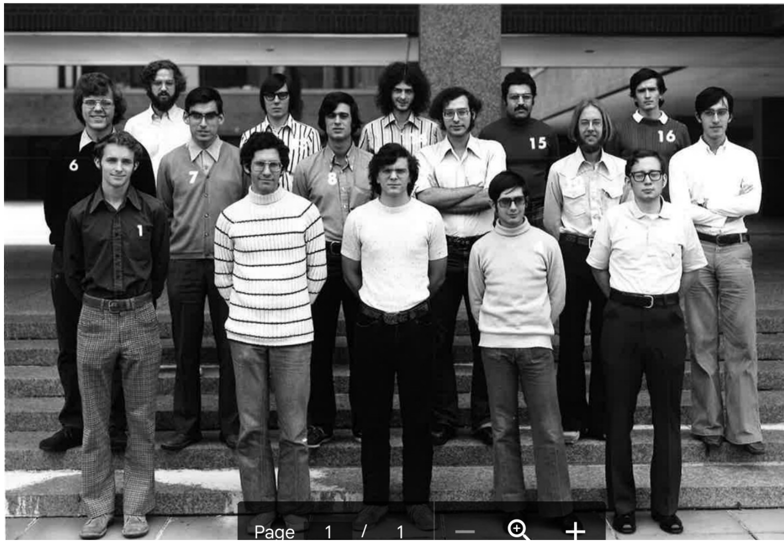
Five amuse-bouches

To celebrate QCD’s 50th birthday

a·muse-bouche:
/əˌmo͞ozˈbo͞oSH/

a small savory item of food served as an appetizer before a meal.

First amuse-bouche: instantons at T $\neq 0$



Princeton, class of '75: me, D. Stein, D. Haldane...

Under David, my thesis followed
Zamalodchikov & Zamalodchikov,
exact S-matrix, 2-index O(N) tensor '78. 28 cit's

Then: instantons@1-loop order. T=0: 't Hooft '76

Gross, RDP, Yaffe '81: T $\neq$ 0: constant fields $A_0 \neq 0$ ($\rightarrow$ holonomous HTL's)

instanton @ 1-loop: fund. & adj. rep's. Result for *all* instanton scale size, ρ

Carvalho '81: $\mu \neq 0$, T = 0: *only* for large ρ.
Nogradi, Papavassiliou, RDP: 2310.?: all ρ.

At large ρ, color E field in instantons Debye screened (J. Collins, unpub'd):

$$\mathcal{W}_{\rm instanton} = \int d^4x \int \frac{d\rho}{\rho^5}\ \exp\left(-\frac{8\pi^2}{g^2(\rho T)} + \#\, g^2 T^2 \rho^2\right)\ldots$$

When does it work?

So what? At $T \neq 0$, perturbation theory fails at $T \sim 100$ *GeV*.

Static sector = QCD_3 → pert thy an expansion not in g^2, but in $\sqrt{g^2}$(Linde'79)

Using resummed Hard Thermal Loop perturbation theory @ NNLO (!)

Haque & Strickland, 2011.06938: N^2L0 HTL valid down to ~ 300 *MeV*

From β-function at 1-loop, $g^2 \sim \#/\log(\rho T)$, so topological susceptibility

$$\chi(T) = \frac{\partial^2}{\partial\theta^2} p(\theta, T) \sim \exp\left(\frac{-8\pi^2}{g^2(T)}\right) \sim \exp\left(\frac{-8\pi^2}{\#/\log(T)}\right) \sim \frac{\kappa}{T^\lambda} + \ldots$$

λ: just from classical action & 1-loop β-fnc! κ: from fluc's at 1-loop order

So what? *Surely* fails at T ~ 100 GeV unless one resums with NNLO HTL!

Lattice!

For a dilute gas of instantons (DGI), $\chi(T) \sim \frac{\kappa}{T^\lambda} + \ldots$

Difficult computations from lattice: λ ~ DGI down to *300 MeV!*

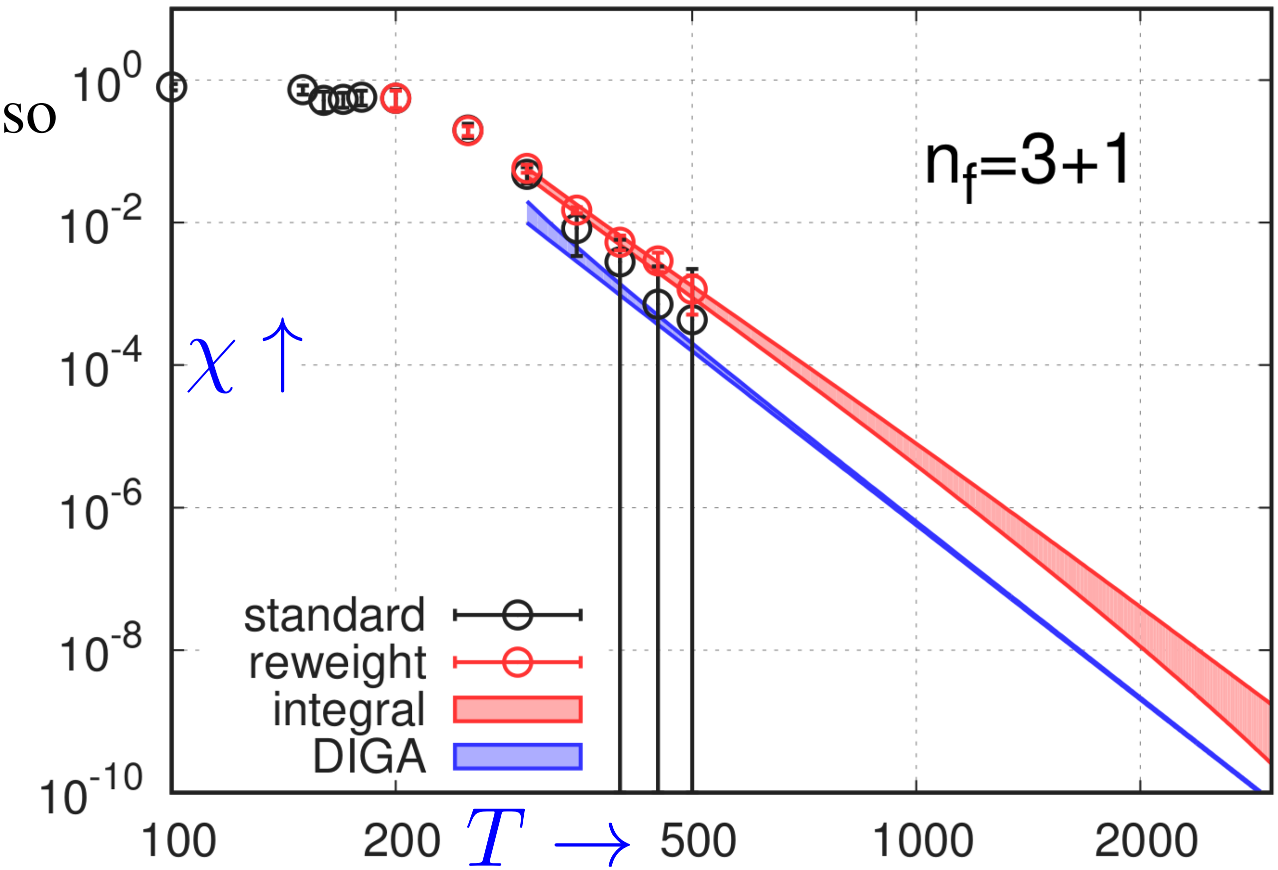
κ: lattice ~ 10 x 1-loop. Need the full 2-loop result to compare! (meh...)

