September 2023

50+ years of lepton pair production

Keith Ellis
IPPP, Durham

What sort of a history is this anyway?

- What better way to uncover the history of lepton pair production than in the words of a main protagonist?

Figure 1 is then a thumbnail ten year

Dilepton Production in Hadron Collisions

Brief History

1968: Columbia-BNL Proposal to Probe Small Distances Using Virtual Photons and to Look for Bumps.

1978: Tokyo

Bumps Have Been Found: J/ψ, ψ', Υ, ...

"Small Distance Probe" Has Found a Constituent (Quark-Gluon) Model Which is Completely Consistent with Lepton Scattering.

Fig. 1.

history and summary of my talk after which the reader can skip to the bibliography to see if I have referred to him properly. In 1978

her/him

Leon Lederman: Proceedings of the 19th ICHEP Tokyo (1978)

The beginning…

Volume 25, Number 21 PHYSICAL REVIEW LETTERS 23 November 1970

Observation of Massive Muon Pairs in Hadron Collisions*

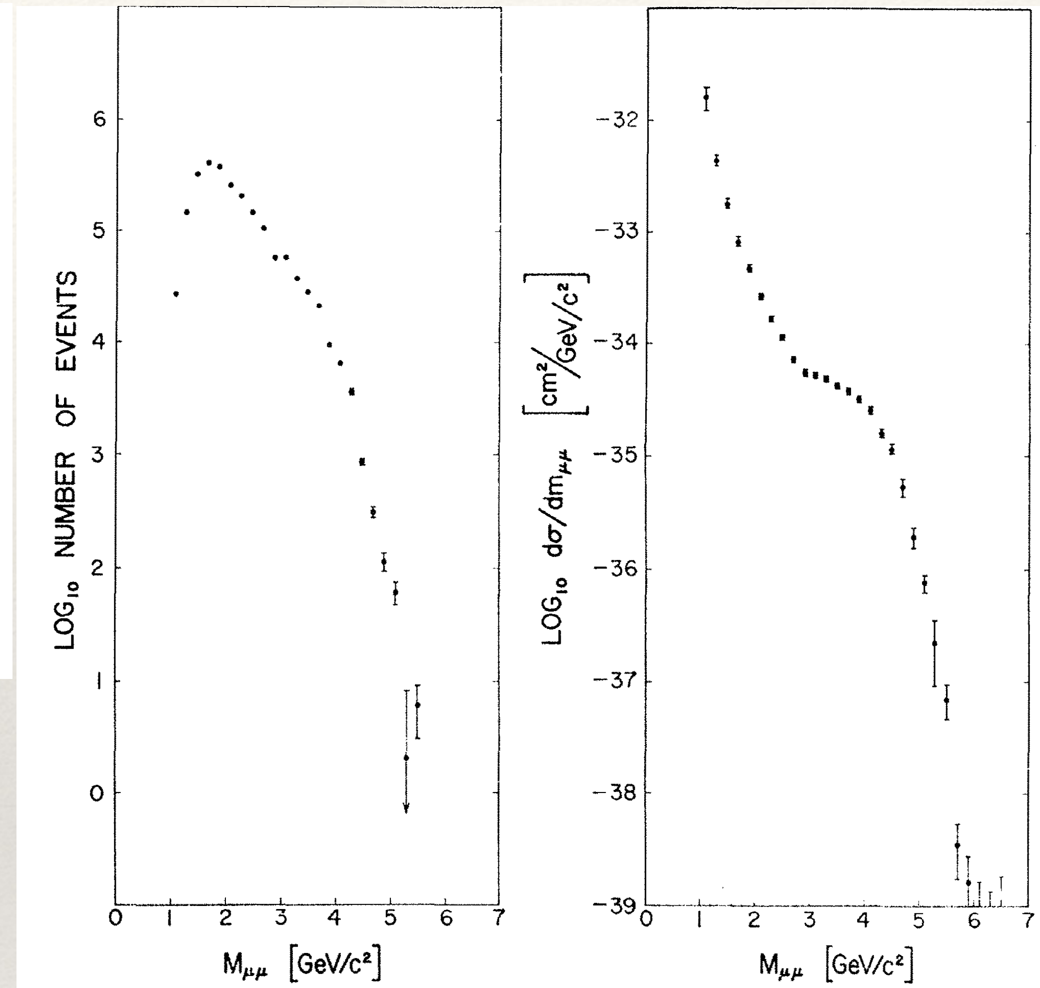
J. H. Christenson, G. S. Hicks, L. M. Lederman, P. J. Limon, and B. G. Pope
Columbia University, New York, New York 10027, and Brookhaven National Laboratory, Upton, New York 11973

and

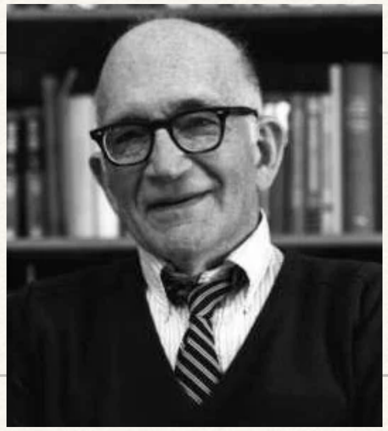
E. Zavattini
CERN Laboratory, Geneva, Switzerland
(Received 8 September 1970)

8th September 1970

Muon pairs in the mass range $1 < m_{\mu\mu} < 6.7$ GeV/c^2 have been observed in collisions of high-energy protons with uranium nuclei. At an incident energy of 29 GeV, the cross section varies smoothly as $d\sigma/dm_{\mu\mu} \approx 10^{-32}/m_{\mu\mu}^{5}$ cm^2 (GeV/c)$^{-2}$ and exhibits no resonant structure. The total cross section increases by a factor of 5 as the proton energy rises from 22 to 29.5 GeV.



- Leon credits Yamaguchi and Okun for suggesting lepton pair processes.
- "As seen both in the mass spectrum and the resultant cross section there is no forcing evidence of any resonant structure."
- "Indeed, in the mass region near 3.5 GeV/c², the observed spectrum may be reproduced by a composite of a resonance and a steeper continuum."
- Drell and Yan had seen the Christenson et al data at the spring APS meeting



Drell-Yan

- Drell and Yan showed that the parton model could be derived if the impulse approximation was valid.
- To accomplish this, they had to impose a transverse momentum cut-off for the particles that appeared in the quantum field theory.

Assumed anti-parton distributions= parton distributions!

$$\frac{d\sigma}{dQ^2} = \frac{4\pi\alpha}{3Q^2}\frac{1}{Q^2}\mathscr{F}(\tau) = \frac{4\pi\alpha}{3Q^2}\frac{1}{Q^2}\int_0^1 dx_1 \int_0^1 dx_2\, \delta(x_1 x_2 - \tau) \sum_a \lambda_a^{-2} F_{2a}(x_1) F'_{2\bar{a}}(x_2)$$

No color factor!

Unknown! parton charges

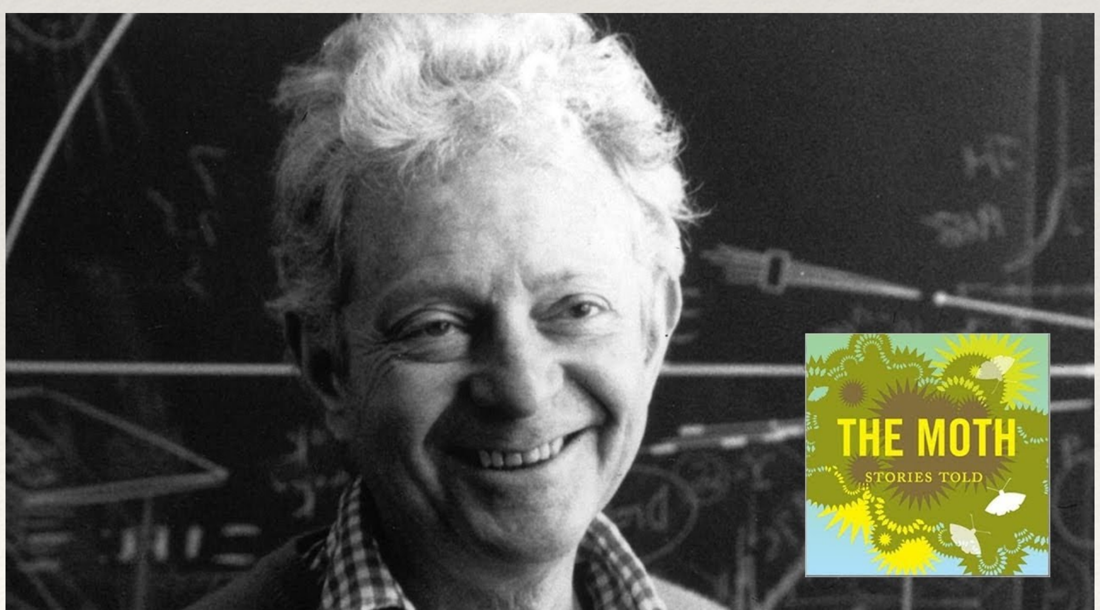
- Rapid fall-off of the cross section, despite the fact that the partons were point-like particles (in contrast to DIS).

cf, Altarelli, Brandt & Preparata, PRL (1970)

Leon on “Drell-Yan”

> I come now to the Drell-Yan process i.e. dilepton production in hadronic collisions, (sigh!) named by Feynman after an experiment at BNL by Christenson. Here there is consid-

Lederman, Batavia Conference, 9th International Symposium on Lepton and Photon Interactions at High Energy, (1979)



13:50

The first Drell Yan prediction

MASSIVE LEPTON-PAIR PRODUCTION IN HADRON-HADRON COLLISIONS AT HIGH ENERGIES*

Sidney D. Drell and Tung-Mow Yan

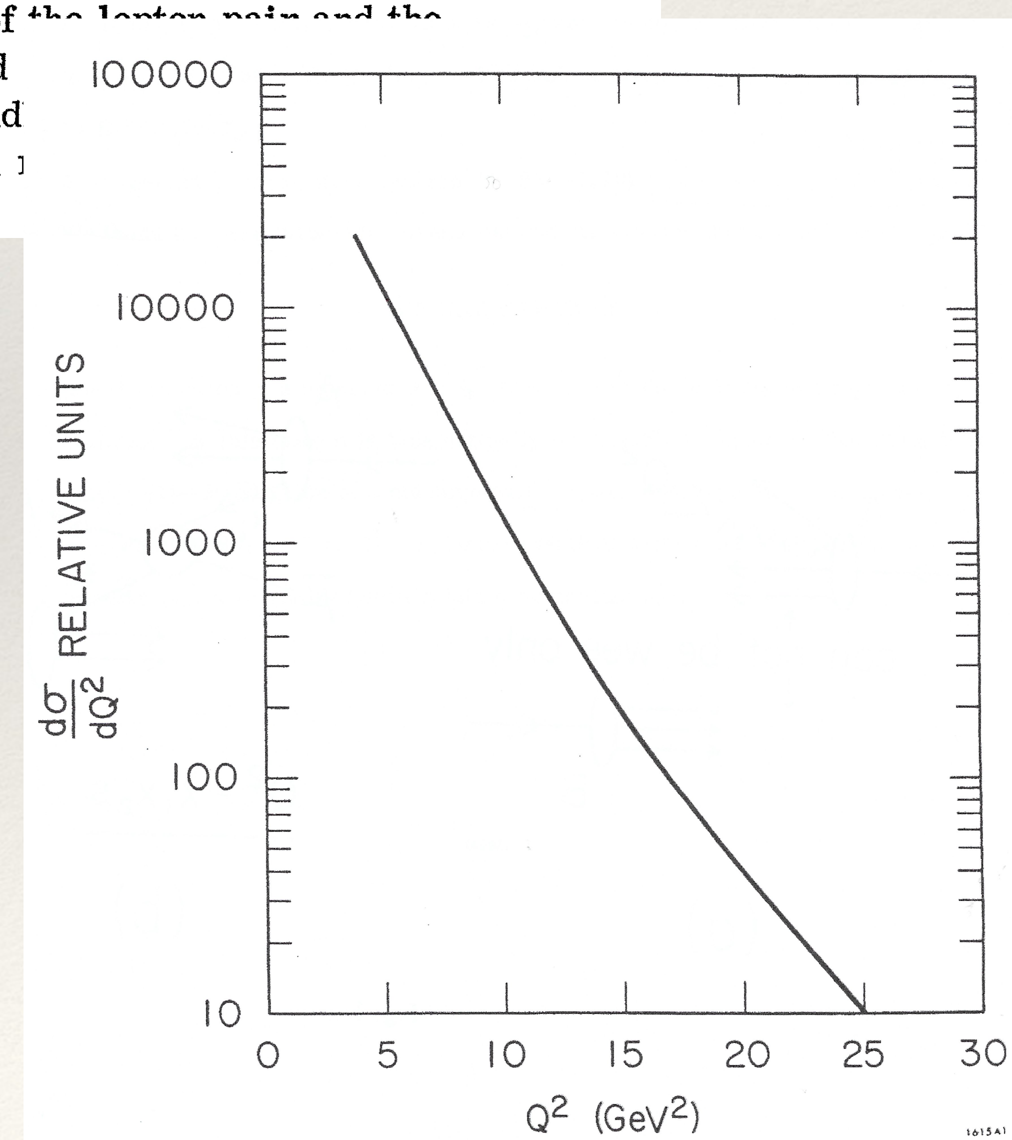
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305

(Received 25 May 1970)

May1970!

