50 Years of QCD @ UCLA

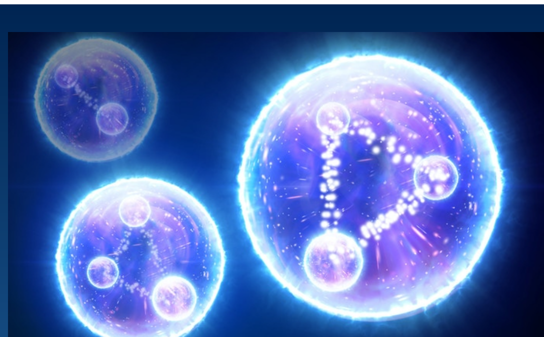
Higgs-Confinement Phase Transitions in QCD

Thomas Dumitrescu (UCLA)

13 September 2023, 14:50-15:40

[Work to appear with Po-Shen Hsin]

SIMONS FOUNDATION



UCLA Mani L. Bhaumik Institute for Theoretical Physics

Part 1: Gauge Theory Phases

Phases of Gauge Theory

Play a role in many physical systems:

- **QCD:** confining phase of $SU(3)_c$ gauge theory
- **Electroweak theory:** Higgs phase of

$$SU(2)_W \times U(1)_Y \to U(1)_{\text{e.m.}}$$

- **Condensed matter physics:** conventional BCS superconductor is a Higgs phase with $U(1)_{\text{e.m.}} \to \mathbb{Z}_2$

Here "phase" is used casually to mean "regime".

Order Parameters and Symmetries

Sharp notion of phases and transitions typically requires order parameters and global symmetries **[Landau]**:

- **Order parameters:** large electric/magnetic loops **[Wilson; Polyakov; Susskind; 't Hooft]**

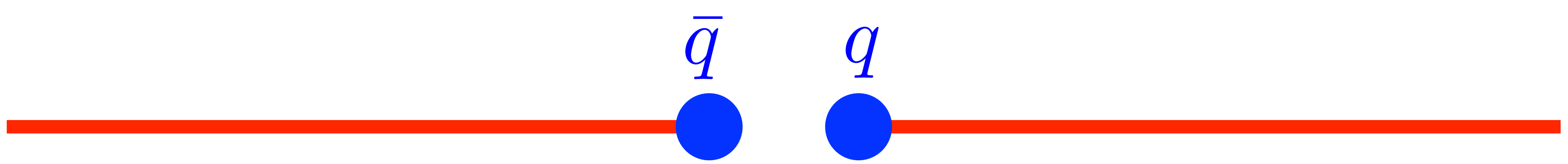
- **Symmetries:** generalized, one-form global symmetries **[Gaiotto, Kapustin, Seiberg, Willett]**, e.g. center symmetry.

- **Example:** pure $SU(3)_c$ gauge theory has a $\mathbb{Z}_3$ one-form center symmetry, unbroken in vacuum, broken at high T.

Fundamental Matter

One-form symmetries: always broken by fundamental matter. (They rely on non-generic matter.)

Example: QCD with fundamental quarks has no one-form center symmetry. Wilson lines can end; confining strings can break.



No sharp notion of confinement in QCD!
Finite T lattice simulations (no sign problem) show a smooth crossover for non-zero quark masses.

Higgs-Confinement Continuity

Fundamental scalar fields can lead to complete Higgsing with a gapped vacuum, much like in the confining regime.

There are no symmetries that distinguish Higgsing and confinement: **they can be continuously connected, without a phase transition**.

- Rigorous lattice results [Fradkin, Shenker; also Banks, Rabinovici]
- True in examples, especially with SUSY [Intriligator, Seiberg]
- **Continuity dogma is standard lore** [Dimopoulos, Raby, Susskind]

Today: Higgs-Confinement Transitions

- Examples where Higgs-confinement continuity fails (QCD + Higgs fields + Yukawa couplings).
- Unexpected Higgs-Confinement phase transitions.
- The reason is that Higgs and confining regimes are different symmetry protected topological (SPT) phases.
- Applications to QCD at finite baryon density: confining at low densities, color superconducting Higgs phase at high densities.

Part 2: What are Symmetry Protected Topological (SPT) Phases?

What are SPTs?

