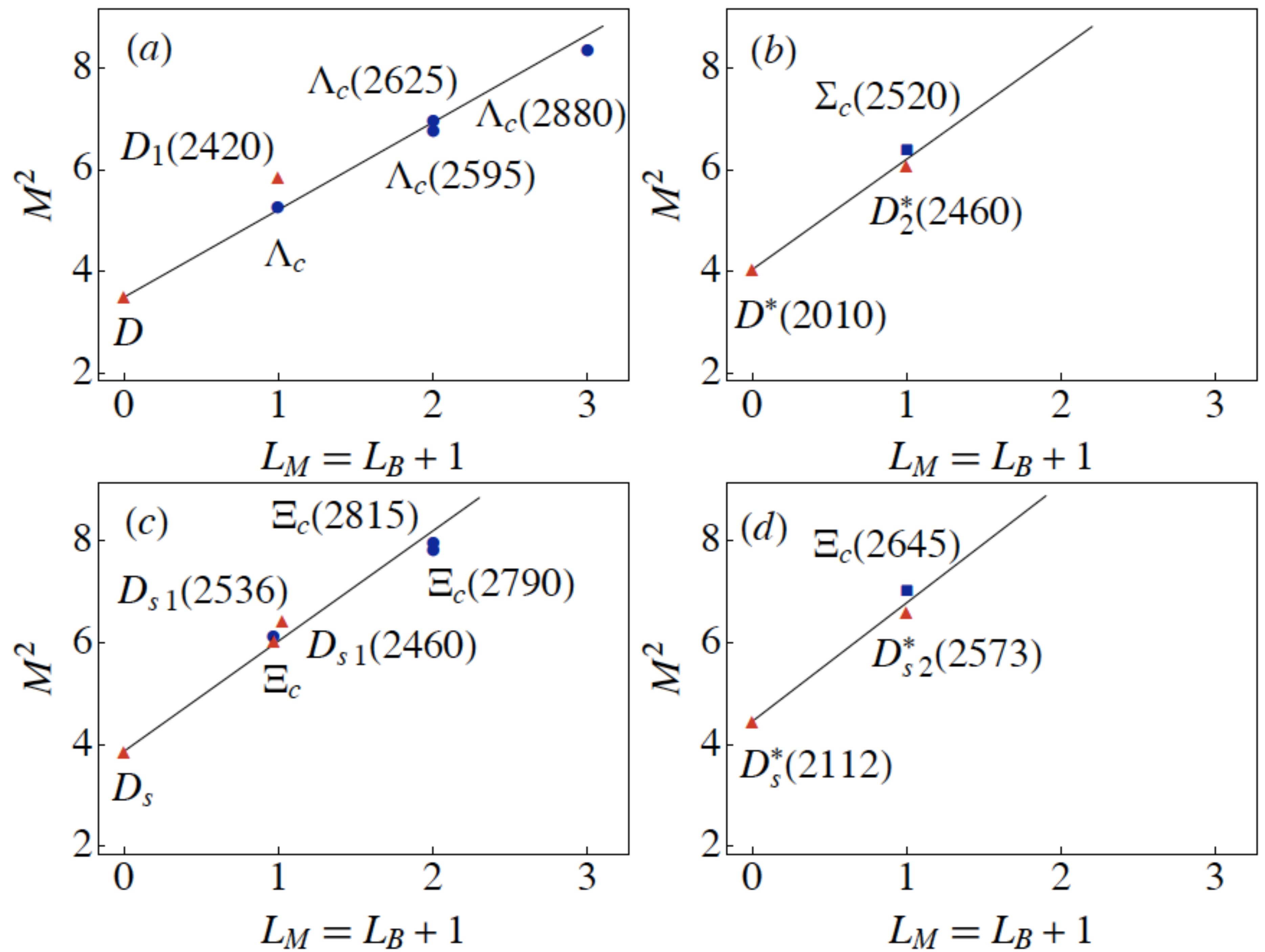
de Téramond, Dosch, Lorcé, sjb

Supersymmetry across the light and heavy-light spectrum



de Téramond, Dosch, Lorcé, sjb

Supersymmetry across the light and heavy-light spectrum

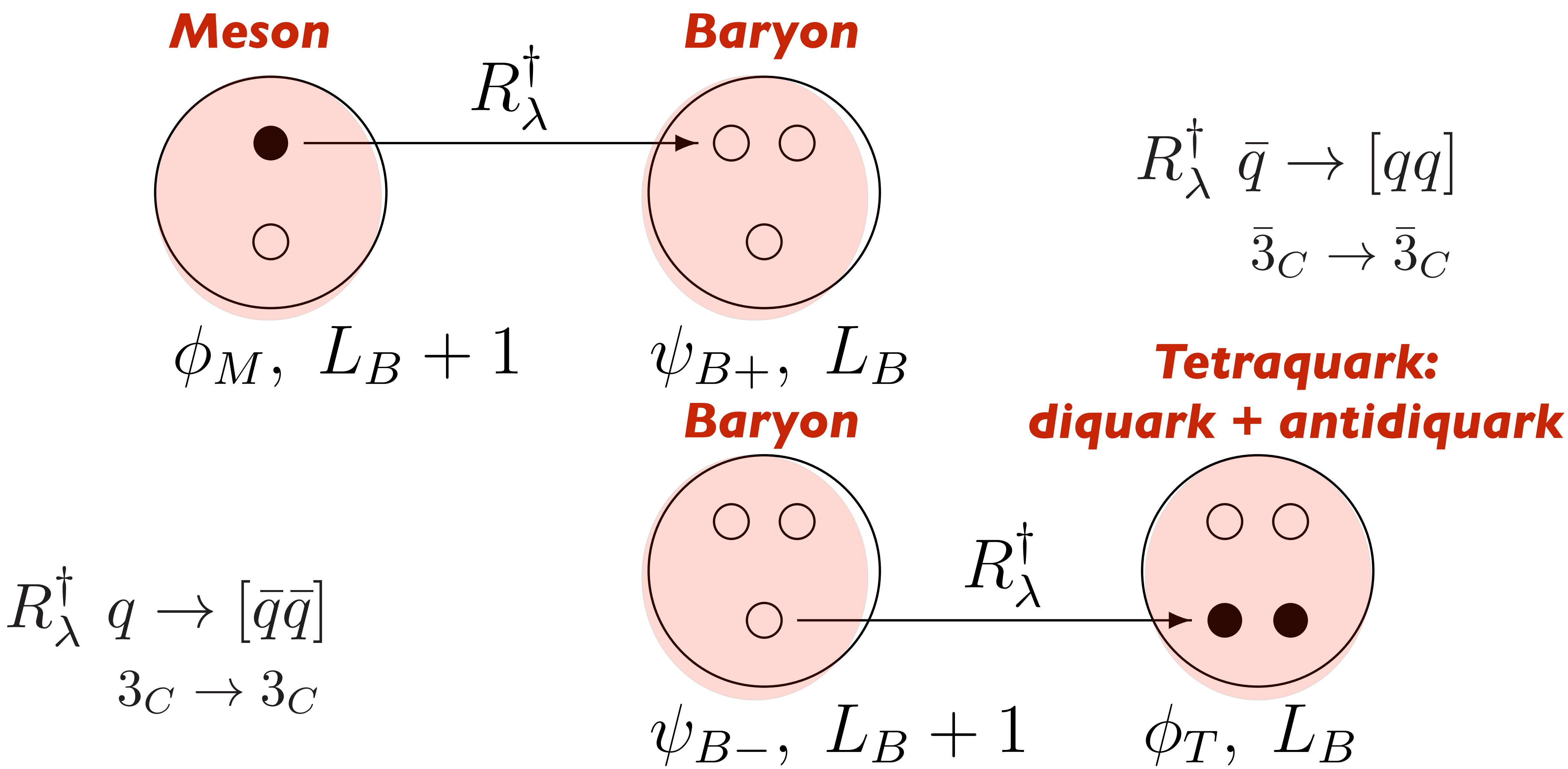


Heavy charm quark mass does not break supersymmetry

Superconformal Algebra

Four-Plet Representations

Bosons, Fermions with Equal Mass!



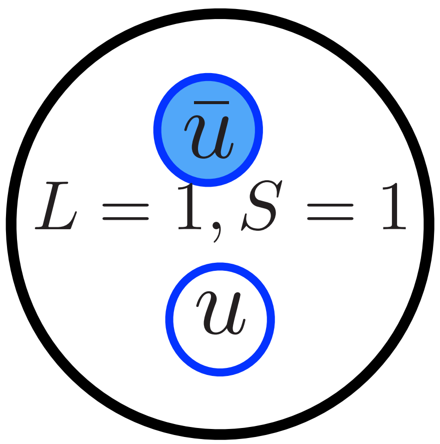
Proton: |u[ud]> Quark + Scalar Diquark
Equal Weight: L=0, L=1

Superconformal Algebra 4-Plet

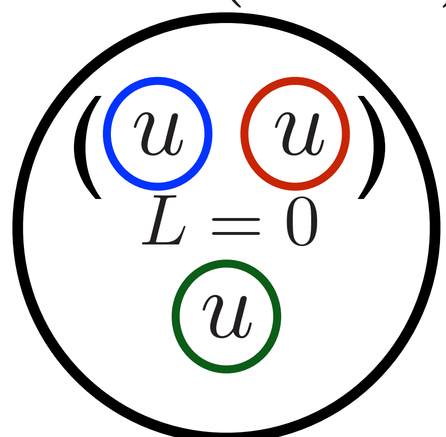
$$R^\dagger_\lambda \quad \bar{q} \rightarrow (qq) \quad S = 1$$
$$\bar{3}_C \rightarrow \bar{3}_C$$

Vector ()+ Scalar [] Diquarks

$f_2(1270)$



$\Delta^+(1232)$



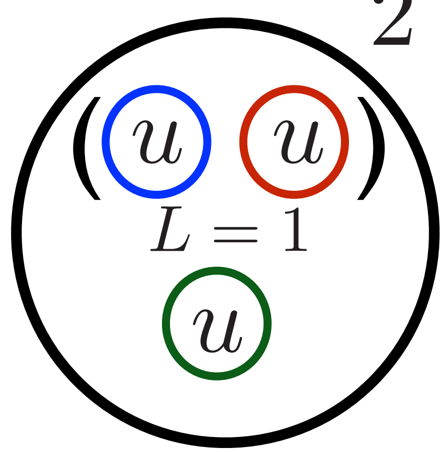
Tetraquark

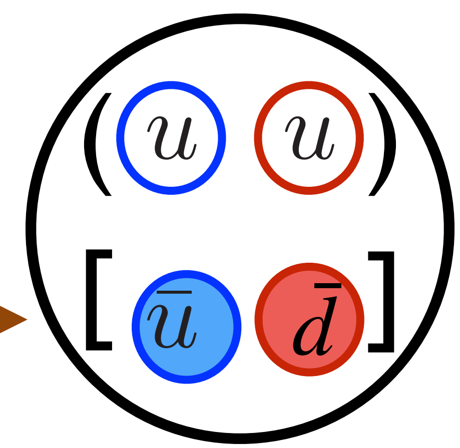
$J^{PC} = 1^{++}$

$a_1(1260)$

$$J^P = \frac{3}{2}^+$$

$J^{PC} = 2^{++}$





$S = 0$
$L = 0$

$$R^\dagger_\lambda \quad q \rightarrow [\bar{q}\bar{q}]$$
$$3_C \rightarrow 3_C$$

Meson

Baryon

Meson			Baryon			Tetraquark		
q-cont	$J^{P(C)}$	Name	q-cont	J^P	Name	q-cont	$J^{P(C)}$	Name
$\bar{q}q$	0^{-+}	$\pi(140)$	—	—	—	—	—	—
$\bar{q}q$	1^{+-}	$b_1(1235)$	$[ud]q$	$(1/2)^+$	$N(940)$	$[ud][\bar{u}\bar{d}]$	0^{++}	$f_0(980)$
$\bar{q}q$	2^{-+}	$\pi_2(1670)$	$[ud]q$	$(1/2)^-$	$N_{\frac{1}{2}^-}(1535)$	$[ud][\bar{u}\bar{d}]$	1^{-+}	$\pi_1(1400)$
				$(3/2)^-$	$N_{\frac{3}{2}^-}(1520)$			$\pi_1(1600)$
$\bar{q}q$	1^{--}	$\rho(770),\ \omega(780)$	—	—	—	—	—	—
$\bar{q}q$	2^{++}	$a_2(1320),\ f_2(1270)$	$[qq]q$	$(3/2)^+$	$\Delta(1232)$	$[qq][\bar{u}\bar{d}]$	1^{++}	$a_1(1260)$
$\bar{q}q$	3^{--}	$\rho_3(1690),\ \omega_3(1670)$	$[qq]q$	$(1/2)^-$	$\Delta_{\frac{1}{2}^-}(1620)$	$[qq][\bar{u}\bar{d}]$	2^{--}	$\rho_2(\sim 1700)?$
				$(3/2)^-$	$\Delta_{\frac{3}{2}^-}(1700)$			
$\bar{q}q$	4^{++}	$a_4(2040),\ f_4(2050)$	$[qq]q$	$(7/2)^+$	$\Delta_{\frac{7}{2}^+}(1950)$	$[qq][\bar{u}\bar{d}]$	3^{++}	$a_3(\sim 2070)?$
$\bar{q}s$	$0^{-(+)}$	$K(495)$	—	—	—	—	—	—
$\bar{q}s$	$1^{+(-)}$	$\bar{K}_1(1270)$	$[ud]s$	$(1/2)^+$	$\Lambda(1115)$	$[ud][\bar{s}\bar{q}]$	$0^{+(+)}$	$K_0^*(1430)$
$\bar{q}s$	$2^{-(+)}$	$K_2(1770)$	$[ud]s$	$(1/2)^-$	$\Lambda(1405)$	$[ud][\bar{s}\bar{q}]$	$1^{-(+)}$	$K_1^*(\sim 1700)?$
				$(3/2)^-$	$\Lambda(1520)$			
$\bar{s}q$	$0^{-(+)}$	$K(495)$	—	—	—	—	—	—
$\bar{s}q$	$1^{+(-)}$	$K_1(1270)$	$[sq]q$	$(1/2)^+$	$\Sigma(1190)$	$[sq][\bar{s}\bar{q}]$	0^{++}	$a_0(980)$
								$f_0(980)$
$\bar{s}q$	$1^{-(-)}$	$K^*(890)$	—	—	—	—	—	—
$\bar{s}q$	$2^{+(+)}$	$K_2^*(1430)$	$[sq]q$	$(3/2)^+$	$\Sigma(1385)$	$[sq][\bar{q}\bar{q}]$	$1^{+(+)}$	$K_1(1400)$
$\bar{s}q$	$3^{-(-)}$	$K_3^*(1780)$	$[sq]q$	$(3/2)^-$	$\Sigma(1670)$	$[sq][\bar{q}\bar{q}]$	$2^{-(-)}$	$K_2(\sim 1700)?$
$\bar{s}q$	$4^{+(+)}$	$K_4^*(2045)$	$[sq]q$	$(7/2)^+$	$\Sigma(2030)$	$[sq][\bar{q}\bar{q}]$	$3^{+(+)}$	$K_3(\sim 2070)?$
$\bar{s}s$	0^{-+}	$\eta(550)$	—	—	—	—	—	—
$\bar{s}s$	1^{+-}	$h_1(1170)$	$[sq]s$	$(1/2)^+$	$\Xi(1320)$	$[sq][\bar{s}\bar{q}]$	0^{++}	$f_0(1370)$
								$a_0(1450)$
$\bar{s}s$	2^{-+}	$\eta_2(1645)$	$[sq]s$	$(?)^?$	$\Xi(1690)$	$[sq][\bar{s}\bar{q}]$	1^{-+}	$\Phi'(1750)?$
$\bar{s}s$	1^{--}	$\Phi(1020)$	—	—	—	—	—	—
$\bar{s}s$	2^{++}	$f_2'(1525)$	$[sq]s$	$(3/2)^+$	$\Xi^*(1530)$	$[sq][\bar{s}\bar{q}]$	1^{++}	$f_1(1420)$
$\bar{s}s$	3^{--}	$\Phi_3(1850)$	$[sq]s$	$(3/2)^-$	$\Xi(1820)$	$[sq][\bar{s}\bar{q}]$	2^{--}	$\Phi_2(\sim 1800)?$
$\bar{s}s$	2^{++}	$f_2(1950)$	$[ss]s$	$(3/2)^+$	$\Omega(1672)$	$[ss][\bar{s}\bar{q}]$	$1^{+(+)}$	$K_1(\sim 1700)?$

Meson Baryon Tetraquark

New Organization of the Hadron Spectrum

M. Nielsen, sjb

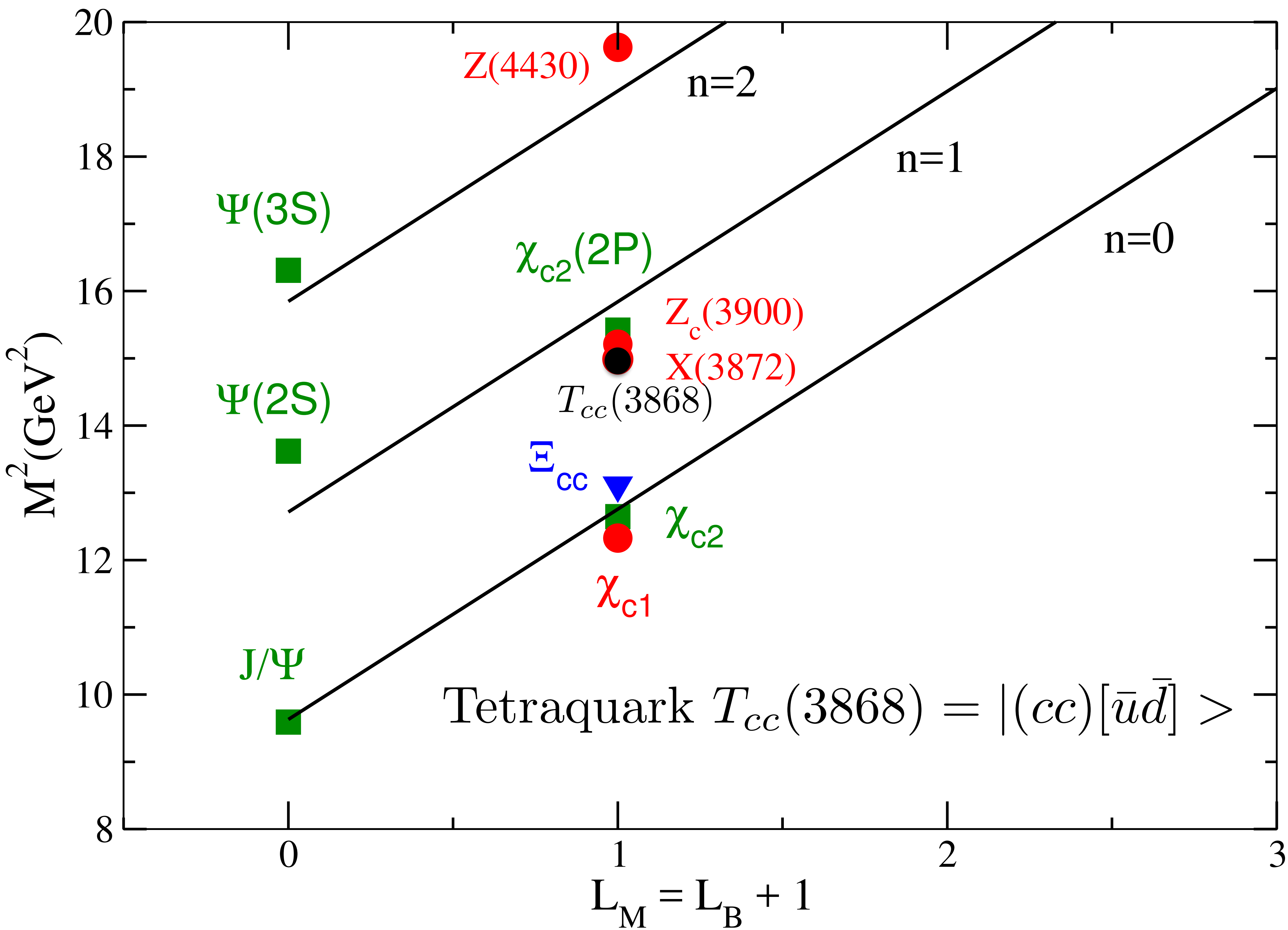
Superpartners for states with one c quark

