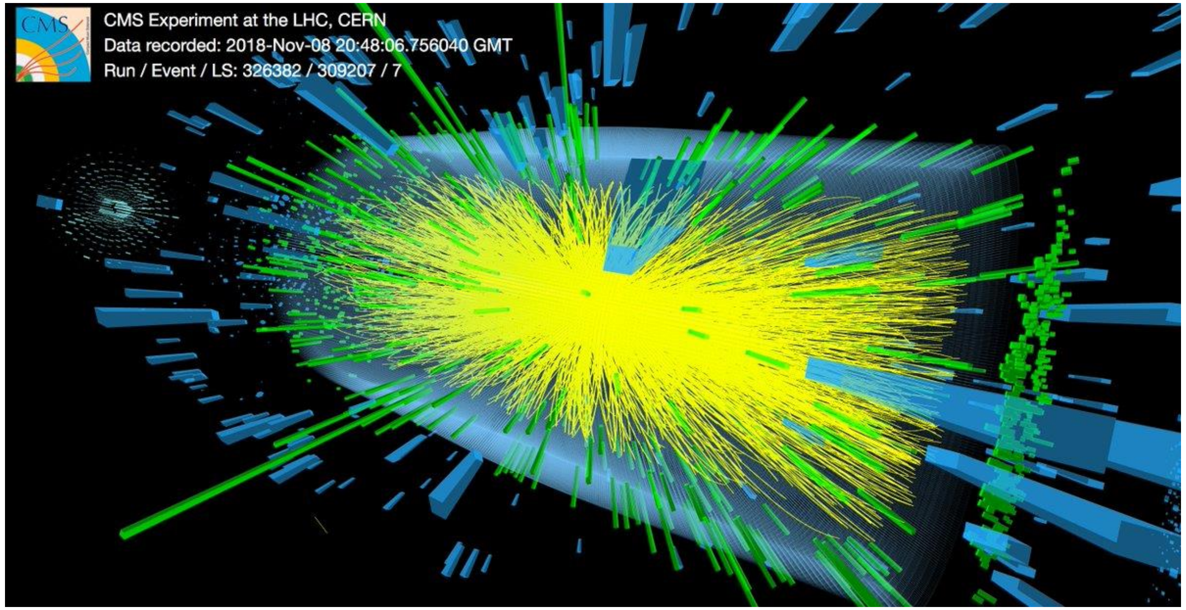
Classical lumps* and their quantum descendants in QCD's Regge limit



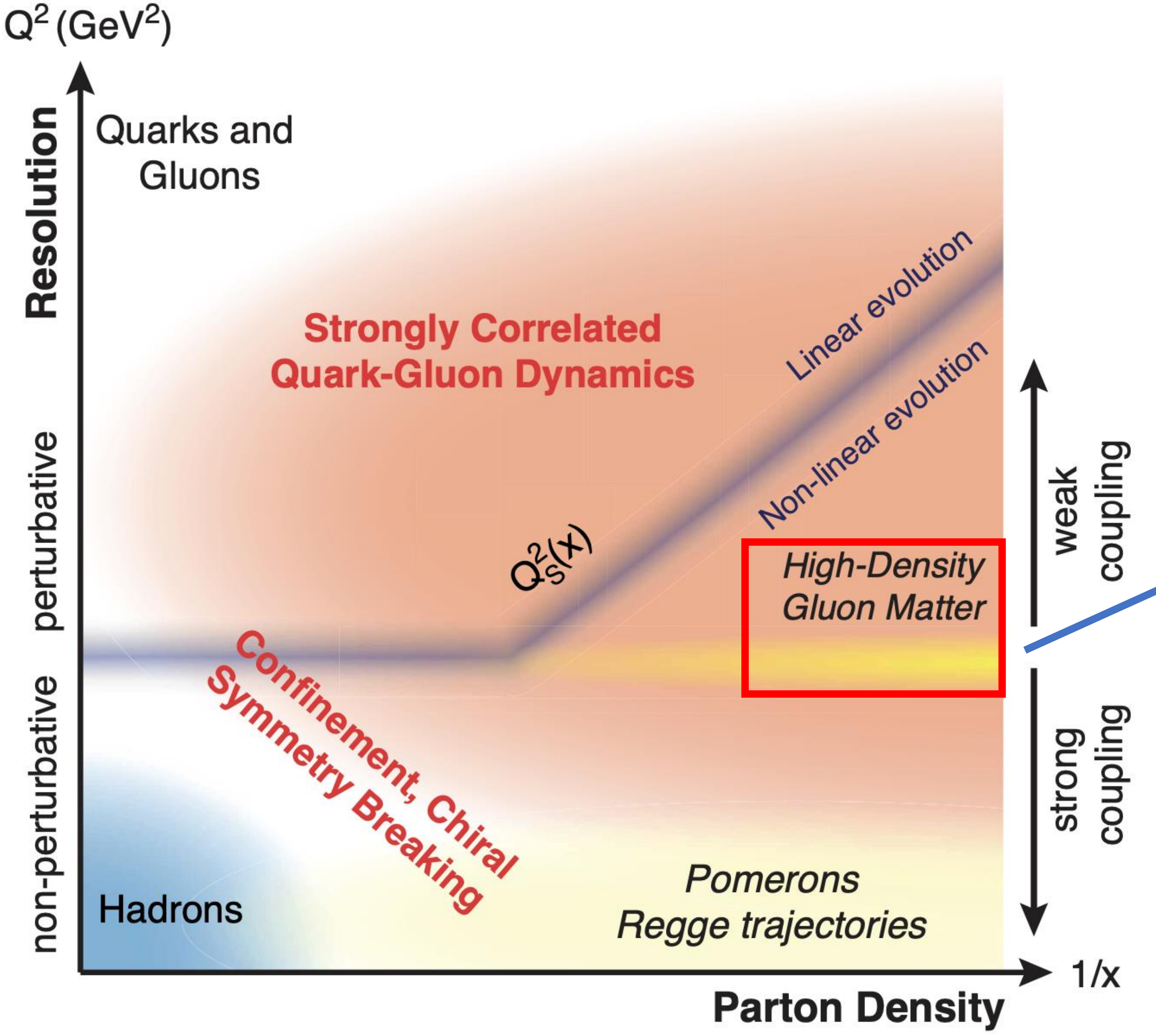
Raju Venugopalan

Brookhaven National Lab and CFNS, Stony Brook

QCD@50, UCLA, Sept. 11-15, 2023

* In homage to Sidney Coleman's Erice Lectures

Exploring Terra incognita in QCD's Regge limit

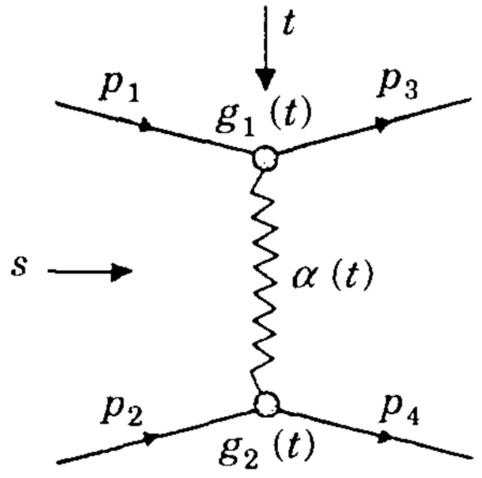


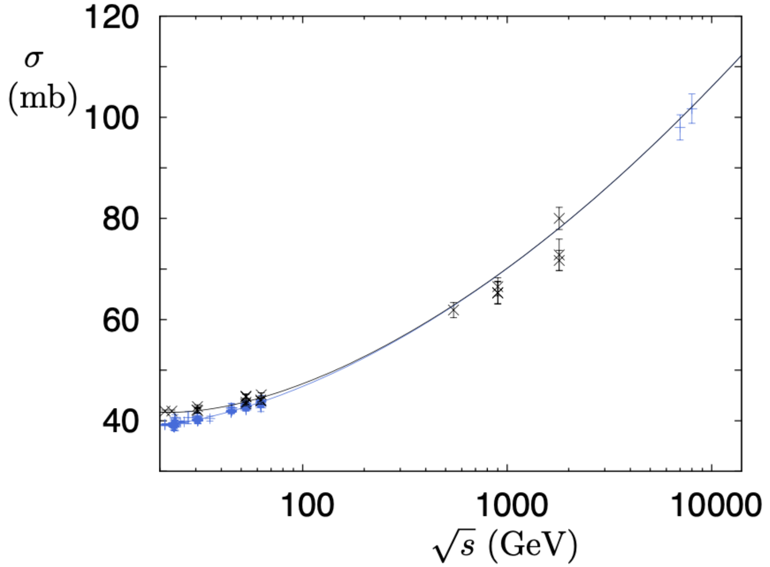
Dominates QCD cross-sections.

Physics of multi-particle production

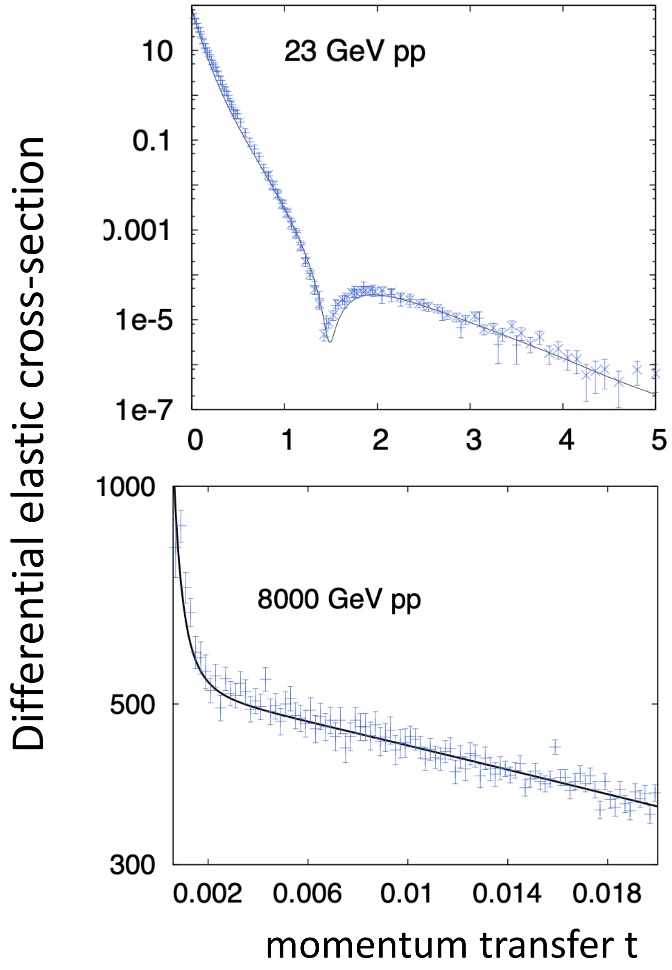
Aschenauer et al., arXiv:1708.01527
Rep.Prog. Phys. 82, 024301 (2019)

What is the Pomeron in QCD?



