50 years of QCD, UCLA, 11-15/09/2023

QCD and I:
a long and complex relation

Gabriele Veneziano

Dedicated to Marcello Ciafaloni and to his contributions to PQCD

COLLÈGE
DE FRANCE
1530

CERN

Preamble

Throughout my career I have oscillated many times between string theory (ST) and QCD.

If I'm sometimes referred to as the father of ST, QCD has killed my 5y old baby…to my great disappointment.

But I have been one of the first to concede defeat and to switch to QCD… until ST's comeback in 1984.

Nonetheless, even while working mostly on quantum string gravity after 1984, I have been repeatedly distracted by interesting problems in strong interactions/QCD.

This talk is about some of those switches and distractions

Outline

- Before QCD (1965-1973)
  - A soft spot for strong interactions in Florence and Rehovot
  - Birth, rise and fall of the hadronic string
  - QCD takes over: 't Hooft's large-N_c limit
- Entering QCD through the large-N door
  - Topology vs perturbation theory in the DRM
  - The topological expansion in QCD

- Perturbative QCD
  - Factorization theorem
  - Jet calculus, pre-confinement
  - Fracture functions
- Non perturbative QCD
  - String junction and multi-quark states
  - U(1) problem & large-N effective Lagrangian
  - EDMN and θ
  - The proton "spin" crisis
  - SUSY variants and yet another large-N limit
- More recent involvements in QCD
  - Spont. CP breaking and axion potential
  - A model for π-π scattering in large-N QCD
  - Can we bootstrap large-N QCD?

Before QCD

(1965-1973)

- A soft spot for strong interactions in Florence and Rehovot
- Birth, rise and fall of the hadronic string
- QCD takes over: 't Hooft's large-N_c limit

As a student of Raoul Gatto in the mid sixties in Florence I was attracted by the strong interaction problem, in spite of the fact that Gatto himself and his “gattini” (Altarelli, Maiani, Preparata, ..) were mostly focussed on the weak interactions.

Early encounters with Sergio Fubini reinforced that tendency and got me interested in current-algebra (CA) and its hadronic saturation.

I continued on this path as a grad. student at the Weizmann Institute with Hector Rubinstein and a bit later also with Miguel Virasoro and Marco Ademollo.

We turned from CA to superconvergence and finite-energy sum rules. Imposing Dolen-Horn-Schmit duality led to some interesting bootstrap constraints (coming back now? See de Rham, Jaitly, Tolley, PRD 2023).

Birth, rise and fall of the hadronic string

That phase culminated in the construction of the dual resonance model (DRM), the analysis of its (Hagedorn-like) spectrum, the operator formalism, the conditions for ghost elimination, Virasoro algebra, and suggestions of an underlying string theory.

The connection was eventually fully established through a precise formulation of the classical relativistic string (Nambu & Goto, 1970) and through its first correct (light-cone) quantization (Goddard, Goldstone, Rebbi &Thorn, 1972).

Paradoxically, as soon as the DRM had been raised to the level of a respectable Theory, it became apparent that it was not the right one for strong interactions!

It was too revolutionary a theory to be inside any QFT framework!

QCD

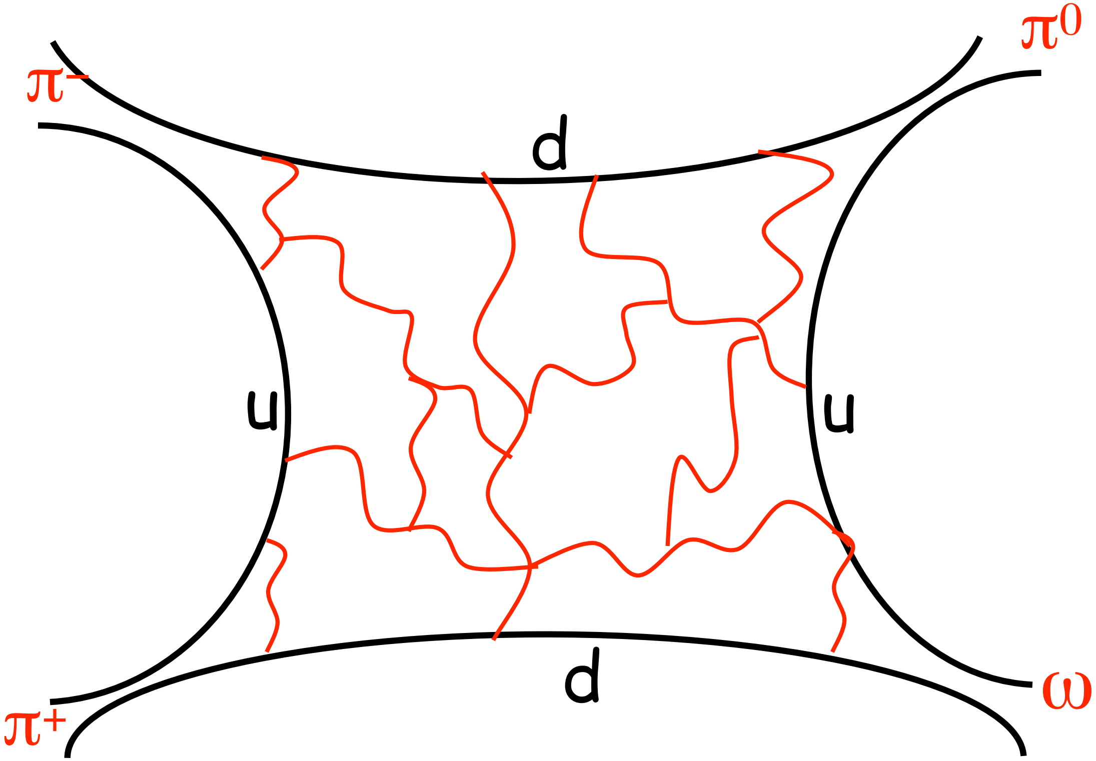
(also revolutionary… ma non troppo)

with its

1. Proven ultraviolet freedom
2. Conjectured (and later proven) infrared slavery (confinement) leading to string-like excitations via chromo-electric flux tubes.
3. A reinterpretation of the duality diagrams (and their higher topologies) in terms of a large-N_c expansions ('t Hooft 1974)

takes over

In large-N_c QCD resonances have zero width, the scattering amplitude is meromorphic, obeys (most likely) DHS duality, generates a scale ($\Lambda^{-2} \sim \alpha'$) via dimensional transmutation, fixes the hadronic coupling.



This was for me, quite regretfully, a point of no return … to the hadronic string!

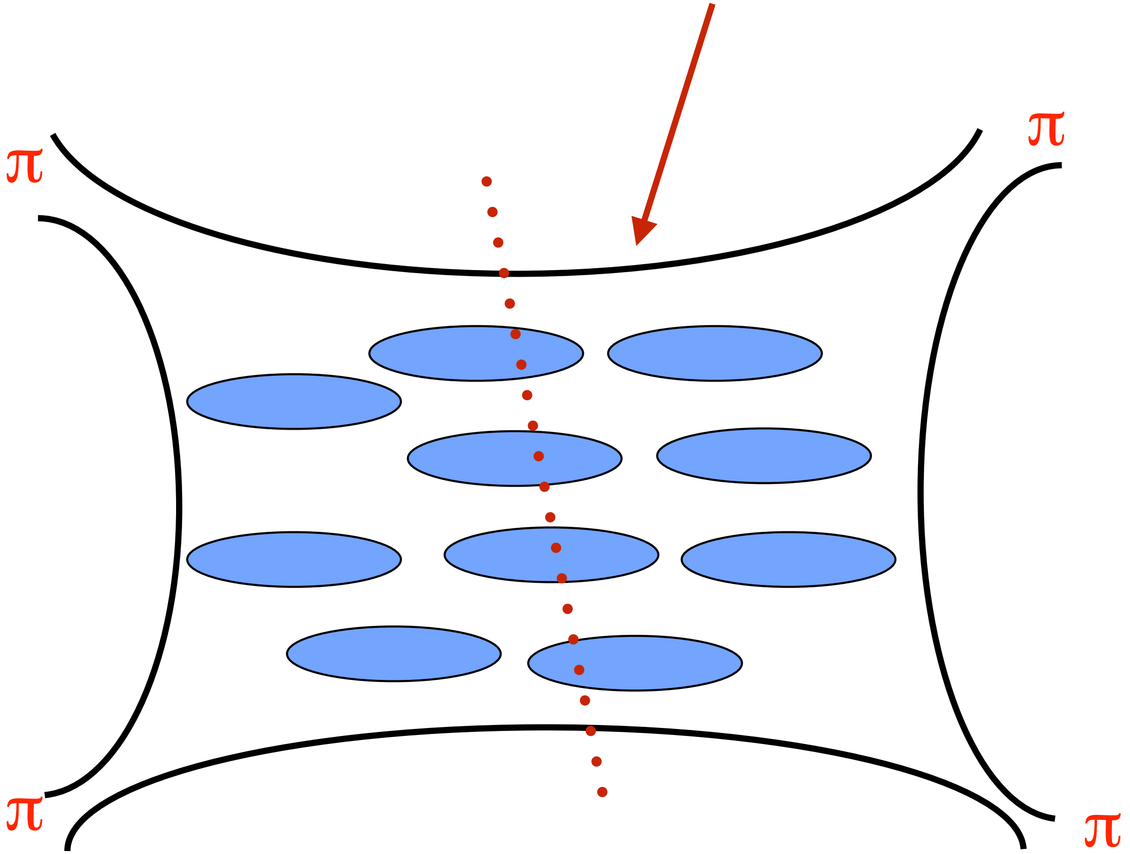
Entering QCD through the large-N door

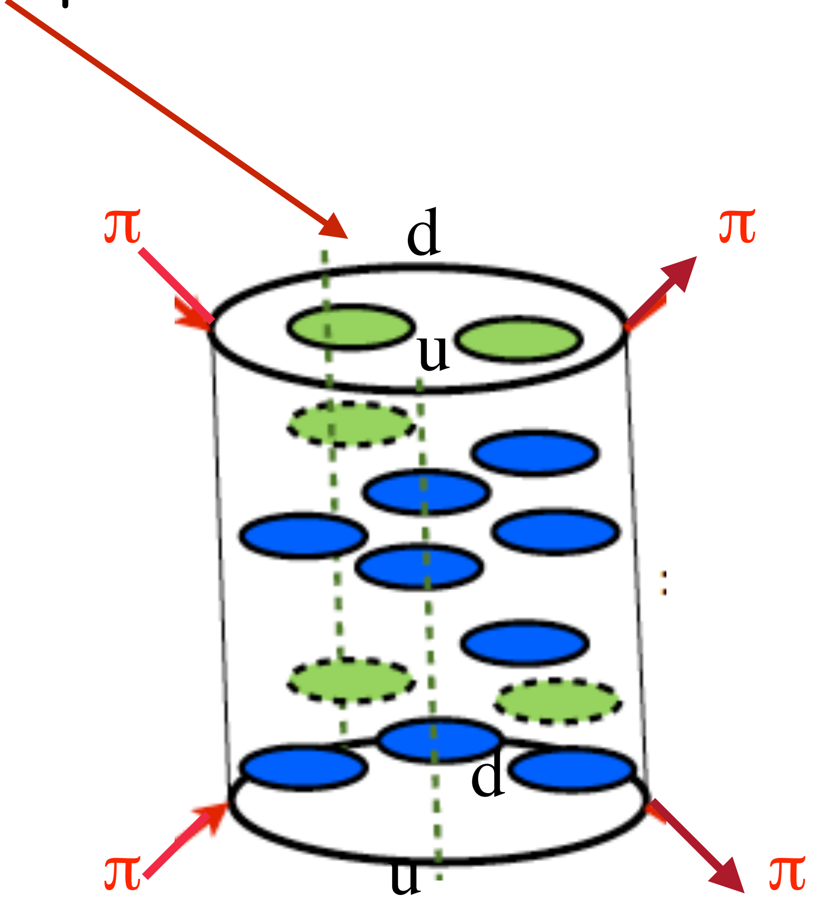
- Topology vs perturbation theory in the DRM
- The topological expansion in QCD

Topology vs perturbation theory in the DRM

In 1970 I wrote a paper with Di Giacomo, Fubini and Sertorio on DRM unitarization.