Meson			Baryon			Tetraquark		
q-cont	$J^{P(C)}$	Name	q-cont	J^P	Name	q-cont	$J^{P(C)}$	Name
$\bar{q}c$	0^-	$D(1870)$	—	—	—	—	—	—
$\bar{q}c$	1^+	$D_1(2420)$	$[ud]c$	$(1/2)^+$	$\Lambda_c(2290)$	$[ud][\bar{c}\bar{q}]$	0^+	$\bar{D}_0^*(2400)$
$\bar{q}c$	2^-	$D_J(2600)$	$[ud]c$	$(3/2)^-$	$\Lambda_c(2625)$	$[ud][\bar{c}\bar{q}]$	1^-	—
$\bar{c}q$	0^-	$\bar{D}(1870)$	—	—	—	—	—	—
$\bar{c}q$	1^+	$\bar{D}_1(2420)$	$[cq]q$	$(1/2)^+$	$\Sigma_c(2455)$	$[cq][\bar{u}\bar{d}]$	0^+	$D_0^*(2400)$
$\bar{q}c$	1^-	$D^*(2010)$	—	—	—	—	—	—
$\bar{q}c$	2^+	$D_2^*(2460)$	$(qq)c$	$(3/2)^+$	$\Sigma_c^*(2520)$	$(qq)[\bar{c}\bar{q}]$	1^+	$D(2550)$
$\bar{q}c$	3^-	$D_3^*(2750)$	$(qq)c$	$(3/2)^-$	$\Sigma_c(2800)$	$(qq)[\bar{c}\bar{q}]$	—	—
$\bar{s}c$	0^-	$D_s(1968)$	—	—	—	—	—	—
$\bar{s}c$	1^+	$D_{s1}(2460)$	$[qs]c$	$(1/2)^+$	$\Xi_c(2470)$	$[qs][\bar{c}\bar{q}]$	0^+	$\bar{D}_{s0}^*(2317)$
$\bar{s}c$	2^-	$D_{s2}(\sim 2860)?$	$[qs]c$	$(3/2)^-$	$\Xi_c(2815)$	$[sq][\bar{c}\bar{q}]$	1^-	—
$\bar{s}c$	1^-	$D_s^*(2110)$	—	—	—	—	—	—
$\bar{s}c$	2^+	$D_{s2}^*(2573)$	$(sq)c$	$(3/2)^+$	$\Xi_c^*(2645)$	$(sq)[\bar{c}\bar{q}]$	1^+	$D_{s1}(2536)$
$\bar{c}s$	1^+	$D_{s1}(\sim 2700)?$	$[cs]s$	$(1/2)^+$	$\Omega_c(2695)$	$[cs][\bar{s}\bar{q}]$	0^+	??
$\bar{s}c$	2^+	$D_{s2}^*(\sim 2750)?$	$(ss)c$	$(3/2)^+$	$\Omega_c(2770)$	$(ss)[\bar{c}\bar{s}]$	1^+	??

predictions

beautiful agreement!

M. Nielsen, sjb



$Mesons : GreenSquare, Baryons(BlueTriangle), Tetraquarks(RedCircle)$

Connection to the Linear Instant-Form Heavy Quark Potential

Harmonic Oscillator $U(\zeta) = \kappa^4 \zeta^2$ LF Potential for relativistic light quarks

$\Updownarrow$

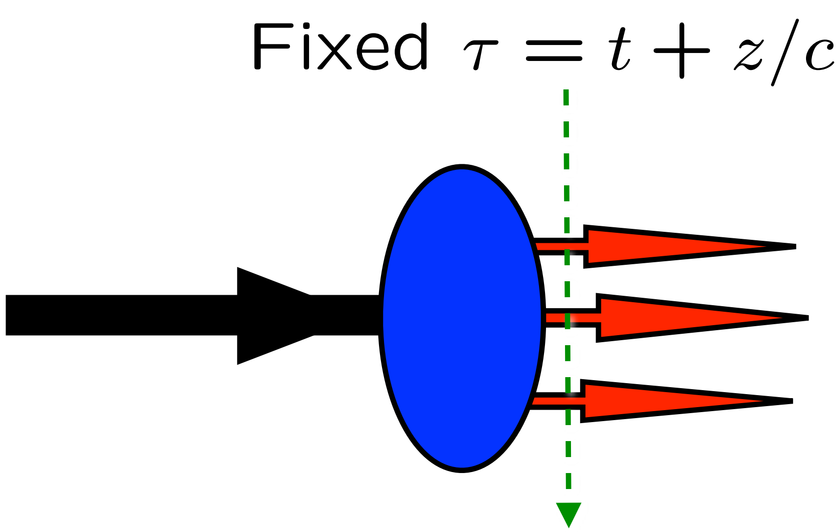
Linear instant nonrelativistic form $V(r) = Cr$ for heavy quarks

A.P. Trawinski, S.D. Glazek, H. D. Dosch, G. de Teramond, sjb

Bound States in Relativistic Quantum Field Theory:

Light-Front Wavefunctions

Dirac's Front Form: Fixed $\tau = t + z/c$



Boost invariant, Lorentz frame independent, Causal

$$\psi(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$

Invariant under boosts. Independent of P^μ

$$\boxed{\mathrm{H}^{QCD}_{LF} |\psi> = M^2 |\psi>}$$

Direct connection to QCD Lagrangian

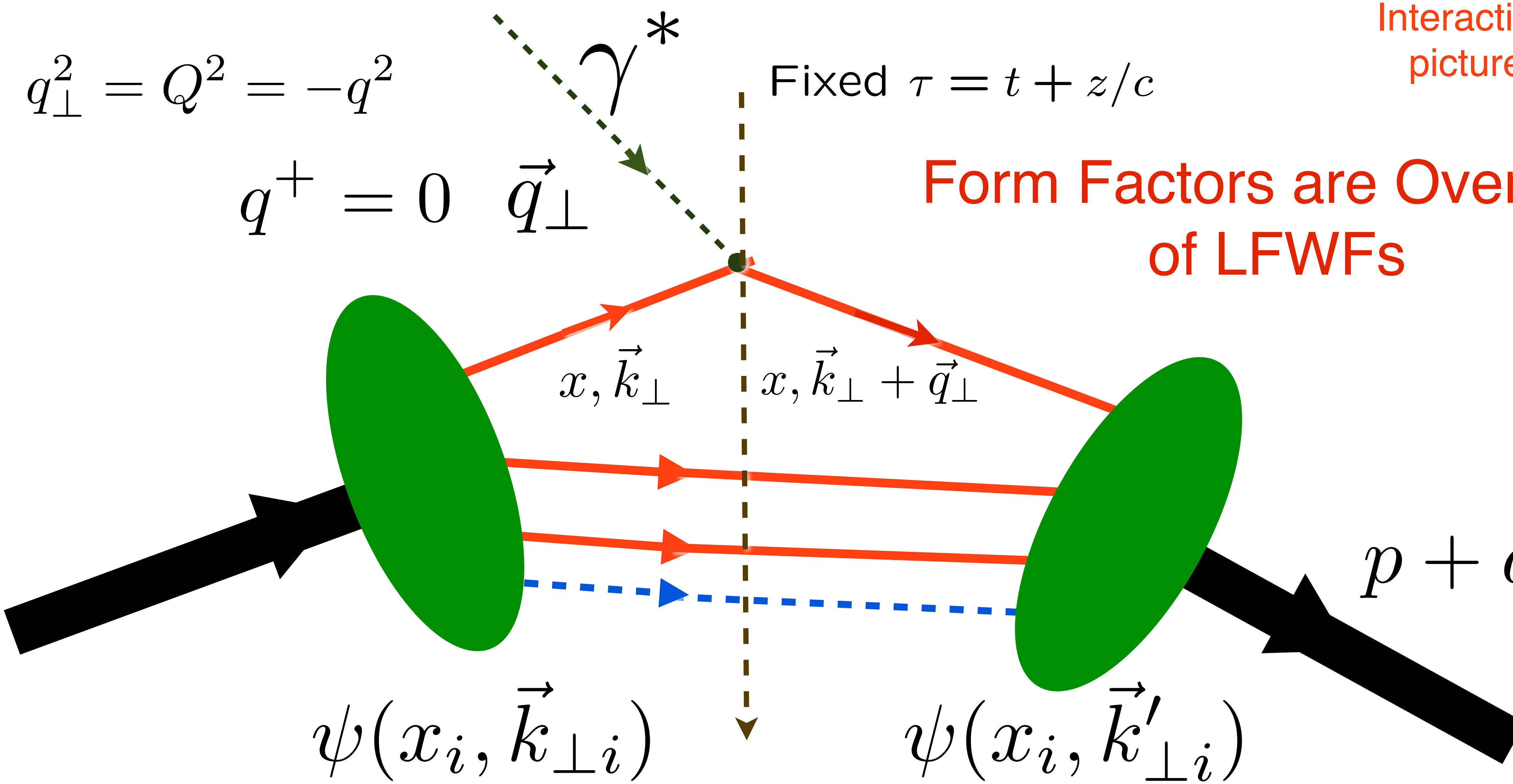
LF Wavefunction: off-shell in invariant mass

Remarkable new insights from AdS/CFT, the duality between conformal field theory and Anti-de Sitter Space

$$< p+q|j^+(0)|p >= 2p^+F(q^2)$$

Front Form

Interaction picture



Form Factors are Overlaps of LFWFs

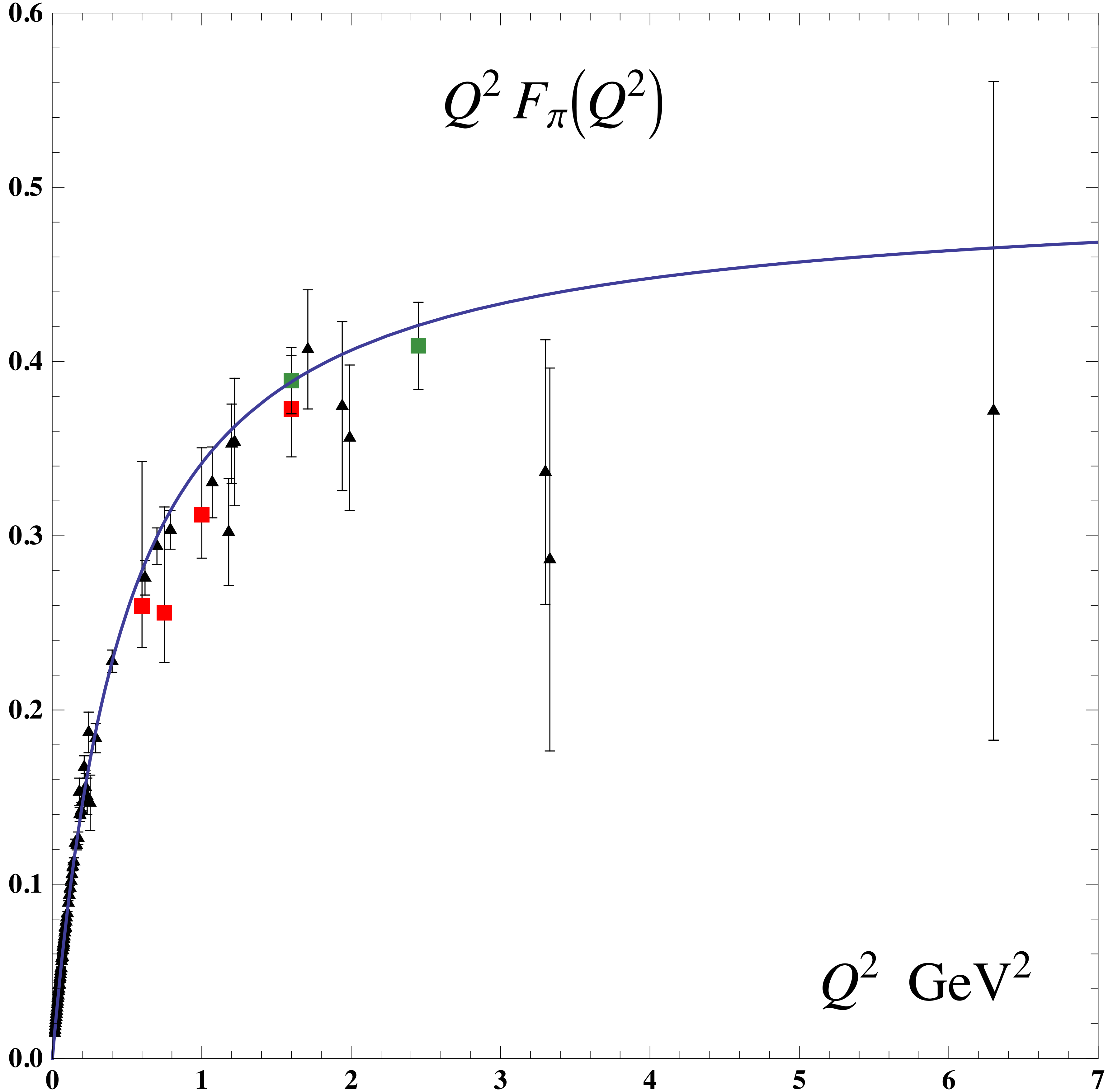
struck $\quad \vec{k}'_{\perp i} = \vec{k}_{\perp i} + (1 - x_i)\vec{q}_\perp$

spectators $\quad \vec{k}'_{\perp i} = \vec{k}_{\perp i} - x_i\vec{q}_\perp$

Drell &Yan, West
Exact LF formula!

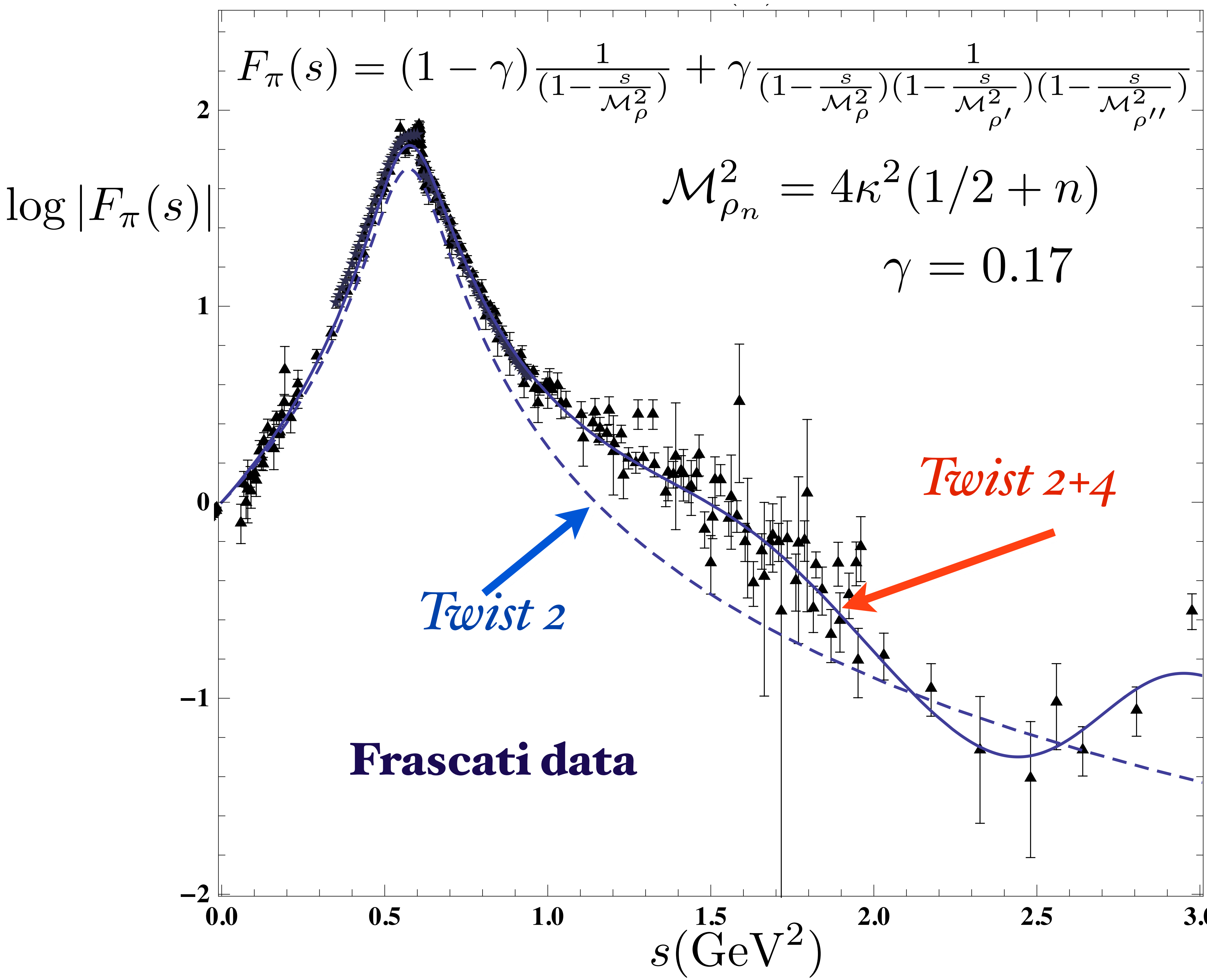
Drell, sjb

Transverse size $\propto \frac{1}{Q}$



$Q^2 F_\pi(Q^2)$
Q^2 GeV^2
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0
1
2
3
4
5
6
7