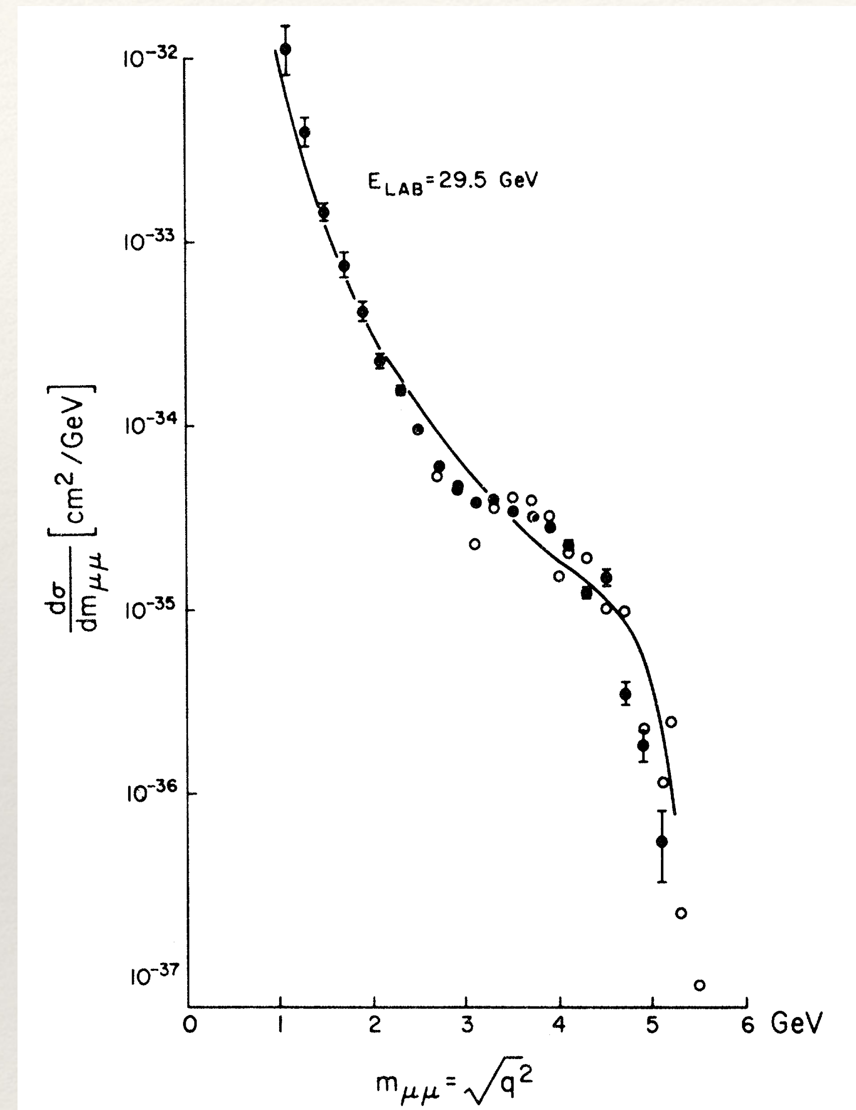
On the basis of a parton model studied earlier we consider the production process of large-mass lepton pairs from hadron-hadron inelastic collisions in the limiting region, $s \to \infty$, Q^2/s finite, Q^2 and s being the squared invariant masses of two initial hadrons, respectively. General scaling properties and inelastic electron scattering are discussed. In particular, a rapid section as $Q^2/s \to 1$ is predicted as a consequence of the observed elastic scattering structure function νW_2 near threshold.

- Predictions are
  - approximate scaling $\frac{Q^3 \, d\sigma}{dQ} = F(\tau), \ \tau = Q^2/s,$
  - angular dependence, $(1 + \cos^2\theta)$
  - A^1 dependence on nucleon number.



Two explanations of Leon's shoulder

thanks to C. Quigg



Altarelli, Brandt and Preparata, 9th September 1970
Light cone analysis of massive pair production,

Follow up experiment at Fermilab

- Fermilab proposal E288 (1974)
- A Study of Di-Lepton Production in Proton Collisions at NAL

NAL PROPOSAL # 288

Scientific Spokesman:

L. M. Lederman
Physics Department
Columbia University
New York, New York 10027

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280-1754

A Study of Di-Lepton Production in Proton Collisions at NAL

J. A. Appel, M. H. Bourquin, D. C. Hom, L. M. Lederman, J. P. Repellin, H. D. Snyder, J. K. Yoh (Columbia University); B. C. Brown, P. Limon, T. Yamanouchi (NAL).

(Formerly #70 Phase III)

2. Intermediate Boson Production

The reaction is

$$p + "N" \to W^{\pm} + \text{anything} \quad \lfloor\!\to e^{\pm} + \nu \qquad (4)$$

Historically, such experiments have been carried out at BNL and at Argonne but suffered from the inability of theorists to predict the cross section. Thus a negative result was useless since no statement could be made concerning the W-mass. In contrast, neutrino production (or lack of it) led to the one firm number we have: $M_W > 2$ GeV.

However, the recent BNL dimuon experiment[4] demonstrated an easily measurable continuum of lepton pairs emerging from proton-uranium collisions. The arguments of Chilton[5] and Yamaguchi[6] related reaction (4) to the reaction:

$$p + "N" \to "\gamma" + \text{anything} \quad \lfloor\!\to \mu^{+} + \mu^{-} \text{ or } e^{+} + e^{-} \qquad (5)$$

The prediction for Intermediate Boson production is

Asymptotic freedom expands it scope

- The publication of the DGLAP equation Altarelli-Parisi 1977, Dokshitser (Sov. Phys. JETP, 46,641) with its physical picture of parton evolution, raised the issue of whether the Drell-Yan model could be extended to QCD.
- Politzer (1977) deserves credit for outlining the factorization idea.
- Unlike in the parton model, the transverse momentum is now unbounded.
- Transverse momentum in Drell-Yan processes (APP) and AEM (1979) followed Politzer's lead regulating collinear/soft singularities by continuing off-shell, (which turned out to be a tricky procedure).

cf, Sachrajda, 2/1978 - Lepton pair production and the Drell-Yan formula in QCD

Radiative corrections to Drell-Yan

Altarelli, RKE, Martinelli had written a previous paper mainly on radiative corrections to DIS, including corrections to DY as a (erroneous) postscript

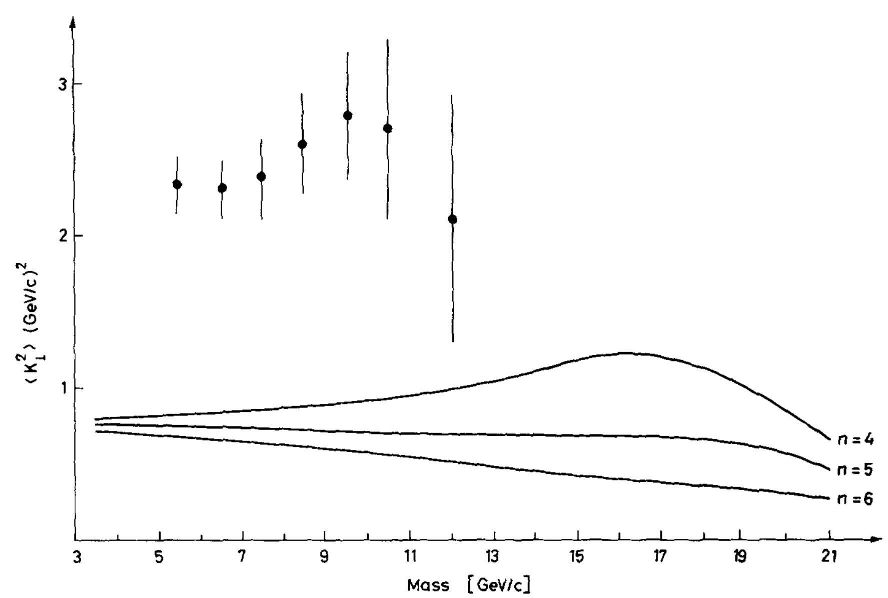

Fig. 3. The hard component of the $\langle k_\perp^2 \rangle$ of the muon pair as a function of their invariant mass is compared with the experimental points taken from ref. [9] for three different powers $n = 4, 5, 6$ of the gluon distribution, following the procedure described in the text.

- QCD predicts an approximate linear rise of $\langle k_T^2 \rangle$ with s or Q^2, but only at fixed τ.
- Intrinsic k_T needed.

Transverse momentum in DY processes, Altarelli, Parisi and Petronzio (1977)

LARGE PERTURBATIVE CORRECTIONS TO THE DRELL-YAN PROCESS IN QCD *

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Istituto di Fisica dell' Università,
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Istituto Nazionale di Fisica Nucleare,
Laboratori Nazionali di Frascati,
Frascati 00044, Italy

Received 17 April 1979

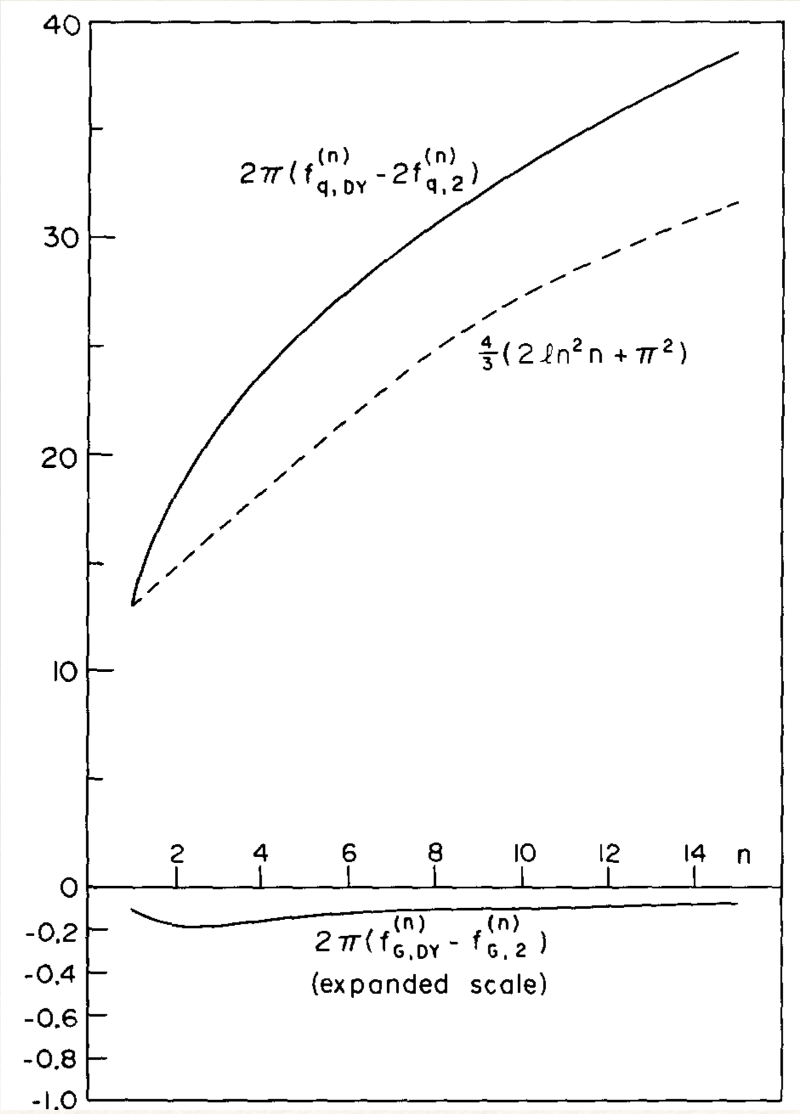
Marciano(1975) - Dimensional Regularization and Mass singularities

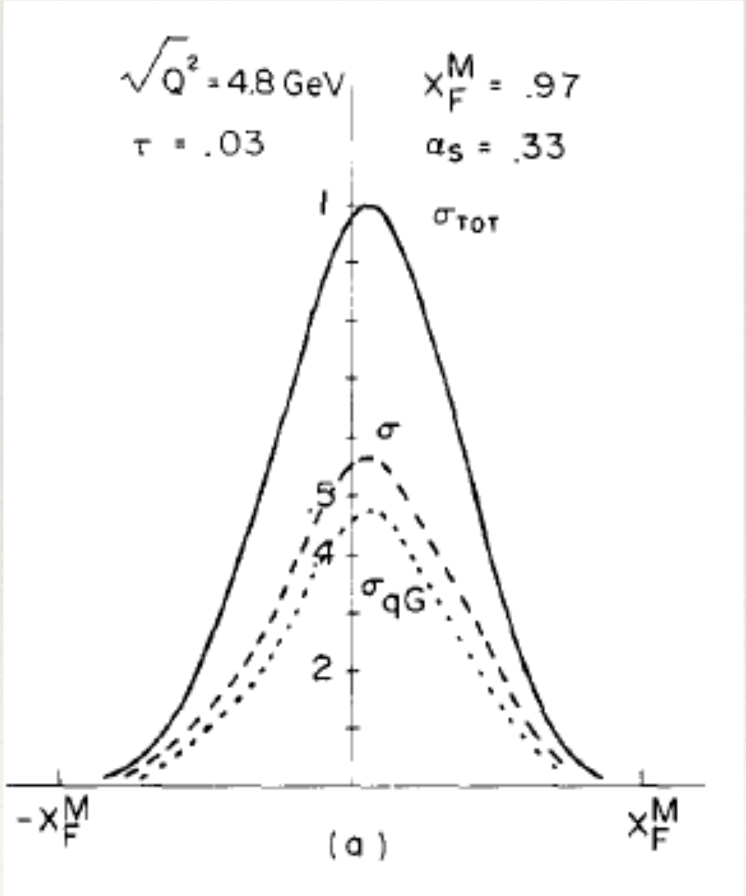
QCD corrections for hadron-hadron interactions

$$\alpha_s f_q(z) = C_F \frac{\alpha_s}{2\pi}\Big[\big(1 + \frac{4\pi^2}{3}\big)\delta(1-z) + 2(1+z^2)\Big(\frac{\ln(1-z)}{1-z}\Big)_+ + \frac{3}{(1-z)_+} - 6 - 4z\Big]$$

$$\alpha_S f_G(z) = \frac{1}{2}\frac{\alpha_s}{2\pi}\Big[(z^2 + (1-z)^2)\ \ln(1-z) + \frac{9}{2}z^2 - 5z + \frac{3}{2}\Big]$$

- Correction relative to DIS
- $\frac{\alpha_S}{2\pi} \approx \frac{1}{20}$
- Simple origin for the large size of the corrections;
- Phenomenology, x_F distribution;





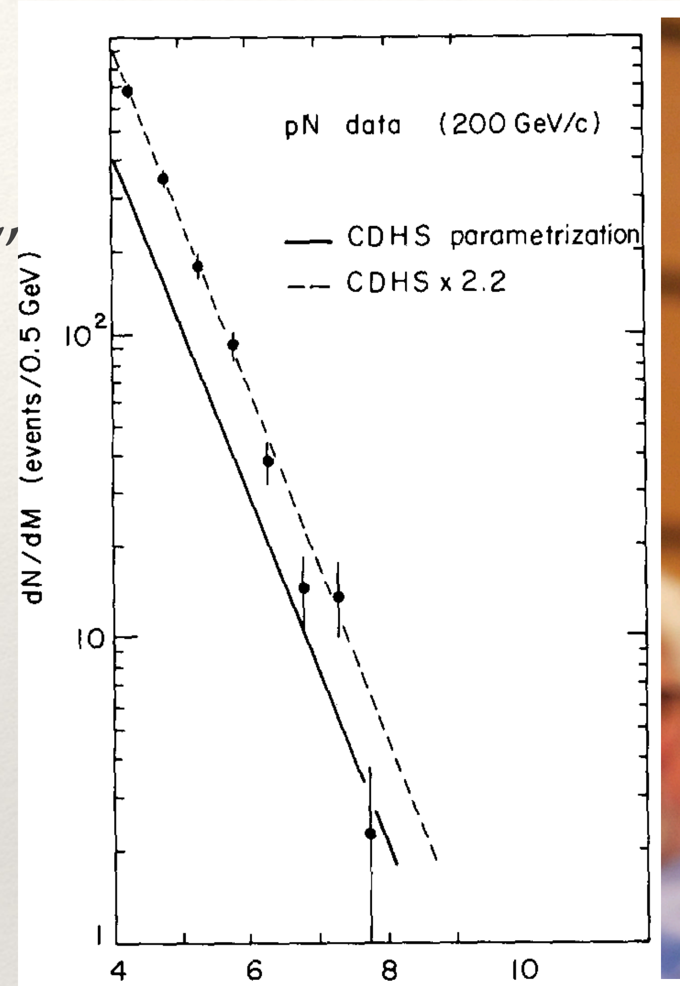
Altarelli, Ellis, Martinelli, see also Kubar-Andre and Paige, and Abad and Humpert

Drell-Yan data and K-factor

- Data lay above the naive DY prediction, leading to the introduction of a “K-factor”

$$\left(\frac{d\sigma}{dQ^2}\right)_{EXP} = K \left(\frac{d\sigma}{dQ^2}\right)_{NAIVE\ D.Y.}$$

- From ~4 experiments $K \geq 2$
- Telegdi question (N_c or not?)