Topology, rather than the order in g_s, was the organizing criterion. First two terms: the Reggeon and Pomeron poles.





I later argued that, under some reasonable assumption, the Pomeron intercept had to be near (and possibly higher than) one, irrespectively of the Reggeon intercept, itself constrained below one by planar unitarity.

The argument used Feynman's analog-gas model and Dalton's law ($p = p_1 + p_2$ for two non interacting species) and follows from a simple dynamical assumption:

$$\sigma_P = \sum_{X,Y} \sigma(2 \to X + Y) \; ; \; \sigma_R = \sum_{X} \sigma(2 \to X + 0) \; ; \; \sigma_{el} = \sigma(2 \to 1 + 1)$$

$$\boxed{\sigma_{el} \times \sigma_P \sim (\sigma_R)^2 \Rightarrow (\alpha_P - 1) \sim 2(\alpha_R - 1) + 2(1 - \alpha_R) \sim 0}$$

The idea did not quite work in the DRM/string context, because of the presence of massless particles.

The topological expansion in DRM and QCD

After 't Hooft's paper I realized that what I had been advocating in the DRM since 1970 was a large-N_f expansion (@fixed $g_s^2 N_f$).

Later on, converted to QCD, I reformulated it as a 1/N expansion in QCD with a fixed N_f/N_c ratio (thus reinterpreting the planar and cylinder topologies in terms of QCD diagrams). This is the topological expansion of QCD, nowadays sometimes used in connection with holography (see e.g. Kiritsis et al.^{s} V-QCD recent papers)

I still believe there is some truth in all that because QCD, unlike the NG-string, has a mass gap (=> supercritical Pomeron @ LHC?)

It has been used phenomenologically in the so-called "dual-parton-model" approach to soft HE-physics (see e.g. Phys. Rep by Capella et al. 1994)

Perturbative QCD

- Factorization theorem and QCD parton model
- Jet calculus, preconfinement
- Fracture functions

Factorization theorem and QCD parton Model

The original (GW) OPE derivation of scaling violations in QCD had been reformulated in a physically more transparent way by DGLAP.

Roberto Petronzio, who had just arrived from Rome as a CERN post-doc in 1978, was familiar with that reinterpretation.

- Amati, Petronzio and I asked ourselves whether the DGLAP approach could allow for a derivation of the QCD-improved parton model.
- To this purpose we had to show that collinear (mass) singularities, when present, can be lumped/ factorized into some universal (non perturbative) quantities such as structure and fragmentation functions.
- Their evolution, instead, would be calculable in PT.

- We made use of general theorems on IR and mass singularities due to Kinoshita (1962) and to T.D. Lee & Nauenberg (1964)
- We were facing competition from an American group (Harvard-MIT-Caltech) and had many exchanges with Georgi who was visiting CERN
- Eventually, also thanks to work by Al Mueller and others, the QCD-parton model was established.

Relating Hard QCD Processes Through Universality of Mass Singularities

D. Amati, R. Petronzio, G. Veneziano (CERN)

Mar 1978 - 19 pages

Nucl.Phys. B140 (1978) 54-72

(1978)

DOI: 10.1016/0550-3213(78)90313-9

CERN-TH-2470

Abstract (Elsevier)

Hard QCD processes involving final jets are studied and compared by means of a simple approach to mass singularities. This is based on the Lee-Nauenberg-Kinoshita theorem and on a rather subtle use of gauge invariance in hard collinear gluon bremsstrahlung. One-loop results are easily derived for processes involving any number of initial quarks and/or currents. The method greatly simplifies the computation of higher-order loops at the leading log level and our preliminary results allow us to conclude that the crucial features encountered at the one-loop level will persist. We are thus able to relate different hard processes and to show that suitable ratios of cross sections, being free from mass singularities, can be computed perturbatively, as usually assumed in QCD-inspired parton models. We are also able to relate our universal leading mass singularities to leading scaling violations and to extend therefore the results of the operator product expansion method to processes outside the range of the light-cone analysis. Some delicate points caused by confinement-related singularities (e.g., narrow resonance poles) are also discussed.

Relating Hard QCD Processes Through Universality of Mass Singularities. 2.

D. Amati, R. Petronzio, G. Veneziano (CERN)

Jul 1978 - 21 pages

Nucl.Phys. B146 (1978) 29-49

(1978)

DOI: 10.1016/0550-3213(78)90430-3

CERN-TH-2527

Abstract (Elsevier)

Extending previous techniques we obtain at all orders the factorization of mass singularities for every hard QCD process. These appear in a universal factor that can be reabsorbed into the standard parton density. Thus suitable ratios of cross sections can be computed by a perturbative expansion in the running coupling constant. Moreover, at the leading log level we obtain, after explicit cancellation of infrared divergences, the scaling violation of the operator product expansion.

Physics Letters B

Volume 78, Issues 2–3, 25 September 1978, Pages 281-284

Factorization and the parton model in QCD

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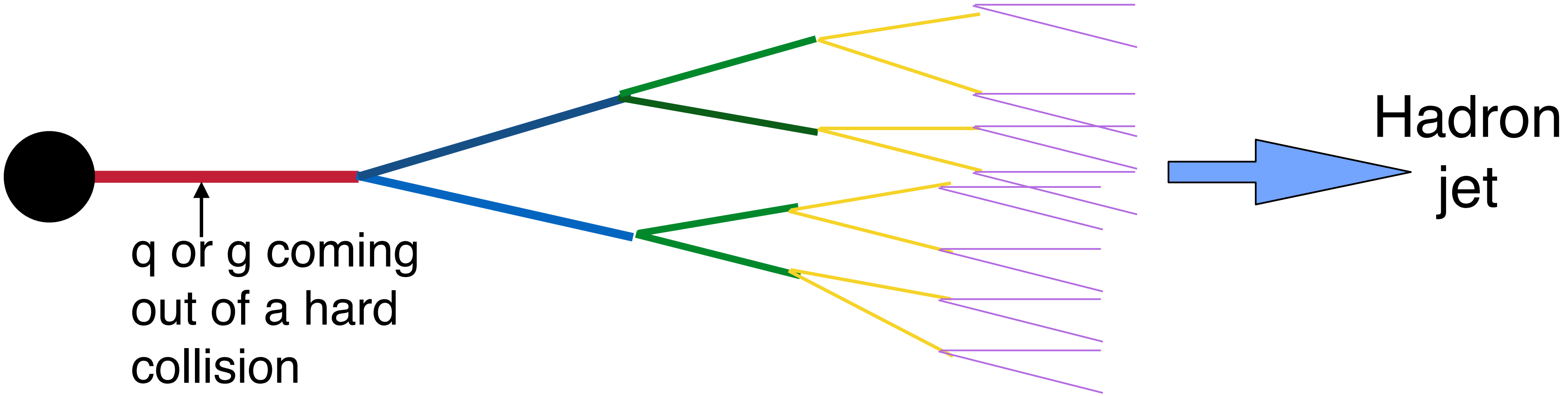
Received 19 June 1978, Available online 10 October 2002.

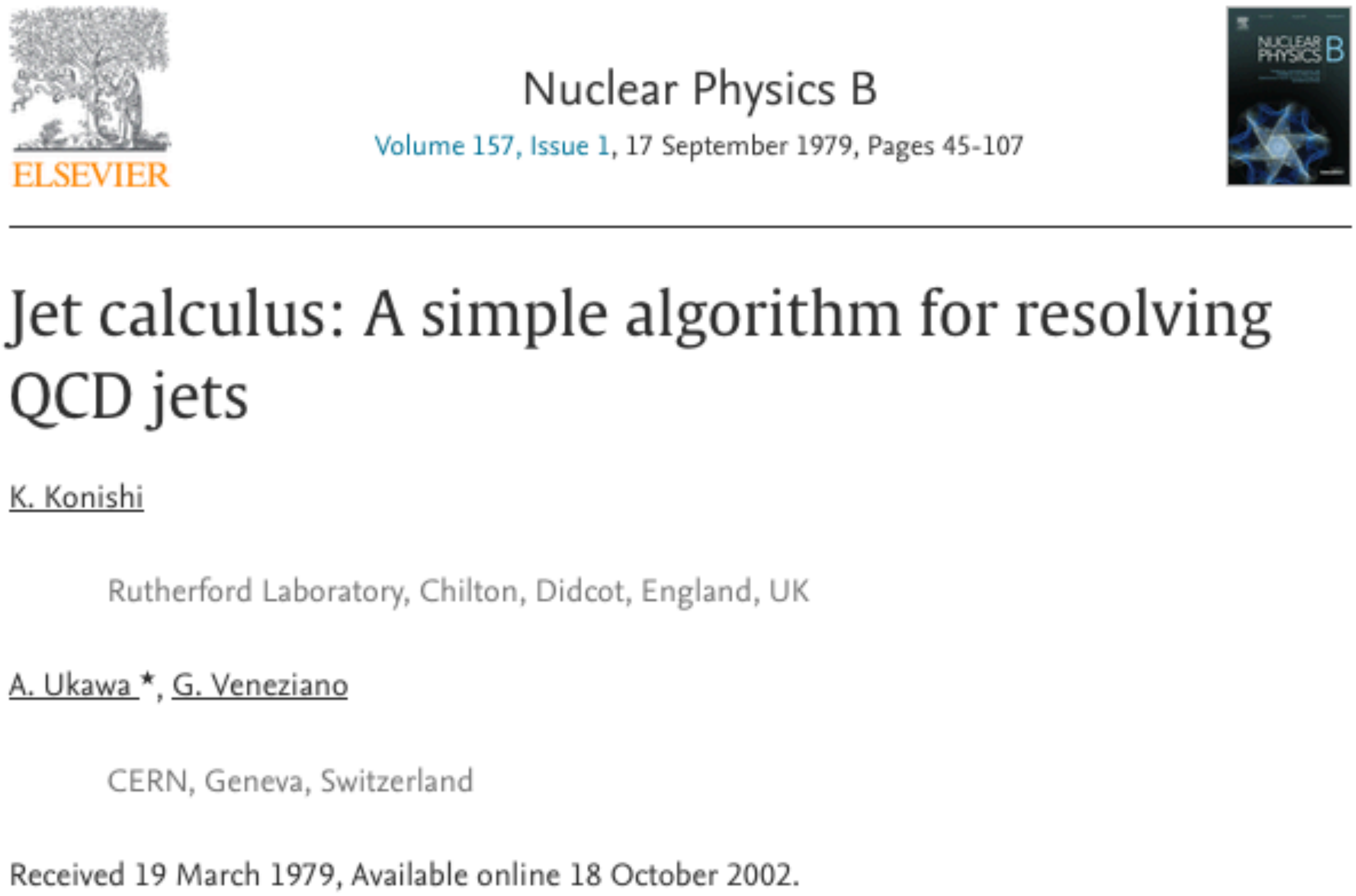
Some of this work has been done while one of us (H.G.) was a guest at CERN. We gratefully acknowledge useful conversations with many colleagues in the CERN theory division, particularly D. Amati, A. De Rújula, R. Petronzio and G. Veneziano.

