From QCD to Gravitational Waves

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QCD at 50

Mani L. Bhaumik Institute, UCLA, Sept 14, 2023

European Research Council

AGENCE NATIONALE DE LA RECHERCHE
ANR

The Universe had a very violent origin

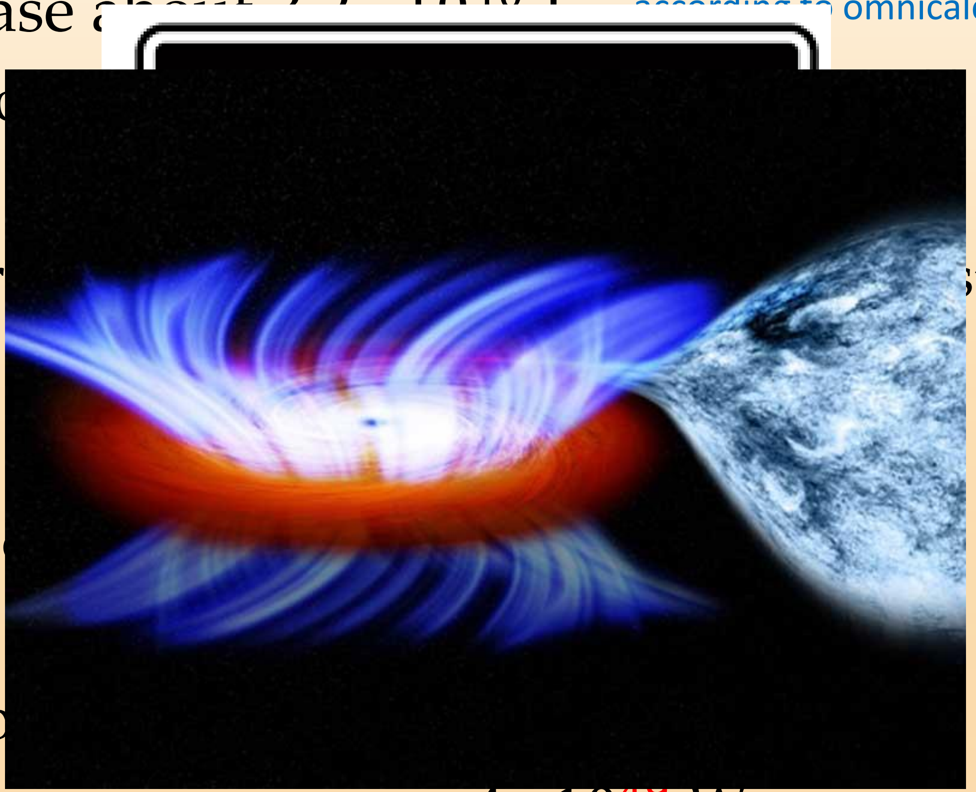
It remains a violent place

Much of that violence is associated with black holes

Imagine a small black hole — 3 $M_\odot$ — eating the Earth

– It's about 8.9 km in size, a bit smaller than Paris

Would release a … 10^{40} J … omnicalculator.com/physics/black-hole



Sun's lumin… J over its lifetime

Supernova r… …tly in neutrinos

Peak visible …

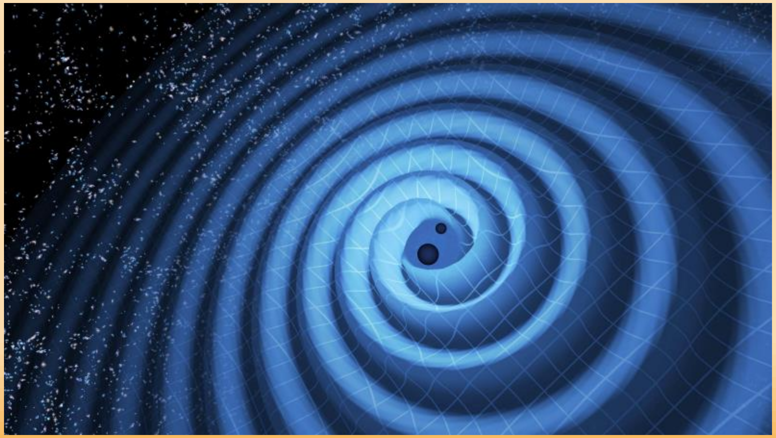
Luminosity …

Peak lumino…

$\sim 4 \cdot 10^{49}$ W

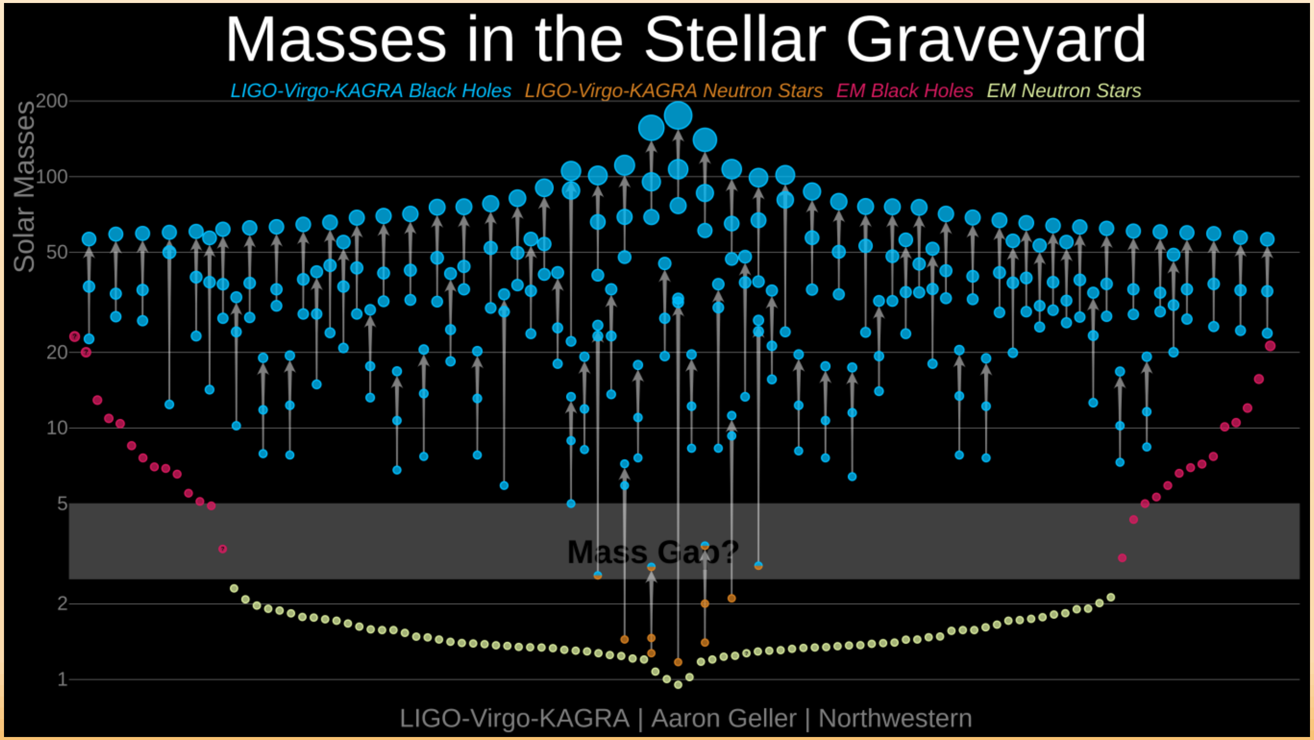
Until very recently, this violence was invisible