Timelike Pion Form Factor from AdS/QCD and Light-Front Holography



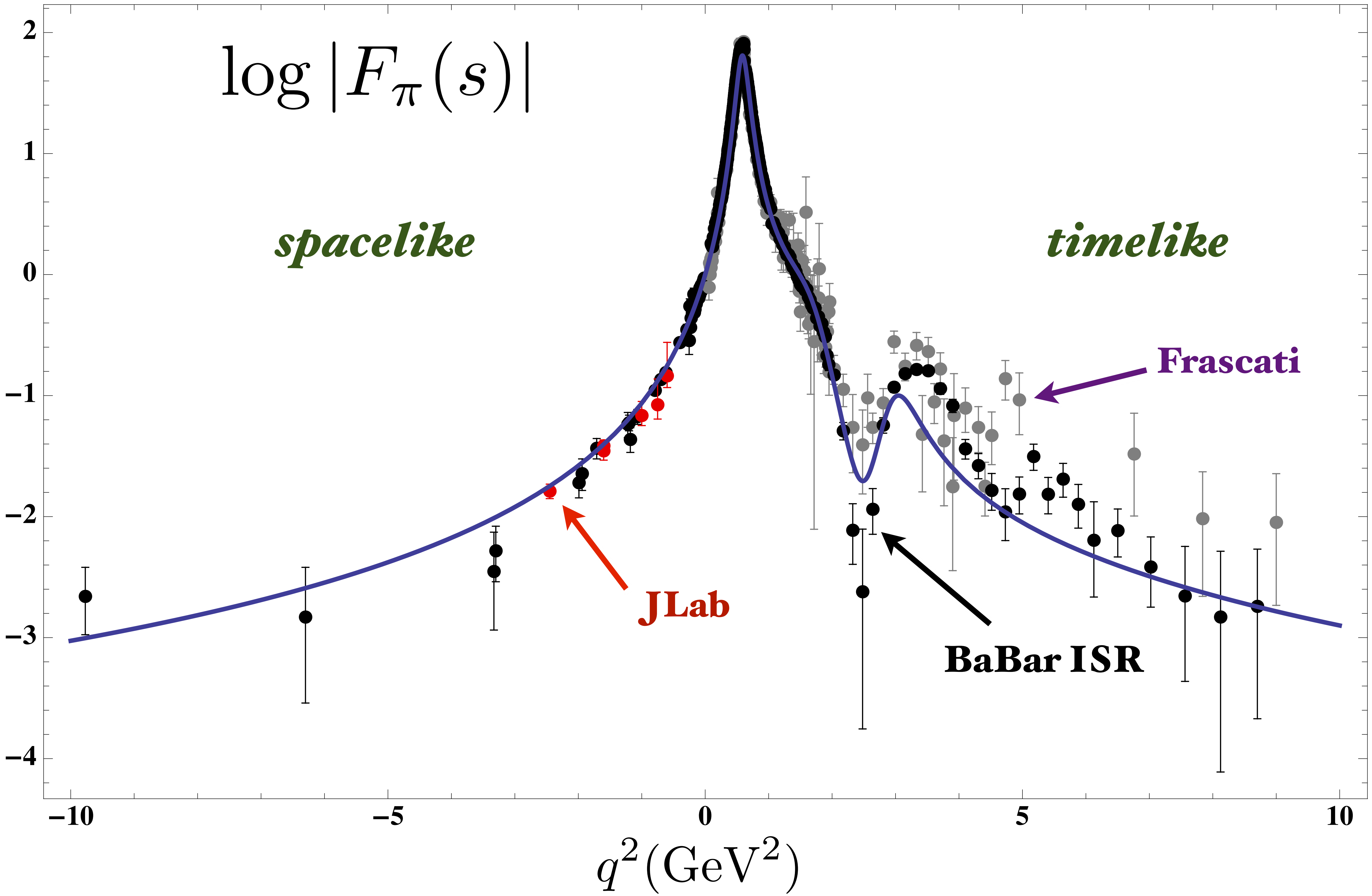
Prescription for Timelike poles :

$$\frac{1}{s - M^2 + i\sqrt{s}\Gamma}$$

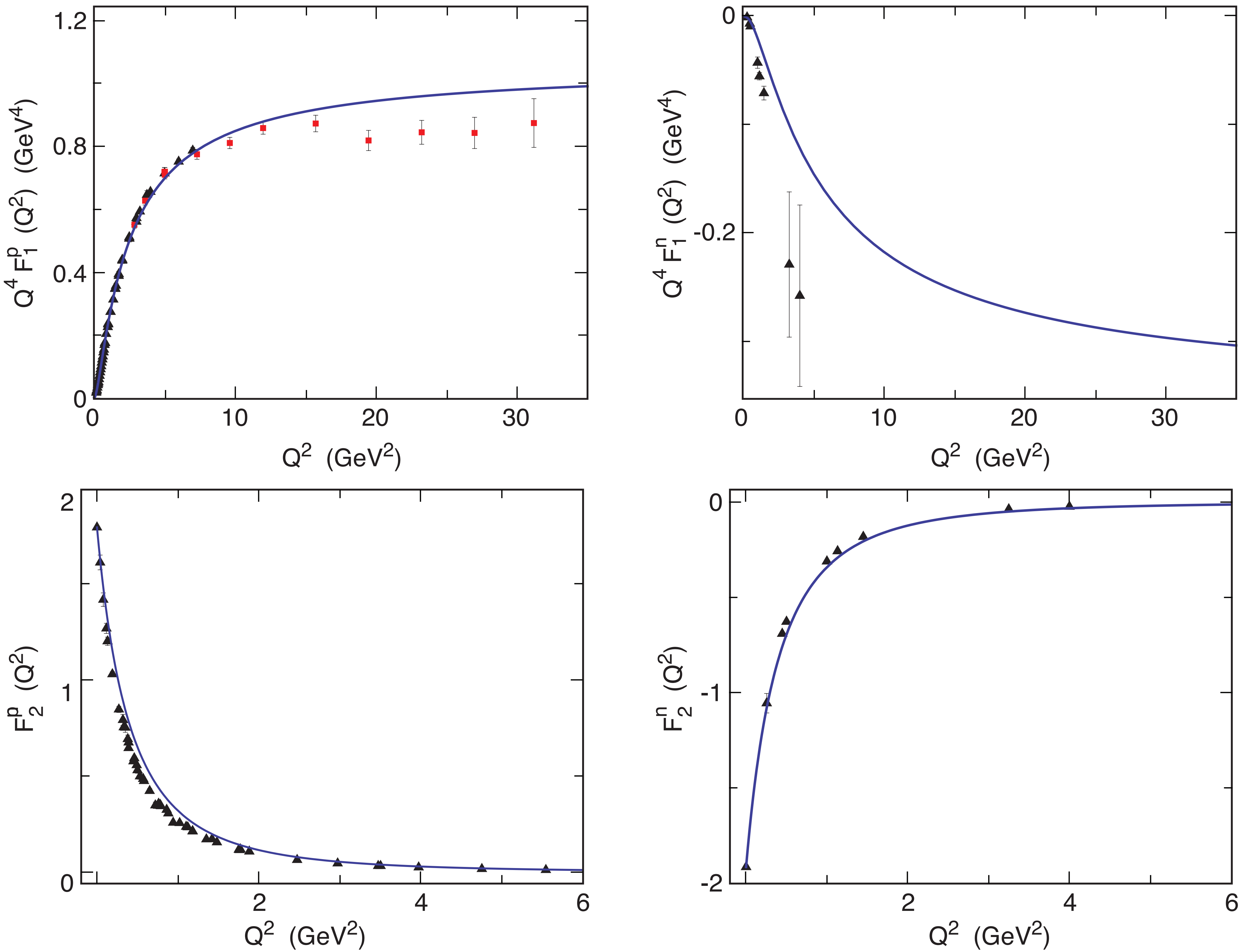
14% four-quark probability

G. de Teramond & sjb

Pion Form Factor from AdS/QCD and Light-Front Holography



Using $SU(6)$ flavor symmetry and normalization to static quantities



Exact LF Formula for Pauli Form Factor

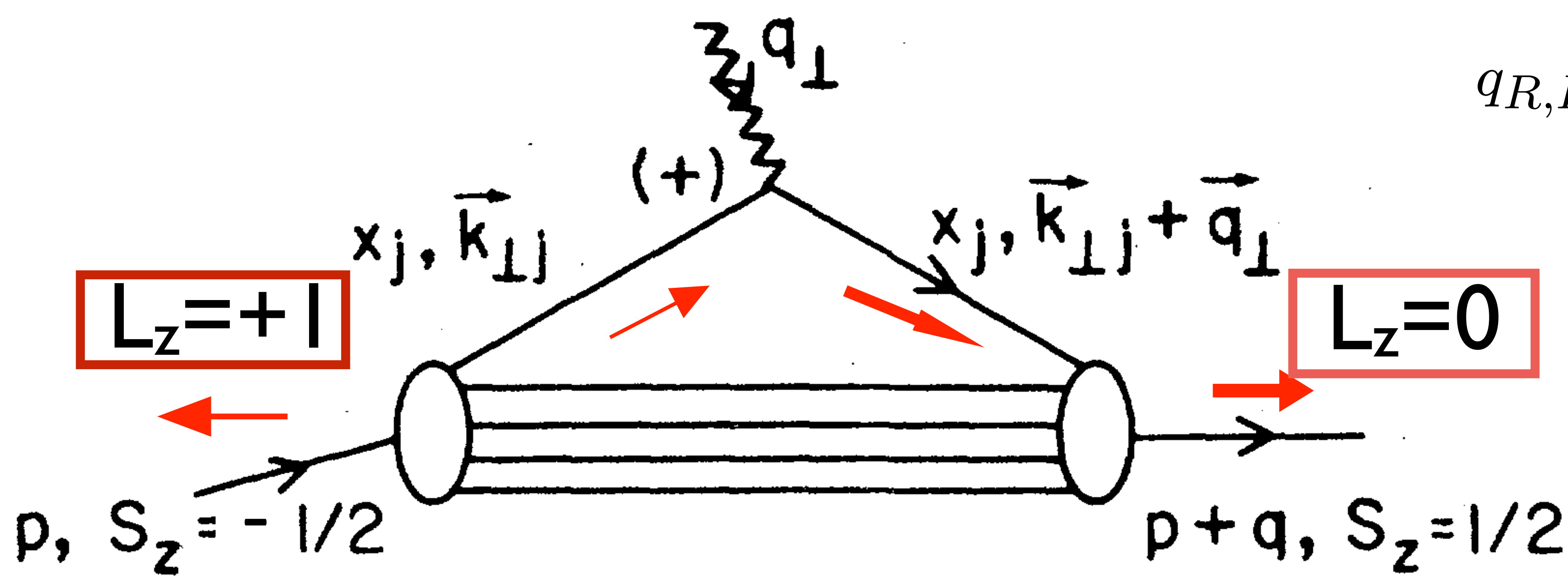
$$\frac{F_2(q^2)}{2M} = \sum_a \int [\mathrm{d}x][\mathrm{d}^2\mathbf{k}_\perp] \sum_j e_j \, \frac{1}{2} \; \times$$

Drell, sjb

$$\Big[-\frac{1}{q^L}\psi_a^{\uparrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\,\psi_a^{\downarrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) + \frac{1}{q^R}\psi_a^{\downarrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\,\psi_a^{\uparrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) \Big]$$

$$\mathbf{k}'_{\perp i} = \mathbf{k}_{\perp i} - x_i \mathbf{q}_\perp \qquad \mathbf{k}'_{\perp j} = \mathbf{k}_{\perp j} + (1 - x_j)\mathbf{q}_\perp$$

$$q_{R,L} = q^x \pm i q^y$$

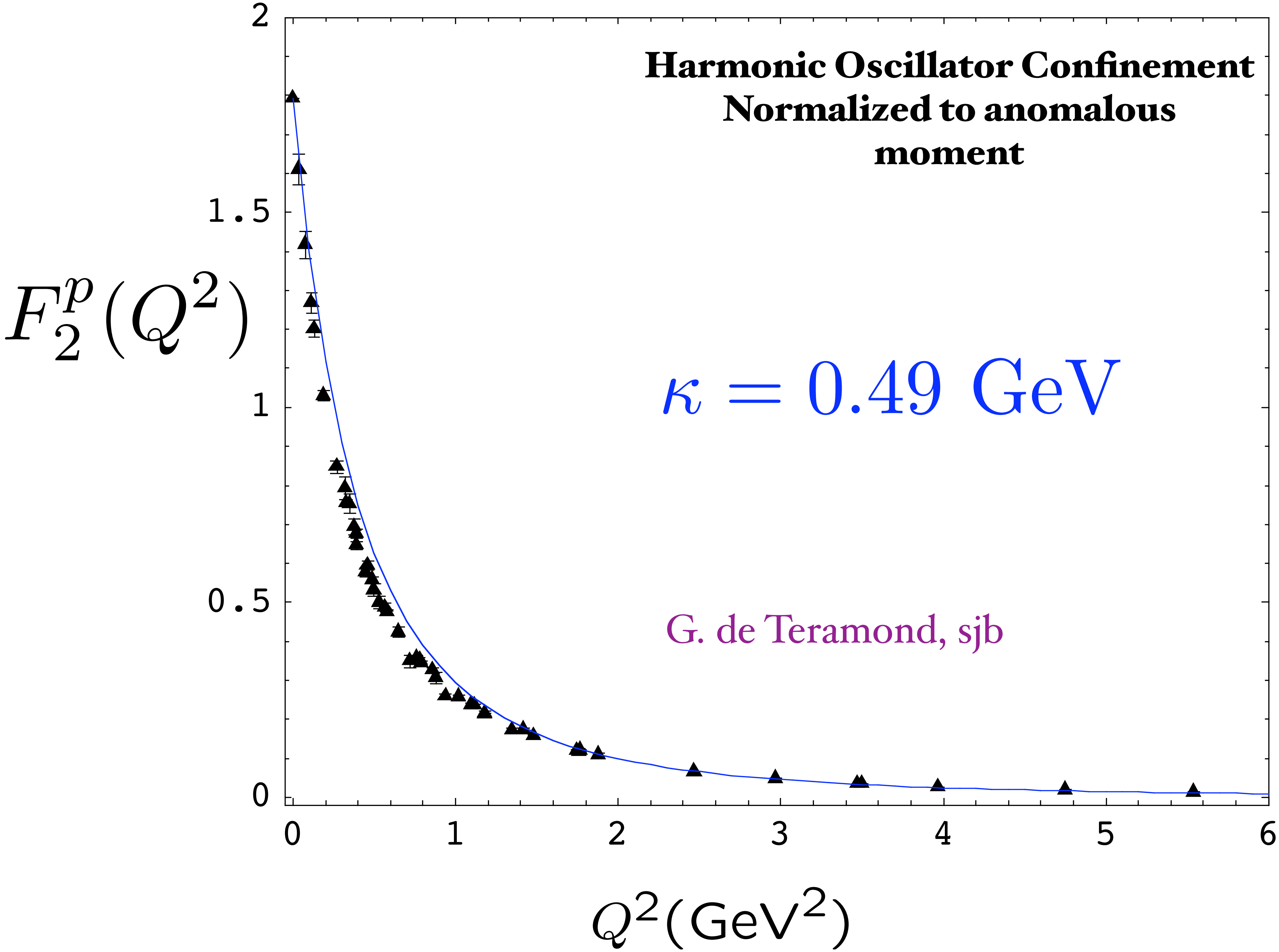


Must have $\Delta\ell_z = \pm 1$ to have nonzero $F_2(q^2)$

Nonzero Proton Anomalous Moment -->
Nonzero orbital quark angular momentum

Spacelike Pauli Form Factor

From overlap of $L = 1$ and $L = 0$ LFWFs



Deur, de Tèramond, sjb

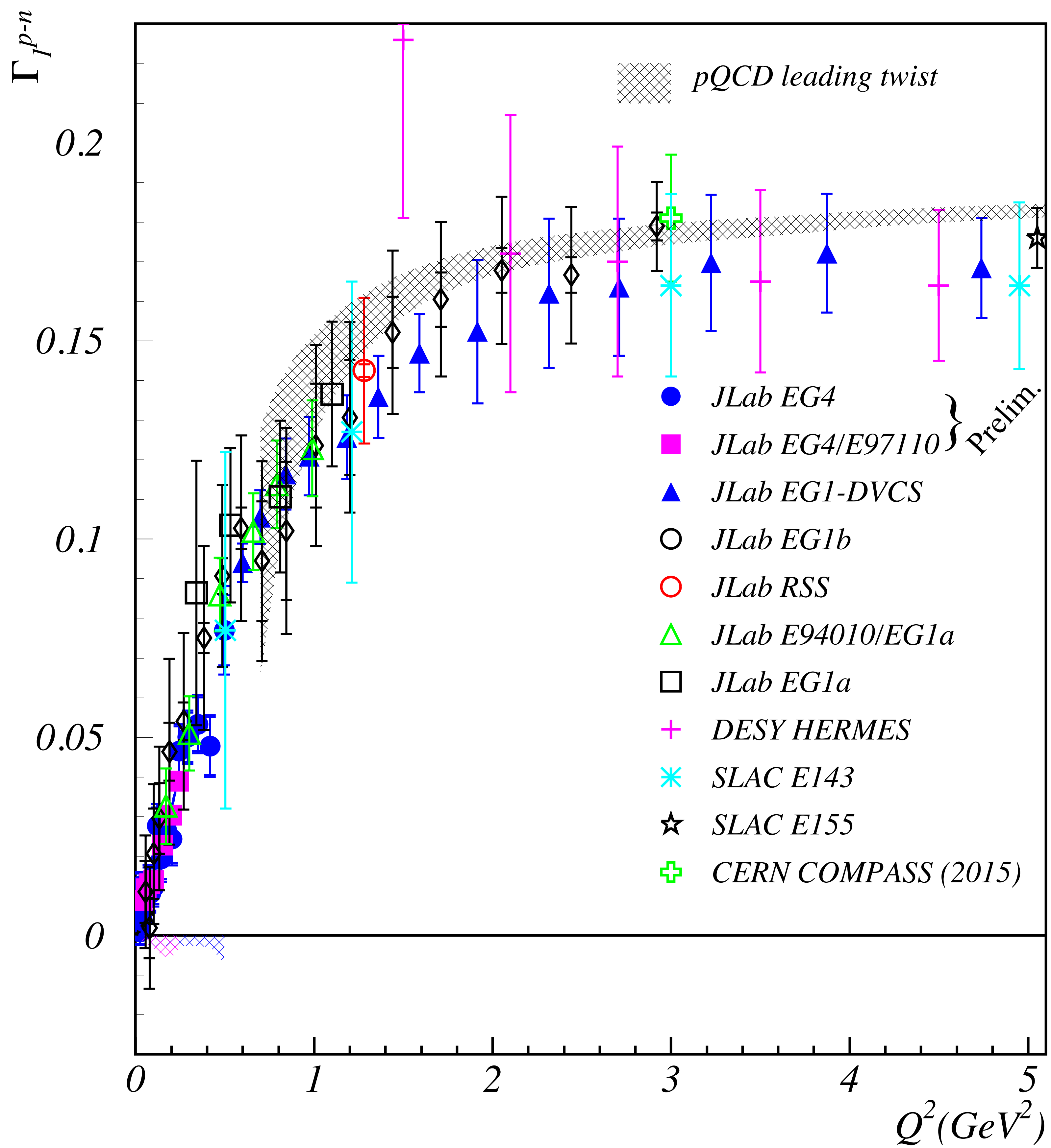
Bjorken sum rule defines effective charge: $\alpha_{g1}(Q^2)$

$$\int_0^1 dx[g_1^{ep}(x,Q^2) - g_1^{en}(x,Q^2)] \equiv \frac{g_a}{6}[1 - \frac{\alpha_{g1}(Q^2)}{\pi}]$$