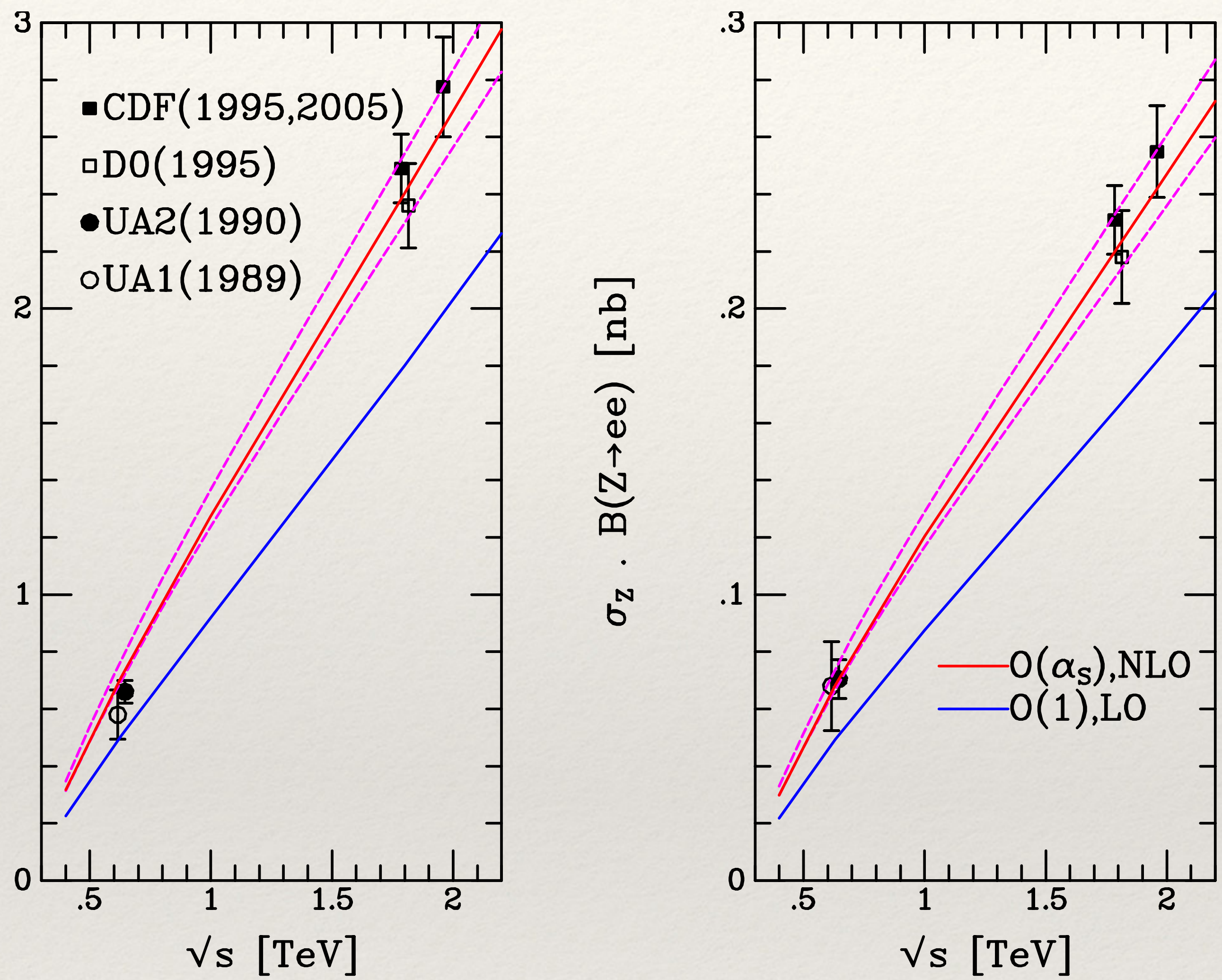


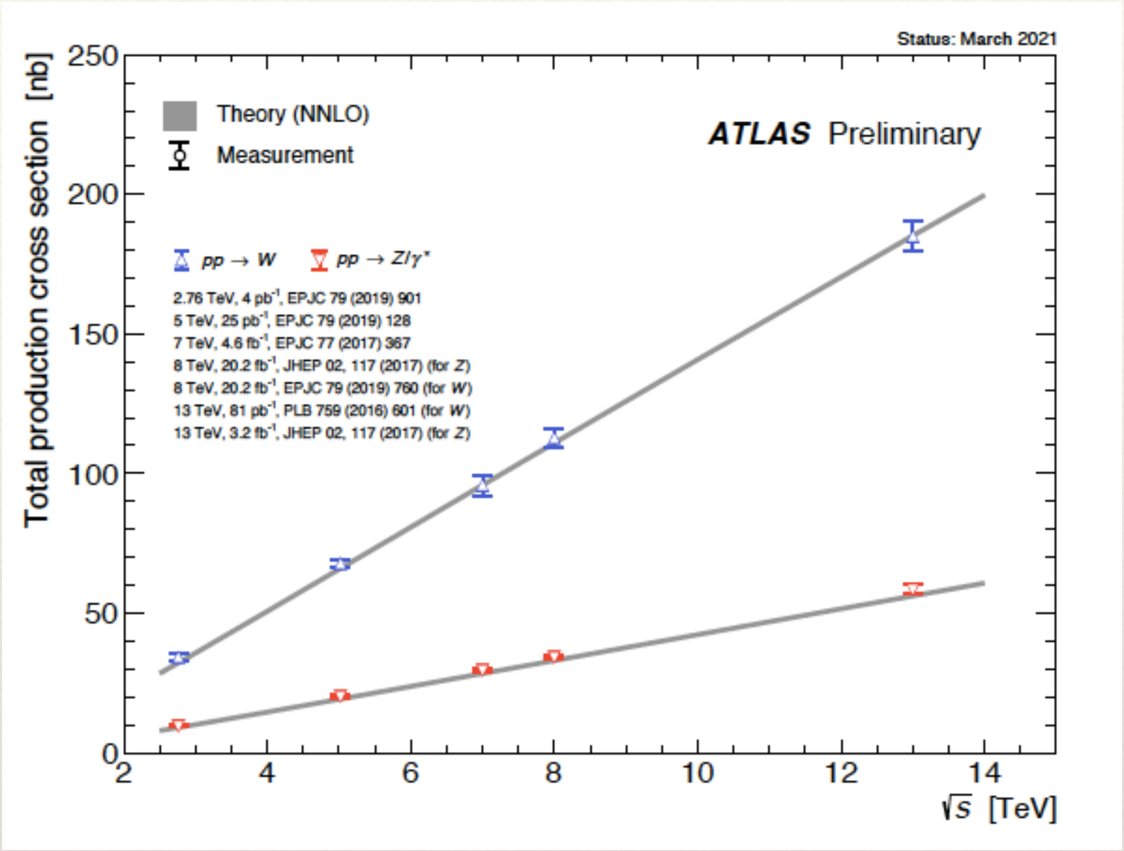
$K = (d^2\sigma/dx_1dx_2)_{exp}/(d^2\sigma/dx_1dx_2)_{DY\ model}$.

Reaction	pN	$\bar{p}$N	π^-N	π^+N	π^-H_2	$(\pi^- - \pi^+)$N
K	2.2 ± 0.4	2.4 ± 0.5	2.2 ± 0.3	2.4 ± 0.4	2.4 ± 0.4	2.2 ± 0.4
Events	960	44	5607	2073	138	–

NA3, Badier et al,

Experimental Situation for massive boson prediction





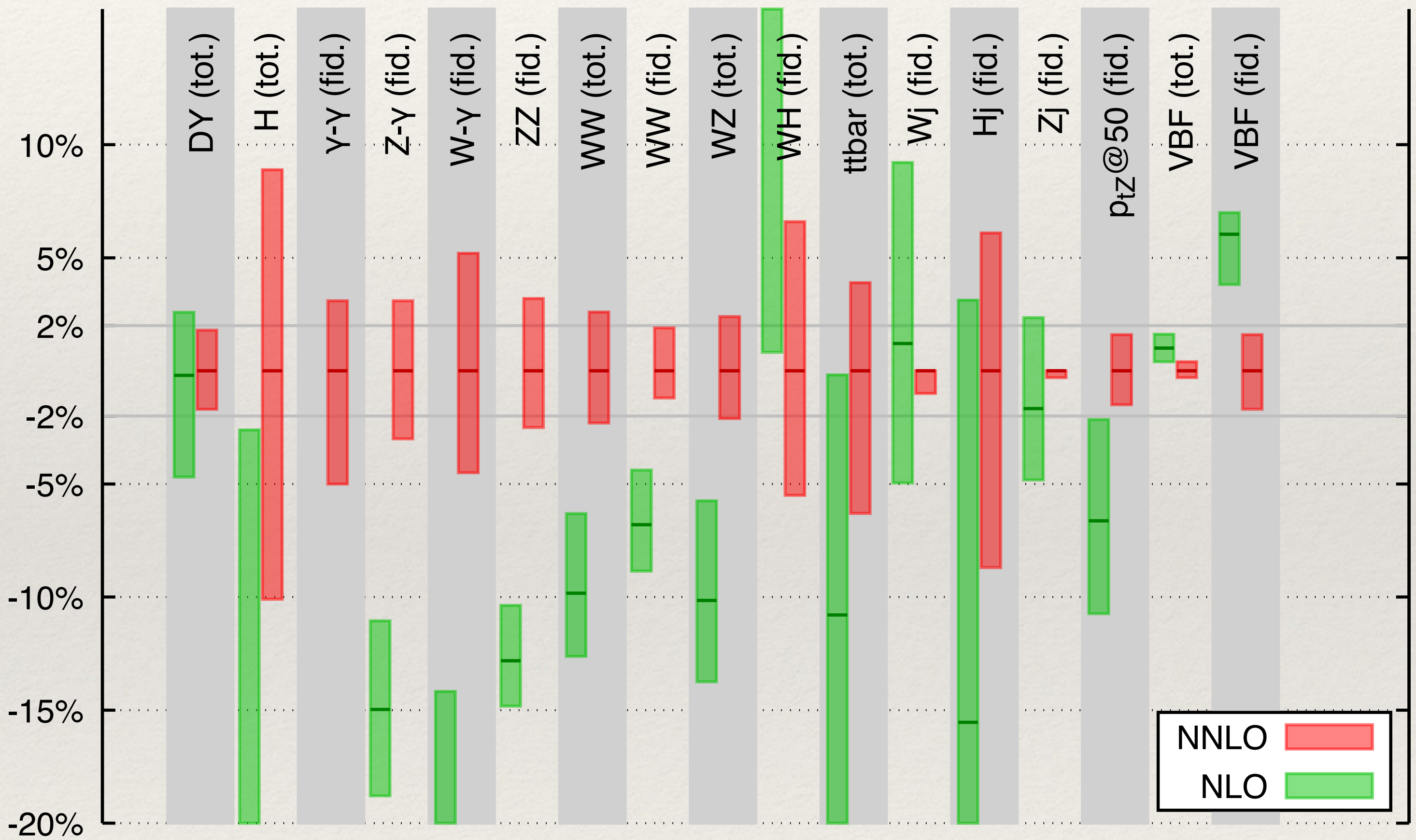
- Plots show the necessity of NLO corrections, and current ATLAS results compared with NNLO calculations.

NLO QCD solved!

- NLO order is a solved problem numerically, (with the exception of processes first occurring at one-loop level, and processes with a large number of external partons). NLO electroweak corrections also often included. In some cases matched with parton shower.
- MadGraph5_aMC@NLO, Recola, Openloops 2, Gosam, POWHEG(Box)
- Ingredients required -
  - Tree-level and one-loop diagram generation;
  - Subtraction procedure to cancel soft and collinear divergences between real and virtual (ERT, Catani-Seymour, FKS);
  - Reduction to known integrals (Generalized Unitarity, OPP, Tensor reduction to scalar integrals, Passarino&Veltman Collier, On the fly reduction);
  - Complete basis set of one-loop scalar integrals (‘tHooft & Veltman, Denner Nierste & Scharf, RKE & Zanderighi).