- Condensed matter origins: IQHE, Haldane/AKLT chain, topological insulators/superconductors [David Kaplan's talk]

- Closely related to anomaly inflow (esp. global anomalies)

- Most robust setting: fully gapped phases of matter, no symmetry breaking, no topological order (naively trivial).

- More recently: gapless SPTs (more delicate and less well understood, we will encounter an example)

- SPTs = non-trivial topological actions for background fields associated with unbroken global symmetries.

Chern-Simons SPTs in 2+1d

- Gapped system with unbroken $U(1)$ flavor symmetry and corresponding non-dynamical background gauge field A.

- Effective action can have quantized Chern-Simons term:

$$\frac{ik}{4\pi}\int AdA \qquad k \in \mathbb{Z}$$

- Contributes phase to Euclidean partition function $Z[A]$

- A jump in k necessarily signals a phase transition (first order, unless the gap closes). Example: free massive Dirac fermion **[David Kaplan's talk]**.

SPTs in 3+1d

- Prototypical example of SPT-enforced phase transition: free 2-component Weyl fermion ψ_α with time-reversal T

$$\mathcal{L} = -i\bar{\psi}\bar{\sigma}^\mu\partial_\mu\psi - \frac{m}{2}(\psi\psi + \bar{\psi}\bar{\psi}) \qquad m \in \mathbb{R}$$

- For either sign of m, the theory is gapped and trivial. Related by $U(1)_{\text{axial}}$, which has mixed gravity anomaly. This leads to the following SPTs in the two phases:

$$\frac{\theta_g}{384\pi^2}\int_{M_4} \text{tr}(R \wedge R) \qquad \theta_g = 0, \pi$$

- T-symmetry quantizes θ_g. SPT jump at $m = 0$ enforces a phase transition. Without T: θ_g is not quantized (no SPT), and complex mass $m \in \mathbb{C}$ can avoid phase transition.

Vafa-Witten Positivity and SPTs

QCD with a positive (bare) quark mass $m_q > 0$ can be regulated in such a way that its path-integral measure is positive. This amounts to setting all theta-angles to zero. This is key assumption in the derivation of QCD inequalities [Vafa, Witten; Weingarten].

Measure positivity holds on arbitrary four-manifolds M_4 and for arbitrary-vector like background gauge fields A .

The resulting Euclidean partition functions are then also positive: $Z[M_4, A] > 0$ (trivial SPT). Our SPT examples will all violate positivity (Yukawas, finite chemical potential).

Part 3: Higgs-Confinement Phase Transitions from SPTs

Example 1: SU(2) Higgs-Yukawa QCD

- Gauge group $SU(2)_c$, flavor symmetry $SU(2)_f$
- **Matter:** 1 Dirac = 2 Weyl Quarks $(\psi_\alpha)^{a=1,2}_{i=1,2}$

Real bi-fundamental Higgs: $h^i_a = \left(h^i_a\right)^\dagger$

$$\mathcal{L}_{\text{HYQCD}} = \mathcal{L}_{\text{QCD}} + \mathcal{L}_{\text{Higgs}} + \mathcal{L}_{\text{Yukawa}}$$

$$\mathcal{L}_{\text{QCD}} \supset -\frac{m_q}{2}\varepsilon_{ab}\varepsilon^{ij}\psi^a_i\psi^b_j \qquad \mathcal{L}_{\text{Yukawa}} \supset \frac{1}{2}\left(y_1 h^i_a h^j_b + y_2 h^j_a h^i_b\right)\psi^a_i\psi^b_j$$

- Time reversal T requires $m_q, y_{1,2} \in \mathbb{R}$. We take them > 0.

Phases of SU(2) HYQCD

(C) $M_h^2 \to +\infty$: integrate out Higgs. Leaves QCD with $m_q > 0$. Gapped, trivial SPT.

(H) $M_h^2 \to -\infty$: large color-flavor locking Higgs vev (AF):

$$\langle h_a^i \rangle = v \delta_a^i \ , \qquad v > 0$$

$SU(2)_c$ fully Higgsed, $SU(2)_f$ unbroken by identifying $a = i$.

$$\psi_i^a \to \psi_i^j = \mathbf{1} \oplus \mathbf{3}$$

Yukawas: $\mathbf{3}$ fermions flip sign, inducing $\theta_g = \pi$

Comments

- The $\theta_g = \pi$ SPT jump between the Higgs and the confining phase forces an **unexpected phase transition**.