Jet calculus, pre-confinement

That same year Konishi, Ukawa and I introduced a description of QCD jet evolution in terms of a branching process & dubbed it Jet Calculus: It is now being extended to N^{n}L order (Dixon...)



1979: Amati & I proposed "pre-confinement", as a description of the "final" stage of the pert. process in terms of a set of limited-mass color-singlet combinations of quarks & gluons. After adding coherence and hadronization models (w/ contributions by Mueller, Bassetto, Ciafaloni, Marchesini, Webber), leads to popular event generators, like HERWIG.



Nuclear Physics B

Volume 157, Issue 1, 17 September 1979, Pages 45-107

Jet calculus: A simple algorithm for resolving QCD jets

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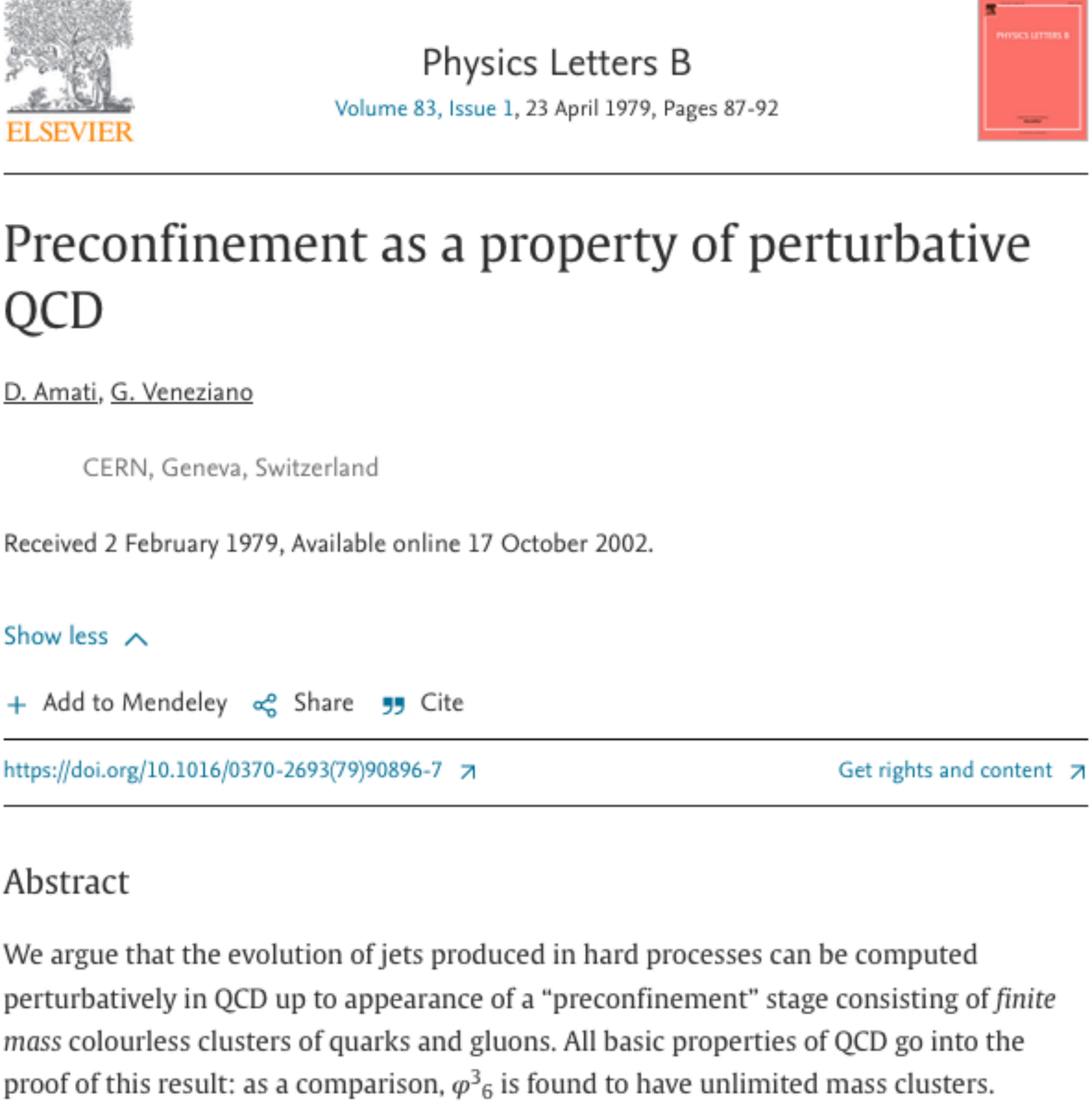
Received 19 March 1979, Available online 18 October 2002.



ELSEVIER

Physics Letters B

Volume 83, Issue 1, 23 April 1979, Pages 87-92

Preconfinement as a property of perturbative QCD

D. Amati, G. Veneziano

CERN, Geneva, Switzerland

Received 2 February 1979, Available online 17 October 2002.

https://doi.org/10.1016/0370-2693(79)90896-7

Abstract

We argue that the evolution of jets produced in hard processes can be computed perturbatively in QCD up to appearance of a “preconfinement” stage consisting of *finite mass* colourless clusters of quarks and gluons. All basic properties of QCD go into the proof of this result: as a comparison, $\varphi^3{}_6$ is found to have unlimited mass clusters.

This time our competitors were Feynman, Field & Fox

The naive jet picture
(Feynman-Field 1977)

FF1 1977 (preQCD)

Feynman quote from FF1

“The model we shall choose is not a popular one, so that we will not duplicate too much of the work of others who are similarly analyzing various models (e.g. constituent interchange model, multiperipheral models, etc.). We shall assume that the high P_T particles arise from direct hard collisions between constituent quarks in the incoming particles, which fragment or cascade down into several hadrons.”

The QCD jet picture
(Feynman-Field-Fox 1978)

FFF2 1978

Feynman quote from FFF2

“We investigate whether the present experimental behavior of mesons with large transverse momentum in hadron-hadron collisions is consistent with the theory of quantum-chromodynamics (QCD) with asymptotic freedom, at least as the theory is now partially understood.”

A Conference & a beer at Caltech

- In 1979 I was invited to a QCD conference at Caltech. Among the speakers: Wilczek & Feynman teasing each other on AF and Nobel prizes…
- RF must have been impressed by my talk about KUV +AV. In his talk, he quoted mine (as well as Petronzio's) referring to the two of us as Veneziano & Petronziano…
- At the end I told him I had enjoyed his talk. His reply was: of course, I treated you so well!
- Then, he proposed to have a beer in a local pub: you have to explain me better, he said.

- He kept asking questions and I did my best to answer. At some point he stopped me shouting: but then you have been cheating me! This is just Altarelli Parisi!

  I tried to convince him that there was more in Jet Calculus than DGLAP and he looked more or less happy…
- I also learned the meaning of "fraying" (in his talk

he mentioned repeatedly "the fraying jet")

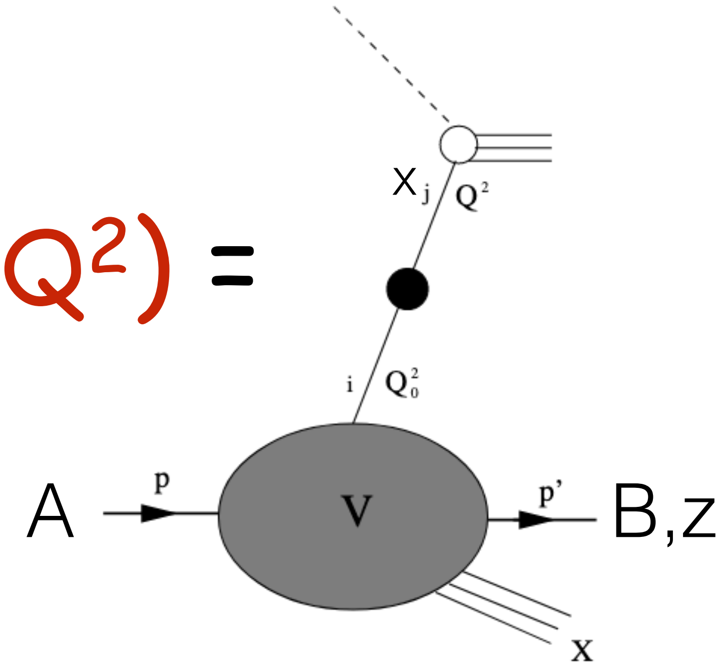
- Too bad we never managed to have another beer together…

Fracture functions

(L. Trentadue & GV, Phys.Lett.B 323 (1994))

- Another (pre-QCD) idea of Feynman (see “Photon-hadron interactions”, Benjamin, 1972) as we learned later.
- Can we define/factorize in QCD a pdf after having selected a particular trigger?

$$M^{j}_{AB}(x,z;Q^2) =$$



- A way to make precise what is meant, e.g., by the structure function of the Pomeron (or by a diffractive pdf) by taking $A = B = p$ and $z \sim 1$.
- Q: Is the Pomeron gluon-rich? Since the Higgs production rate is dominated by the gluon density, a diffractive trigger might enhance the signal/bcknd ratio for Higgs production (D. Graudenz & GV hep-ph/9508401). As it turned out you lose more than you gain…
- Also, usual factorization proof fails (Grazzini, Trentadue & GV hep-ph/9709452), no universality.
- See e.g. 2308.11251 for a recent use of Fracture Functions.