Precision QCD

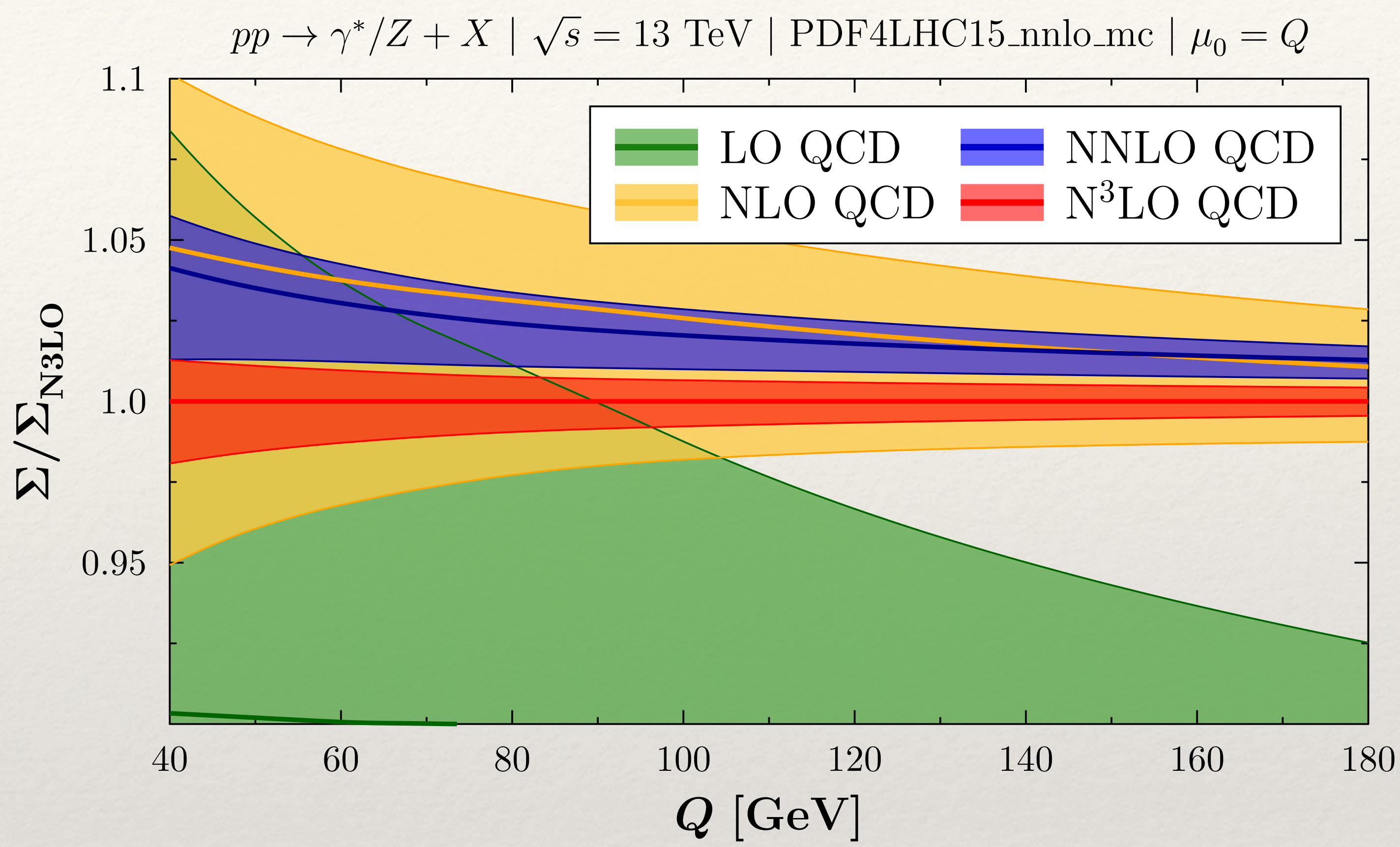
- We compute higher orders in QCD to increase the precision of our predictions i.e. to reduce the theoretical error.
- As we accumulate higher order terms we can ask how our error estimates in lower order perform.
- The NNLO central value lies within the NLO error band in only 4 out of the 17 cases shown.



Gavin Salam, (LHCP2016)

N^{3}LO results for inclusive Z/γ^* etc

- Results for $Z, W^{\pm}, H, WH, ZH$ normalized to N^{3}LO.
- Both μ_R and μ_F are varied by a factor 2 about their central values respecting the constraint $\frac{1}{2} < \frac{\mu_R}{\mu_F} < 2$, "7-point scale variation"
- In most of the analyzed cases the seven point scale variation at NNLO does not capture the N3LO central value.



Baglio et al, 2209.06138, c.f. Mistlberger

Differential distributions

Transverse momentum distribution in DY

- DDT wrote down a very beautiful formula (8/78)
- $$\frac{d\sigma}{dq^2 dq_T^2 dy} = \frac{4\pi\alpha^2}{9sq^2q_T^2} \times \frac{\partial}{\partial \ln q_T^2} \sum_{F=q,\bar{q}} e_F^2\, D_a^F(x_1, \ln\frac{q_T^2}{\mu^2})\, D_b^F(x_2, \ln\frac{q_T^2}{\mu^2})\, T^2(q_T^2, q^2)$$
- Parisi & Petronzio (2/79), based on arguments from electrodynamics, correct the form factor T. Similar conclusion by Curci et al, (3/79).
- The formulations are in b-space, (Fourier conjugate to q_T to make transverse momentum conservation multiplicative) and there is the additional result, that the shrinkage of the intercept at q_T is calculable. $$\frac{\left.\frac{d\sigma}{dp_T^2}\right|_{p_T=0}}{\int dp_T^2 \frac{d\sigma}{dp_T^2}} = \left(\frac{\Lambda}{Q}\right)^\eta, \quad \eta \approx 0.6$$

  (Balancing semi-hard gluons).

All orders result for q_T distribution

$$\frac{d\sigma}{dQ^2 dy dq_T^2} = \frac{4\pi\alpha^2}{9Q^2 s} \int d^2b \ \exp(iq_T . b) \sum_j e_j^2$$

$$\times \sum_a \int_{x_a}^{1} \frac{d\xi_A}{\xi_A} f_{a/A}(\xi_a; 1/b) \frac{d\xi_B}{\xi_B} f_{b/B}(\xi_b; 1/b)$$

$$\times \exp\Big\{ - \int_{1/b^2}^{Q^2} \frac{d\bar{\mu}^2}{\bar{\mu}^2} \Big[\ln \frac{Q^2}{\bar{\mu}} A(\alpha_S(\bar{\mu}) + B(\alpha_S(\bar{\mu}) \Big] \Big\}$$

$$+ \frac{4\pi^2\alpha^2}{9Q^2 s} Y(q_T; Q, x_a, x_b)$$

$$A(\alpha_S(\mu) = \sum_{n=0}^{\infty} A^{(n)} \left(\frac{\alpha_S}{2\pi}\right)^n, \quad A^{(1)} = C_F, A^{(2)} = 2C_F \Big\{ C_A(\frac{67}{18} - \frac{\pi^2}{6}) - \frac{10T_F n_f}{9} \Big\}$$

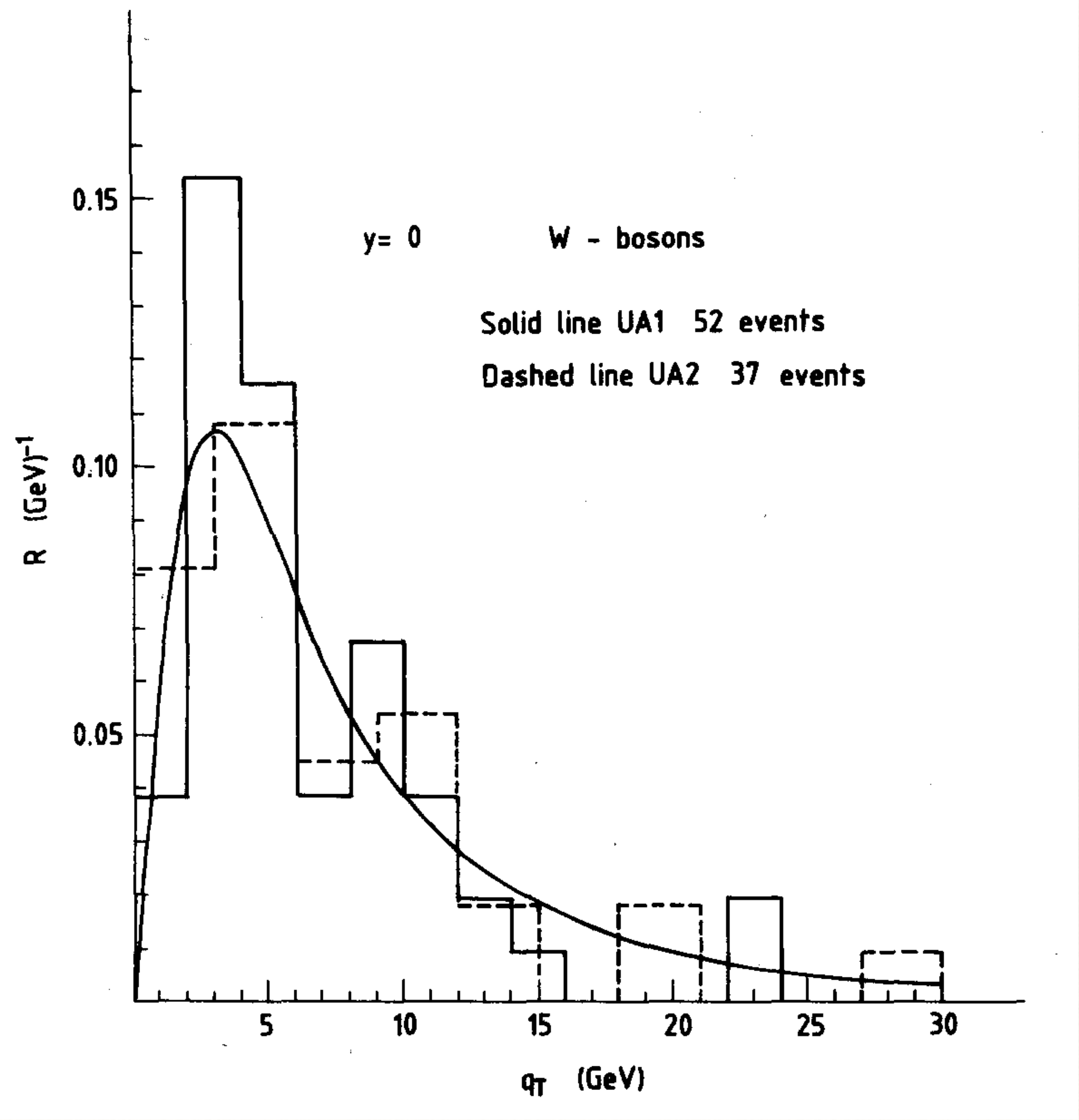
$$B(\alpha_s(\mu)) = = \sum_{n=0}^{\infty} B^{(n)} \left(\frac{\alpha_S}{2\pi}\right)^n, \quad B^{(1)} = -3C_F,$$

$$B^{(2)} = C_F \Big[C_F(\pi^2 - \frac{3}{4} - 12\zeta_3) + C_A(\frac{11\pi^2}{9} - \frac{193}{12} + 6\zeta_3) + T_R n_f(\frac{17}{3} - \frac{4\pi^2}{9}) \Big]$$

Collins, Sterman and Soper (1984)

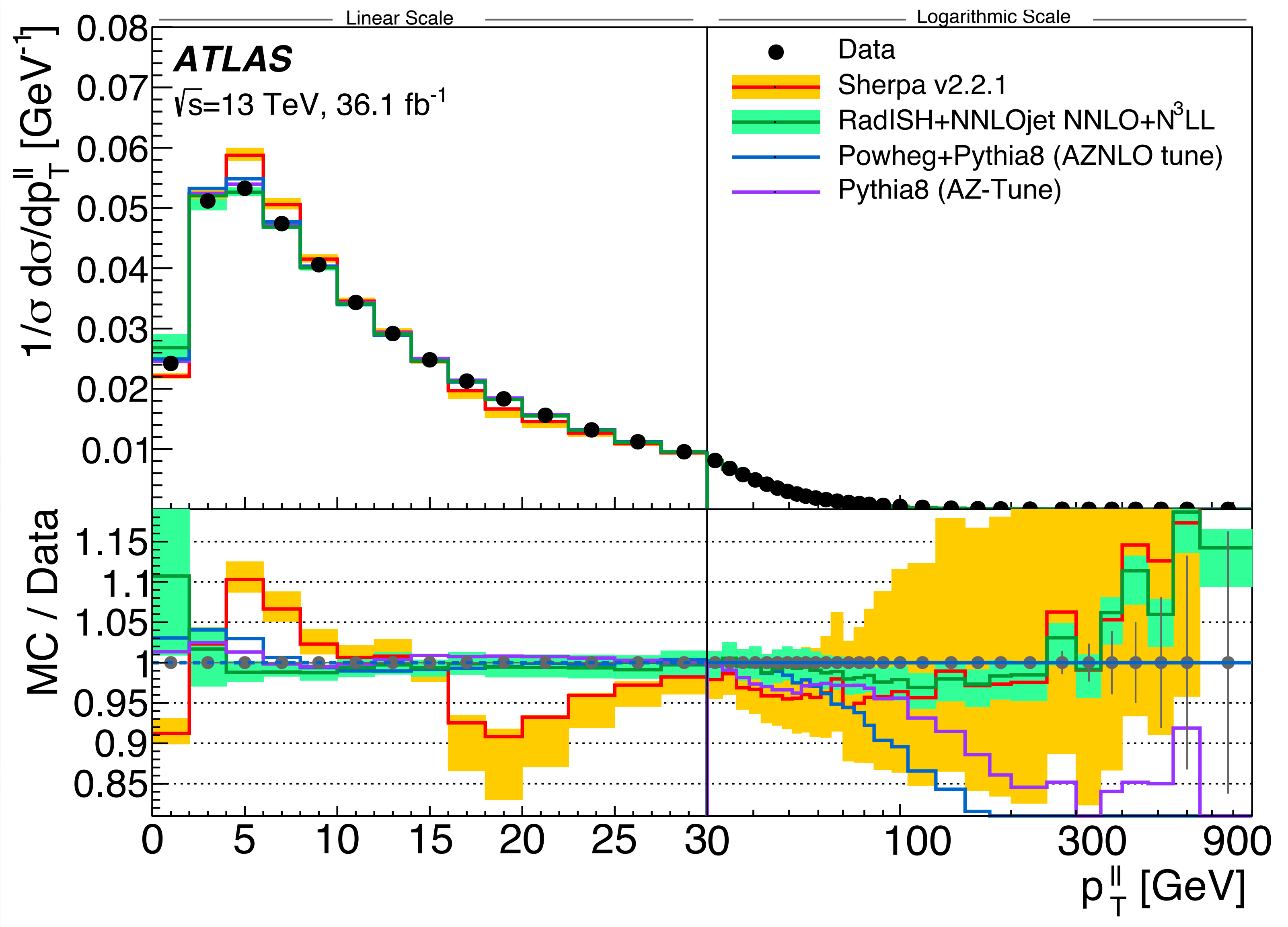
W Discovery(1983)!