Moral: sometimes “useless” computations need not remain so

Right: Borsanyi+...1606.07494

Petreczky+...1606.03145

...Athenodorou+...2208.08921



Second amuse-bouche: at T = 0, it *ain't* instantons

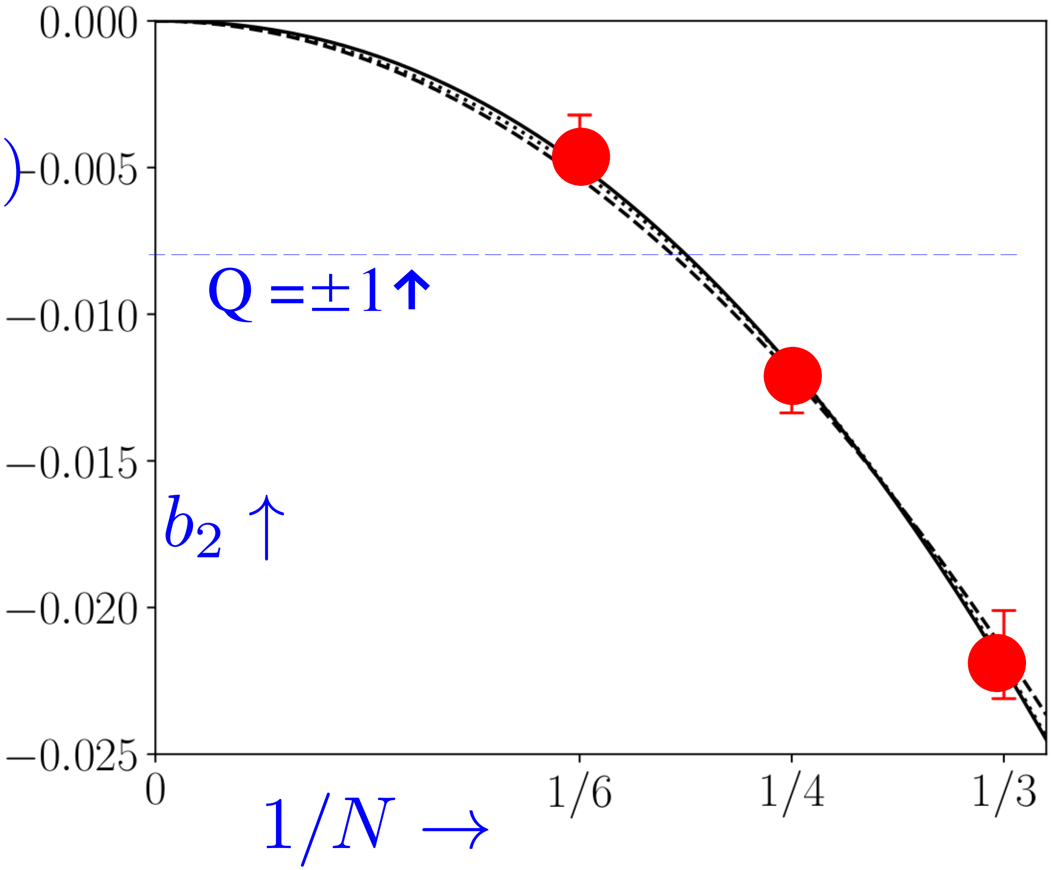
Veneziano & Witten '78: topological effects persist as N → ∞.
But g^2 N held fixed → instantons *vanish* as exp(- # N) = exp(- 8 $\pi^2/(g^2 N)$ N)

What if there are objects with fractional topological charge 1/N?
Old story: 't Hooft '81, van Baal...Gonzalez-Arroyo...Unsal, Poppitz...

Bonanno, Bonati, d'Elia 2012.14000: *pure* SU(N), *no* quarks, N = 3, 4, 6

Compute energy at nonzero θ:

$$E(\theta) - E(0) = \theta^2/2(1 + b_2\theta^2 + \ldots)$$



Dilute gas, Q = ± 1: b_2 = -1/12 ~ -.08

Dilute gas, Q = ± 1/N: b_2 = - .08 $1/N^2$

Lattice: b_2 ~ $-.19/N^2$ →

dense gas of frac'y chgd objects!

Nair & RDP 2206.11284: Z(N) dyons?

Third amuse-bouche: the axial anomaly at nonzero spin

An *old* story: the η', with spin zero, is heavy because of the axial anomaly.

How does the axial anomaly affect mesons with *higher* spin?

For “heterochiral” mesons with nonzero spin, *many* anomalous couplings:

F. Giacosa, A. Koenigstein, & RDP, 1709.07454

Only classified the possible couplings: how *big* are these couplings?

Values for anomalous couplings in a dilute gas of instantons (DGI):

F. Giacosa, Shahriyar Jafarzade, & RDP, 2309.00086

Couplings in DGI are *small*, & decrease as J increases.

In *vacuum, T=0*: yes, instantons don't work, but we can compute; *start* with DGI

Chiral symmetry

Quarks in QCD:

$$\overline{q}(\not{D} + m)q = \overline{q}_L \not{D} q_L + \overline{q}_R \not{D} q_R + m_{qk}(\overline{q}_L q_R + \overline{q}_R q_L)$$

When $m_{qk} = 0$, classically a global symmetry of $SU(3)_L$ x $SU(3)_R$ x $U(1)_A$.

$$q_{\mathrm{L,R}} \longrightarrow \mathrm{e}^{\mp \mathrm{i} \alpha_A / 2} U_{\mathrm{L,R}} \, q_{\mathrm{L,R}}$$

Because of the axial anomaly,

$$\partial^\mu \, \overline{q}^a \gamma_\mu \gamma_5 q^a = 3 g^2 \, \mathrm{tr} \, G_{\mu\nu} \, \widetilde{G}_{\mu\nu} / (16\pi^2)$$

Quantum mechanically the symmetry reduces to $SU(3)_L$ x $SU(3)_R$ x $Z(3)_A$.
$Z(3)_A$ because of the zero modes for each flavor.

Construct effective Lagrangians:
All terms invariant under $SU(3)_L$ x $SU(3)_R$ (+ soft breaking from $m_{qk} \neq 0$)

Most terms are invariant under $U(1)_A$.

Anomalous terms violate $U(1)_A$, and are invariant *only* under $Z(3)_A$.

Scalars, usual linear sigma model

For spin zero, form mesons in the usual way

$$\Phi^{ij} = \overline{q}^{j}_{L}\, q^{i}_{R}\ ,\ \Phi \to e^{i\alpha_A}\, U^{\dagger}_{L}\, \Phi\, U_{R}$$

$J^P = 0^-$: π, K, η, η' , obviously.

Less certainty about $J^P = 0^+$:

$\sigma(600)$, $a_0(980)$ + ...; *or* $a_0(1450) f_0(1370)$, $f_0(1710)$?

Doesn't really matter for us.

Potential terms invariant under $SU(3)_L$ x $SU(3)_R$ x $U(1)_A$:

$$\mathcal{V}^{U(1)} = m^2\, \mathrm{tr}\Phi^\dagger\Phi + \lambda_1(\mathrm{tr}\Phi^\dagger\Phi)^2 + \lambda_2 \mathrm{tr}(\Phi^\dagger\Phi)^2 + \ldots$$

Anomalous couplings

Anomalous terms only invariant under $SU(3)_L$ x $SU(3)_R$ x $Z(3)_A$

The anomalous term of lowest order is ('t Hooft '76 +...)

$$\mathcal{V}^{Z(3)} = \kappa_0 \det(\Phi) + \text{c.c.} \sim \Phi^3 \qquad\qquad \Phi \to e^{2\pi i/3}\Phi$$

$Z(3)_A$ invariant terms of higher order include

$$\kappa_0' \, \text{tr}(\Phi^\dagger \Phi)\det(\Phi) + \kappa_0'' \, (\det \Phi)^2 + \text{c.c.}$$

Zero modes of a single instanton, $Q_{topological} = \pm 1$, generate κ_0 & κ_0', $Z(3)_A$ inv.

