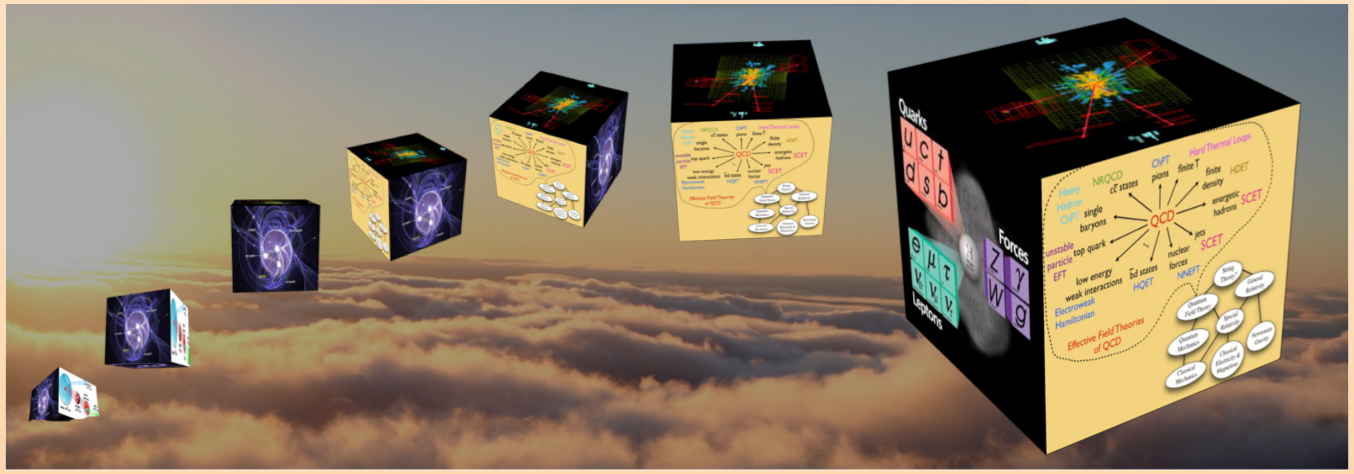
Effective Field Theories of QCD

Iain Stewart

50 Years of Quantum Chromodynamics, UCLA

September 13, 2023

Massachusetts Institute of Technology

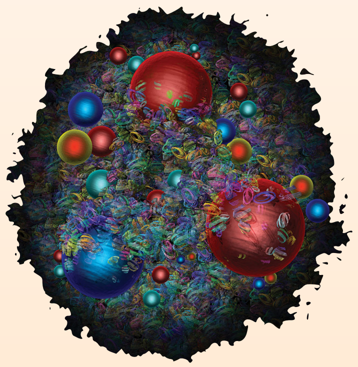


QCD is the richest known QFT

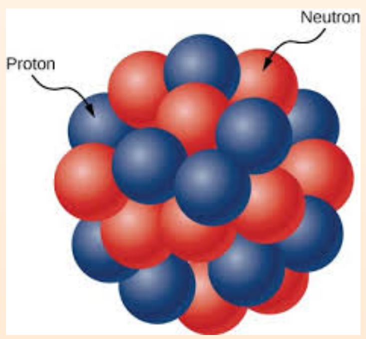
$$\mathcal{L}_{\mathrm{QCD}} = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu a} + \bar{\psi}_i (i \not{D} - m_i) \psi_i$$

Responsible for a plethora of interesting states, phenomena, and fields of physics

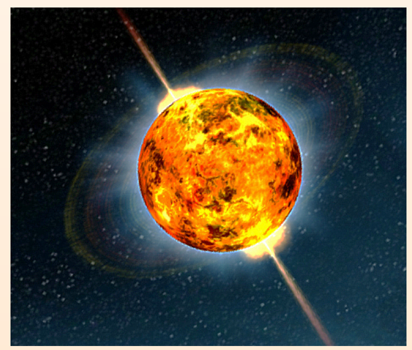
Hadrons



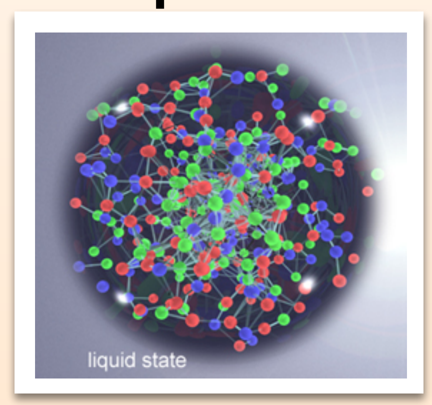
Nuclei



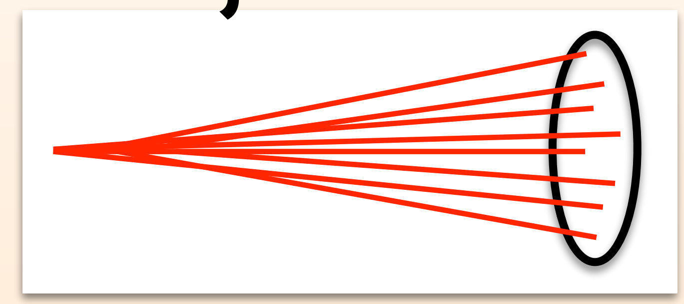
Neutron stars



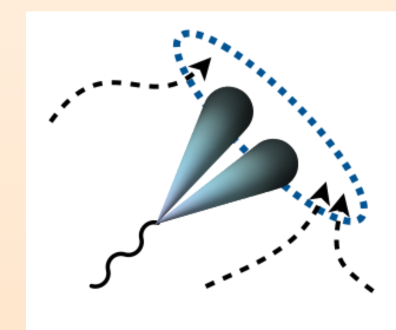
Quark-gluon plasma



jets



jet substructure



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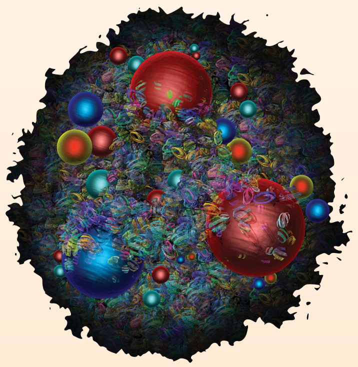

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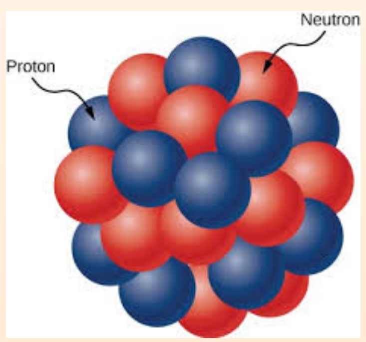

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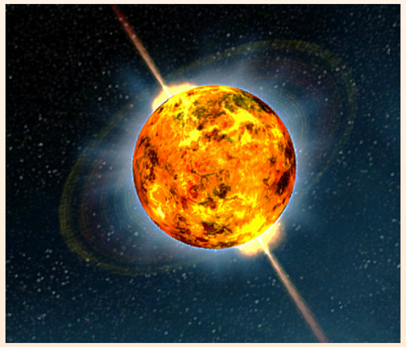

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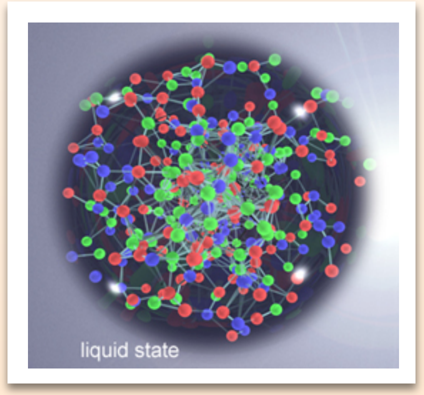
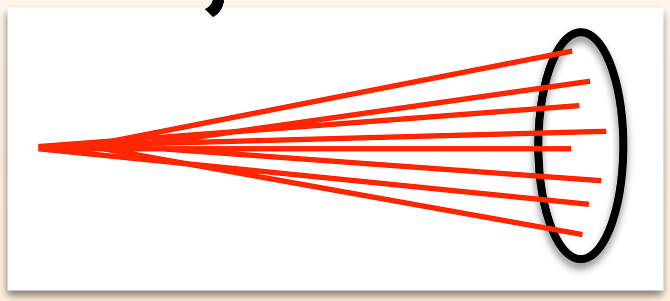


jets



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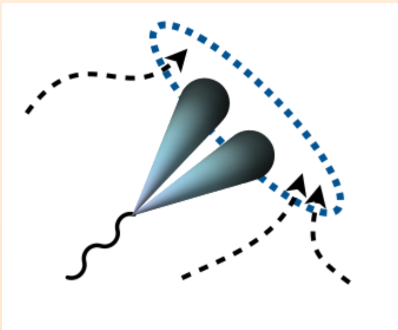

- Asymptotic Freedom   Gross, Wilczek, & Politzer
- Confinement
- Chiral Symmetry breaking, pions
- QCD phases and phase transitions
- Non-relativistic confined quarks $Q\bar{Q}$
- Exotic bound states, X, T, Z …
- large Nc …

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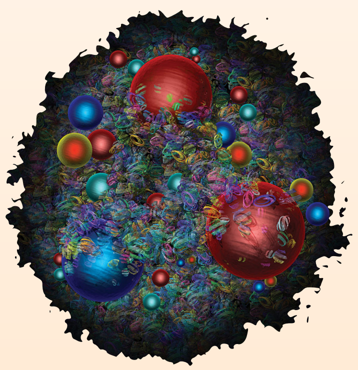

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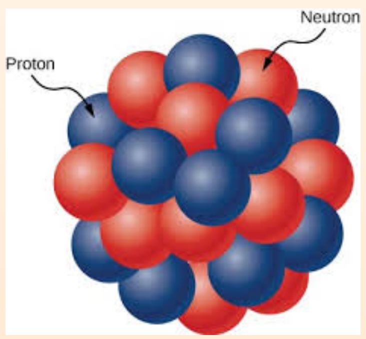

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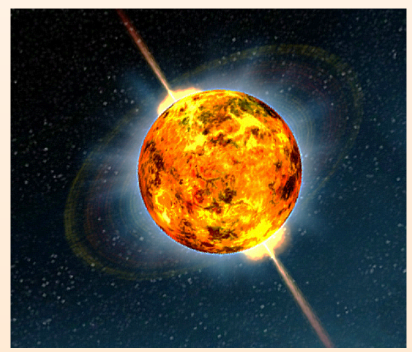

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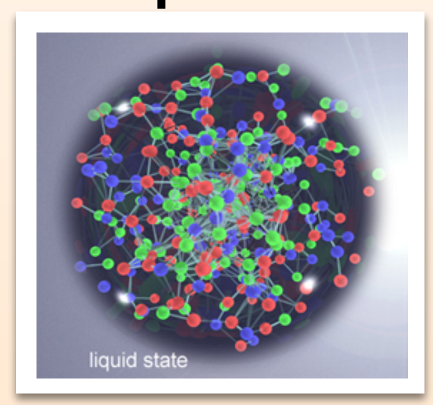

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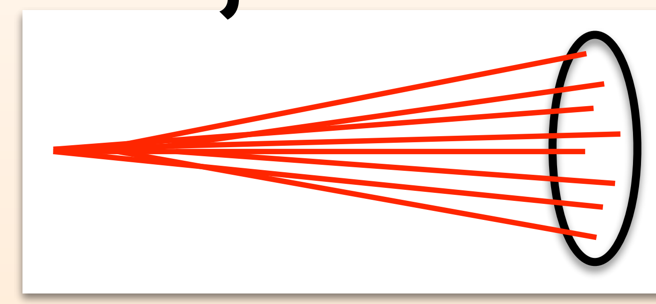

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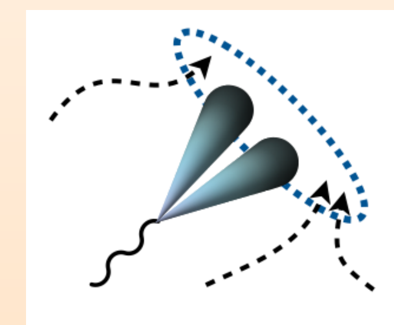

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- Collider physics
- Factorization & Resummation
- Gluon saturation
- Multiloop QFT, Amplitudes
- Flavor physics
- Lattice QCD
- Models

…

Effective Field Theory

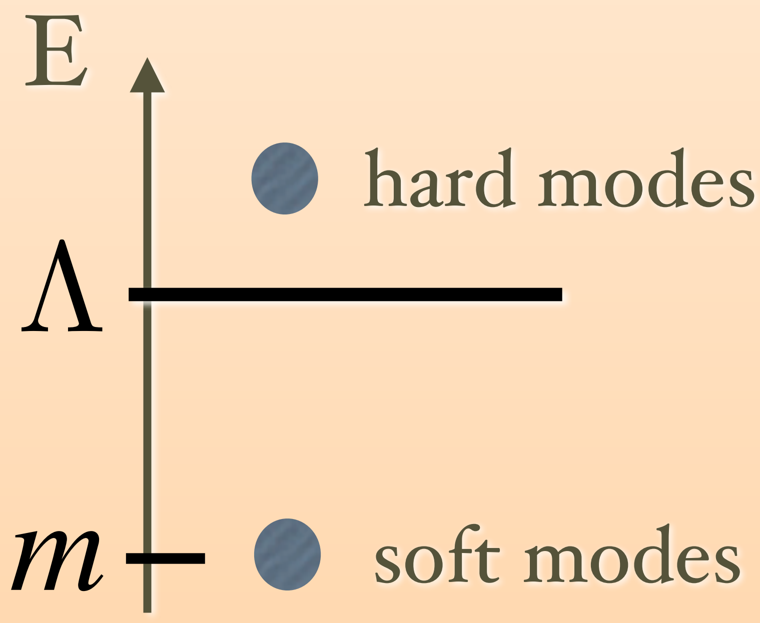
$$\mathscr{L} = \mathscr{L}^{(0)} + \mathscr{L}^{(1)} + \mathscr{L}^{(2)} + \mathcal{O}(\epsilon^3) \qquad \epsilon \ll 1$$

Goals:

- Find simplest framework that captures the essential physics $\mathscr{L}^{(0)}$, while identifying suitable expansion parameters ϵ
- Focus on IR dynamics, simplify the description of UV physics
- Organize in a manner that can be corrected to arbitrary precision

Key Idea:

- Decoupling. To describe physics at an IR scale m we do not need to know the detailed dynamics of what is going with heavy or off shell particles



$$\Lambda^2 \gg m^2$$

Ken Wilson (Nobel Prize '82)

Effective Field Theory

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Method: Determine relevant

- degrees of freedom — what fields?
- symmetries — what interactions?
- expansion parameters — power counting

Why?

- Simplifies calculations, eliminates baggage of more general theory
- Makes approximations explicit. Forced to consider uncertainties.

Modern attitude: every QFT is an EFT.

"a new and cooler view", Weinberg

my free EFT online course: https://courses.mitxonline.mit.edu/learn/course/course-v1:MITxT+8.EFTx+3T2022/home

Effective Field Theory

$$\mathscr{L} = \mathscr{L}^{(0)} + \mathscr{L}^{(1)} + \mathscr{L}^{(2)} + \mathcal{O}(\epsilon^3) \qquad\qquad \epsilon \ll 1$$

Concepts:

- Renormalization order by order in ϵ
- Field Freedom (field redefinitions)
- Top-Down EFT versus Bottom-Up EFT
- Matching and Decoupling
- Power counting equivalent to operator dimension, or more general
- Renormalization Group Evolution
- Universality of short and long distance parameters (functions)

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QCD very often in the vanguard for formulating these concepts

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Understood early on for tree level analyses

Also true with loops, spont. broken theories, etc.

H. Georgi "Onshell EFT", `91

C. Arzt, `93

also CCWZ `69

Effective Field Theory

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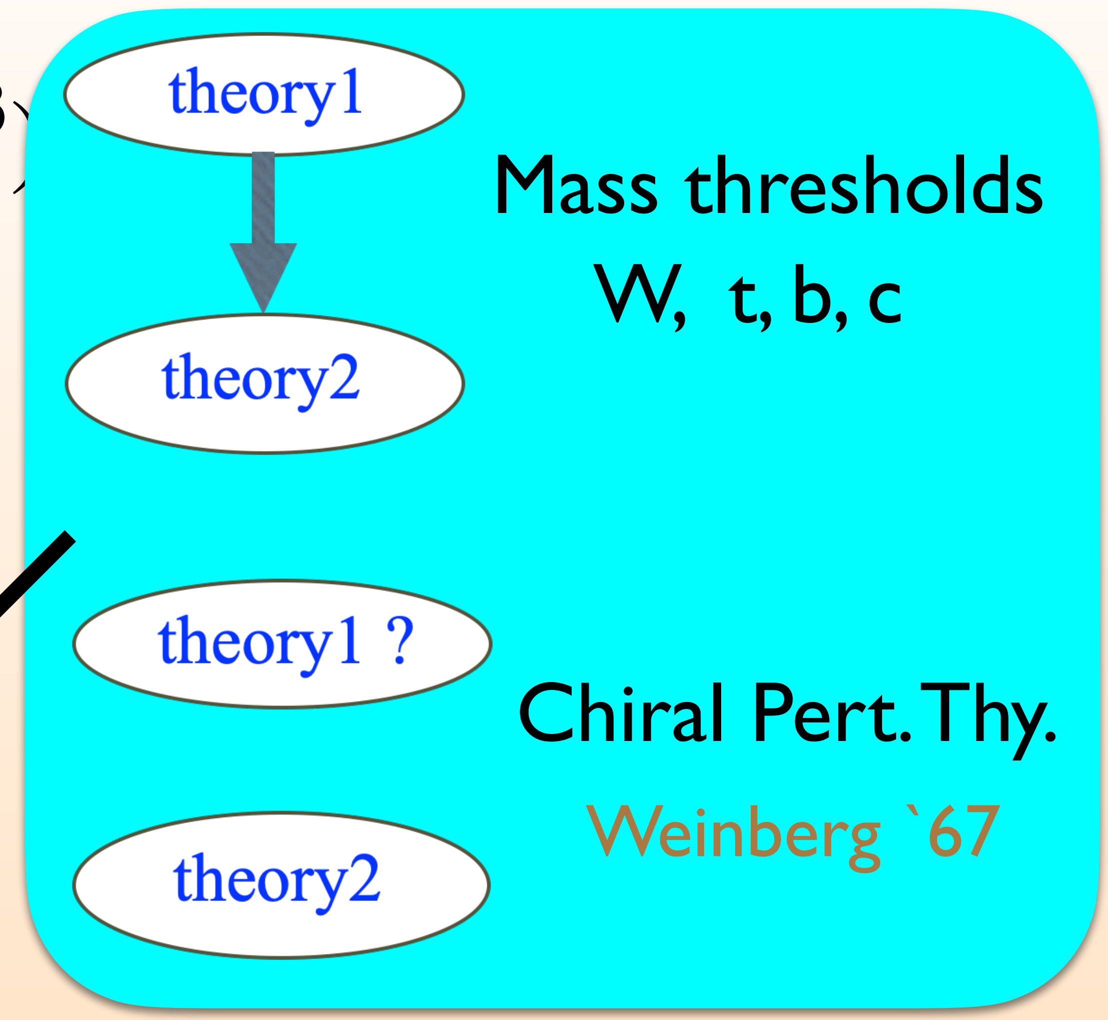

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Effort to carry out higher order calculations drove field to mass independent schemes (like $\overline{\mathrm{MS}}$)

Decouple by hand with "matching" relations

Effective Field Theory

$$\mathscr{L} = \mathscr{L}^{(0)} + \mathscr{L}^{(1)} + \mathscr{L}^{(2)} + \mathcal{O}(\epsilon^3) \qquad \epsilon \ll 1$$

Concepts:

p.c. = Op. mass dim. needs coordinate homogeneity (Lorentz invariance)

Many examples where p.c. more involved (nonrelativistic, hard collisions, ...)