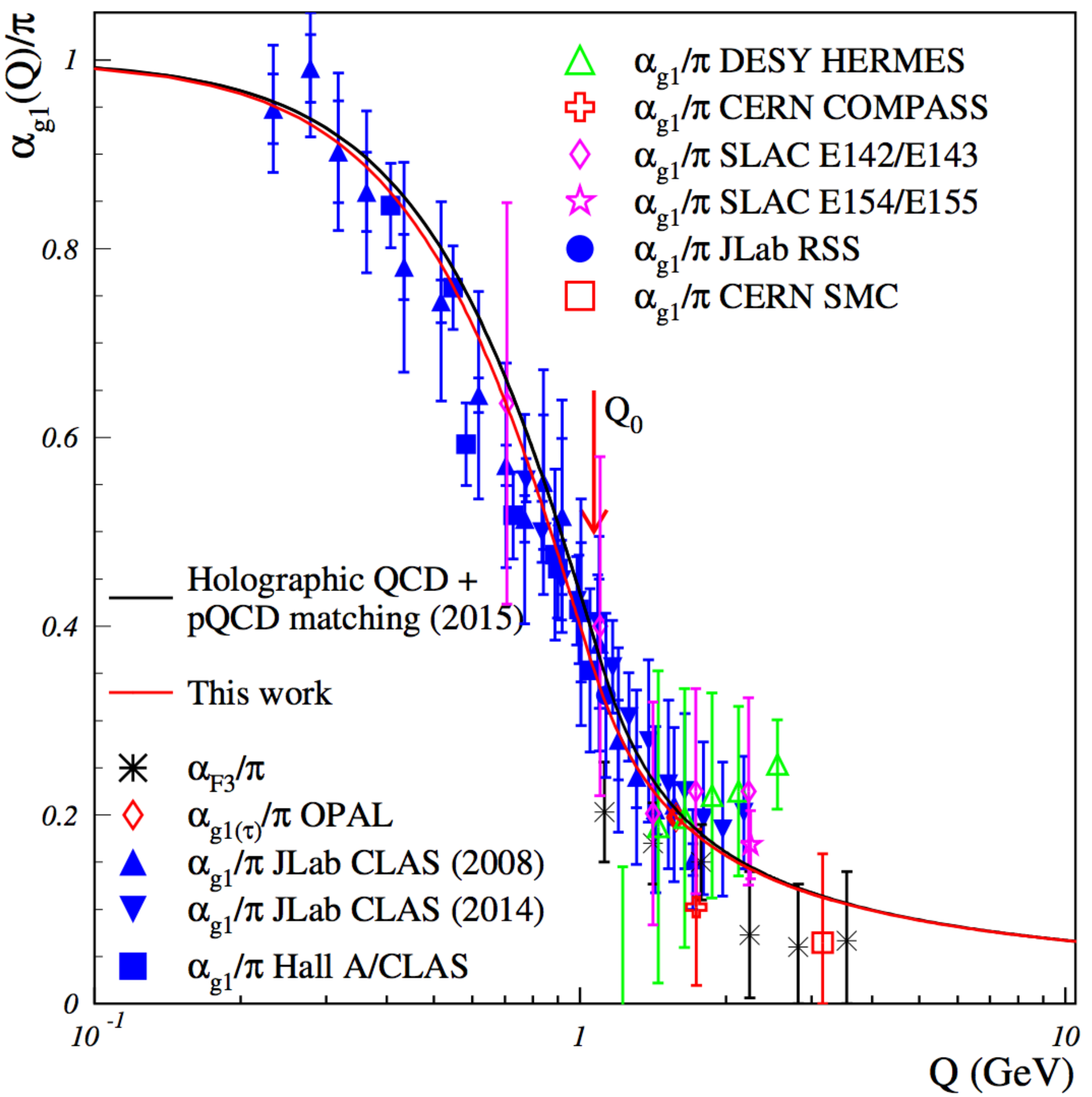
- ***Can be used as standard QCD coupling***
- ***Well measured***
- ***Asymptotic freedom at large Q^2***
- ***Computable at large Q^2 in any pQCD scheme***
- ***Universal β_0, β_1***
- ***Analytic connection to other schemes:***

Commensurate scale relations

Bjorken sum $\Gamma_1^{p\text{-}n}$ measurements



Running Coupling from AdS/QCD



Bjorken sum rule:

$$\frac{\alpha_{g_1}(Q^2)}{\pi} = 1 - \frac{6}{g_A}\int_0^1 dx\, g_1^{p-n}(x, Q^2)$$

Effective coupling in LFHQCD (valid at low-Q^2)

$$\alpha_{g_1}^{AdS}(Q^2) = \pi \exp\left(-Q^2/4\kappa^2\right)$$

Imposing continuity for α and its first derivative

A. Deur, S.J. Brodsky, G.F. de Téramond,
Phys. Lett. B 750, 528 (2015); J. Phys. G 44, 105005 (2017).

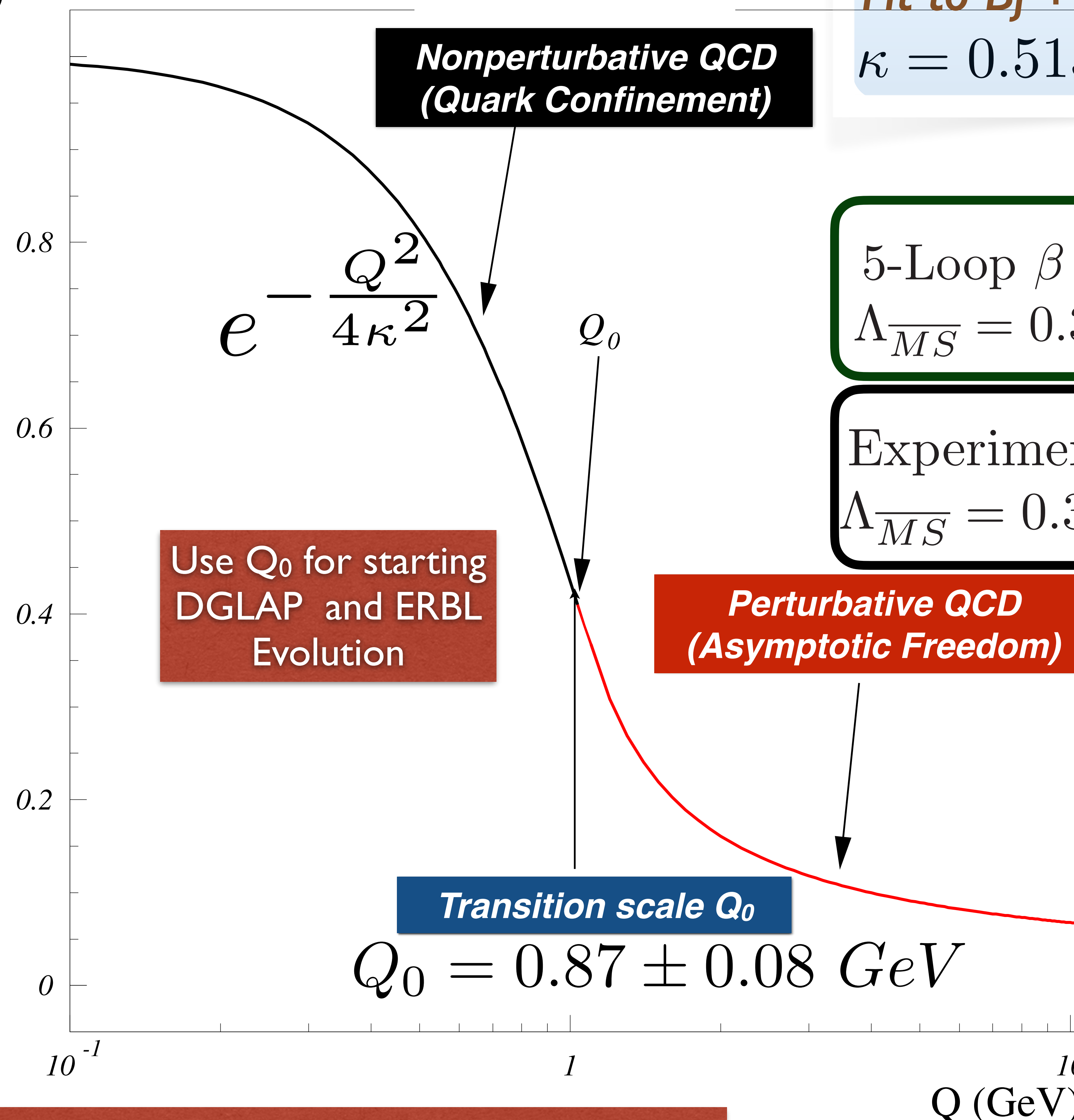
Analytic, defined at all scales, IR Fixed Point

$$m_\rho = \sqrt{2}\kappa$$
$$m_p = 2\kappa$$

All-Scale QCD Coupling

Deur, de Tèramond, sjb

Fit to Bj + DHG Sum Rules:
$\kappa = 0.513 \pm 0.007 \; GeV$

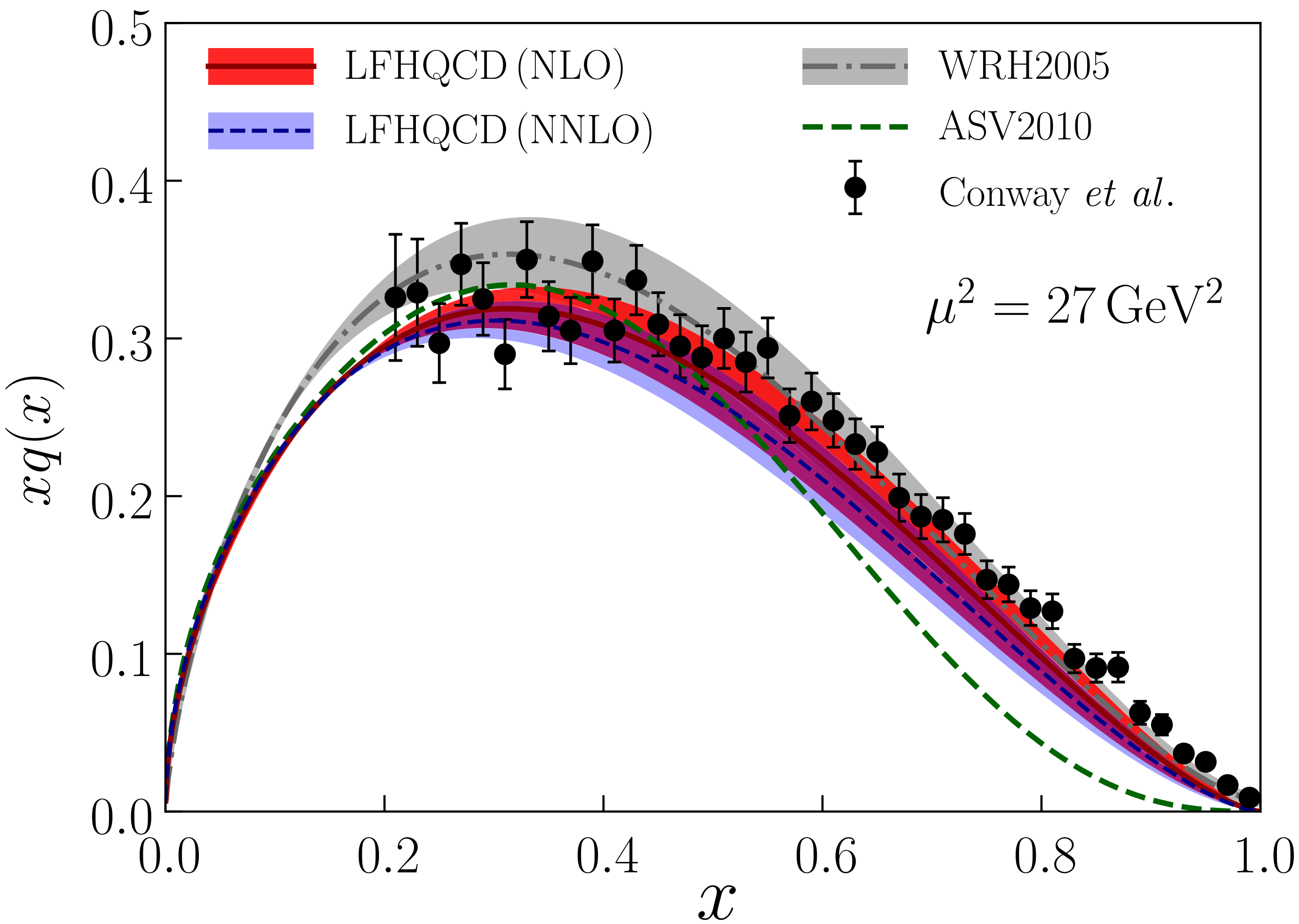


5-Loop β Prediction:
$\Lambda_{\overline{MS}} = 0.339 \pm 0.019 \; GeV$

Experiment:
$\Lambda_{\overline{MS}} = 0.332 \pm 0.017 \; GeV$

$\lambda \equiv \kappa^2$

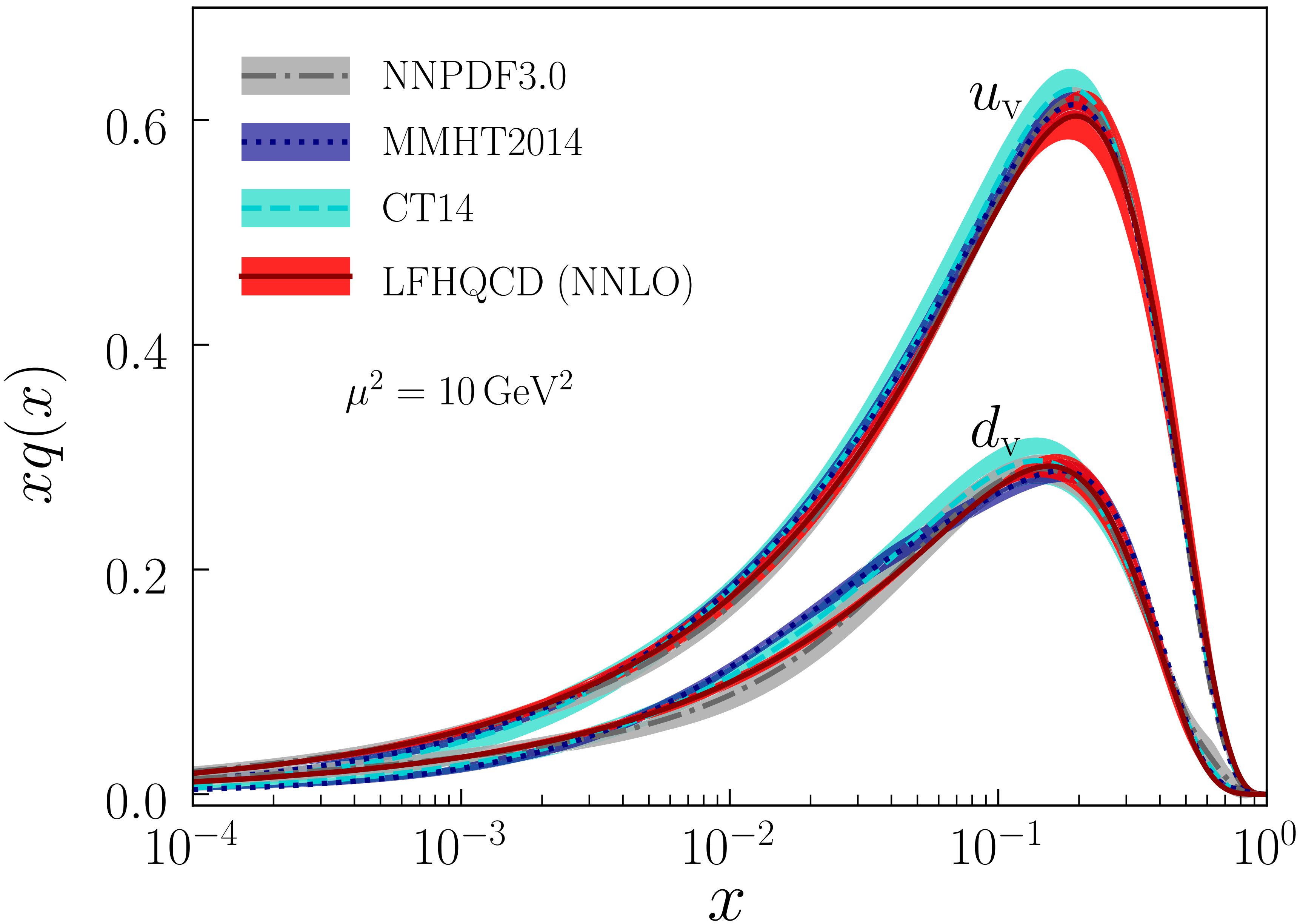
Reverse Dimensional Transmutation!



Comparison for $xq(x)$ in the pion from LFHQCD (red band) with the NLO fits [82,83] (gray band and green curve) and the LO extraction [84]. NNLO results are also included (light blue band). LFHQCD results are evolved from the initial scale $\mu_0 = 1.1 \pm 0.2$ GeV at NLO and the initial scale $\mu_0 = 1.06 \pm 0.15$ GeV at NNLO.

Universality of Generalized Parton Distributions in Light-Front Holographic QCD

Guy F. de Té́ramond, Tianbo Liu, Raza Sabbir Sufian, Hans Günter Dosch, Stanley J. Brodsky, and Alexandre Deur ***PHYSICAL REVIEW LETTERS 120, 182001 (2018)***



Comparison for $xq(x)$ in the proton from LFHQCD (red bands) and global fits: MMHT2014 (blue bands) [5], CT14 [6] (cyan bands), and NNPDF3.0 (gray bands) [77]. LFHQCD results are evolved from the initial scale $\mu_0 = 1.06\pm0.15$ GeV.

Universality of Generalized Parton Distributions in Light-Front Holographic QCD

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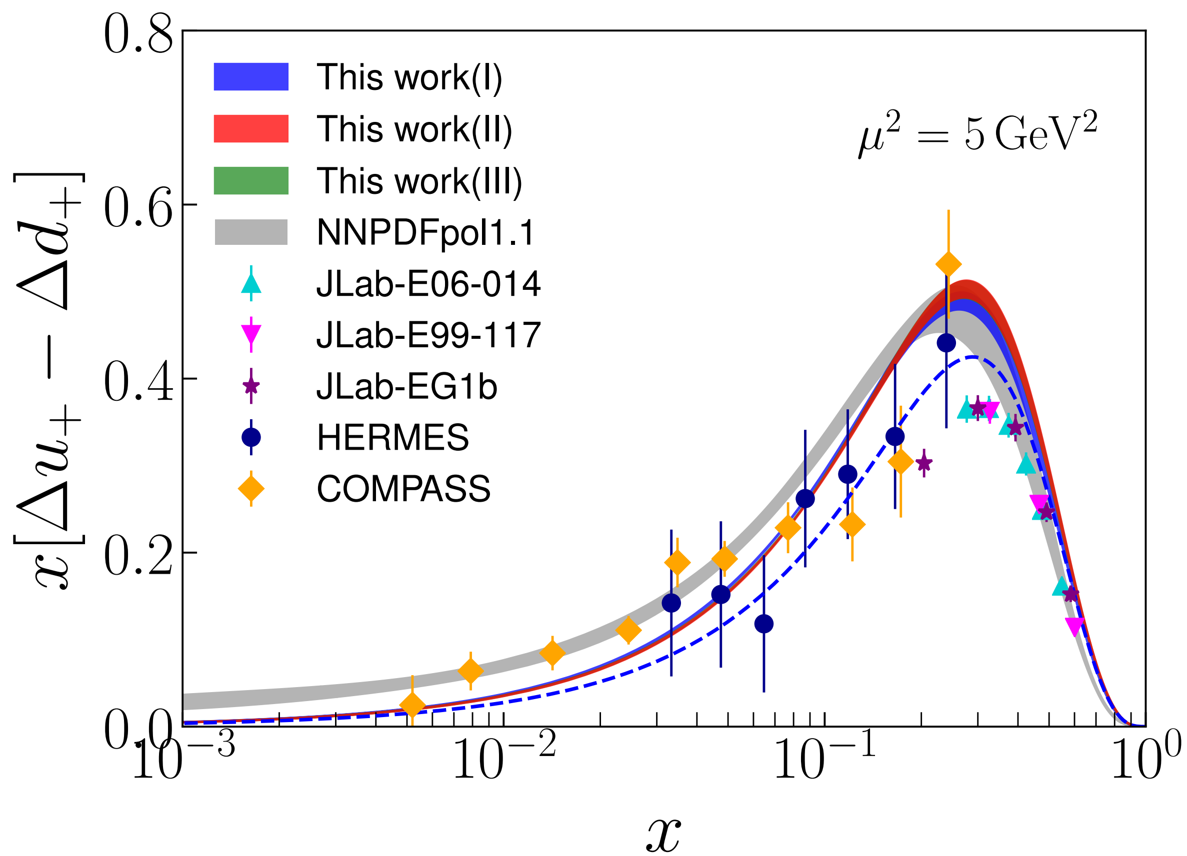
PHYSICAL REVIEW LETTERS 120, 182001 (2018)