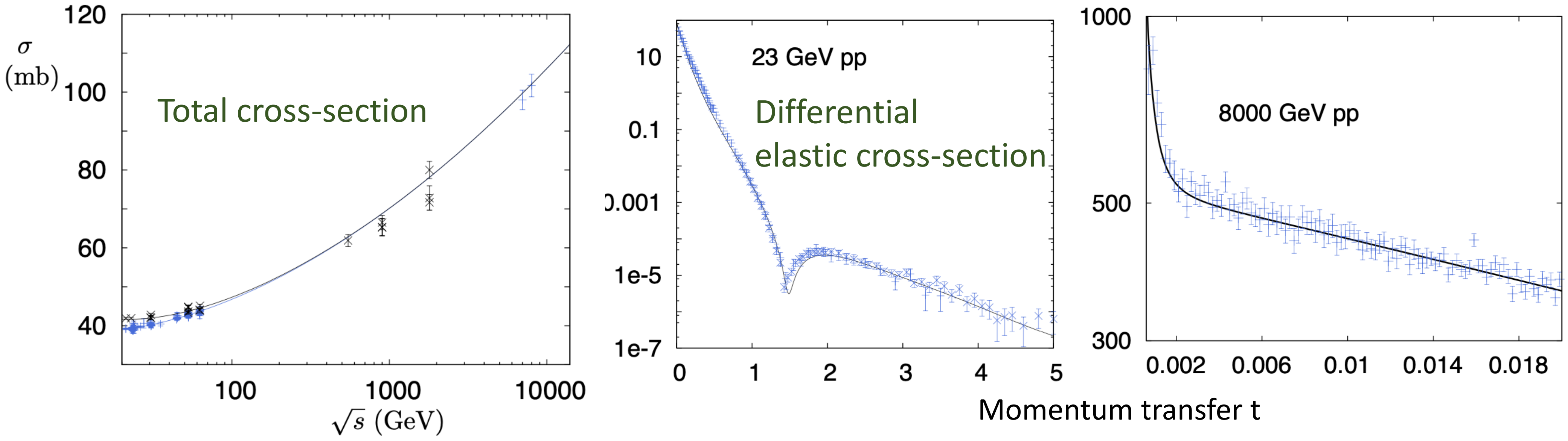


Donnachie, Landshoff, Phys. Lett. B750 (2015) 669



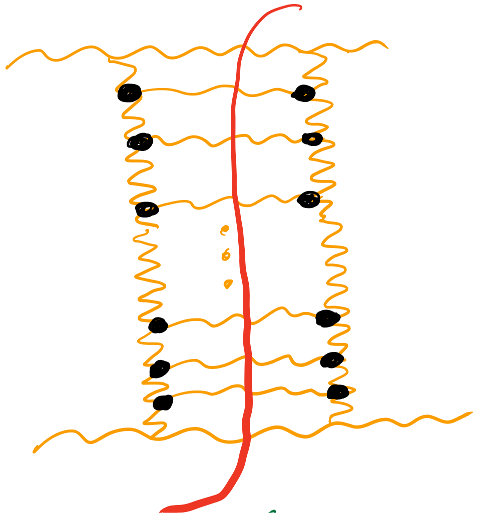
Total cross-sections across three orders of magnitude in energy (SPS -> LHC) simply described in terms of Pomeron and Reggeon trajectories

What is the Pomeron in QCD?



Donnachie, Landshoff, Phys. Lett. B750 (2015) 669

QCD Pomeron



Pomeron: t-channel exchange with vacuum quantum numbers
In pQCD: compound color singlet exchange of two dressed ("reggeized") gluons

Odderon: responsible for difference in proton-proton and proton-anti-proton cross-sections In pQCD: compound color singlet exchange of three reggeized gluons

Odderon discovery claim, TOTEM+D0, PRL 127 (2021) 062003

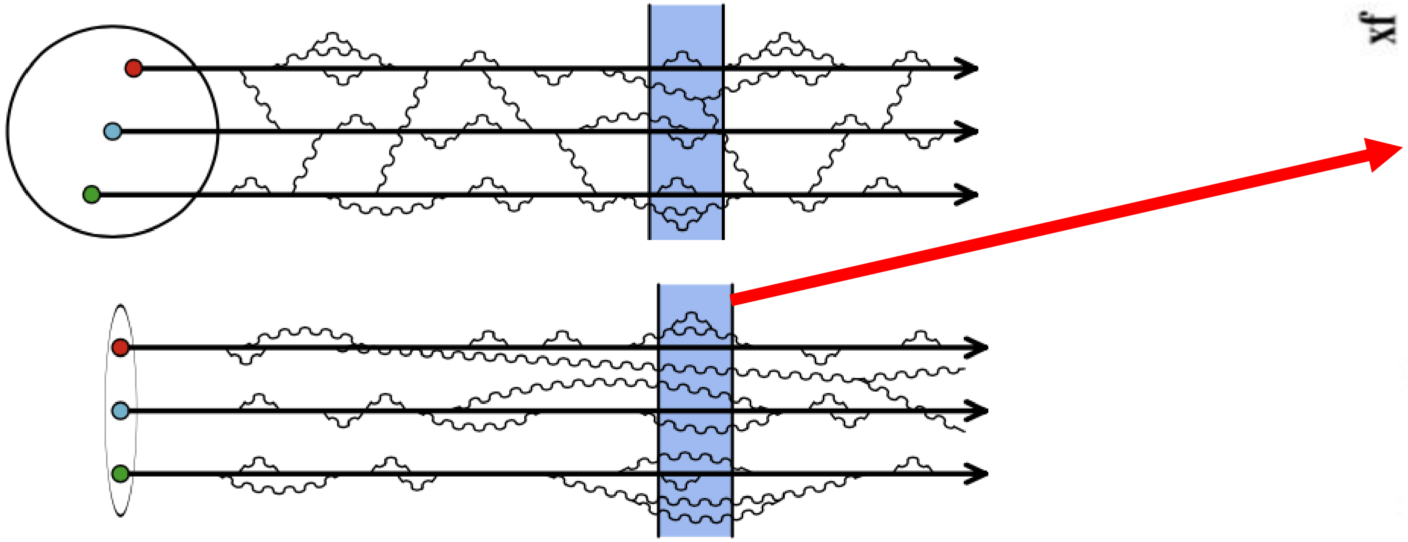
The Pomeron (+Odderon) in QCD

Unsolved problem: Is the Pomeron a robust object in QCD?

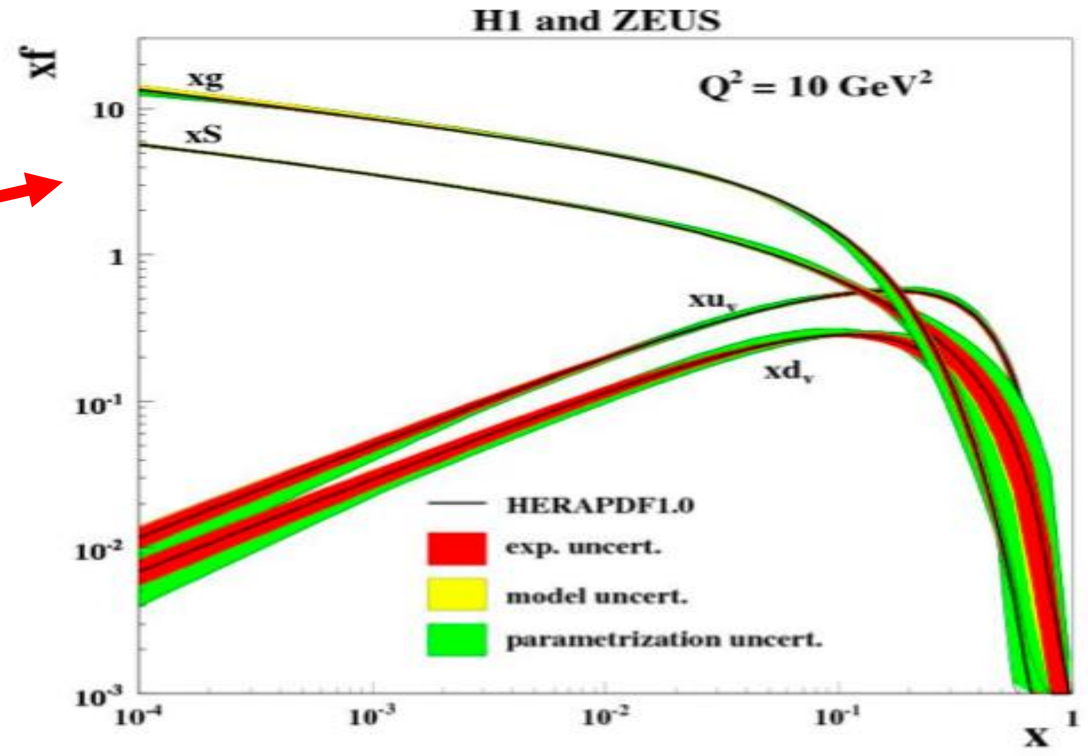
If so, unitarity (Froissart bound) + (more mundane) diffractive dissociation demand multi-Pomeron dynamics/exchanges

All evidence points to Pomeron dynamics being dominated by glue

Parton distributions
HERA DIS collider

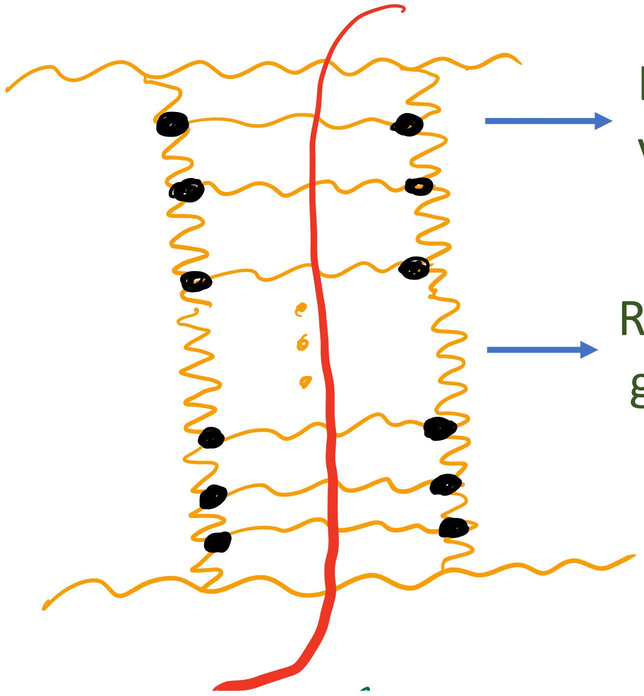


Parton model picture: from arXiv:0708.0047

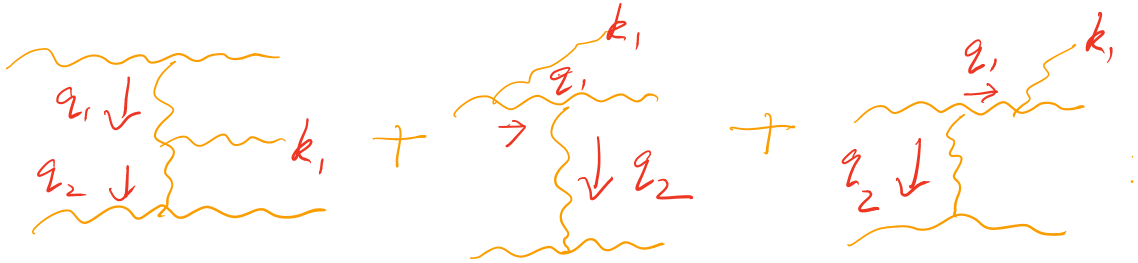


The BFKL Pomeron

Sophisticated construction to describe 2→ N scattering in multi − Regge kinematics

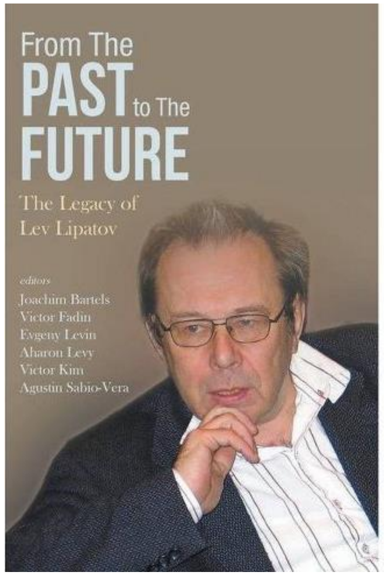


Lipatov vertex



Reggeized gluon

$$\equiv \frac{1}{t_i} \rightarrow \frac{1}{t_i} e^{\alpha(t_i)(y_{i-1} - y_i)}$$