It’s all in gravitational radiation



Gravitational Wave Events

- LIGO–VIRGO–KAGRA
  - 90 confirmed events since Sept 2015 gracedb.ligo.org/latest/
  - In GWTC-1,2,3, runs O1–O3
  - Run O4 began in May
  - Mostly binary black holes, some neutron-star mergers, some mixed



Gravitational Wave Observatories

- LIGO: 4km arms

Hanford, Washingon

Livingston, Lousiana

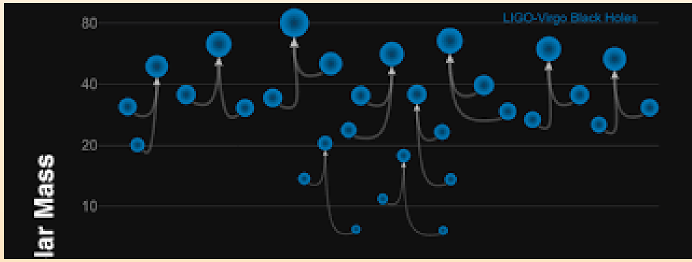
- Virgo: 3km arms
  - Pisa, Italy

- KAGRA: underground

To be joined by
IndiGO, Aundha Nagnath, Maharashtra, India (?)

Science Goals

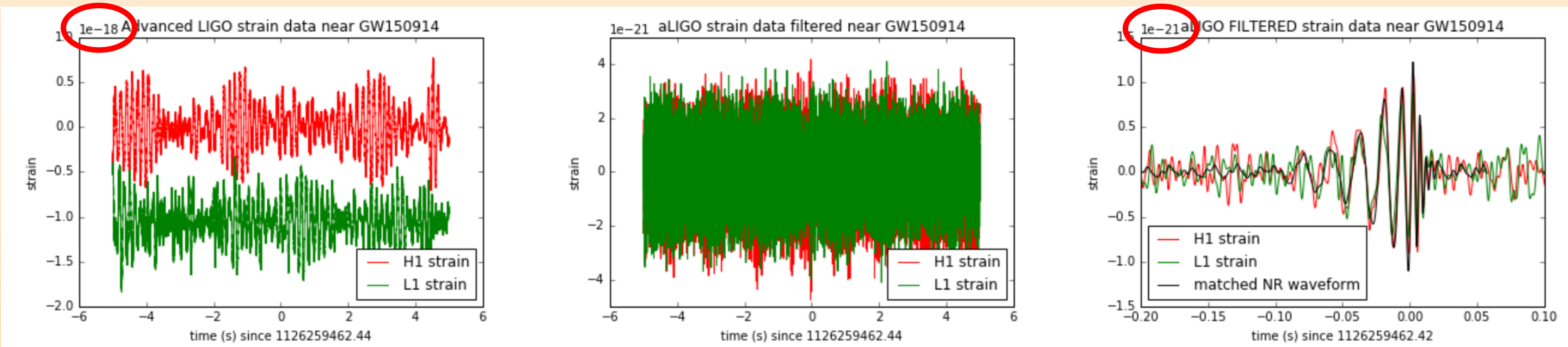
- Determine presence and distribution of heavy compact objects
  - Already discovered unexpected class of black holes, tens of solar masses, indeed binaries of such black holes
  - More surprises?



- Neutron stars
  - Determine equation of state
  - Contribution to transferric element production
  - Measure Hubble constant?
- Limits on exotic compact objects
- Precision strong-field tests of Einstein gravity
- Insight into quantum gravity [in the LISA era]?

Theorists' Role

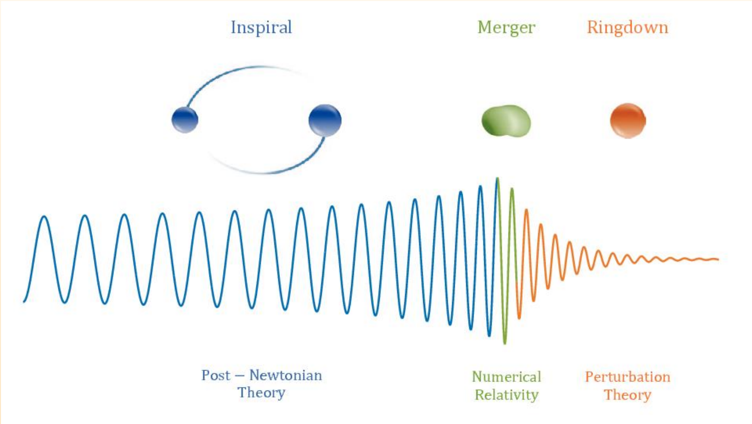
- One aspect of detection & analysis
- High noise level: 400 times larger than signal! Rehman [1711.07421]



- To extract the signal, we need theoretical templates
- That's where theorists come into the picture
- Also need theoretical calculations to measure parameters of mergers once seen — masses, spins, etc.

Basic Setup in Mergers

- Two slowly inspiralling compact objects
- Ringdown
  - Perturbation theory: normal modes of final black hole
- Merger
  - Strong fields: numerical relativity
- Inspiral
  - Weak-field perturbation theory
- Waveform templates
  - For detection
  - For measurements



Antelis et al

Gravitational Waves

- General relativists have worked hard for many results
  - Using an Effective One-Body formalism

Buonanno, Pan, Taracchini, Barausse, Bohé, Cotesta, Shao, Hinderer, Steinhoff, Vines; Damour, Nagar, Bernuzzi, Bini, Balmelli, Messina; Iyer, Sathyaprakash; Jaranowski, Schaefer

  - Numerical Relativity Pretorius; Campanelli *et al.*; Baker *et al.*

- Joined by Effective Field Theorists
  - Take advantage of $r_{\mathrm{BH}} \ll r_{\mathrm{sep}} \ll \lambda_{\mathrm{GW}}$

Goldberger, Rothstein; Goldberger, Li, Prabhu, Thompson; Chester; Porto,… Kol; Levi,…; Foffa, Sturani, …

You're All Wondering

- This all seems quite entertaining
- But it appears to be a heading towards a talk about General Relativity
- What is it doing in a conference celebrating 50 years of QCD?