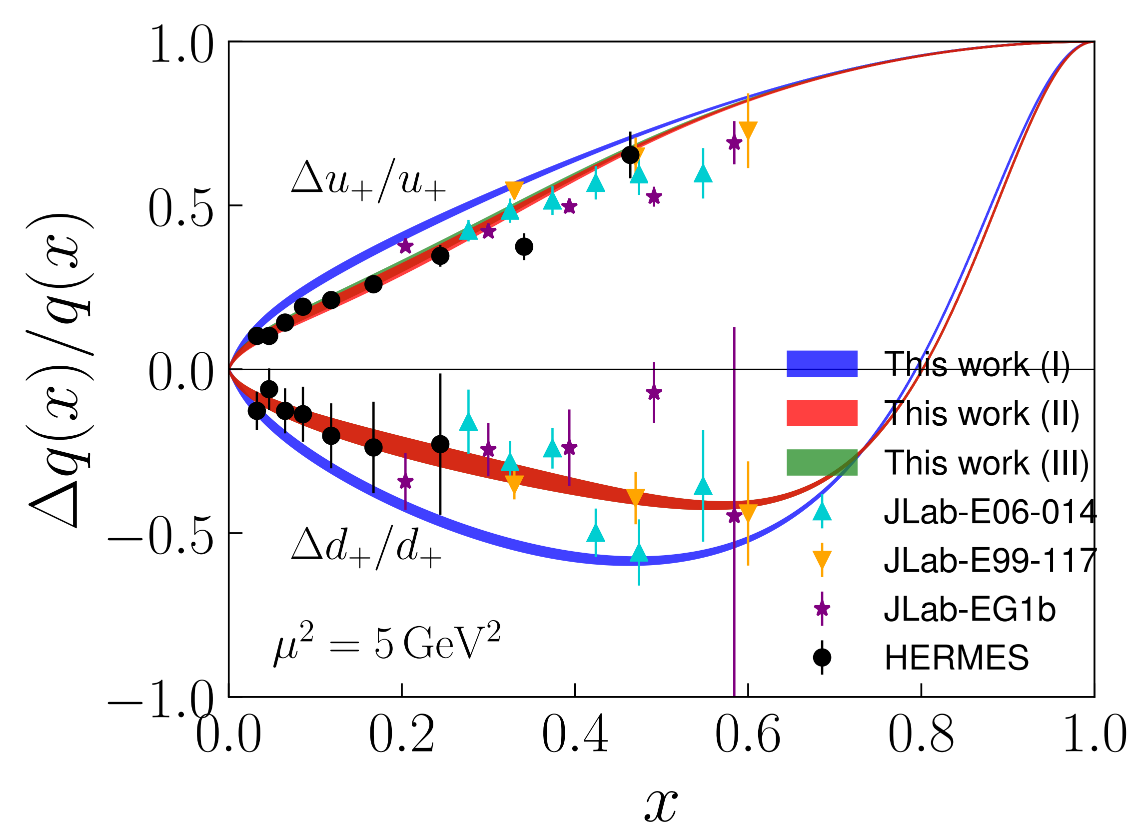
Tianbo Liu, Raza Sabbir Sufian, Guy F. de T´eramond, Hans Gunter Dösch, Alexandre Deur, sjb



Polarized distributions for the isovector combination $x[\Delta u_+(x) - \Delta d_+(x)]$

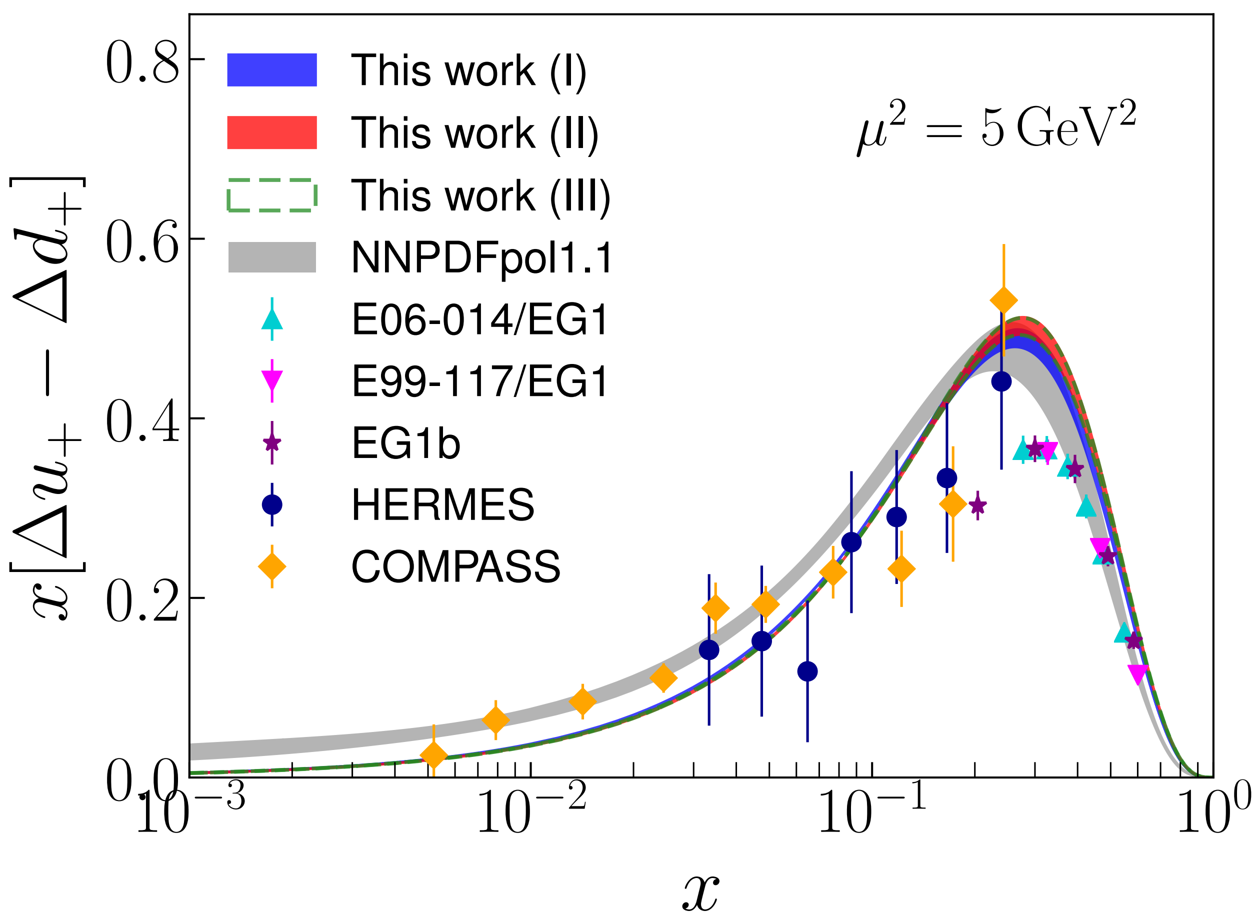
$$d_+(x) = d(x) + \bar{d}(x) \qquad u_+(x) = u(x) + \bar{u}(x)$$

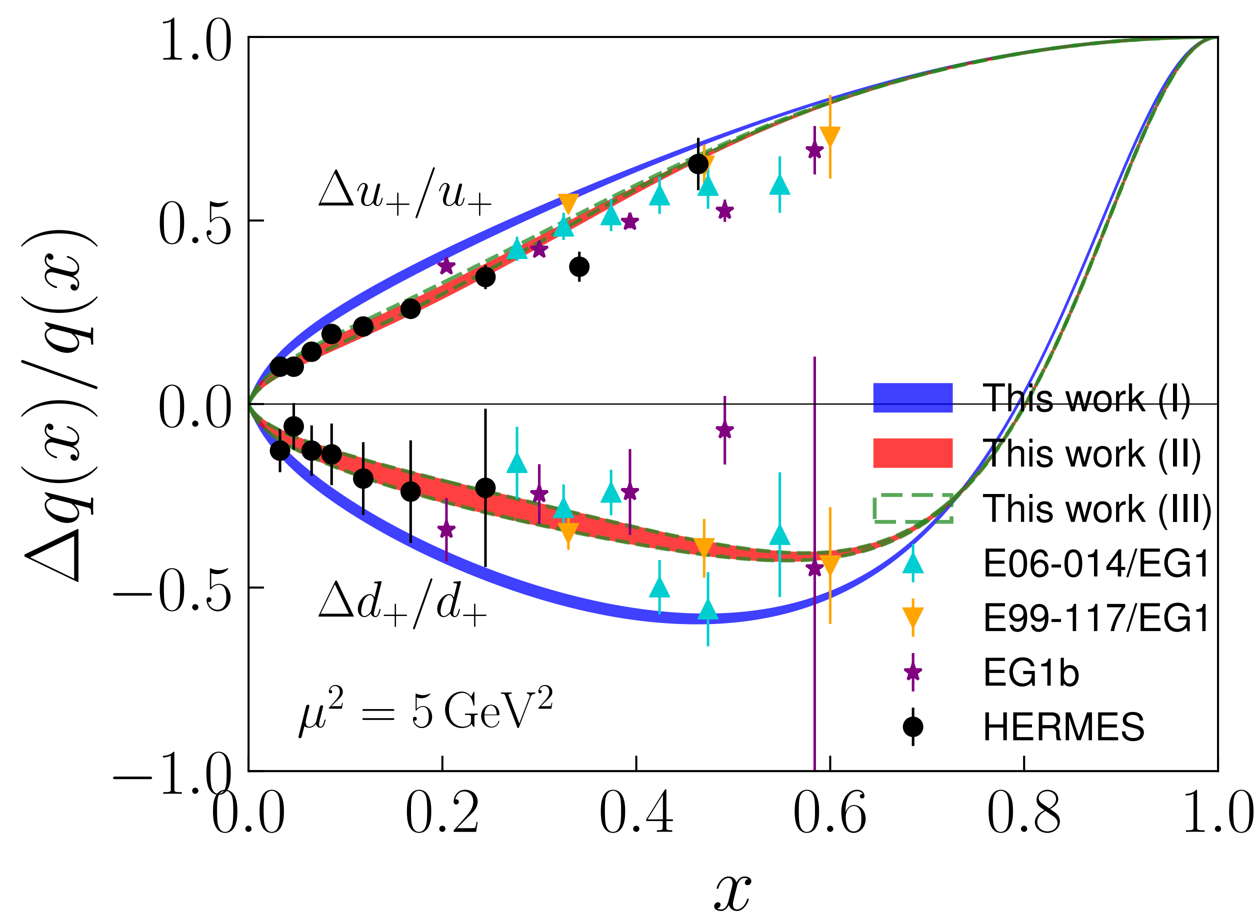
$$\Delta q(x) = q_\uparrow(x) - q_\downarrow(x)$$



Polarized GPDs and PDFs (HLFHS Collaboration, 2019)

- Separation of chiralities in the AdS action allows computation of the matrix elements of the axial current including the correct normalization, once the coefficients c_τ are fixed for the vector current
- Helicity retention between quark and parent hadron (pQCD prediction): $\lim_{x\to 1} \frac{\Delta q(x)}{q(x)} = 1$
- No spin correlation with parent hadron: $\lim_{x\to 0} \frac{\Delta q(x)}{q(x)} = 0$





An analytic first approximation to QCD

AdS/QCD + Light-Front Holography

- **As Simple as Schrödinger Theory in Atomic Physics**
- **LF radial variable ζ conjugate to invariant mass squared**
- **Relativistic, Frame-Independent, Color-Confining**
- **Unique confining potential!**
- **QCD Coupling at all scales: Essential for Gauge Link phenomena**
- **Hadron Spectroscopy and Dynamics from one parameter**
- **Wave Functions, Form Factors, Hadronic Observables, Constituent Counting Rules**
- **Insight into QCD Condensates**
- **Systematically improvable with DLCQ-BLFQ Methods**

Supersymmetric Features of Hadron Physics from Superconformal Algebra and Light-Front Holography

Light-Front Holography: First Approximation to QCD

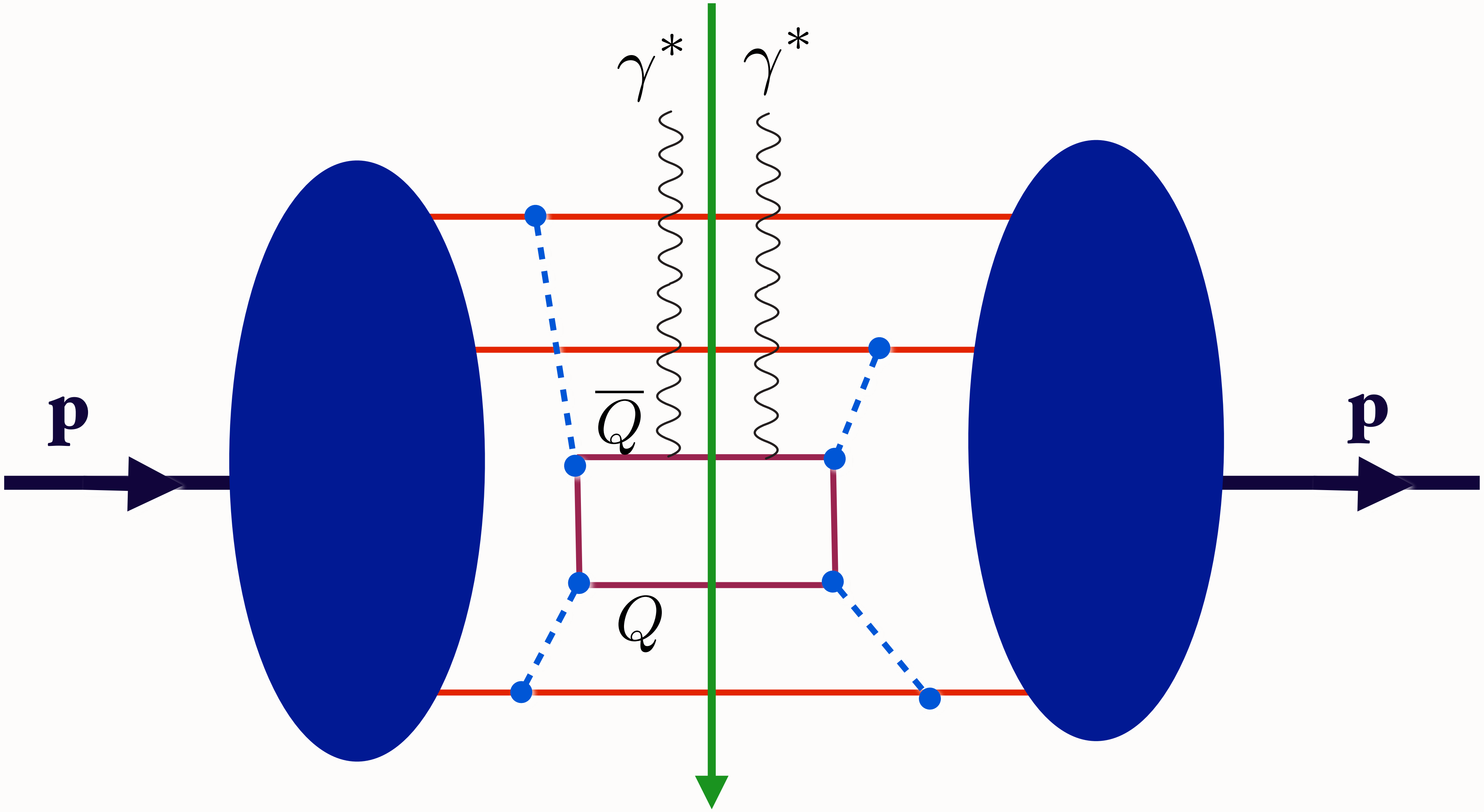
- **Color Confinement, Analytic form of confinement potential**

de Téramond, Dosch, Lorcé, sjb

- **Retains underlying conformal properties of QCD despite mass scale (DeAlfaro-Fubini-Furlan Principle)**
- **Massless quark-antiquark pion bound state in chiral limit, GMOR**
- **QCD coupling at all scales**
- **Connection of perturbative and nonperturbative mass scales**
- **Poincarè Invariant**
- **Hadron Spectroscopy-Regge Trajectories with universal slopes in n, L**
- **Supersymmetric 4-Plet: Meson-Baryon -Tetraquark Symmetry**
- **Light-Front Wavefunctions**
- **Form Factors, Structure Functions, Hadronic Observables**
- **OPE: Constituent Counting Rules**
- **Hadronization at the Amplitude Level: Many Phenomenological Tests**
- **Systematically improvable: Basis LF Quantization (BLFQ)**

Supersymmetric Features of Hadron Physics from Superconformal Algebra and Light-Front Holography

Cut of Proton Self Energy:
QCD predicts
Intrinsic Heavy Quarks!