The term ~ κ_0'' is $Z(6)_A$ invariant, generated by $Q_{topological} = \pm 2$

RDP & F. Rennecke, 1910.14052; F. Rennecke, 2003.13876

For now just the anomalous terms of lowest mass dimension, ~ κ_0.

Soft breaking of chiral symmetry

Add

$$\mathcal{L}_{mass} = \mathrm{tr} H(\Phi + \Phi^\dagger) \qquad H = \# \begin{pmatrix} m_{up} & 0 & \\ 0 & m_{down} & 0 \\ 0 & 0 & m_{strange} \end{pmatrix}$$

When $\langle \Phi \rangle = \phi_0 \neq 0$,

$$m_\pi^2 \sim m_u + m_d \ , \ m_K^2 \sim m_{u,d} + m_s$$

With the anomaly, π, K, & η light; η´ heavy, GB's eigenstates of $SU(3)_V$

η mainly octet, η´ mainly singlet

N.B.: with*out* the anomaly, GB's eigenstates of flavor, *not* $SU(3)_V$:

Gross, Wilczek & Treiman '78; RDP & Wilczek, '82

$$\pi^0 \sim \overline{u}u \ , \ \eta \sim \overline{d}d \ , \ \eta' \sim \overline{s}s$$

The anomaly makes the η´ heavy, *and* prevents massive isospin violation

Vectors, $J^{PC} = 1^{-+}$.

As γ_μ flips chirality, can only pair LL and RR, n*eutral* under $U(1)_A$:

$$L^{ij}_\mu = \overline{q}^j_{\rm L}\,\gamma_\mu\, q^i_{\rm L}\ ,\ R^{ij}_\mu = \overline{q}^j_{\rm R}\,\gamma_\mu\, q^i_{\rm R}\ ;\ L_\mu \longrightarrow U^\dagger_{\rm L}\, L_\mu\, U_{\rm L}\ ,\ R_\mu \longrightarrow U^\dagger_{\rm R}\, R_\mu\, U_{\rm R}$$

Obvious mixing and mass terms, invariant under $U(1)_A$:

$$\beta\,{\rm tr}(L_\mu \Phi^\dagger \partial_\mu \Phi + R_\mu \Phi \partial_\mu \Phi^\dagger) + m_V^2\,{\rm tr}(L_\mu^2 + R_\mu^2) + \kappa\,{\rm tr} H(L_\mu^2 + R_\mu^2)$$

Anomalous terms start with 3rd order in ∂'s, Wess-Zumino-Novikov-Witten term

$J^P = 1^-$: $V_\mu = L_\mu + R_\mu$, ϱ(770), ω(782), K*(892) & ϕ(1020)

Anomaly does *not* contribute to mass terms, so ϱ, ω, & ϕ are *flavor* eigenstates:

$$\rho_\mu\ ,\ \omega_\mu \sim \bar{l}\gamma_\mu l\ ,\ l = u, d\ ;\ \phi_\mu \sim \overline{s}\gamma_\mu s$$

Higher spin

Classify multiplets according to the *un*broken $SU(3)_L$ x $SU(3)_R$ x $Z(3)_A$.

As usual, form mesons by inserting some Γ, ~ γ's and D's, between q and q-bar.

Heterochiral:

$$\Phi_{\mu\nu\ldots} = \overline{q}_L \overleftrightarrow{D}_\mu \overleftrightarrow{D}_\nu \ldots q_R \ , \ \Phi_{\mu\nu\ldots} \to \mathrm{e}^{i\alpha_A} U_L^\dagger \Phi_{\mu\nu\ldots} U_R$$

$Z(3)_A$ inv.→ anomalous terms: affect mixing of higher spin analogies of η & η'.
And many new terms, new decays...

Homochiral:

$$L_{\mu\nu\lambda\ldots} = \overline{q}_L \gamma_\mu \overleftrightarrow{D}_\nu \overleftrightarrow{D}_\lambda \ldots q_L \ ; \ R_{\mu\nu\lambda\ldots} = \overline{q}_R \gamma_\mu \overleftrightarrow{D}_\nu \overleftrightarrow{D}_\lambda \ldots q_R$$

$$L_{\mu\nu\lambda\ldots} \to U_L^\dagger L_{\mu\nu\lambda\ldots} U_L \ ; \ R_{\mu\nu\lambda\ldots} = U_R^\dagger R_{\mu\nu\lambda\ldots} U_R$$

Invariant under $U(1)_A$, so no anomalous mass terms.
Masses close to eigenstates of flavor, as in the usual quark model.
Anomalous terms ~ Wess-Zumino-Novikov-Witten, ignore.

Generalized anomalous interactions

For a single matrix, the determinant is (i,j,k = $SU(3)_L$; i' j'k' = $SU(3)_R$ indices)

$$\det \Phi = \epsilon^{ijk} \epsilon^{i'j'k'} \Phi^{ii'} \Phi^{jj'} \Phi^{kk'} / 6!$$

As all indices are summed over, invariant under $SU(3)_L$ x $SU(3)_R$,

With three Φ's, invariant under $Z(3)_A$, and not $U(1)_A$.

Since *all* $\Phi_{\mu\nu\ldots}$ transform the same,

$$\Phi_{\mu\nu\ldots} \rightarrow \mathrm{e}^{i\alpha_A} U_L^\dagger \Phi_{\mu\nu\ldots} U_R$$

Consider:

$$\epsilon[\Phi_1 \Phi_2 \Phi_3] = \epsilon^{ijk} \epsilon^{i'j'k'} \Phi_1^{ii'} \Phi_2^{jj'} \Phi_3^{kk'} / 6! \; ; \; \epsilon[\Phi^3] = \det \Phi$$

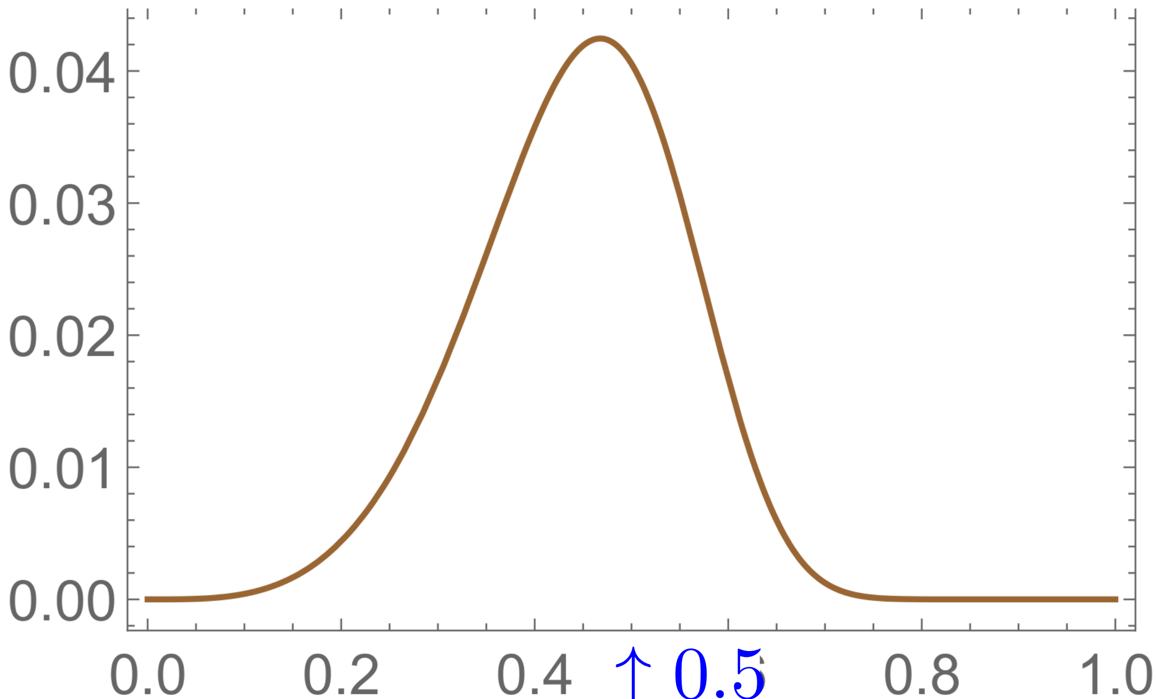
Type of generalized determinant, obviously $SU(3)_L$ x $SU(3)_R$ x $Z(3)_A$ invariant