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★ $\alpha_s^{(k)}(\mu)$ for k light flavors

★ $\mathscr{L}^{(0)} = \sum_i C_i(\mu, \Lambda)\, \mathcal{O}_i(\mu, m)$

eg. crucial for Electroweak H

★ $\alpha_s \ln \frac{\Lambda}{m}$, $\alpha_s \ln^2 \frac{\Lambda}{m}$

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Reduce, Reuse & Recycle:

short distance couplings C_i

long distance m.elts. $\langle \mathcal{O}_i \rangle$

could be numbers or functions

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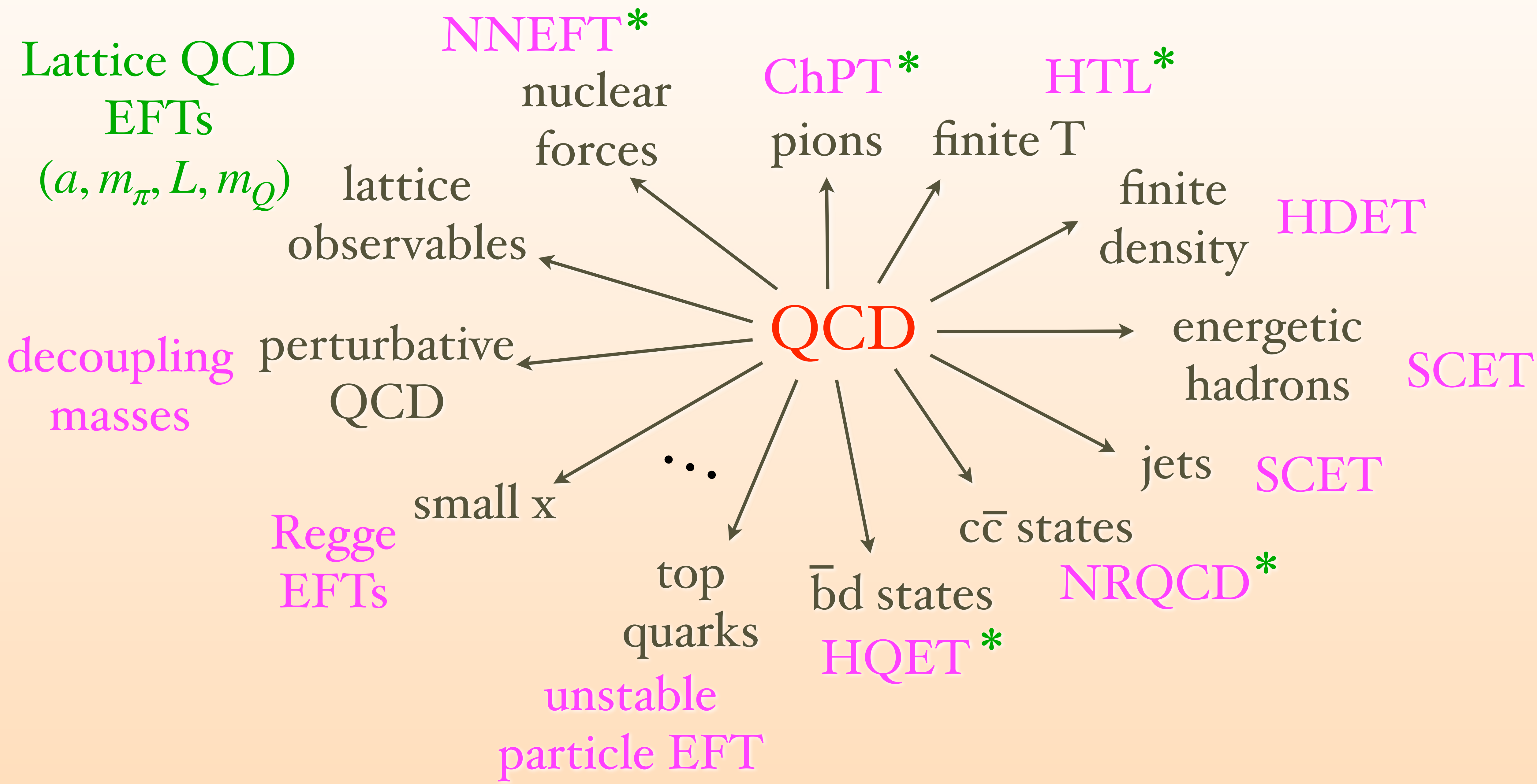
- Universality of short and long distance parameters (functions)
- Systematic symmetry breaking. Make emergent symmetries explicit.

Effective Field Theories of QCD



Chiral Perturbation Theory for Nuclear Forces

ChPT-π

Weinberg

$SU(2)_L \times SU(2)_R \to SU(2)_V$ $\quad \langle \bar{\psi}\psi \rangle \neq 0$ $\quad \frac{p^2}{\Lambda_\chi^2}, \frac{m_\pi^2}{\Lambda_\chi^2} \ll 1$

$$\mathscr{L}_\chi = \frac{f^2}{8}\mathrm{tr}\left[\, \partial^\mu \Sigma^\dagger \partial_\mu \Sigma \,\right] + v_0 \mathrm{tr}\left[\, m_q^\dagger \Sigma + m_q \Sigma^\dagger \,\right] + \mathscr{O}(p^4)$$

← Gasser & Leutwyler `85

Derivatively coupled

Nonlinear (fluctuations on vacuum manifold), $\Sigma = \exp\left(\frac{2i}{f}\frac{\vec{\pi}\cdot\vec{\tau}}{\sqrt{2}}\right)$

Naive dimensional analysis, count 4π s, $\Lambda_\chi = 4\pi f$ (Georgi & Manohar `84)

Chiral loops predict non-analytic dependence on quark masses, $\ln m_q$

Pheno: multi-π processes, predict $\pi\pi$ scattering lengths,
calculate pion effects on e.m. & weak currents

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ChPT-Nπ One nucleon with pions $\mathscr{L} = \mathscr{L}_{N\pi} + \mathscr{L}_\chi$

Must also consider $\frac{p}{m_N} \ll 1$ expansion

Gasser, Sainio, Svarc `88
Jenkins, Manohar `91
…

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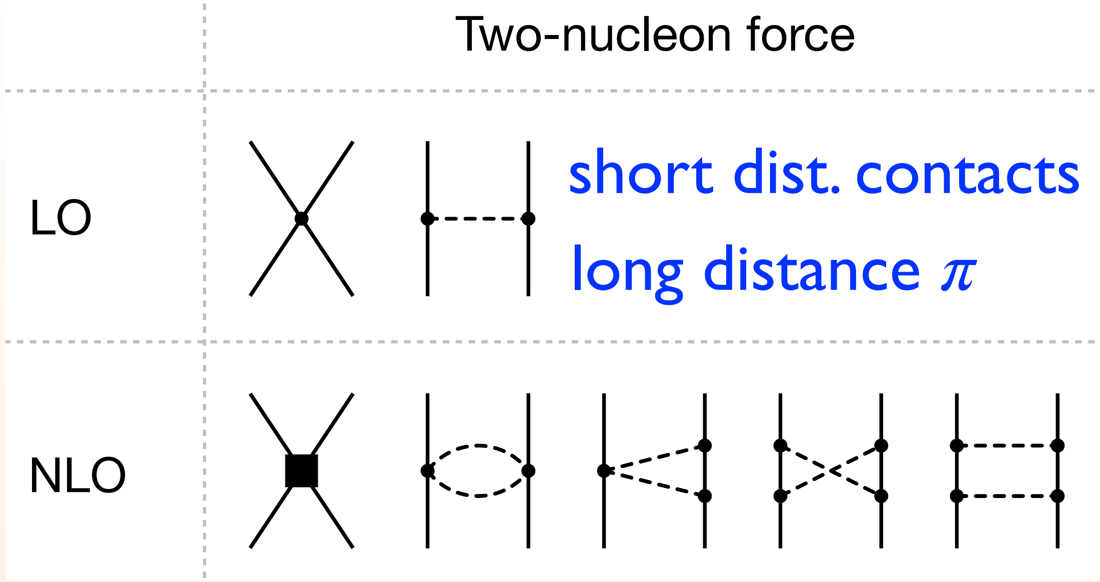
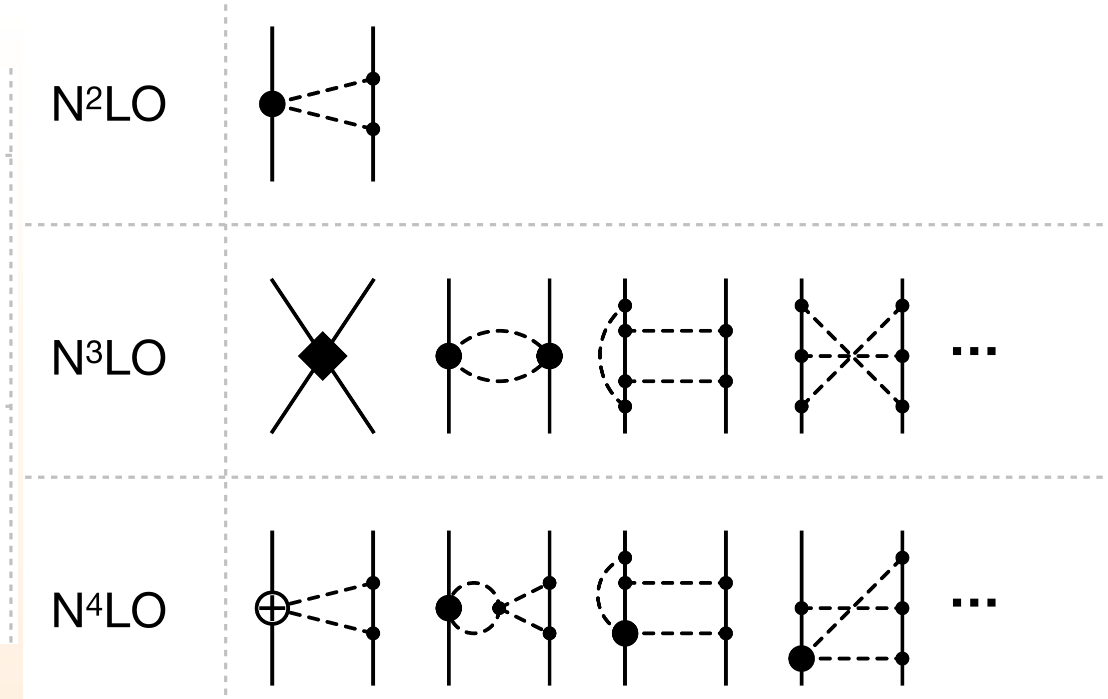
...

NNChPT Two or more nucleons $\mathscr{L} = \mathscr{L}_{NN} + \mathscr{L}_{NN\pi} + \mathscr{L}_{N\pi} + \mathscr{L}_\chi$

short distance contact interactions $\sum_i C_i\,(\bar{\psi}\psi)_i^2 + \ldots$ (→ $\mathscr{L}_{NN}$)

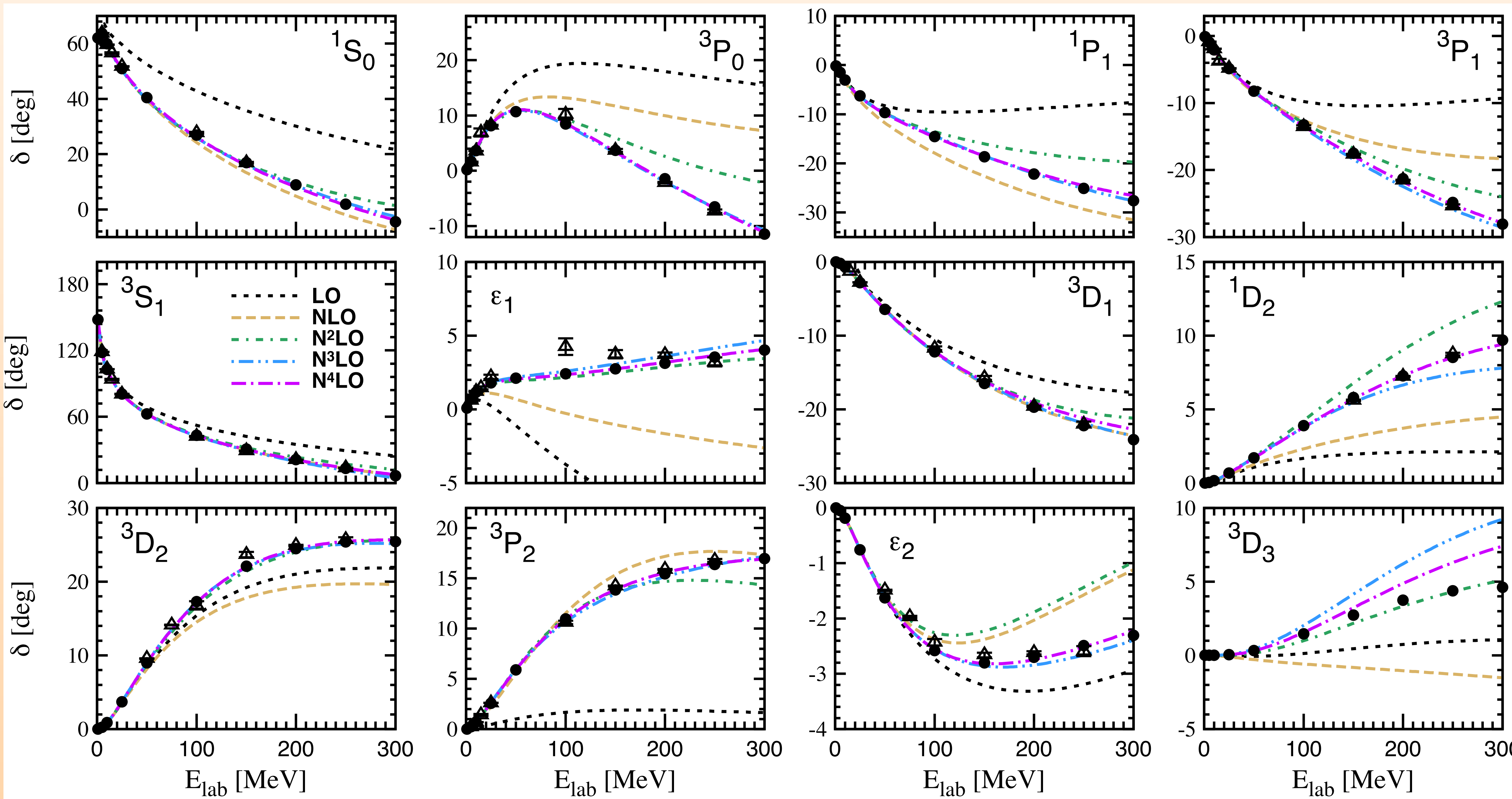
Weinberg \`90
Kaplan, Savage, Wise \`98
van Kolck, Bedaque, Hammer
...



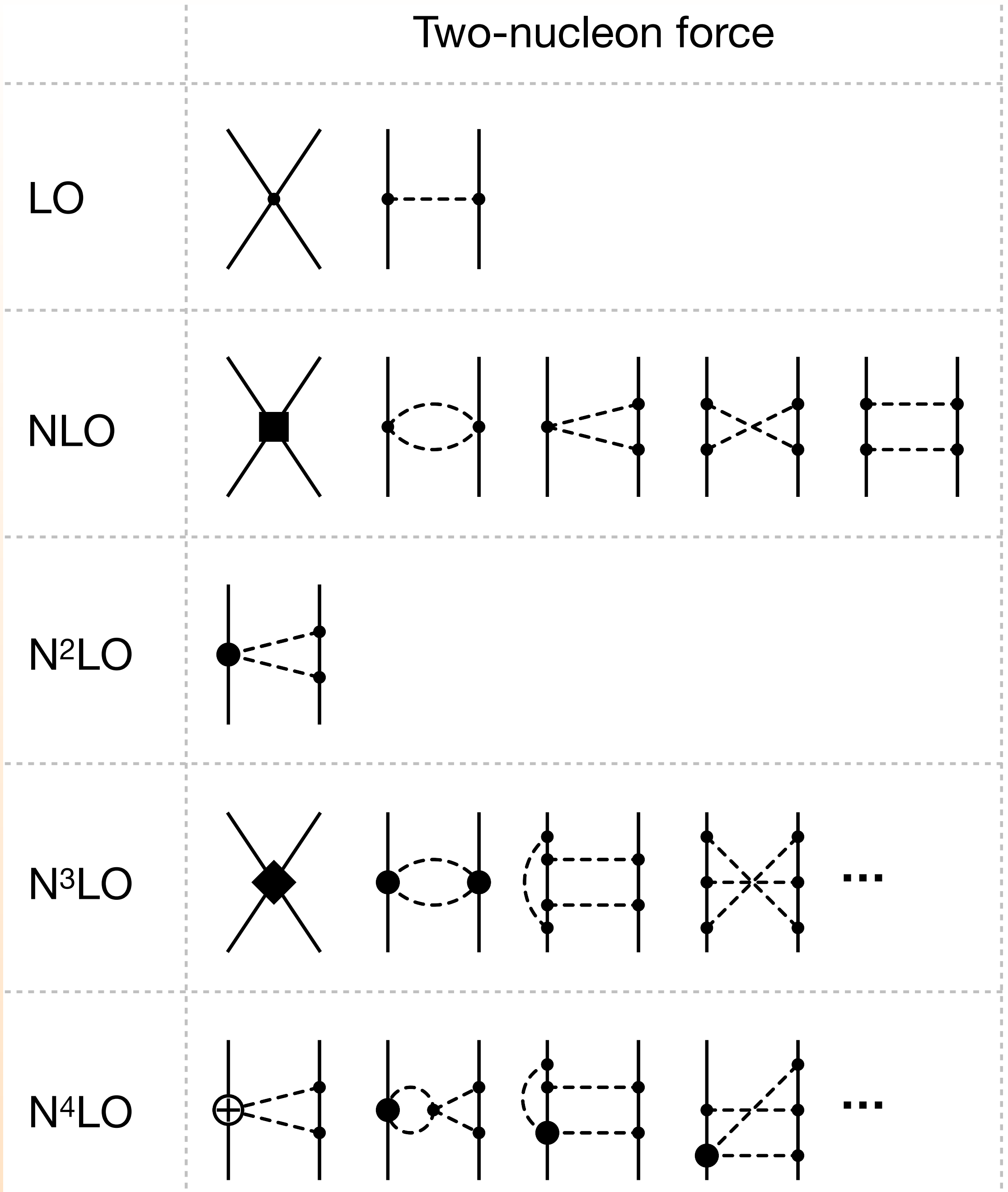


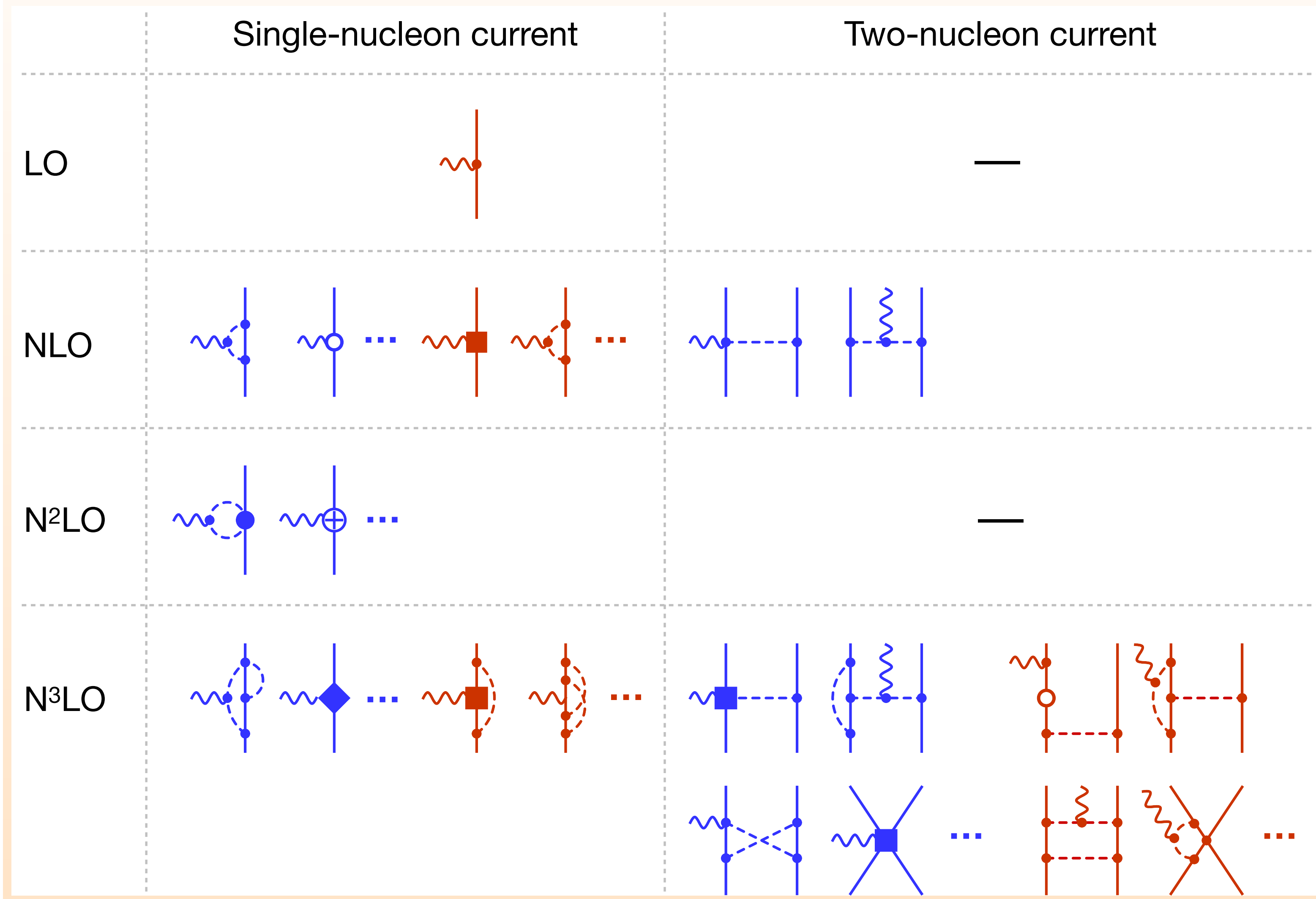
Reinert, Krebs, Epelbaum (1711.08821)

eg. NN phase shifts



Filin, Möller, Baru, Epelbaum, Krebs, Reinert (2009.08911)