- To understand that, we need to look back at more recent history than the early years of QCD

- The early successes of QCD were followed by perturbative calculations of basic processes and corrections

Altarelli, Ellis, Martinelli;
Amati, Bassetto, Ciafaloni, Marchesini, Veneziano;
…

- Multi-jet processes also attracted attention in the 1980s, up to $2 \to 4$ processes

Kunszt & Gunion; Parke & Taylor; Kunszt & Stirling

Unexpected Simplicity

- Look at helicity amplitudes — Bjorken in the 1960s

High-Energy Trident Production with Definite Helicities*

J. D. Bjorken†
Stanford Linear Accelerator Center, Stanford University, Stanford, California

AND

M. C. Chen
Department of Physics, Idaho State University, Pocatello, Idaho
(Received 18 July 1966)

- Many ways of implementing them in Yang–Mills theories, but the "Chinese magic" of Xu, Zhang, Chang endures
- The celebrated Parke–Taylor amplitude

$$\frac{\langle m_1 m_2\rangle^4}{\langle 12\rangle\langle 23\rangle\cdots\langle (n-1)n\rangle\langle n1\rangle}$$

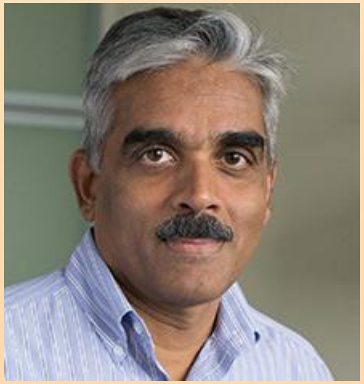
which form was actually written down by Mangano, Parke & Xu

A Seed

"Simple results should have a simple derivation" — Feynman (*attr*)

The start of a new era of computing QCD amplitudes

In spite of early work on QCD at Harvard, no-one was doing perturbative QCD when I arrived



V. Parameswaran Nair

Understood Parke–Taylor amplitudes as correlators on $\mathbb{CP}^1$ — underpinning of Witten's twistor string theory

Onwards to Loop Amplitudes

- New technology: the unitarity method, marked the birth of *Scattering Amplitudes* as a subfield Bern, Dixon, Dunbar, DAK
- Focus on physical on-shell quantities even in intermediate steps
- Generalized unitarity: cut any propagator & match analytic structure extract gauge-invariant coefficients of basis integrals
  Bern, Dixon, DAK; Britto, Cachazo, Feng
- We had to traverse a long desert before the utility became clear to the wider community
- Was important to have a protector
- And enthusiastic senior colleagues

Al Mueller

Zoltan Kunszt

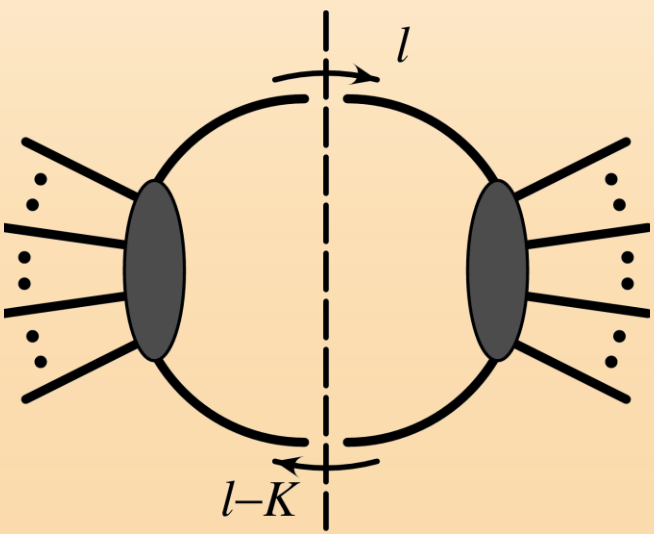
$\mathcal{N} = 4$

It was too hard to do full QCD, so we went into the theorists' laboratory: $\mathcal{N} = 4$ SUSY

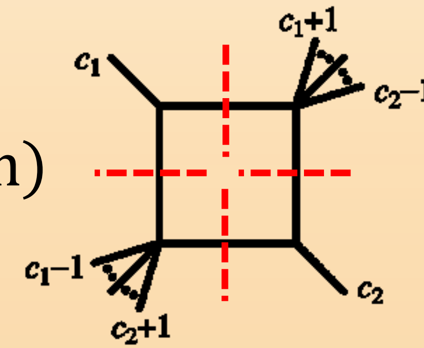
- It has remained a great laboratory & development environment ever since