Lev Lipatov

Fadin, hep-ph/9807528

$$\alpha^{(2)}(t) = \kappa_\Gamma^2 \left(\frac{\mu^2}{-t}\right)^{2\epsilon} \left(\frac{\beta_0}{\epsilon^2} + \frac{\gamma_K^{(2)}}{8\epsilon} + \frac{\gamma_\Lambda^{(2)}}{2} + \zeta_2 \beta_0\right) + \mathcal{O}(\epsilon)$$

$\gamma_K^{(2)}$: Two loop cusp anomalous dimension

$\gamma_\Lambda^{(2)}$: Two loop wedge anomalous dimension

$$\sigma_{tot} = 2 \, \mathrm{Im} A(s, t=0)$$

$$= s^\lambda \text{ with } \lambda = \frac{4\alpha_s N_c \ln 2}{\pi}$$

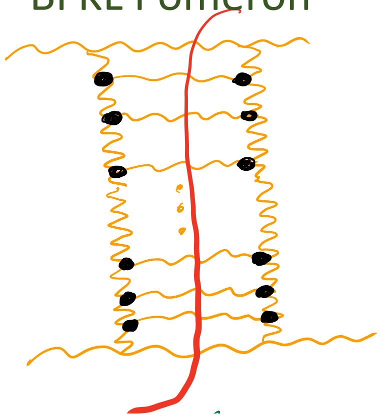
$$\simeq 0.5 \text{ for } \alpha_s = 0.2$$

RG described by BFKL Hamiltonian:

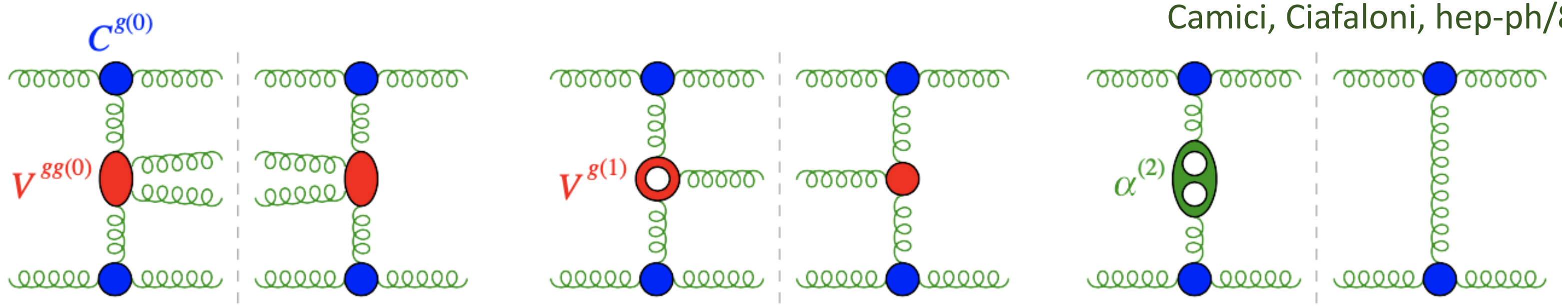
Remarkable properties: holomorphic separability; generalization to an integrable model; Large body of beautiful work in N=4 SUSY

The NLL BFKL equation

BFKL Pomeron

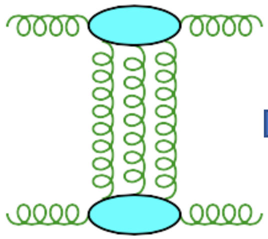


Fadin, Lipatov, hep-ph/9802290
Camici, Ciafaloni, hep-ph/8903389



Regge factorization at NLL $[\alpha_S(\alpha_S \text{ Ln}(s/t))^n]$: Includes one loop corrections to the Lipatov vertex ($V^{g(1)}$) and two loop corrections to the Regge trajectory ($\alpha^{(2)}$)

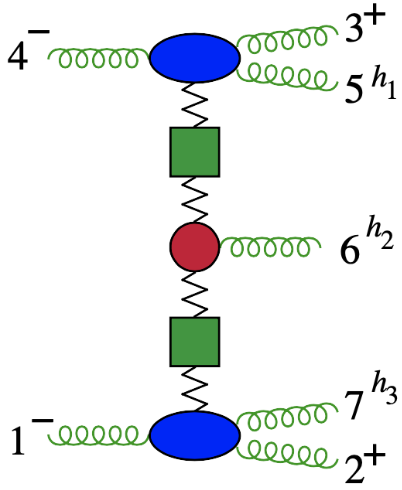
Beyond NLL:



Three reggeized gluon exchange corresponds to Regge cut in angular momentum plane – this can be computed

Falcioni et al., arXiv: 2111.10664, arXIv:2112.11098

Multi-Regge limit of planar SYM $\mathcal{N} = 4$:



At large t'Hooft coupling, AdS/CFT duality between amplitudes and minimal area surfaces with closed light-like polygon boundaries

Dual conformal tranformations → BDS ansatz; rich mathematical structure of MHV amplitudes in multi-Regge kinematics

BDS: Bern, Dixon, Smirnov
See for example, Dixon, Liu, Miczajka, arXiv:2110.11388

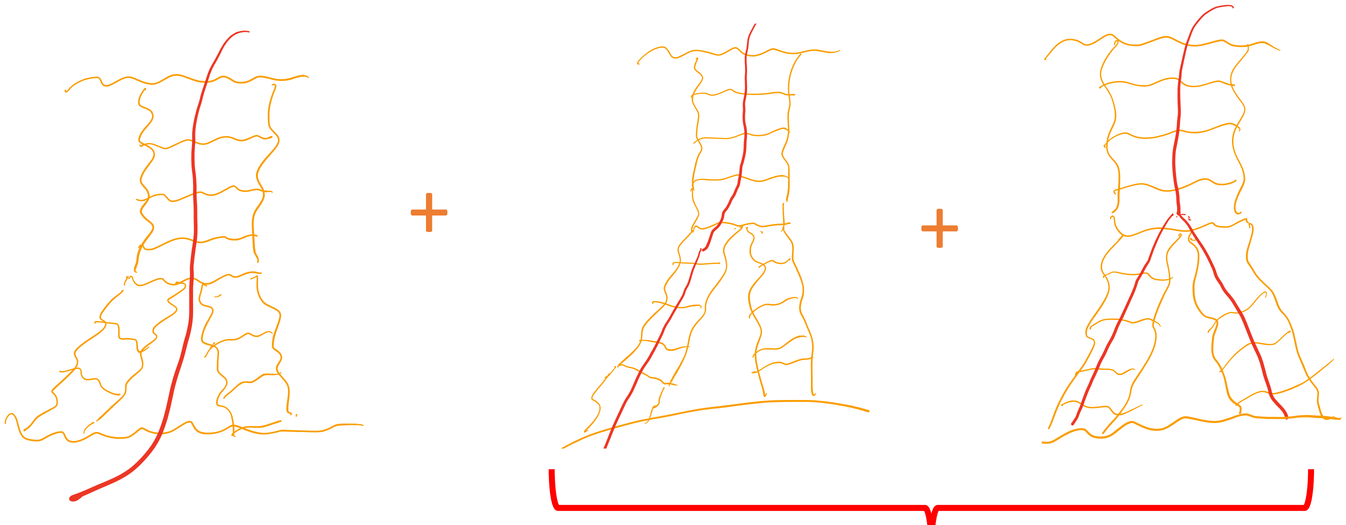
Figures from excellent review of state-of-the art:
Del Duca, Dixon, arXiv:2203.13026

Breakdown of the OPE: Multi-Pomeron and Reggeon exchange

Rapid BFKL growth leads to large phase-space occupancy N at high energies
→ novel many-body gluodynamics

Gribov,Levin,Ryskin (1983)
Mueller, Qiu (1986)

Partons recombine and screen – many-body “shadowing”



Recombination

Screening

$$= -\frac{[\alpha_s G(x,Q^2)]^2}{Q^2 R^2}$$

A fascinating equilibrium of splitting and recombination should eventually result.
It is a considerable theoretical challenge to calculate this equilibrium in detail…

F. Wilczek, Nature (1999)

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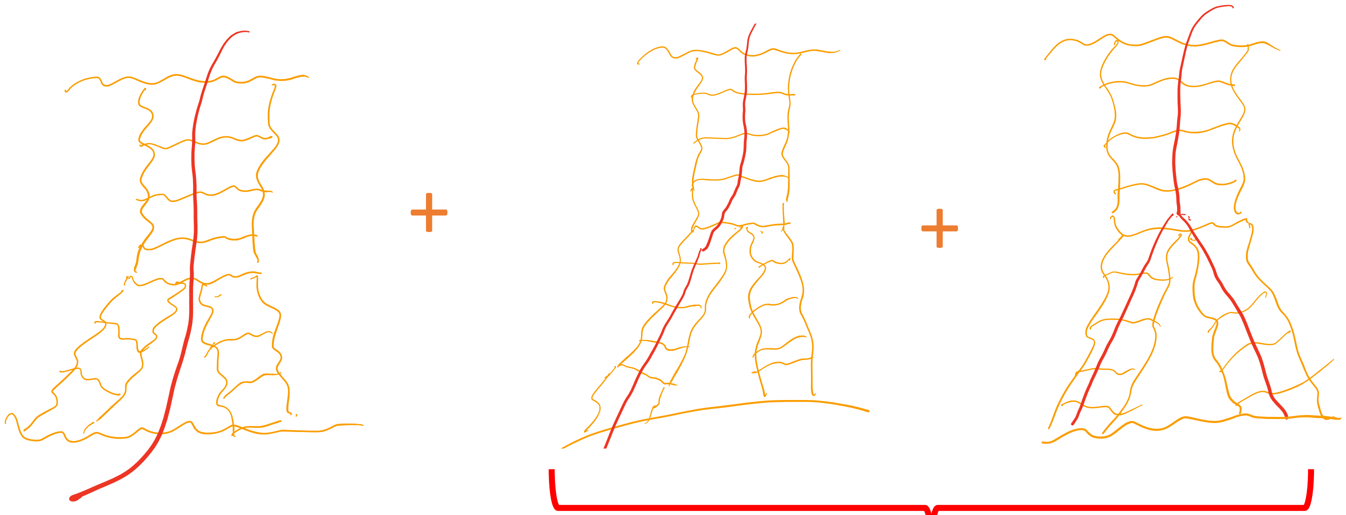


$$= -\frac{\left[\alpha_s G(x,Q^2)\right]^2}{Q^2 R^2}$$

Recombination

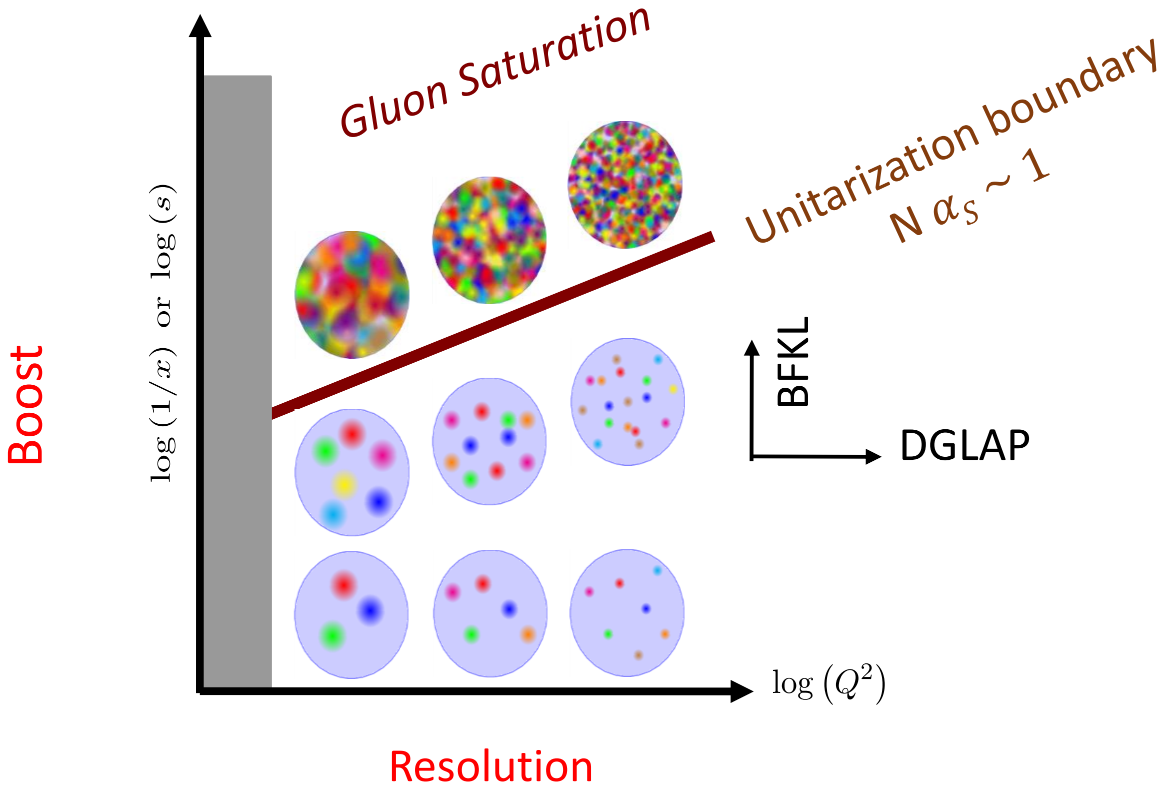
Screening

”Death by a million cuts”: All-twist power suppressed contributions equally important for

$$N \equiv \frac{xG_A(x,Q_S^2)}{2(N_c^2-1)\pi R_A^2 Q_S^2} = \frac{1}{\alpha_S(Q_S)}$$

$N \to \frac{1}{\alpha_S}$ = classicalization!