- As for fermion mass: can avoid transition by braking P and T. Then theta-angles are not quantized and no transition is needed. This partially explains why the two phases still look very similar, even with Yukawas.

- [Thorngren et.al.] recently considered Higgs SPTs involving one-form symmetries (absent in QCD). We only use SPTs for unbroken, ordinary (zero-form) symmetries.

Example 2: SU(3) Higgs-Yukawa QCD

Start with three-flavor QCD: $SU(3)_c$ gauge theory with

3 Dirac = 6 Weyl quarks: $\Psi_i^a = (\psi_i^a, \bar{\chi}_i^a) \qquad a, i = 1, 2, 3$

quark mass $m_q > 0$, flavor symmetry $U(1)_B \times SU(3)_f$

Add Higgs field h_a^i (with potential) and Yukawa couplings:

$$\mathcal{L}_{\text{Yukawa}} = y \varepsilon_{abc} \varepsilon^{ijk} \bar{h}_i^a \left(\psi_j^b \psi_k^c + \bar{\chi}_j^b \bar{\chi}_k^c \right) \qquad y > 0$$

Preserves T, P.

Pairs quarks in color/flavor anti-symmetric channel.

Phases of SU(3) HYQCD

(C) $M_h^2 \to +\infty$: integrate out Higgs. Leaves QCD with $m_q > 0$. Gapped, trivial SPT.

(H) $M_h^2 \to -\infty$: again take color-flavor locking Higgs vev:

$$h_a^i = v\delta_a^i \qquad v \in \mathbb{C}$$

$SU(3)_c$ fully Higgsed, $SU(3)_f$ unbroken by identifying $a = i$.

New feature: $U(1)_B$ spontaneously broken: $\langle \det(h_a^i) \rangle = v^3$

Massless Nambu-Goldstone boson; otherwise gapped.

The Higgs Phase as a Gapless SPT

Massless $U(1)_B$ Nambu-Goldstone boson can in principle render the $\theta_g = \pi$ SPTs meaningless (remove via field redefinition). Subtle; believed to require a mixed anomaly between $U(1)_B$ and gravity, which is absent. Thus we can still meaningfully ask whether $\theta_g = 0, \pi$ in Higgs phase.

Fermions transform under unbroken $SU(3)_f$:

$$\psi_i^a \to \psi_i^j = \mathbf{1} \oplus \mathbf{8} \qquad \chi_a^i \to \chi_j^i = \mathbf{1} \oplus \mathbf{8}$$

Masses: $M_{\mathbf{1}} = m_q \pm 4yv \qquad M_{\mathbf{3}} = m_q \pm 2yv$

At large vevs, 9 fermions flip sign: $\theta_g = \pi$ (and also $\theta_f = \pi$)

Breaking Baryon Number

Deform theory by adding $B = 2$ flavor singlet operator:

$$\Delta\mathcal{L} = \varepsilon \det(h_a^i) \qquad \varepsilon \in \mathbb{R}$$

Preserves P, T . Lifts the Nambu-Goldstone boson: Higgs phases now gapped and naively indistinguishable from the confining phase. Still find $\theta_g = \pi$ jump, forcing a transition.

This is an important sanity check that the SPT indeed has robust consequences, even in the $\varepsilon \to 0$ gapless case.