Amazingly simple result

$$A^{\text{tree}}(1^+ \cdots m_1^- \cdots m_2^- \cdots n^+)$$



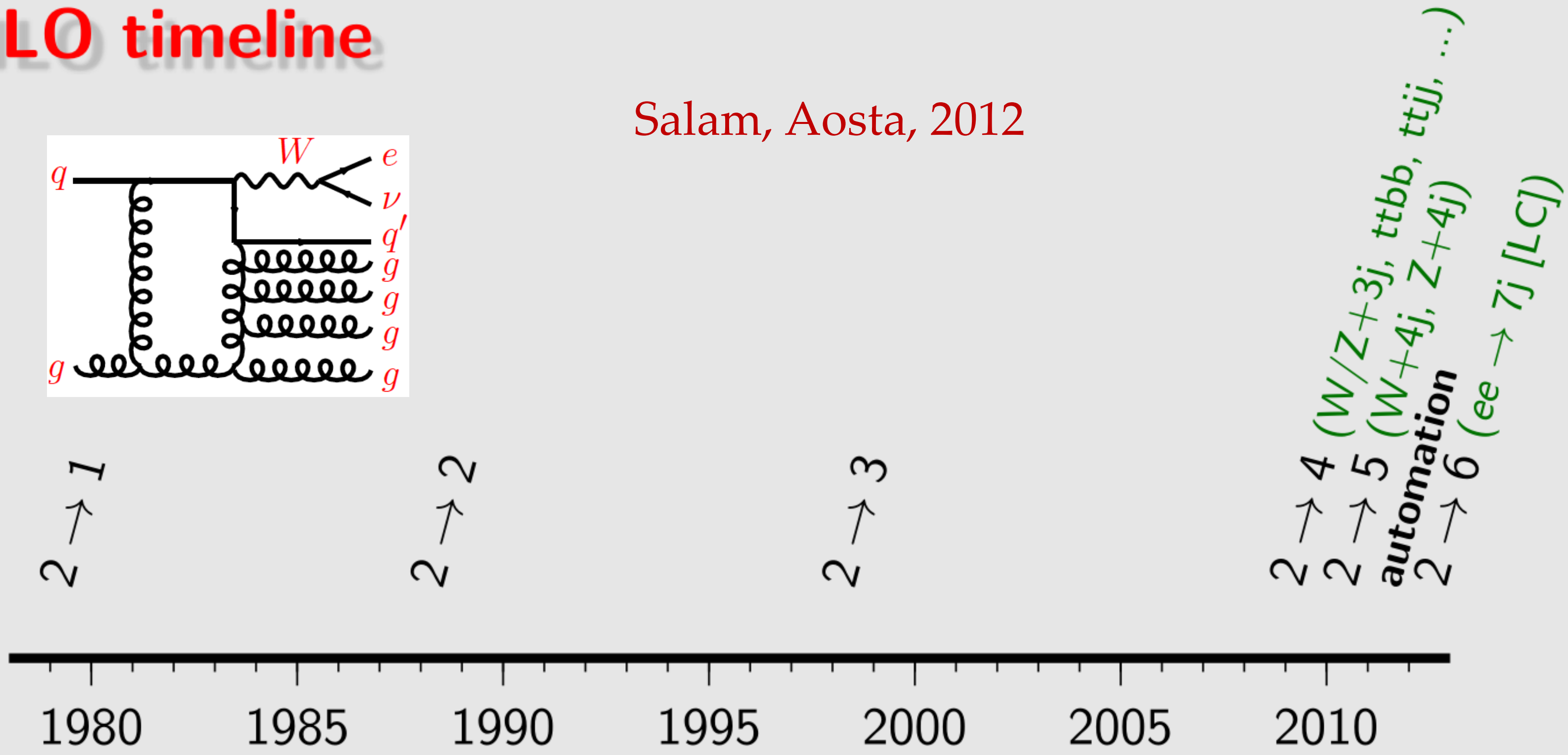
$$\times \sum \quad \left(-\frac{1}{2}\ \text{denom}\right)$$



Today, a student exercise not because the students are better, but because the technology is

NLO timeline

Salam, Aosta, 2012



2010: NLO $W+4j$ [BlackHat+Sherpa: Berger et al] [unitarity]
2011: NLO $WWjj$ [Rocket: Melia et al] [unitarity]
2011: NLO $Z+4j$ [BlackHat+Sherpa: Ita et al] [unitarity]
2011: NLO $4j$ [BlackHat+Sherpa: Bern et al] [unitarity]
2011: first automation [MadNLO: Hirschi et al] [unitarity + feyn.diags]
2011: first automation [Helac NLO: Bevilacqua et al] [unitarity]
2011: first automation [GoSam: Cullen et al] [feyn.diags(+unitarity)]
2011: $e^+e^- \to 7j$ [Becker et al, leading colour] [numerical loops]

Taking Landau Seriously

…the only observable quantities are the momenta and polarizations of freely moving particles. Therefore if we don't want to introduce unobservables, we may introduce in the theory as fundamental quantities only the scattering amplitudes.

The physical basis of this technique [are] the unitarity conditions and the principle of locality of interaction which expresses itself in the analytic properties of the fundamental quantities of the theory

Lev Landau (Pauli Memorial) 1961, as quoted by Bob Jaffe

- Adopt these ideas not as a rejection of field theory
- But as a distillation of its essence

You're allowed to calculate unphysical quantities,
but you're not obliged to.

The only quantities you're obliged to calculate are observables

Growing Community

Over 200 attendees
Lots of enthusiastic younger physicists

- Recursion for trees Berends, Giele; Britto, Cachazo, Feng, Witten
- Integration by Parts identities Chetyrkin, Tkachov; Laporta; Smirnov; Lee; Usovitch
- Differential equations for higher-loop Feynman integrals Kotikov; Bern, Dixon, DAK; Gehrmann, Remiddi; Henn
- Rational reconstruction Peraro
- Understanding of structures underlying amplitudes

Approaches to Gravitational Waves

- Traditional: solve General Relativity perturbatively
- Effective Field Theory: use separation of scales to compute better in General Relativity
- An idea: use scattering amplitudes

Thibault Damour*
Institut des Hautes Etudes Scientifiques, 35 route de Chartres, 91440 Bures-sur-Yvette, France
(Dated: October 31, 2017)

We finally indicate a way to connect our classical results to the quantum gravitational scattering amplitude of two particles, and we urge amplitude experts to use their novel techniques to compute the 2-loop scattering amplitude of scalar masses, from which one could deduce the third post-Minkowskian effective one-body Hamiltonian

Scattering Amplitudes

- It's a bound-state classical problem

- Why might quantum scattering amplitudes help?
- Calculate only what's needed for physical quantities
  - No auxiliary Hamiltonians or potentials
  - No confusing or ambiguous separation between "conservative" and "radiation reaction"
- Powerful toolkit developed for QCD-targeted calculations
- Double copy: amplitude calculations in gravity are vastly simplified by the observation that

$$\text{Gravity} \sim (\text{Yang–Mills})^2$$

Kawai, Lewellen, Tye; Bern, Carrasco, Johansson; Cachazo, He, Yuan

Feynman Vertices

- De Witt:

$$\frac{\delta^3 S}{\delta\varphi_{\mu\nu}\delta\varphi_{\sigma'\tau'}\delta\varphi_{\rho''\lambda''}} \longrightarrow$$

$$\begin{aligned}&\mathrm{Sym}\Big[-\tfrac{1}{4}P_3(p\cdot p'\eta^{\mu\nu}\eta^{\sigma\tau}\eta^{\rho\lambda})-\tfrac{1}{4}P_6(p^\sigma p^\tau\eta^{\mu\nu}\eta^{\rho\lambda})+\tfrac{1}{4}P_3(p\cdot p'\eta^{\mu\sigma}\eta^{\nu\tau}\eta^{\rho\lambda})+\tfrac{1}{2}P_6(p\cdot p'\eta^{\mu\nu}\eta^{\sigma\rho}\eta^{\tau\lambda})+P_3(p^\sigma p^\lambda\eta^{\mu\nu}\eta^{\tau\rho})\\&-\tfrac{1}{2}P_3(p^\tau p'^\mu\eta^{\nu\sigma}\eta^{\rho\lambda})+\tfrac{1}{2}P_3(p^\rho p'^\lambda\eta^{\mu\sigma}\eta^{\nu\tau})+\tfrac{1}{2}P_6(p^\rho p^\lambda\eta^{\mu\sigma}\eta^{\nu\tau})+P_6(p^\sigma p'^\lambda\eta^{\tau\mu}\eta^{\nu\rho})+P_3(p^\sigma p'^\mu\eta^{\tau\rho}\eta^{\lambda\nu})\\&\qquad-P_3(p\cdot p'\eta^{\nu\sigma}\eta^{\tau\rho}\eta^{\lambda\mu})\Big],\end{aligned} \quad (2.6)$$

