UCLA Mani L. Bhaumik Institute for Theoretical Physics

50 Years of QCD

Welcome to UCLA

50 Years of QCD: 1973 - Asymptotic freedom

Ultraviolet Behavior of Non-Abelian Gauge Theories*

David J. Gross† and Frank Wilczek
Joseph Henry Laboratories, Princeton University, Princeton, New Jersey 08540

(Received 27 April 1973)

It is shown that a wide class of non-Abelian gauge theories have, up to calculable logarithmic corrections, free-field–theory asymptotic behavior. It is suggested that Bjorken scaling may be obtained from strong-interaction dynamics based on non-Abelian gauge symmetry.

Non-Abelian gauge theories have received much attention recently as a means of constructing unified and renormalizable theories of the weak and electromagnetic interactions.[1] In this note we report on an investigation of the ultraviolet (UV) asymptotic behavior of such theories. We have found that they possess the remarkable feature, perhaps unique among renormalizable theories, of asymptotically approaching free-field theory. Such asymptotically free theories will exhibit, for matrix elements of currents between on-mass-shell states, Bjorken scaling. We therefore suggest that one should look to a non-Abelian gauge theory of the strong interactions to provide the explanation for Bjorken scaling, which has so far eluded field-theoretic understanding.

The UV behavior of renormalizable field theories can be discussed using the renormalization-group equations,[2,3] which for a theory involving one field (say $g\varphi^4$) are

$$[m\partial/\partial m+\beta(g)\,\partial/\partial g-n\gamma(g)]\Gamma_{\text{asy}}^{(n)}(g;P_1,\ldots,P_n)=0. \tag{1}$$

$\Gamma_{\text{asy}}^{(n)}$ is the asymptotic part of the one-particle–irreducible renormalized n-particle Green's function, $\beta(g)$ and $\gamma(g)$ are finite functions of the renormalized coupling constant g, and m is either the renormalized mass or, in the case of massless particles, the Euclidean momentum at which the theory is renormalized.[4] If we set $P_i=\lambda q_i^0$, where q_i^0 are (nonexceptional) Euclidean momenta, then (1) determines the λ dependence of $\Gamma^{(n)}$:

$$\Gamma^{(n)}(g;P_i)=\lambda^D\Gamma^{(n)}(\bar{g}(g,t);q_i)\exp\left[-n\int_0^t\gamma(\bar{g}(g,t'))\,dt'\right], \tag{2}$$

where $t=\ln\lambda$, D is the dimension (in mass units) of $\Gamma^{(n)}$, and $\bar{g}$, the invariant coupling constant, is the solution of

$$d\bar{g}/dt=\beta(\bar{g}),\quad \bar{g}(g,0)=g. \tag{3}$$

The UV behavior of $\Gamma^{(n)}$ $(\lambda\to+\infty)$ is determined by the large-t behavior of $\bar{g}$ which in turn is controlled by the zeros of β: $\beta(g_f)=0$. These fixed points of the renormalization-group equations are said to be UV stable [infrared (IR) stable] if $\bar{g}\to g_f$ as $t\to+\infty$ $(-\infty)$ for $\bar{g}(0)$ near g_f. If the physical coupling constant is in the domain of attraction of a UV-stable fixed point, then

$$\Gamma^{(n)}(g;P_i)\underset{\lambda\to\infty}{\approx}\lambda^{D-n\gamma(g_f)}\Gamma^{(n)}(g_f;q_i)\exp\left\{-n\int_0^\infty[\gamma(\bar{g}(g,t))-\gamma(g_f)]\,dt\right\}, \tag{4}$$

so that $\gamma(g_f)$ is the anomalous dimension of the field. As Wilson has stressed, the UV behavior is determined by the theory at the fixed point ($g=g_f$).[5]

In general, the dimensions of operators at a fixed point are not canonical, i.e., $\gamma(g_f)\neq 0$. If we wish to explain Bjorken scaling, we must assume the existence of a tower of operators with canonical dimensions. Recently, it has been argued for all but gauge theories, that this can only occur if the fixed point is at the origin, $g_f=0$, so that the theory is asymptotically free.[6,7] In that case the anomalous dimensions of all operators vanish, one obtains naive scaling up to finite and calculable powers of $\ln\lambda$, and the structure of operator products at short distances is that of free-field theory.[7] Therefore, the existence of such a fixed point, for a theory of the strong interactions, might explain Bjorken scaling and the success of naive light-cone or parton-model relations. Unfortunately, it appears that the fixed point at the origin, which is common to all theories, is not UV stable.[8,9] The only exception would seem to be non-Abelian gauge theories, which hitherto have not been explored in this re-

[14] Y. Nambu and G. Jona-Lasino, Phys. Rev. 122, 345 (1961); S. Coleman and E. Weinberg, Phys. Rev. D 7, 1888 (1973).