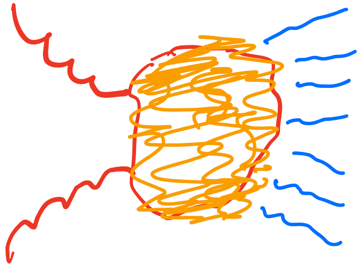
Classicalization + perturbative unitarization: gluon saturation



Unitarization boundary defined by emergent close packing scale

$Q_S(x) >> \Lambda_{QCD}$, $\alpha_S(Q_S) << 1$
- defines lump scale within hadron

$2 \to N$



$$P_{2\to N} \sim e^{S} \alpha_s^N N!$$

If $N \sim \frac{1}{\alpha_s}$

$$P_{2\to N} \sim e^{S} \alpha_s^N \left(\frac{1}{\alpha_s}\right)^N e^{-1/\alpha_s}$$

Exponential suppression of high occupancy states (classical lumps) unless

$S \sim 1/\alpha_s \sim N$

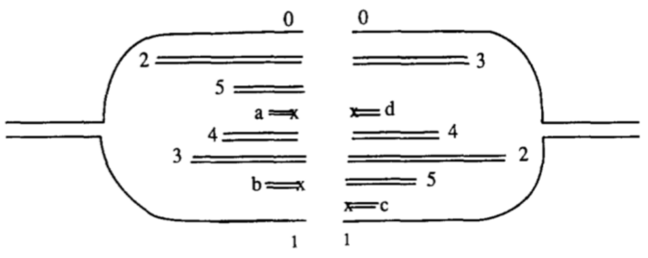
$$\Rightarrow P_{2\to N} \sim O(1)$$

Dvali, RV, arXiv:2106.11989, PRD (2022)

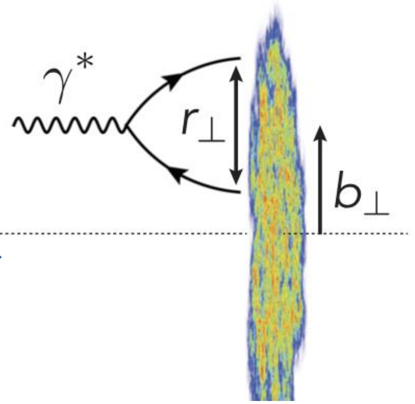
Classicalization and perturbative unitarization: gluon saturation

s-channel ”dipole” scattering picture

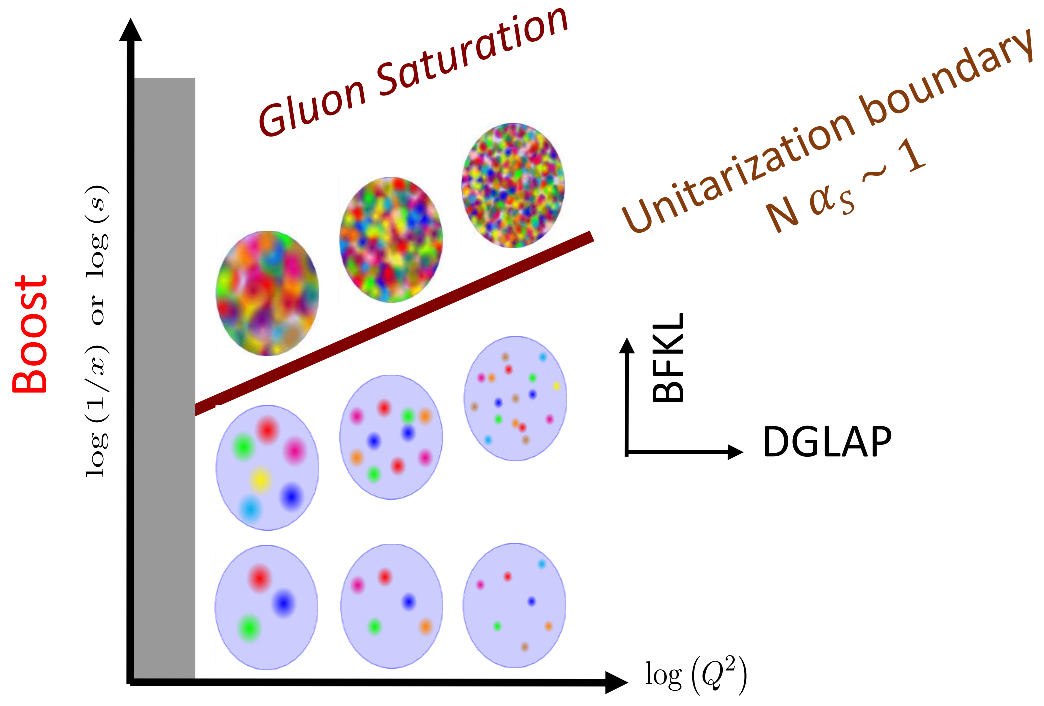
Mueller, NPB415 (1994) 373
Mueller, Patel, hep-ph/9403256



Boosted projectile



Boosted target



$$\sigma_{q\bar{q}P}(r_\perp, x) = \sigma_0\,\left[1 - \exp\left(-r_\perp^2\, Q_s^2(x)\right)\right]$$

Emergent semi-hard scale $Q_s^2(x) = Q_0^2\left(\frac{x_0}{x}\right)^{\lambda}$ → BFKL eigenvalue

Color transparency for $r_\perp^2 Q_S^2$ << 1 ($\sigma \propto A$)

Color opacity (”black disk”) for $r_\perp^2 Q_S^2$ >> 1 ($\sigma \propto A^{2/3}$)

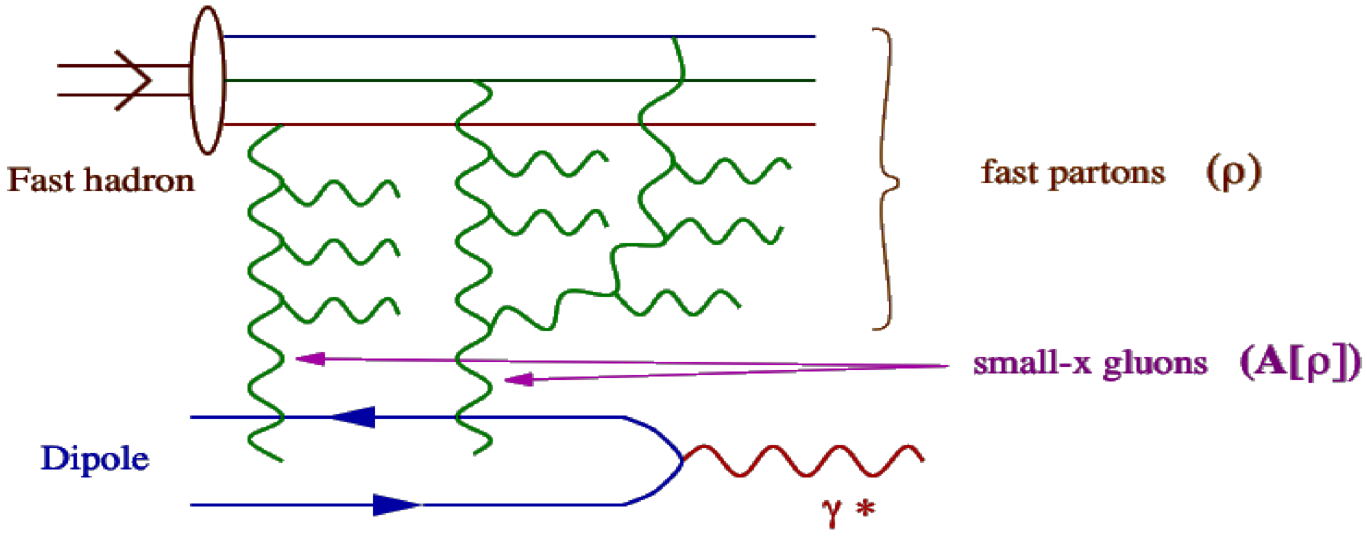
QCD picture of observed “shadowing” at small x

The Color Glass Condensate: classical EFT for Regge asymptotics

Born-Oppenheimer separation between fast and slow light-front modes

Large x (P^+) modes: static, strong ($\sim 1/g$) color sources ρ^a

Small x ($k^+ << P^+$) modes: fully dynamical gauge fields A_μ^a



$$\mathcal{Z}[j] = \int [d\rho] W_{\Lambda^+}[\rho] \left\{ \frac{\int^{\Lambda^+} [dA]\delta(A^+) e^{iS_{\Lambda^+}[A,\rho] - \int j\cdot A}}{\int^{\Lambda^+} [dA]\delta(A^+) e^{iS_{\Lambda^+}[A,\rho]}} \right\}$$

$W_{\Lambda^+}[\rho]$: nonpert. gauge inv. weight functional defined at initial $x_0 = \Lambda^+ / P^+$

$S_{\Lambda^+}[A,\rho]$: Yang-Mills action + gauge-inv. coupling of sources to fields (Wilson line)

CGC review: Gelis,Iancu,Jalilian-Marian,RV:arXiv 1002.0333

The Color Glass Condensate: classical EFT for Regge asymptotics

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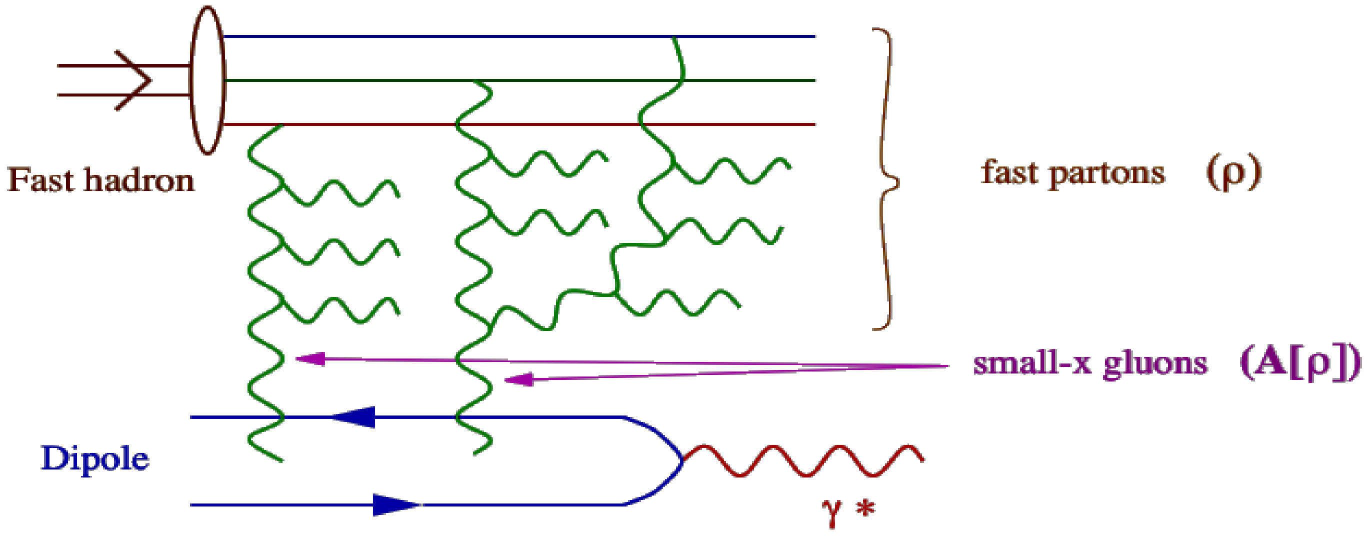


