High Energy Collider Observables at Ultimate Precision in QCD

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50 Years of QCD, UCLA, 11.9.-15.9.2023

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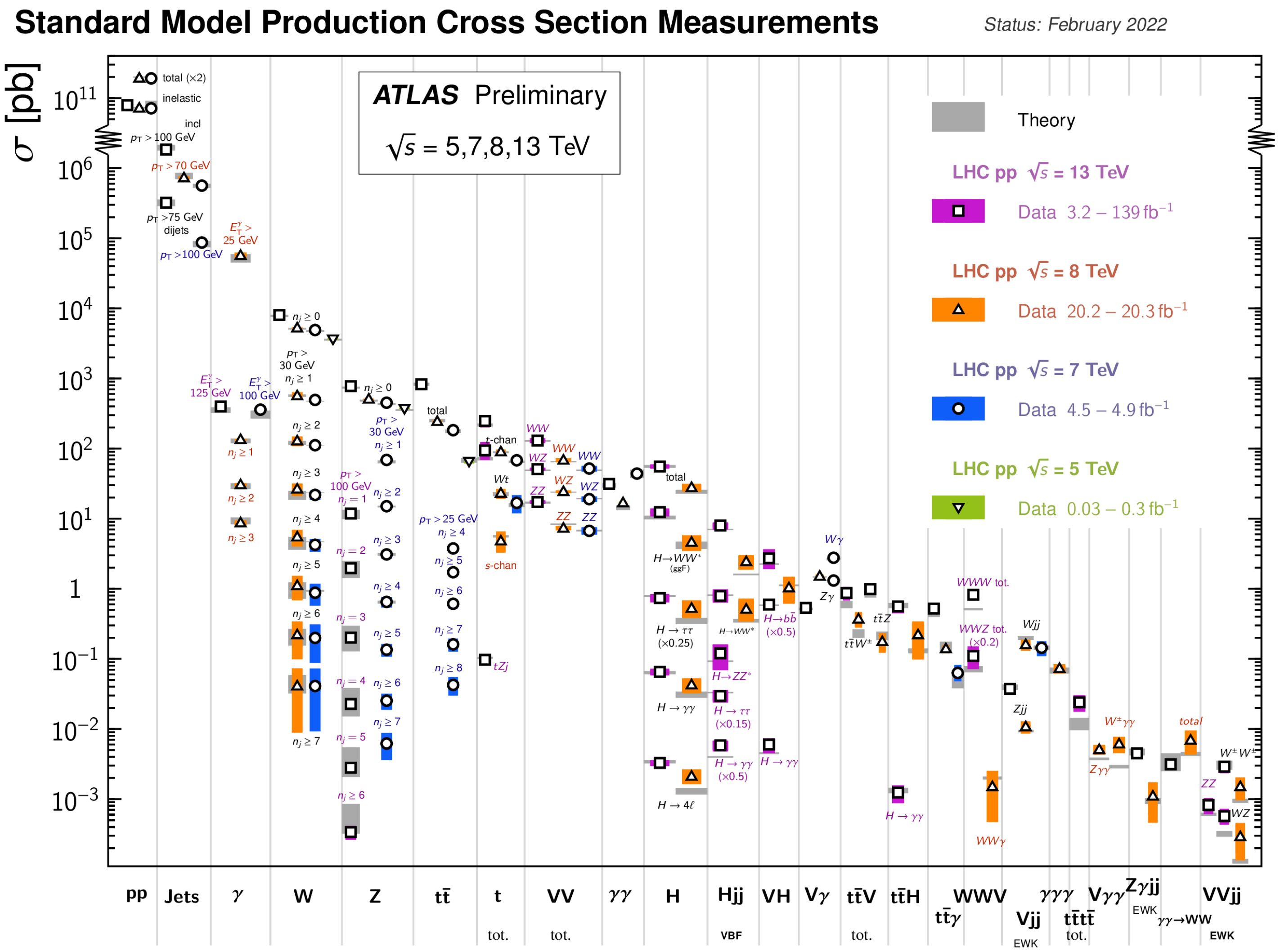
Precision physics at hadron colliders

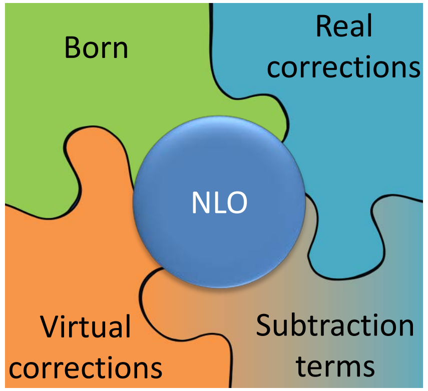
- Precision tests of the Standard Model
  - Measurements of masses and couplings
- Interplay of calculations and measurements
  - Accuracy on many cross sections now ≈(1..5)%
- Ultimate precision frontier at hadron colliders: 1%
  - Require theory predictions accurate at this level



State-of-the-art

- Precise predictions: perturbation theory expansion of observables
- Experimental measurements: fiducial cross sections
  - theory predictions account for experimental cuts and definition of final state
- First NLO QCD predictions in 1980's

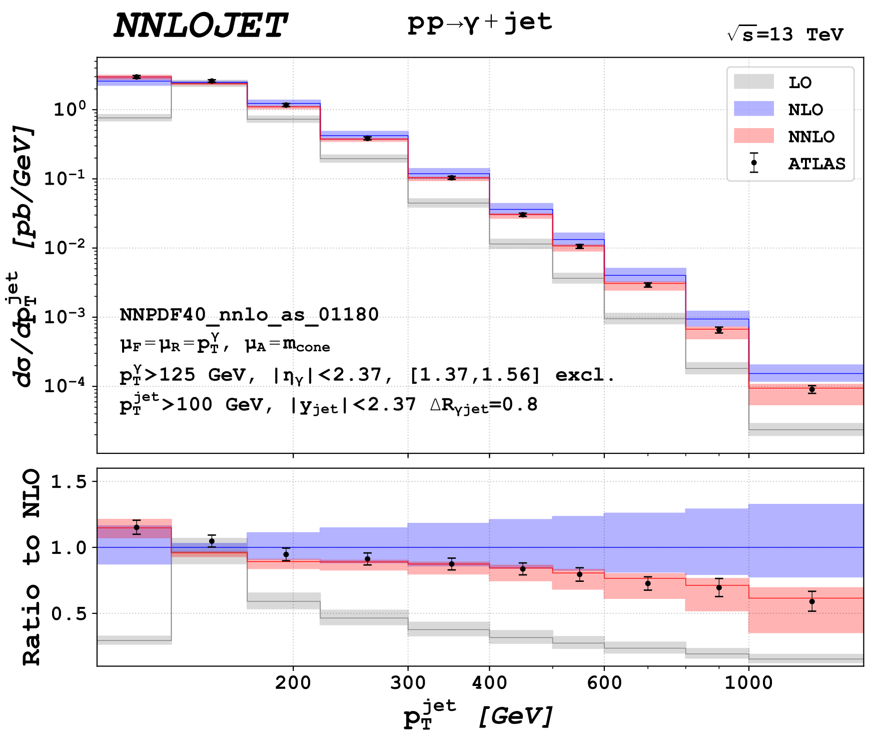
  [K.Ellis, D.Ross, A.Terrano; Z.Kunszt, D.Soper; W.Giele, N.Glover, D.Kosower]
- Automated tools for LO and NLO QCD and electroweak (2010's)
  - infrastructure from event generator programs
    - HERWIG, PYTHIA, SHERPA, aMC@NLO
  - standard interface to one-loop amplitude providers
    - BlackHat, GoSam, Recola, OpenLoops, NJet, MadLoop, CutTools



- Combined with parton shower
  - full event properties with NLO accuracy on differential cross sections