Instantons

Instanton density as function of scale size ρ with $\Lambda_{\overline{MS}}$ regularization:
Boccaletti & D. Nogradi, 2001.03383

$$n(\rho) = \exp\left(-\frac{8\pi^2}{g^2(\rho\Lambda_{\overline{\rm MS}})} - 7.07534 \right) \frac{1}{\pi^2\rho^5} \left(\frac{16\pi^2}{g^2(\rho\Lambda_{\overline{\rm MS}})} \right)^6$$

Instanton density is peaked at relatively *small* size: Schaefer & Shuryak, 9610451



$n(\rho) \uparrow$

$\uparrow 0.5$

$\rho\Lambda_{\overline{MS}} \rightarrow$

Of *course* the density blows up at larger ρ.

Vacuum is *not* a Dilute Gas of Instantons!

But we can compute...

Anomalous interactions

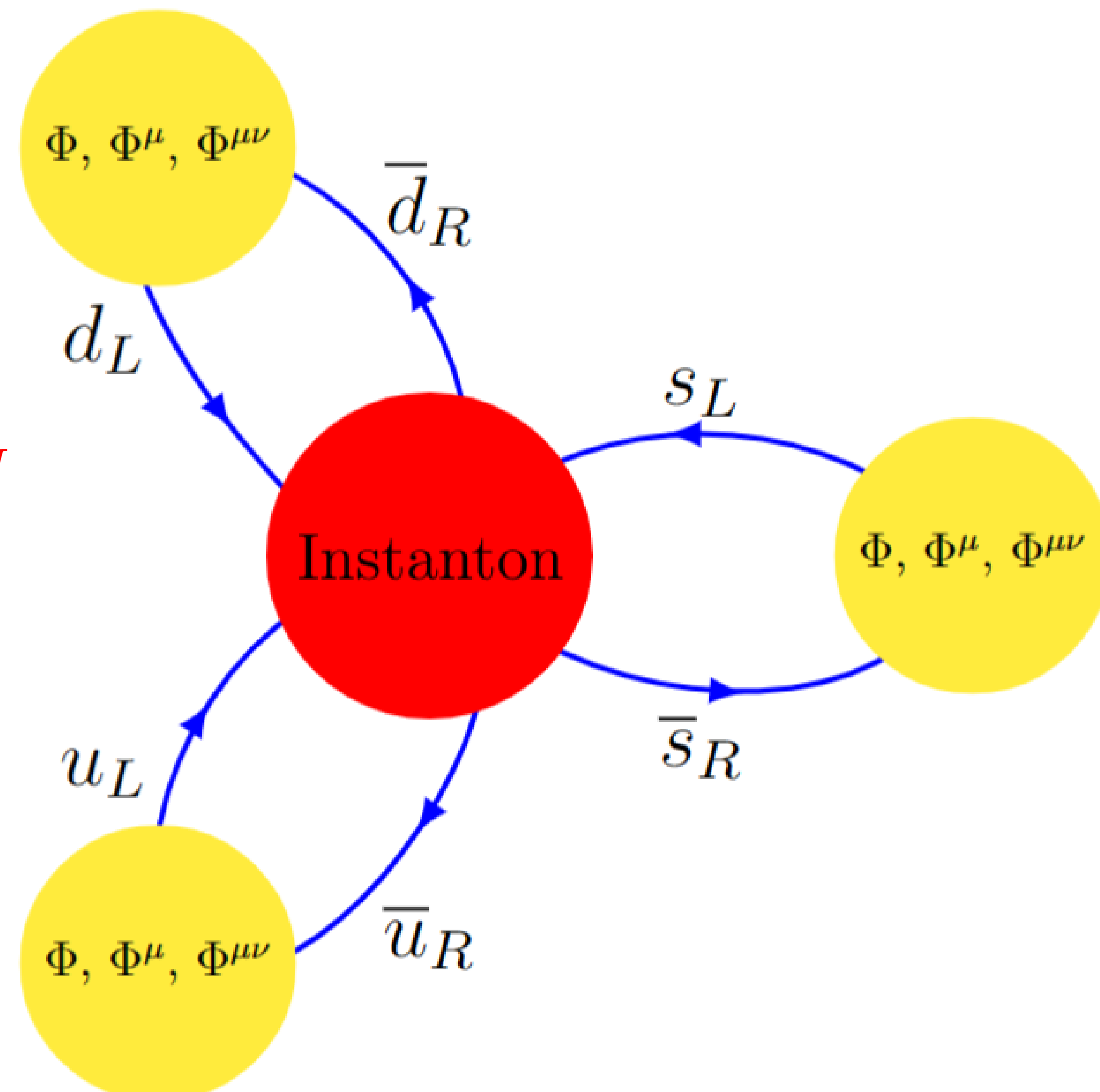
Compute in chiral limit, so *all* instanton zero modes enter, *not* suppressed
't Hooft '76, Grossman '77, Jackiw & Rebbi '77, Atiyah, Hitchin, Singer '77

At large distances, match mesonic to free quark operators.
Assuming all mesonic operators have mass dimension = 1,
need *phenomenological* constants M_J:

$$\Phi_{\mu\nu\ldots} = \overline{q}_L D_\mu D_\nu \ldots q_R / M_J^{2+J}$$

We *assume* $M_0 \sim M_1 \sim M_2$.

Need not be true, testable.



Anomalous interactions: spin zero

In terms of quarks, zero modes generate the anomalous interaction ('t Hooft '76)

$$-\left(\det\left(\overline{q}_L q_R\right) + \det\left(\overline{q}_R q_L\right)\right)/3!$$

In a Dilute Gas of Instantons (DGI), with $\Lambda_{\overline{MS}} = 300\ \mathrm{MeV}$

$$k_0 = (8\pi^2)^3 \int_0^{\Lambda^{-1}_{\overline{\mathrm{MS}}}} d\rho\, n(\rho)\, \rho^9 = 2.6\ 10^6/\mathrm{GeV}^5$$

In terms of mesons,

$$-\kappa_0\left(\det\Phi + \det\Phi^\dagger\right)\ ;\ \kappa_0 = k_0 M_0^6/48$$

Fitting to a linear sigma model & the η-η' mixing angle, $\theta_{PV} = -43.4^o$ gives

$$\kappa_0 = 1.3\ \mathrm{GeV}\ ;\ M_0 = 170\ \mathrm{MeV}$$

Which is reasonable.

Details of η-η' mixing

Start with $SU(3)_V$ flavor basis:

$$\phi_N = (\overline{u}u + \overline{d}d)/\sqrt{2}\ ;\ \phi_s = \overline{s}s$$

Rotation to $SU(3)_V$ basis

$$\begin{pmatrix} \eta(547) \\ \eta'(958) \end{pmatrix} = \begin{pmatrix} \cos\beta_0 & \sin\beta_0 \\ -\sin\beta_0 & \cos\beta_0 \end{pmatrix} \begin{pmatrix} \eta_N \\ \eta_S \end{pmatrix}$$

In terms of the model parameters,

$$\beta_0 = \frac{1}{2}\tan^{-1}\left(\frac{2.67\sqrt{2}(-\kappa_0)\phi_N}{m^2_{\eta_N} - m^2_{\eta_s}}\right) < 0$$

$$\phi_N = \langle\eta_N\rangle/\sqrt{2} = 113 \text{ GeV}\ ;\ \phi_s = \langle\eta_s\rangle = 130 \text{ GeV}$$

Smaller κ_0 gives smaller β_0.