$$\frac{\delta^4 S}{\delta\varphi_{\mu\nu}\delta\varphi_{\sigma'\tau'}\delta\varphi_{\rho''\lambda''}\delta\varphi_{\iota'''\kappa'''}} \longrightarrow$$

$$\begin{aligned}&\mathrm{Sym}\Big[-\tfrac{1}{8}P_6(p\cdot p'\eta^{\mu\nu}\eta^{\sigma\tau}\eta^{\rho\lambda}\eta^{\iota\kappa})-\tfrac{1}{8}P_{12}(p^\sigma p^\tau\eta^{\mu\nu}\eta^{\rho\lambda}\eta^{\iota\kappa})-\tfrac{1}{4}P_6(p^\sigma p'^\mu\eta^{\nu\tau}\eta^{\rho\lambda}\eta^{\iota\kappa})+\tfrac{1}{8}P_6(p\cdot p'\eta^{\mu\sigma}\eta^{\nu\tau}\eta^{\rho\lambda}\eta^{\iota\kappa})\\&+\tfrac{1}{4}P_6(p\cdot p'\eta^{\mu\nu}\eta^{\sigma\tau}\eta^{\rho\iota}\eta^{\lambda\kappa})+\tfrac{1}{4}P_{12}(p^\sigma p^\tau\eta^{\mu\nu}\eta^{\rho\iota}\eta^{\lambda\kappa})+\tfrac{1}{2}P_6(p^\sigma p'^\mu\eta^{\nu\tau}\eta^{\rho\iota}\eta^{\lambda\kappa})-\tfrac{1}{4}P_6(p\cdot p'\eta^{\mu\sigma}\eta^{\nu\tau}\eta^{\rho\iota}\eta^{\lambda\kappa})\\&+\tfrac{1}{4}P_{24}(p\cdot p'\eta^{\mu\nu}\eta^{\sigma\rho}\eta^{\tau\lambda}\eta^{\iota\kappa})+\tfrac{1}{4}P_{24}(p^\sigma p^\tau\eta^{\mu\rho}\eta^{\nu\lambda}\eta^{\iota\kappa})+\tfrac{1}{4}P_{12}(p^\rho p'^\lambda\eta^{\mu\sigma}\eta^{\nu\tau}\eta^{\iota\kappa})+\tfrac{1}{2}P_{24}(p^\sigma p'^\rho\eta^{\tau\mu}\eta^{\nu\lambda}\eta^{\iota\kappa})\\&-\tfrac{1}{2}P_{12}(p\cdot p'\eta^{\nu\sigma}\eta^{\tau\rho}\eta^{\lambda\mu}\eta^{\iota\kappa})-\tfrac{1}{2}P_{12}(p^\sigma p'^\mu\eta^{\tau\rho}\eta^{\lambda\nu}\eta^{\iota\kappa})+\tfrac{1}{2}P_{12}(p^\sigma p^\rho\eta^{\tau\lambda}\eta^{\mu\nu}\eta^{\iota\kappa})-\tfrac{1}{2}P_{24}(p\cdot p'\eta^{\mu\nu}\eta^{\tau\rho}\eta^{\lambda\iota}\eta^{\kappa\sigma})\\&-P_{12}(p^\sigma p^\tau\eta^{\nu\rho}\eta^{\lambda\iota}\eta^{\kappa\mu})-P_{12}(p^\rho p'^\lambda\eta^{\nu\iota}\eta^{\kappa\sigma}\eta^{\tau\mu})-P_{24}(p_\sigma p'^\rho\eta^{\tau\iota}\eta^{\kappa\mu}\eta^{\nu\lambda})-P_{12}(p^\rho p'^\iota\eta^{\lambda\sigma}\eta^{\tau\mu}\eta^{\nu\kappa})\\&+P_6(p\cdot p'\eta^{\nu\rho}\eta^{\lambda\sigma}\eta^{\tau\iota}\eta^{\kappa\mu})-P_{12}(p^\sigma p^\rho\eta^{\mu\nu}\eta^{\tau\iota}\eta^{\kappa\lambda})-\tfrac{1}{2}P_{12}(p\cdot p'\eta^{\mu\rho}\eta^{\nu\lambda}\eta^{\sigma\iota}\eta^{\tau\kappa})-P_{12}(p^\sigma p^\rho\eta^{\tau\lambda}\eta^{\mu\iota}\eta^{\nu\kappa})\\&\qquad-P_6(p^\rho p'^\iota\eta^{\lambda\kappa}\eta^{\mu\sigma}\eta^{\nu\tau})-P_{24}(p^\sigma p'^\rho\eta^{\tau\mu}\eta^{\nu\iota}\eta^{\kappa\lambda})-P_{12}(p^\sigma p'^\mu\eta^{\tau\rho}\eta^{\lambda\iota}\eta^{\kappa\nu})+2P_6(p\cdot p'\eta^{\nu\sigma}\eta^{\tau\rho}\eta^{\lambda\iota}\eta^{\kappa\mu})\Big].\end{aligned} \quad (2.7)$$

Three-point

Four-point

⋮

Double Copy

- Yang–Mills three-point amplitude

$$\frac{\langle 1\,2\rangle^3}{\langle 2\,3\rangle\langle 3\,1\rangle}$$

- Gravity three-point amplitude

$$\left(\frac{\langle 1\,2\rangle^3}{\langle 2\,3\rangle\langle 3\,1\rangle}\right)^2 = (\mathrm{YM})^2$$

- Field-theory equivalent of Kawai–Lewellen–Tye expression for closed-string amplitudes in terms of open-string ones
- Generalization to higher multiplicity is known and proven
- Useful for loops via generalized unitarity

State of the Art

$$G(1 + v^2 + v^4 + v^6 + v^8 + v^{10} + v^{12} + \cdots)$$
$$G^2(1 + v^2 + v^4 + v^6 + v^8 + v^{10} + v^{12} + \cdots)$$
$$G^3(1 + v^2 + v^4 + v^6 + v^8 + v^{10} + v^{12} + \cdots)$$
$$G^4(1 + v^2 + v^4 + v^6 + v^8 + v^{10} + v^{12} + \cdots)$$
$$G^5(1 + v^2 + v^4 + v^6 + v^8 + v^{10} + v^{12} + \cdots)$$
$$G^6(1 + v^2 + v^4 + v^6 + v^8 + v^{10} + v^{12} + \cdots)$$
$$G^7(1 + v^2 + v^4 + v^6 + v^8 + v^{10} + v^{12} + \cdots)$$