State-of-the-art

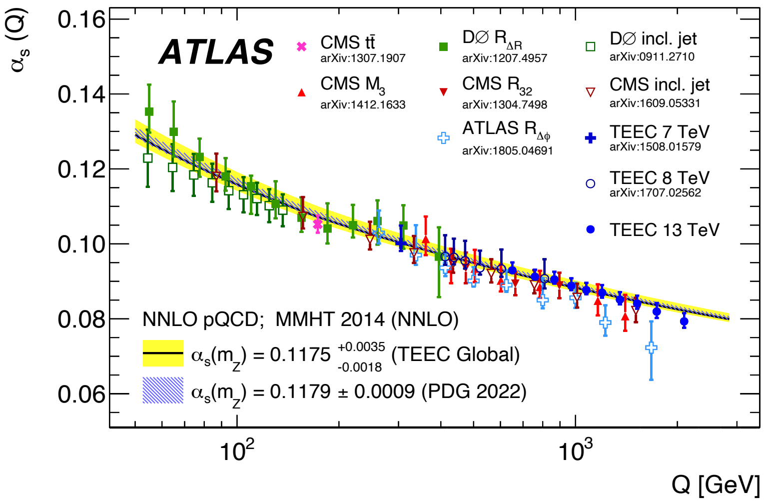
- NNLO QCD predictions for 2 ⟶ 2 processes (NNLO revolution, 2015 ⟶)
  - accomplished during past 10 years on case-by-case basis
  - as parton-level event generators (full final state information)
  - computationally expensive
  - current frontier at NNLO: 2 ⟶ 3

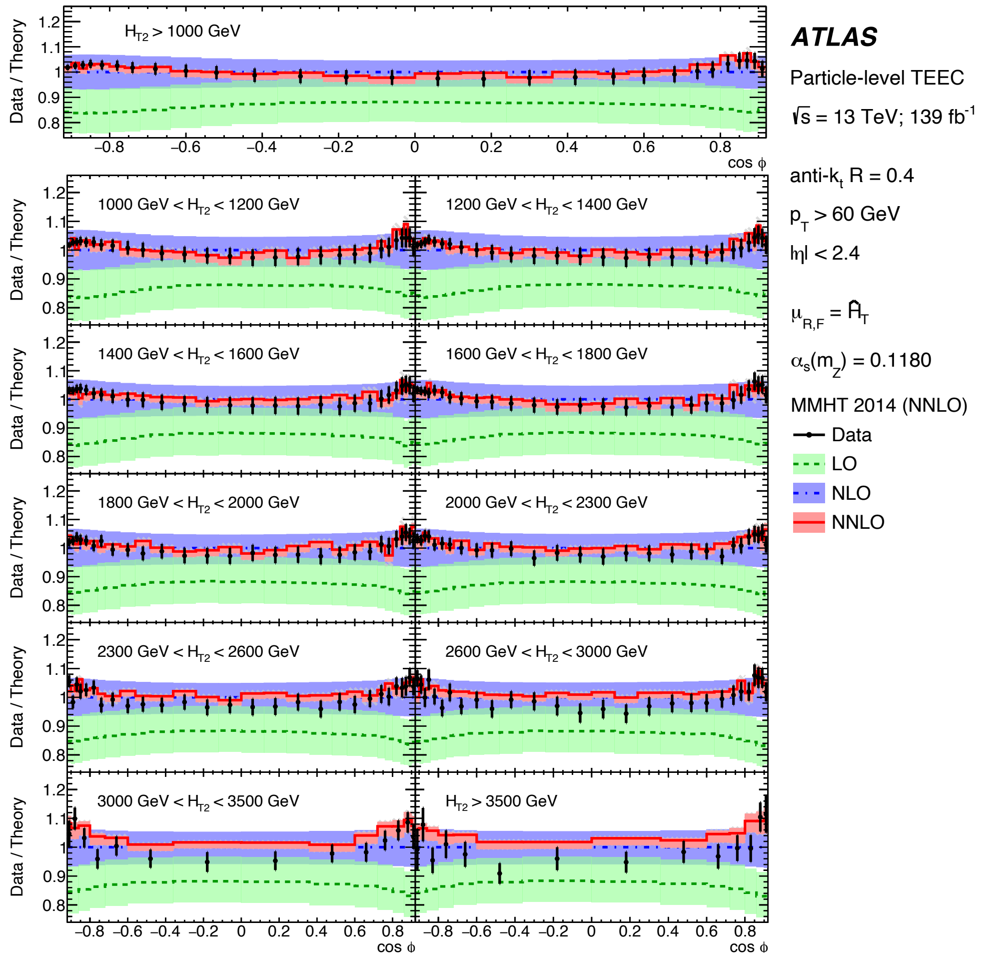


- Typical size of corrections and uncertainty
  - NLO corrections: 10..100%, uncertainty: 10..30%
  - NNLO corrections: 2..15%, uncertainty: 3..8%
  - expect N3LO to yield uncertainty at level of 1%.

State-of-the-art

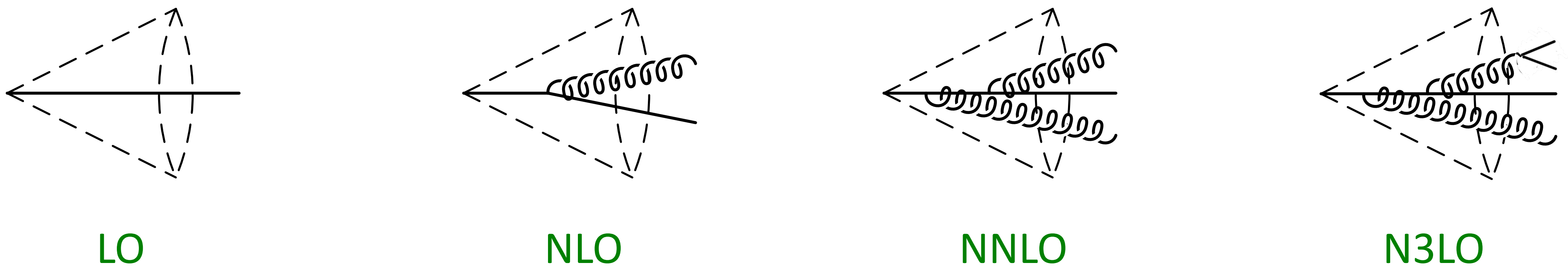
- Precision phenomenology with NNLO QCD predictions
  - 3-jet production and related event shapes at hadron colliders [M.Czakon, A.Mitov, R.Poncelet]
  - Enabled ATLAS α_s determination based on energy-energy correlation functions





Fixed-order perturbation theory

- One extra parton per order in perturbation series
- Partons are combined into jets using same algorithm as in experiment



  - No algorithm dependence at leading order
  - Theoretical description more accurate with increasing order
- Parton shower: multiple emissions, approximate description

Ingredients to fixed order calculations

- Matrix elements with extra real (R) or virtual (V) partons

	Matrix elements	Parton evolution
LO	Born	1-loop
NLO	R, V	2-loop
NNLO	RR, RV, VV	3-loop
N3LO	RRR, RRV, RVV, VVV	4-loop

- Infrared singularities in all R-type and V-type subprocesses
  - sum of all subprocesses finite
  - require procedure to arrange IR cancellations between subprocesses
- Incoming hadrons: parton distributions
  - mass factorization of initial-state radiation and parton evolution

Ingredients to fixed order calculations

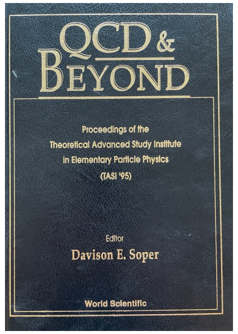
- Different final state multiplicity for real and virtual corrections
  - R: n+1 particles; V: n particles
  - application of event selection, fiducial cuts: evaluate separately
- Upcycling of lower-order calculations
  - only purely virtual correction (V, VV, VVV, ...) genuinely new
  - real radiation corrections from higher-multiplicity calculations at lower order
  - e.g. Higgs boson production: NNLO RV contribution = NLO V contribution to H+jet
  - stability: use analytic one-loop amplitudes if available [Z.Bern, L.J.Dixon, D.Kosower]