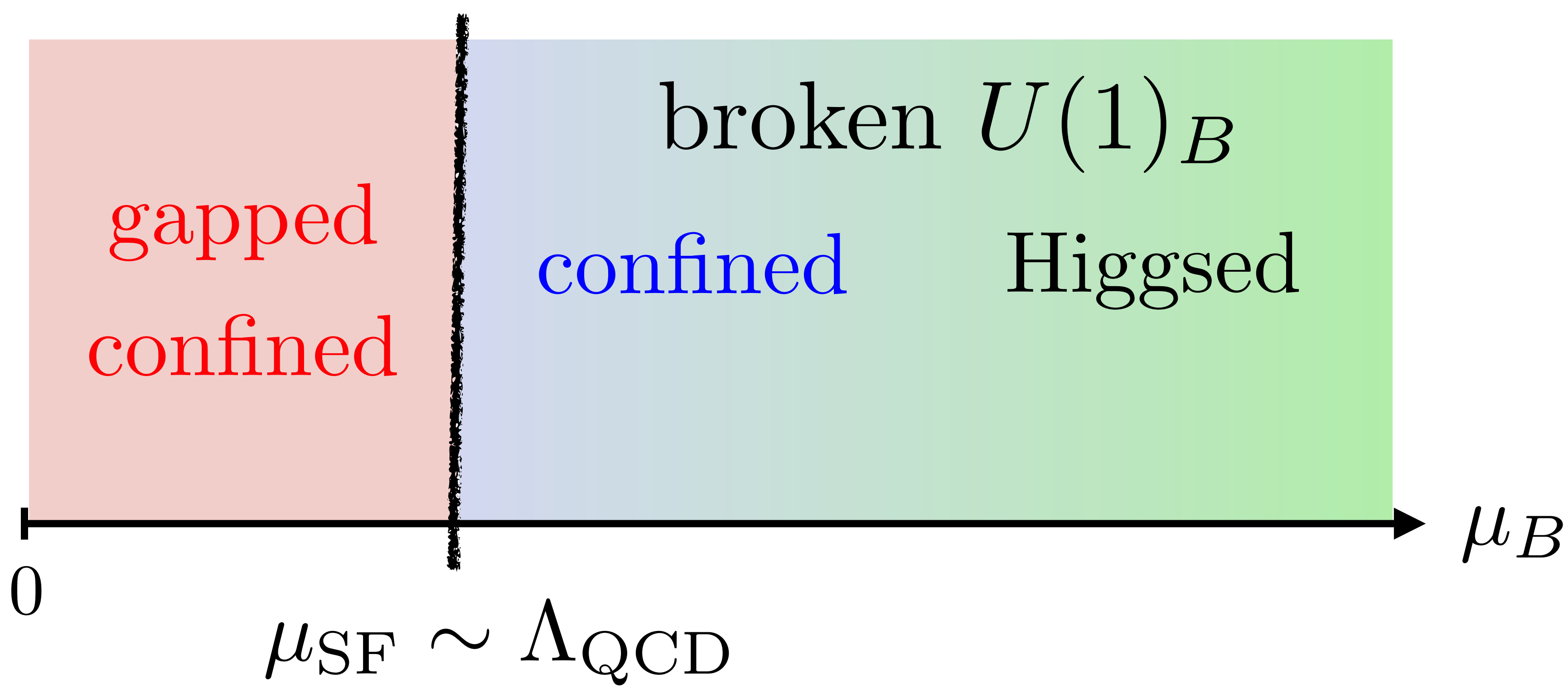
Similarly, $\theta_g = \pi$ is robust against breaking $SU(3)_f$ via generic quark masses.

Part 4: QCD at Finite Baryon Density

Finite Density QCD

Ordinary QCD: 3 degenerate flavors, quark mass $m_q > 0$
$U(1)_B$ chemical potential μ_B (preserves P, T; sign problem)

$SU(3)_f$
symmetric
case



[Schäfer, Wilczek] conjecture: Higgs-confinement (or quark-hadron) continuity in the $U(1)_B$ breaking superfluid phase.

The High-Density Higgs Regime

When $\mu_B \gg \Lambda_{\rm QCD}$ QCD is weakly coupled (AF): 1-gluon exchange in color/flavor anti-symmetric channel leads to a CFL vev for composite Higgs field [Alford, Rajagopal, Wilczek; …]

$$\varepsilon_{abc}\varepsilon^{ijk}\langle\psi_j^b\psi_k^c + \bar{\chi}_j^b\bar{\chi}_k^c\rangle \sim \mu_B\delta_a^i$$

Same quantum numbers as fundamental h_a^i in HYQCD, with same consequences. Thus the two Higgs phases are qualitatively identical, but exact gap sizes are different [Son]:

$$\Delta_{\mathbf{1}} = 2\Delta_{\mathbf{8}} \sim \mu_B e^{-1/g(\mu_B)}$$

Is High-Density QCD a Gapless SPT?

Proposal: the SPTs are also the same, i.e. $\theta_g = \pi$.