Probability (QED) $\propto \frac{1}{M_\ell^4}$ Probability (QCD) $\propto \frac{1}{M_Q^2}$

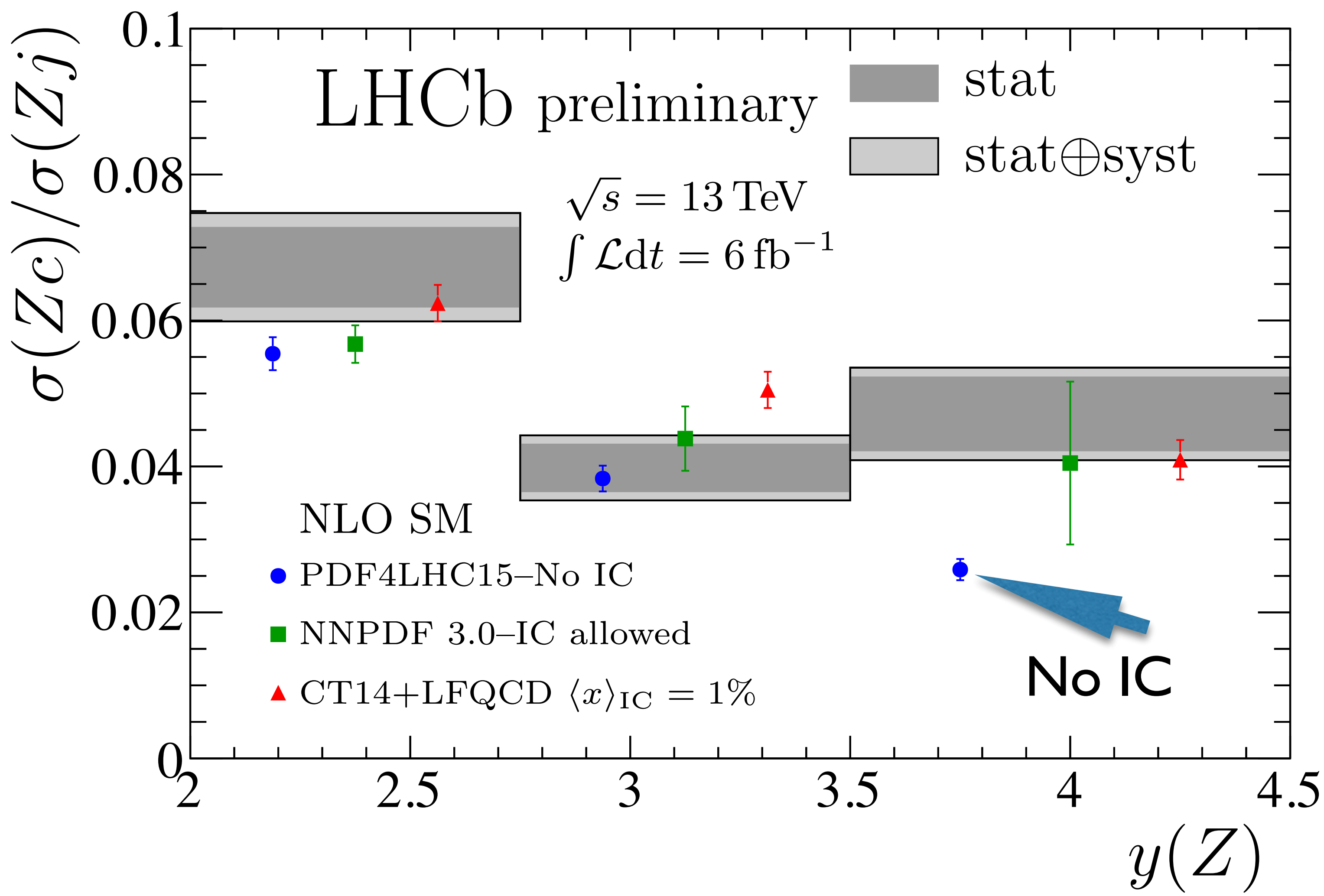
$$x_Q \propto (m_Q^2 + k_\perp^2)^{1/2}$$

Hoyer, Peterson, Sakai, Collins, Ellis, Gunion, Mueller, sjb
Polyakov, et al.

$$pp \to Z + c + X \qquad g + c \to Z + c$$

$Z + c$: results

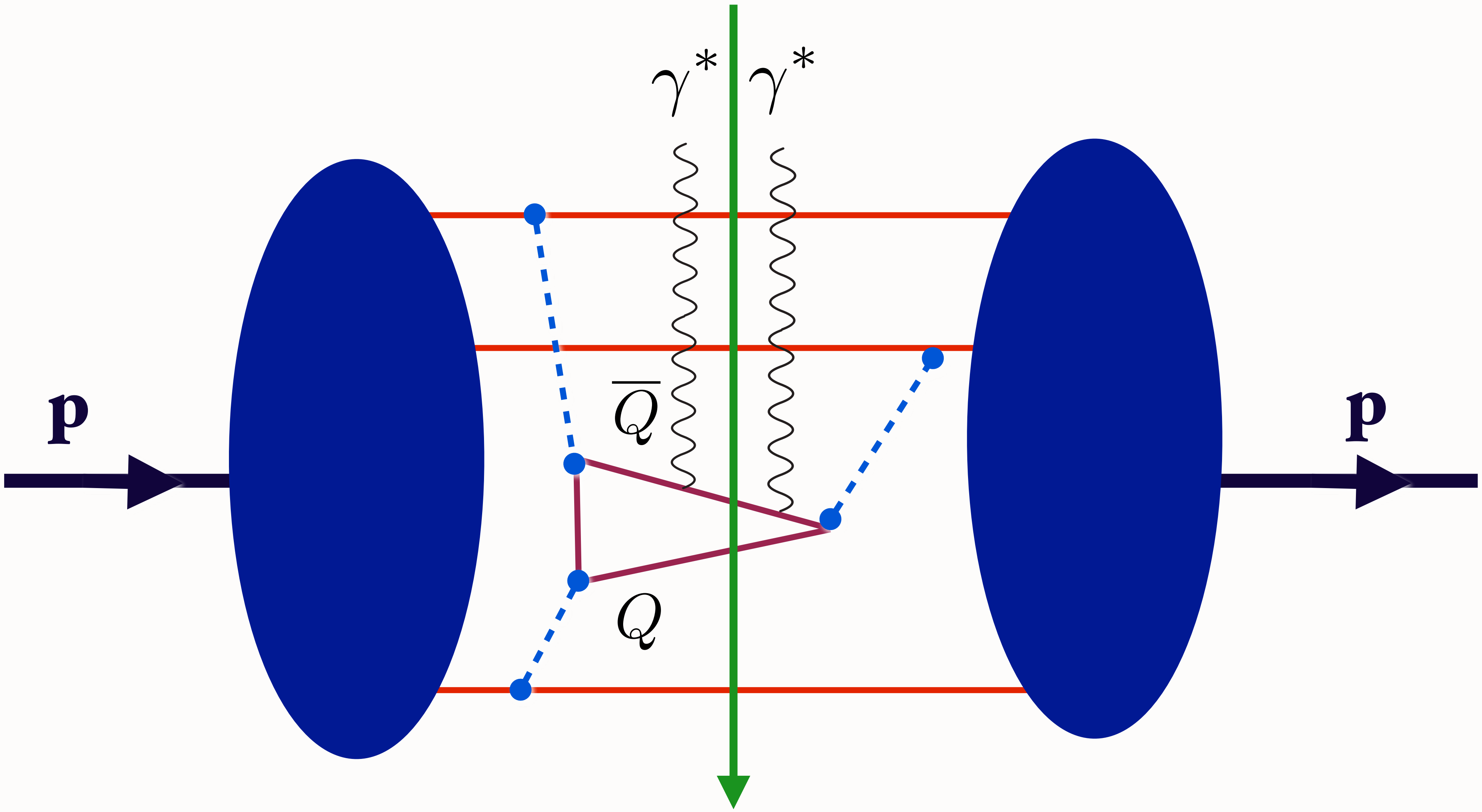
LHCb-PAPER-2021-029



- Clear enhancement in highest-y bin
- Consistent with expected effect from $|uudc\bar{c}\rangle$ component predicted by LFQCD
- Inconsistent with No-IC theory at $\sim$ 3 standard deviations
- Global PDF analysis required to determine true significance

I.A. Schmidt, V. Lyubovitskij, sjb

Interference of Intrinsic and Extrinsic Heavy Quark Amplitudes



Interference predicts $Q(x) \neq \bar{Q}(x)$

$$\frac{d\sigma}{dy dp_T^2}(pp \to D^+ c\bar{d}X) \neq \frac{d\sigma}{dy dp_T^2}(pp \to D^- \bar{c} dX)$$

QED Analog: J. Gillespie, sjb (1968)

Constraints on charm-anticharm asymmetry in the nucleon from lattice QCD

Raza Sabbir Sufian[a], Tianbo Liu[a], Andrei Alexandru[b,c], Stanley J. Brodsky[d], Guy F. de Téramond[e], Hans Günter Dosch[f], Terrence Draper[g], Keh-Fei Liu[g], Yi-Bo Yang[h]

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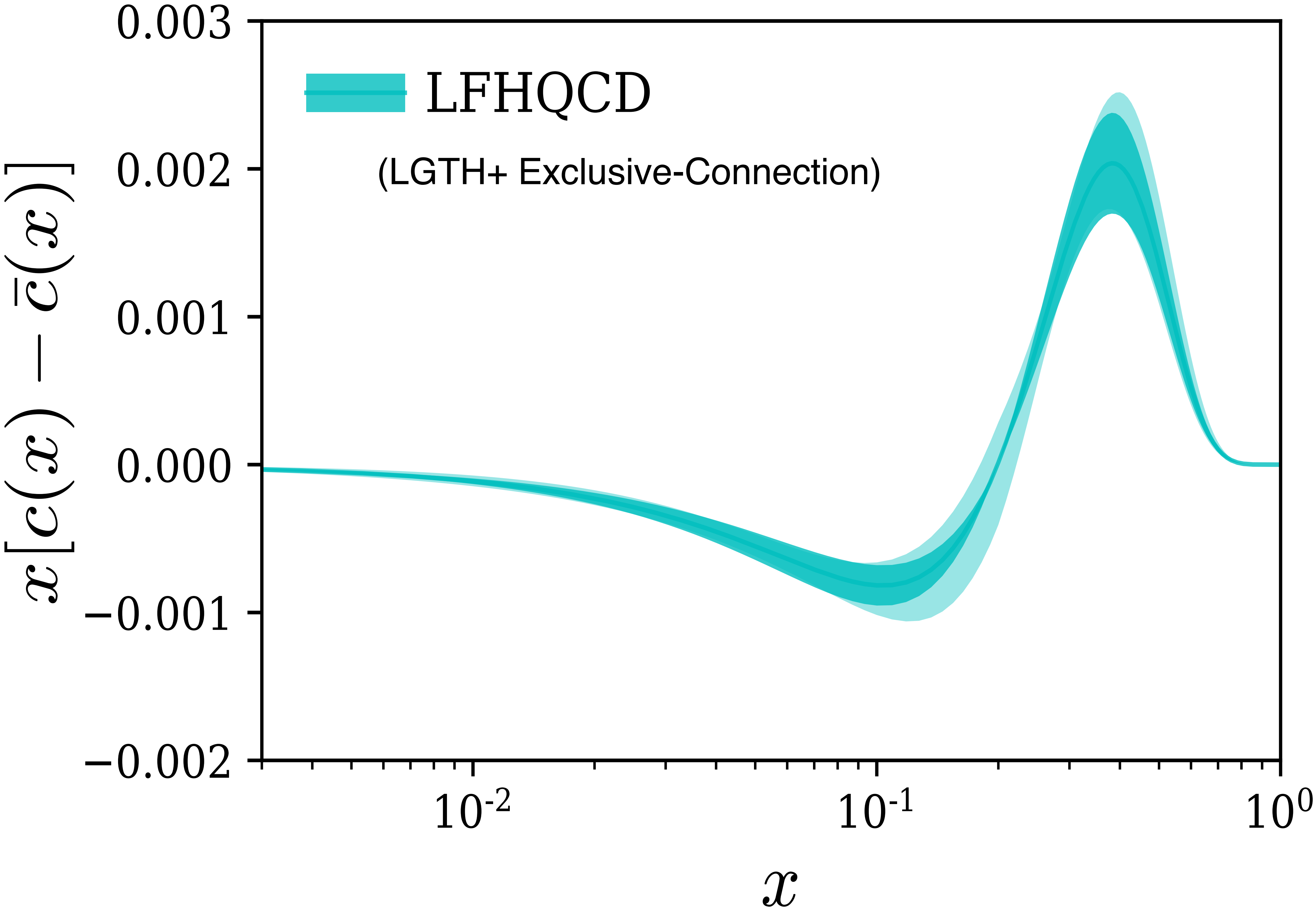
Abstract

We present the first lattice QCD calculation of the charm quark contribution to the nucleon electromagnetic form factors $G^c_{E,M}(Q^2)$ in the momentum transfer range $0 \leq Q^2 \leq 1.4\ \mathrm{GeV}^2$. The quark mass dependence, finite lattice spacing and volume corrections are taken into account simultaneously based on the calculation on three gauge ensembles including one at the physical pion mass. The nonzero value of the charm magnetic moment $\mu^c_M = -0.00127(38)_{\mathrm{stat}}(5)_{\mathrm{sys}}$, as well as the Pauli form factor, reflects a nontrivial role of the charm sea in the nucleon spin structure. The nonzero $G^c_E(Q^2)$ indicates the existence of a nonvanishing asymmetric charm-anticharm sea in the nucleon. Performing a nonperturbative analysis based on holographic QCD and the generalized Veneziano model, we study the constraints on the $[c(x) - \bar{c}(x)]$ distribution from the lattice QCD results presented here. Our results provide complementary information and motivation for more detailed studies of physical observables that are sensitive to intrinsic charm and for future global analyses of parton distributions including asymmetric charm-anticharm distribution.

Keywords: Intrinsic charm, Form factor, Parton distributions, Lattice QCD, Light-front holographic QCD, JLAB-THY-20-3155, SLAC-PUB-17515

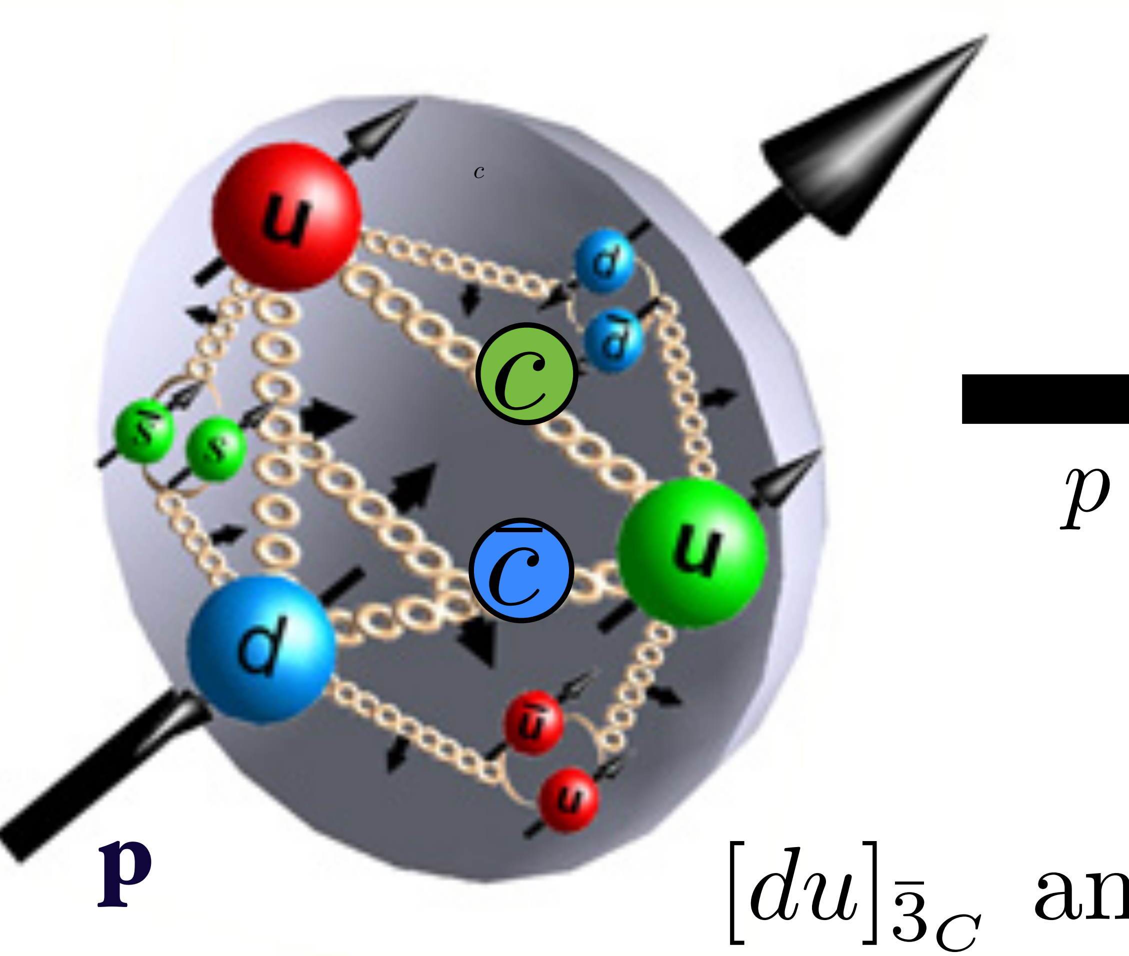


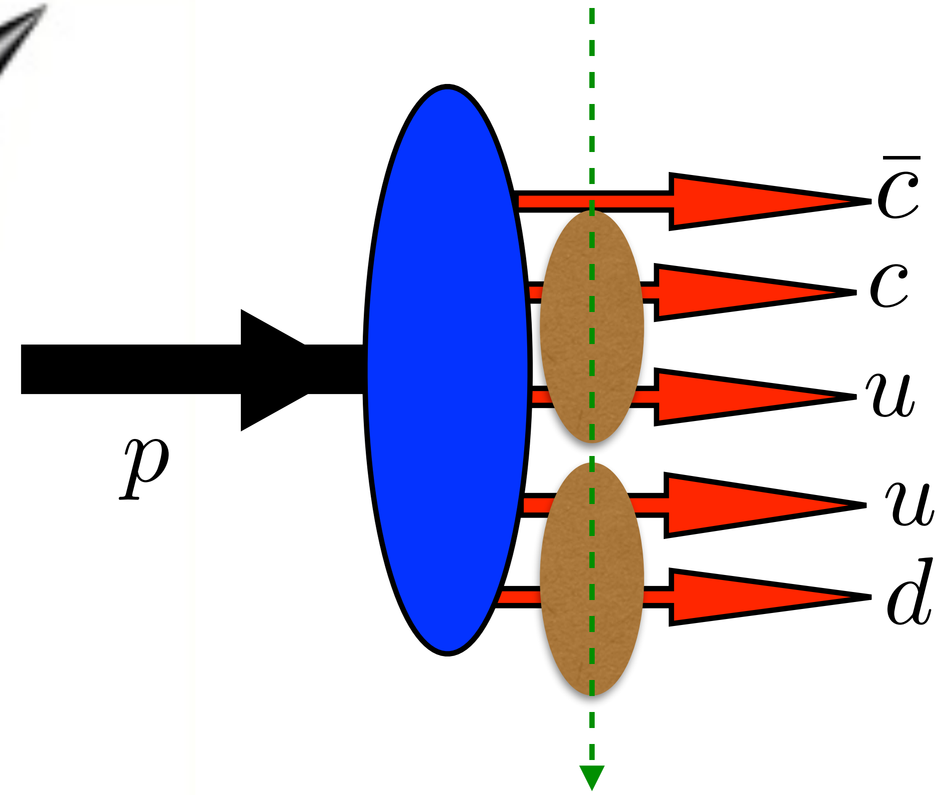
The distribution function $x[c(x) - \bar{c}(x)]$ obtained from the LFHQCD formalism using the lattice QCD input of charm electromagnetic form factors $G^c_{E,M}(Q^2)$. The outer cyan band indicates an estimate of systematic uncertainty in the $x[c(x) - \bar{c}(x)]$ distribution obtained from a variation of the hadron scale κ_c by 5%.