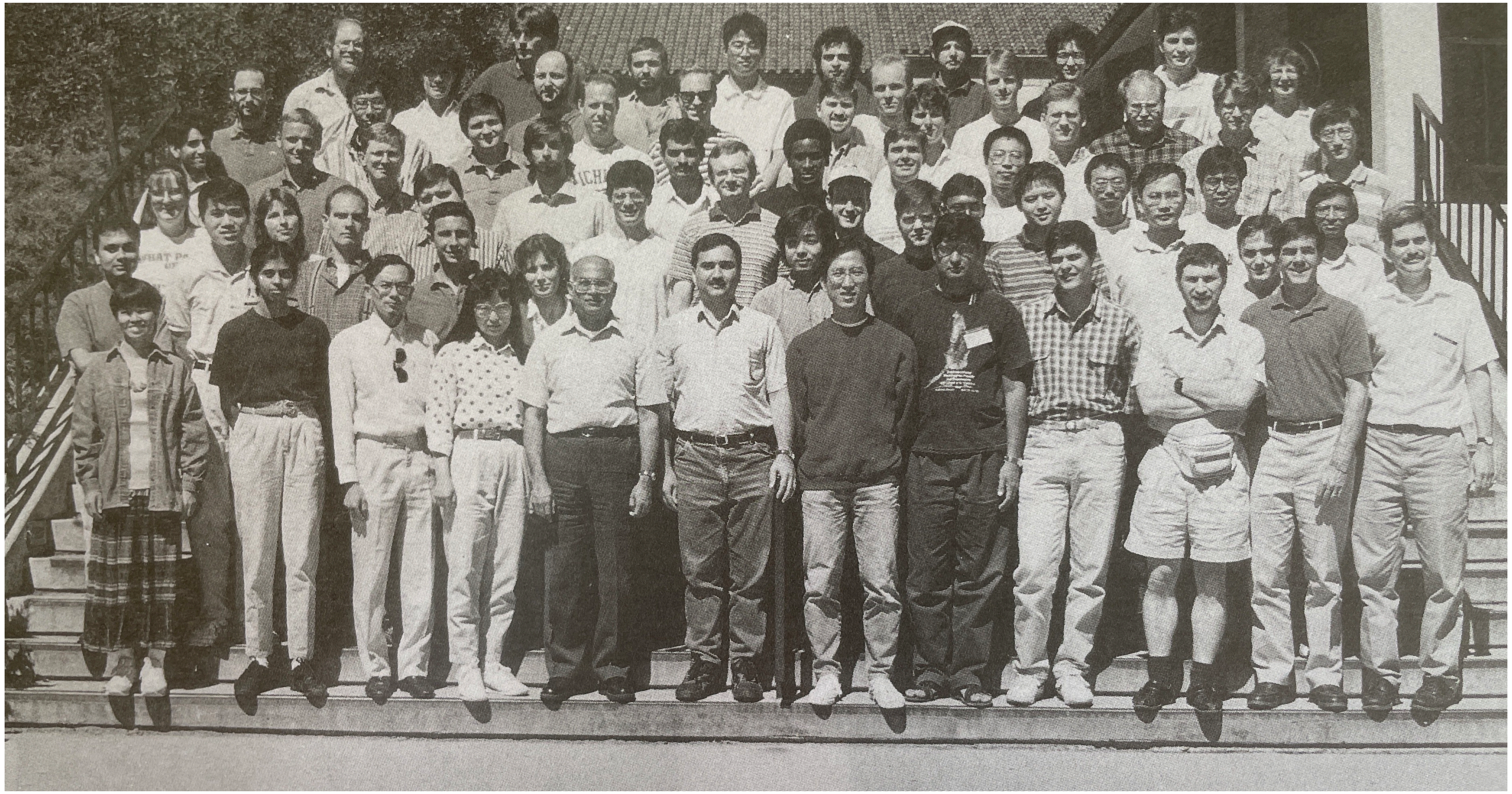
- Cancellation of infrared singularities between subprocesses
  - must evaluate integrals of type [Z.Kunszt, D.Soper: 1995 TASI lectures]

$$\mathcal{I} = \lim_{\epsilon \to 0} \left[\int_0^1 \frac{dx}{x} x^{\epsilon} F(x) - \frac{1}{\epsilon} F(0) \right]$$

TASI '95: QCD and beyond

Boulder (CO), June 1995





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Methods

$$\mathcal{I} = \lim_{\epsilon\to 0}\left[\int_0^1 \frac{dx}{x} x^\epsilon F(x) - \frac{1}{\epsilon}F(0)\right]$$

- Subtraction
  - subtract singular (soft and/or collinear behavior) from R, integrate and add back

$$\mathcal{I} = \lim_{\epsilon\to 0}\left[\int_0^1 \frac{dx}{x} x^\epsilon \left(F(x) - F(0)\right) + F(0)\int_0^1 \frac{dx}{x} x^\epsilon - \frac{1}{\epsilon}F(0)\right]$$

  - many variants at NLO and NNLO: dipole, FKS, antenna, residue, sector-improved,.....
    [S.Catani, M.Seymour; S.Frixione, Z.Kunszt, A.Signer; A.Gehrmann-De Ridder, N.Glover, TG; M.Czakon; F.Caola, K.Melnikov, R.Röntsch; V.del Duca, C.Duhr, A.Kardos, Z.Trocsanyi, G.Somogyi; G.Bertolotti, L.Magnea, G.Pelliccioli, A.Ratti, C.Signorile-Signorile, P.Torrielli, S.Uccirati]
- Slicing
  - cut off singular region from phase space integral, add integrated below-cut contribution

$$\mathcal{I} \approx \lim_{\epsilon\to 0}\left[\int_\delta^1 \frac{dx}{x} x^\epsilon F(x) + F(0)\int_0^\delta \frac{dx}{x} x^\epsilon - \frac{1}{\epsilon}F(0)\right] = \int_\delta^1 \frac{dx}{x} x^\epsilon F(x) + F(0)\ln\delta$$

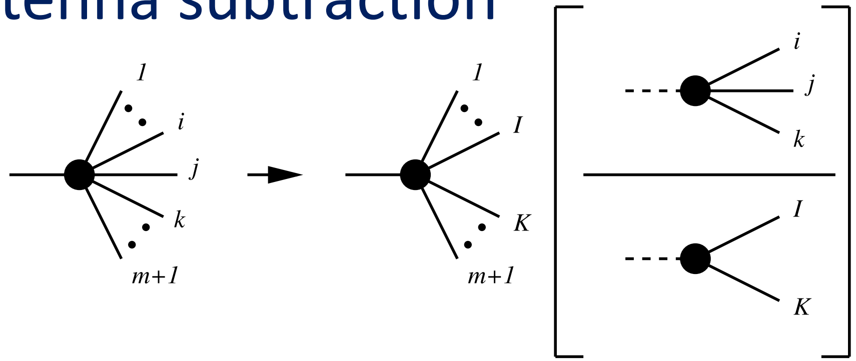
  - variants up to N3LO, depending on slicing variable: q_T, N-jettiness
    [S.Catani, M.Grazzini; R.Boughezal, X.Liu, F.Petriello; J.Gaunt, M.Stahlhofen, F.Tackmann, J.Walsh]

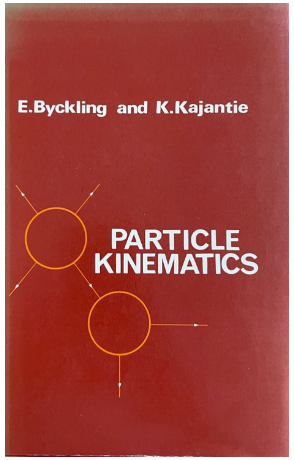
NNLO subtraction

- Structure of NNLO cross section

$$\begin{aligned}\mathrm{d}\sigma_{\mathrm{NNLO}} = &\int_{n+2} \left(\mathrm{d}\sigma^{RR} - \mathrm{d}\sigma^{S}\right) \\ &+ \int_{n+1} \left(\mathrm{d}\sigma^{RV} - \mathrm{d}\sigma^{T} + \mathrm{d}\sigma^{MF,1}\right) \\ &+ \int_{n} \left(\mathrm{d}\sigma^{VV} + \mathrm{d}\sigma^{MF,2} + \int_2 \mathrm{d}\sigma^{S} + \int_1 \mathrm{d}\sigma^{T}\right)\end{aligned}$$