Non-Perturbative QCD

- String junction and multiquark states
- U(1) problem & large-N effective Lagrangian
- EDMN and θ
- The proton "spin" crisis
- SUSY variants and another large-N limit

String junction and multiquark states

(G.C. Rossi and GV, 1977 + Phys. Rep. 1980)

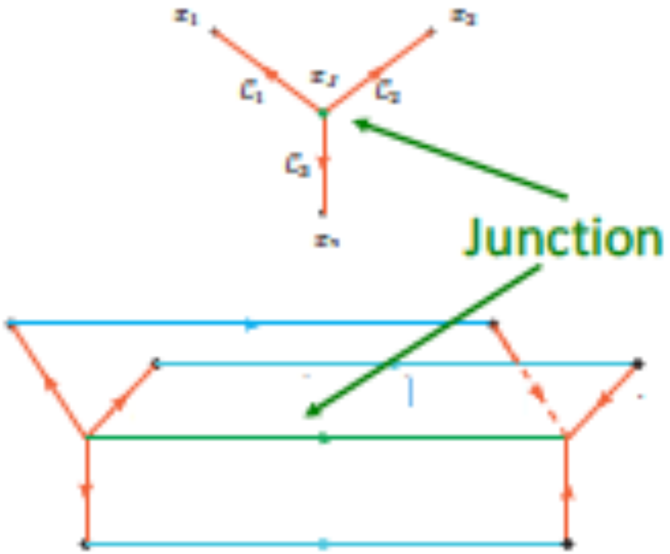
- Basic idea: associate different "elementary" hadrons with irreducible (~"single trace") gauge invariant operators.
- For mesons and glueballs it's quite trivial in terms of Wilson lines, loops.
- For baryons one needs to introduce the notion of a "string junction"
- Recently made more precise in LQCD @ strong 'tHooft coupling (G.C.Rossi and GV, 1603.05830)

Meson → $M(x_1,x_2)=\frac{1}{\sqrt{3}}\bar{q}_k(x_1)C(x_1,x_2)^k_j q^j(x_2)$

$$C(x_1,x_2)^k_j = P\exp(ig\int_{x_1}^{x_2} A_\mu(z)dz_\mu)\Big|^k_j$$

Propagator → $\langle M(x_1,x_2)M^{+}(y_1,y_2)\rangle$

gauge invariance
+
planarity



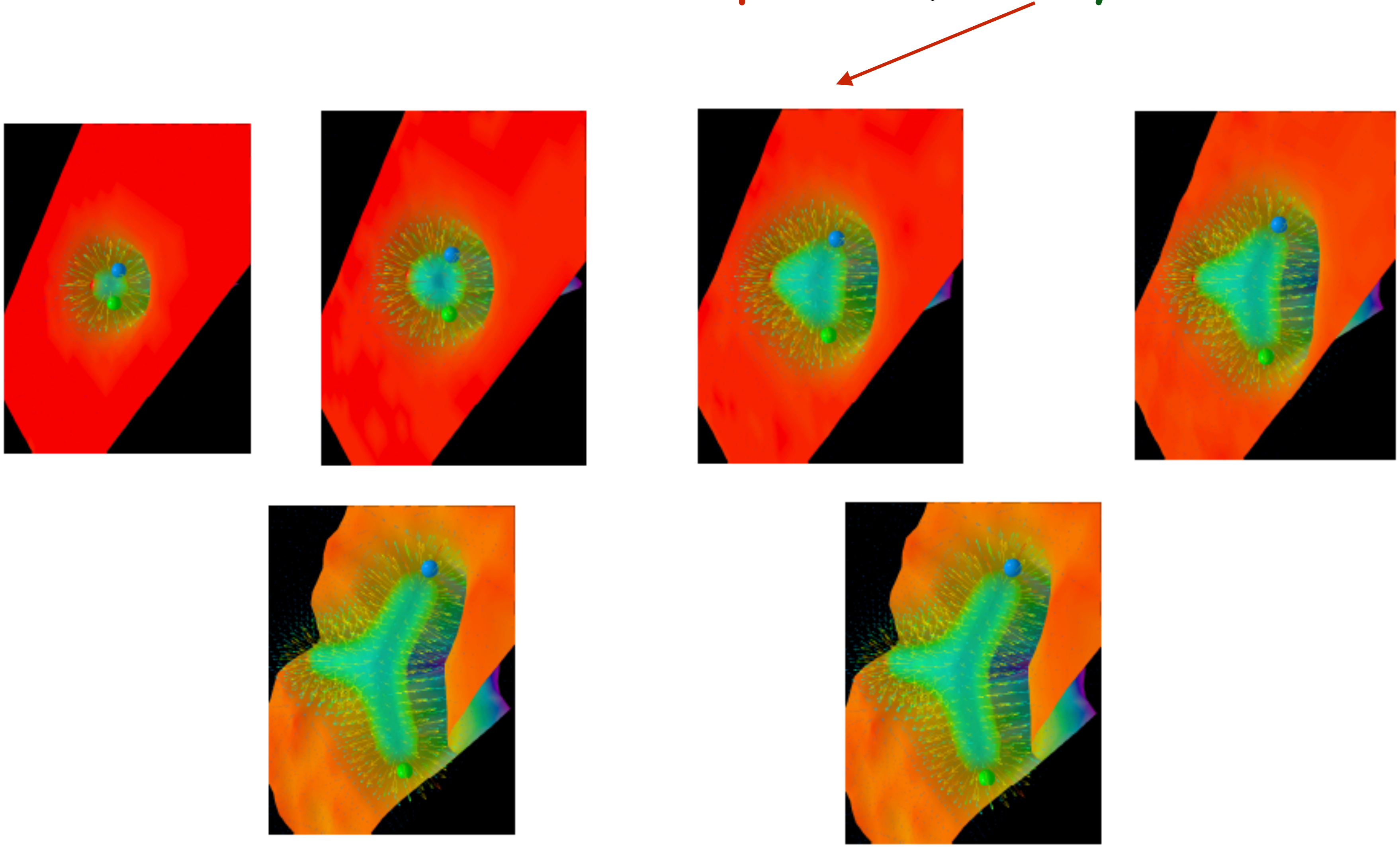
Baryon → $B(x_1,x_2,x_3)=$

$$\frac{\varepsilon^{i_1 i_2 i_3}}{\sqrt{3!}}q_{k_1}(x_1)C(x_1,\underline{x_J})^{k_1}_{i_1}q_{k_2}(x_2)C(x_2,\underline{x_J})^{k_2}_{i_2}q_{k_3}(x_3)C(x_3,\underline{x_J})^{k_3}_{i_3}$$

Propagator → $\langle B(x_1,x_2,x_3)B^{+}(y_1,y_2,y_3)\rangle$

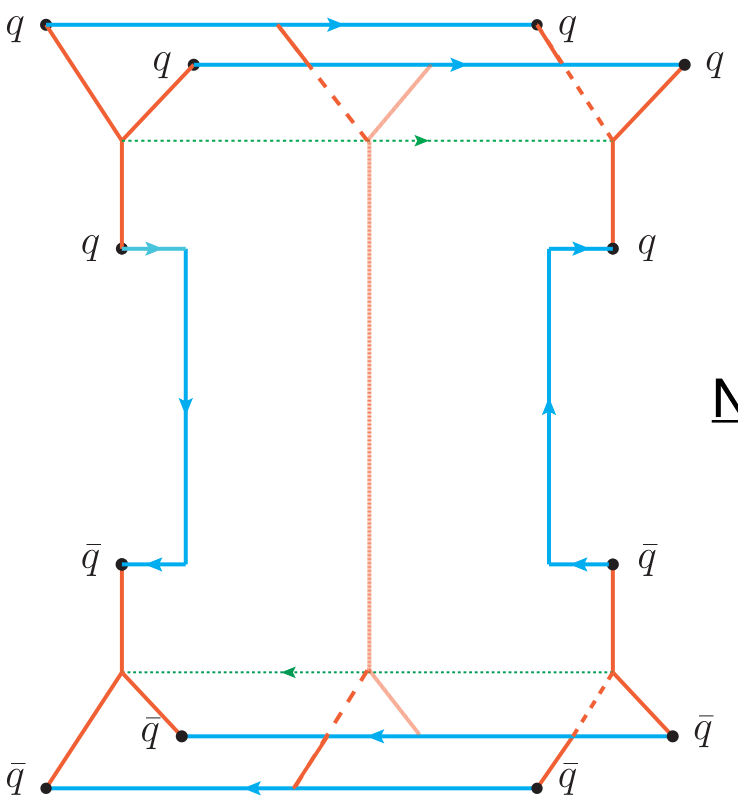
The junction propagates like a 4th constituent!

Junction clearly seen in lattice simulations when the three (static) quarks are pushed apart. Junction sits at the Fermat-Torricelli point! (Bissey et al. 2006)

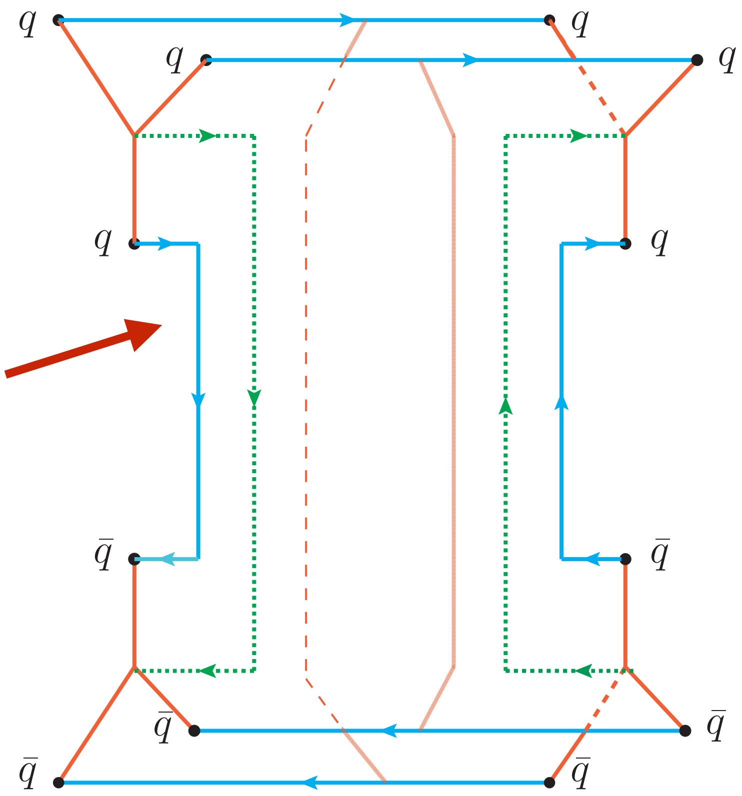


Tracking the junction's flow is important!

Rosner's diagram & and one w/ same quark-flow



Not Rosner!



s-channel scattering thru tetraquark states, dual to qqbar mesons

s-channel annihilation into 2-meson states, dual to qqbar states with junctions

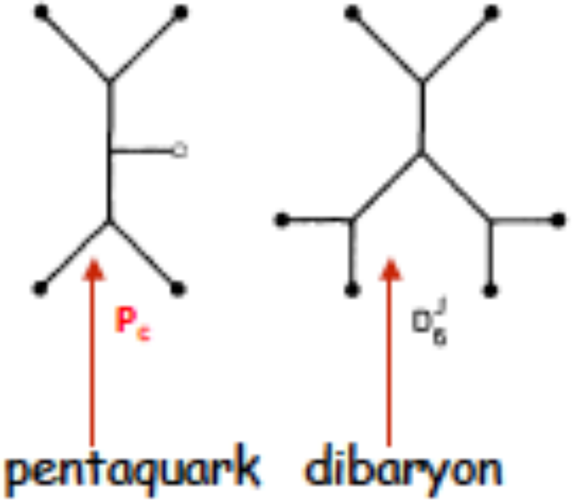
Junction implies existence of exotic (multiquark) hadrons

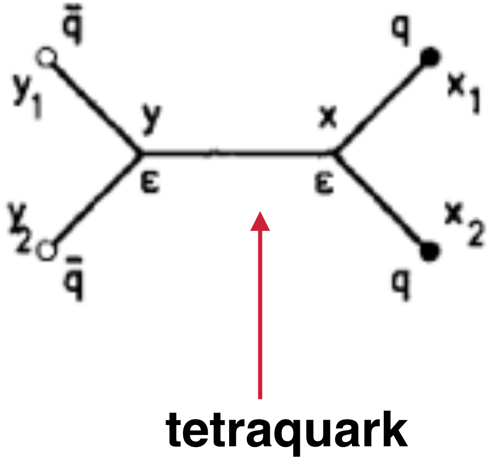
- (Rossi and GV 1977 & 1980)

M_4^J = baryonium
with $qq\bar{q}\bar{q}$
quantum numbers

$$\varepsilon_{j_1j_2j_3}\varepsilon^{k_1k_2k_3}\left[\bar{q}(y_1)\exp\int_y^{y_1}\right]^{j_1}\left[\bar{q}(y_2)\exp\int_y^{y_2}\right]^{j_2}$$

$$\times\left[\exp\int_x^y\right]^{j_3}_{k_1}\left[\exp\int_{x_1}^{x} q(x_1)\right]_{k_2}\left[\exp\int_{x_2}^{x} q(x_2)\right]_{k_3}$$





Premature claim: exp. "confirmation" had to wait for heavy quark discovery (LHCb..)… unless they are molecules.