- At the same time as CSS, we in AEM+Mario Greco produced q_T plots using all the theoretical information available at the time.
- A similar plot using our prediction, with 68 UA1 events, (and without the UA2 data!) was presented by Carlo Rubbia in his Nobel lecture.



Z-p_T (2019)

- LHC results at $\sqrt{s} = 13$ TeV rather more impressive, e.g from ATLAS.
- The theoretically most evolved calculation Radish+NNLOJet (c.f. Gehrmann) gives the best representation of the data.



1912.02844

If this were a proper history….

- First NNLO calculation of Drell-Yan process Hamberg, Van Neerven, Matsuura
- Issue of whether initial state interactions compromise factorization raised Brodsky, Bodwin and Lepage
    - Low order demonstration of factorization for Drell-Yan process,Lindsay, Ross, Sachrajda (1983)
    - Situation was summarized in 2004 by Collins, Sterman,Soper "recent work has, we believe, established its validity at all orders. Nevertheless, …. there is plenty of room for improvement in our understanding."

Focus of the rest of the talk

- This concludes the historical part of the talk.
- For the rest of the talk I shall focus on the results at NNLO and in re-summed perturbation theory using MCFM.
- Subsequent talks will (presumably) address QCD at colliders and further efforts in NNLO QCD and first results at N^3LO.
  - Seeing quarks and gluons: following QCD from initial to final states (Sterman)
  - From partons to jets and back - Simulating QCD interactions at highest energies(Hoeche)
  - High-Energy Collider Observables at Ultimate Precision in QCD(Gehrmann)
  - Perturbative techniques for precision collider physics and cosmology(Anastasiou)
  - The evolution of the precision program: from QCD to SMEFT(Boughezal)

NNLO cross sections in MCFM

MCFM (mcfm.fnal.gov)

- MCFM 10.3 (January 30th, 2023) contains about 350 processes at hadron-colliders evaluated at NLO.
- We have tried to improve the documentation by giving a web-page and a specimen input file for every process.
- Since matrix elements are calculated using analytic formulae, one can expect better performance, in terms of stability and computer speed, than fully numerical codes.
- In addition MCFM contains many processes evaluated at NNLO using both the jettiness and the q_T slicing schemes. Non-local slicing approaches for NNLO QCD in MCFM, Campbell, RKE and Seth 2202.07738
- NNLO results for $pp \to X$, require process $pp \to X + 1$ parton at NLO, and two loop matrix elements for $pp \to X$, (all provided by other authors, mainly Gehrmann et al).
- MCFM also includes transverse momentum resummation at N^3LL+NNLO for W,Z,H,WW,ZZ,WH and ZH processes.

Fiducial qT resummation of color-singlet processes at N^3LL+NNLO, CuTe-MCFM 2009.11437, Becher and Neumann

Transverse momentum resummation at N^3LL+NNLO for diboson processes,Campbell, RKE, Neumann and Seth, 2210.10724

Web-page for every process, with specimen input files.

15:41 4G

1 $f(-p_1) + f(-p_2) \to W^+(\to \nu(p_3) + e^+(p_4))$

1.1 W-boson production, processes 1,6

These processes represent the production of a W boson which subsequently decays leptonically. This process can be calculated at LO, NLO, and NNLO. NLO calculations can be performed by dipole subtraction, zero-jettiness slicing and q_T-slicing. NNLO calculations can be performed by zero-jettiness slicing and q_T-slicing.

When `removebr` is true, the W boson does not decay.

Input files for these 6 possibilities, as used plots for 'Non-local slicing approaches for NNLO QCD in MCFM', ref. [1] are given in the link below.

1.2 Input files as used for NNLO studies, ref. [1]

- ./lo/input_W+.ini
- ./nlo/input_W+.ini
- ./nlo/input_W+_qt.ini
- ./nlo/input_W+_scet.ini
- ./nnlo/input_W+_qt.ini
- ./nnlo/input_W+_scet.ini

1.3 Input file for transverse momentum resummed cross-sections, ref. [2]

- input_W+.ini

1.4 Input files for jet-vetoed cross-sections, ref. [3]

- vetowp30nlo.ini
- vetowp30nnlo.ini
- vetowp30nnll.ini
- vetowp30n3ll.ini
- vetowp30nlomc.ini
- vetowp30nnlomc.ini

1.5 Plotter

nplotter_W_only.f is the default plotting routine.

1.6 Example input and output file(s)

input1.ini process1.out

References

[1] J.M. Campbell, R.K. Ellis and S. Seth, *Non-local slicing approaches for NNLO QCD in MCFM*, 2202.07738.

[2] T. Becher and T. Neumann, *Fiducial q_T resummation of color-singlet processes at N^3LL+NNLO*, JHEP **03** (2021) 199 [2009.11437].

[3] J.M. Campbell, R.K. Ellis, T. Neumann and S. Seth, *Jet-veto resummation at N^3LL_p+NNLO in boson production processes*, 2301.11768.

NNLO results

- In a recent paper (2202.07738) we tried to document all the processes calculated at NNLO.
- About 50% are available in MCFM.
- We use both q_T slicing and jettiness slicing.

Process	MCFM	Process	MCFM
$H + 0$ jet [8–14]	✓ [15]	$W^{\pm} + 0$ jet [16–18]	✓ [15]
$Z/\gamma^* + 0$ jet [11, 17–19]	✓ [15]	ZH [20]	✓ [21]
$W^{\pm}\gamma$ [18, 22, 23]	✓ [24]	$Z\gamma$ [18, 25]	✓ [25]
$\gamma\gamma$ [18, 26–28]	✓ [29]	single top [30]	✓ [31]
$W^{\pm}H$ [32, 33]	✓ [21]	WZ [34, 35]	✓
ZZ [1, 18, 36–40]	✓	W^+W^- [18, 41–44]	✓
$W^{\pm} + 1$ jet [45, 46]	[3]	$Z + 1$ jet [47, 48]	[4]
$\gamma + 1$ jet [49]	[5]	$H + 1$ jet [50–55]	[6]
$t\bar{t}$ [56–61]		$Z + b$ [62]	
$W^{\pm}H$+jet [63]		ZH+jet [64]	
Higgs WBF [65, 66]		$H \to b\bar{b}$ [67–69]	
top decay [31, 70, 71]		dijets [72–74]	
$\gamma\gamma$+jet [75]		$W^{\pm}c$ [76]	
$b\bar{b}$ [77]		$\gamma\gamma\gamma$ [78]	
HH [79]		HHH [80]	

Most apart from heavy quark and jet production are generalizations of Drell-Yan

NNLO by slicing

$$\sigma_{NNLO} = \int d\Phi_N |\mathscr{M}_N|^2 + \int d\Phi_{N+1} |\mathscr{M}_{N+1}|^2 \theta_N^< + \int d\Phi_{N+2} |\mathscr{M}_{N+2}|^2 \theta_N^<$$

Unresolved

$$+ \int d\Phi_{N+1} |\mathscr{M}_{N+1}|^2 \theta_N^> + \int d\Phi_{N+2} |\mathscr{M}_{N+2}|^2 \theta_N^>$$

Resolved

$$\equiv \sigma_{NNLO}(\tau < \tau_{cut}) + \sigma_{NNLO}(\tau > \tau_{cut})\,.$$

$$\theta_N^< = \theta(\tau_{cut} - \tau) \text{ and } \theta_N^> = \theta(\tau - \tau_{cut})$$

- Unresolved is subject to a factorization formula and power corrections.
- Resolved radiation contribution obtained from NLO calculation with one additional jet, available by subtraction in MCFM.
- As the cut on the resolved radiation becomes smaller, neglected power corrections are also smaller, but cancellation between resolved and unresolved is bigger.

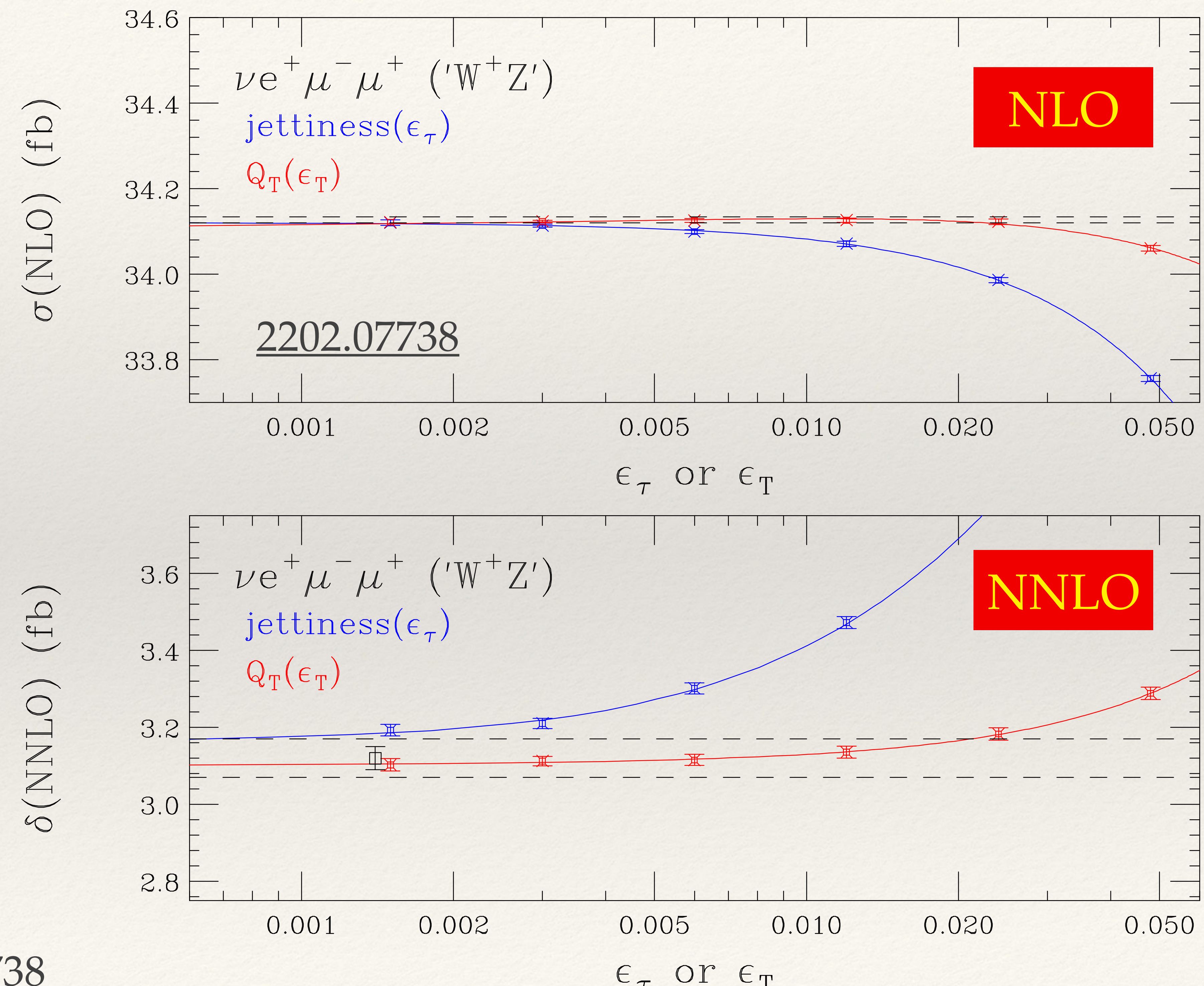
$$\sigma(\tau < \tau_{cut}) = \int H \otimes B \otimes B \otimes S \otimes \left[\prod_n^N J_n\right] + \cdots\,.$$

Slicing parameters

- For color singlet production, "q_T" of produced color singlet object, (Catani et al hep-ph/0703012v2)
- "N-jettiness" (Boughezal et al) 1505.03893 $\mathcal{T}_N = \sum_k \min_i \left\{ \frac{2p_i \cdot q_k}{Q_i} \right\}$
  - The p_i are light-like reference vectors for each of the initial beams and final-state jets in the problem
  - q_k denote the four-momenta of any final-state radiation.
  - $Q_i = 2E_i$ is twice the lab-frame energy of each jet
  - Can handle coloured final states, e.g. H+jet
- Recent new parameter "Jet veto" (Gavardi et al), 2308.11577

NNLO results: dependence on slicing procedure

- For most (but not all) processes the power corrections are smaller for Q_T slicing than for jettiness.
- Factor of two in the exponent difference between the leading form factors for q_T and jettiness
- removed by defining $\epsilon_T = q_T^{cut}/Q$ and $\epsilon_\tau = (\tau^{cut}/Q)^{\frac{1}{\sqrt{2}}}$



Campbell et al, 2202.07738

Examples of NNLO results from MCFM

Process		target		MCFM		
	$\sigma_{NLO^\star}$	σ_{NNLO}	δ_{NNLO}	σ_{NNLO}	δ_{NNLO}	
$pp \to H$	29.78(0)	39.93(3)	10.15(3)	39.91(5)	10.13(5)	nb
$pp \to Z$	56.41(0)	55.99(3)	−0.42(3)	56.03(3)	−0.38(3)	nb
$pp \to W^-$	79.09(0)	78.33(8)	−0.76(8)	78.41(6)	−0.68(6)	nb
$pp \to W^+$	106.2(0)	105.8(1)	−0.4(1)	105.8(1)	−0.4(1)	nb
$pp \to \gamma\gamma$	25.61(0)	40.28(30)	14.67(30)	40.19(20)	14.58(20)	pb
$pp \to e^-e^+\gamma$	2194(0)	2316(5)	122(5)	2315(5)	121(5)	pb
$pp \to e^-\bar{\nu}_e\gamma$	1902(0)	2256(15)	354(15)	2251(2)	349(2)	pb
$pp \to e^+\nu_e\gamma$	2242(0)	2671(35)	429(35)	2675(2)	433(2)	pb
$pp \to e^-\mu^-e^+\mu^+$	17.29(0)	20.30(1)	3.01(1)	20.30(2)	3.01(2)	fb
$pp \to e^-\mu^+\nu_\mu\bar{\nu}_e$	243.7(1)	264.6(2)	20.9(3)	264.9(9)	21.2(8)	fb
$pp \to e^-\mu^-e^+\bar{\nu}_\mu$	23.94(1)	26.17(2)	2.23(3)	26.18(3)	2.24(2)	fb
$pp \to e^-e^+\mu^+\nu_\mu$	34.62(1)	37.74(4)	3.12(5)	37.78(4)	3.16(3)	fb
$pp \to ZH$	780.0(4)	846.7(5)	66.7(6)	847.3(7)	67.3(6)	fb
$pp \to W^\pm H$	1446.5(7)	1476.1(7)	29.6(10)	1476.7(8)	30.2(4)	fb

Table 4. NLO results, computed using MCFM with NNLO PDFs (denoted $\sigma_{NLO^\star}$), total NNLO cross sections from `vh@nnlo` ($W^\pm H$ and ZH only) and MATRIX (remaining processes, using the extrapolated result from Table 6 of Ref. [24]) and the target NNLO coefficients (δ_{NNLO}, with $\delta_{NNLO} = \sigma_{NNLO} - \sigma_{NLO^\star}$). The result of the MCFM calculation (0-jettiness, fit result b_0 from Eq. (3.9)) is shown in the final column.