- Each line finite and free of poles → numerical implementation
- For example: using antenna subtraction



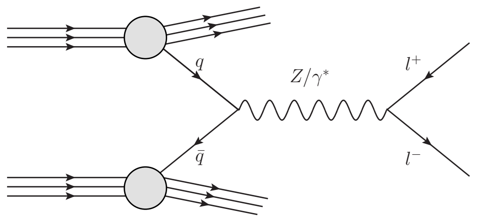


NNLOJET code

- NNLO parton level event generator
  - Based on antenna subtraction
- Provides infrastructure
  - Process management
  - Phase space, histogram routines
  - Validation and testing
  - Parallel computing (MPI) support for warm-up and production
  - ApplGrid/fastNLO interfaces in development
- Processes implemented at NNLO
  - Z+(0,1)jet, γ+1 jet, H+(0,1)jet, W+(0,1)jet, H+2jet (VBF)
  - DIS-2j, LHC-2j
  - Typical runtimes: 60’000-250’000 core-hours

NNLOJET project:
X. Chen, J. Cruz-Martinez, J, Currie, R. Gauld, A. Gehrmann-De Ridder, E.W.N. Glover, M. Höfer, A. Huss, I. Majer, M. Marcoli, J. Mo, T. Morgan, J. Niehues, J. Pires, C.Preuss, A. Rodriguez-Gracia, R. Schürmann, G. Stagnitto, D. Walker, J. Whitehead, TG

Application: Drell-Yan process



- Drell-Yan lepton pair (neutral-current or charged-current) production
  - Benchmark observable: multi-differential measurements
  - Precision measurements of EW parameters and parton distributions
- Standard Model theory well understood
  - NLO EW [C.Carloni Calame, G.Motagna, A.Nicrosini, A.Vicini; S.Dittmaier, M.Huber]
  - NNLO QCD and NNLO QCD+EW (+ total cross section to N3LO QCD) [C.Anastasiou, L.J.Dixon, K.Melnikov, F.Petriello; S.Catani, L.Cieri, G.Ferrera, D.de Florian, M.Grazzini; C.Duhr, F.Dulat, B.Mistlberger; R.Bonciani, L.Buonocore, M.Grazzini, S.Kallweit, N.Rana, F.Tramontano, A.Vicini; F.Buccioni, F.Caola, H.Chawdhry, F.Devoto, M.Heller, A.von Manteuffel, K.Melnikov, R.Röntsch, C.Signorile-Signorile]
  - transverse momentum resummation to N3LL QCD [W.Bizon, P.F.Monni, E.Re, L.Rottoli, P.Torrielli]
- Precision Tools: FEWZ, DYNNLO, DYturbo, POWHEG, ...

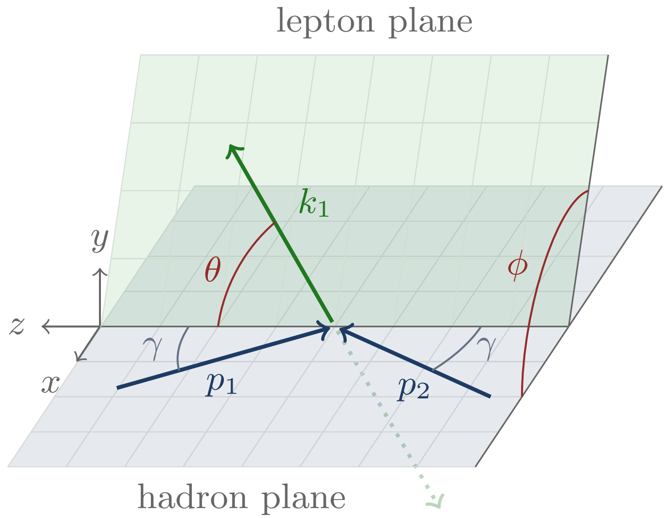
Triple-differential Drell-Yan cross section

- Lepton pair production: EW precision observable

$$\frac{d^3\sigma}{dm_{ll}dy_{ll}d\cos\theta^*} = \frac{\pi\alpha^2}{3m_{ll}s}\sum_q P_q(\cos\theta^*)\left[f_q(x_1,Q^2)f_{\bar{q}}(x_2,Q^2) + (q \leftrightarrow \bar{q})\right]$$

- ATLAS 8 TeV measurement [1710.05167]

Observable	Central-Central	Central-Forward
m_{ll} [GeV]	[46,66,80,91,102,116,150,200]	[66,80,91,102,116,150]
$\|y_{ll}\|$	[0,0.2,0.4,0.6,0.8,1,1.2, 1.4,1.6,1.8,2,2.2,2.4]	[1.2,1.6,2,2.4,2.8,3.6]
$\cos\theta^*$	[-1,-0.7,-0.4,0,0.4,0.7,1]	[-1,-0.7,-0.4,0,0.4,0.7,1]
Total Bin Count:	504	150



Triple-differential Drell-Yan cross section

- Measured with fiducial event selection cuts (on single leptons)

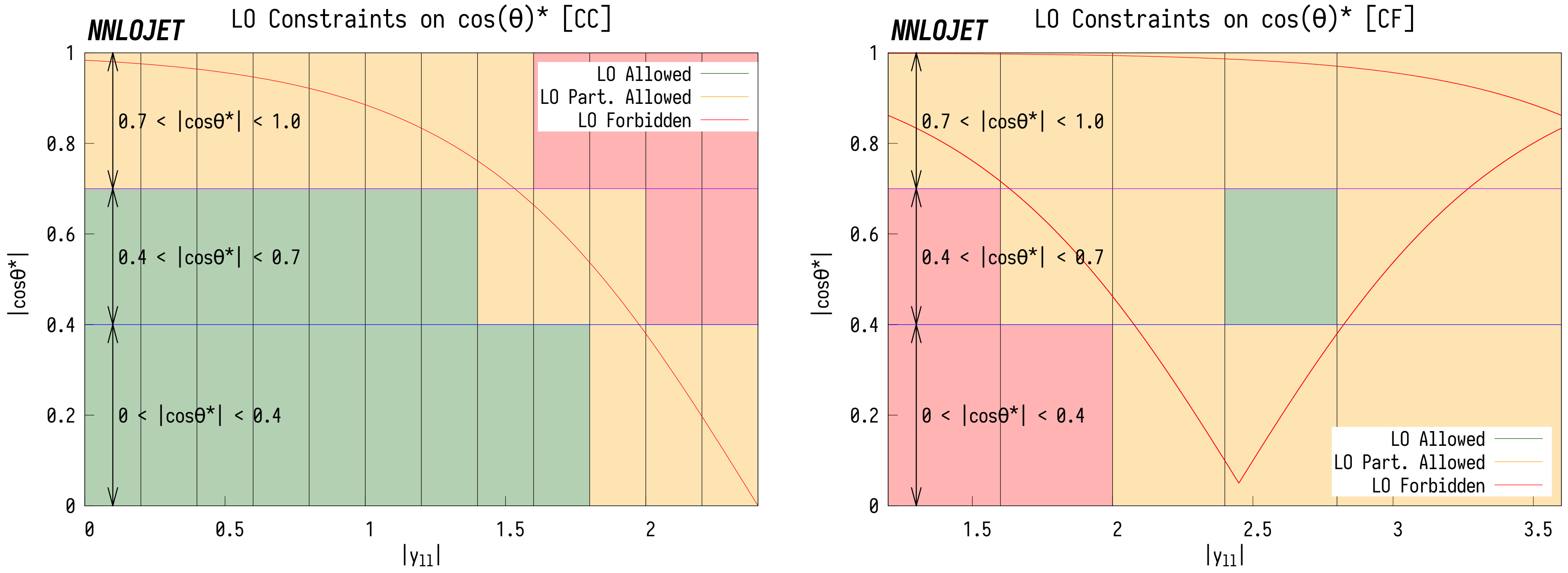
Central-Central	Central-Forward	
$p_T^l > 20$ GeV	$p_{T,F}^l > 20$ GeV	$p_{T,C}^l > 25$ GeV
$\|y^l\| < 2.4$	$2.5 < \|y_F^l\| < 4.9$	$\|y_C^l\| < 2.4$
$46 \text{ GeV} < m_{ll} < 200 \text{ GeV}$	$66 \text{ GeV} < m_{ll} < 150 \text{ GeV}$	