Explicit construction for large nuclei (large number of coherent sources of color charge at small x-large "Ioffe time")

$$W_{\Lambda^+}[\rho] \to \int [d\rho] \exp\left(-\int d^2x_\perp \left[\frac{\rho^a\rho^a}{2\mu_A^2} - \frac{d_{abc}\rho^a\rho^b\rho^c}{\kappa_A}\right]\right)$$

Pomeron configurations

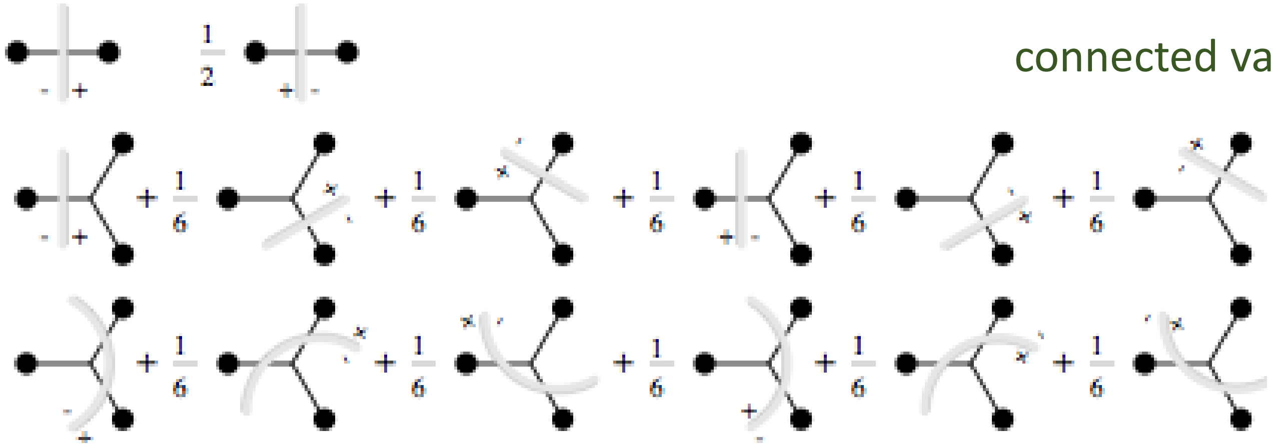
Odderon configurations

For A>>1, $\mu_A^2 \sim Q_S^2 \propto A^{1/3} >> \Lambda_{QCD}^2$
weak coupling EFT for large parton densities!

McLerran,RV (1994)

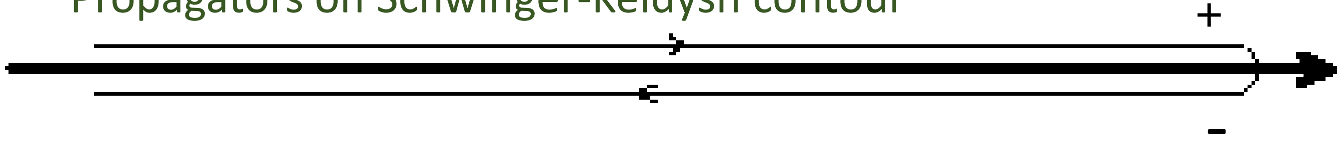
General all-order formalism: Cutkosky's rules in strong fields

$$2\,\mathrm{Im} \sum_{\text{conn.}} V =$$



connected vacuum graphs in $\lambda\phi^3$

Propagators on Schwinger-Keldysh contour



Well-known example: Schwinger pair production in strong field QED

Simple understanding of "AGK cutting rules" of Reggeon Field Theory: combinatorics of cut and uncut sub-graphs contributing to a given multiplicity

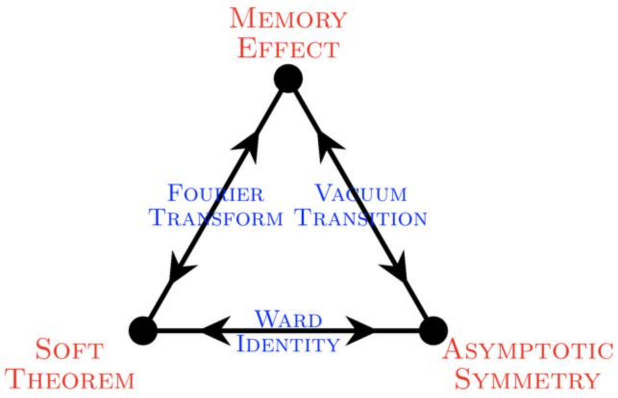
- Very general consequence of unitarity in strong fields
- Independent of language of Pomerons and Reggeons

AGK: Abramovsy,Gribov,Kancheli

Paradigm shift? Perhaps Pomerons best viewed as simplest constructions enforcing strong field unitarity rather than fundamental objects

Gelis,RV: hep-ph/0601209, hep-ph/0608117

Color memory in the CGC



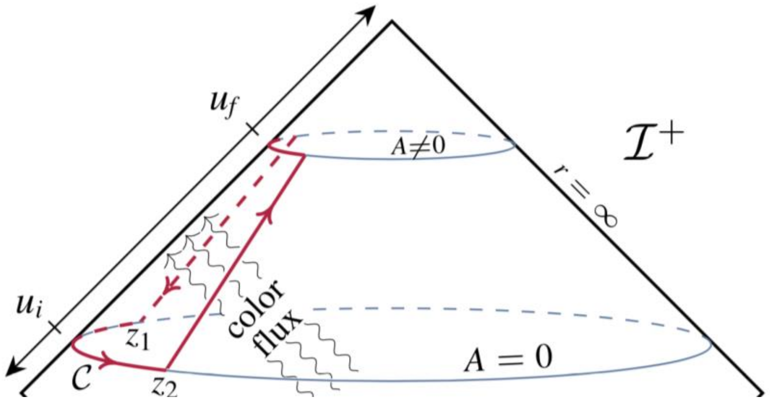
Strominger,arXiv:1703.05448

Static Yang-Mills shockwave wave solution in LC gauge

$A_i = 0$ | $A_i = -\frac{-1}{ig} U \partial_i U^\dagger$

$x^- = 0$

Transverse dynamics can be mapped on to celestial sphere at null infinity:



$(r, u, z, \bar{z}) \to (\lambda r, \lambda^{-1} u, \lambda^{-1} z, \lambda^{-1} \bar{z})$

Map: $x^+ = \sqrt{2r}$, $x^- = \frac{1}{\sqrt{2}}(u + rz\bar{z})$,

$x^1 + i x^2 = 2 r z$

For $\lambda \to \infty$:

$r \to \infty \;\to\; x^+ \to \infty, x^- \to 0$

Kick Q_S suffered by dipole is the color memory effect

Pate, Raclariu, Strominger, PRL (2017)

Ball,Pate,Raclariu,Strominger,RV, Ann. Phys. 407 (2019) 15

The Wilson lines $U = P \exp\left(i\int_y^\infty dy' \frac{\rho(x_t, y')}{\nabla_t^2} \right)$ are vertex operators on the celestial sphere

Satisfy a 2-D conformal Kac-Moody algebra

V.P. Nair (1988)

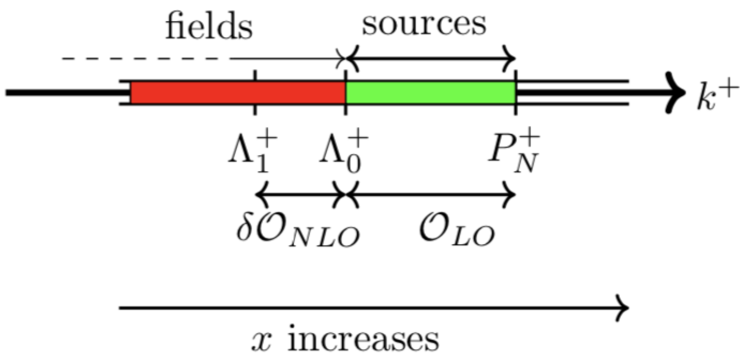
He,Mitra,Strominger (2015)

CGC EFT: RG hierarchy of many-body correlators

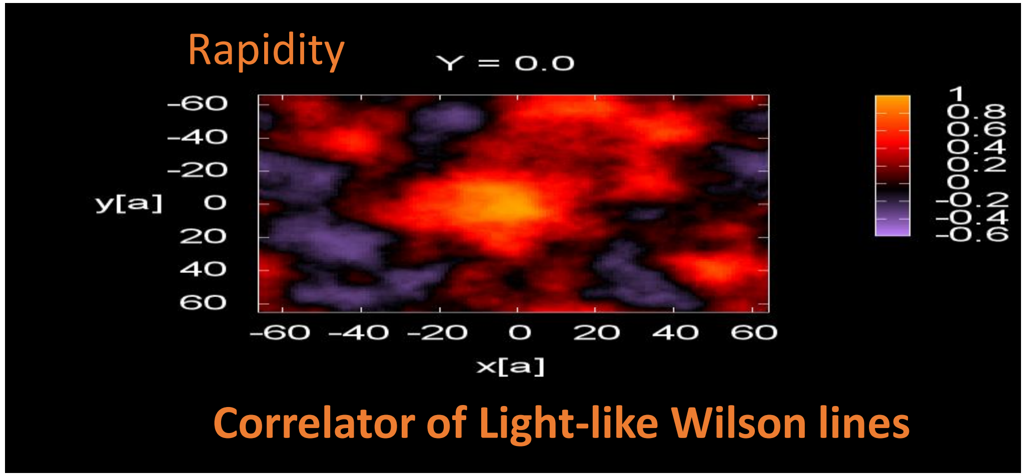
$$\frac{\partial}{\partial Y}\langle \mathcal{O}[\rho]\rangle_Y = \frac{1}{2}\langle \int_{x,y} \frac{\partial}{\partial \rho^a(x)} \chi^{ab}_{x,y} \frac{\partial}{\partial \rho^b(y)} \mathcal{O}[\rho]\rangle_Y$$

Rapidity"
→ time"