	LO	NLO	N^{2}LO	N^{3}LO	N^{4}LO
Q_d [fm^2]	0.24 ± 0.10	0.26 ± 0.01	0.282 ± 0.006	0.2854 ± 0.0017	0.2854 ± 0.0005

eg. Deuteron Quadrupole Moment

Chiral NNEFT $Q_d = 0.2854^{+0.0038}_{-0.0017}\ \mathrm{fm}^2$

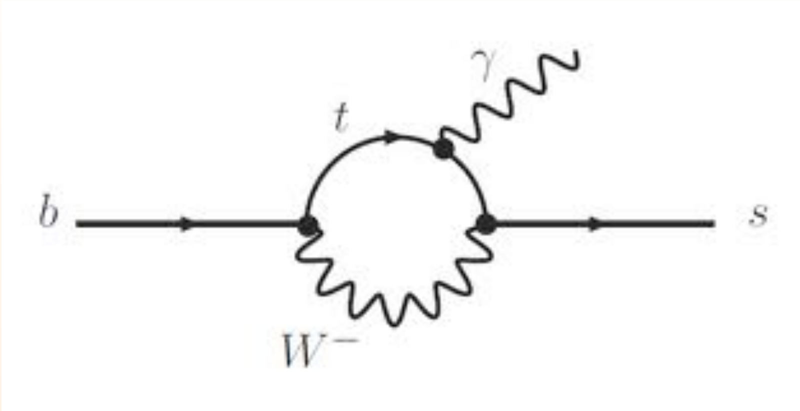
Hyperfine spectroscopy $Q_d = 0.285699(15)(18)\ \mathrm{fm}^2$

QCD effects for Weak Interactions

$B \to X_s \gamma$ Importance of RGE for electroweak-H Wilson coefficients

$m_b \ll m_{W,Z,t,H}$

FCNC



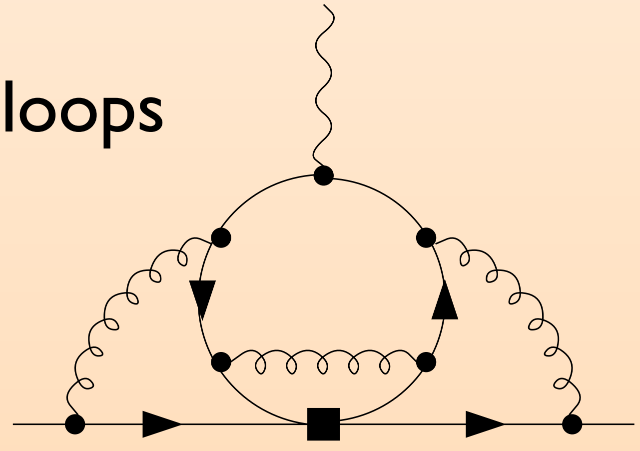
$$H_W = \sum_i C_i O_i \quad , \qquad O_7 = \frac{e}{4\pi^2} m_b \left[\bar{s}\sigma^{\mu\nu} P_R b\right] F_{\mu\nu} \quad , \ldots$$

Bertonni, Borzumati, Masiero \`87
Grinstein, Springer, Wise \`88

$$C_7^{\mathrm{LO}} = -0.20$$

$$C_7^{\mathrm{LL}}(\mu = m_b) = -0.30$$ enhances Branching ratio by more than factor of 2!

Calculated up to NNLO, with anomalous dimensions up to 4-loops



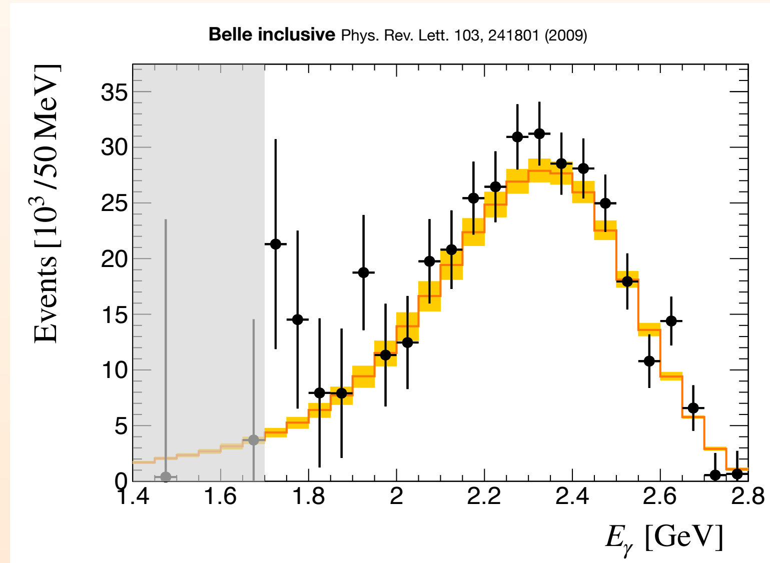
Misiak, Asatrian, Bieri, Czakon, Czarnecki, Ewerth, Ferroglia, Gambino, Gorbahn, Greub, Haish, Hovhannisyan, Hurth, Mitov, Poghosyan, Slusarczyk, Steinhauser \`06

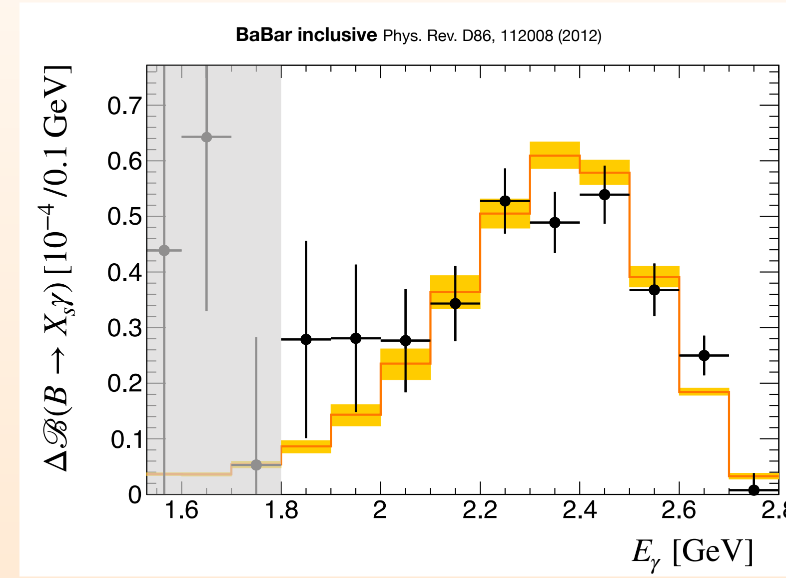
Misiak, Asatrian, Boughezal et.al. (update in \`15)

$B \to X_s \gamma$ Importance of RGE for electroweak-H Wilson coefficients

$m_b \ll m_{W,Z,t,H}$

$$H_W = \sum_i C_i \, O_i \quad , \qquad O_7 = \frac{e}{4\pi^2} m_b \left[\bar{s}\sigma^{\mu\nu} P_R \, b\right] F_{\mu\nu} \quad , \ldots$$





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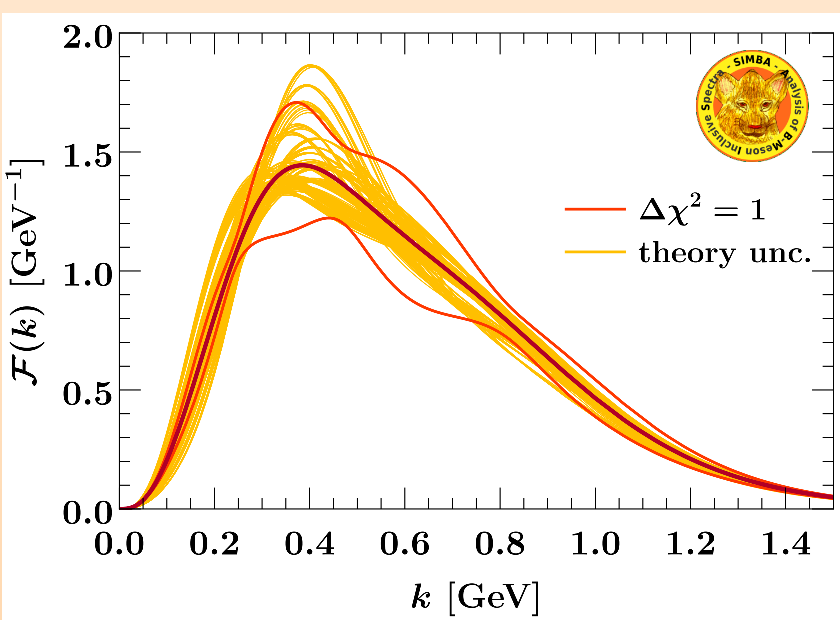
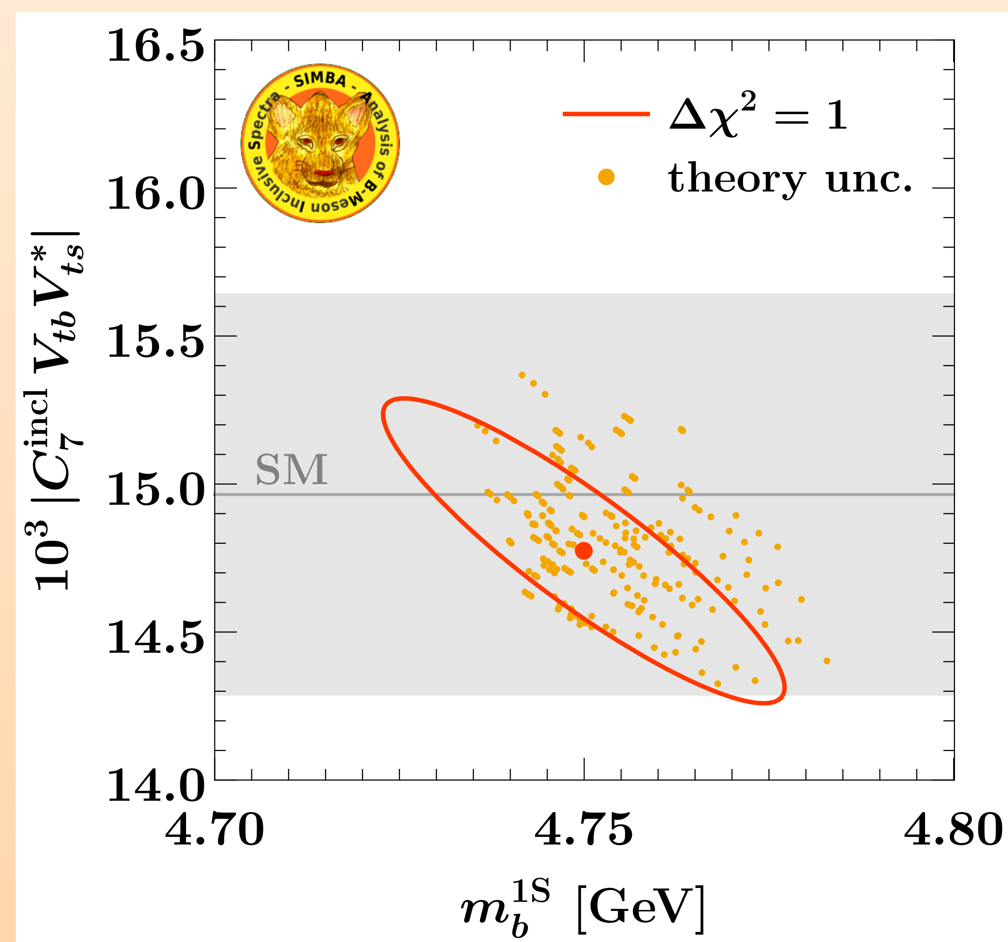
<u>Global Fit to Data</u>

$|C_7|$ from norm

$\mathscr{F}$ = b-quark PDF from shape

SIMBA collab (Bernlochner, Lacker, Ligeti, IS, Tackman, Tackman, 2007.04320)





Inclusive (non-pert corrections treated with HQET, SCET, …)

Heavy Quark Effective Theory

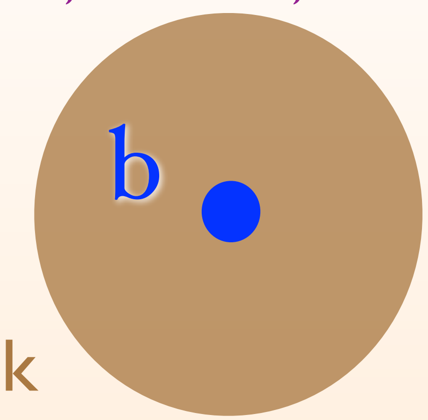
HQET

$k^\mu \sim \Lambda_{\rm QCD} \ll m_Q = m_{b,c}$

Eichten, Hill, Isgur, Wise, Chay, Georgi, Grinstein, Shifman, Vainshtein, Voloshin, Neubert, Luke, Uraltsev, …

$$\mathscr{L}_{\rm HQET} = \bar{h}_v \, iv \cdot D \, h_v + \mathscr{L}_{\rm QCD}^{q,g} + \mathscr{O}\left(\frac{1}{m_Q}\right)$$

heavy quark is static color source



brown muck
= light quarks, gluons

v^μ velocity:

$$p_Q^\mu = m_Q v^\mu + k^\mu$$

U(4) Heavy Quark Spin-Flavor symmetry (b & c)

<u>Pheno:</u>

- $B \to D^{(*)} \ell \bar{\nu}$ single leading "Isgur-Wise" form factor
  no $\mathscr{O}(1/m_Q)$ corrections at zero-recoil
  dedicated program for form factors on lattice

$$\to V_{cb}$$

HQET

$k^\mu \sim \Lambda_{\rm QCD} \ll m_Q = m_{b,c}$

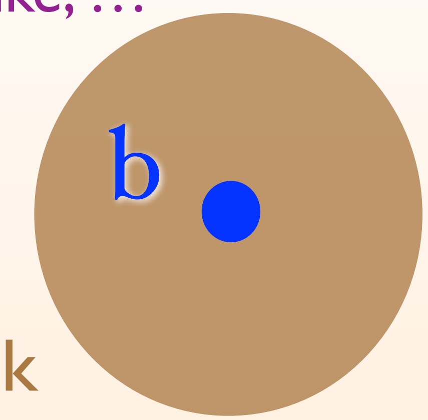
Eichten, Hill, Isgur, Wise, Chay, Georgi, Grinstein, Shifman, Vainshtein, Voloshin, Neubert, Luke, …

$$\mathscr{L}_{\rm HQET} = \bar{h}_v \, iv \cdot D \, h_v + \mathscr{L}^{q,g}_{\rm QCD} + \mathcal{O}\left(\frac{1}{m_Q}\right)$$

heavy quark is static color source

v^μ velocity:

$$p_Q^\mu = m_Q v^\mu + k^\mu$$



brown muck = light quarks, gluons

U(4) Heavy Quark Spin-Flavor symmetry (b & c)

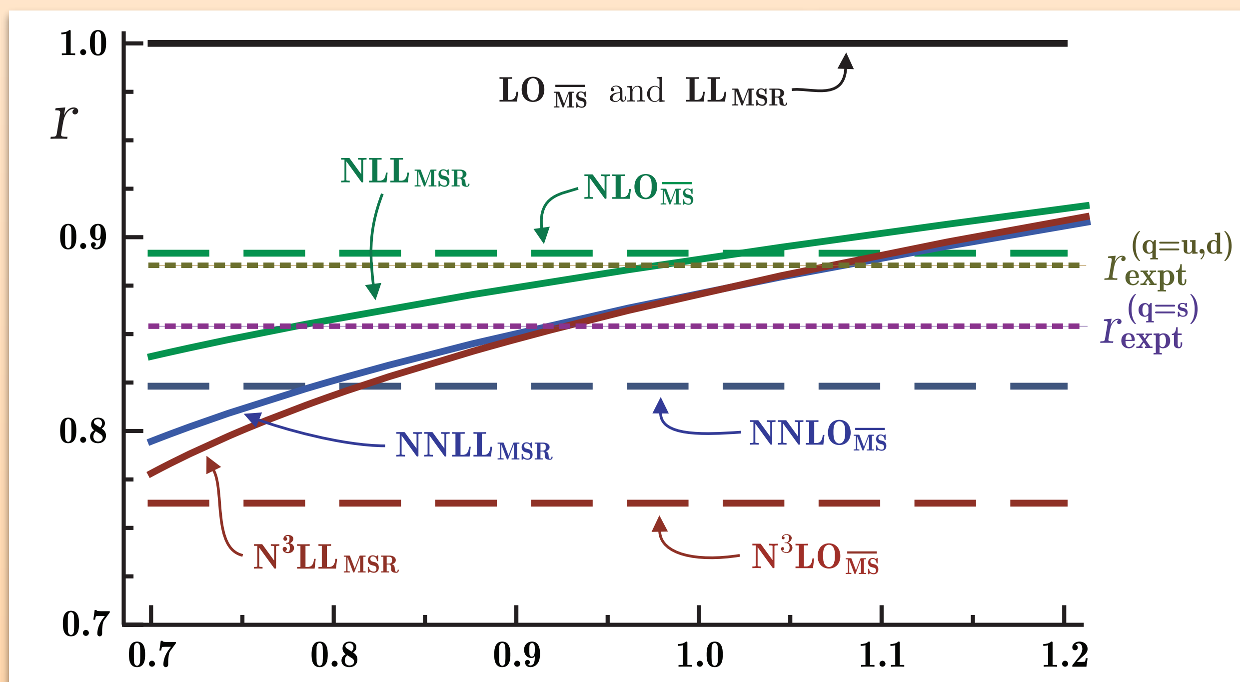
<u>Pheno:</u>

- universal non-pert. corrections: kinetic energy $\langle Q_v | \bar{h}_v \, D_T^2 \, h_v | Q_v \rangle$ and chromomagnetic term $\mu_G^2 = -\langle Q_v | \bar{h}_v g \sigma_{\mu\nu} G^{\mu\nu} h_v | Q_v \rangle / 3$

eg. Same matrix elements appear in inclusive $B \to X_c \ell \bar{\nu}$ OPE and in Hadron masses

calculable! $\mathcal{O}(\alpha_s^3)$: Grozin`08

$$r = \frac{m_{B^*}^2 - m_B^2}{m_{D^*}^2 - m_D^2} = \frac{C_G(m_b, \mu)}{C_G(m_c, \mu)} + \frac{\Sigma_p}{\mu_G^2}\left(\frac{1}{m_b} - \frac{1}{m_c}\right) + \ldots$$



Hoang, Jain, Scimemi, IS `09

Soft Collinear Effective Theory

Soft Collinear Effective Theory

"EFT for Collider Physics"

EFT for hard interactions which produce energetic (collinear) and soft particles.