Snowmass Review [2204.05194] – Buonanno, Khalil, O'Connell, Roiban, Solon, Zeng

A Flourishing of New Ideas…

…that I lack time to discuss

- EFT Matching

  Cheung, Rothstein, Solon; Bern, Kosmopoulos, Luna, Roiban, Teng; Bern, Parra-Martinez, Roiban, Ruf, Shen, Solon, Zeng

- Eikonal Phase

  Amati, Ciafaloni, Veneziano; Di Vecchia, Heissenberg, Russo, Veneziano

- Amplitude analysis

  Bjerrum-Bohr, Damgaard, Plante, Vanhove

- Heavy mass field theory

  Brandhuber, Chen, Travaglini, Wen; Damgaard, Haddad, Helset

- World line formalisms

  Goldberger, Rothstein; Levi, Steinhoff; Dlapa, Kälin, Liu, Porto; Jakobson, Mogull, Plefka, Steinhoff; Shen; Edison, Levi

- Spin Exponentiation

  Arkani-Hamed, Huang, O'Connell; Guevara, Ochirov, Vines; Chen, Huang, Kim, Lee; Bautista, Guevara, Kavanagh

- EFT post-Newtonian

  Foffa, Mastrolia, Sturani, Sturm; Blümlein, Maier, Marquard, Schäfer

Some Background

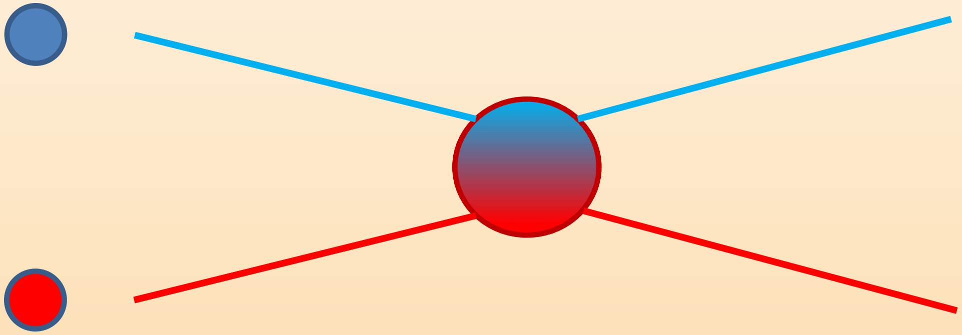
Effective Field Theory

- In the weak-field regime, well before merger, there are three distinct scales
  - Size of compact objects r
  - Separation of compact objects R
  - Wavelength of emitted radiation λ
  - Hierarchy: $\lambda \gg R \gg r$

- Hierarchy of scales can be used to write down an effective field theory
  - Originated with Goldberger and Rothstein
  - Intellectual debt to Steven Weinberg

Set-up

- Scatter two 'things'



- If they're both massive, look at point particles

Direct Route

- Compute corrections to the potential

$$H^{\mathrm{PN}} = H_{\mathrm{Newton}} + H_{1\mathrm{PN}} + H_{2\mathrm{PN}} + \cdots$$

$$= m_1 + m_2 + \frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2} + V^{\mathrm{PN}}(r_{12}, p_1, p_2)$$

$$V^P = -\frac{G m_1 m_2}{|r_{12}|} + \text{corrections}$$

- Bound state ↔ scattering connection
  - Via Hamiltonian
  - Analytic continuation in J

Porto, Kälin

Direct Route

- Compute scattering amplitude in effective theory built out of the potential

- Match coefficients to classical scattering amplitude
  - Compute quantum scattering amplitude
  - Take its classical limit

Cheung, Rothstein, Solon (2018)

- Infrared singularities cancel in matching
- Effective theory calculation is just summing bubbles

A Frontier-Piercing Result

Potential to 3rd order

$$V(\boldsymbol{p},\boldsymbol{r}) = \sum_{i=1}^{\infty} c_i(\boldsymbol{p}^2)\left(\frac{G}{|\boldsymbol{r}|}\right)^i$$

$$\begin{aligned} c_3 = \frac{\nu^2 m^4}{\gamma^2 \xi}\Bigg[& \frac{1}{12}\left(3-6\nu+206\nu\sigma-54\sigma^2+108\nu\sigma^2+4\nu\sigma^3\right) - \frac{4\nu\left(3+12\sigma^2-4\sigma^4\right)\operatorname{arcsinh}\sqrt{\frac{\sigma-1}{2}}}{\sqrt{\sigma^2-1}} \\ & - \frac{3\nu\gamma\left(1-2\sigma^2\right)\left(1-5\sigma^2\right)}{2(1+\gamma)(1+\sigma)} - \frac{3\nu\sigma\left(7-20\sigma^2\right)}{2\gamma\xi} - \frac{\nu^2\left(3+8\gamma-3\xi-15\sigma^2-80\gamma\sigma^2+15\xi\sigma^2\right)\left(1-2\sigma^2\right)}{4\gamma^3\xi^2} \\ & + \frac{2\nu^3(3-4\xi)\sigma\left(1-2\sigma^2\right)^2}{\gamma^4\xi^3} + \frac{\nu^4(1-2\xi)\left(1-2\sigma^2\right)^3}{2\gamma^6\xi^4}\Bigg] \end{aligned}$$

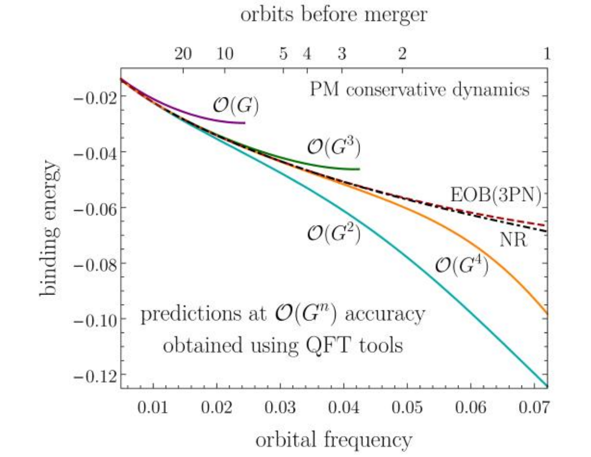
Bern, Cheung, Roiban, Shen, Solon, Zeng (2019)

- $O(G^4)$ double copy and generalized unitarity are crucial

Bern, Parra-Martinez, Roiban, Ruf, Shen, Solon, Zeng (2021)