"diffusion coefficient": retarded Green function in strong field background



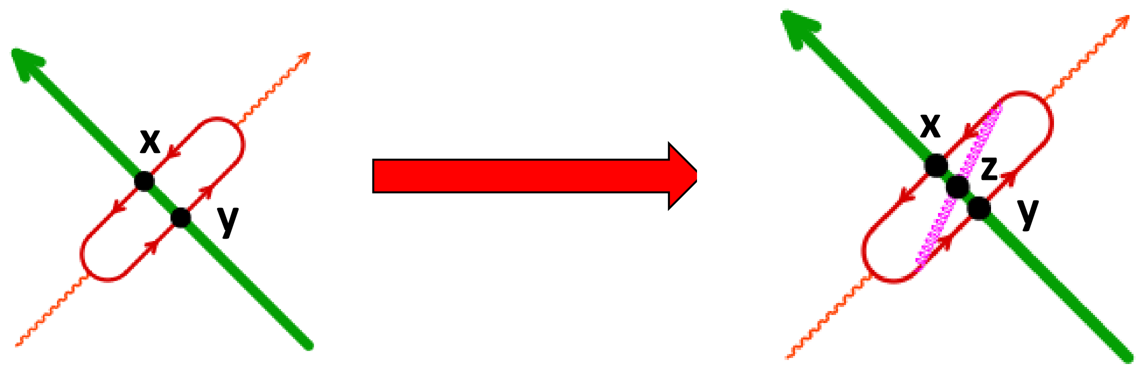
Langevin diffusion "wee" partons
in functional space of color fields
B-JIMWLK hierarchy of n-point Wilson line correlators

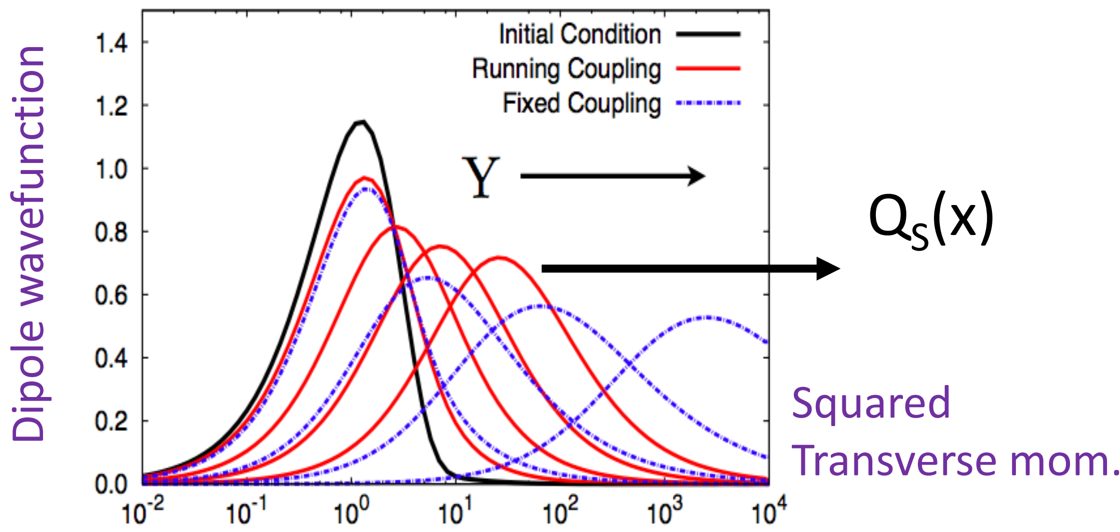


Dumitru,Jalilian-Marian,Lappi,Schenke,RV
PLB706 (2011)219

Balitsky, hep-ph/9509348
Jalilian-Marian,Kovner, Leonidov, Weigert, hep-ph/9706377
Iancu, Leonidov, McLerran, hep-ph/0011241

Inclusive DIS: dipole evolution in gluon shockwave background

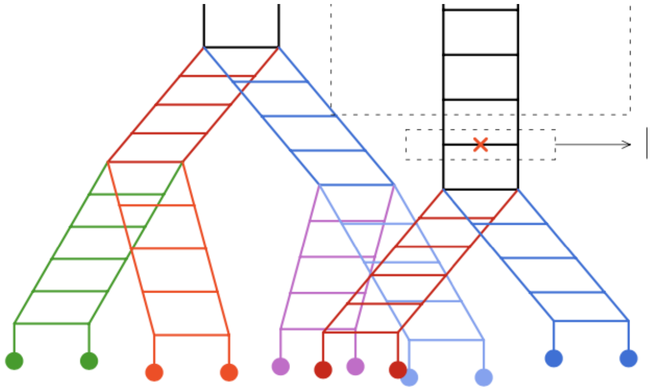




$$\frac{\partial}{\partial Y}\langle \mathrm{Tr}(V_x V_y^\dagger)\rangle_Y = -\frac{\alpha_S N_c}{2\pi^2}\int_{z_\perp} \frac{(x_\perp - y_\perp)^2}{(x_\perp - z_\perp)^2 (z_\perp - y_\perp)^2}\,\langle \mathrm{Tr}(V_x V_y^\dagger) - \frac{1}{N_c}\mathrm{Tr}(V_x V_z^\dagger)\,\mathrm{Tr}(V_z V_y^\dagger)\rangle_Y$$

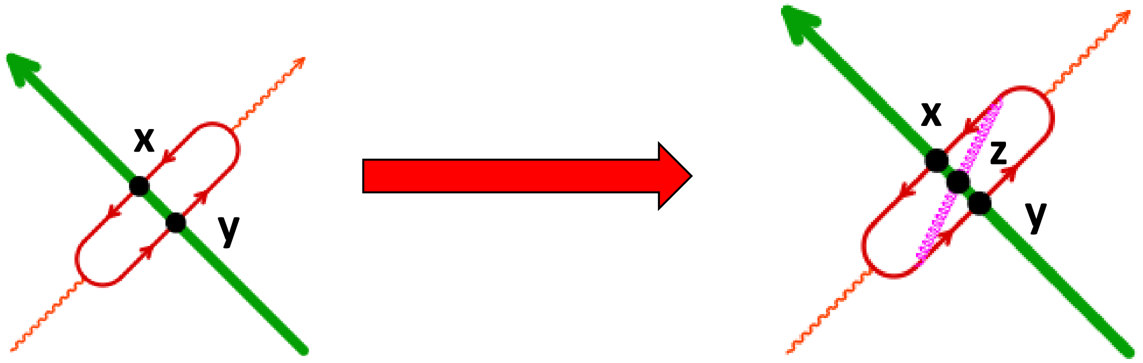
Y = Ln(1/x)

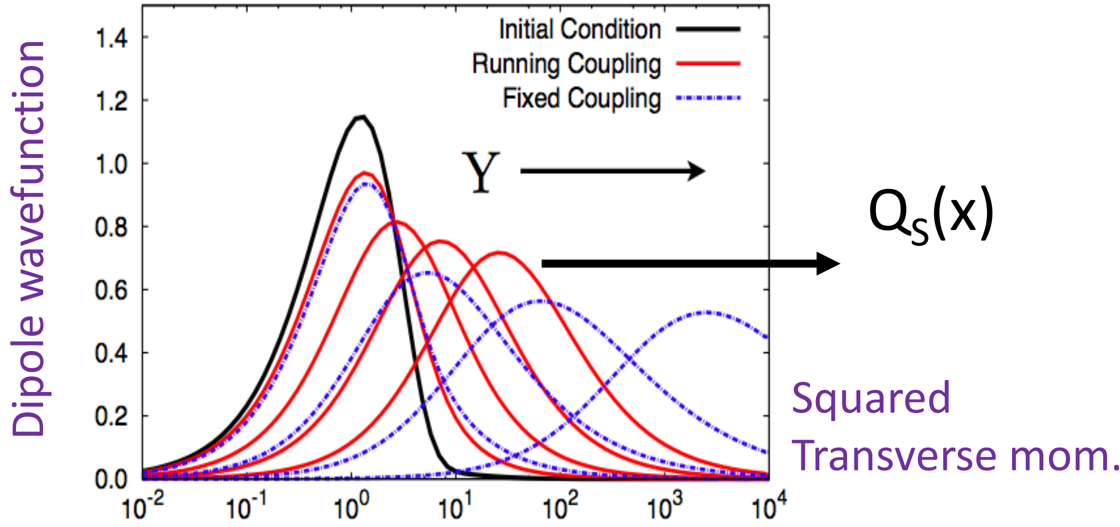
Closed form expression for A >>1, Nc→ ∞: non-linear Balitsky-Kovchegov (BK) eqn.



"Fan multi-Pomeron" diagrams

Inclusive DIS: dipole evolution in gluon shockwave background





B-JIMWLK RG eqn. for dipole Wilson-line correlator:

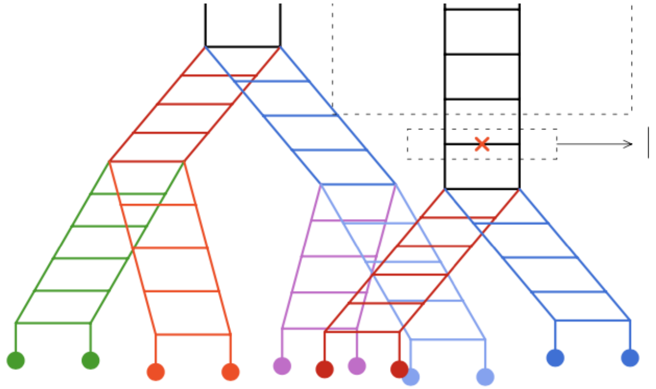
$$\frac{\partial}{\partial Y}\langle \mathrm{Tr}(V_x V_y^\dagger)\rangle_Y = -\frac{\alpha_S N_c}{2\pi^2}\int_{z_\perp} \frac{(x_\perp - y_\perp)^2}{(x_\perp - z_\perp)^2 (z_\perp - y_\perp)^2}\,\langle \mathrm{Tr}(V_x V_y^\dagger) - \frac{1}{N_c}\mathrm{Tr}(V_x V_z^\dagger)\,\mathrm{Tr}(V_z V_y^\dagger)\rangle_Y$$

Closed form expression for A >>1, $N_c \to \infty$: non-linear Balitsky-Kovchegov (BK) eqn.

Fixed point of evolution saturates cross-section for fixed impact parameter

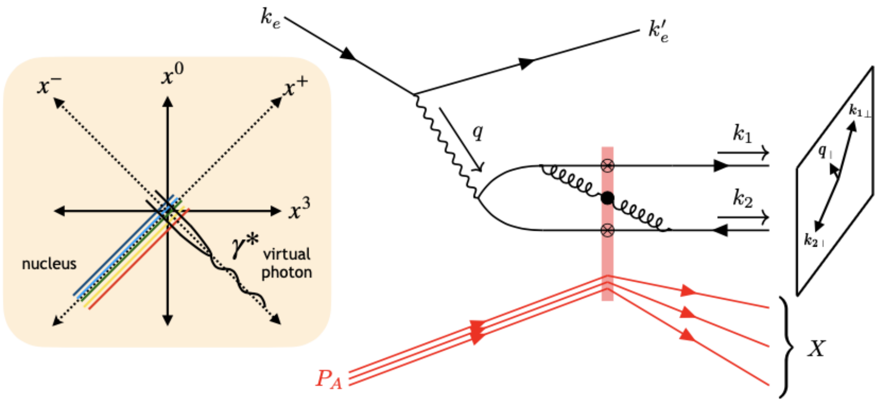
- this defines the close packing scale $Q_S(x)$

The BFKL equation is the low density $V \approx 1 - ig\rho/\nabla T^2$ limit of the BK equation…