Bauer, Fleming, Luke, Pirjol, IS `00, `01

Higgs production, DY, ...

Jet Physics

Jet Substructure

B-Decays and CP violation

Quarkonia Production

TMDs / Nuclear Physics

(Heavy Ion collisions)

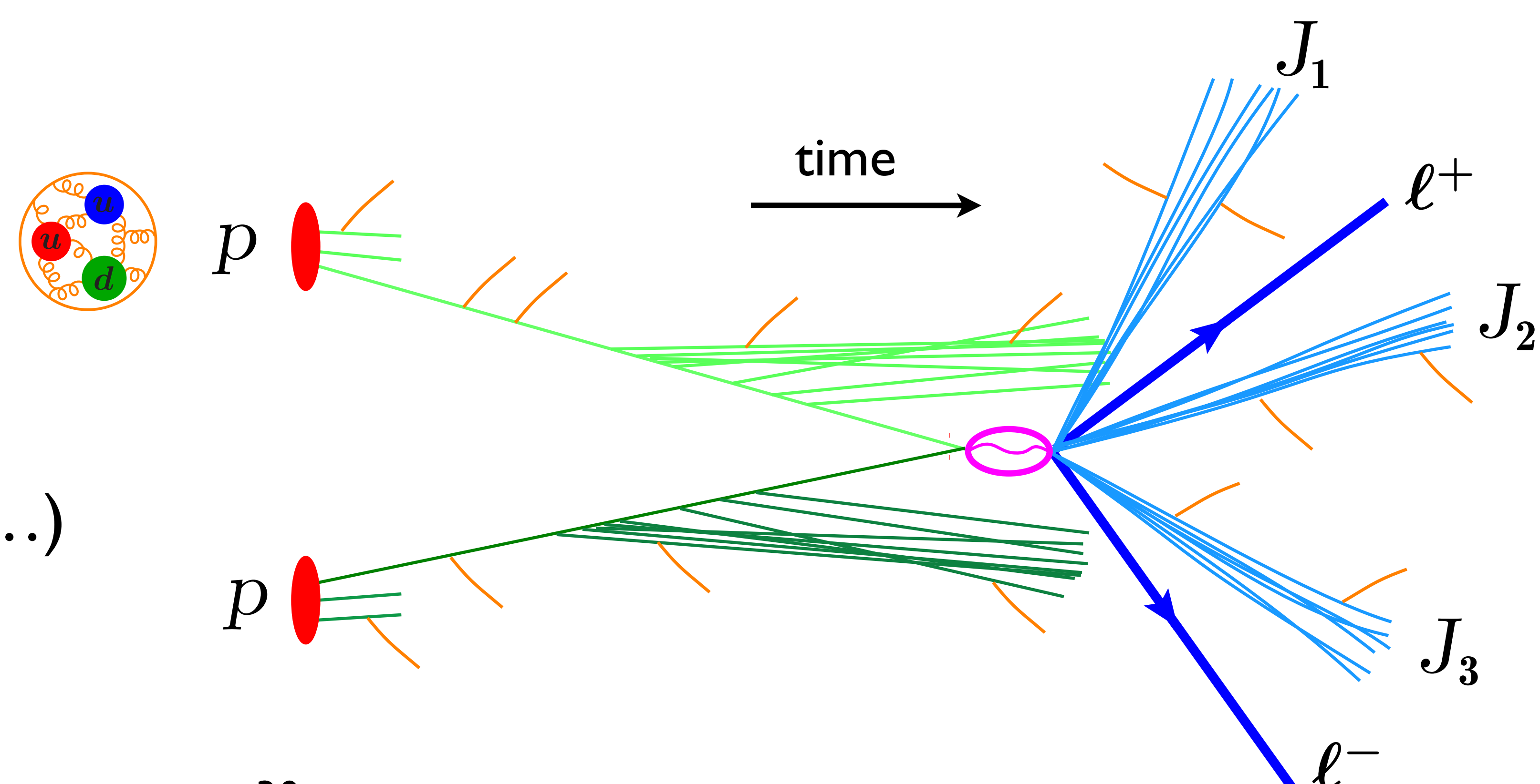
Higher order Resummation

Infrared Structure of Gauge Theory

Subtractions for Fixed Order QCD

Gauge theory at Subleading Power

High Energy Limit / Regge phenomena



builds on extensive past literature (CSS factorization, exclusive fact, ...)

For guide to SCET literature see my review in 50 yrs of QCD, 2212.11107

Non-perturbative Factorization:

parton distributions

$$d\sigma = f_a f_b \otimes \hat{\sigma} \otimes F$$

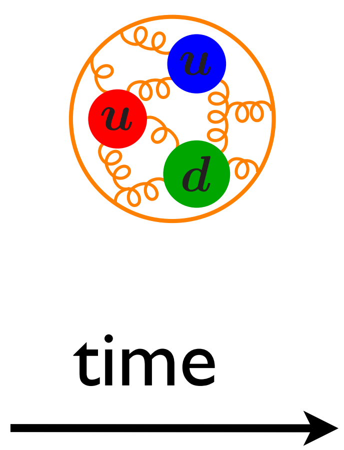
hadronization:

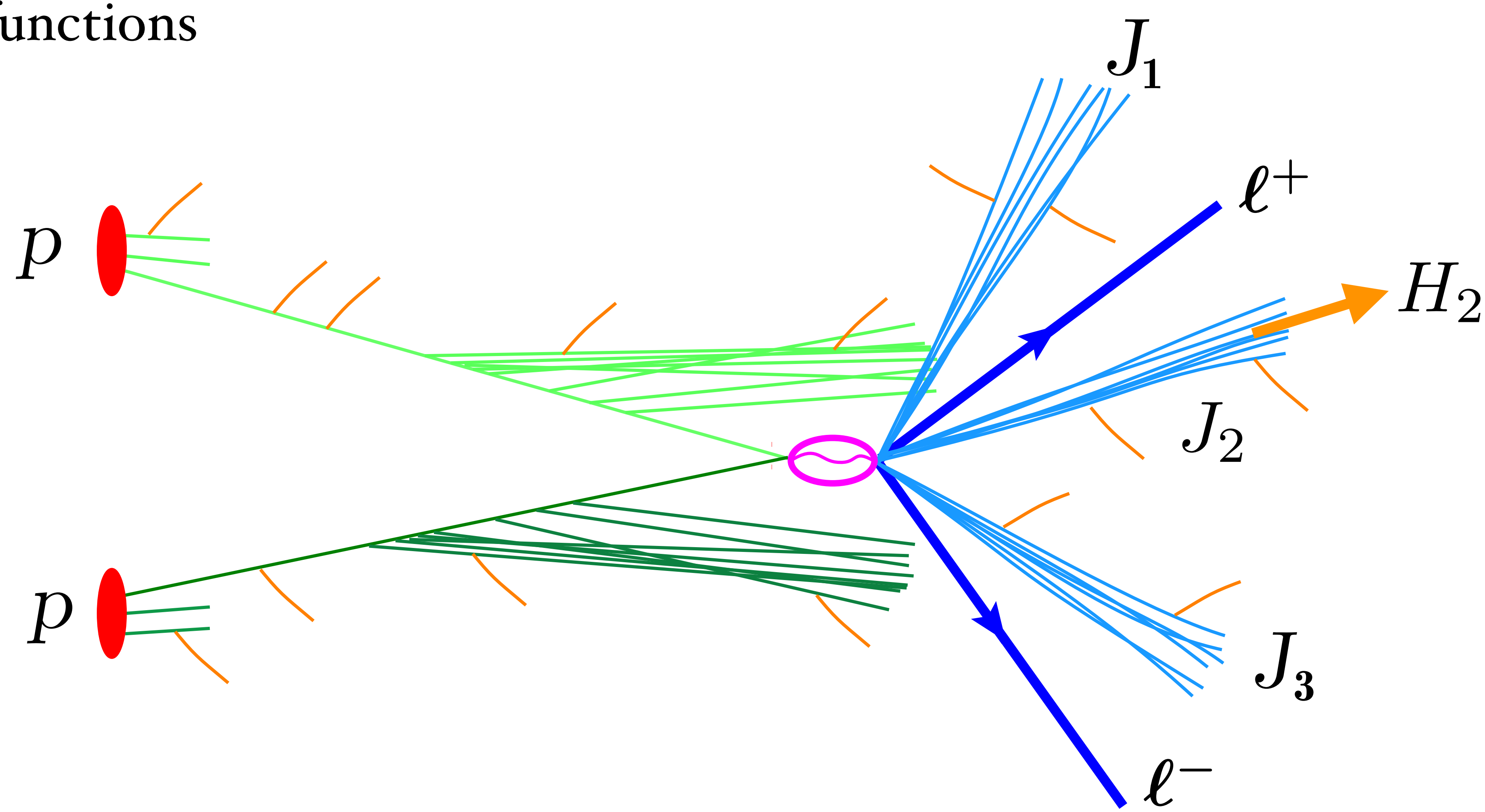
fragmentation fns.,
soft hadronization, ...

(QFT operators)

perturbative cross section

universal hadronic dynamics
via
universal hadronic functions





Non-perturbative Factorization:

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hadronization: fragmentation fns., soft hadronization, ... (QFT operators)

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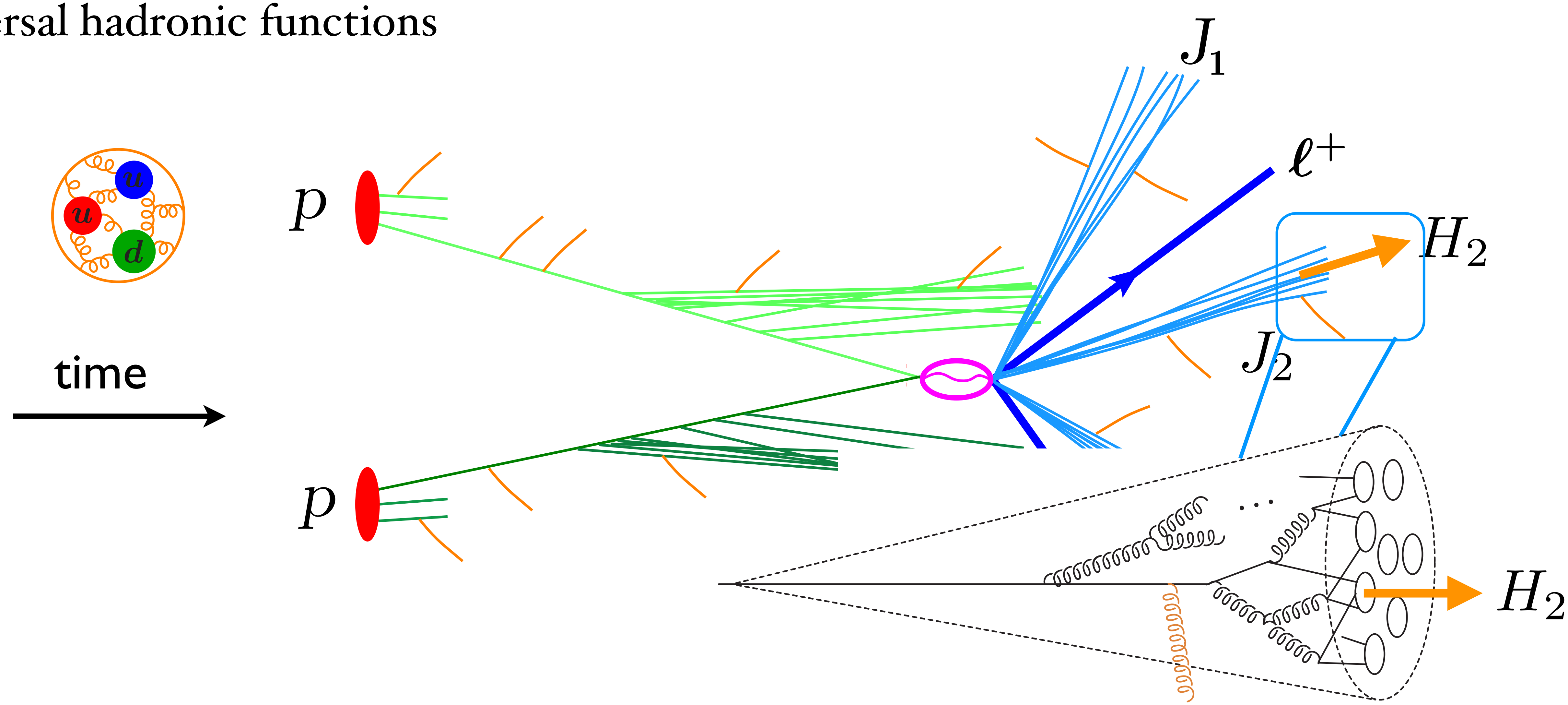
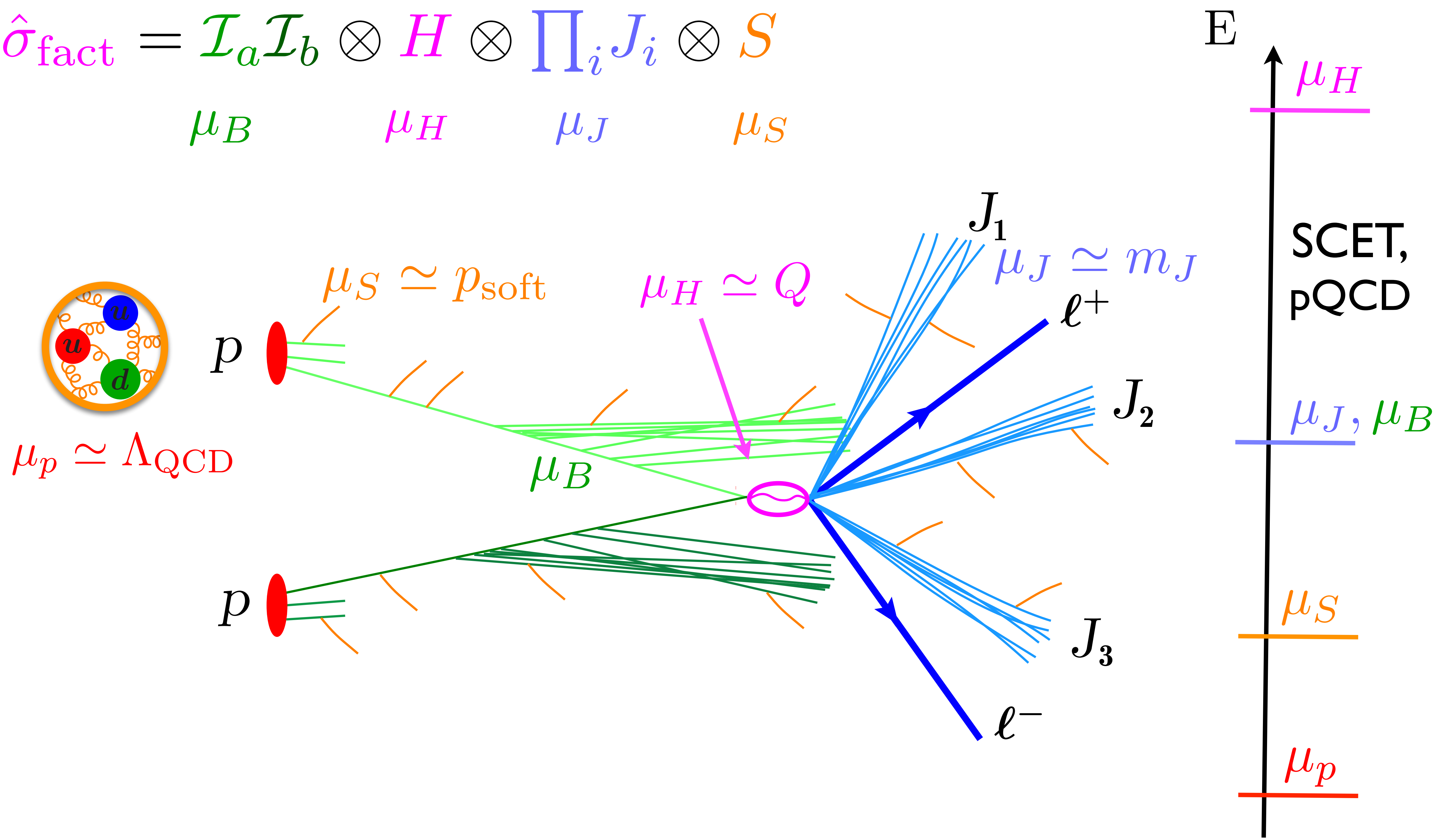


Perturbative Factorization: for multi-scale problems with fixed # jets

beam hard jet pert. soft

$$\hat{\sigma}_{\rm fact} = \mathcal{I}_a \mathcal{I}_b \otimes H \otimes \prod_i J_i \otimes S$$

μ_B μ_H μ_J μ_S



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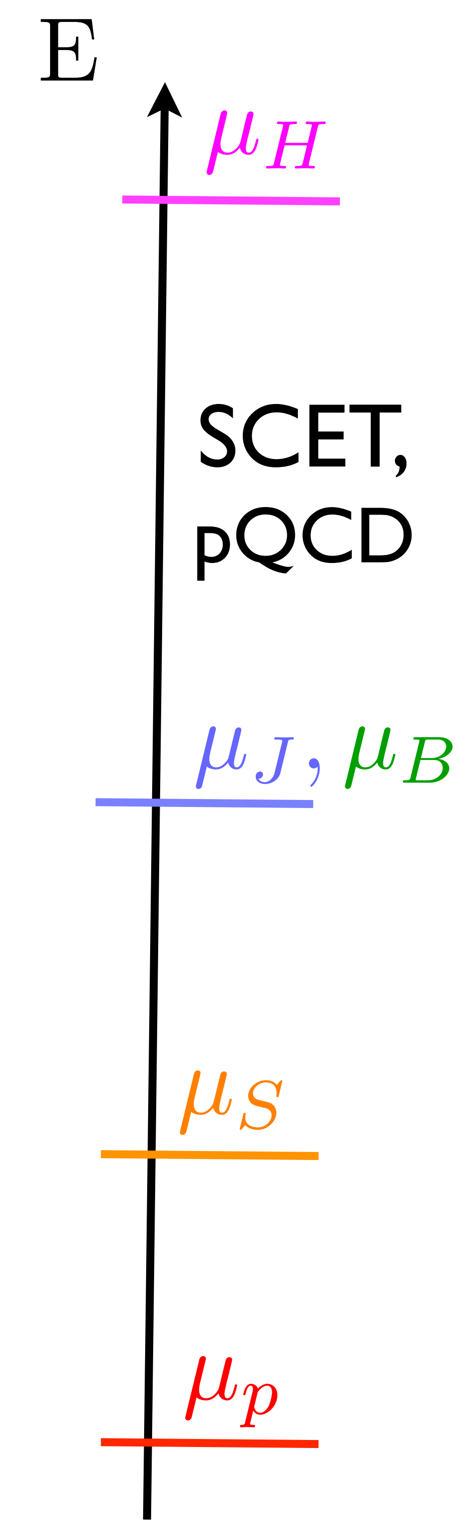


Perturbative Universality

- H determined by hard process, independent of jet radius, etc.
- J_i , $\mathcal{I}_{a,b}$ splitting and virtual effects for parton i, encode jet dynamics, independent of H — universal collinear dynamics
- S soft radiation, all partons contribute, eikonal Feynman rules — universal soft dynamics

Scale dependence $\longleftrightarrow$ RGE sums up logarithms $\log\left(\frac{\mu_H}{\mu_S}\right), \ldots$

Perturbative QCD Results:

fixed order:

$$\hat{\sigma} = \sigma_0 \left[1 + \alpha_s + \alpha_s^2 + \dots\right]$$
$$= \text{LO} + \text{NLO} + \text{NNLO} + \dots$$