- Fiducial cuts influence acceptances in triple-differential bins

Triple-differential Drell-Yan cross section

- Leading order: fiducial cuts intersect bin definitions

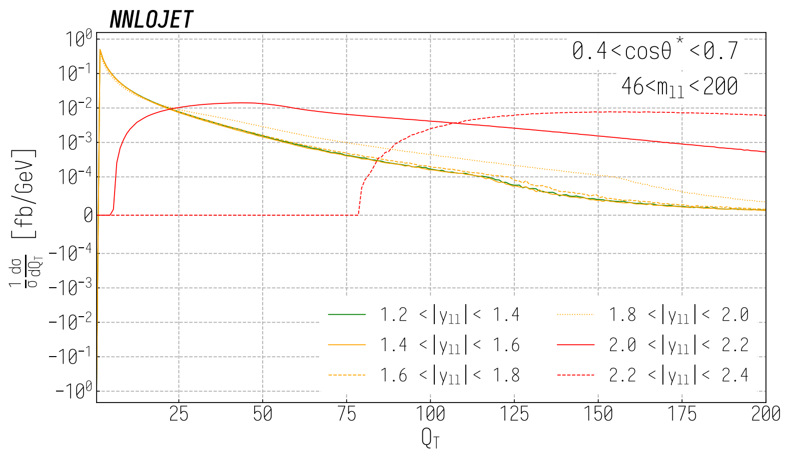
  [A.Gehrmann-De Ridder, E.W.N.Glover, A.Huss, C.Preuss, D.Walker, TG]

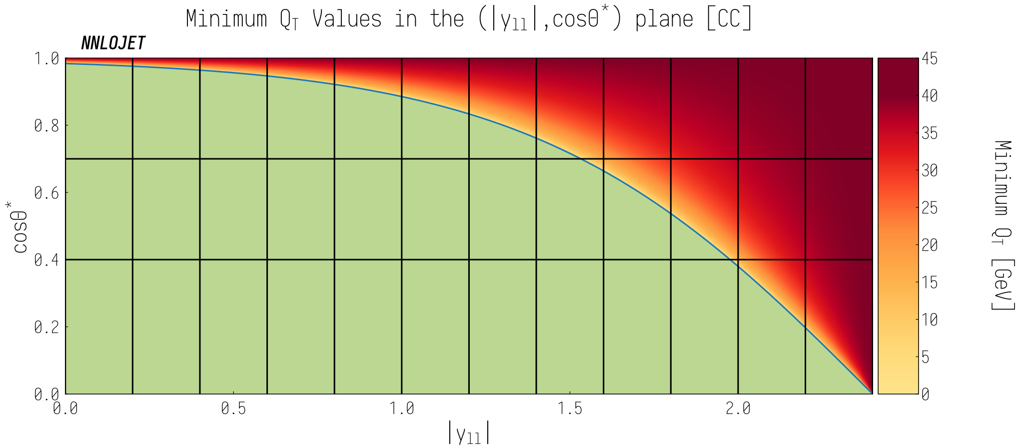


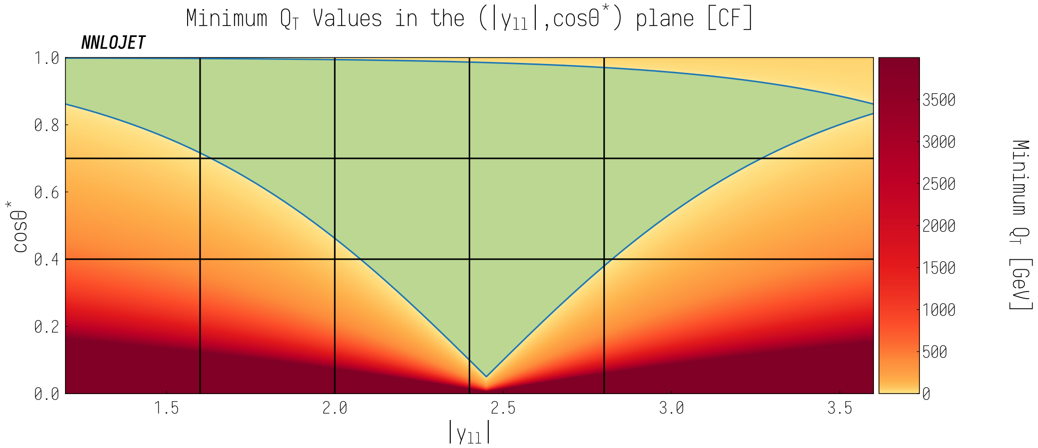
Triple-differential Drell-Yan cross section

- Leading-order forbidden bins
  - require finite Q_T of lepton pair
  - shown here: symmetric lepton pair

→ prediction starts only at NLO
  - lower accuracy
  - potential perturbative instabilities