[15] K. Symanzik (to be published) has recently suggested that one consider a $\lambda\varphi^4$ theory with a negative λ to achieve UV stability at $\lambda=0$. However, one can show, using the renormalization-group equations, that in such theory the ground-state energy is unbounded from below (S. Coleman, private communication).

[16] W. A. Bardeen, H. Fritzsch, and M. Gell-Mann, CERN Report No. CERN-TH-1538, 1972 (to be published).

[17] H. Georgi and S. L. Glashow, Phys. Rev. Lett. 28, 1494 (1972); S. Weinberg, Phys. Rev. D 5, 1962 (1972).

[18] For a review of this program, see S. L. Adler, in Proceedings of the Sixteenth International Conference on High Energy Physics, National Accelerator Laboratory, Batavia, Illinois, 1972 (to be published).

Reliable Perturbative Results for Strong Interactions?*

H. David Politzer
Jefferson Physical Laboratories, Harvard University, Cambridge, Massachusetts 02138

(Received 3 May 1973)

An explicit calculation shows perturbation theory to be arbitrarily good for the deep Euclidean Green's functions of any Yang-Mills theory and of many Yang-Mills theories with fermions. Under the hypothesis that spontaneous symmetry breakdown is of dynamical origin, these symmetric Green's functions are the asymptotic forms of the physically significant spontaneously broken solution, whose coupling could be strong.

Renormalization-group techniques hold great promise for studying short-distance and strong-coupling problems in field theory.[1,2] Symanzik[2] has emphasized the role that perturbation theory might play in approximating the otherwise unknown functions that occur in these discussions. But specific models in four dimensions that had been investigated yielded (in this context) disappointing results.[3] This note reports an intriguing contrary finding for any generalized Yang-Mills theory and theories including a wide class of fermion representations. For these one–coupling-constant theories (or generalizations involving product groups) the coefficient function in the Callan-Symanzik equations commonly called $\beta(g)$ is negative near $g=0$.

The constrast with quantum electrodynamics (QED) might be illuminating. Renormalization of QED must be carried out at off-mass-shell points because of infrared divergences. For small e^2, we expect perturbation theory to be good in some neighborhood of the normalization point. But what about the inevitable logarithms of momenta that grow as we approach the mass shell or as some momenta go to infinity? In QED, the mass-shell divergences do not occur in observable predictions, when we take due account of the experimental situation. The renormalization-group technique[4] provides a somewhat opaque analysis of this situation. Loosely speaking,[5] the effective coupling of soft photons goes to zero, compensating for the fact that there are more and more of them. But the large-p^2 divergence represents a real breakdown of perturbation theory. It is commonly said that for momenta such that $e^2\ln(p^2/m^2)\sim 1$, higher orders become comparable, and hence a calculation to any finite order is meaningless in this domain. The renormalization group technique shows that the effective coupling grows with momenta.

The behavior in the two momentum regimes is reversed in a Yang-Mills theory. The effective coupling goes to zero for large momenta, but as p^2's approach zero, higher-order corrections become comparable. Thus perturbation theory tells *nothing* about the mass-shell structure of the symmetric theory. Even for arbitrarily small g^2, there is no sense in which the interacting theory is a small perturbation on a free multiplet of massless vector mesons. The truly catastrophic infrared problem makes a symmetric particle interpretation impossible. Thus, though one can well approximate asymptotic Green's functions, to what particle states do they refer?