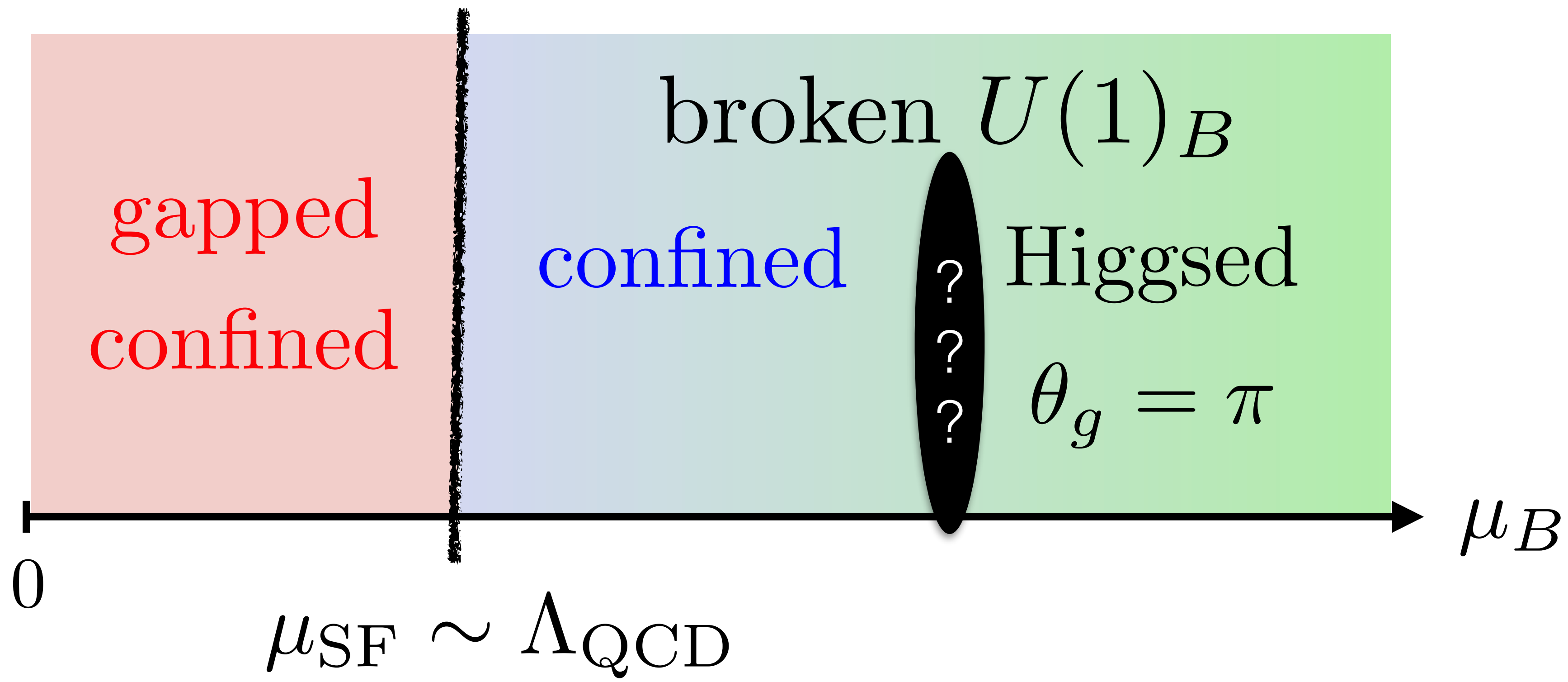
Both are weakly-coupled Higgs phases. SPT only sensitive to fermions, which have the same fate (in reverse order):

- QCD at $\mu_B \gg \Lambda_{\mathrm{QCD}}$: 1-gluon exchange $\Longrightarrow$ pairing $\Longrightarrow$ fermion gaps
- HYQCD at $\mu_B = 0$: Yukawas $\Longrightarrow$ pairing in same channel $\Longrightarrow$ similar gaps. Turning on μ_b maintains fermion gaps.

Related by a deformation that maintains the fermion gap.

New QCD Phase Transitions?