Multi-Pomeron diagrams → BFKL ladder

Extracting the gluon Weizsäcker-Williams dist. at small x



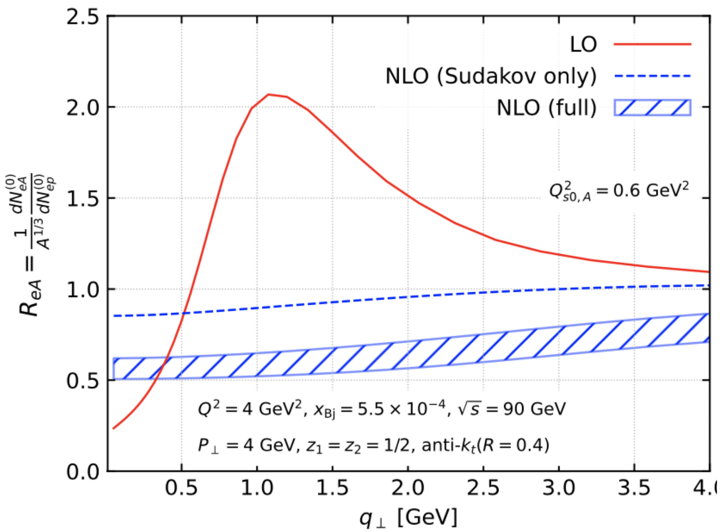
Back-to-back di-jets in DIS

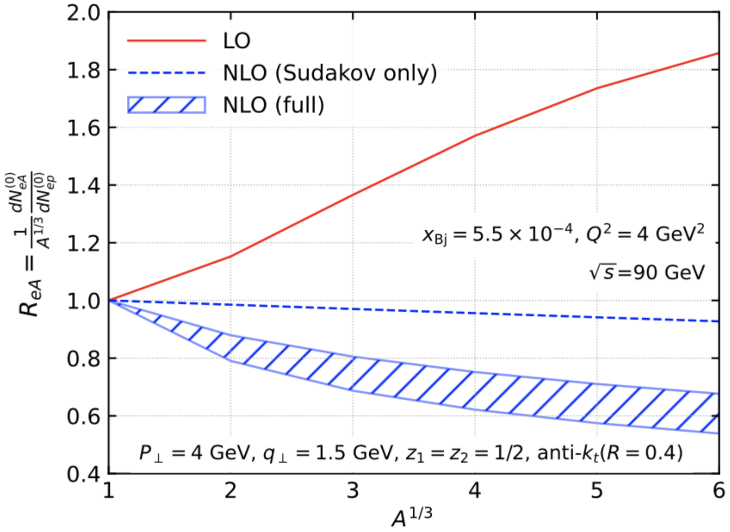
Factorization of small-x TMDs to NLO accuracy

$$\mathrm{d}\sigma^{(0),\lambda=\mathrm{T}} = \mathcal{H}_{\mathrm{LO}}^{0,\lambda=\mathrm{T}} \int \frac{\mathrm{d}^2\boldsymbol{B}_\perp}{(2\pi)^2} \int \frac{\mathrm{d}^2\boldsymbol{r}_{bb'}}{(2\pi)^2} e^{-i\boldsymbol{q}_\perp\cdot\boldsymbol{r}_{bb'}} \hat{G}^0_{\eta_c}(\boldsymbol{r}_{bb'},\mu_0)\mathcal{S}(\boldsymbol{P}_\perp^2,\mu_0^2)$$
$$\times\left\{1+\frac{\alpha_s(\mu_R)N_c}{2\pi}f_1^{\lambda=\mathrm{T}}(\chi,z_1,R)+\frac{\alpha_s(\mu_R)}{2\pi N_c}f_2^{\lambda=\mathrm{T}}(\chi,z_1,R)+\alpha_s(\mu_R)\beta_0\ln\left(\frac{\mu_R^2}{P_\perp^2}\right)\right\}$$
$$+\mathcal{H}_{\mathrm{LO}}^{0,\lambda=\mathrm{T}} \int \frac{\mathrm{d}^2\boldsymbol{B}_\perp}{(2\pi)^2} \int \frac{\mathrm{d}^2\boldsymbol{r}_{bb'}}{(2\pi)^2} e^{-i\boldsymbol{q}_\perp\cdot\boldsymbol{r}_{bb'}} \hat{h}^0_{\eta_c}(\boldsymbol{r}_{bb'},\mu_0)\mathcal{S}(\boldsymbol{P}_\perp^2,\mu_0^2)$$
$$\times\frac{-2\chi^2}{1+\chi^4}\left\{\frac{\alpha_s(\mu_R)N_c}{2\pi}\left[1+\ln(R^2)\right]+\frac{\alpha_s(\mu_R)}{2\pi N_c}\left[-\ln(z_1z_2R^2)\right]\right\}+\mathcal{O}\left(\frac{q_\perp}{P_\perp},\frac{Q_s}{P_\perp},\alpha_sR^2,\alpha_s^2\right)$$

$\hat{G}^0$ and $\hat{h}^0$ respectively are unpolarized and linearly polarized WW distributions, $\mathcal{S}$ the Sudakov soft factor resumming double+single logs in P_T/q_T

Caucal,Salazar,Schenke,Stebel,RC, arXiv:2308.00022





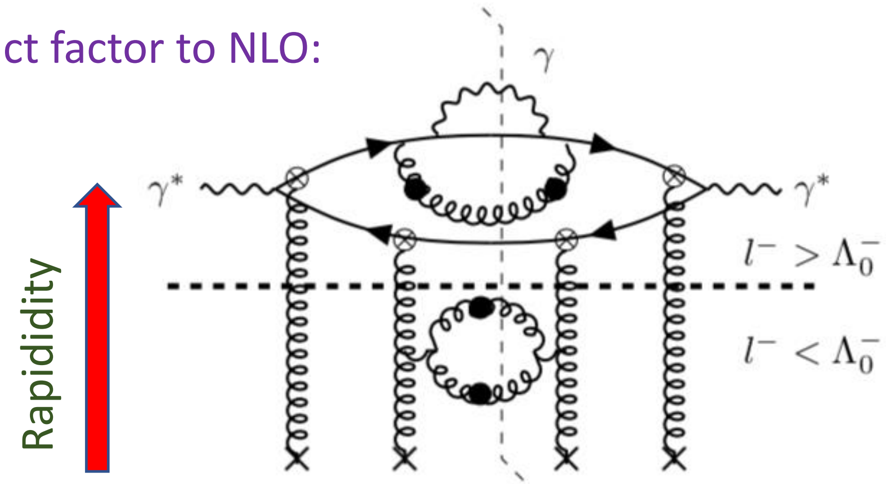
Global analyses to extract "universal" TMDs from p+A collisions at the LHC and e+A collisions from the EIC

Large # of inclusive, semi-inclusive, exclusive and diffractive final states

From LO+LLx to NLO+NLLx

State of the art:

Impact factor to NLO:



Evolution to NLLx:
$O(\alpha_S^2 \ \mathrm{Ln}(\frac{1}{x}))$

(Dressed "shockwave" propagators include coherent multiple scatterings to all orders)

Small x evolution:

NLO BFKL: Fadin, Lipatov (1998)
NLO JIMWLK: Balitsky, Chirilli, arXiv:1309.7644, Grabovsky, arXiv:1307.5414
Caron-Huot, arXiv:1309.6521, Kovner,Lublinsky,Mulian, arXiv:1310.0378,
Lublinsky, Mulian, arXiv:1610.03453
NNLO BK (SYM): Caron-Huot, Herranen (2018)

Resummed NLLx:

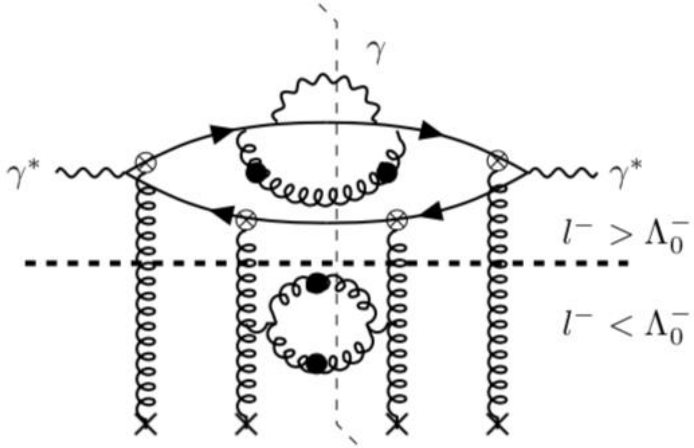
Salam (1999); Ciafaloni,Colferai,Salam,Stasto (1999-2004)
Ducloue,Iancu,Madrigal,Mueller,Soyez,Triantaffyllopoulos (2015-2019)

NLO impact factors:

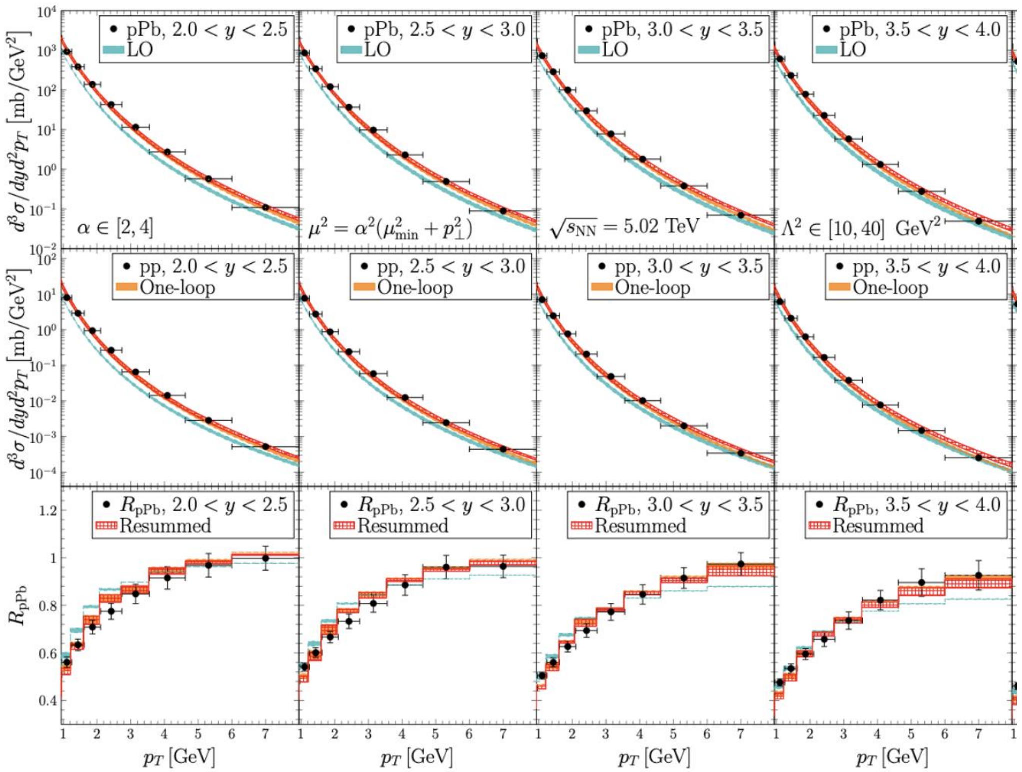
Inclusive DIS: Balitsky,Chirilli (2013)
Diffractive DIS: Boussarie,Szymanowski,Wallon (2016)
Massive quarks: Beuf,Lappi,Paatelainen (2021)
p+A forward di-jets: Iancu,Mulian (2021)
Photon+di-jet in DIS: Roy,RV (2020)
DIS di-jets/di-hadrons: Caucal,Salazar,RV (2021); Caucal, Salazar, Schenke, RV (2022)
Taels, Altinoluk,Beuf, Marquet, arXiv:2204.11650; Bergabo, Jalilian-Marian, arXiv:2207.03606
+ 20 odd papers this year