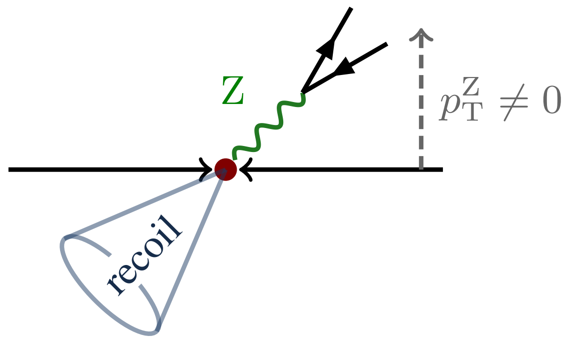




Triple-differential Drell-Yan cross section

Forbidden bins at leading order

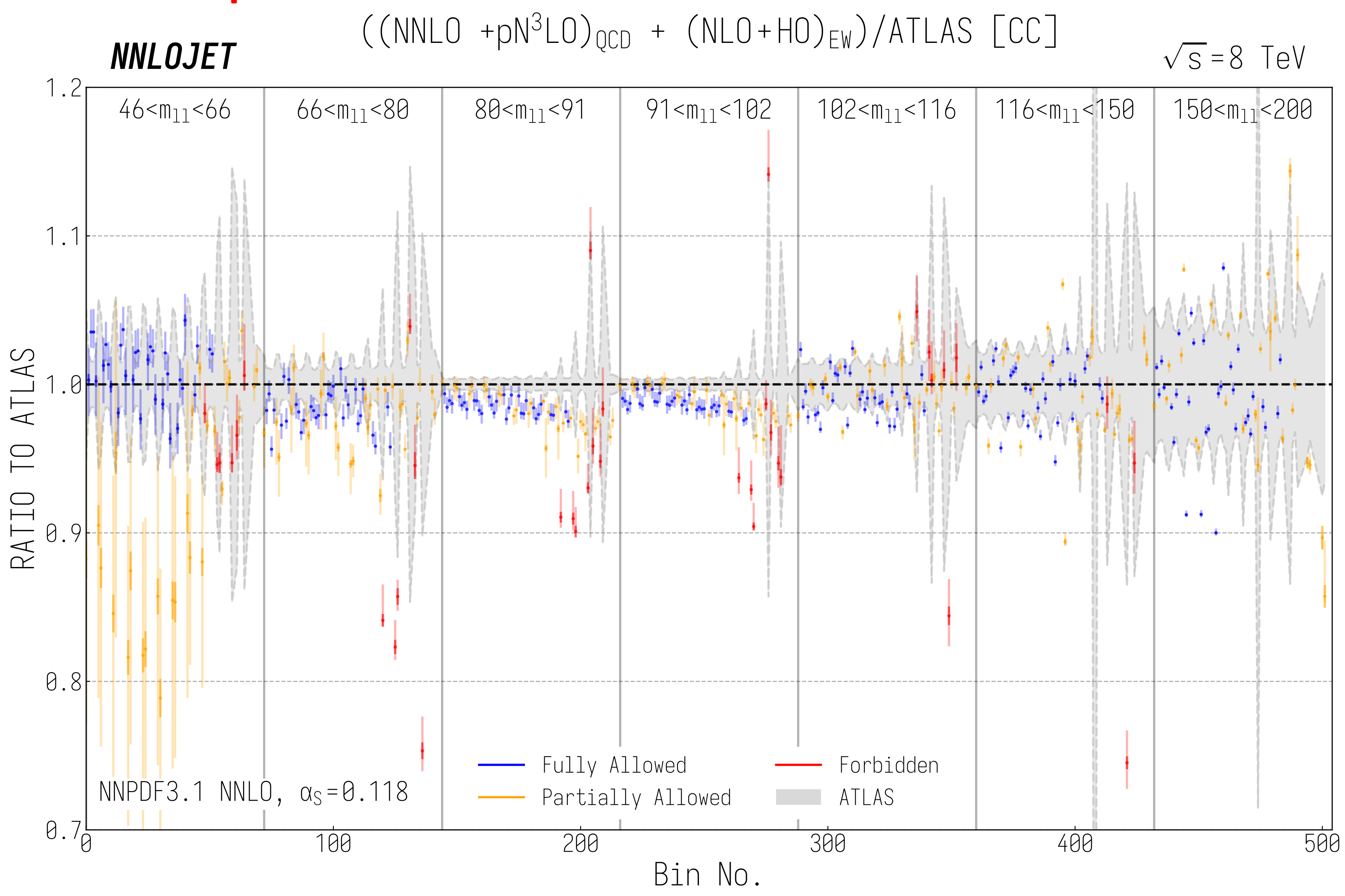
- large theory uncertainty, poor agreement with data
- $O(\alpha_s^3)$ corrections (Drell-Yan N^3LO) obtained from V+jet at NNLO
  [R.Boughezal, J.Campbell, K.Ellis, C.Focke, W.Giele, X.Liu, F.Petriello; MCFM: T.Neumann, J.Campbell; NNLOJET: A.Gehrmann-De Ridder, N.Glover, A.Huss, T.Morgan, D.Walker, TG]
  - use NNLOJET implementation
  - replace jet requirement by (small) Q_T cut
  - numerical convergence at small Q_T challenging

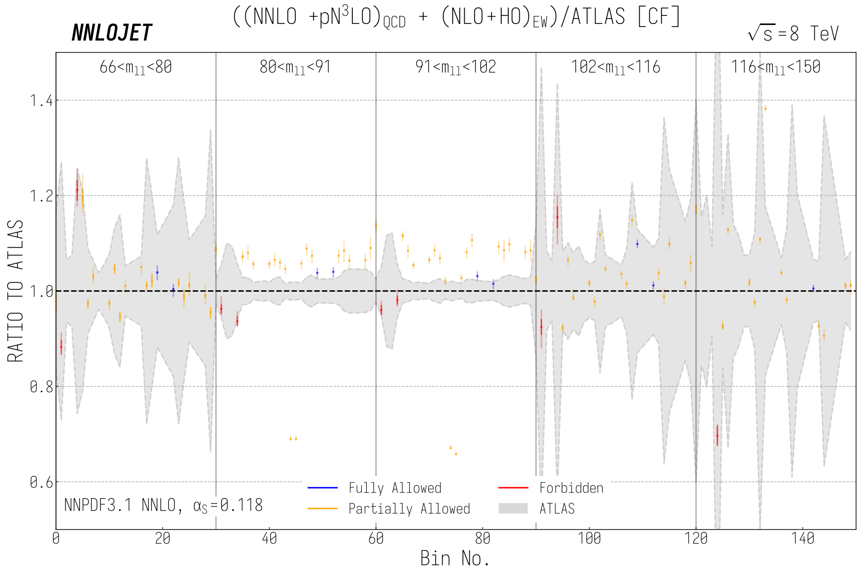


State-of-the-art theory prediction

- QCD NNLO (α_s^2) plus N3LO (α_s^3) in LO-forbidden bins
- combined with (NLO+HO) EW corrections [C.Carloni Calame, G.Motagna, A.Nicrosini, A.Vicini]

Triple-differential Drell-Yan cross section





Future applications

- measurement of $\sin^2\Theta_W$
- determination of parton distributions

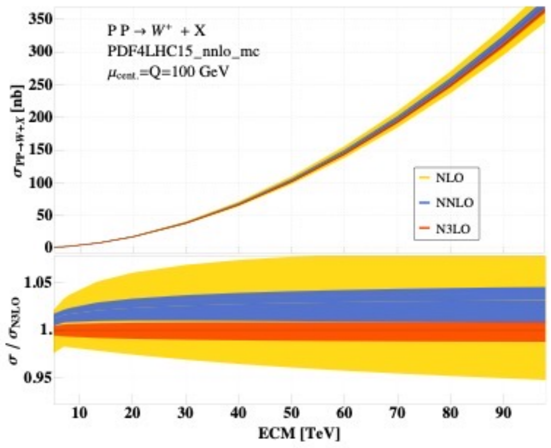
Towards N3LO for fiducial cross sections

Inclusive coefficient functions (total cross section) at N3LO

- computed analytically
- three-loop form factors (VVV)
- inclusive phase space up to triple emission (RRR,RRV,RVV)
- 100s of loop and phase-space master integrals

Results

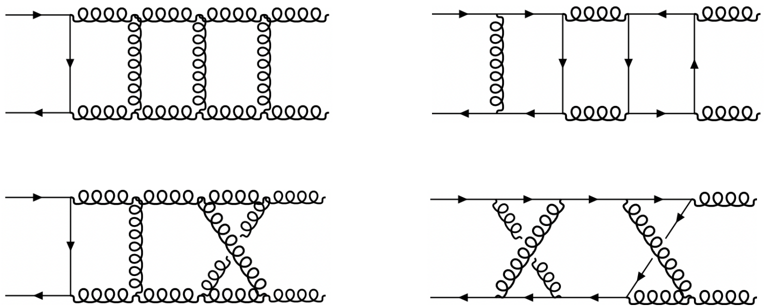
- Deep inelastic structure functions [S.Moch, J.Vermaseren, A.Vogt]
- Higgs boson production [C.Anastasiou, C.Duhr, F.Dulat, F.Herzog, B.Mistlberger]
- Higgs boson rapidity distribution [B.Mistlberger]
- Drell-Yan production: $\gamma^*/Z^0,W^\pm$ [C.Duhr, F.Dulat, B.Mistlberger]
- associated VH production [n3loxs: J.Baglio, C.Duhr, B.Mistlberger, R.Szafron]



Towards N3LO for fiducial cross sections

Three-loop amplitudes for 2 $\rightarrow$ 2 processes (VVV)

- algebraic complexity of integral reduction, computation of master integrals
- recent innovations
  - finite-field methods [A.von Manteuffel, R.Schabinger; T.Peraro]
  - canonical integral basis [J.Henn]
  - minimal tensor decomposition [T.Peraro, L.Tancredi]
- first results
  - four-parton amplitudes
    [F.Caola, A.Chakraborty, G.Gambuti, A.von Manteuffel, L.Tancredi]
  - parton-photon amplitudes
    [P.Bargiela, F.Caola, A.Chakraborty, G.Gambuti, A.von Manteuffel, L.Tancredi]
  - V+3-parton amplitudes (planar)
    [P.Jakubcik, C.Mella, N.Syrrakos, L.Tancredi, TG]