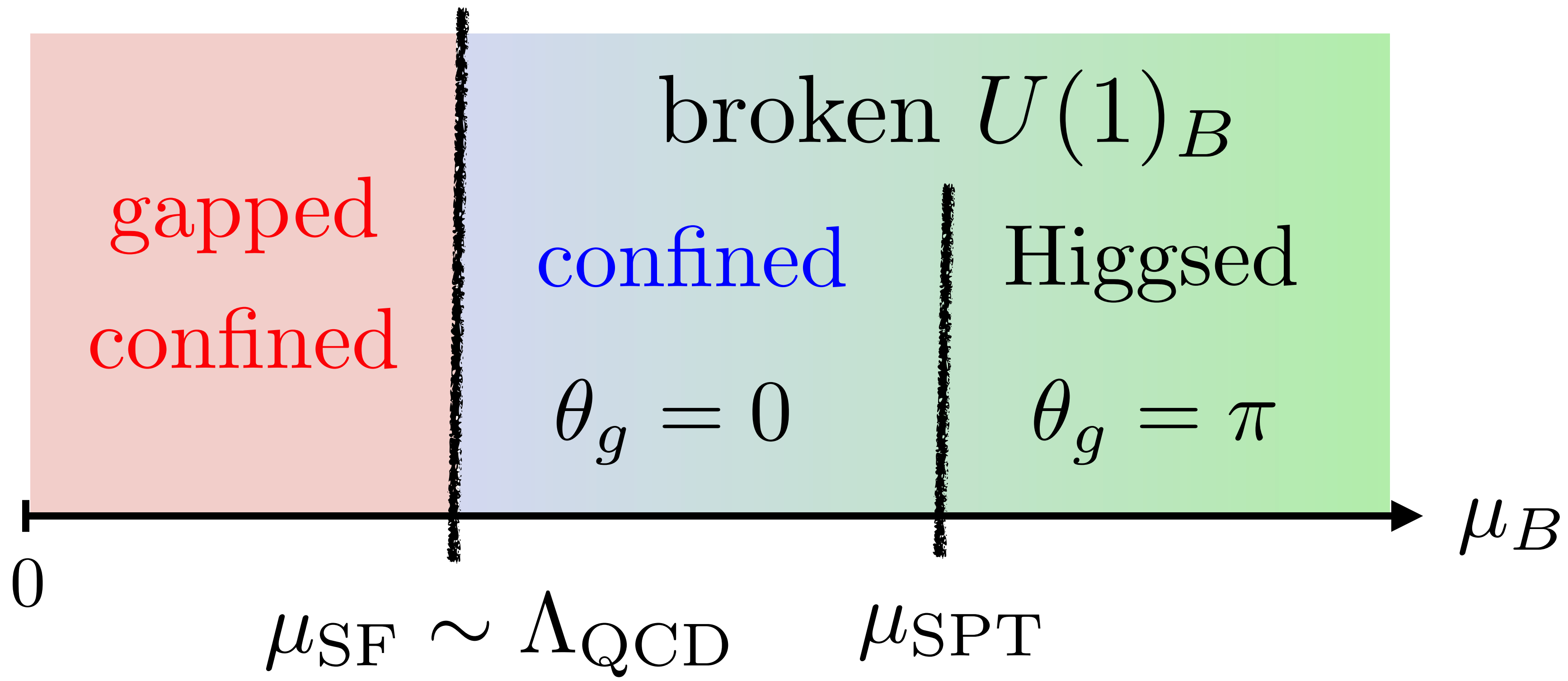
Can $\theta_g = \pi$ persist down to $\mu_{\rm SF}$, or is there a new transition within the superfluid phase?



[Cherman, Sen, Yaffe] have imagined such a transition, motivated by $U(1)_B$ vortices. Unrelated to our SPTs.

Candidate SPT Transition

Expectation: superfluid transition driven by pairing of light spin-$\frac{1}{2}$ octet baryons into spin-0 H-dibaryon [Jaffe], which condenses to break $U(1)_B$. This gives $\theta_g = 0$.



If a 9th spin-$\frac{1}{2}$ singlet baryon exists, it is a heavy excited/bound state. Natural for it to kick in at higher densities, leading to new SPT-driven transition at $\mu_{\rm SPT} > \mu_{\rm SF}$.

Conclusions and Outlook

- Higgs-confinement continuity can fail if the two regimes are in different SPT phases (general, simple examples).
- High-density QCD is a gapless SPT with $\theta_g = \pi$ (robust)
- Motivates searches for unexpected phase transitions.
- Next: realistic quark masses, consequences of $\theta_g = \pi$ for neutron star interiors (EOS, boundary layers).
- On the theoretical side, it is desirable to sharpen our toolbox for gapless SPTs (delicate; much to understand).

Thank You for Your Attention!

Happy Birthday QCD!