Spin one heterochiral, $h_1(1170)$ & $h_1(1415)$

$$\Phi^{ij}_\mu = \overline{q}^i_L \overleftrightarrow{D}_\mu \, q^j_R = \Phi_\mu / M_1^3$$

$\Phi_\mu = S_\mu + i\, P_\mu$:

P_μ : $J^{PC} = 1^{+-}$: $b_1(1235)$, $K_{1,B}$, $h_1(1170)$, $h_1(1415)$ $\;{}^{2S+1}L_J = {}^1S_1$ *h_1's like η & η'*

S_μ: $J^{PC} = 1^{--}$, $\varrho(1700)$, $K^*(1680)$, $\omega(1650)$, $\phi(?)$ ${}^{2S+1}L_J = {}^1P_1$.

Anomalous term:

$$-k_1(\epsilon[(\overline{q}_L q_R)(\overline{q}_L D_\mu q_R)^2] + R \leftrightarrow L)/6$$

$$k_1 = (8\pi^2)^3 \int_0^{\Lambda^{-1}_{\overline{\rm MS}}} d\rho \, n(\rho) \, \rho^{9+2} = 9.9 \, 10^6 \ {\rm GeV}^{-7}$$

Versus k_0, extra factor of ρ^2 in k_1 because of the D_μ in Φ_μ.

Spin one heterochiral: mixing angle

In terms of mesonic fields:

$$\kappa_1(\epsilon[\Phi\Phi_\mu\Phi_\mu] + \text{c.c.}) \; ; \; a_1 = -k_1 M_1^6 M_0^2/48 = -0.14 \text{ GeV} < 0$$

Calculate mixing angle between $h_1(1170)$ & $h_1(1415)$, like between η & η'

$$\beta_1 \simeq \frac{1}{2}\tan^{-1}\left(\frac{\sqrt{2}\,(-\kappa_1)\,\phi_N/3}{2(m^2_{K_{1B}} - m^2_{b_1}) - \sqrt{2}a_1\phi_S/6}\right) = +\,0.75^o > 0$$

Assuming $M_1 = M_0$, *β_1 is small and positive.*

Spin one mixing angle vs experiment

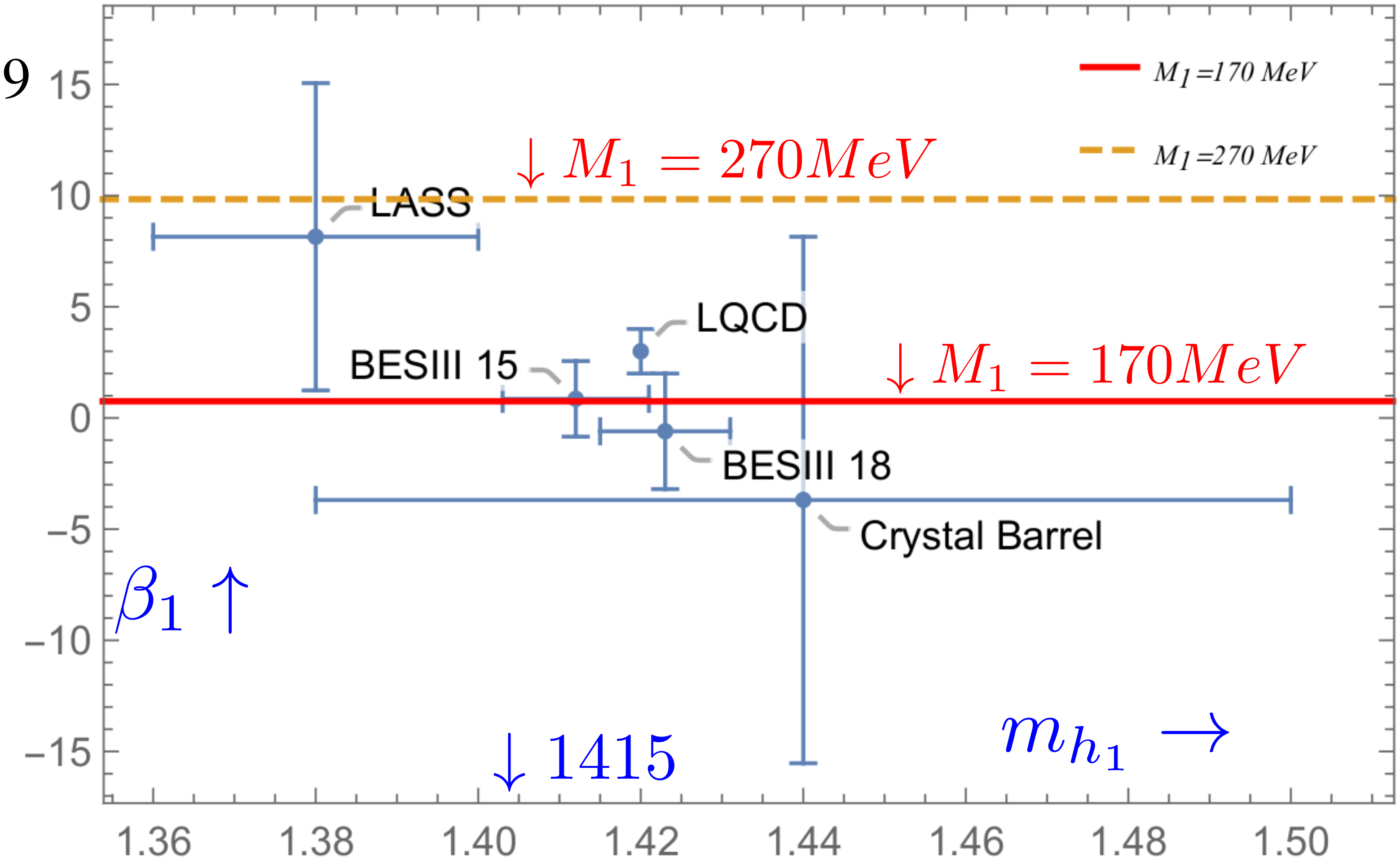
Mixing angle $\beta_1 = 0.75^o$ is small *if* $M_1 = M_0$.

But the anomalous coupling $a_1 \sim M_1^6$, and so *very* sensitive to M_1 vs M_0.:
For $M_1 = 270$ Mev, $\beta_1 = 10^o$

Experimentally, mass of $h_1(1415)$ uncertain; width ~ 90 MeV

Also lattice, LQCD
Dudek+… 1102.4299

Experiment
& LQCD
can measure M_1.