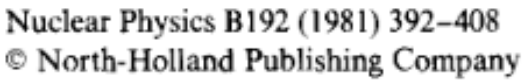
U(1) problem @ large N and the WV relation

- Weinberg's old argument that strong-interactions automatically respect CP invalidated by instantons: can't have the cake (solve U(1) prob.) and eat it (solve strong-CP prob.). From what we know QCD chooses to solve the U(1) problem and to be stuck with CP.
- Furthermore, it seems to be able to do so at leading order in a small N_f/N_c expansion giving (WV relation)

$$m_s^2 = \frac{2N_f}{F_\pi^2}\ \chi_t^{YM} + \mathcal{O}(m_i)$$

Agreement with data for $\chi_t^{YM} \sim (180\ \mathrm{MeV})^4$

Topological susceptibility on the lattice



Nuclear Physics B192 (1981) 392–408
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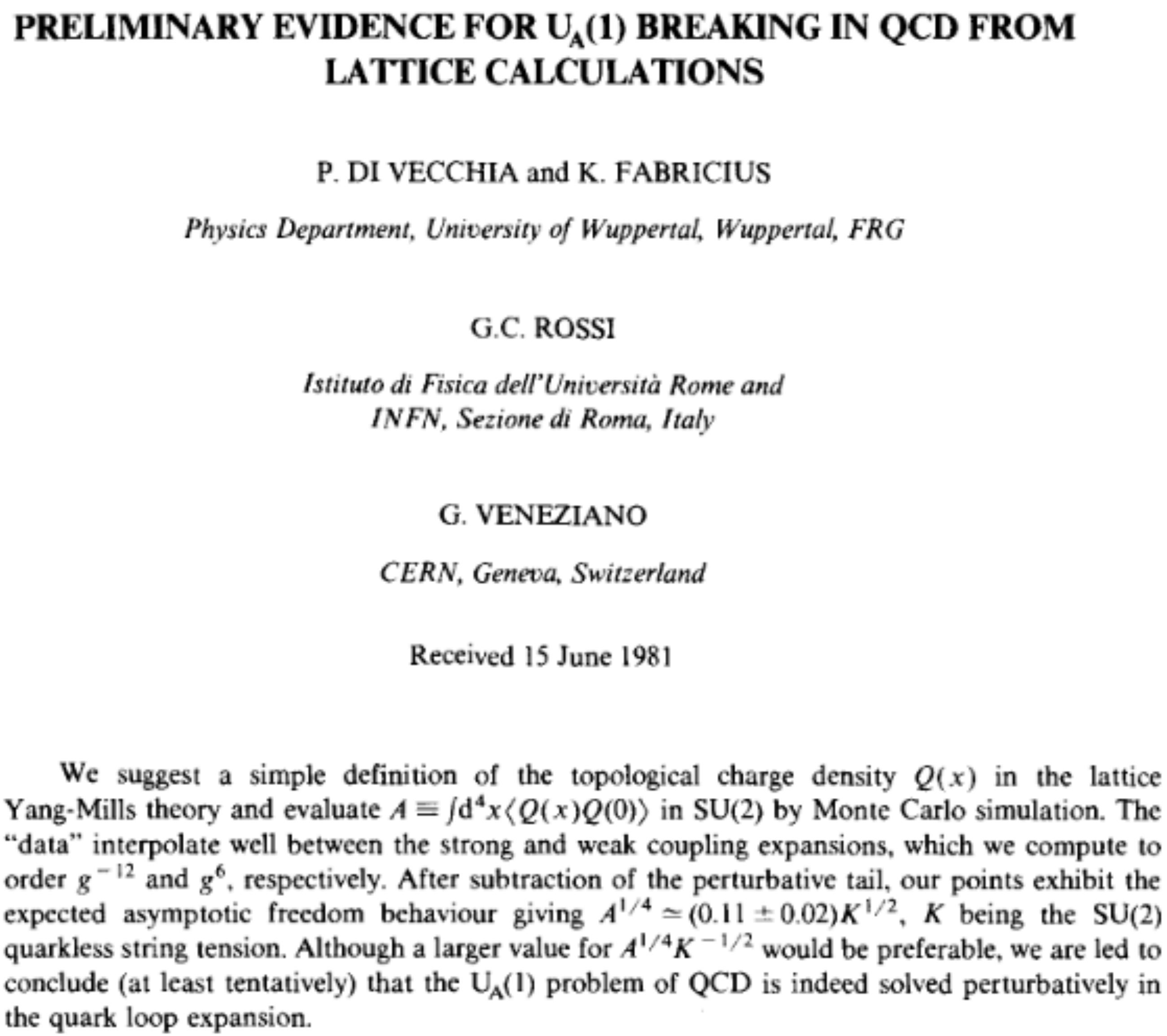
Nucl.Phys.B 192 (1981) 392, Phys.Lett.B 108 (1982) 323



PRELIMINARY EVIDENCE FOR $U_A(1)$ BREAKING IN QCD FROM LATTICE CALCULATIONS

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Received 15 June 1981

We suggest a simple definition of the topological charge density $Q(x)$ in the lattice Yang-Mills theory and evaluate $A \equiv \int d^4x \langle Q(x)Q(0) \rangle$ in SU(2) by Monte Carlo simulation. The "data" interpolate well between the strong and weak coupling expansions, which we compute to order g^{-12} and g^6, respectively. After subtraction of the perturbative tail, our points exhibit the expected asymptotic freedom behaviour giving $A^{1/4} \simeq (0.11 \pm 0.02)K^{1/2}$, K being the SU(2) quarkless string tension. Although a larger value for $A^{1/4}K^{-1/2}$ would be preferable, we are led to conclude (at least tentatively) that the $U_A(1)$ problem of QCD is indeed solved perturbatively in the quark loop expansion.

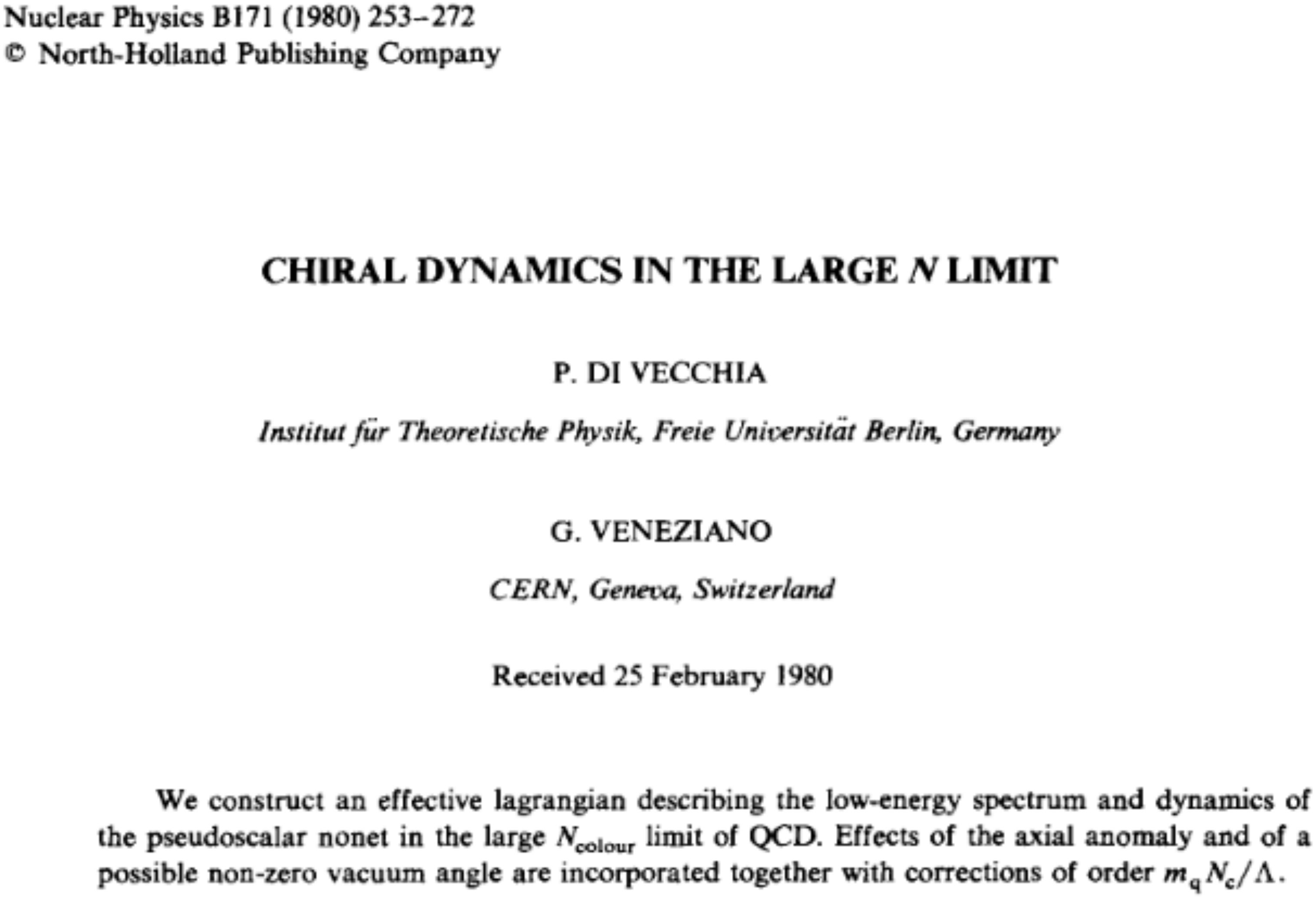
More recent computations of χ_t^{YM} use overlap fermions obeying the Ginsparg-Wilson relation (Del Debbio, Giusti, Pica, hep-th/0407052)

$$\chi_t^{YM} = (191 \pm 5\ \text{MeV})^4 \text{ at } \beta = 6.0$$

i.e. well within what's needed for the pseudosc. spectrum.

Also, it has been checked that χ_t^{YM} is roughly independent of N_c in sharp contrast with dilute-instanton expectations (Del Debbio et al. 0706.1479)

The large-N Lagrangian story



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CHIRAL DYNAMICS IN THE LARGE *N* LIMIT

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Received 25 February 1980

We construct an effective lagrangian describing the low-energy spectrum and dynamics of the pseudoscalar nonet in the large N_{colour} limit of QCD. Effects of the axial anomaly and of a possible non-zero vacuum angle are incorporated together with corrections of order $m_q N_c/\Lambda$.