Consider theories defined by the Lagrangian

$$\mathfrak{L}=-\tfrac{1}{4}F_{\mu\nu}{}^aF^{a\mu\nu}+i\bar{\psi}_i\gamma\cdot D_{ij}\psi_j, \tag{1}$$

where

$$F_{\mu\nu}{}^a=\partial_\mu A_\nu{}^a-\partial_\nu A_\mu{}^a+gf^{abc}A_\mu{}^bA_\nu{}^c,$$

Roadmap for QCD:

PHYSICAL REVIEW D VOLUME 8, NUMBER 10 15 NOVEMBER 1973

Asymptotically Free Gauge Theories. I[*]

David J. Gross[†]
National Accelerator Laboratory, P. O. Box 500, Batavia, Illinois 60510
and Joseph Henry Laboratories, Princeton University, Princeton, New Jersey 08540

Frank Wilczek
Joseph Henry Laboratories, Princeton University, Princeton, New Jersey 08540

(Received 23 July 1973)

Asymptotically free gauge theories of the strong interactions are constructed and analyzed. The reasons for doing this are recounted, including a review of renormalization-group techniques and their application to scaling phenomena. The renormalization-group equations are derived for Yang-Mills theories. The parameters that enter into the equations are calculated to lowest order and it is shown that these theories are asymptotically free. More specifically the effective coupling constant, which determines the ultraviolet behavior of the theory, vanishes for large spacelike momenta. Fermions are incorporated and the construction of realistic models is discussed. We propose that the strong interactions be mediated by a "color" gauge group which commutes with SU(3) × SU(3). The problem of symmetry breaking is discussed. It appears likely that this would have a dynamical origin. It is suggested that the gauge symmetry might not be broken and that the severe infrared singularities prevent the occurrence of noncolor singlet physical states. The deep-inelastic structure functions, as well as the electron-positron total annihilation cross section are analyzed. Scaling obtains up to calculable logarithmic corrections, and the naive light-cone or parton-model results follow. The problems of incorporating scalar mesons and breaking the symmetry by the Higgs mechanism are explained in detail.

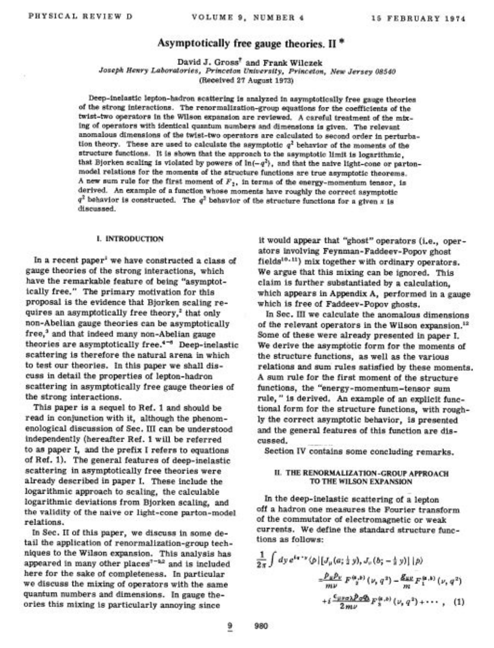
I. INTRODUCTION

In recent years the renormalization group has played an increasingly important role in the study of the asymptotic behavior of renormalizable field theories. This approach has acquired new importance due to the recent discovery that non-Abelian gauge theories are asymptotically free.[1-3] In this paper we shall amplify and extend the results reported in Ref. 1.

The renormalization group dates from the fundamental work of Gell-Mann and Low,[4] who studied the asymptotic behavior of the photon propagator in quantum electrodynamics. The remarkable discovery of Gell-Mann and Low was that the asymptotic form of the photon propagator was determined by the zeros of a certain, calculable, function of the coupling constant and not by the actual value of the charge. The renormalization-group equations were extended by Bogolubov and Shirkov to the vertex function[5] and employed to analyze the ultraviolet and infrared behavior of quantum electrodynamics and other field theories.[6,7] (For a review of this work see Ref. 8.)