Color confinement potential from AdS/QCD

$$U(\zeta^2) = \kappa^4 \zeta^2, \zeta^2 = b_\perp^2 x(1-x)$$

Fixed $\tau = t + z/c$





Intrinsic Charm

$|\bar{c}[cu][ud]>$

$[du]_{\bar{3}_C}$ and $[cu]_{\bar{3}_C}$ $J=0$ diquark dominance

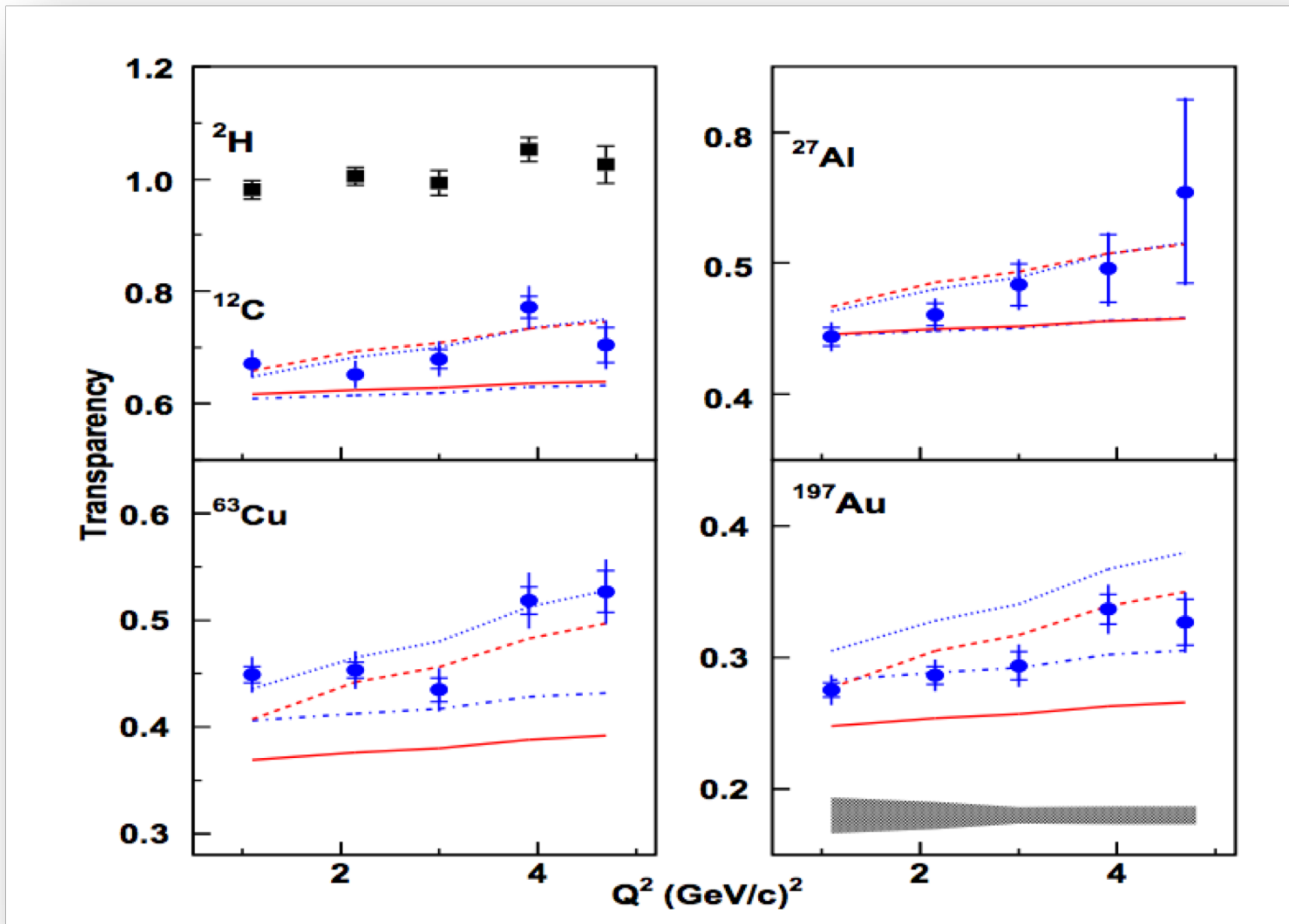
$$\psi_n(\vec{k}_{\perp i}, x_i) \propto \frac{1}{\kappa^{n-1}} e^{-\mathcal{M}_n^2/2\kappa^2} \Pi_{j=1}^n \frac{1}{\sqrt{x_j}}$$

$$\mathcal{M}_n^2 = \sum_{i=1}^n \left(\frac{k_\perp^2 + m^2}{x}\right)_i$$

Color Transparency verified for π^+ and ρ electroproduction

Hall C E01-107 pion electro-production

A(e,e'π^+)

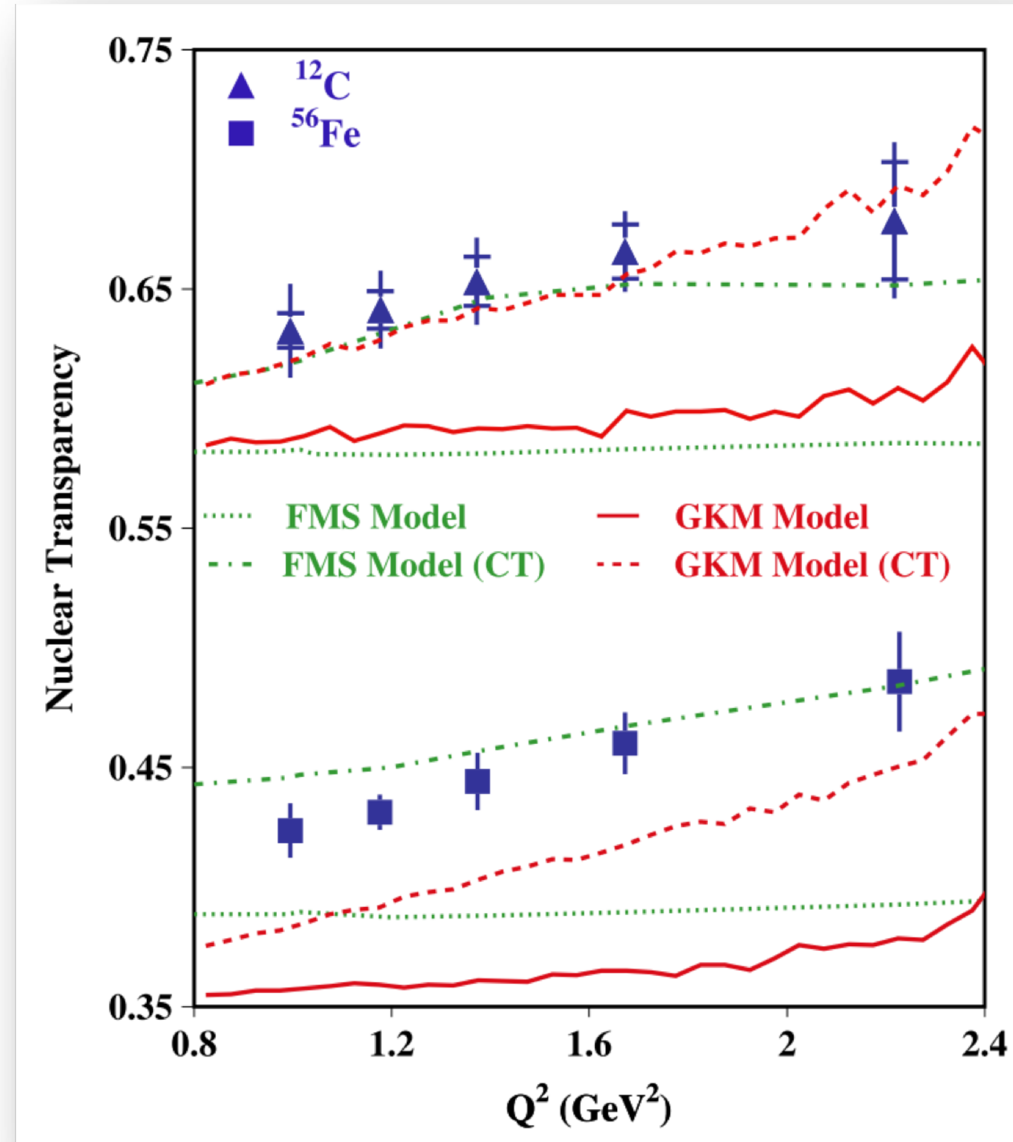


B.Clasie *et al.* PRL 99:242502 (2007)

X. Qian *et al.* PRC81:055209 (2010)

CLAS E02-110 rho electro-production

A(e,e'ρ^0)



L. El Fassi *et al.* PLB 712,326 (2012)

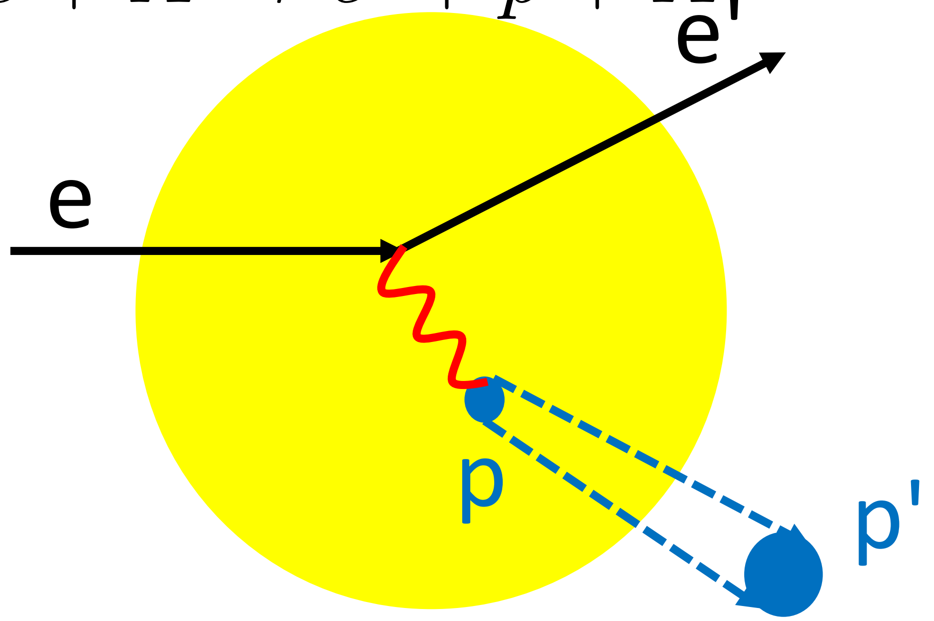
$$T_A = \frac{\frac{d\sigma}{dQ^2}(pA \to \pi^+ X)}{\frac{d\sigma}{dQ^2}(pp \to \pi^+ X)}$$

$$T_A = \frac{\frac{d\sigma}{dQ^2}(pA \to \rho^0 X)}{\frac{d\sigma}{dQ^2}(pp \to \rho^0 X)}$$

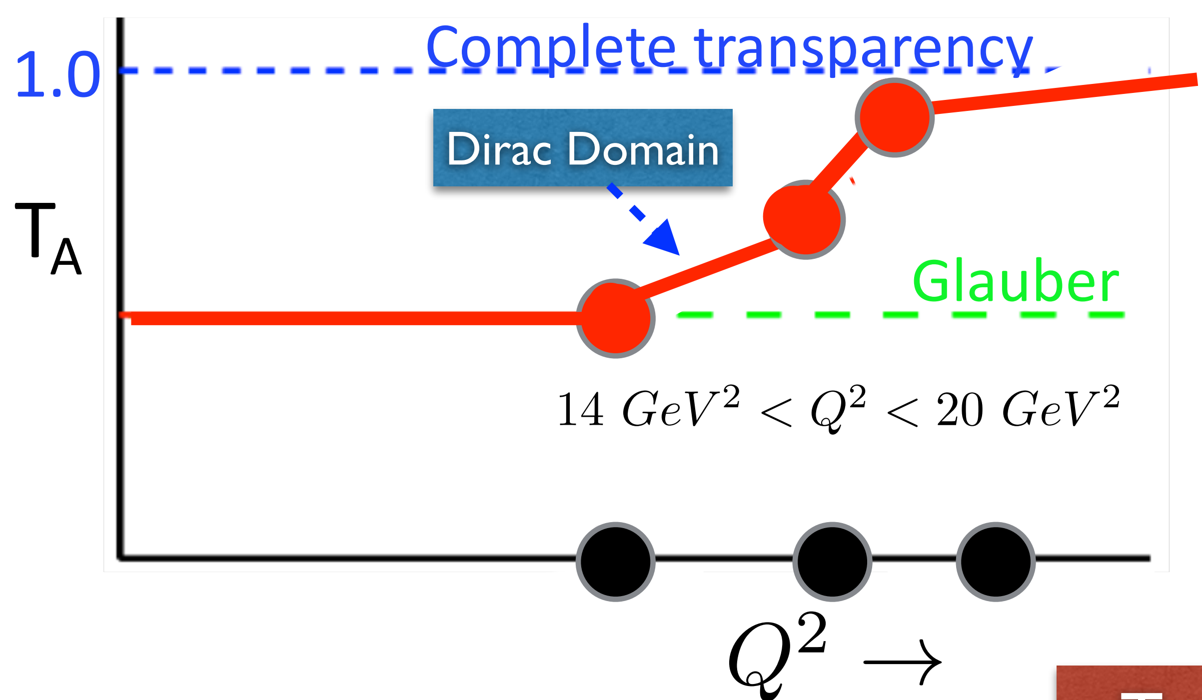
A.H. Mueller, sjb

Color transparency: fundamental prediction of QCD

$$e + A \to e' + p + X$$



- Not predicted by strongly interacting hadronic picture → arises in picture of quark-gluon interactions
- QCD: color field of singlet objects vanishes as size is reduced
- Signature is a rise in nuclear transparency, T_A, as a function of the momentum transfer, Q^2



$$T_A = \frac{\sigma_A}{A\ \sigma_N}$$ (nuclear cross section) (free nucleon cross section)

G. de Teramond, sjb

Two-Stage Color Transparency for Proton

Drell-Yan-West Formula in Impact Space

$$F(q^2) = \sum_n \prod_{i=1}^{n} \int dx_i \int \frac{d^2\mathbf{k}_{\perp i}}{2(2\pi)^3}\, 16\pi^3\, \delta\Big(1 - \sum_{j=1}^{n} x_j\Big)\, \delta^{(2)}\Big(\sum_{j=1}^{n} \mathbf{k}_{\perp j}\Big)$$

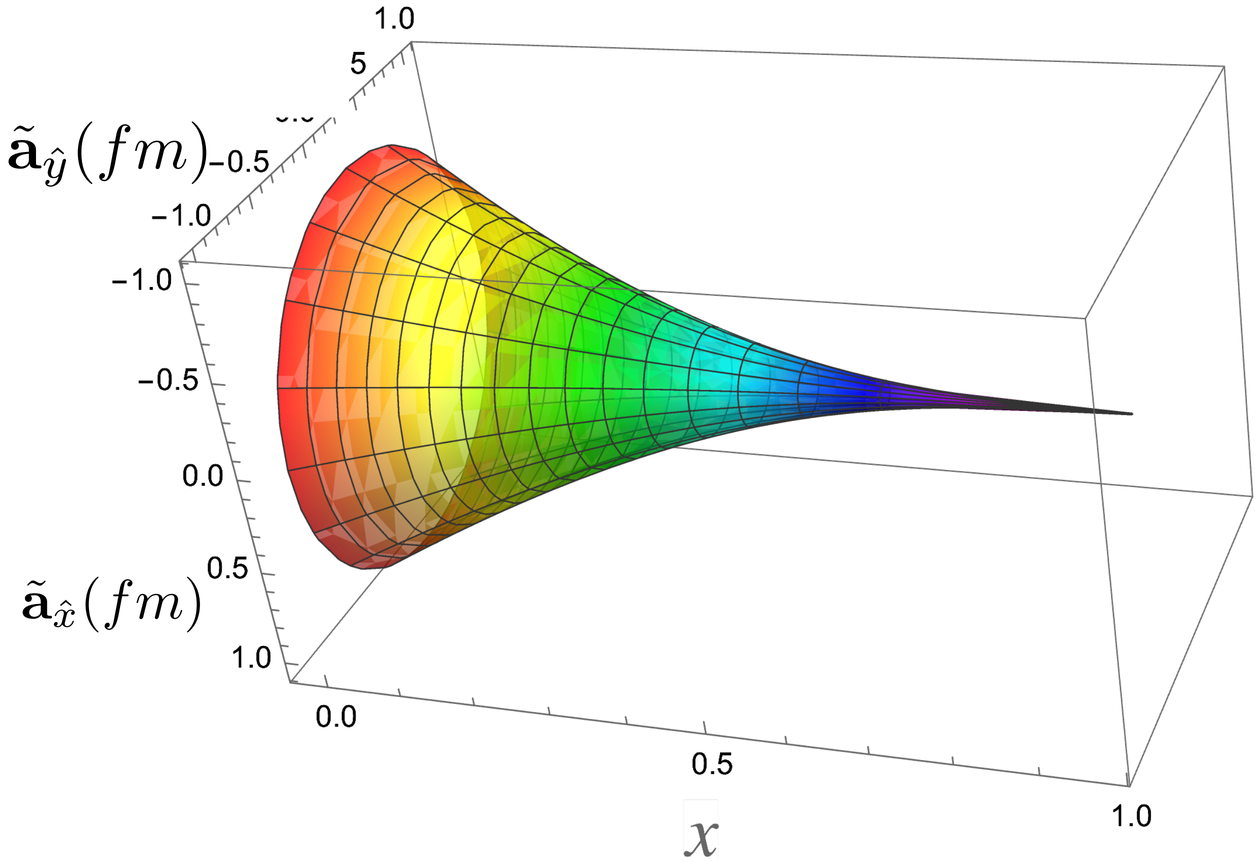
$$\sum_j e_j \psi_n^*(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\psi_n(x_i, \mathbf{k}_{\perp i}, \lambda_i),$$

$$= \sum_n \prod_{j=1}^{n-1} \int dx_j \int d^2\mathbf{b}_{\perp j} \exp\Big(i\mathbf{q}_\perp \cdot \sum_{j=1}^{n-1} x_j \mathbf{b}_{\perp j}\Big) \left|\psi_n(x_j, \mathbf{b}_{\perp j})\right|^2$$

$$\textstyle\sum_{i=1}^{n} x_i = 1 \text{ and } \sum_{i=1}^{n} \mathbf{b}_{\perp i} = 0.$$

$$F(q^2) = \int_0^1 dx \int d^2\mathbf{a}_\perp e^{i\mathbf{a}_\perp \cdot \mathbf{q}_\perp} q(x, \mathbf{a}_\perp),$$

where $\boxed{\mathbf{a}_\perp = \sum_{j=1}^{n-1} x_j \mathbf{b}_{\perp j}}$ is the x-weighted transverse position coordinate of the $n-1$ spectators.



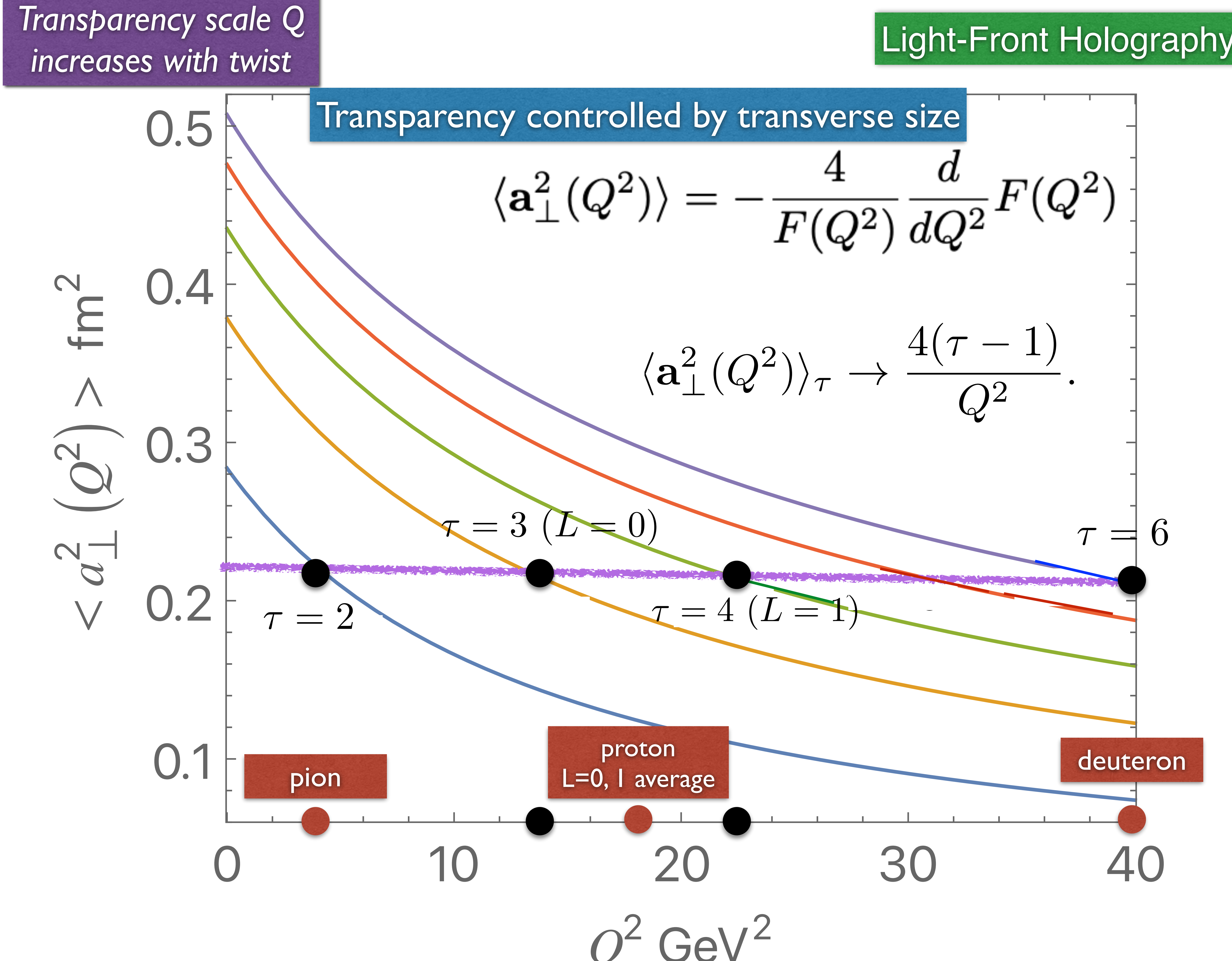
$$< \tilde{\mathbf{a}}_\perp^2(x) >= \frac{\int d^2\mathbf{a}_\perp \mathbf{a}_\perp^{\mathbf{2}} q(x,\mathbf{a}_\perp)}{\int d^2\mathbf{a}_\perp q(x,\mathbf{a}_\perp)}$$

At large light-front momentum fraction x, and equivalently at large values of Q^2, the transverse size of a hadron behaves as a point-like color-singlet object. This behavior is the origin of color transparency in nuclei.