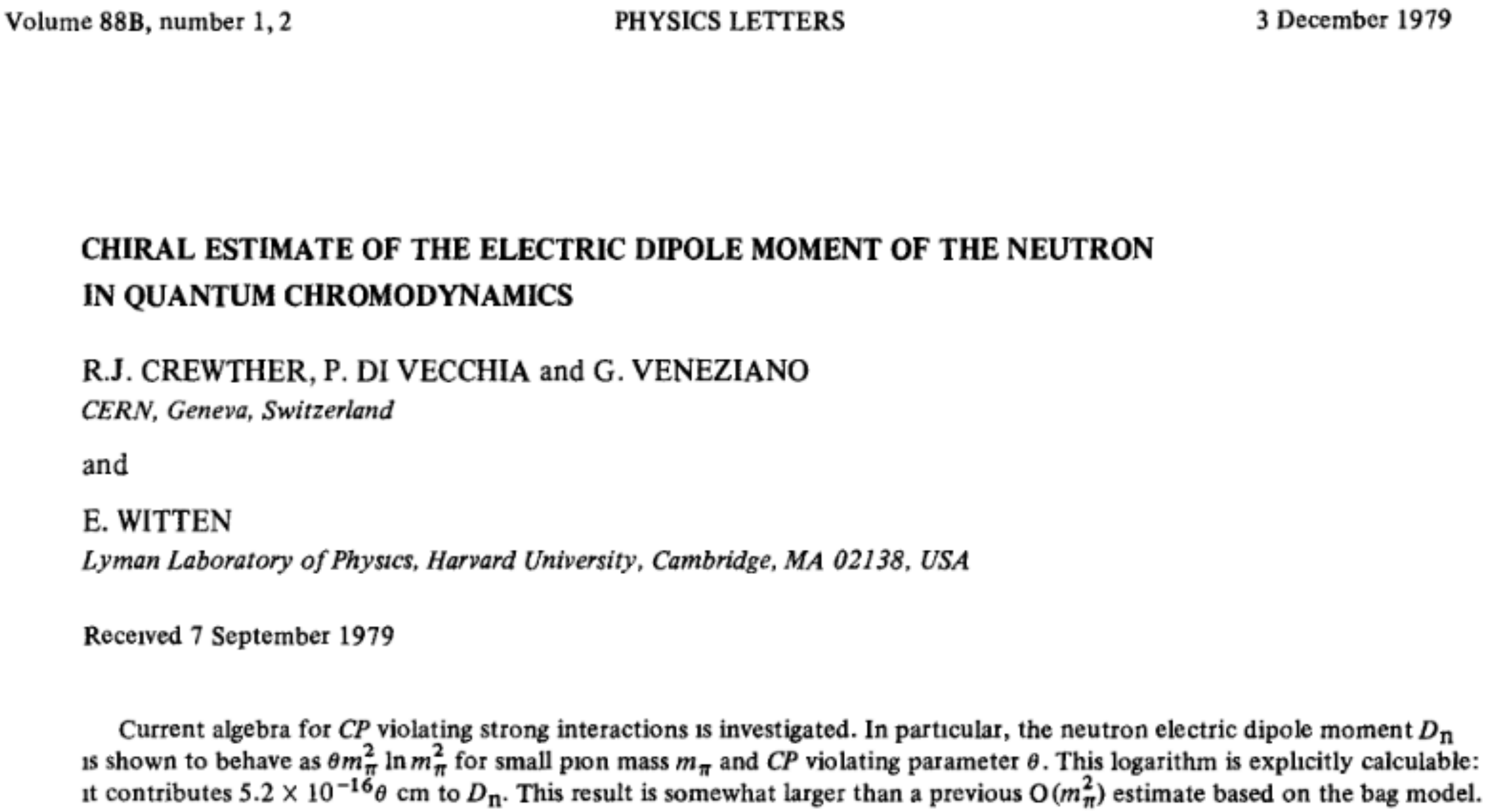
Also: Rosenzweig, Schechter & Trahern, Nath & Arnowitt, Witten

$$\mathcal{L}(U, U^+, q) = \mathcal{L}_0(U, U^+) + \tfrac{1}{2} i q(x) \mathrm{Tr}[\log U - \log U^+]$$
$$+ \frac{N}{a F_\pi^2} q^2(x) - \theta q(x) + \frac{F_\pi}{2\sqrt{2}} \mathrm{Tr}(MU + M^+ U^+)$$

The large-N effective Lagrangian summarizes all the basic properties that follow from SSB, the explicit breaking due to quark masses, the effect of the strong anomaly, and those of the θ angle.

It gives the WV relation, shows how different quantities depend on θ and how θ-dependence disappears if one quark is massless. It also connects the topological susceptibilities of pure YM theory to the one in QCD. It shows how periodicity in θ is recovered thanks to some level crossing at θ = π . It can be extended to include an axion field and determine its potential.

The EDMN story



Volume 88B, number 1, 2 PHYSICS LETTERS 3 December 1979

CHIRAL ESTIMATE OF THE ELECTRIC DIPOLE MOMENT OF THE NEUTRON IN QUANTUM CHROMODYNAMICS

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and

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Received 7 September 1979

Current algebra for CP violating strong interactions is investigated. In particular, the neutron electric dipole moment D_n is shown to behave as $\theta m_\pi^2 \ln m_\pi^2$ for small pion mass m_π and CP violating parameter θ. This logarithm is explicitly calculable: it contributes $5.2 \times 10^{-16}\theta$ cm to D_n. This result is somewhat larger than a previous $O(m_\pi^2)$ estimate based on the bag model.

Di Vecchia and Witten were visiting CERN-TH while Crewther was there on a junior staff position.

We had a few discussions about θ-dependence and decided to look for strong-CP violations in hadronic physics as a result of a non-vanishing value of θ.

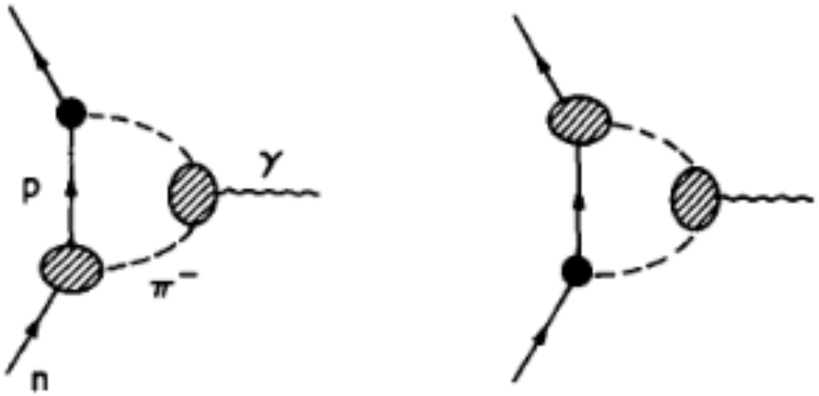
We wanted to make sure that such effects would be unavoidable and (at least in principle) observable.

We first computed the contribution of θ to $\eta \to 2\pi$. That was theoretically simple to work out from Current Algebra, but could hardly put strong bounds on θ.

We knew that a much more sensitive quantity was the neutron's EDM, D_n and were aware of a computation by Baluni giving an estimate on the proportionality constant between D_n and θ. It had a fudge factor and no proof that it could not be zero by some unknown reason.

The first thing to do was to find a CP violating pion nucleon coupling (without γ_5). Again a straightforward Current Algebra calculation.

The crucial point, however, was the realization that coupling a photon to the neutron through a pion loop would produce (thanks to the above coupling) a distinctive logarithmically enhanced (~ $\log m_\pi$) contribution to D_n.



The "proton-spin" crisis

(EMC-SMC exp. on polarized μp DIS)

Not a real crisis (the true spin of the proton is 1/2!) but yet another manifestation of the U(1) anomaly.

What's small is the matrix element of (a moment of the unquenched) topological charge in the nucleon w.r.t. its naive (OZI) value. Narison, Shore & GV 9404277:

$$\frac{\Sigma_1}{\Sigma_8} = \left(\frac{g_{\eta' NN}}{\sqrt{2}\, g_{\eta NN}}\right) \sqrt{\frac{2N_f \chi_t'(0)}{F_\pi^2}} \to 1 \quad (\text{OZI limit})$$

RGI ~ 1

Sc. dep. ~ 0.60 from QCDSR

=> a universal screening of topological charge as the explanation for its small value? Can be tested...

SUSY variants and another large-N limit

For quite a while I worked on SUSY extensions of Yang-Mills and QCD. In particular, effective lagrangians and instanton calculations.

Of course SUSY allows to get (analytic) results that are not available in QCD itself.

One particular calculable quantity is the gluino condensate in SYM.

Much later, however, that work turned out to be possibly useful for QCD itself: this is the story of planar equivalence…

Generalize QCD to $N \neq 3$ ($N = N_c$ here) in new ways by **playing with matter rep**.

A possibility, called for stringy reasons QCD_{OR}, is to assign quarks to the 2-index-antisym. rep. of SU(N) (+ its c.c.)

As in 't Hooft's expansion, N_f is kept fixed ($N_f < 6$, or else AF lost at large N) but fermions are dynamical.

NB: For $N = 3$ this is ordinary **QCD!**

Armoni-Shifman-GV claim of Planar Equivalence (2003)

At large-N a **bosonic C-even sector** of **QCD_{OR}** is equivalent to a **corresponding sector** of **QCD_{Adj}** i.e. of QCD with N_f **Majorana** fermions in the adjoint

Corollary:

For **$N_f = 1$ and $m = 0$**, QCD_{OR} is planar-equivalent to supersymmetric Yang-Mills (**SYM**) theory

Some properties of the latter should show up in one flavor QCD … if N=3 is large enough

ASV gave both perturbative and NP arguments.

Proven (Unsal & Yaffe, hep-th/0608180) provided C is not spont. broken (it is at small comp. radius)

The quark condensate in N_f=1 QCD (Armoni Shifman GV 0309013)

Using $\langle\bar{\lambda}\lambda\rangle_\mu = -\frac{9}{2\pi^2}\mu^3\lambda_\mu^{-2} exp\left(-\frac{1}{\lambda_\mu}\right)$ $\qquad \lambda_\mu = \alpha_s(\mu)N/2\pi$