Resummed calculations at small-q_T

Transverse momentum resummation at small q_T

- Transverse momentum resummation (*à la* DDT) is nowadays often performed in SCET language.
- Current state of the art has NNLO matched to N^3LL
- Table shows the perturbative results needed at each nominal order, $L \sim 1/\alpha_s$

Approximation	Nominal order	Accuracy $\sim \alpha_s^n L_\perp^k$	Γ_{cusp}	$\gamma_{\text{coll.}}$	H
LL	α_s^{-1}	$2n \geq k \geq n+1$	Γ_0	tree	tree
NLL+LO	α_s^0	$2n \geq k \geq n$	$\Gamma_1,$	γ_0	tree
N^2LL+NLO	α_s^1	$2n \geq k \geq \max(n-1, 0)$	Γ_2	γ_1	1-loop
N^3LL +NNLO	α_s^2	$2n \geq k \geq \max(n-2, 0)$	Γ_3	γ_2	2-loop

Table adapted from Becher, Neubert and Pecjak

Small-q_T in SCET language

Cross-section for Born level process

Hard function contains virtual corrections

$$d\sigma_{ij}(p_1, p_2, \{\underline{q}\}) = \int_0^1 d\xi_1 \int_0^1 d\xi_2 \, d\sigma^0_{ij}(\xi_1 p_1, \xi_2 p_2, \{\underline{q}\}) \, \mathscr{H}_{ij}(\xi_1 p_1, \xi_2 p_2, \{\underline{q}\}, \mu)$$

$$\times \frac{1}{4\pi} \int d^2 x_\perp \, e^{-iq_\perp x_\perp} \left(\frac{x_T^2 Q^2}{b_0^2} \right)^{-F_{ij}(x_\perp, \mu)} B_i(\xi_1, x_\perp, \mu) \cdot B_j(\xi_2, x_\perp, \mu)$$

Collinear anomaly:vestige of rapidity divergences

Beam functions encode soft and collinear emission at low transverse momentum.

Collinear Anomaly

- In SCET the beam functions and the soft function have light-cone divergences which are not regulated by dimensional regularization;
- These are not soft divergences; they are due to gluons at large rapidity;
- This requires an additional regulator, which can be removed at the end of the calculation;
- However a vestige of this regulator remains. The product of the two beam functions depends on the large scale of the problem, Q ;
- This has been called the "collinear factorization anomaly" of SCET. Quantum effects modify a classical symmetry, $p \to \lambda p, \bar{p} = \bar{\lambda}\bar{p}$ with only $\lambda\bar{\lambda} = 1$ unbroken.

Becher, Neubert, 1007.4005

SCET-based resummation: New information on the constants

- The more recent information on the constants in this formula will be used later on.

$$\beta(\alpha_s) = -2\alpha_s \sum_{n=0}^{\infty} \beta_n \left(\frac{\alpha_s}{4\pi}\right)^{n+1} = -0.12 - 0.015 - 0.0018 - 0.0012 - 0.000095$$

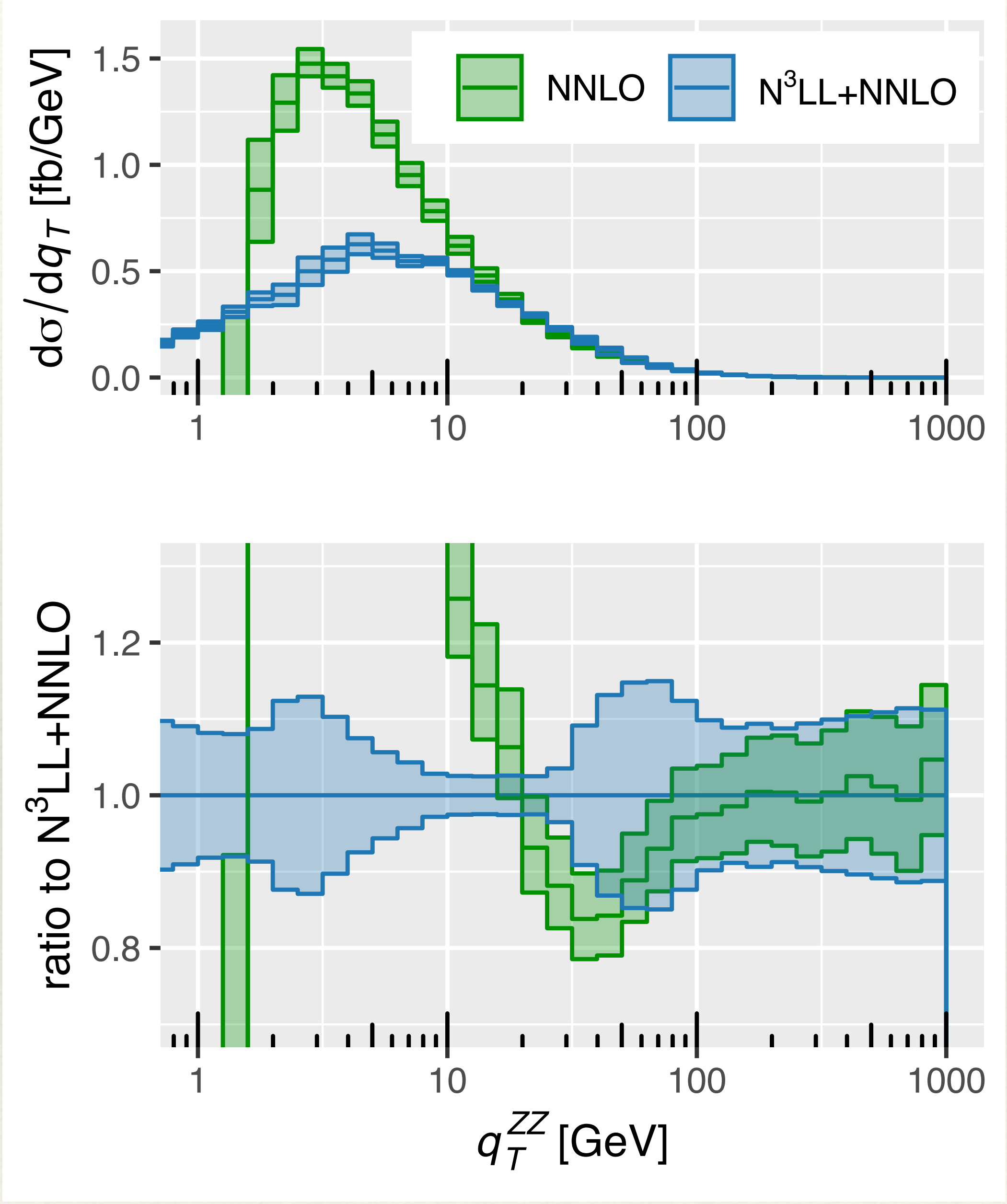
$$\Gamma^i_{\rm cusp}(\alpha_s) = \sum_{n=0}^{\infty} \Gamma^i_n \left(\frac{\alpha_s}{4\pi}\right)^{n+1} = 0.133 + 0.023 + 0.0037 + 0.00058 + 0.00065$$

$$\gamma(\alpha_s) = \sum_{n=0}^{\infty} \gamma_n \left(\frac{\alpha_s}{4\pi}\right)^{n+1} = -0.1 + 0.00035 - 0.0019 + 0.00000029$$

- Numerical values are in the MSbar scheme, with $n_f = 5$ and $\alpha_S = \pi/10$

Vector boson pair production at small q_T

- Resummation effects are potentially more important for vector boson pair production at the same q_T since Q is larger.
- Resummation at N³LL+NNLO becomes important below $\sim 50 - 100$ GeV.

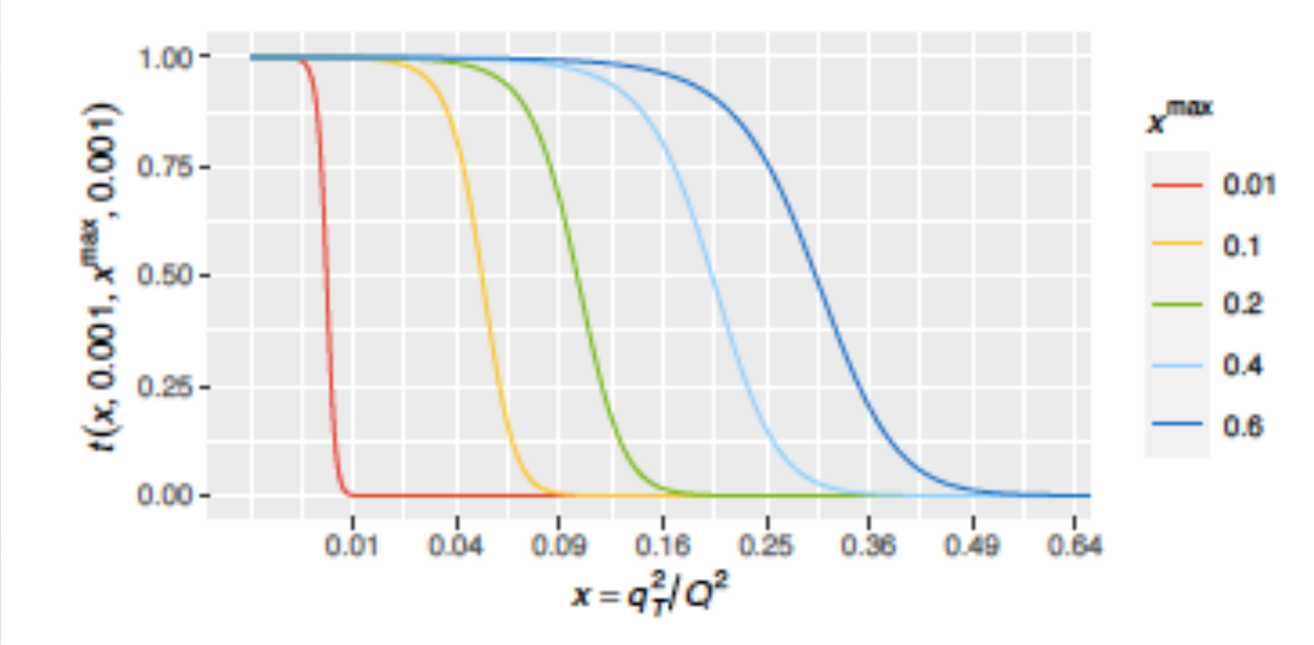


Transverse momentum distribution of the ZZ pair at NNLO and NNNLL+NNLO using CMS cuts at $\sqrt{s} = 13.6$ TeV

Matching to fixed order

$$\left.\frac{d\sigma^{\mathrm{N^3LL}}}{dq_T}\right|_{\text{naively matched to NNLO}} = \frac{d\sigma^{\mathrm{N^3LL}}}{dq_T} + \Delta\sigma, \text{ where } \Delta\sigma = \left[\frac{d\sigma^{\mathrm{NNLO}}}{dq_T} - \frac{d\sigma^{\mathrm{N^3LL}}}{dq_T}\right]_{\text{expanded to NNLO}}$$

- Fixed order result recovered up to higher order terms, (which can induce unphysical behavior).



- Also problems at small q_T, introduce cutoff q_0;
- So we need to implement a transition function, and choose its parameters on a case-by-case basis.

$$\left.\frac{d\sigma^{\mathrm{N^3LL}}}{dq_T}\right|_{\text{matched to NNLO}} = t(x)\left[\frac{d\sigma^{\mathrm{N^3LL}}}{dq_T} + \Delta\sigma|_{q_t>q_0}\right] + (1-t(x))\,\frac{d\sigma^{\mathrm{NNLO}}}{dq_T}$$