Spin two heterochiral: η(1645) & η(1870)

$$\Phi^{ij}_{\mu\nu} = \overline{q}^i_{\rm L}(\overleftrightarrow{D}_\mu \overleftrightarrow{D}_\nu + \overleftrightarrow{D}_\nu \overleftrightarrow{D}_\mu) q^j_{\rm R} - (g^{\mu\nu}/4)\, \overline{q}^i_{\rm L}(\overleftrightarrow{D}^2) q^j_{\rm R}$$

$\Phi_{\mu\nu} = S_{\mu\nu} + iP_{\mu\nu}$:

$P_{\mu\nu}$: $J^{PC} = 2^{-+}$: $\pi_2(1670)$, $K_2(1670)$, $\eta_2(1645)$, $\eta_2(1870)$; *η_2's like η & η'*

$S_{\mu\nu}$: $J^{PC} = 2^{++}$, a_2, K_2^*, f_2, f_2' ; $^{2S+1}L_J = {}^1P_1$. Not clear exp.'y

Anomalous term:

$$-k_2(\epsilon[(\overline{q}_L q_R)(\overline{q}_L(D_\mu D_\nu - g_{\mu\nu}D^2)q_R))^2] + R \leftrightarrow L)/6$$

$$k_2 = (8\pi^2)^3 \int_0^{\Lambda^{-1}_{\overline{\rm MS}}} d\rho\, n(\rho)\, \rho^{9+4} = 4.05\, 10^7\ {\rm GeV}^{-9}$$

Spin two heterochiral: mixing

In terms of mesonic fields:

$$\kappa_2(\epsilon[\Phi\Phi_{\mu\nu}\Phi_{\mu\nu}] + \text{c.c.}) \; ; \; \kappa_2 = -k_2 M_2^8 M_0^2/48 = +0.017 \text{ GeV} > 0$$

The mixing angle between the $\eta_2(1645)$ & $\eta_2(1870)$:

$$\beta_2 \simeq \tan^{-1}\left(\frac{\sqrt{2}\,(-\kappa_2)\,\phi_N/3}{2(m^2_{K_{2P}} - m^2_{\pi_2}) - \sqrt{2}a_2\phi_S/6}\right)/2 \sim -0.05^o < 0$$

Why do the mixing angles, β_J, decrease with J?

Two reasons: the quark anomalous coupling is ~ ρ^{2J} in k_J's, from D_μ's in $\Phi_{\mu\nu}$....

$$k_J = (8\pi^2)^3 \int_0^{\Lambda^{-1}_{\overline{MS}}} d\rho \, n(\rho) \, \rho^{9+2J}$$

The instanton density peaks at *small* $\rho\Lambda_{MS}$ ~ 0.5. Thus the a_J's decrease with J:

$$\kappa_0 = 1.3 \text{ GeV} \; ; \; \kappa_1 = -0.14 \text{ GeV} \; ; \; \kappa_2 = 0.017 \text{ GeV}$$

This *may* be an artifact from assuming a DGI's, *and/or* assuming $M_2 = M_1 = M_0$.

Second, $\tan(\beta_J)$ ~ 1/(difference meson mass(J))^2

For J=0, pseudo GB's are *much* ligher than "ordinary" mesons, with J=1 & 2,
So $|\beta_0| \gg |\beta_2| \gg |\beta_2|$:

$$\beta_0 = -43.6^o \; ; \; \beta_1 = +0.75^o \; ; \; \beta_2 = -0.05^o$$

New anomalous decays

Fun with effective Lagrangians! Couple spin zero, one, & two, all heterochiral:

$$-b_2\left(\epsilon\left[\left(\partial_\mu \Phi\right)\Phi_\nu\, \Phi^{\mu\nu}\right] + \text{c.c.}\right)\ ;\ b_2 = k_2 M_0^2 M_1^3 M_2^4/48 \approx 0.099$$

Coupling two spin zero particles to one spin two, all heterochiral:

$$-c_2\left(\epsilon\left[\left(\partial_\mu \Phi\right)\left(\partial_\nu \Phi\right)\Phi^{\mu\nu}\right] + \text{c.c.}\right)\ ;\ |c_2| = k_2 M_0^2 M_2^4/48 = 0.474\,\text{GeV}^{-1}$$

Generate rare decays: from first term,

$$\phi(2170) \to b_1(1235)\pi\ ;\ \Gamma = 0.071\ \text{MeV}$$

Generate rare decays: from second term,

$$f_2(2300) \to \pi\pi\ ;\ \Gamma = 0.05\ \text{MeV}$$

Width φ(2170)~83 MeV; width f_2~149 MeV. Useful (but hard) to measure exp.y!

Fourth amuse-bouche:

WHAT the **** is going on with

the chiral phase transition?

Or: how,sometimes, being “wrong” can be right...

The anomaly for two flavors

The spin zero fields are $\Phi = \sigma + i\eta + (\vec{a}_0 + i\vec{\pi}) \cdot \vec{\sigma}$

The $U(1)_A$ invariant mass term is

$$\mathrm{tr}\, \Phi^\dagger \Phi = \sigma^2 + \eta^2 + \vec{a}_0^2 + \vec{\pi}^2$$

For two flavors, the determinent is also a mass term,

$$-(\det \Phi + \mathrm{c.c.}) = -\sigma^2 + \eta^2 - \vec{\pi}^2 + \vec{a}_0^2$$

With*out* the anomaly, symmetry = $SU(2)_L$ x $SU(2)_R$ x $U(1)_A$ = O(4) x O(2)

(The $U(1)_A$ invariant mass is invariant under O(8),
but $U(1)_A$ invariant quartic terms reduce this to O(4) x O(2))

With the anomaly, $SU(2)_L$ x $SU(2)_R$ x $Z(2)_A$ = O(4) x O(2) ~ O(4)

The anomaly makes the η meson heavy & helps the σ to condense

For three flavors, the anomaly only contributes to the η' mass when $\langle\sigma\rangle \neq 0$.

Chiral phase transition: two flavors

RDP & Wilczek '84: consider the chiral phase transition at a temperature T_χ.

$$\mathcal{V} = m^2\, \mathrm{tr}\Phi^\dagger\Phi + \kappa_0(\det\Phi + \mathrm{c.c.}) + \lambda_1(\mathrm{tr}\Phi^\dagger\Phi)^2 + \lambda_2\mathrm{tr}(\Phi^\dagger\Phi)^2 + \ldots$$

Two flavors: the determinant from the anomaly is a mass term.

If $m^2(T_\chi)=0$ & $\kappa_0\,(T_\chi)\neq 0$, (χ trans.=2nd order) universality class = O(4).

If $m^2(T_\chi)=0$ *and* $\kappa_0\,(T_\chi)=0$ (!), universality class = O(4)xO(2).

RDP & D. Stein '81: transition at T ≠ 0 ~ theory in 3 dimensions.

To *leading* order in ε about 4 - ε dimensions, (RDP & D. Stein, PRB '81),

there is *no* infrared stable fixed point → *first* order transition, when $N_f > \sqrt{2}$

fluctuation induced first order (= Coleman-Weinberg). But ε = 1!

Expect $\kappa_0\,(T_\chi) \neq 0$, so if 2nd order, the universality class is that of O(4)

Chiral phase transition: three flavors

For three flavors, consider just the potential as a function of σ:

$$\mathcal{V}(\sigma) = m^2\,\sigma^2 - \kappa_0\,\sigma^3 + \lambda\,\sigma^4 + \ldots$$

If $\kappa_0\,(T_\chi) \neq 0$, then the potential has a cubic term, and cannot be "flat"

→ for massless quarks, the chiral transition *must* be first order.

If $\kappa_0\,(T_\chi) = 0$ (!), the universality class is SU(3) x SU(3) x U(1)

To leading order in ε in 4 - ε dimensions, fluctuation ind'd 1st order

For four flavors, to leading order in ε, fluc ind'd 1st order (neglecting κ_0).

For > four flavors, the det term irrelevant; ~ $O(\varepsilon)$, fluctuation induced 1st order

A little history

‘80’s: expected a first order deconfining transition dominates quarks,

chiral transition *completely* irrelevant.

‘90’s: Lattice QCD with dynamical quarks: the deconfining transition weakened,

chiral symmetry matters more.