CGC state of the art: global analysis of DIS+hadron-hadron collisions

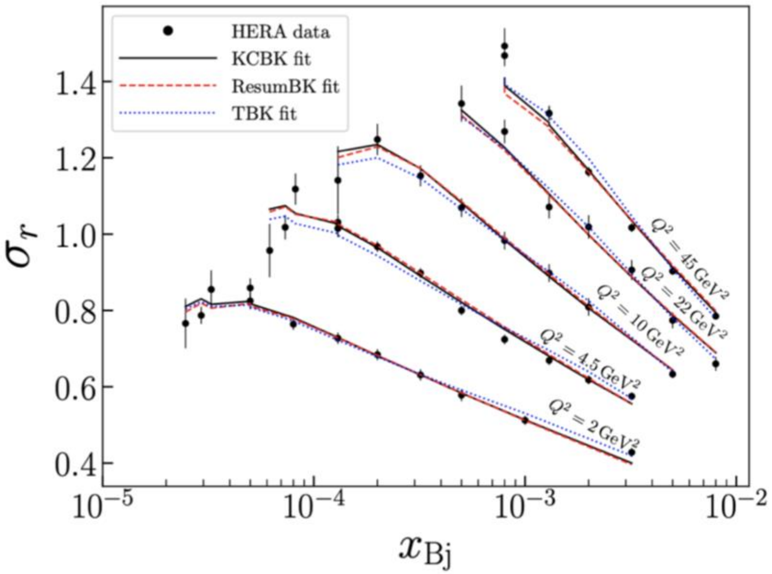
Precision CGC EFT computations



Single inclusive hadron distributions at the LHC



HERA DIS structure functions



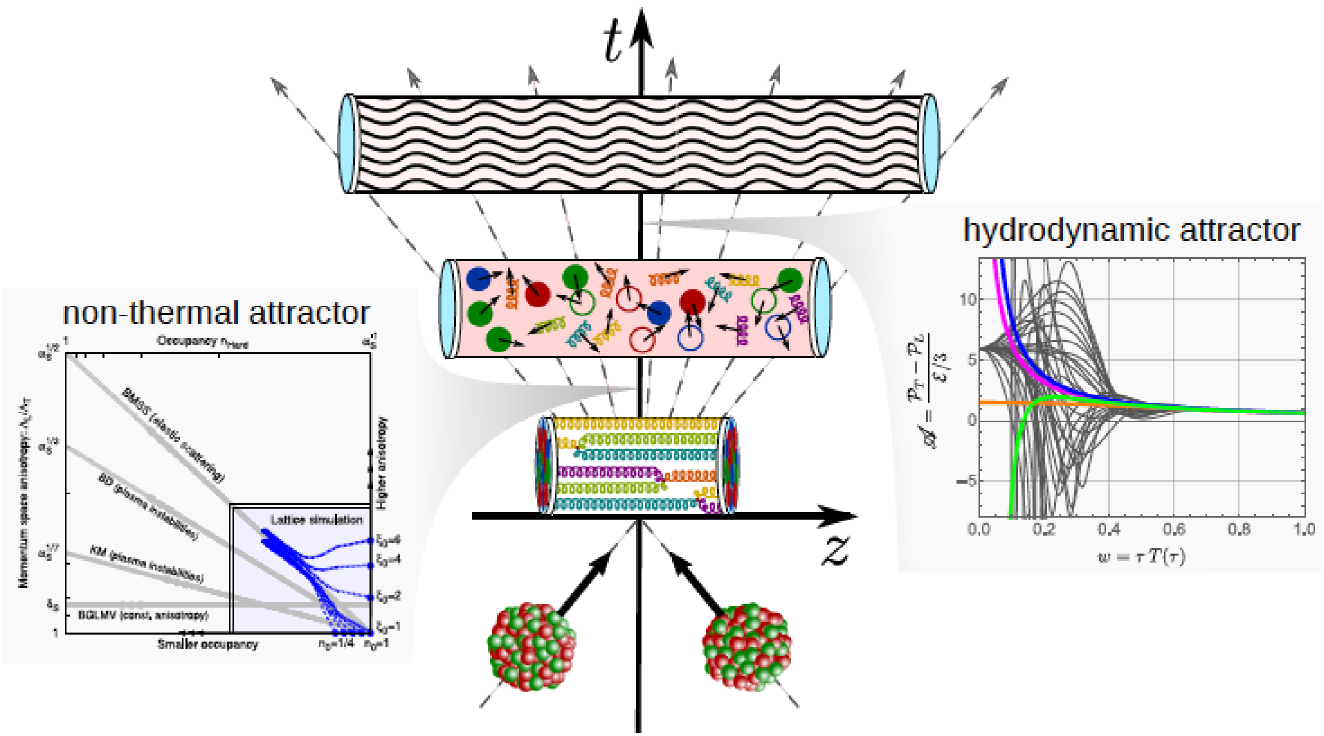
Beuf,Hanninen,Lappi,Mantysaari, arXiv:2007.01645

Shi,Wang,Wei,Xiao, arXiv:2112.06975

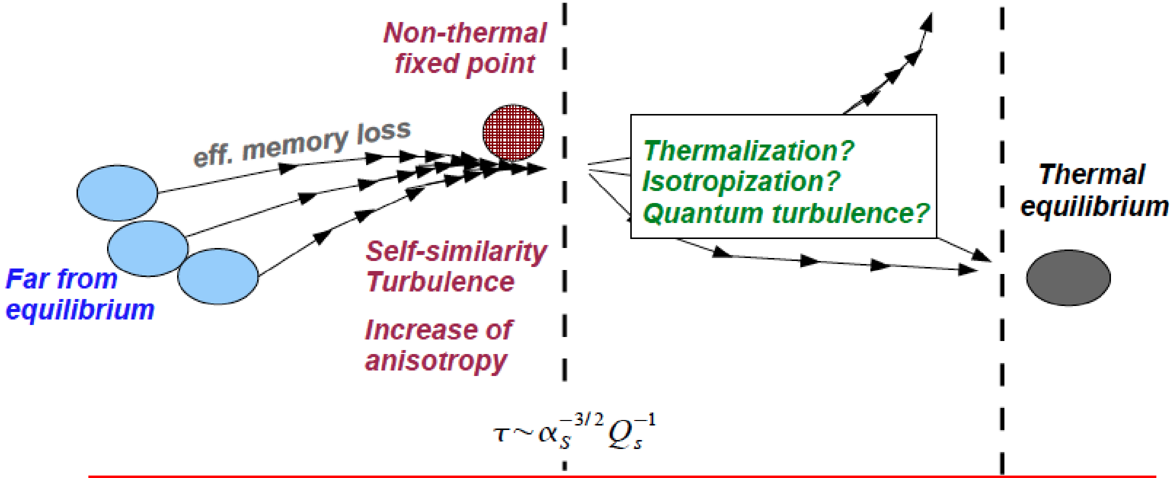
SURGE DOE Topical Theory Collaboration: 22 PI's from 16 institutions (2022-2027)

Spacetime evolution of a heavy-ion collision

Quark-Gluon Plasma undergoing hydrodynamic expansion



Collision of overoccupied Color Glass Condensate shockwaves



Thermal soft gluon bath for $\tau > \frac{1}{\alpha_S^{5/2}} \frac{1}{Q_S}$

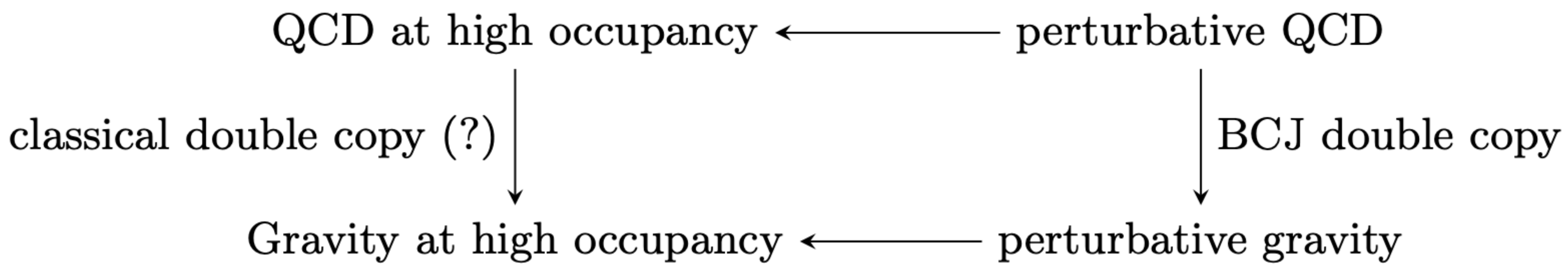
Thermalization temperature: $T_i = \alpha_S^{2/5} Q_S$

Very rapid thermalization
as $\alpha_S(Q_S) \rightarrow 0$ and $Q_S \rightarrow \infty$

Baier,Mueller,Schiff,Son, hep-ph/0009237

QCD thermalization: Ab initio approaches and interdisciplinary connections
Jürgen Berges, Michal P. Heller, Aleksas Mazeliauskas, and RV
Rev. Mod. Phys. **93**, 035003 (2021)

Double Copy: gluon → gravitational radiation in shockwave collisions



Monteiro,O'Connell,White, arXIv:1410.0239
Goldberger, Ridgeway, arXiv:1611.03493

Bern, Carrasco, Johannson,
arXiv: 1004.0476

In QCD, in the CGC EFT, strong field semi-classical methods powerful alternative to amplitudes approach
- RG equations in rapidity allow for quantitative study of approach to gluon saturation

Can we do the same for gravity in the strong field regime of trans-Planckian scattering?
Can we compute gravitational wave radiation with varying frequency and impact parameter to extract quantum features of GR, and obtain insight into BH formation?

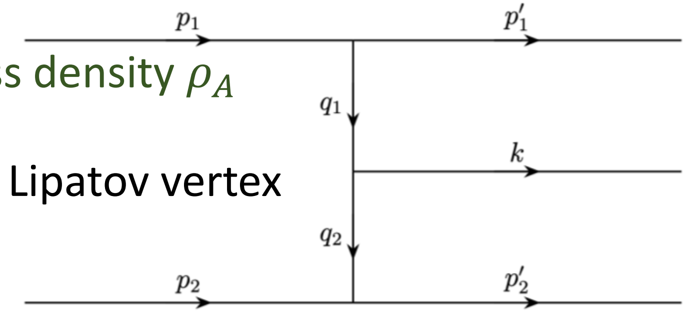
Eg., Amati, Ciafaloni, Veneziano, et al.

Derivation of Lipatov double copy from Einstein's eqns.

Solve Einstein's eqns. for linearized perturbations $h_{\mu\nu}$ around strong field shockwave metric for $R_S < b$

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$$

Shockwave A with mass density ρ_A



GR Lipatov vertex

Shockwave B with mass density ρ_B

Radiation amplitude:

$$-\boldsymbol{k}^2 \tilde{h}_{ij} = \int \frac{d\boldsymbol{q}_2}{(2\pi)^2} \frac{\rho_A(\boldsymbol{q}_1)}{\boldsymbol{q}_1^2} \frac{\rho_B(\boldsymbol{q}_2)}{\boldsymbol{q}_2^2} \Gamma_{ij}$$

where

$$\Gamma_{ij} = 2\left[\left(q_{2i} - k_i \frac{\boldsymbol{q}_{2\perp}^2}{\boldsymbol{k}_\perp^2}\right)\left(q_{2j} - k_j \frac{\boldsymbol{q}_{2\perp}^2}{\boldsymbol{k}_\perp^2}\right) - k_i k_j \frac{\boldsymbol{q}_{1\perp}^2 \boldsymbol{q}_{2\perp}^2}{\boldsymbol{k}_\perp^4}\right]$$

Lipatov double copy

$$\Gamma_{\mu\nu} \equiv \frac{1}{2} C_\mu C_\nu - \frac{1}{2} N_\mu N_\nu$$

C_μ is the QCD Lipatov vertex and
N_μ is the QED Bremsstrahlung vertex

A semi-classical computation in GR (completely analogous to prior QCD YM demonstration) can reproduce Lipatov's result obtained by Feynman diagram computations in multi-Regge kinematics

Himanshu Raju, RV, in preparation.
In QCD, analogous derivation of Lipatov vertex from gluon shock wave collisions:
Blaizot, Gelis, RV, hep-ph/0402256, Gelis, Mehtar-Tani, hep-ph/0512079

Concluding remarks

Significant progress in understanding realtime dynamics in QCDs Regge limit
using strong field weak coupling methods.

May inspire a novel way to think about strong field dynamics at large coupling

Rich interdisciplinary connections – heavy-ion collisions, cold atoms, and now GR.

Another interesting small-x phenomenon I don't have time to discuss:
the quenching of proton's spin due to sphaleron-like transitions
– can be ruled out (or not) at the EIC
Tarasov, RV, arXIv:2109.10370 (PRD 2022)