Towards N3LO for fiducial cross sections

Infrared singularity structure of real radiation understood

- RRR: four-parton collinear factors [V.del Duca, C.Duhr, R.Haindl, A.Lazopoulos, M.Michel]
- RRR: triple-soft current [S.Catani, L.Cieri, D.Colferai, F.Coradeschi, A.Torrini; V.del Duca, C.Duhr, R.Haindl, Z.Liu]
- RRV: three-parton collinear factors at one loop [S.Catani, D.de Florian, G.Rodrigo; M.Czakon, S.Sapeta]
- RRV: one-loop double-soft current [S.Catani, L.Cieri; Y.Zhu; M.Czakon, F.Eschment, T.Schellenberger]
- RVV: simple collinear factors at two loops [C.Duhr, M.Jacquier, TG]
- RVV: two-loop soft current [Y.Li, H.X.Zhu; C.Duhr, TG; L.Dixon, E.Herrmann, K.Yan, H.X.Zhu]

Require scheme for infrared cancellations

Towards N3LO for fiducial cross sections

Infrared cancellations: challenges

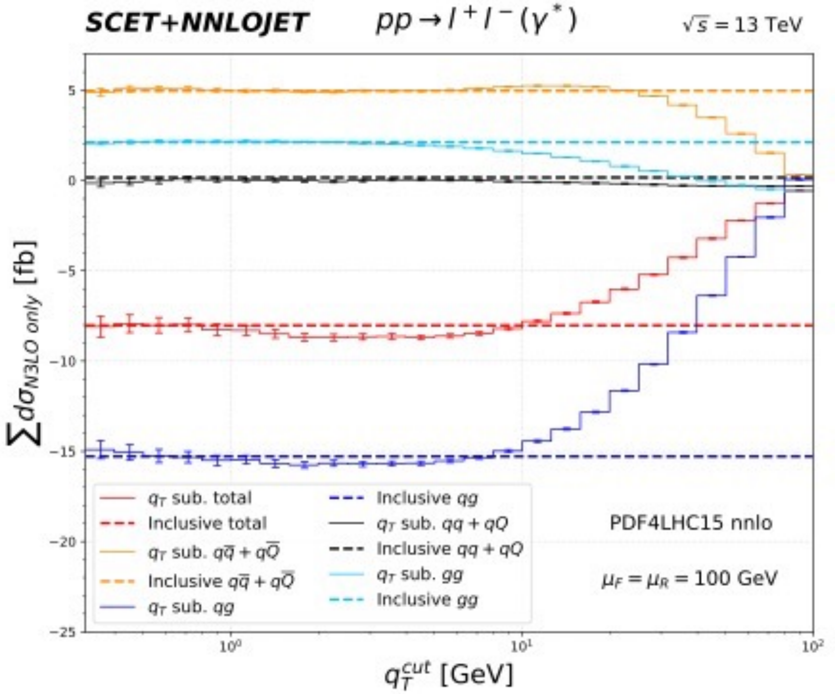
- subtraction $$\mathcal{I} = \lim_{\epsilon\to 0}\left[\int_0^1 \frac{dx}{x} x^\epsilon \left(F(x) - F(0)\right) + F(0)\int_0^1 \frac{dx}{x} x^\epsilon - \frac{1}{\epsilon}F(0)\right]$$
  - construction of subtraction term (completeness, overcompensation)
  - integration of building blocks (analytical or numerical)
- slicing $$\mathcal{I} \approx \lim_{\epsilon\to 0}\left[\int_\delta^1 \frac{dx}{x} x^\epsilon F(x) + F(0)\int_0^\delta \frac{dx}{x} x^\epsilon - \frac{1}{\epsilon}F(0)\right]$$
  - analytic computation of below-cut contribution
  - numerical importance of power-suppressed terms, value of slicing parameter

N3LO for Drell-Yan observables

Slicing parameter: transverse momentum (q_T slicing) [S.Catani, M.Grazzini]

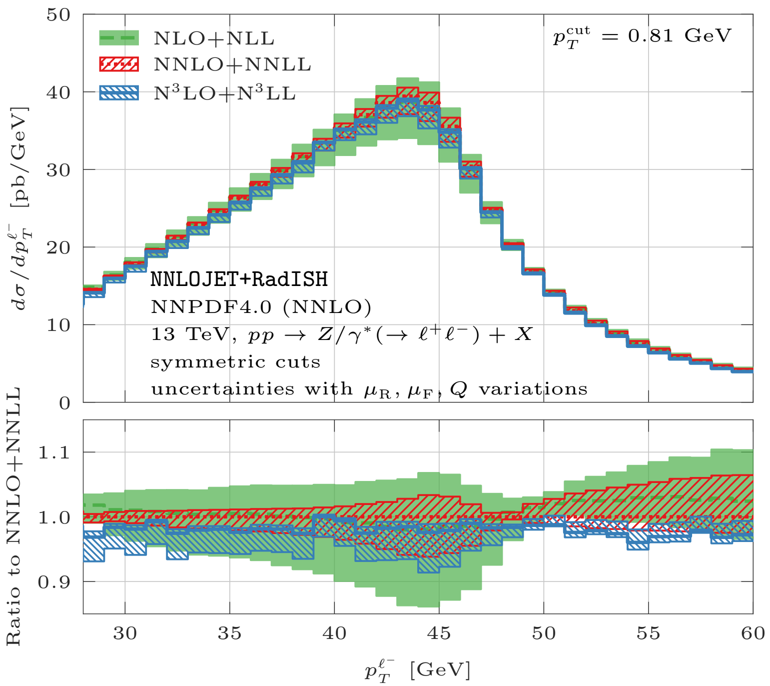
$$\frac{d\sigma_X^{N3LO}}{dO} = \mathcal{H}_{N3LO} \otimes \frac{d\sigma_X^{LO}}{dO} + \left[\int_{q_{T,X}} \frac{d\sigma_{X+j}^{NNLO}}{dO} - \frac{d\sigma_{X,CT}^{NNLO}}{dO}(q_T) \right]$$

- below-cut contribution from expansion of N3LL q_T resummation to $O(\alpha_s^3)$ [W.Bizon, P.Monni, E.Re, P.Torrielli; S.Camrada, L.Cieri, G.Ferrera;T.Becher, T.Neumann; W.L.Ju, M.Schönherr]
- ingredients: three-loop soft and beam functions [Y.Li, H.X.Zhu; M.Ebert, B.Mistlberger, G.Vita; M.X.Luo, H.X.Zhu, T.Z.Yang, Y.J.Zhu; D.Baranowski, A.Behring, K.Melnikov, L.Tancredi, C.Wever]
- check: independence on $q_{T,cut}$ slicing parameter
- check: reproduce inclusive coefficient functions (no ingredients or methodology in common!) [X.Chen, E.W.N.Glover, A.Huss, T.Z.Yang, H.X.Zhu, TG]