The basic idea underlying the renormalization-group equations is very simple. A renormalizable field theory contains two types of parameters—masses or coupling constants with positive dimensions of mass (i.e., due to $M\bar{\psi}\psi$ or $\lambda\phi^3$ terms in the Lagrangian) and dimensionless coupling constants (i.e., due to $\lambda\phi^4$ or $\bar{\psi}\lambda^a\psi A_\mu$ terms in the Lagrangian). Coupling constants with negative dimensions of mass give nonrenormalizable theories. If one considers a Green's function for large and spacelike momenta (so as to exclude any Landau singularities), then one would expect that the generalized mass terms in the Lagrangian ($M\bar{\psi}\psi$ or $\lambda\phi^3$) could be neglected. In other words the leading asymptotic behavior of the Green's functions should be the same as would be calculated in a massless theory. This can be proved, to any finite order in perturbation theory, by using Weinberg's theorem.[9] The massless theory contains no dimensional parameters to set the scale of momenta, therefore one might expect that the asymptotic behavior of the amplitudes would be determined by pure dimensional analysis. This is called naive or cannonical scaling. It does not occur in practice, since the massless theory does contain a hidden dimensional parameter. This parameter, μ, must be introduced in order to perform the subtractions necessary to renormalize the theory and render it finite. Due to infrared singularities these subtractions, for the massless theory, must be performed off shell, say at some spacelike momenta $p^2=-\mu^2$. The subtractions then define the physical coupling constants and the scale of the fields (which are determined by the wave-function renormalization constants). The subtraction point, μ, is arbitrary. If we change the subtraction point the net effect is to change the value of the coupling constants and the scale of the

---



PHYSICAL REVIEW D VOLUME 9, NUMBER 4 15 FEBRUARY 1974

Asymptotically free gauge theories. II [*]

David J. Gross[†] and Frank Wilczek
Joseph Henry Laboratories, Princeton University, Princeton, New Jersey 08540

(Received 27 August 1973)

Deep-inelastic lepton-hadron scattering is analyzed in asymptotically free gauge theories of the strong interactions. The renormalization-group equations for the coefficients of the twist-two operators in the Wilson expansion are reviewed. A careful treatment of the mixing of operators with identical quantum numbers and dimensions is given. The relevant anomalous dimensions of the twist-two operators are calculated to second order in perturbation theory. These are used to calculate the asymptotic q^2 behavior of the moments of the structure functions. It is shown that the approach to the asymptotic limit is logarithmic, that Bjorken scaling is violated by powers of $\ln(-q^2)$, and that the naive light-cone or parton-model relations for the moments of the structure functions are true asymptotic theorems. A new sum rule for the first moment of F_2, in terms of the energy-momentum tensor, is derived. An example of a function whose moments have roughly the correct asymptotic q^2 behavior is constructed. The q^2 behavior of the structure functions for a given x is discussed.

I. INTRODUCTION

In a recent paper[1] we have constructed a class of gauge theories of the strong interactions, which have the remarkable feature of being "asymptotically free." The primary motivation for this proposal is the evidence that Bjorken scaling requires an asymptotically free theory,[2] that only non-Abelian gauge theories can be asymptotically free,[3] and that indeed many non-Abelian gauge theories are asymptotically free.[4-6] Deep-inelastic scattering is therefore the natural arena in which to test our theories. In this paper we shall discuss in detail the properties of lepton-hadron scattering in asymptotically free gauge theories of the strong interactions.

This paper is a sequel to Ref. 1 and should be read in conjunction with it, although the phenomenological discussion of Sec. III can be understood independently (hereafter Ref. 1 will be referred to as paper I, and the prefix I refers to equations of Ref. 1). The general features of deep-inelastic scattering in asymptotically free theories were already described in paper I. These include the logarithmic approach to scaling, the calculable logarithmic deviations from Bjorken scaling, and the validity of the naive or light-cone parton-model relations.

In Sec. II of this paper, we discuss in some detail the application of renormalization-group techniques to the Wilson expansion. This analysis has appeared in many other places[7-12] and is included here for the sake of completeness. In particular we discuss the mixing of operators with the same quantum numbers and dimensions. In gauge theories this mixing is particularly annoying since it would appear that "ghost" operators (i.e., operators involving Feynman-Faddeev-Popov ghost fields[10,11]) mix together with ordinary operators. We argue that this mixing can be ignored. This claim is further substantiated by a calculation, which appears in Appendix A, performed in a gauge which is free of Faddeev-Popov ghosts.