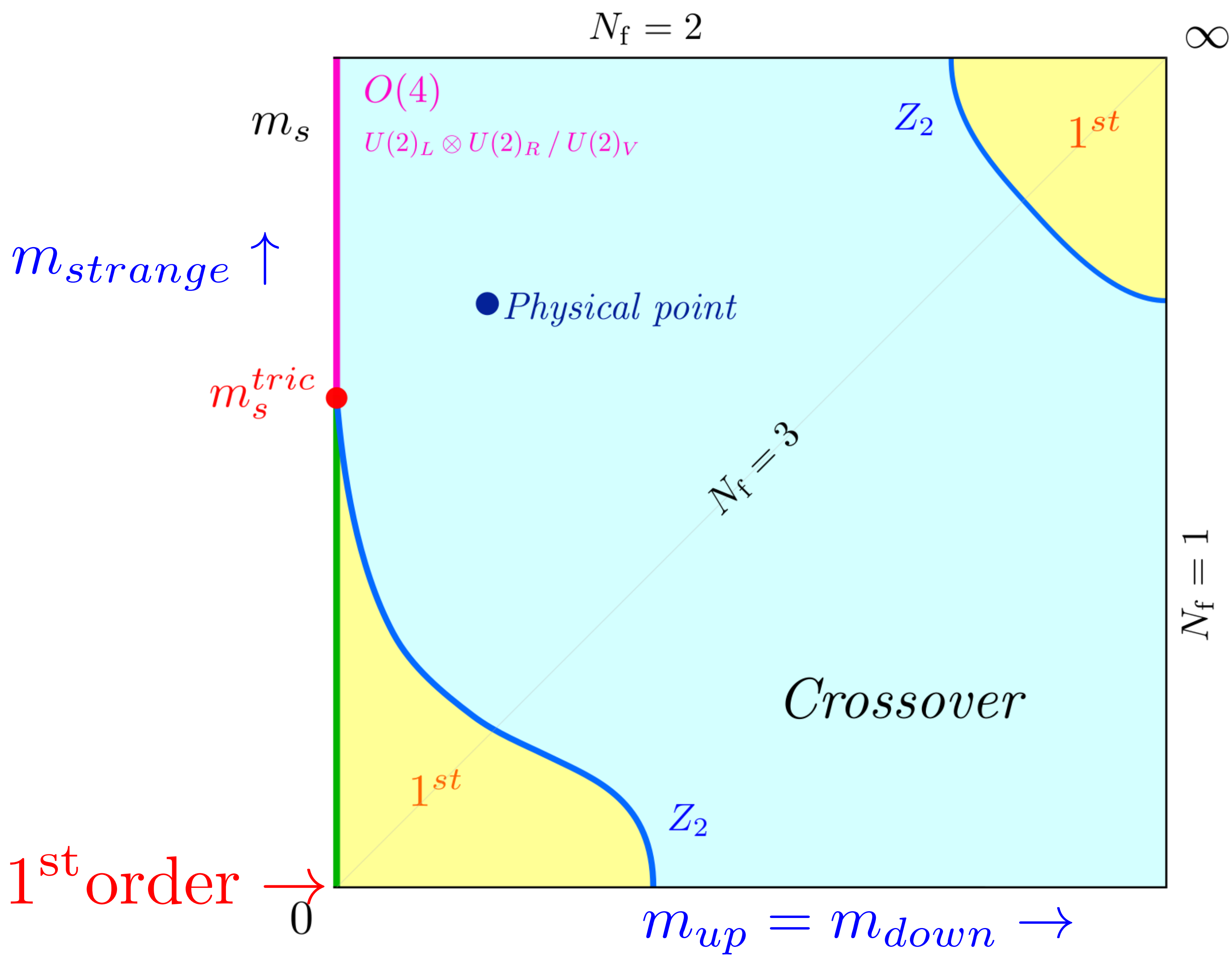
”Columbia” phase diagram, as a function of $m_u = m_d$ and m_s.

‘23: Lattice: consensus, QCD is crossover, $T_\chi = 156 \pm 2$ MeV

What is the order of the chiral transition for *massless* quarks?

Lattice: "Columbia" phase diagram for 3 flavors

Surely $\kappa_0(T_\chi) \neq 0$, chiral transition *1st* order in the chiral limit
Lattice QCD: Columbia group, Brown + … PRL 65, 2491 (1990)



Lattice: "Frankfurt" phase diagram for 3 flavors

Cuteri, Philipsen, Sciarra: 2107.12739: chiral transition *2nd* order in χ limit!

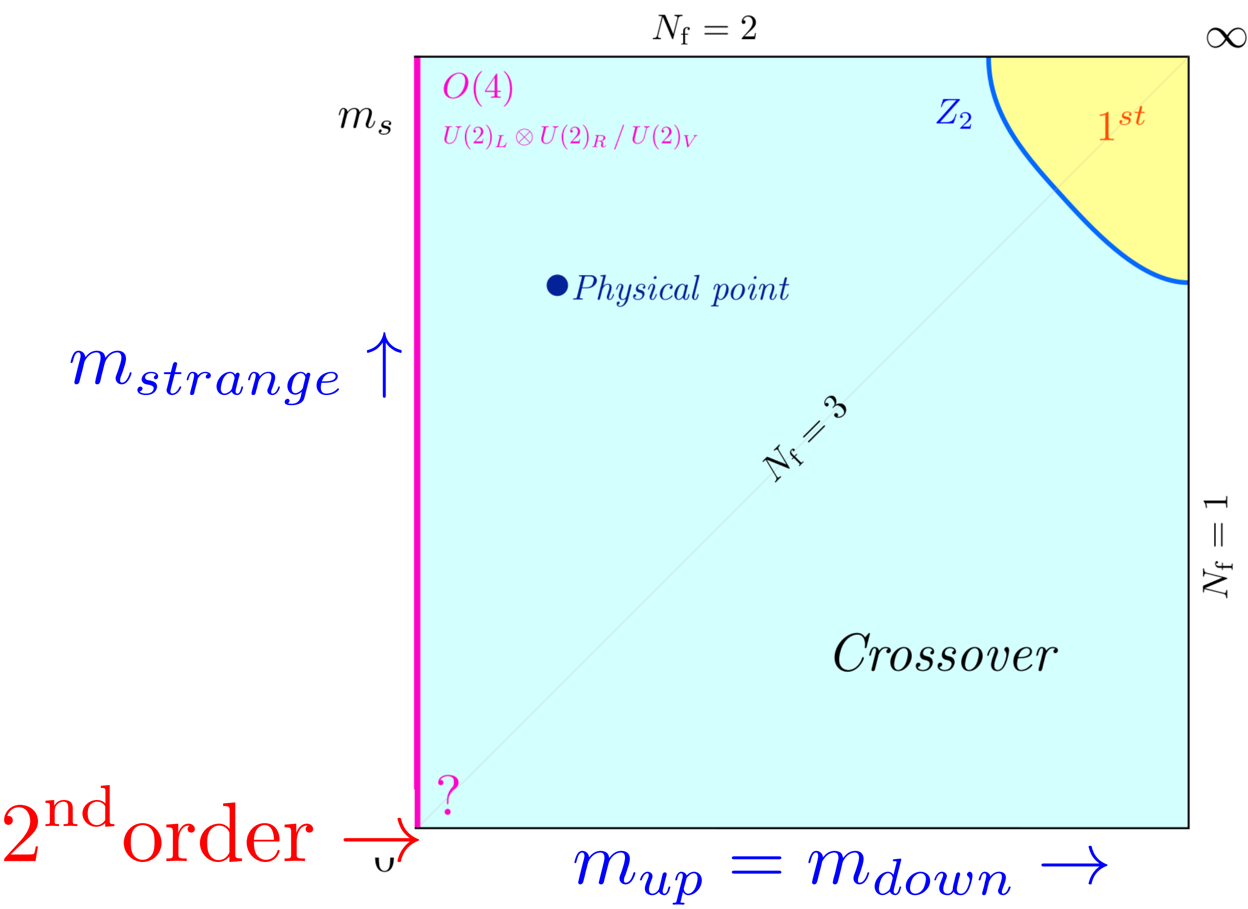
$$\kappa_0(T_\chi) = 0$$

If true, absolutely astonishing.