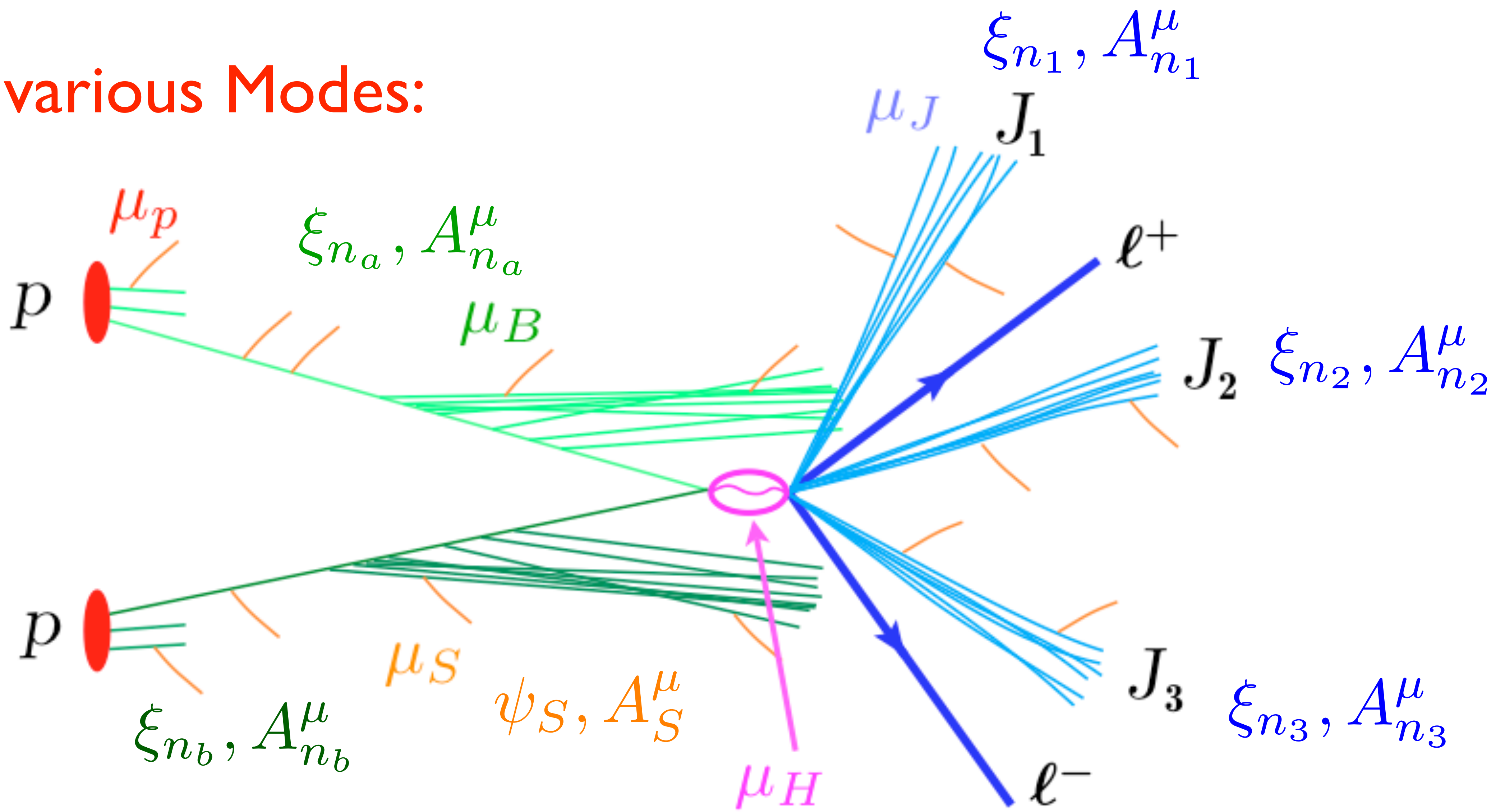
SCET anomalous dimensions:

resummation of large (double) logs $\quad L = \log(\dots)$ $\quad \log\left(\frac{\Lambda_{\text{QCD}}}{Q}\right)$, $\log\left(\frac{p_T}{Q}\right)$, ...

$$\ln \hat{\sigma}(y) = \sum_k L(\alpha_s L)^k + \sum_k (\alpha_s L)^k + \sum_k \alpha_s (\alpha_s L)^k + \sum_k \alpha_s^2 (\alpha_s L)^k + \dots$$
$$= \text{LL} + \text{NLL} + \text{NNLL} + \text{N}^3\text{LL} + \dots$$

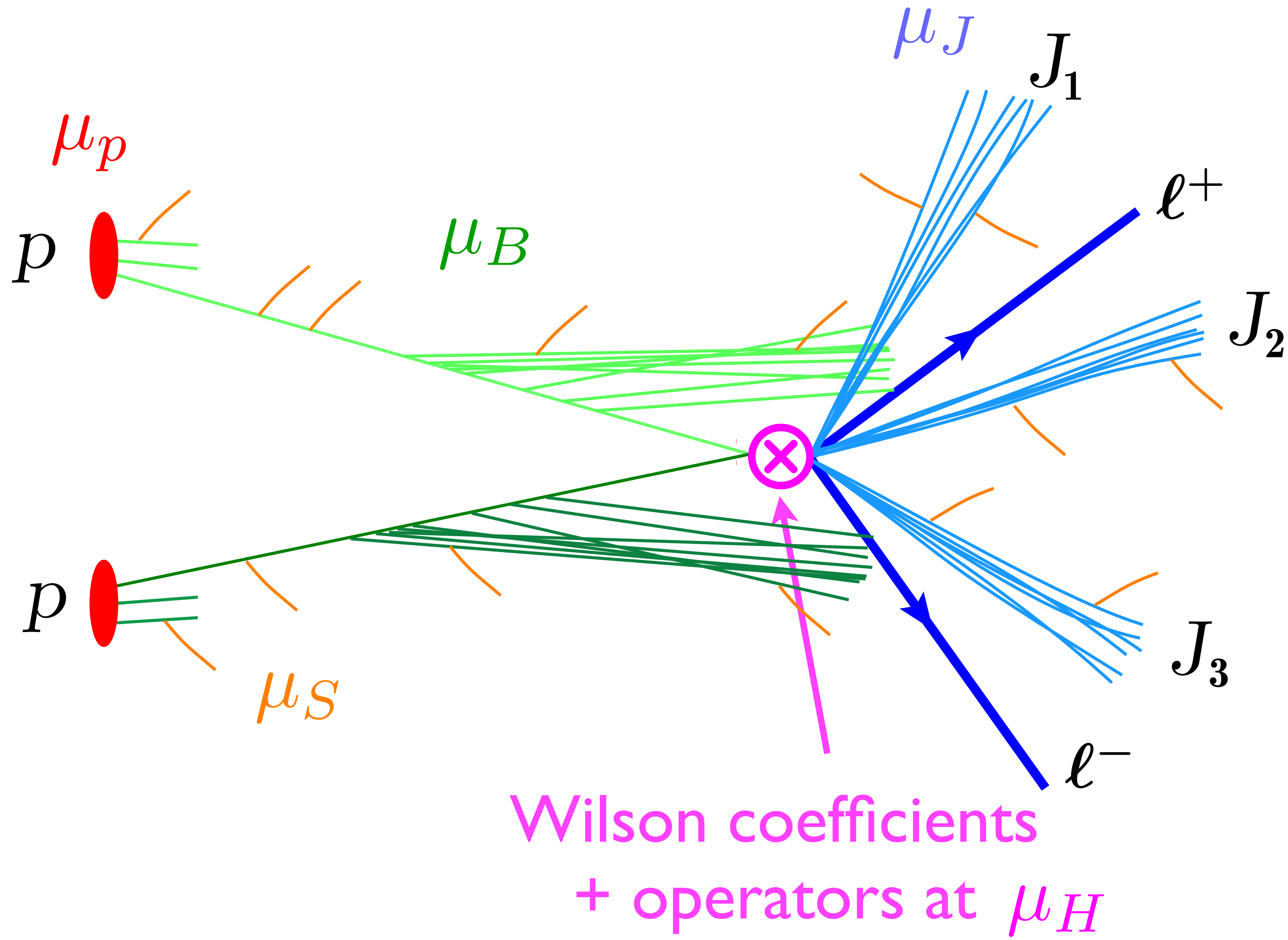
Soft Collinear Effective Theory

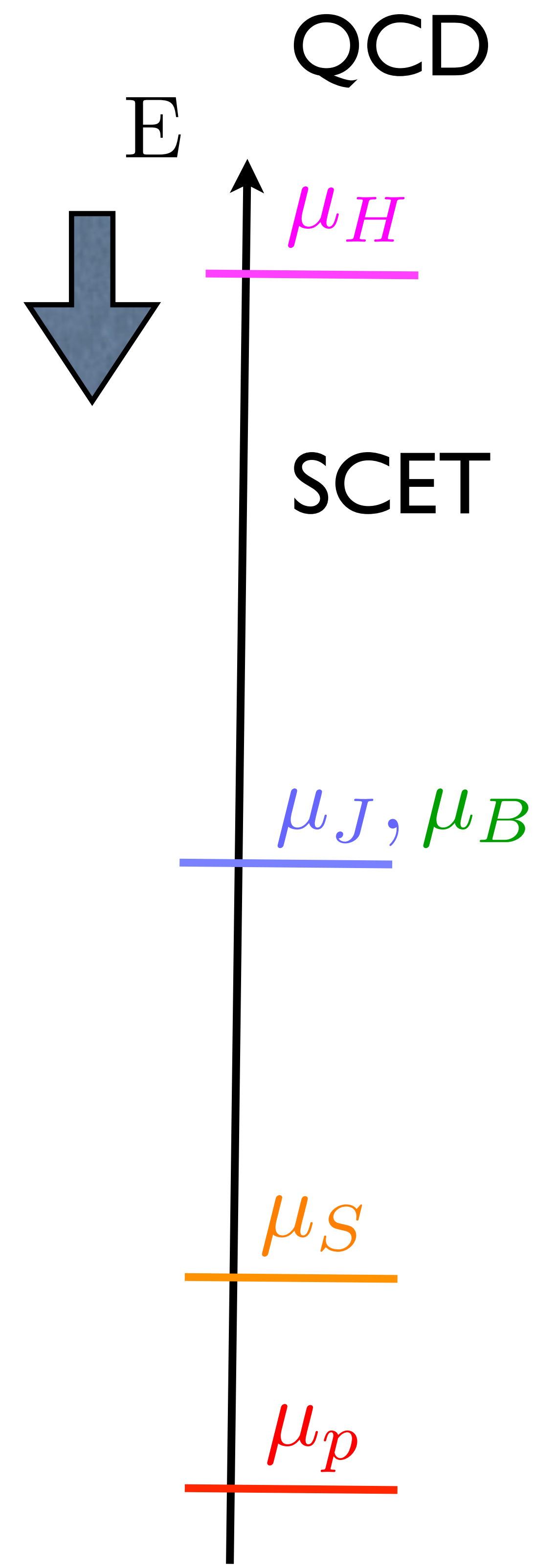
Fields for various Modes:



- dominant contributions from isolated regions of momentum space

Key Simplifying Principle is to Exploit the Hierarchy of Scales

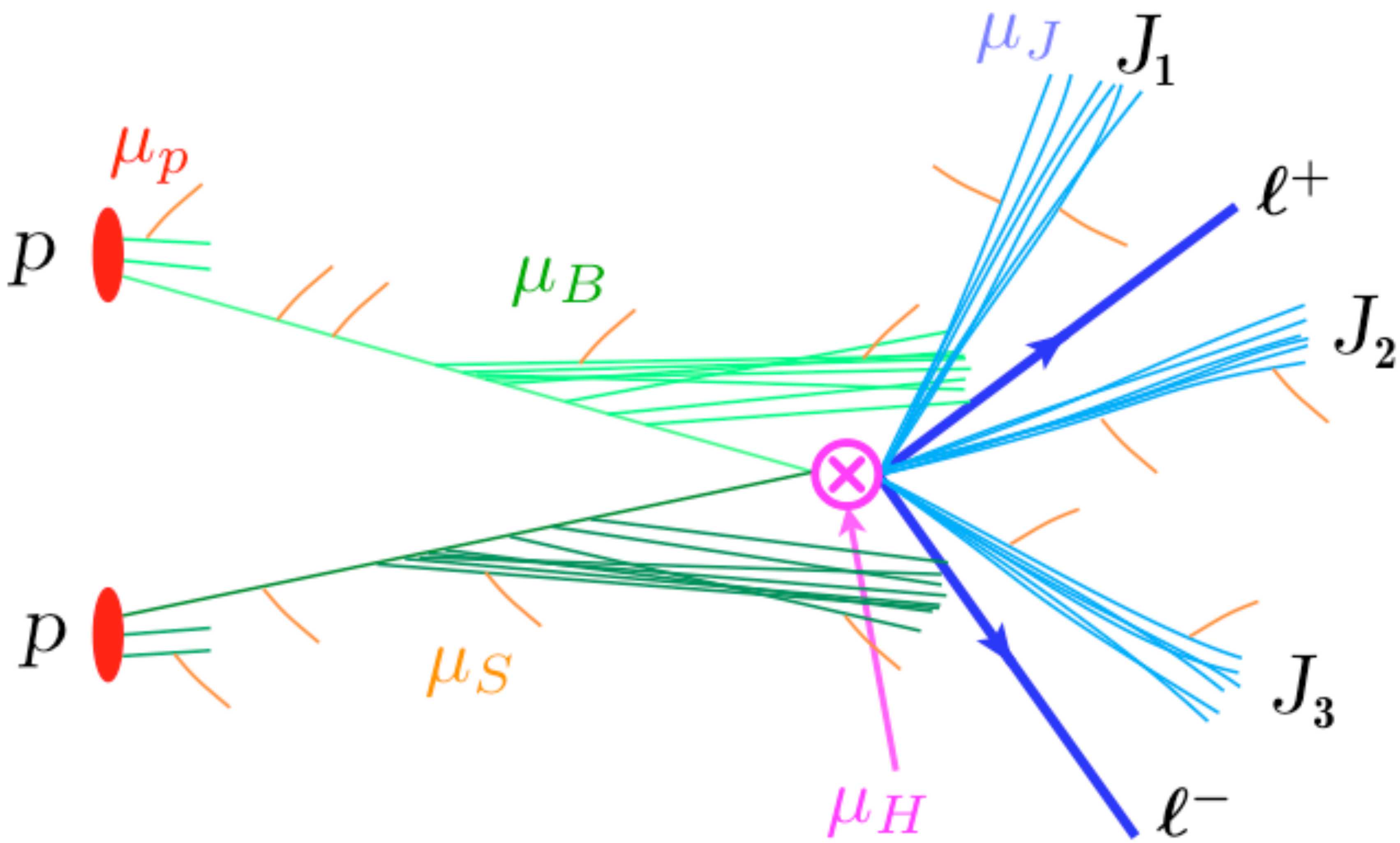


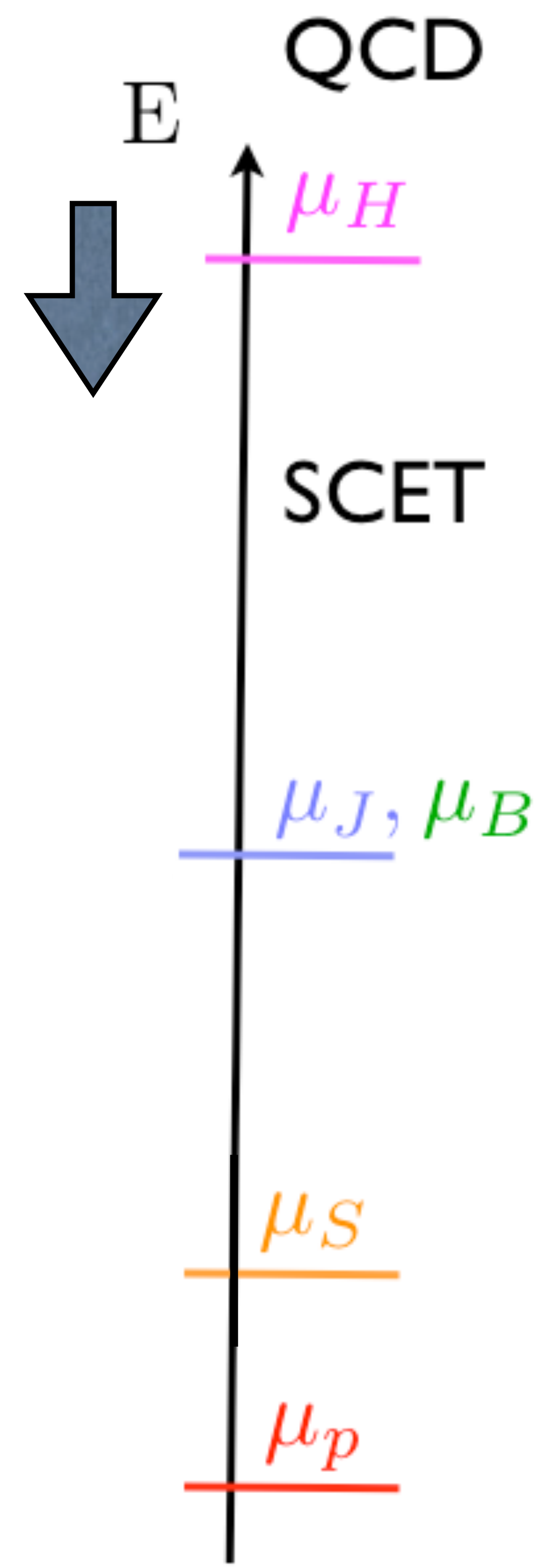


$$\mathcal{L} = \sum_i C_i O_i$$

$$d\sigma = \int (\text{phase space}) \Big| \sum_i C_i \langle O_i \rangle \Big|^2 = \sum_j H_j \otimes (\text{longer distance dynamics})_j$$

Hard-collinear factorization





Operators are built of building block fields:

$$\mathcal{O} = (\mathcal{B}_{n_a\perp})(\mathcal{B}_{n_b\perp})(\mathcal{B}_{n_1\perp})(\bar{\chi}_{n_2})(\chi_{n_3})$$

"quark jet" $\chi_n = (W_n^\dagger \xi_n)$

"gluon jet" $\mathcal{B}_{n\perp}^\mu = \frac{1}{g}[W_n^\dagger i D_\perp^\mu W_n]$ or $\mathcal{B}_{n\perp}^{A\mu} = \frac{1}{g}\frac{1}{\bar{n}\cdot\partial_n}\bar{n}_\nu G_n^{B\nu\mu}\mathcal{W}_n^{BA}$

SCET Lagrangian

$$\mathcal{L} = \sum_{p\geq 0} \mathcal{L}^{(p)}_{\rm dyn} + \sum_{p} \mathcal{L}^{(p)}_{\rm hard} + \mathcal{L}^{(0)}_{\rm G}$$

Dynamics of infrared modes

Hard Scattering operators (typically once)

Glauber gluon exchange (only factorization violating term)

- $\mathcal{L}^{(0)}_{\rm hard} = \sum_i C_i^{(0)} \mathcal{O}_i^{(0)}$ Leading operators for a given process
- $\mathcal{L}^{(0)}_{\rm dyn} = \sum_n \mathcal{L}^{(0)}_n + \mathcal{L}^{(0)}_{\rm soft}$ Collinear and Soft dynamics (Factorizes after soft-collinear decoupling)

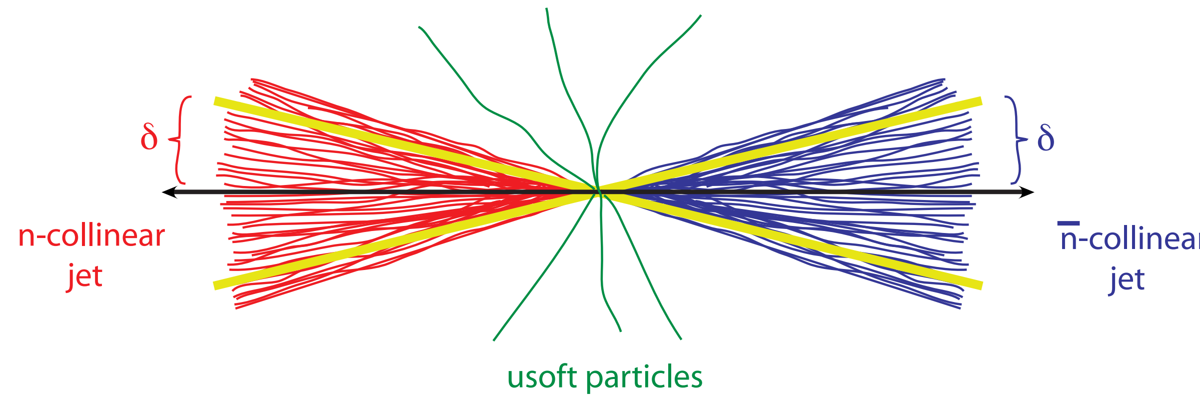
Often the leading power physics Factorizes. ~~$\mathcal{L}^{(0)}_{\rm G}$~~