, the known scale dep. & vanishing of OR at N=2, we get

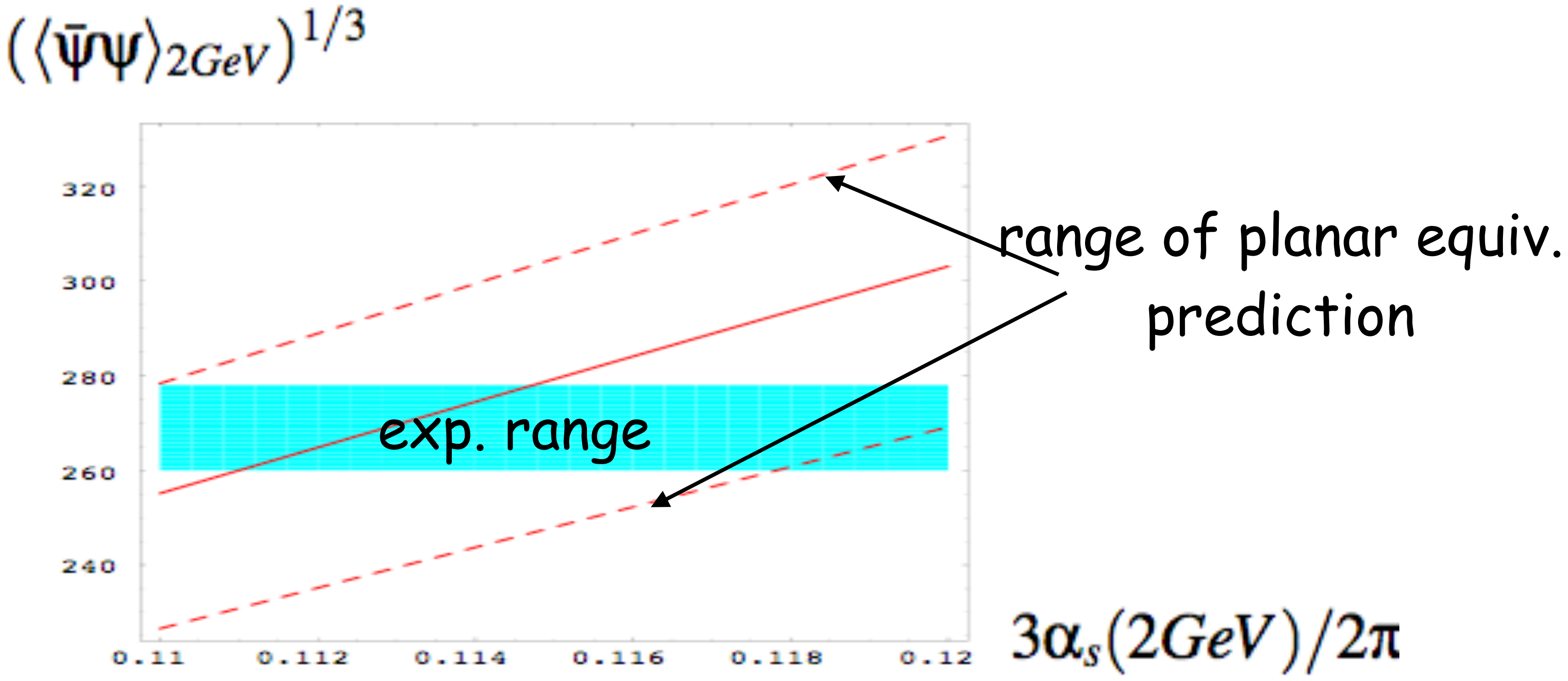
$$\langle\bar{\psi}\psi\rangle_\mu = -\frac{3}{2\pi^2}\mu^3\lambda_\mu^{-1578/961} exp(-\frac{27}{31\lambda_\mu})\; k(1/3)$$

1±0.3?

with K(1/N) = (1- 2/N) k(1/N), k(0) = 1

N_f=1 condensate “measured”?

(DeGrand, Hoffmann , Schaefer & Liu, hep-th/0605147)



range of planar equiv. prediction

exp. range

Extension to N_f > 1 (AShoreV, 0511143, ASSV, 1412.3389) checked by Engel, Giusti, Lottini, Sommer (1406.4987) by comparing directly lattice data works for N_f = 3 and k(1/3) = 1.03(16)

More recent distractions

- Spont. CP breaking and axion potential
- A model for π-π scattering in large-N QCD
- Can we bootstrap large-N QCD?

My attraction to QCD keeps coming back all the time!

Happy Birthday QCD!

ADDITIONAL SLIDES

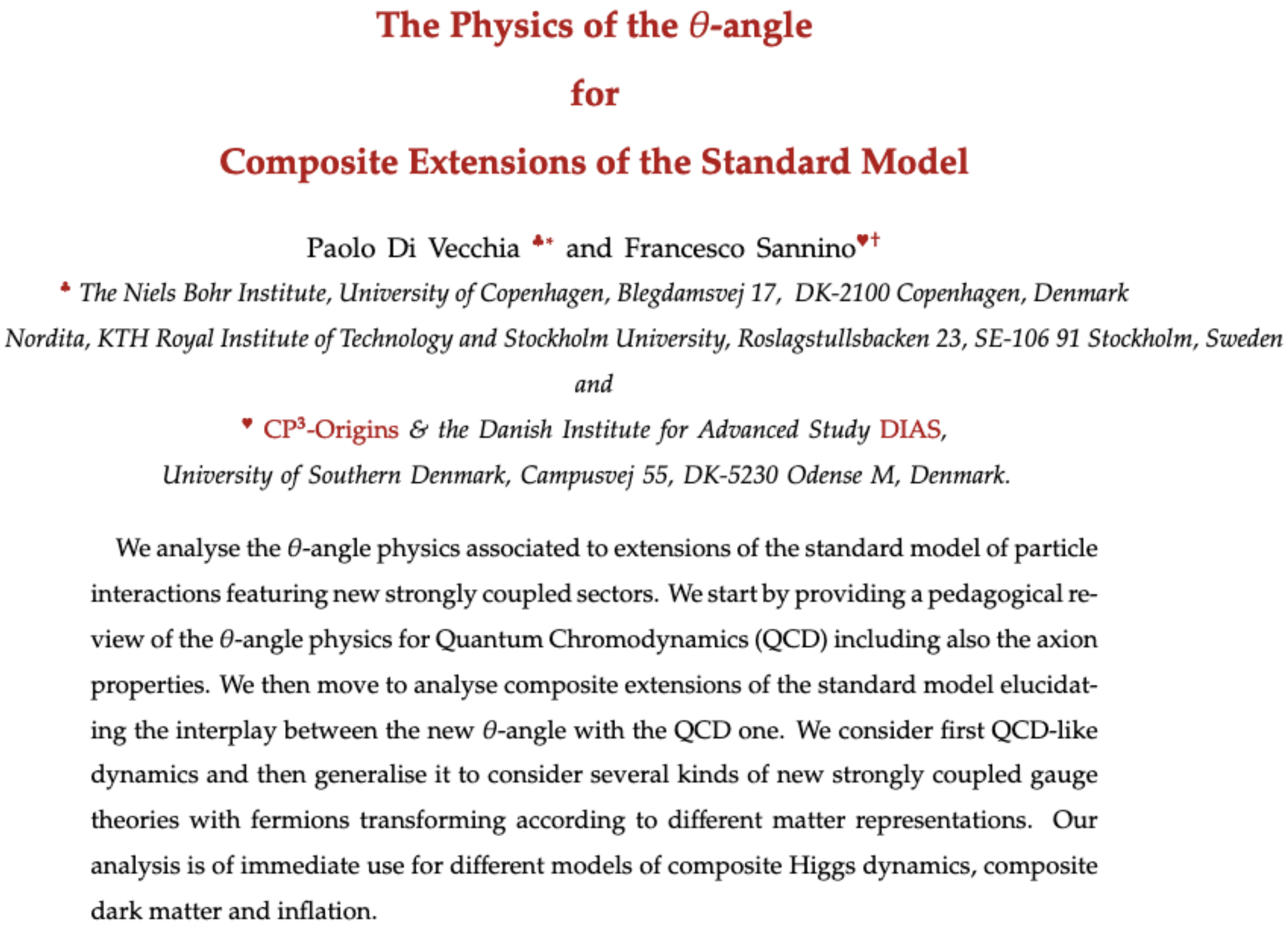
Spont. CP breaking and axion potential

Adding the axion I

Another amusing variation is to add an axion to the Lagrangian…so that the strong-CP problem is automatically solved.

In general an axion is a pseudo-NG boson whose mass is entirely produced by the anomaly (Cf. QCD with a single massless quark). So adding the axion is like adding an extra massless quark but with a much larger condensate ($f_a \gg f_\pi$ implying $m_a \ll m_\pi$).

Details worked out in a paper by Di Vecchia and F. Sannino.



The Physics of the θ-angle for Composite Extensions of the Standard Model

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We analyse the θ-angle physics associated to extensions of the standard model of particle interactions featuring new strongly coupled sectors. We start by providing a pedagogical review of the θ-angle physics for Quantum Chromodynamics (QCD) including also the axion properties. We then move to analyse composite extensions of the standard model elucidating the interplay between the new θ-angle with the QCD one. We consider first QCD-like dynamics and then generalise it to consider several kinds of new strongly coupled gauge theories with fermions transforming according to different matter representations. Our analysis is of immediate use for different models of composite Higgs dynamics, composite dark matter and inflation.

- *Eur.Phys.J.Plus* 129 (2014) 26 *1310.0954 [hep-ph]*

Adding the axion II

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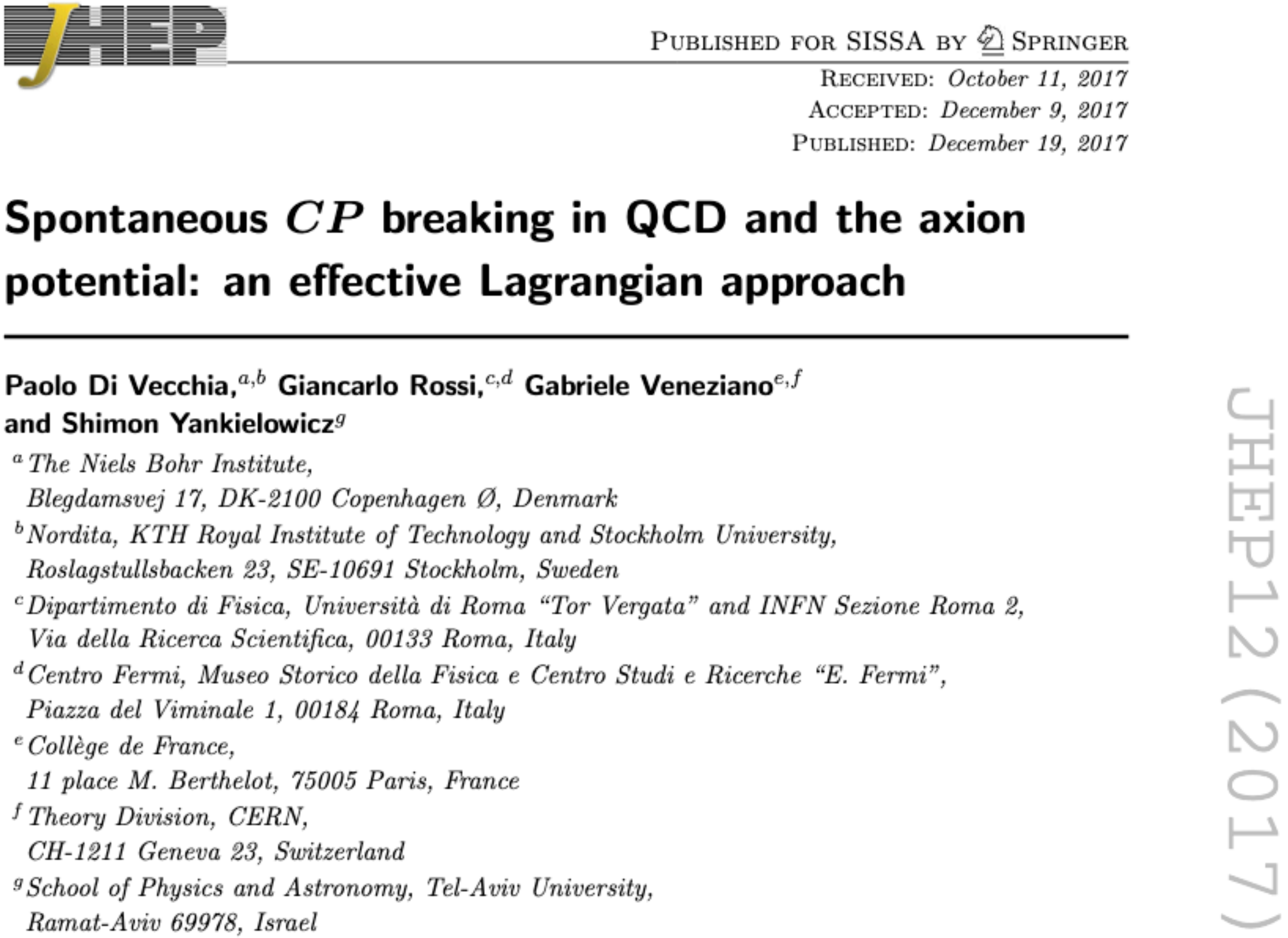

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Spontaneous CP breaking in QCD and the axion potential: an effective Lagrangian approach

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In that paper we studied the possible spontaneous breaking of CP near $\theta = \pi$ and its implications on the axion potential. There are two competing small scales and the physics depends on their ratio:

$$\epsilon = \frac{m_q \langle \bar{q} q \rangle}{\chi_{YM}} \sim \frac{m_\pi^2}{a} \sim \frac{m_q N_c}{\Lambda_{QCD}}$$

At zero temperature with $N_c = 3$, $\varepsilon < 1$ but near the QCD phase transition it can be much larger and this would imply interesting new features of the axion potential near its periodicity value.