I assume instead that

$$\kappa_0(T_\chi) \ll \kappa_0(0)$$

So the 1st order region is *much* smaller than expected.



Bielefeld: 2111.12599: *no* sign of a 1st order transition for m_π > 80 MeV.

JLQCD: Aoki+… 2103.05954; 2212.10021; Lattice '23: Pasztor, Fukaya

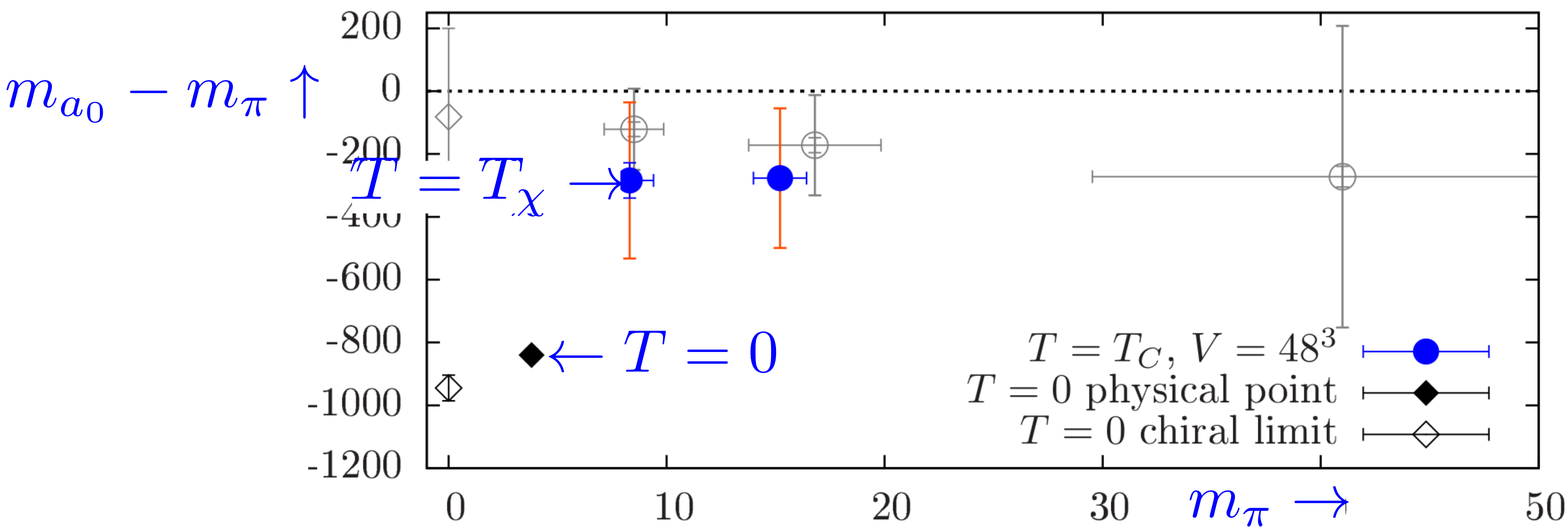
Lattice: anomaly and two flavors

Lattice: chiral transition for two flavors is 2nd order in the chiral limit, O(4)

Mass splitting between the σ and the η directly measure κ_0.; easier to measure $a_0 - \pi$ splitting, $\Delta m = m_{a0}$ - m_π .

Brandt+…1904.02384: $\Delta m(T_\chi)$ - $\Delta m(0)$ = 500 MeV. *Strongly* suggests

$$\kappa_0(T_\chi) \ll \kappa_0(0)$$



Also JLQCD: Aoki+… 2011.01499; 2103.05954

What is the lattice telling us?

The general effective Lagrangian is a sum of $U(1)_A$ invariant terms

$$\mathcal{V}^{U(1)} = m^2\, \mathrm{tr}\Phi^\dagger\Phi + \lambda_1(\mathrm{tr}\Phi^\dagger\Phi)^2 + \lambda_2 \mathrm{tr}(\Phi^\dagger\Phi)^2 + \ldots$$

Include all anomalous terms to dimension four for two flavors, and six to three flavors:

$$\mathcal{V}^{Z(3)} = \kappa_0 \,\det(\Phi) + \kappa_0'\, \mathrm{tr}(\Phi^\dagger\Phi)\det(\Phi) + \kappa_0''\,(\det\Phi)^2 + \mathrm{c.c.}$$

Mean field theory: if only temperature is varied, one only tunes one

parameter in the effective Lagrangian, usually the mass:

$$m^2(T) = m^2(0) + \#\, T^2$$

Tuning $m^2(T_\chi)=0$ *and* $\kappa_0(T_\chi)=0$ is unnatural.

A possible solution

RDP, Rennecke,Skokov,2309.?: *in mean field theory, 1st order for* $m_\pi < 150$ *MeV*

Contradicts lattice. So let $\sigma_0(T) = \langle\sigma\rangle(T)$. The *effective* coupling for det Φ is

$$\kappa_0^{\rm eff}(T) = \kappa_0 + \kappa_0' \, \sigma_0(T)^2 + \kappa_0'' \, \sigma_0(T)^3$$

Work in mean field theory, all κ_0, κ_0', κ_0'' *in*dependent of T, and *assume*

$$\kappa_0 \ll \kappa_0' \sigma_0(0)^2 \; , \; \kappa_0'' \sigma_0(0)^3$$

At T=0, the effective coupling $\kappa_0^{\rm eff}$ is *large*, because κ_0' & κ_0'' are large
But at T_χ, $\kappa_0^{\rm eff}$ is *small* because then only the *small* κ_0 enters.

Clearly unnatural: 1st order transition eventually arises for *some* $m_\pi < 80$ MeV
Testable on the lattice with effort (3-point function)

If so, the restoration of χ symmetry drives the *approximate* restoration of $U(1)_A$ in the entire plane of T and μ, including T=0 for cold, dense quarks!

What if $U(1)_A$ *is* exactly restored at T_χ?

If so, a profound and *very* interesting miracle.

For $SU(N_f)xSU(N_f)xU(1)$, in 4-ε dim's to ~O(ε), fluct ind'd *1st* order for $N_f > \sqrt{2}$.

Recently, evidence for a *new* fixed point at ε=1!

$N_f = 2$, O(4) x O(2): 1st order: Sorokin, 2105.00072; 2205.07199. Monte Carlo

2nd order: Calabrese, Parruccini, 0403140; pert thy in 3 dim's

2nd order: conformal bootstrap, Nakayama & Ohtsuki, 1407.6195;
Henriksson, Kousvos, Stergiou, 2004.14388

$N_f = 3$, SU(3)xSU(3)xU(1):

2nd order: Kousvos, Stergiou, 2209.02837: conformal bootstrap

Adzhemyan + …, 2104.12195: ε-expansion to *six* loop order!

Possibly new fixed points for $SU(N_f)xSU(N_f)xU(1)$;
Or: "pseudocritical" = *weakly* 1st order, Gorbenko+...1807.11512

The future of QCD

Today, five amuse-bouche

At $T \neq 0$, $\mu \leq T$, numerical simulations of lattice QCD are the *bedrock* for

our understanding of heavy ion collisions at very high energy.

At T and $\mu \neq 0$, *especially* $T = 0$, "all" we need to do is solve the sign problem.

Surely requires quantum computers. Can start today with toy models

e.g., "QZD", Z(3) in 1+1 dim.'s with 3 massive flavors, RDP 2101.05813 +…

A. Florio, RDP, S. Valgushev, A. Weichselbaum, 2310.? + S. Economou+….

Solving the sign problem, including in QCD, is one of *the* major problems
for theoretical physics in the 21 st century.

Happy 50th birthday to QCD!

QCD will *undoubtedly* remain cutting edge for its 75th *and* 100th birthdays!

Last amuse-bouche:

A confession…