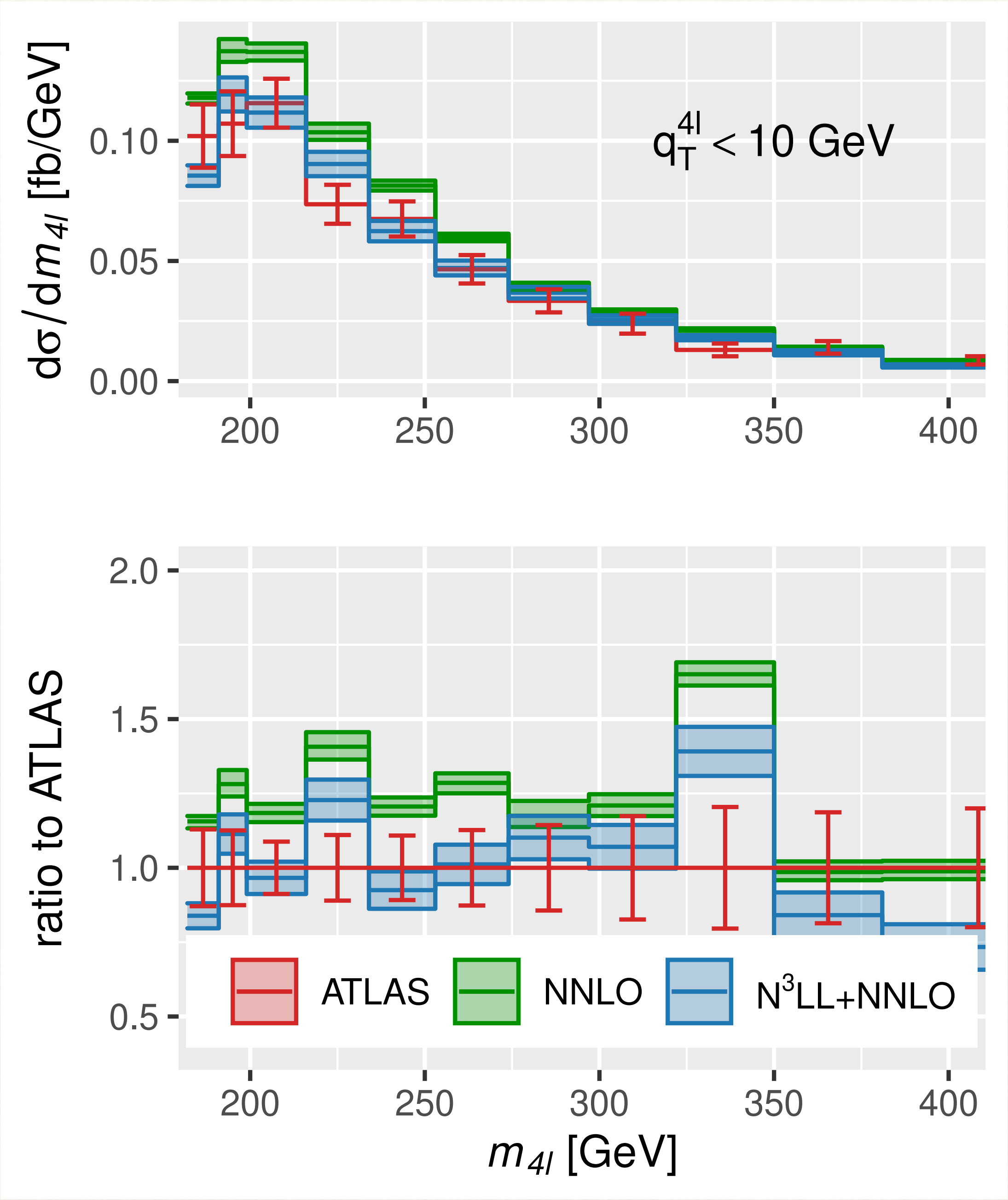
Becher and Neumann, 2009.11437

Example of q_T resummation in four lepton events(ZZ)

- ATLAS $\sqrt{s}$ = 13TeV, 139fb[-1] data, 2103.01918

lepton cuts	$q_T^{\ell_1} > 20\,\mathrm{GeV},\ q_T^{\ell_2} > 10\,\mathrm{GeV},$ $q_T^{\ell_{3,4}} > 5\,\mathrm{GeV},\ q_T^{e} > 7\,\mathrm{GeV},$ $\lvert\eta^\mu\rvert < 2.7,\ \lvert\eta^e\rvert < 2.47$
lepton separation	$\Delta R(\ell, \ell') > 0.05$

- $m_{4l} > 182$ GeV to avoid Higgs region.
- Low q_T data, plotted as a function of m_{4l}
- Agreement with data improves as m_{4l} increases.



Fiducial q_T resummation of color singlet processes at N³LL+NNLO, Becher and Neumann, 2009.11437

Transverse momentum resummation at N³LL+NNLO for diboson processes, Campbell, RKE, Neumann and Seth, 2210.10724

Jet veto cross sections

- It is often important to impose a veto on jets, e.g. in W^+W^- production to veto against top pair background
- Although with $p_T^{\text{veto}} \sim 25$ GeV, logarithms are not as large as in transverse momentum resummation which extends to smaller p_T.
- Resummation is sometimes necessary
- We perform resummation at N3LLp+NNLO, (p=partial, because the coefficient of the collinear anomaly coefficient is only known approximately.

For initial studies see, for example, Becher et al, 1307.0025 , Stewart et al, 1307.1808

New ingredients for jet-veto resummation

- Important step in making SCET results for almost complete N^{3}LL available. For details of the missing piece, see later.

- Formalism applies to jets vetoed over all rapidity, (which is not the case experimentally).

The analytic two-loop soft function for leading-jet p_T

Soft function
Abreu et al,
2204.03987

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Quark and gluon two-loop beam functions for leading-jet p_T and slicing at NNLO

Beam functions
Abreu et al,
2207.07037

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Jet veto cross section

- Jets defined using sequential recombination jet algorithms, (n=1(anti-k_T), n=0(Cambridge-Aachen) n=-1(k_T);
- Jet vetos also generate large logarithms, as codified in factorization formula; however logarithms tend to be smaller than in transverse momentum resummation, since $p_T^{\text{veto}} \sim 25$ GeV;
- Beam and Soft functions for leading jet p_T recently calculated at two-loop order using an exponential regulator by Abreu et al.
- Jet veto cross sections are simpler than the p_T resummed calculation (No b space).

$$d_{ij} = \min(p_{Ti}^n, p_{Tj}^n)\,\frac{\sqrt{\Delta y_{ij}^2 + \Delta\phi_{ij}^2}}{R}\,, \qquad d_{iB} = p_{Ti}^n$$

$$\frac{d^2\sigma(p_T^{veto})}{dM^2 dy} = \sigma_0\,\left|C_V(-M^2,\mu)\right|^2$$
$$\left[\mathscr{B}_c(\xi_1, M, p_T^{veto}, R^2, \mu, \nu)\,\mathscr{B}_{\bar{c}}(\xi_2, M, p_T^{veto}, R^2, \mu, \nu) \times \mathscr{S}(p_T^{veto}, R^2, \mu, \nu)\right]$$

Rapidity regulator ν

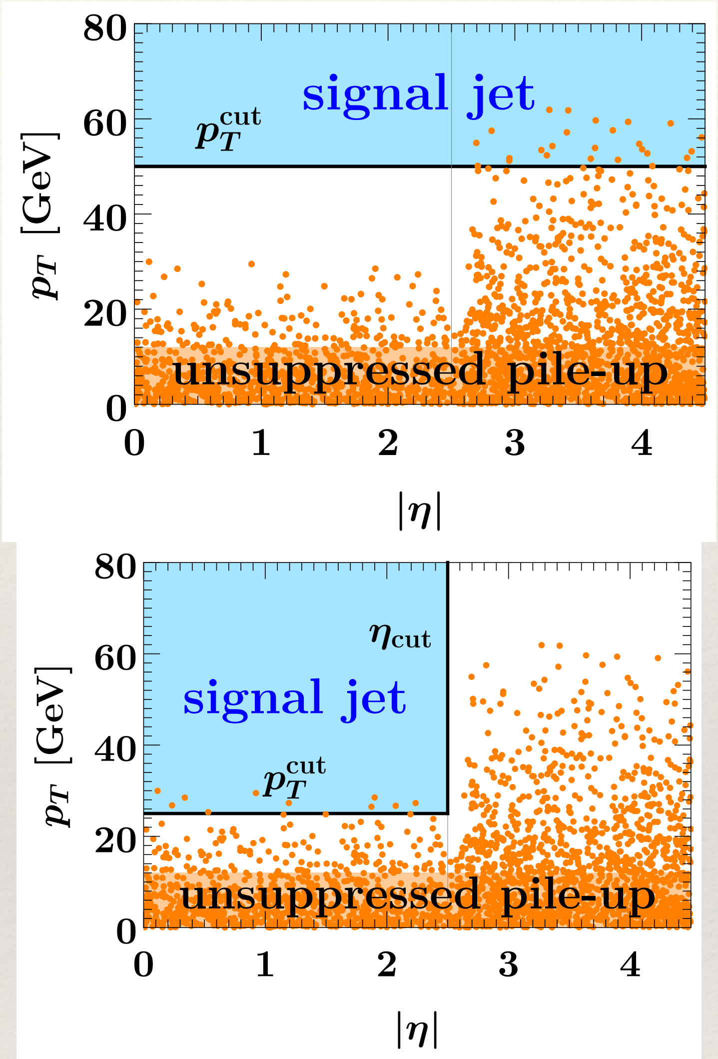
Beam functions Abreu et al, 2207.07037

Soft function Abreu et al, 2204.03987

$$\xi_{1,2} = (M/\sqrt{s})\,e^{\pm y} \qquad \sigma_0 = \frac{4\pi\alpha^2}{3N_c M^2 s}$$

Jet veto cross sections in a limited rapidity range

- Formula so far are valid for jet cross sections which are vetoed for all values of rapidity η_{cut}
- Experimental analyses perform jet cuts for $\eta < \eta_{\text{cut}}$
- To apply the resummed theory we need to be in a region where (see 1810.12911).



Current theory calculation

Typical Experimental cuts

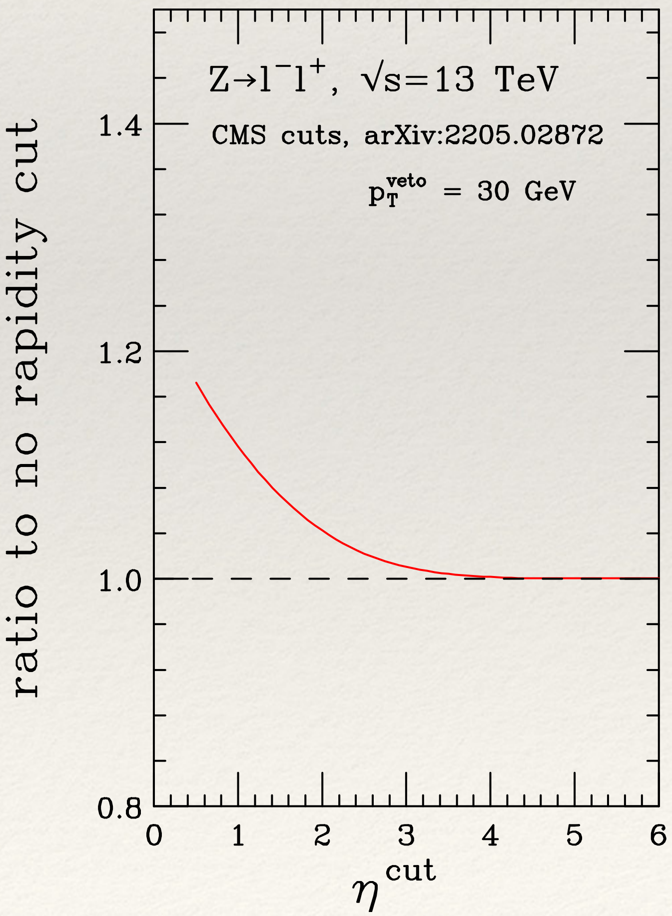
Figure taken from 1810.12911

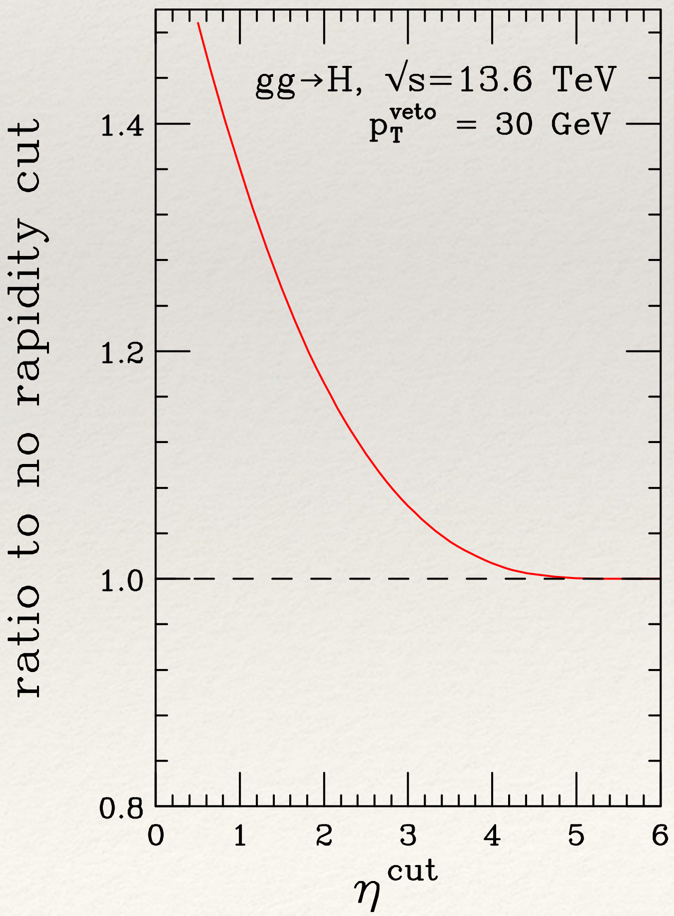
Strategy: determination where resummation is potentially important, before considering limited rapidity range resummation

Effects of rapidity cuts at fixed order

Process	Ref.	y_{cut}
Higgs	–	no study
Z (CMS)	[38]	2.4
W (ATLAS)	[43]	4.4
WW (CMS)	[39]	4.5
WZ (ATLAS)	[44]	4.5
WZ (CMS)	[45]	2.5
ZZ (CMS)	–	no study

- The usual jet veto resummation imposes no cut on the jet rapidity, unlike the experimental analysis.
- To apply this theory we need $\eta_{\text{cut}} \gg \ln(Q/p_T^{\text{veto}})$
- We can address the potential impact by looking at fixed order.
- More important for Higgs (and WW and ZZ) than for Z.