→ Copies of QCD* give dynamics in different sectors, with hard operators providing the only connection between sectors

Examples:

- **Dijet production** $e^+e^- \to 2$ jets

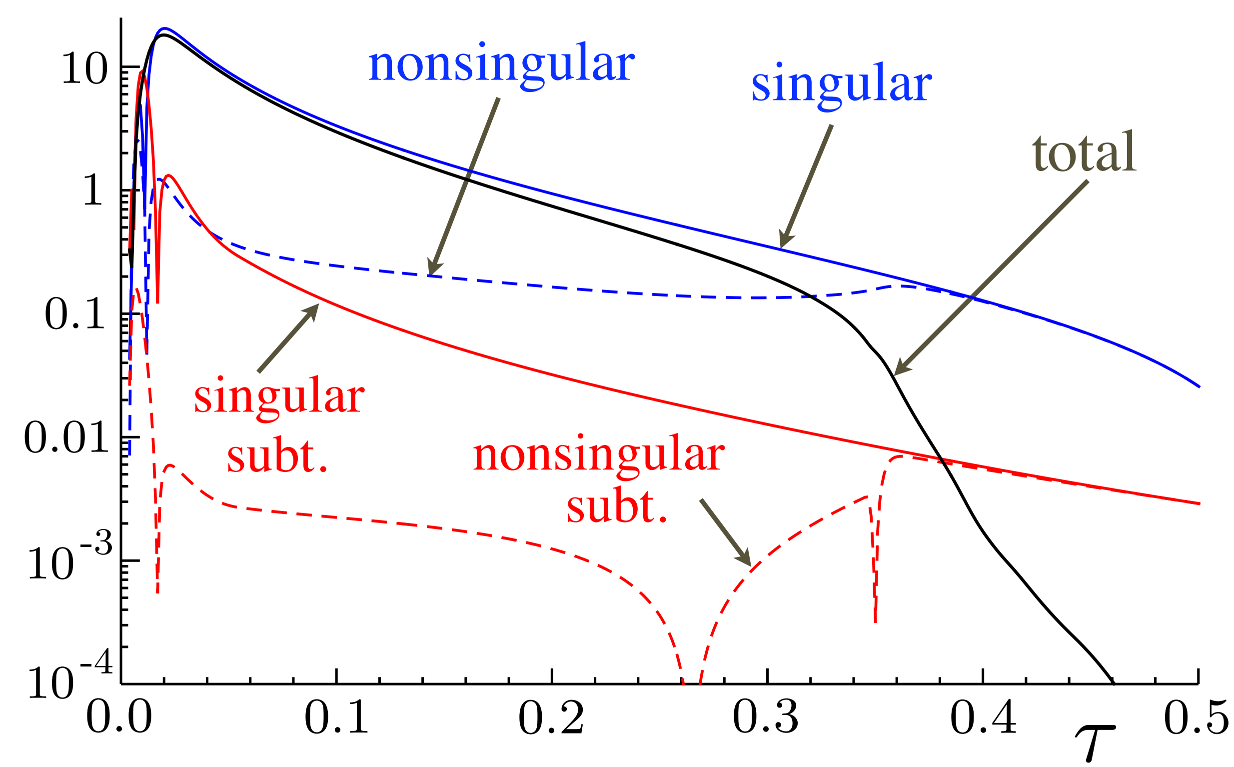
  thrust $\tau \ll 1$



$$\frac{d\sigma}{d\tau} = \sigma_0 H(Q,\mu)\, Q \int d\ell\, d\ell'\, J_T(Q^2\tau - Q\ell, \mu)\, S_T(\ell - \ell', \mu)\, F(\ell')$$

hard function | jet functions (combined) | perturbative soft function | non-perturbative soft function

$$+ \frac{d\sigma^{\text{nonsingular}}}{d\tau}$$



$\tau = 1 - T$

$\alpha_s(m_Z)$ from Thrust

$e^+e^- \to$ jets

Aim at 1% precision

Becher, Schwartz `09

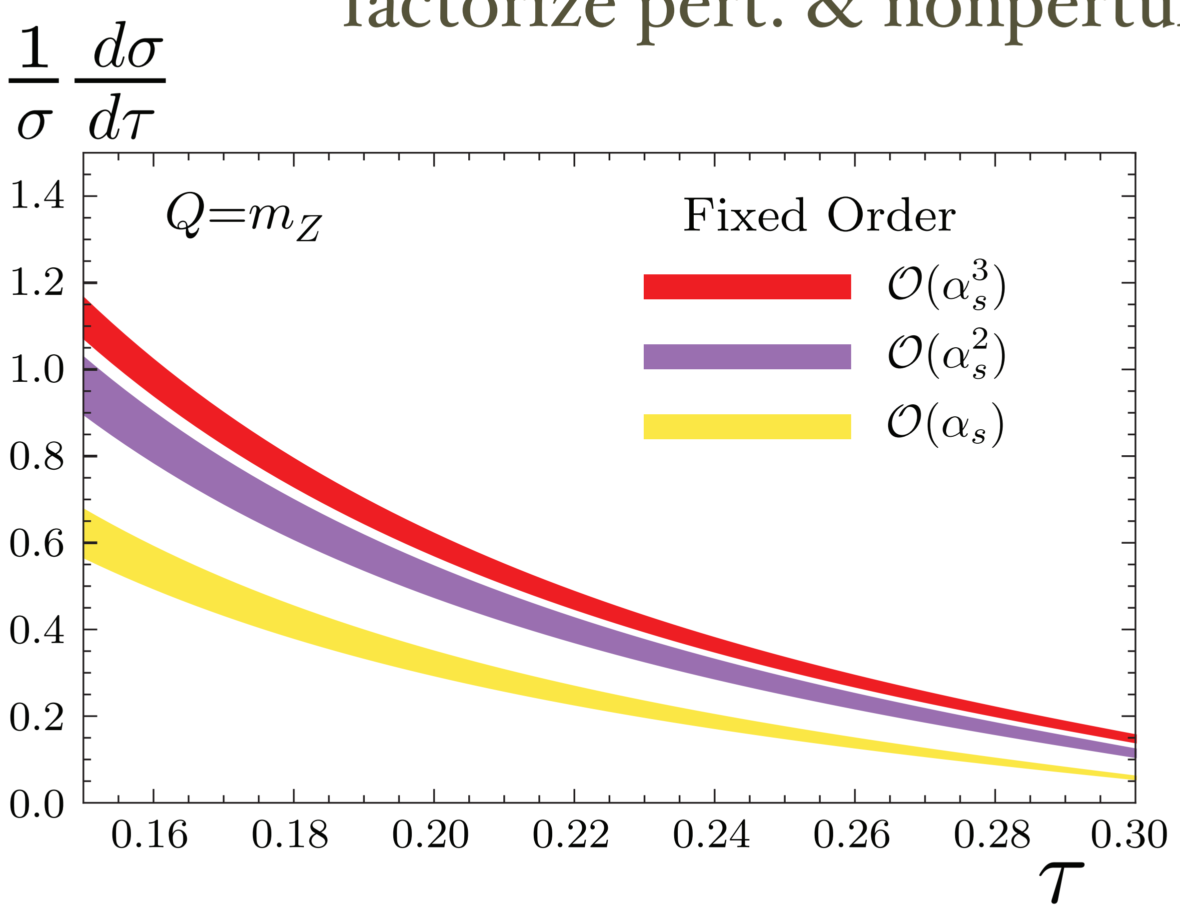
Abbate, Fickinger, Hoang, Mateu, I.S. `10

- $\mathcal{O}(\alpha_s^3)$ + N^{3}LL + $\frac{\Omega_1}{Q\tau}$ power correction + renormalon subtractions, R-RGE

using $\mathcal{O}(\alpha_s^3)$ from Gehrmann et al. & Weinzierl

\+ full treatment of {peak, tail, multijet} + QED effects + b-mass effects + global fit, various Q's

factorize pert. & nonperturbative soft effects: $S = S^{\text{pert}} \otimes S^{\text{mod}}$



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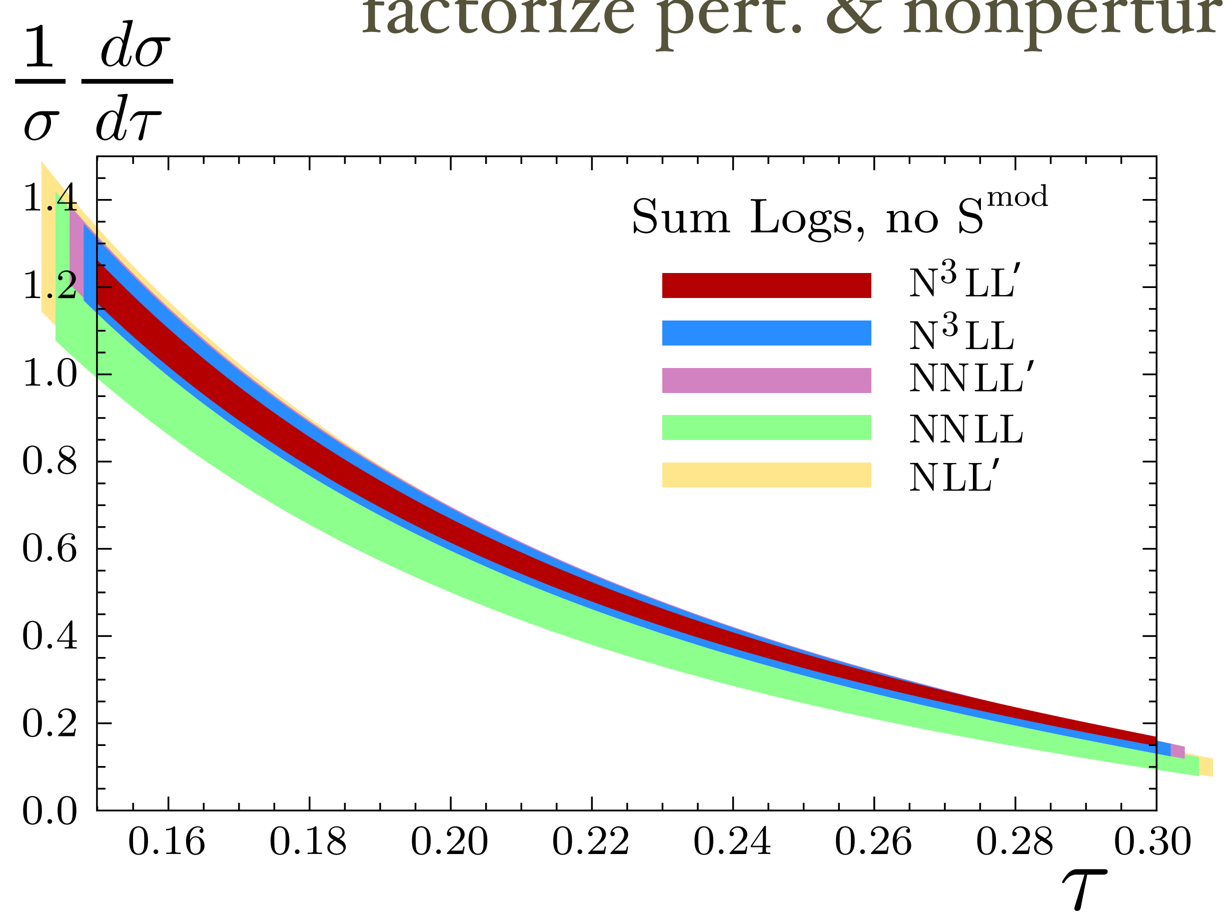

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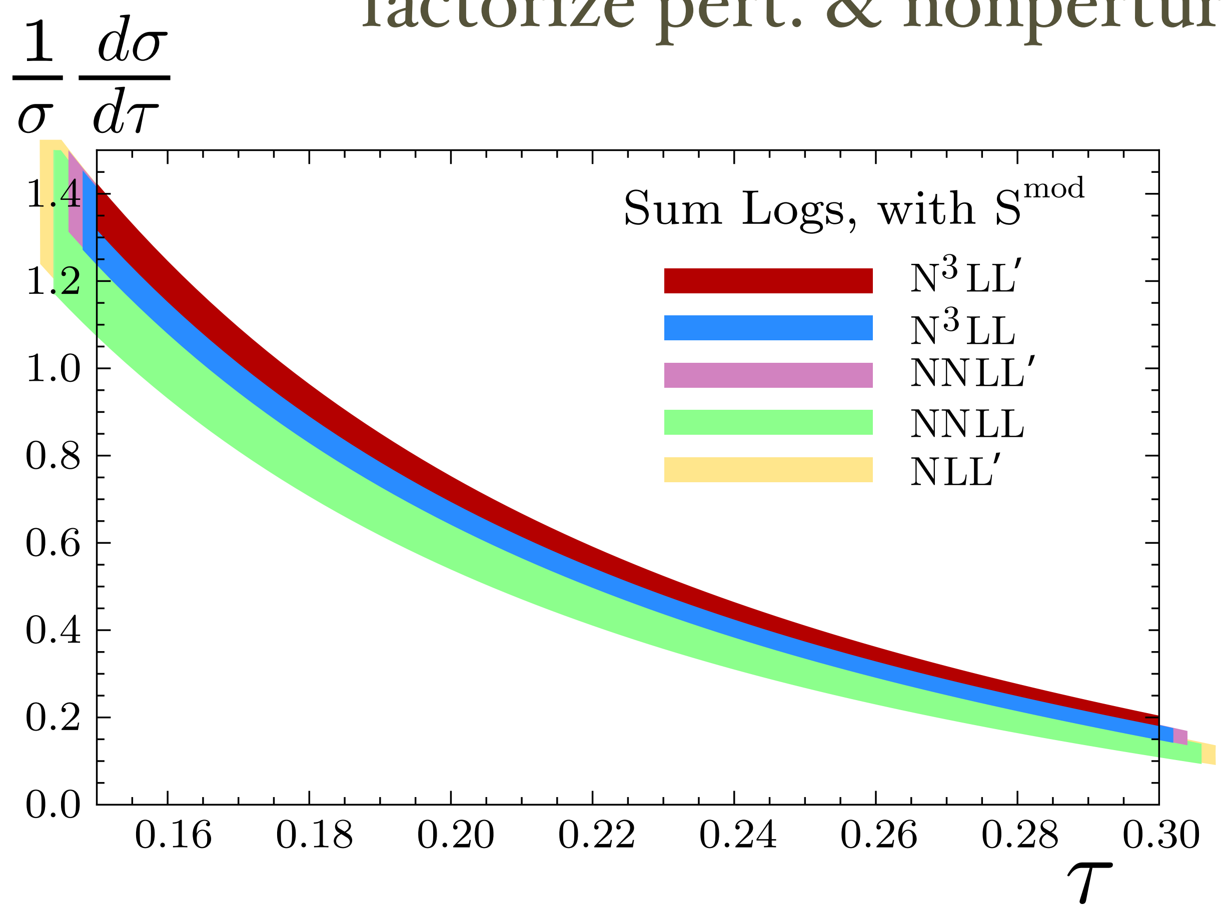

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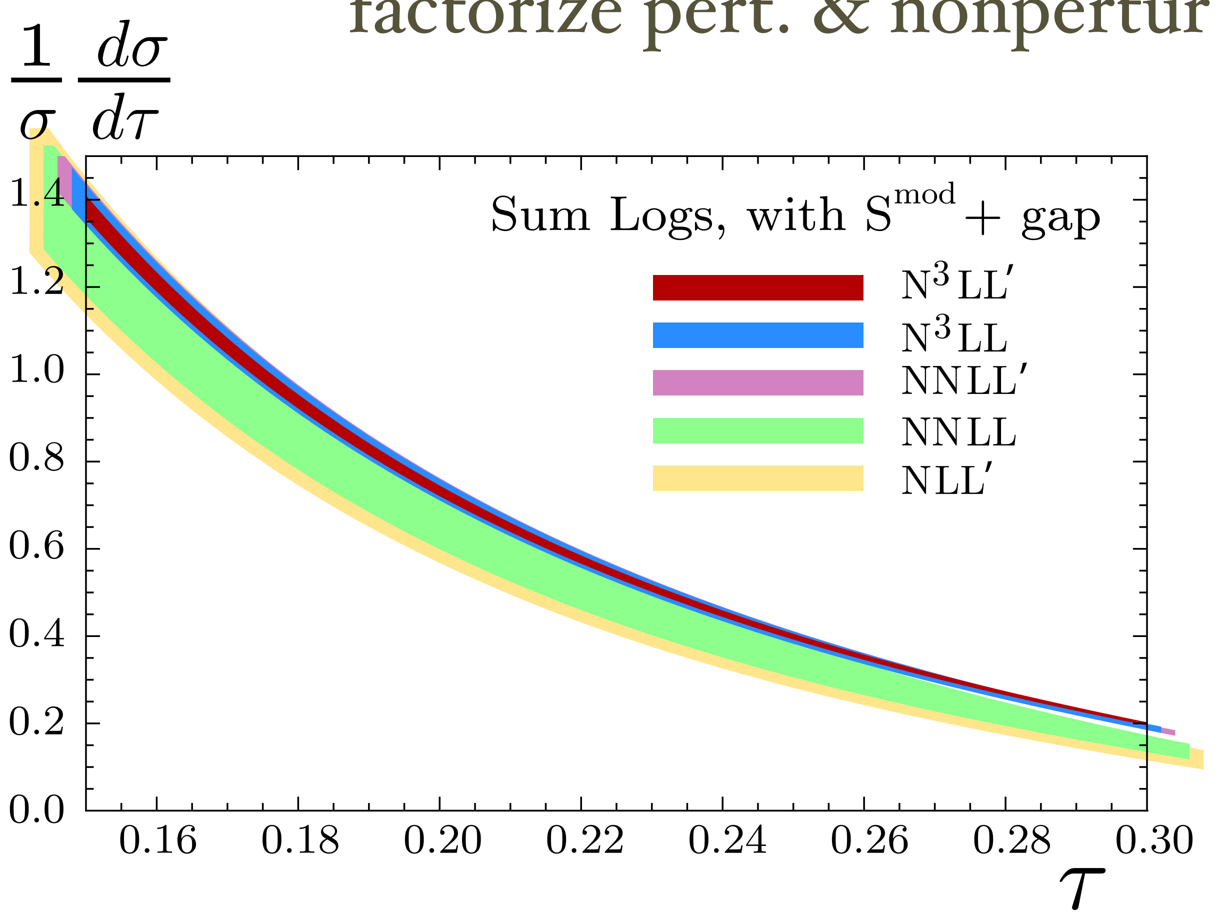