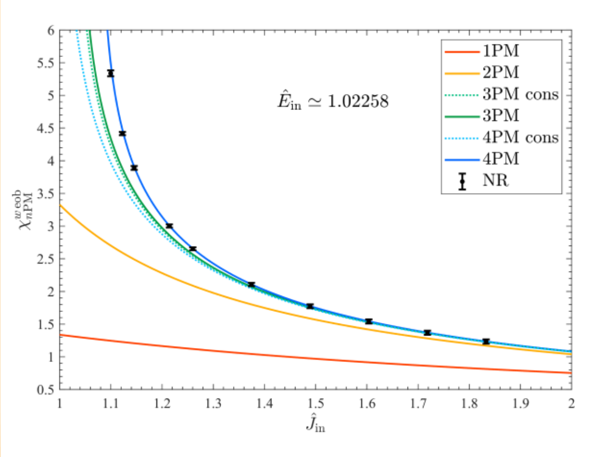
Towards Application

- Binding Energy



Buonanno, Khalil, O'Connell, Roiban, Solon, Zeng

- Combine perturbative results with Effective-One-Body Hamiltonian



Damour, Rettegno

Observables-Based Formalism

with O'Connell, Maybee, Cristofoli, Gonzo; Herrmann, Parra-Martinez, Ruf, Zeng; Manohar, Ridgway, Shen; de la Cruz, Luna, Scheopner

- Pick observables in the quantum theory also relevant classically
- Express in terms of scattering amplitudes in quantum theory
  - Amplitudes are our friends: but they are not directly observable
- Understand how to take the classical limit efficiently
  - Restore $\hbar$, $\hbar \to 0$
  - Couplings: $e \to e/\sqrt{\hbar}$; $\kappa \to \kappa/\sqrt{\hbar}$
  - Messenger wavenumbers: $\overline{\boldsymbol{p}} = \boldsymbol{p}/\hbar$
  - Turn the crank

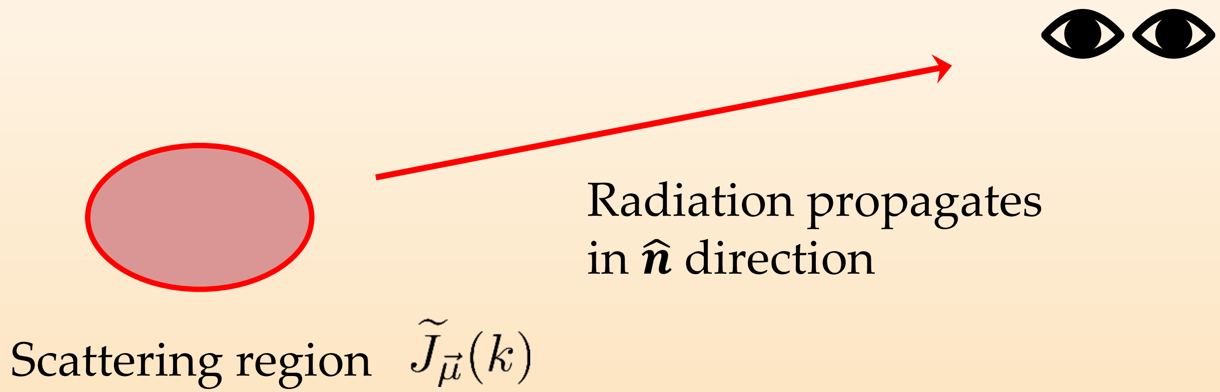
Build Observables

- Localized particles: relativistic wavepackets
  $$|\psi\rangle_{\text{in}} = \int d\Phi(p_1)d\Phi(p_2)\,\phi(p_1)\phi(p_2)\;e^{ib\cdot p_1/\hbar}\,|p_1 p_2\rangle_{\text{in}}$$
- Example: final momentum of (say) particle 1
  $$\langle p^\mu_{1,\text{out}}\rangle = {}_{\text{out}}\langle\psi|\mathbb{P}^\mu_1|\psi\rangle_{\text{out}}$$
- S matrix: evolution operator from far past to far future
  $$|\psi\rangle_{\text{out}} = S|\psi\rangle_{\text{in}}$$
- Then
  $$\langle p^\mu_{1,\text{out}}\rangle = {}_{\text{in}}\langle\psi|S^\dagger\mathbb{P}^\mu_1 S|\psi\rangle_{\text{in}}$$

Impulse (use unitarity)

$$\begin{aligned}\langle\Delta p^\mu_1\rangle &= i\langle\psi|\left[\mathbb{P}^\mu_1, T\right]|\psi\rangle + \langle\psi|T^\dagger\left[\mathbb{P}^\mu_1, T\right]|\psi\rangle \\ &= \quad I^\mu_{(1)} \quad + \quad I^\mu_{(2)} \\ &\quad\;\; \mathcal{O}(g^2) \quad + \quad \mathcal{O}(g^4)\end{aligned}$$

Point-Like Observables



- Local radiation observable

$$R_{\vec{\mu}}(x) = i \int d\Phi(\bar{k}) \left[\widetilde{J}_{\vec{\mu}}(\bar{k}) e^{-i\bar{k}\cdot x} - \widetilde{J}^*_{\vec{\mu}}(-\bar{k}) e^{+i\bar{k}\cdot x}\right]$$

- Waveform is leading large-distance behavior

$$R_{\vec{\mu}}(x) = \frac{1}{|\boldsymbol{x}|} W_{\vec{\mu}}(t, \hat{\boldsymbol{n}}; x)$$

Example: LO Waveform

- Write using *S* matrix & rewrite *S* matrix

$$\langle R^{\text{out}}_{\mu\nu\rho\lambda}(x)\rangle \equiv {}_{\text{out}}\langle\psi|\mathbb{R}_{\mu\nu\rho\lambda}(x)|\psi\rangle_{\text{out}} = {}_{\text{in}}\langle\psi|S^\dagger\mathbb{R}_{\mu\nu\rho\lambda}(x)S|\psi\rangle_{\text{in}}$$

$$\langle R^{\text{out}}_{\mu\nu\rho\lambda}(x)\rangle = 2\,\text{Re}\,i\langle\psi|\mathbb{R}_{\mu\nu\rho\lambda}(x)T|\psi\rangle + \langle\psi|T^\dagger\mathbb{R}_{\mu\nu\rho\lambda}(x)T|\psi\rangle\,.$$