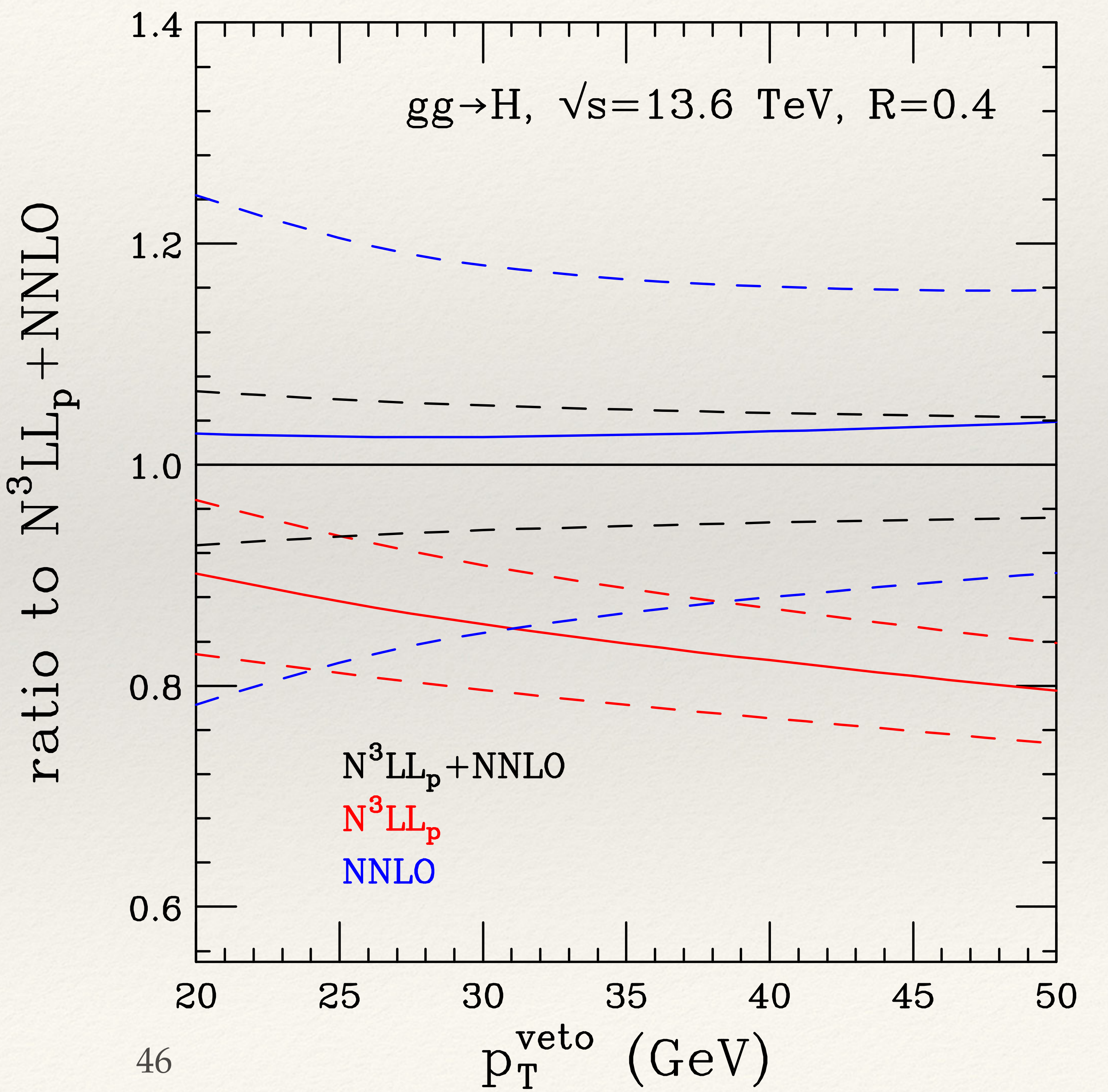


Phenomenological results in N^3LL_p

$N^3LL_p \equiv N^3LL$ with limited information on higher order collinear anomaly coefficient, d_3^{veto}

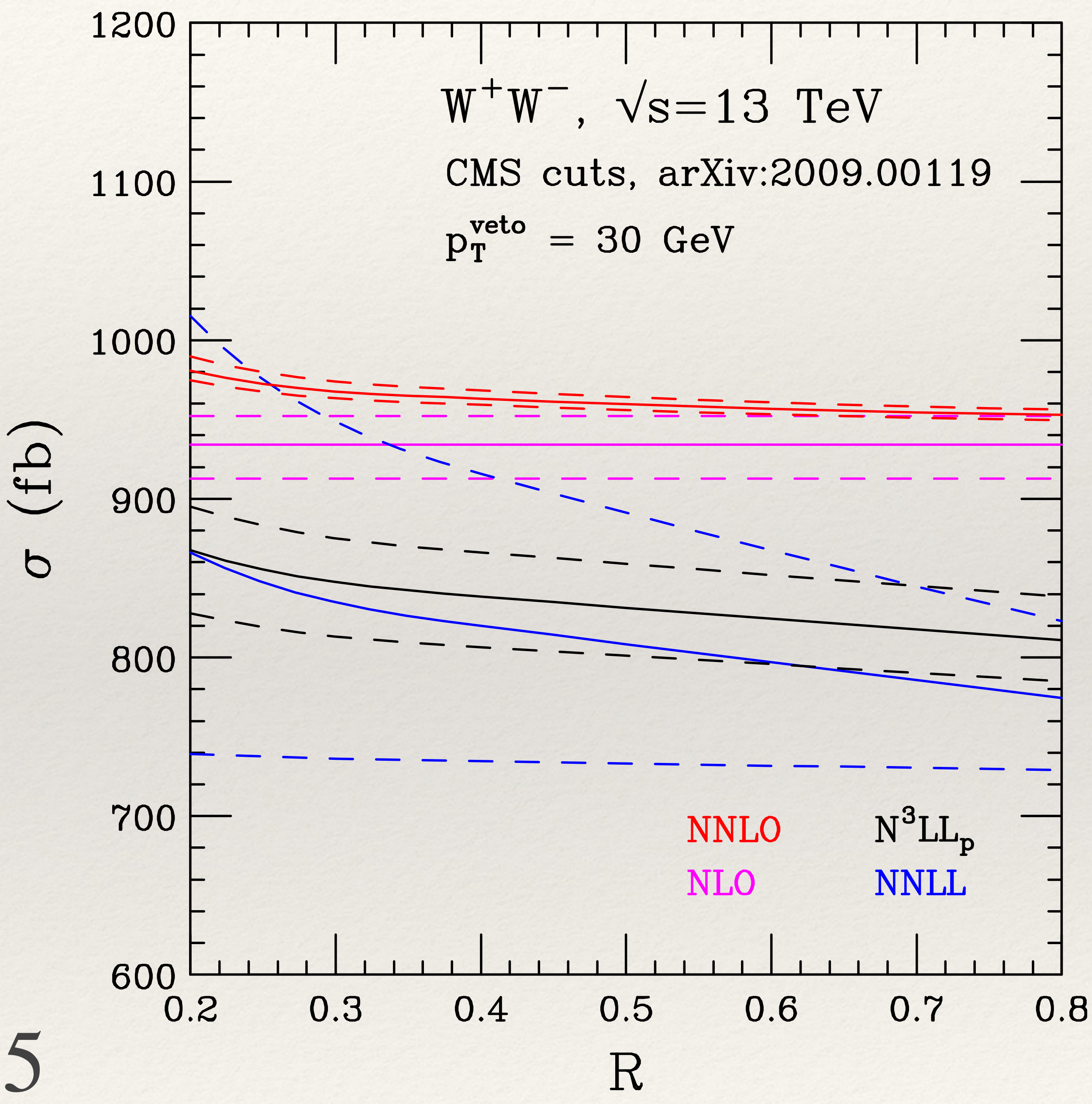
Comparison of NNLO, N^3LL_p and N^3LL_p+NNLO predictions for Higgs production.

- Shown are the ratios of NNLO and N^3LL_P to our best prediction.
- For pTveto < 30 GeV NNLO and N^3LL_p almost overlap, but the combined prediction has the smallest error



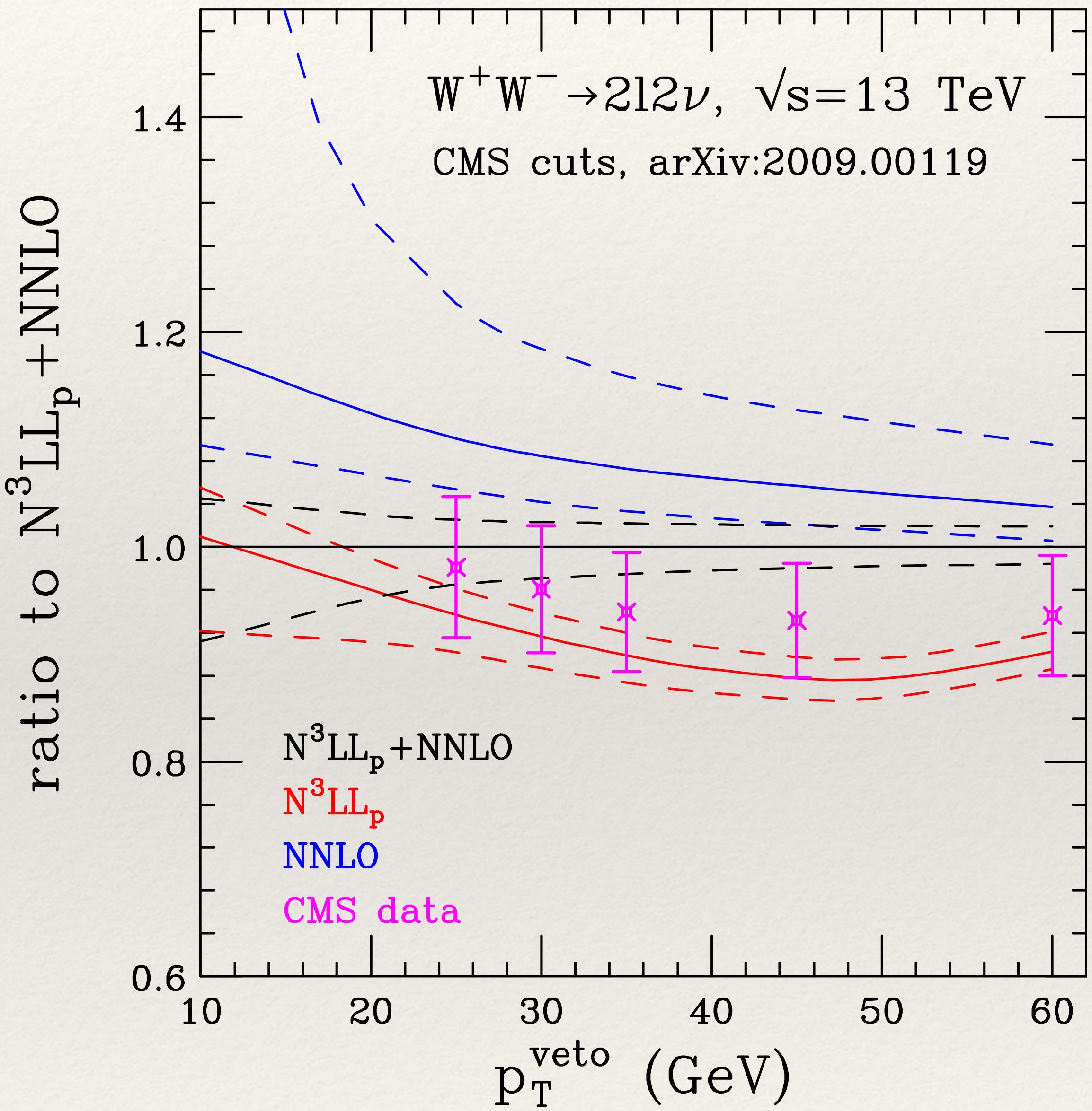
Jet veto in W^+W^- production

- Evidence that neither NNLO nor N^3LL alone is sufficient, especially around $p_T^{\rm veto} = 25 - 30$GeV, R=0.5
- R dependence is modest (zero at NLO!)
- $|\eta_{\rm cut}| < 4.5$, so we can argue that $(\ln(Q/p_T^{\rm veto}) = 1.3 - 2.2) \ll 4.5$



Comparison to data

- The data lies between the N^3LLp and the N^3LLp+NNLO and is marginally inconsistent with the NNLO alone.



Epilogue

- With exception of heavy quark and jet production, the most important high energy processes studied at colliders are really the production of massive bosons, (W,Z/γ*, H, WW, WZ, ZZ, Wγ etc., sometimes in association with jets) which fall under the rubric of Drell-Yan/Lepton pair/Color singlet production.
- The precision QCD community is lucky. Although not necessarily designed as such, the LHC is *de facto* a precision QCD machine.
- Our understanding of these processes, much more sophisticated than 45 years ago, builds on the simple results for lepton pair production.

RGE's for SCET quantities

- $$\frac{d}{d\ln\mu}F_{qq}(L_\perp,\mu) = 2\Gamma^F_{\mathrm{cusp}}$$
- $$\frac{d}{d\ln\mu}\,h^F(L_\perp,\mu) = 2\Gamma^F_{\mathrm{cusp}}(\mu)\,L_\perp - 2\gamma^q(\mu)$$
- $$\frac{d}{d\ln\mu}\,C_V(-M^2\,\mu) = \left[\Gamma^F_{\mathrm{cusp}}(\mu)\,\ln\frac{-M^2}{\mu^2} + 2\gamma^q(\mu)\right]C_V(-M^2\,\mu)$$

Refactorization

- Refactorize

$$\left[\mathscr{B}_q(\xi_1, Q, p_T^{veto}, R, \mu, \nu)\, \mathscr{B}_{\bar{q}}(\xi_2, Q, p_T^{veto}, R, \mu, \nu)\mathscr{S}(p_T^{veto}, R, \mu, \nu)\right]$$

$$= \left(\frac{Q}{p_T^{veto}}\right)^{-2F_{qq}(p_T^{veto}, R, \mu)} e^{2h^F(p_T^{veto}, \mu)}\, \bar{B}_q(\xi_1, p_T^{veto}, R, \mu)\, \bar{B}_{\bar{q}}(\xi_2, p_T^{veto}, R, \mu)$$

"Collinear anomaly"

"Collinear anomaly coefficient"

- In terms of reduced beam function jet vetoed cross section is now given by,

- $$\frac{d^2\sigma(p_T^{veto})}{dQ^2 dy} = \frac{d\sigma_0}{dQ^2}\, \bar{H}(Q, \mu, p_T^{veto})\bar{B}_q(\xi_1, p_T^{veto}, R, \mu)\, \bar{B}_{\bar{q}}(\xi_2, p_T^{veto}, R, \mu) + \mathcal{O}(p_T^{veto}/Q)\,,$$

- The two pieces are separately RG invariant: $\frac{d}{d\mu}\bar{H}(Q, \mu, p_T^{veto}) = \mathcal{O}(\alpha_s^3)$

and $\frac{d}{d\mu}\bar{B}_q(\xi_1, p_T^{veto}, R, \mu)\, \bar{B}_{\bar{q}}(\xi_2, p_T^{veto}, R, \mu) = \mathcal{O}(\alpha_s^3)$

Jet veto in Z production

- At $p_T^{\text{veto}} \sim 25 - 30$ all calculations agree within errors.
- However error estimates differ between NNLO and N^{3}LL +NNLO.
- For $p_T^{\text{veto}} = 30$ GeV, $(\ln(Q/p_T^{\text{veto}} = 1.1) \ll (\eta_{\text{cut}} = 2.4)$
- As expected at (unphysically) small p_T^{veto} resummed calculations show deviations from fixed order.
- Jet veto resummation probably not so necessary at $p_T^{veto} \sim 30\,\text{GeV}$, for W or Z production.