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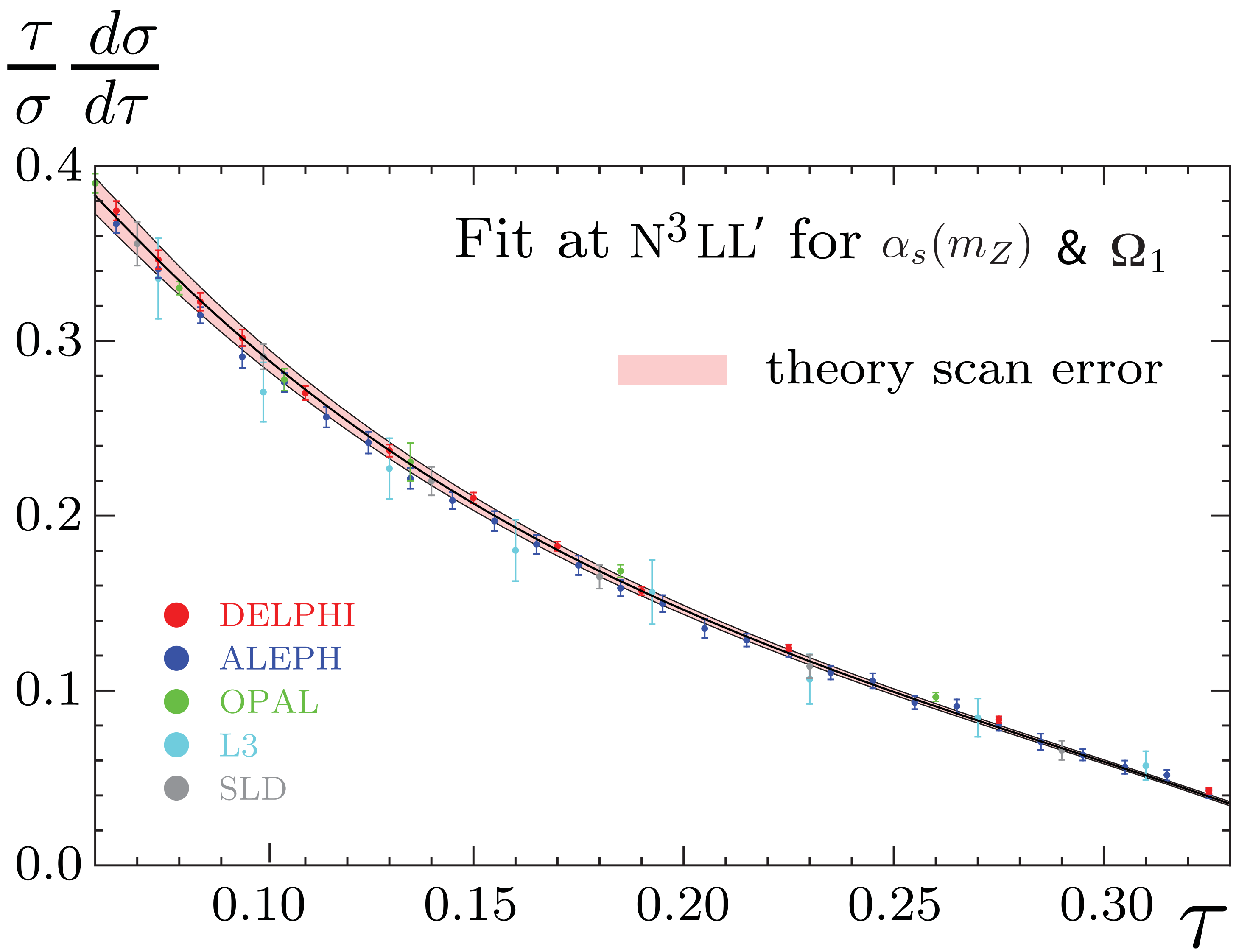

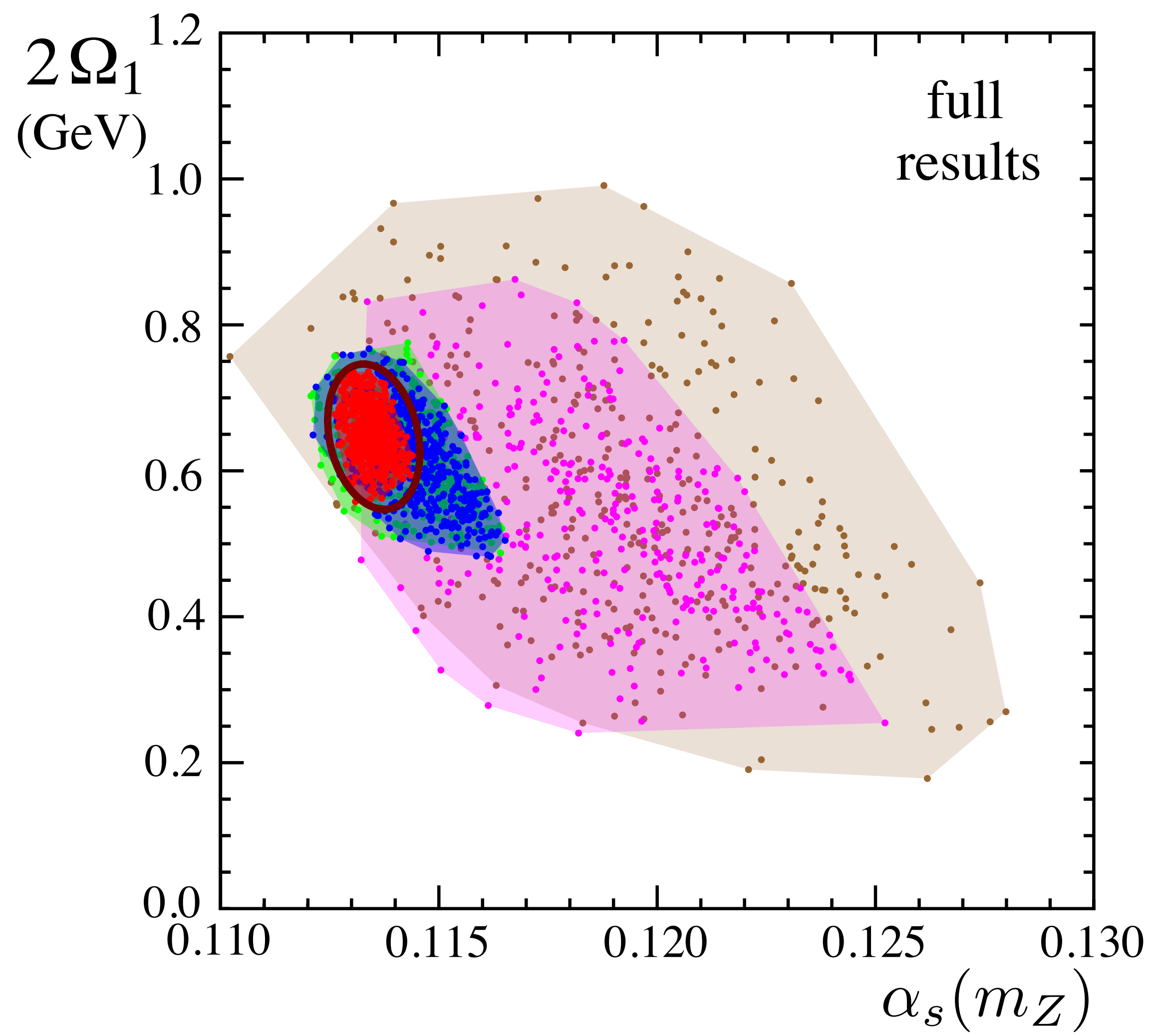

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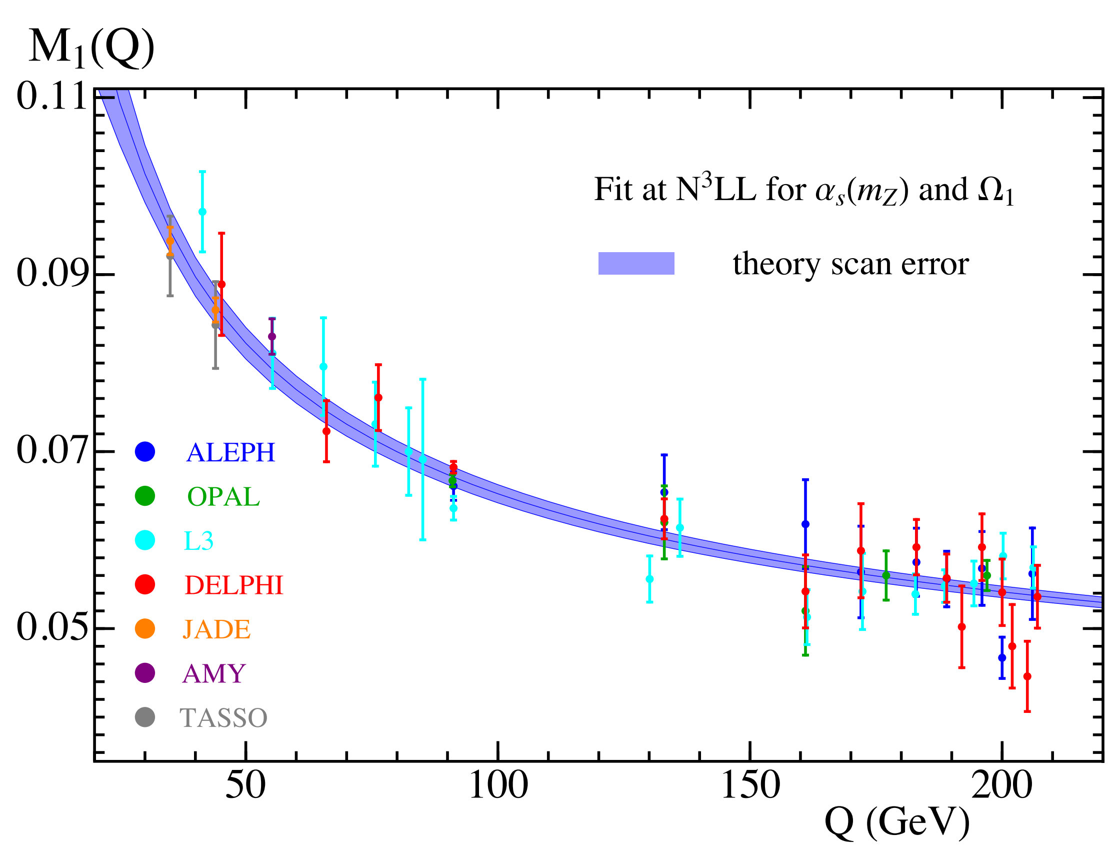
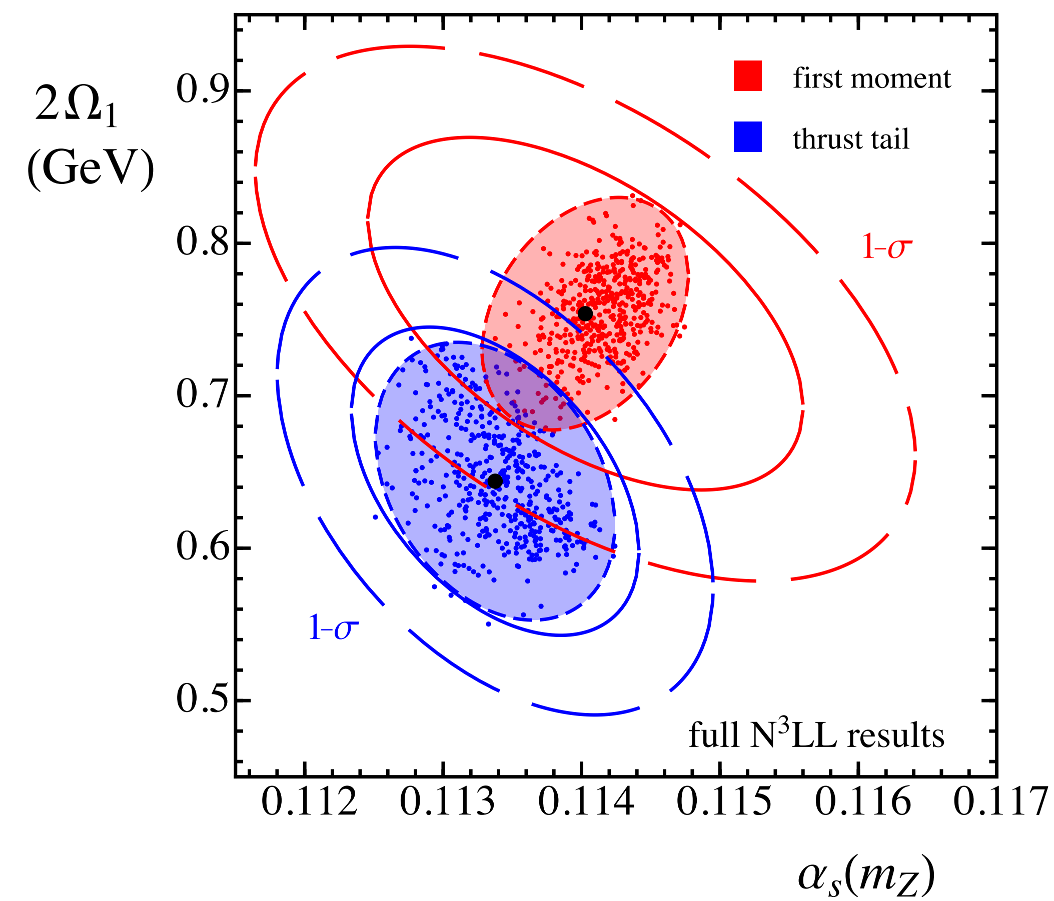
Cross Check with fits for First Moment





Cross check with C-parameter fit (also confirms universality of Ω_1) Hoang et.al \`15

Recent cross check by predicting EEC without a fit (agree with OPAL data) Schindler, IS, Sun \`23

The Higgs p_T Spectrum and Total Cross Section with Fiducial Cuts at N^{3}LL$'$+N^{3}LO

Billis, Dehnadi, Ebert, Michel, Tackmann [2102.08039]

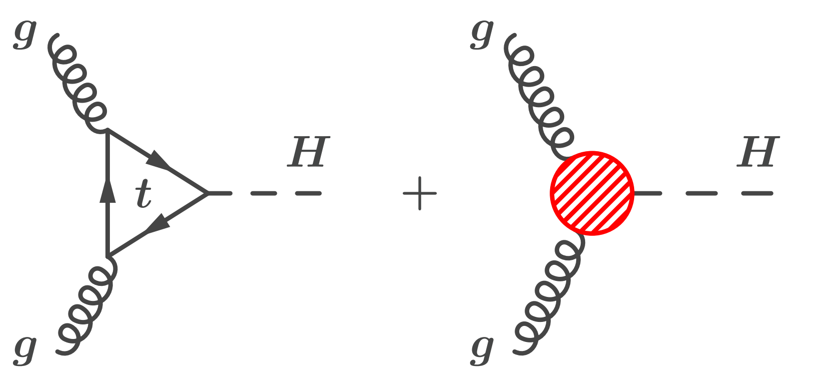
Consider $gg \to H \to \gamma\gamma$ with ATLAS fiducial cuts:

$$p_T^{\gamma 1} \geq 0.35\, m_H\,, \quad p_T^{\gamma 2} \geq 0.25\, m_H\,, \quad |\eta^\gamma| \leq 2.37\,, \quad |\eta^\gamma| \notin [1.37, 1.52]$$

$$\sigma^{\rm fid} = \int dq_T dY\, A(q_T, Y; \Theta)\, W(q_T, Y)$$

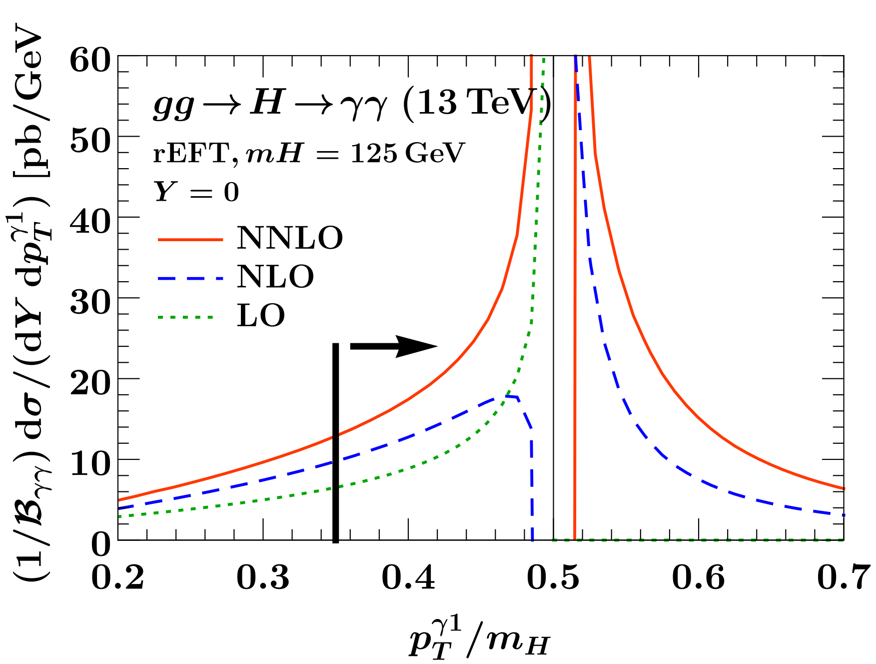
A=acceptance

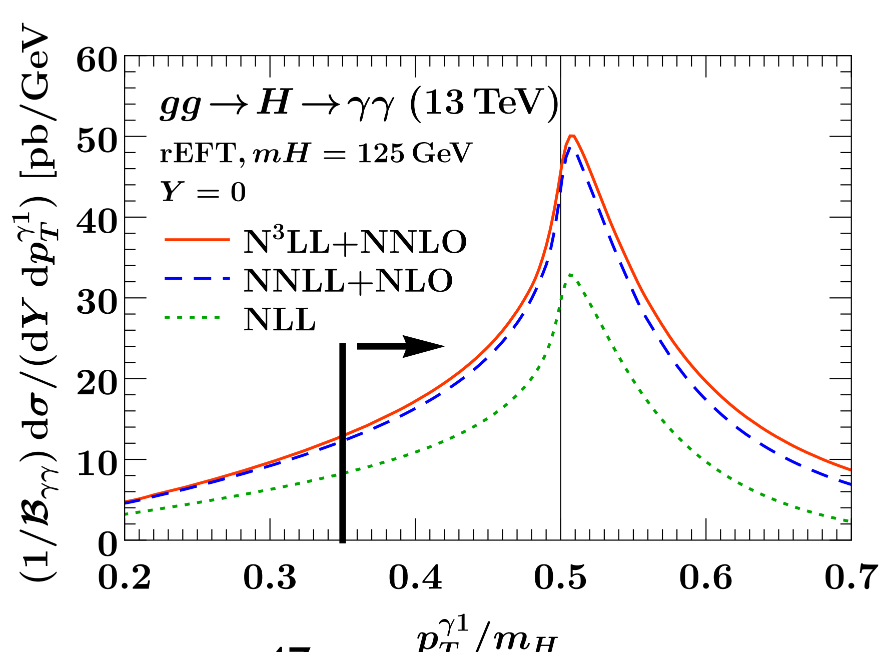
Fiducial cross section measures deviation from SM gluon-fusion:



$$= \Big(\frac{\alpha_s}{12\pi v} C_t + \frac{v}{\Lambda^2} C_{HG}\Big) H\, G^a_{\mu\nu} G^{a,\mu\nu}$$

Acceptance causes a need for resummation to obtain Fiducial cross section





cutting on photon p_T induces large logs

Resumation Inputs

- Three-loop soft and hard function ...includes in particular the three-loop virtual form factor
  [Li, Zhu, '16] [Baikov et al. '09; Lee et al. '10; Gehrmann et al. '10]
- Three-loop unpolarized and two-loop polarized beam functions
  [Ebert, Mistlberger, Vita '20; Luo, Yang, Zhu, Zhu '20]
  [Luo, Yang, Zhu, Zhu '19; Gutierrez-Reyes, Leal-Gomez, Scimemi, Vladimirov '19]
- Four-loop cusp, three-loop noncusp anomalous dimensions
  [Brüser, Grozin, Henn, Stahlhofen '19; Henn, Korchemsky, Mistlberger '20; v. Manteuffel, Panzer, Schabinger '20] [Li, Zhu, '16; Moch, Vermaseren, Vogt '05; Idilbi, Ma, Yuan '06; Vladimirov '16]

Fixed Order Inputs (for non-singular, not discussed here)

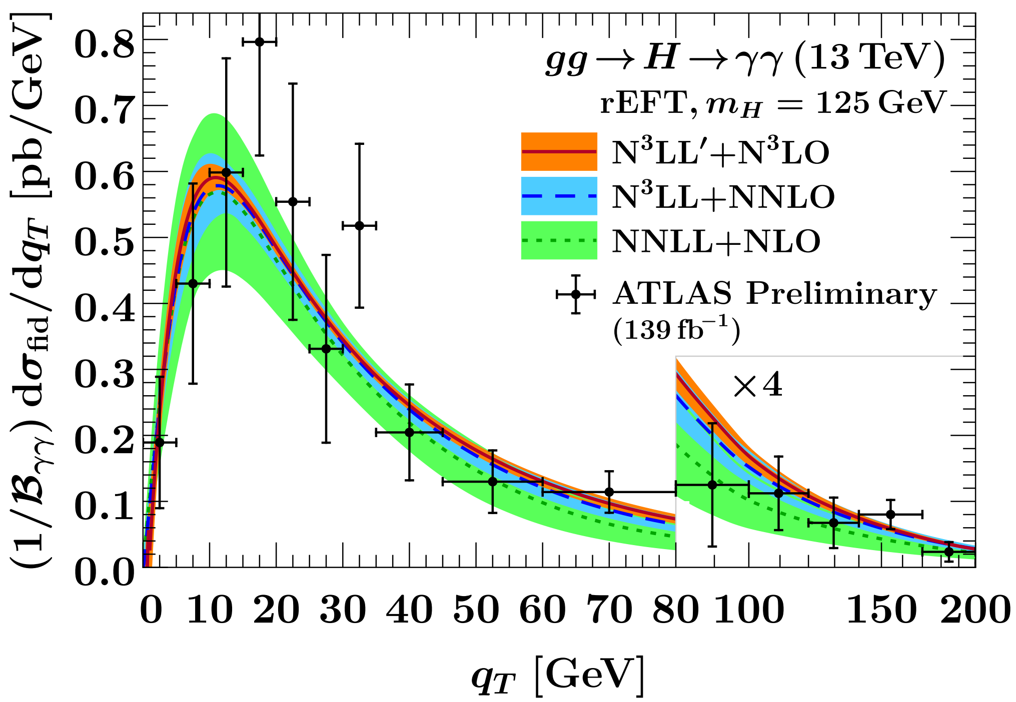
- At NNLO, renormalize & implement bare analytic results for $W(q_T, Y)$
  [Dulat, Lionetti, Mistlberger, Pelloni, Specchia '17]
- At N^3LO, use existing binned NNLO_1 results from `NNLOjet`
  [Chen, Cruz-Martinez, Gehrmann, Glover, Jaquier '15-16; as used in Chen et al. '18; Bizoń et al. '18]
- Use N^3LO total inclusive cross section as additional fit constraint on underflow
  [Mistlberger '18]