- All-orders
- At LO, only the first term contributes

$$\frac{4}{\hbar^{3/2}}\,\text{Re}\sum_\eta\int d\Phi(p_1)d\Phi(p_2)d\Phi(p_1')d\Phi(p_2')d\Phi(k)\;e^{-ib\cdot(p_1'-p_1)/\hbar}$$
$$\times\,\phi(p_1)\phi^*(p_1')\phi(p_2)\phi^*(p_2')k^{[\mu}\varepsilon^{(\eta)\nu]*}k^{[\rho}\varepsilon^{(\eta)\lambda]*}e^{-ik\cdot x/\hbar}\langle p_1'\,p_2'|a_{(\eta)}(k)\,T|p_1\,p_2\rangle$$

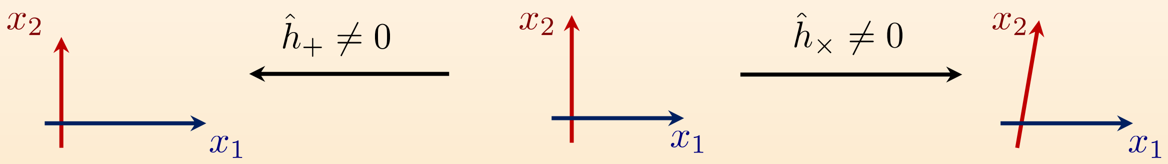
- The matrix element is a five-point amplitude

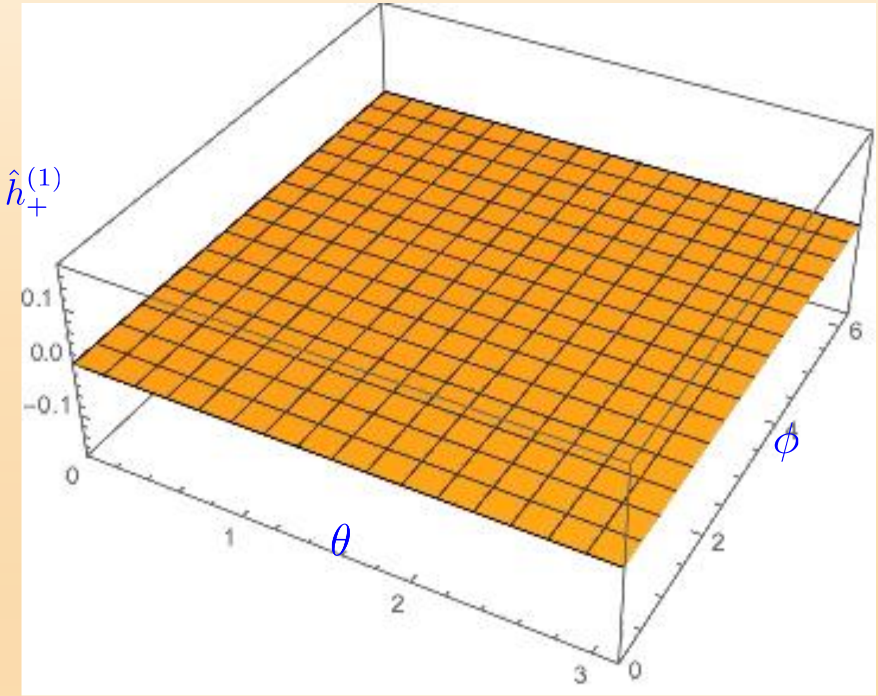
$$\begin{aligned}\langle p_1'\,p_2'|a_{(\eta)}(k)\,T|p_1\,p_2\rangle &= \langle p_1'\,p_2'\,k^\eta|T|p_1\,p_2\rangle\\ &= \mathcal{A}(p_1,p_2\to p_1',p_2',k^\eta)\hat{\delta}^4(p_1+p_2-p_1'-p_2'-k)\end{aligned}$$

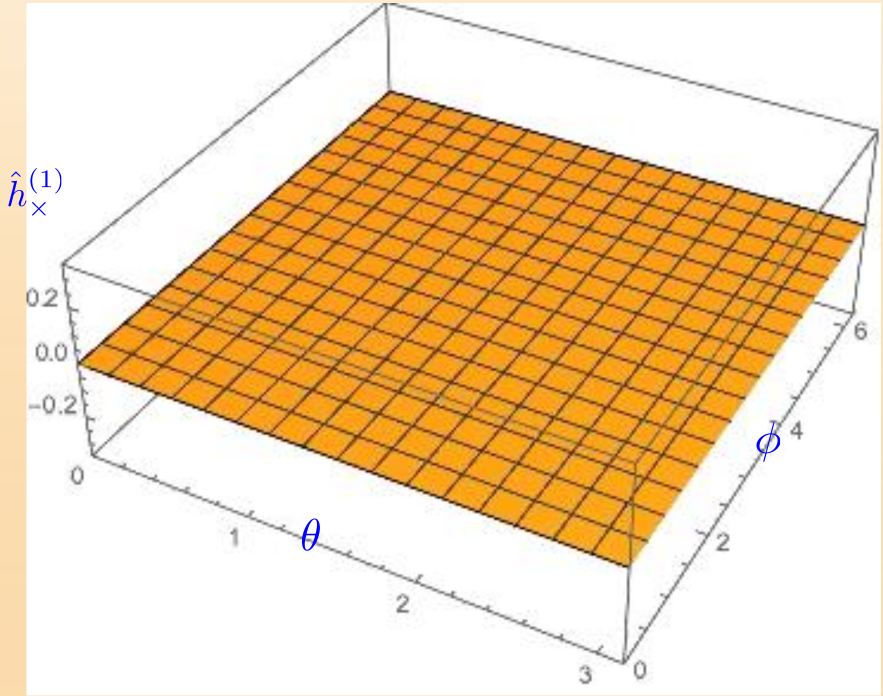
At leading order (courtesy of Radu Roiban)

based on Kovacs, Thorne; Jakobsen, Mogull, Plefka, Steinhoff; Herderschee, Roiban, Teng

$$g_{\mu\nu}\Big|_{|\boldsymbol{x}|\to\infty} = \eta_{\mu\nu} + \frac{\kappa^2 \mathrm{M}}{8\pi|\boldsymbol{x}|}\left[\frac{\kappa^2 \mathrm{M}}{\sqrt{-b^2}}\hat{h}^{(1)}_{\mu\nu} + \left(\frac{\kappa^2 \mathrm{M}}{\sqrt{-b^2}}\right)^2 \hat{h}^{(2)}_{\mu\nu} + \dots\right]$$







Summary

- QCD gave us a window into the rich physics of Yang–Mills theories
- These theories are connected in surprising ways to gravity
- The technologies developed for QCD are bearing fruit even for a subject far removed from the quantum frontier
- Current generation of gravitational-wave observatories will be running until the end of the decade
- A new generation of terrestrial and space-based observatories are planned for the 2030s