Although the dependence of the transverse impact area as a function of x is universal, the behavior in Q^2 depends on properties of the hadron, such as its twist.

$$\langle \mathbf{a}_\perp^2(Q^2)\rangle_\tau \to \frac{4(\tau-1)}{Q^2}.$$

Mean transverse size as a function of Q and Twist



Proton has equal probability for $\tau = 3$ and $\tau = 4$

$$F(q^2) =$$

G. de Teramond, sjb

$$\sum_n \prod_{j=1}^{n-1} \int dx_j \int d^2\mathbf{b}_{\perp j} \exp\left(i\mathbf{q}_\perp \cdot \sum_{j=1}^{n-1} x_j \mathbf{b}_{\perp j}\right) |\psi_n(x_j, \mathbf{b}_{\perp j})|^2$$

$$\sum_i x_i = 1$$

$$\vec{a}_\perp \equiv \sum_{j=1}^{n-1} x_j \vec{b}_{\perp j}$$

$$\vec{a}_\perp^2(Q^2) = -4 \frac{\frac{d}{dQ^2} F(Q^2)}{F(Q^2)}$$

Proton radius squared at $Q^2 = 0$

Color Transparency is controlled by the transverse-spatial size $\vec{a}_\perp^2$ and its dependence on the momentum transfer $Q^2 = -t$:
The scale Q^2_τ required for Color Transparency grows with twist τ

Light-Front Holography:

$$\langle \mathbf{a}_\perp^2(t) \rangle_\tau = \frac{1}{\lambda} \sum_{j=1}^{\tau-1} \frac{1}{j - \alpha(t)},$$

For large Q^2 :

$$\langle \mathbf{a}_\perp^2(Q^2) \rangle_\tau \to \frac{4(\tau - 1)}{Q^2}.$$

Two-Stage Color Transparency

$$\boxed{14\ GeV^2 < Q^2 < 20\ GeV^2}$$

If Q^2 is in the intermediate range, then the twist-3 state will propagate through the nuclear medium with minimal absorption, and the protons which survive nuclear absorption will only have L = 0 (twist-3).

The twist-4 L = 1 state which has a larger transverse size will be absorbed.

Thus 50% of the events in this range of Q^2 will have full color transparency and 50% of the events will have zero color transparency (T = 0).

The $ep \to e'p'$ cross section will have the same angular and Q^2 dependence as scattering of the electron on an unphysical proton which has no Pauli form factor.

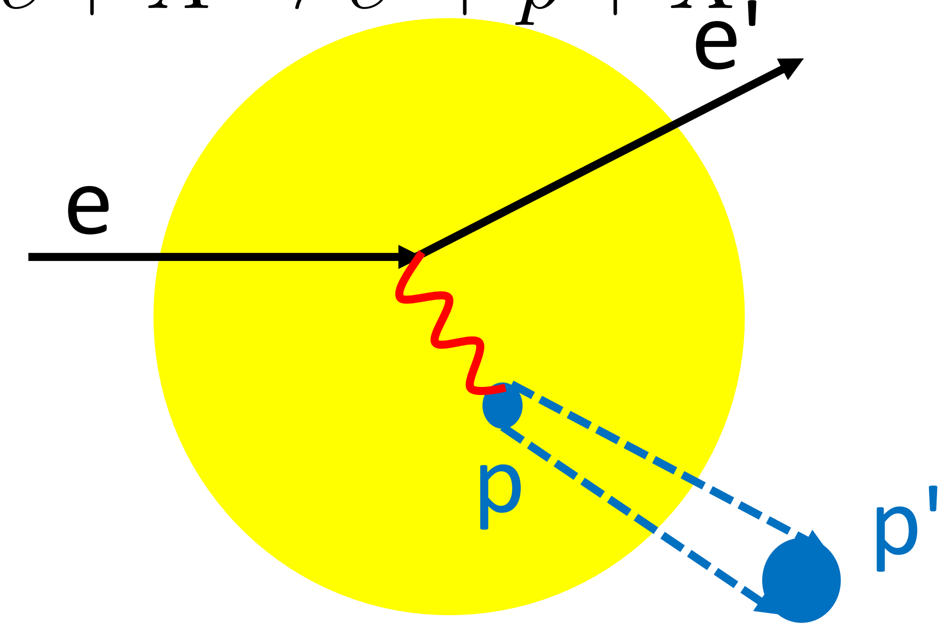
$$\boxed{Q^2 > 20\ GeV^2}$$

However, if the momentum transfer is increased to $Q^2 > 20\ GeV^2$, all events will have full color transparency, and the $ep \to e'p'$ cross section will have the same angular and Q^2 dependence as scattering of the electron on a physical proton eigenstate, with both Dirac and Pauli form factor components.

A.H. Mueller, sjb

Color transparency fundamental prediction of QCD

$$e + A \to e' + p + X$$



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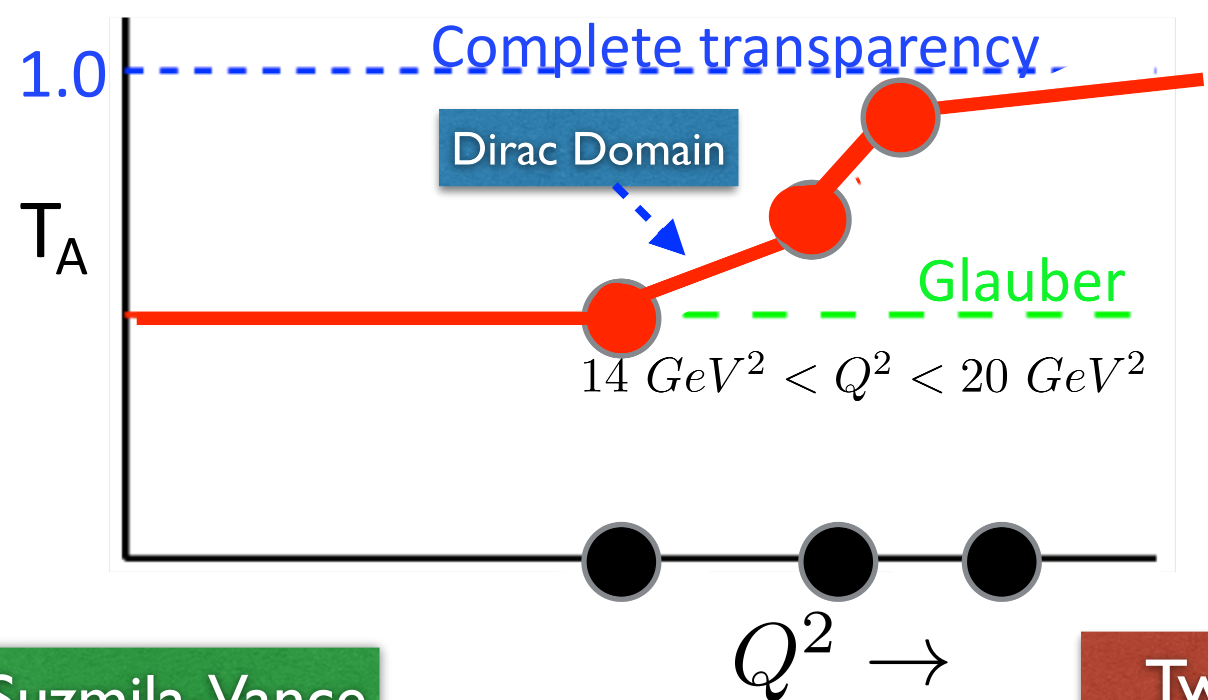


$$T_A = \frac{\sigma_A}{A\ \sigma_N}$$

σ_A (nuclear cross section)

σ_N (free nucleon cross section)

Holly Suzmila-Vance

Two-Stage Color Transparency for Proton

Color Transparency and Light-Front Holography

- Essential prediction of QCD
- LF Holography: Spectroscopy, dynamics, structure
- Transverse size predicted by LF Holography as a function of Q
- Q scale for CT increases with twist, number of constituents
- Two-Stage Proton Transparency: Equal probability L=0,1
- No contradiction with present experiments

$$Q_0^2(p) \simeq 18\ GeV^2 \text{ vs. } Q_0^2(\pi) \simeq 4\ GeV^2 \text{ for onset of color transparency in } {}^{12}C$$

Other Consequences of $[ud]_{\bar{3}_C, I=0, J=0}$ diquark cluster

QCD Hidden-Color Hexadiquark in the Core of Nuclei

J. Rittenhouse West, G. de Teramond, A. S. Goldhaber, I. Schmidt, sjb

$$|\Psi_{HDQ}>=|[ud][ud][ud][ud][ud][ud]>$$
mixes with
$$^4He|npnp>$$

Increases alpha binding energy, EMC effects

Diquarks Can Dominate Five-Quark Fock State of Proton

$$|p>=\alpha|[ud]u>+\beta|[ud][ud]\bar{d}>$$

Natural explanation why $\bar{d}(x) >> \bar{u}(x)$ in proton

Excitations and Decay of HdQ in Alpha-Nuclei
may explain ATOMKI X17 signal

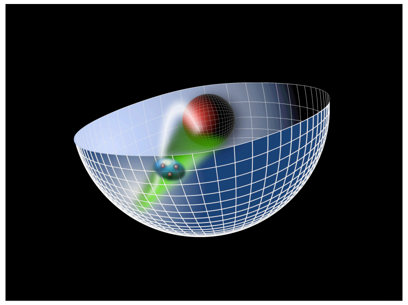
V. Kubarovsky, J. Rittenhouse West, sjb

Underlying Principles

- *PoIncarè Invariance: Independent of the observer's Lorentz frame: Quantization at Fixed Light-Front Time **τ***

- *Causality: Information within causal horizon: Light-Front*

- *Light-Front Holography: AdS₅ = LF (3+1)*



$$z \leftrightarrow \zeta \text{ where } \zeta^2 = b_\perp^2 x(1-x)$$

- *Introduce mass scale **κ** while retaining conformal invariance of the Action (dAFF)*

"Emergent Mass"

- *Unique Dilaton in AdS₅:* $e^{+\kappa^2 z^2}$

- *Unique color-confining LF Potential* $U(\zeta^2) = \kappa^4 \zeta^2$

- *Superconformal Algebra: Mass Degenerate 4-Plet:*

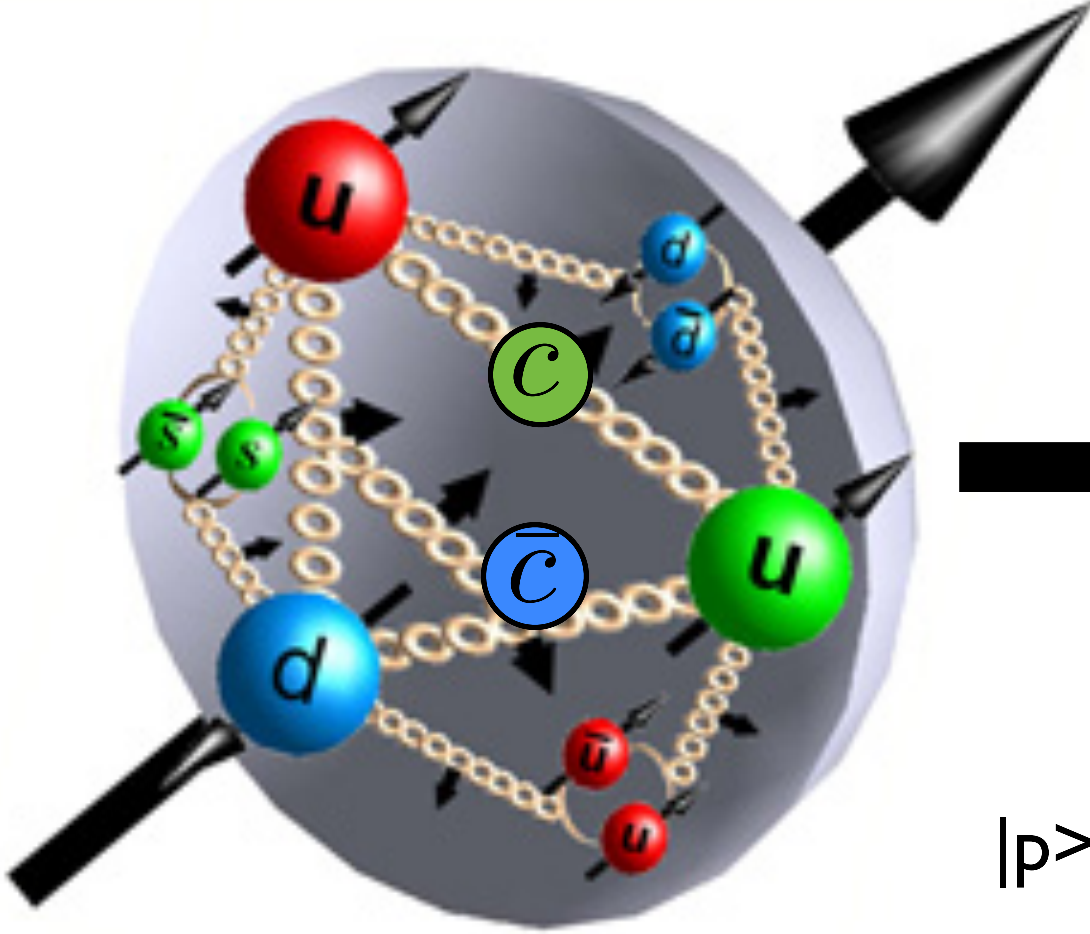
$$\boxed{\text{Meson } q\bar{q} \leftrightarrow \text{Baryon } q[qq] \leftrightarrow \text{Tetraquark } [qq][\bar{q}\bar{q}]}$$

Light-Front Holography: First Approximation to QCD

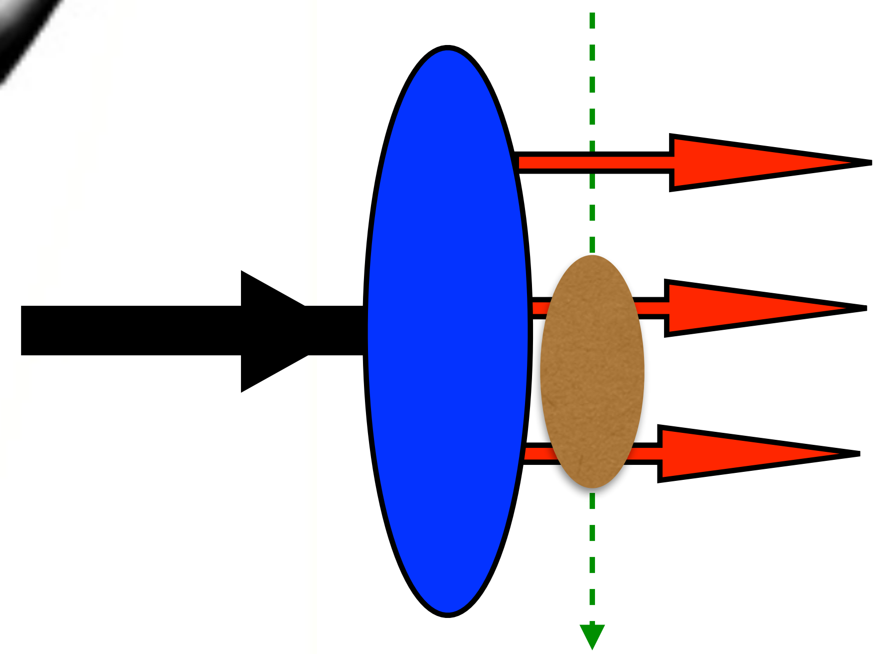
- **Color Confinement, Analytic form of confinement potential**
- **Retains underlying conformal properties of QCD despite mass scale (DeAlfaro-Fubini-Furlan Principle)**
- **Massless quark-antiquark pion bound state in chiral limit, GMOR**
- **QCD coupling at all scales**
- **Connection of perturbative and nonperturbative mass scales**
- **Poincarè Invariant**
- **Hadron Spectroscopy-Regge Trajectories with universal slopes in n, L**
- **Supersymmetric 4-Plet: Meson-Baryon -Tetraquark Symmetry**
- **Light-Front Wavefunctions**
- **Form Factors, Structure Functions, Hadronic Observables**
- **OPE: Constituent Counting Rules**
- **Hadronization at the Amplitude Level: Many Phenomenological Tests**
- **Systematically improvable: Basis LF Quantization (BLFQ)**

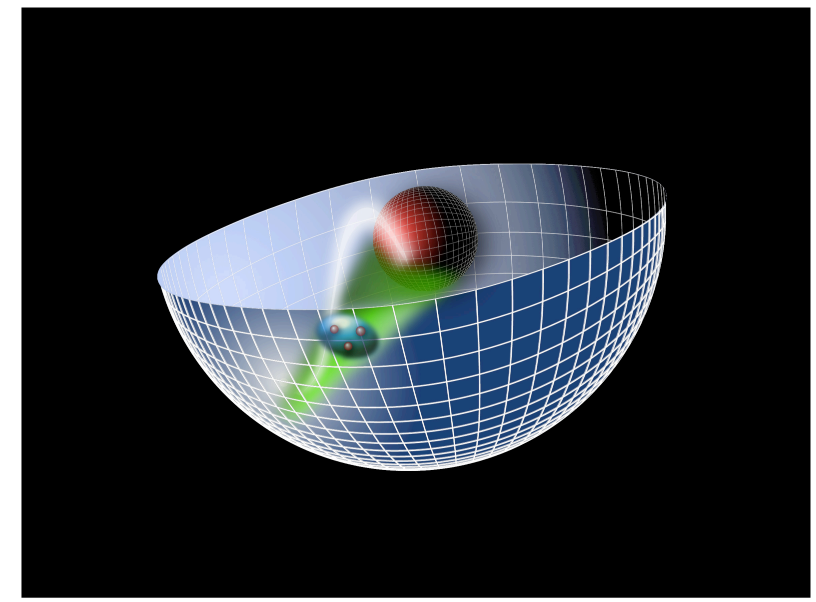
Supersymmetric Features of Hadron Physics
from Superconformal Algebra
and Light-Front Holography

Holographic Light-Front QCD: A Novel Nonperturbative Approach to Color Confinement and Hadron Physics



Fixed $\tau = t + z/c$





|p>=|u[ud]> quark-diquark cluster

with Guy de Tèramond, Hans Günter Dosch, Cèdric Lorcè, Alexandre Deur, and Joshua Erlich

Fifty Years QCD
September 15, 2023
UCLA

Stan Brodsky

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