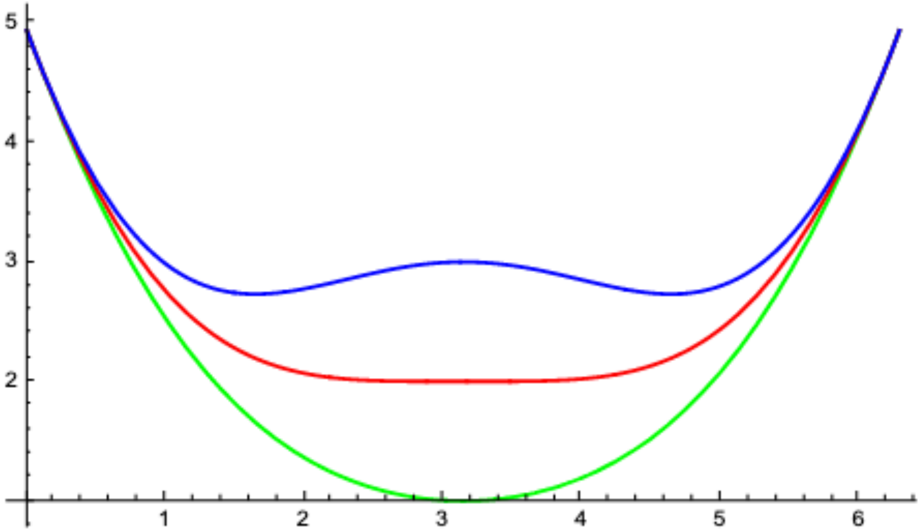

Figure 2: $V(\phi)$ of Eq. (3.1) at $\theta = \pi$, and $\epsilon = 0.5$ (green curve), $\epsilon = 1.0$ (red) and $\epsilon = 2.0$ (blue).

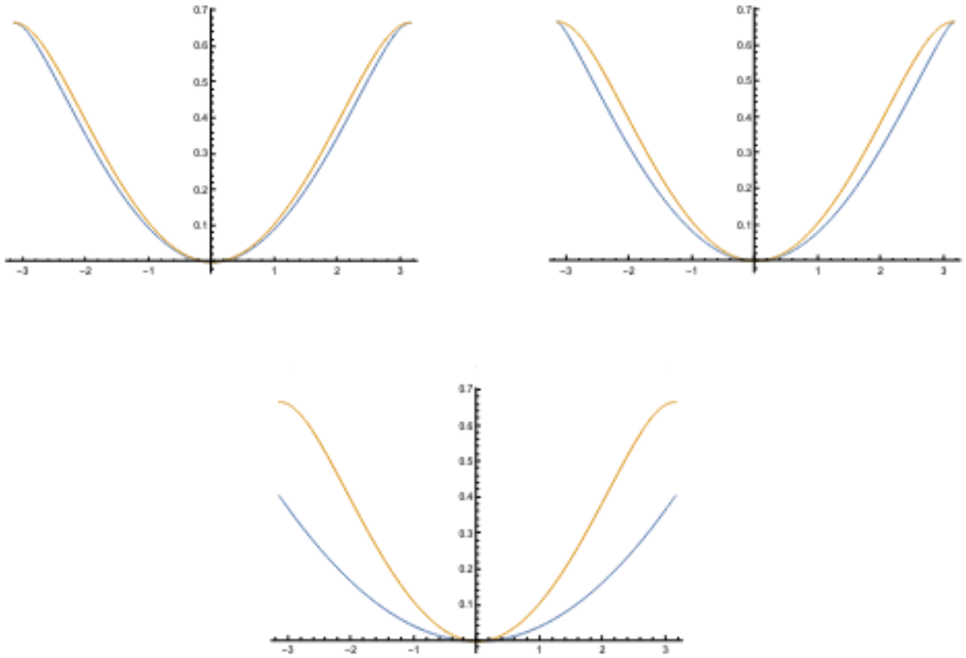

Figure 9: Comparing the conventional axion potential (yellow curves) with the “exact” one (blue curves) for $N_f = 2, \mu_d^2 = 2\mu_u^2$ and at three values of μ_u^2/a: $0.25, 0.5$ (critical value), 2.5. In the first two cases the two potentials (but not necessarily their derivatives) agree at $\zeta = \pm\pi$ while in the third (overcritical) case even the values of the potentials disagree at the boundary of the periodicity interval.

A model for π-π scattering in large-N QCD

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A model for pion-pion scattering in large-N QCD

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A Generalized Lovelace-Shapiro (GLS) model ($\alpha' = 1$)

$$
\begin{aligned}
\frac{A(s,t)}{g^2} &= \sum_{k=1}^{\infty} c_k A_k(s,t) \\
A_k(s,t) &= -\frac{(s+t)}{k} B\left(1-\frac{1-a+s}{k}, 1-\frac{1-a+t}{k}\right)
\end{aligned}
$$

NB: LS corresponds to $a = 1/2$, $k = 1$

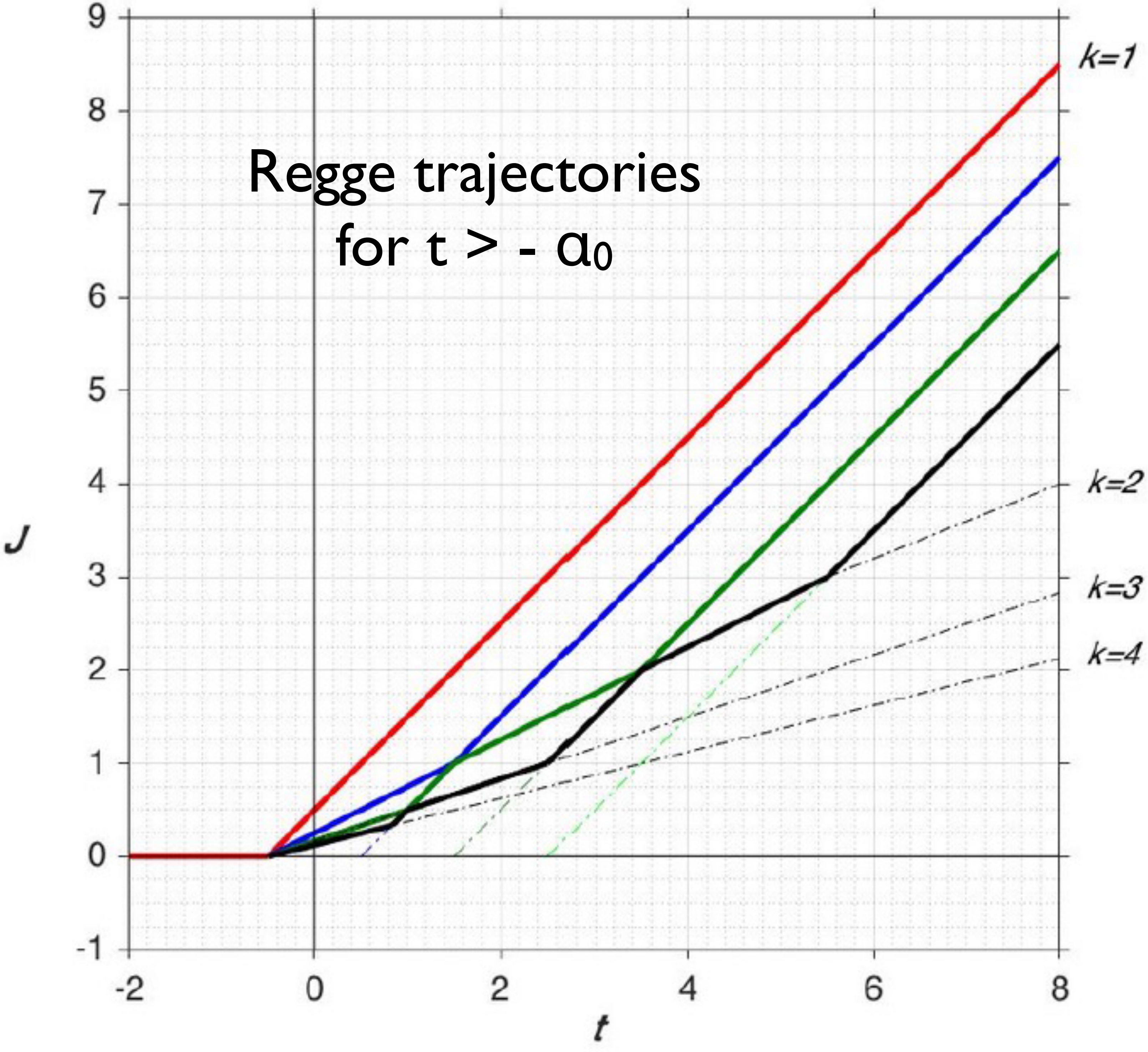
Above: satisfies Adler's condition for any a

See also:

O. Andreev and W. Siegel, Phys. Rev. D **71**, 086001 (2005)

but no unitarity constraint

Regge trajectories for $t > -\alpha_0$



Tree-level Unitarity

(i.e. positivity of residues)

It is believed that the LS model in D=4 is unitary only for $\alpha_0 = 0.5$ or larger (but < 1).

It certainly has negative residues below

Therefore each A_k with $k > 1$ has ghosts

Can the sum be ghost-free?

The answer appears to be: yes, provided the c_k fall off fast enough with k

But that behavior should be provided by AF since it controls fixed angle scattering!

Can we bootstrap large-N QCD?

There is a lot of work going on (see e.g. Amplitudes 2023) about bootstrapping string-like 4-point functions

Even more interestingly one should try to set constraints on a QCD 4-point function (e.g. π-π scattering for massless quarks) in 't-Hooft's limit after listing some of its obvious properties (crossing, positivity, analyticity) and postulating others:

Regge-pole behavior (no cuts in J), factorization, exchange-degeneracy.

DHS duality, i.e. A-> 0 at high energy and a negative range in t.

Fixed angle behavior in space- and time-like regions

Asymptotically linear trajectories.