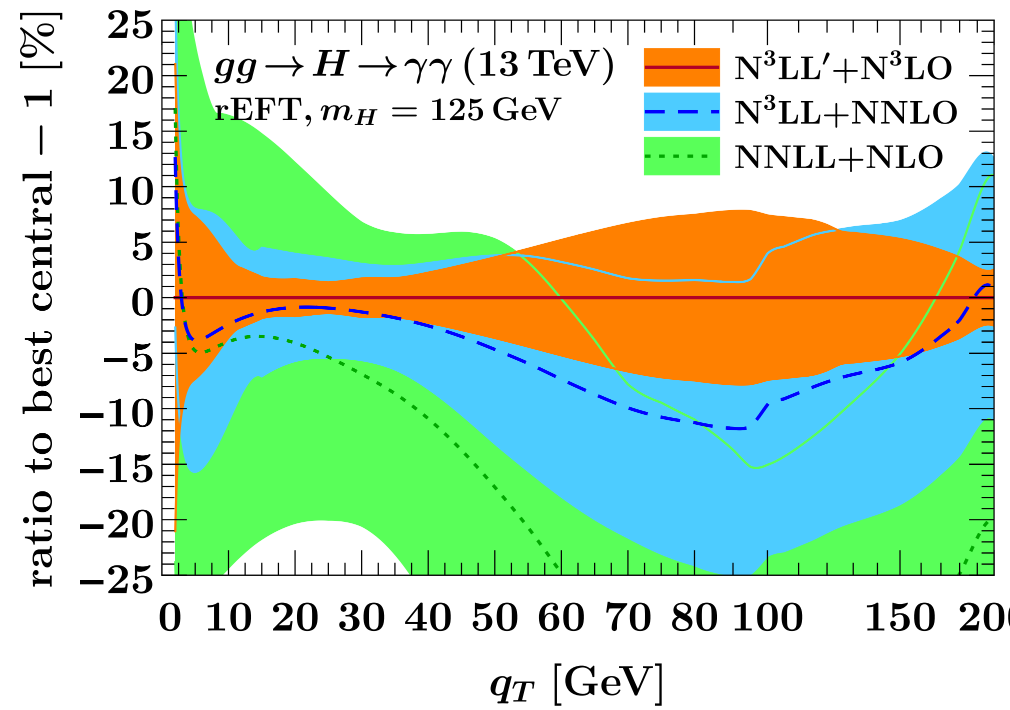
Implemented in C++ Library "SCETlib"

Results

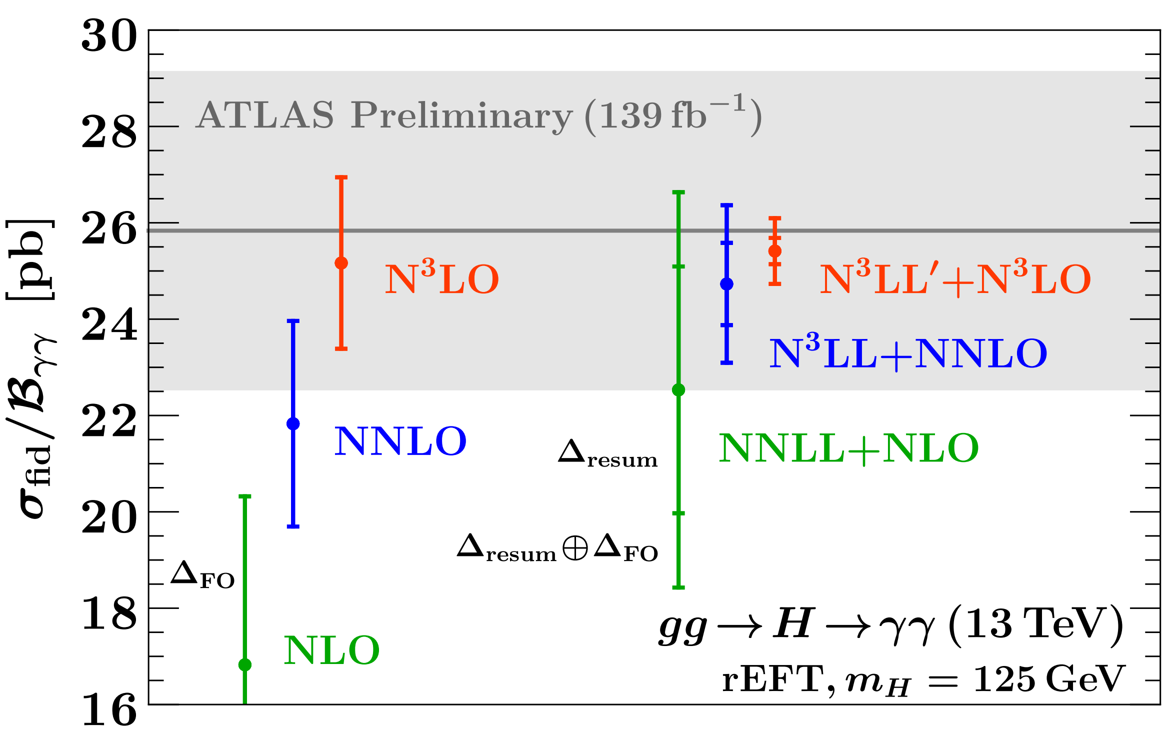
Billis, Dehnadi, Ebert, Michel, Tackmann [2102.08039]

The fiducial q_T spectrum at N^{3}LL′+N^{3}LO





The total fiducial cross section at N^{3}LO and N^{3}LL′+N^{3}LO **(SM)**



Precision and convergence improved

Subleading Power

SCET enables a systematic study of power corrections in various observables

$$\tau \frac{d\sigma}{d\tau} = \sum_{i,j} c_{i,j}^{(0)} \, \alpha_s^i \, \ln^j \tau + \sum_{i,j} c_{i,j}^{(1)} \, \alpha_s^i \, \tau \ln^j \tau + \dots$$

Leading Power — Next to Leading Power

logs generated by power corrections to soft and collinear limits

Interesting:

- Formal questions: Factorization? Universality of functions? Universality of anomalous dimensions?
- Sudakov suppression at subleading power?
- Improve Fixed Order Calculations (subtractions)
- Examples where subleading power is needed (high precision, B's)

Subleading Power in SCET

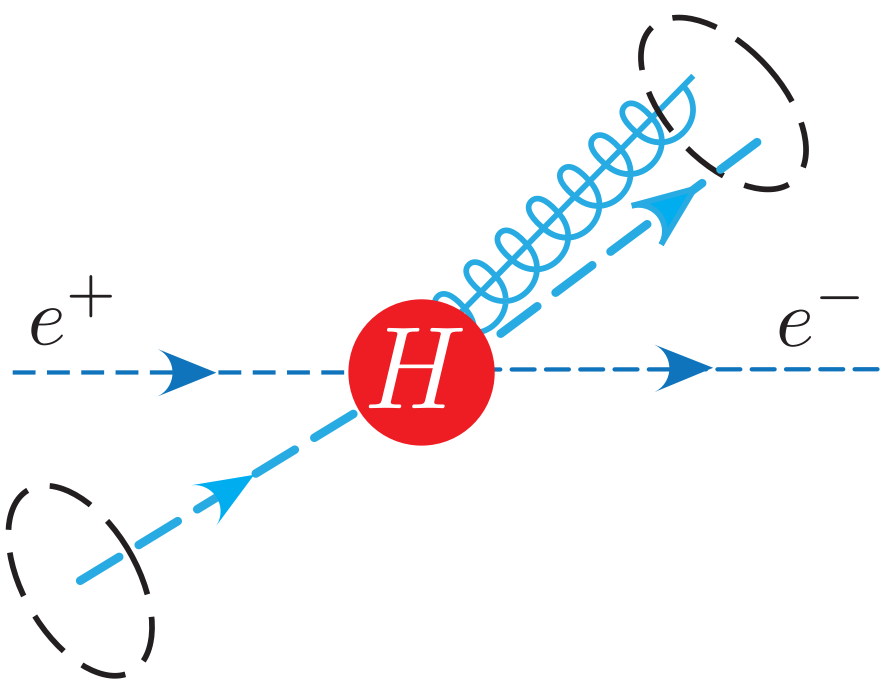
systematic power expansion
about soft & collinear limits

$\lambda \ll 1$

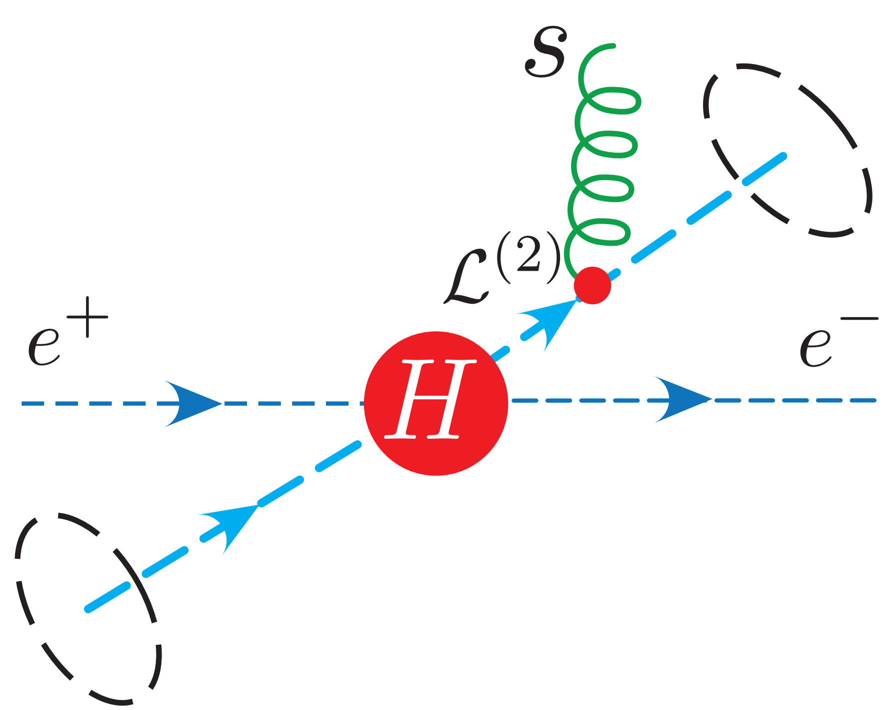
$\mathcal{O}(\lambda^i)$

$$\mathcal{L}_{\mathsf{SCET}} = \mathcal{L}_{\rm hard} + \mathcal{L}_{\rm dyn} = \sum_{i \geq 0} \mathcal{L}^{(i)}_{\rm hard} + \sum_{i \geq 0} \mathcal{L}^{(i)}$$

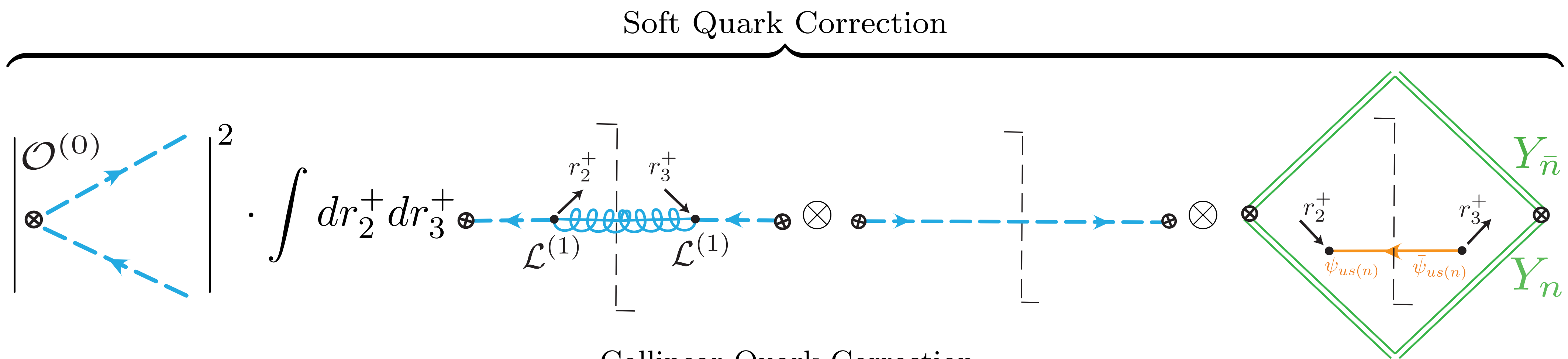
Subleading Hard Scattering Operators

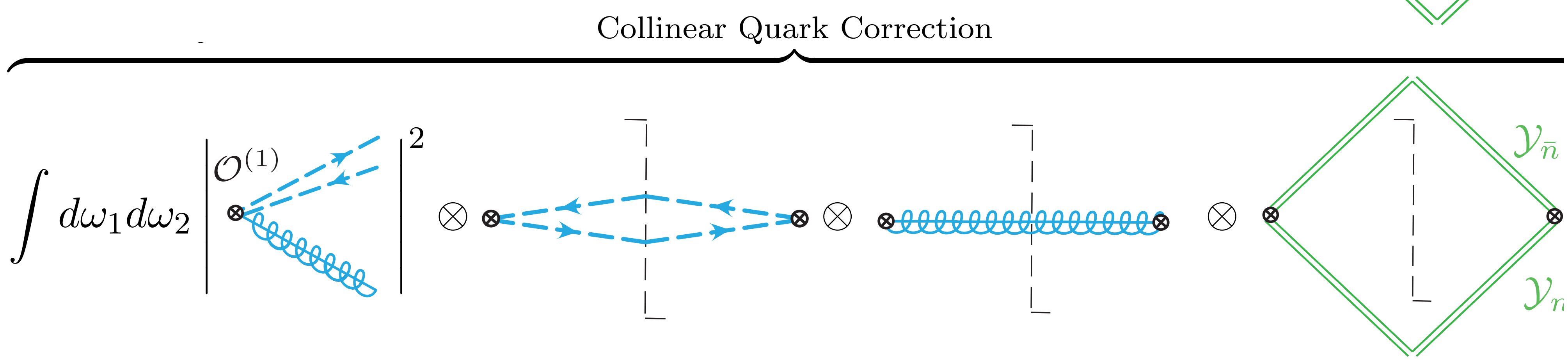


Subleading Lagrangians



Sudakov suppression at subleading power? Eg. in Thrust





$$= \int d\omega_1 \omega_2 \left| C^{(1)}_{\chi\chi n}(\omega_1) C^{(1)}_{\chi\chi n}(\omega_2) \right| J^{(2)}_{\chi\chi}(\omega_1,\omega_2) \otimes J^{(0)}_{g,\bar{n}} \otimes S^{(0)}_q + n \leftrightarrow \bar{n}\,.$$

$$\frac{1}{\sigma_0}\frac{d\sigma^{(2),e^+e^-}_{\rm LL}}{d\tau} = \left(\frac{\alpha_s}{4\pi}\right) 8C_F \log(\tau) e^{-4C_F\left(\frac{\alpha_s}{4\pi}\right)\log^2(\tau)} + \underbrace{\frac{C_F}{(C_F - C_A)\log(\tau)}\left(e^{-4C_F\left(\frac{\alpha_s}{4\pi}\right)\log^2(\tau)} - e^{-4C_A\left(\frac{\alpha_s}{4\pi}\right)\log^2(\tau)}\right)}_{\text{Soft Quark Sudakov}}$$

- Endpoint singularities!

$$\int_0 \frac{dx}{x}$$

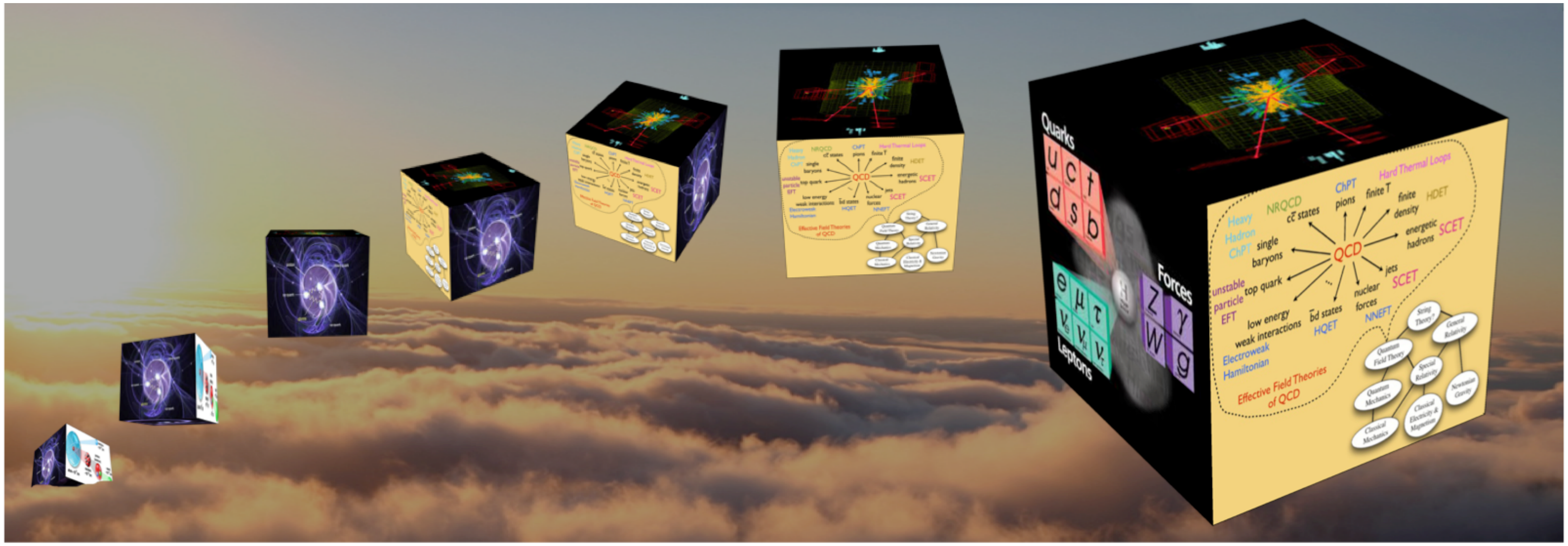
- Conjecture

Moult, IS, Vita, Zhu `19

- Proof (refactorization)

Beneke, Garny, Jaskiewicz, Strohm, Szafron, Vernazza, Wang `22

The End



Quarks
u c t
d s b
Leptons
e μ τ
ν_e ν_μ ν_τ
Forces
Z γ
W g
QCD
Heavy Hadron ChPT
NRQCD
$c\bar{c}$ states
ChPT
pions
finite T
Hard Thermal Loops
finite density
HDET
energetic hadrons
SCET
single baryons
unstable particle EFT
top quark
jets
SCET
low energy weak interactions
Electroweak Hamiltonian
$\bar{b}d$ states
HQET
nuclear forces
NNEFT
Effective Field Theories of QCD
String Theory?
General Relativity
Quantum Field Theory
Special Relativity
Quantum Mechanics
Newtonian Gravity
Classical Electricity & Magnetism
Classical Mechanics