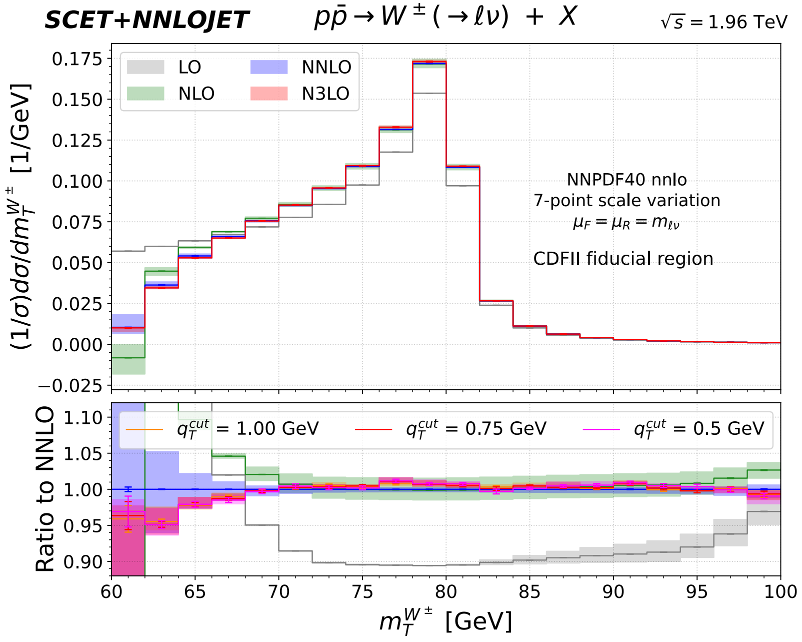
N3LO for Drell-Yan observables

Results: fiducial distributions



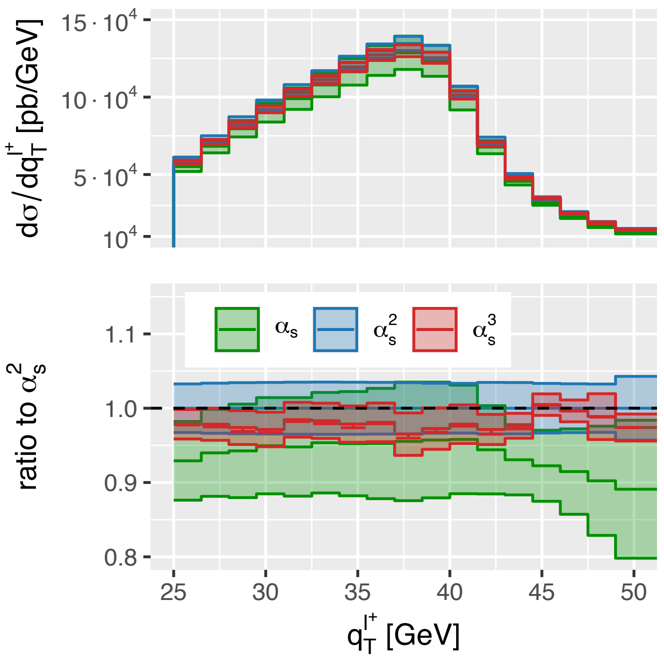
single lepton distribution in NC Drell-Yan, matched to N3LL resummation (RadISH)

[X.Chen, E.W.N.Glover, A.Huss, P.F.Monni, E.Re, L.Rottoli, P.Torrielli, TG]



transverse mass distribution in W boson production (CDF II cuts)

[X.Chen, E.W.N.Glover, A.Huss, T.Z.Yang, H.X.Zhu, TG]



charged lepton distribution in W boson production (ATLAS 5.02 TeV)

[J.Campbell, T.Neumann]

Towards N3LO for fiducial cross sections

Subtraction methods at N3LO: work in progress

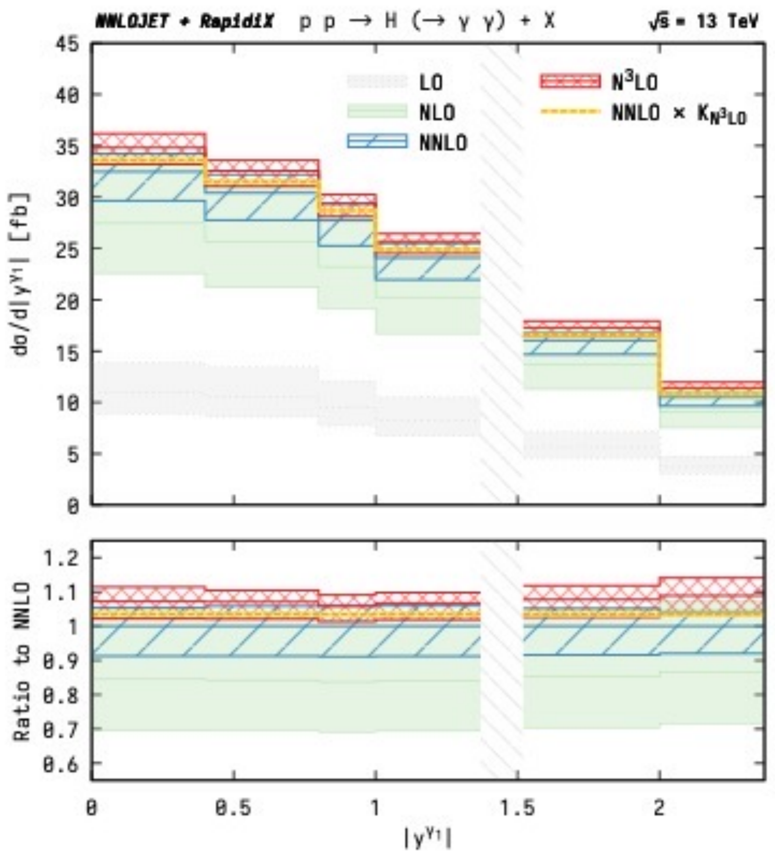
- integrating N3LO antenna functions in all-final kinematics
  [X.Chen, M.Marcoli, P.Jakubcik, G.Stagnitto]

Shortcut for simple processes: Projection to Born

[M.Cacciari, F.Dreyer, A.Karlberg, G.Salam, G.Zanderighi]

$$\frac{d\sigma_X^{N3LO}}{dO} = \frac{d\sigma_{X+j}^{NNLO}}{dO} - \frac{d\sigma_{X+j}^{NNLO}}{dO_B} + \frac{d\sigma_X^{N3LO,\,incl}}{dO_B}$$

- Higgs production in vector boson fusion [F.Dreyer, A.Karlberg]
- Higgs production in gluon fusion, including $H \to \gamma\gamma$
  [X.Chen, N.Glover, A.Huss, B.Mistlberger, A.Pelloni]



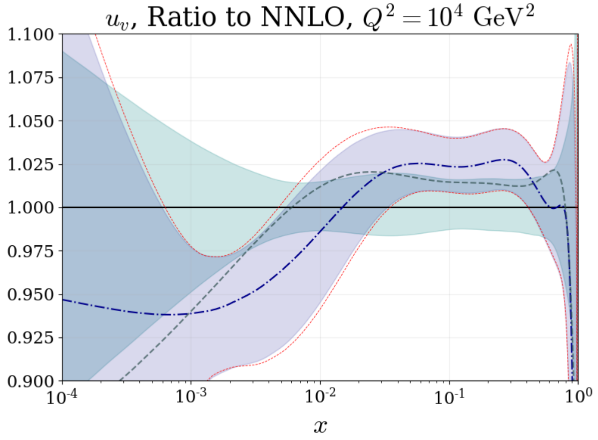
Parton distributions at N3LO

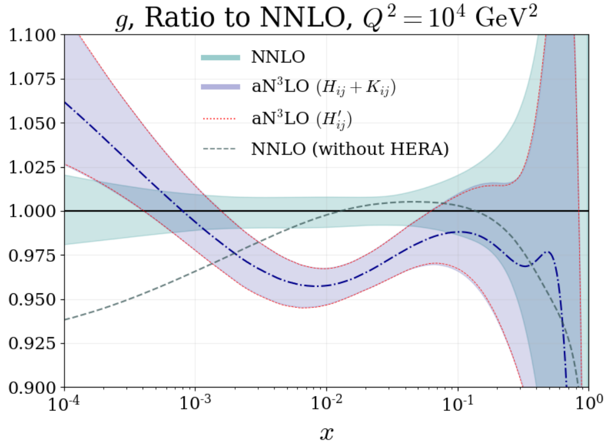
Caveat: current N3LO predictions use NNLO parton distributions

- inherent inconsistency, difficult to quantify

N3LO parton distributions require

- four-loop Altarelli-Parisi splitting functions
  - use four-loop OPE, haunted by ghosts 👻
  - ongoing: lower Mellin moments, specific color and flavor combinations
    [G.Falcioni, F.Herzog, S.Moch, A.Vogt; A.von Manteuffel, V.Sotnikov, T.Z.Yang, TG]
- N3LO coefficient functions for relevant observables
  - structure functions, inclusive cross sections known
    [S.Moch, J.Vermaseren, A.Vogt; C.Duhr, B.Mistlberger]
  - fiducial cross sections in progress





First approximate N3LO parton distribution fits

[MSHT: J.McGowan, T.Cridge, L.Harland-Lang, R.Thorne]

Summary

- LHC embarks on a decade-long program of precision physics
- Ultimate precision challenge for QCD
  - predictions for complex final states at per-cent level accuracy
- Theory ready to face this challenge
  - NNLO predictions becoming the new standard
  - N3LO concepts, techniques and tools developing rapidly
- Stay tuned

Happy 50th anniversary QCD

Image by Freepik