In Sec. III we calculate the anomalous dimensions of the relevant operators in the Wilson expansion.[12] Some of these were already presented in paper I. We derive the asymptotic form for the moments of the structure functions, as well as the various relations and sum rules satisfied by these moments. A sum rule for the first moment of the structure functions, the "energy-momentum-tensor sum rule," is derived. An example of an explicit functional form for the structure functions, with roughly the correct asymptotic behavior, is presented and the general features of this function are discussed.

Section IV contains some concluding remarks.

II. THE RENORMALIZATION-GROUP APPROACH TO THE WILSON EXPANSION

In the deep-inelastic scattering of a lepton off a hadron one measures the Fourier transform of the commutator of electromagnetic or weak currents. We define the standard structure functions as follows:

$$\begin{aligned}\frac{1}{2\pi}\int dy\, e^{iq\cdot y}\langle p|[J_\mu(a;\tfrac{1}{2}y),J_\nu(b;-\tfrac{1}{2}y)]|p\rangle &= \frac{p_\mu p_\nu}{m\nu}F_2^{(a,b)}(\nu,q^2)-\frac{g_{\mu\nu}}{m}F_1^{(a,b)}(\nu,q^2)\\ &\quad+i\frac{\epsilon_{\mu\nu\alpha\lambda}p_\alpha q_\lambda}{2m\nu}F_3^{(a,b)}(\nu,q^2)+\cdots, \qquad (1)\end{aligned}$$

ASYMPTOTIC FREEDOM: AN APPROACH TO STRONG INTERACTIONS[*]

H. David POLITZER
Lyman Laboratory, Harvard University, Cambridge, Mass. 02138, USA

Received 17 June 1974

Contents:

Asymptotic Freedom and the Renormalization Group

Asymptotically Free Theories of the Strong Interactions

Abstract:

The discovery of asymptotic freedom has opened up the possibility of extracting new sorts of detailed, dynamical consequences from a strongly interacting quantum field theory. The necessary tools – perturbation theory, the renormalization group, gauge theories, and the operator product expansion – are not new. To anyone familiar with these field theoretic approaches to strong interactions, the novel feature is a simple fact: there is a unique class of theories in which "the origin is an ultraviolet fixed point". But the consequences are so exciting that it seemed appropriate to review these ideas as they reflect on each other. Many important applications of the renormalization group and the operator product expansion to hadronic physics are omitted; the emphasis here is on recent work based on asymptotically free field theories. No doubt, there are some developments so recent that they are not treated in this article.

The discussion of the basic results concerning short distance behavior is informal, but, hopefully, accurate and complete. The specific applications are treated in varying detail.

Single orders for this issue

PHYSICS REPORTS (Section C of PHYSICS LETTERS) 14, No. 4 (1974) 129–180.

Copies of this issue may be obtained at the price given below. All orders should be sent directly to the Publisher. Orders must be accompanied by check.

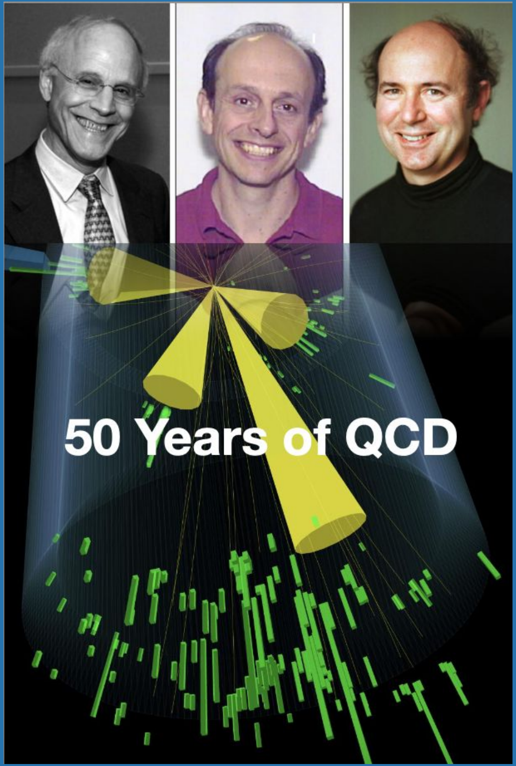
Single issue price Dfl. 20.–, postage included.

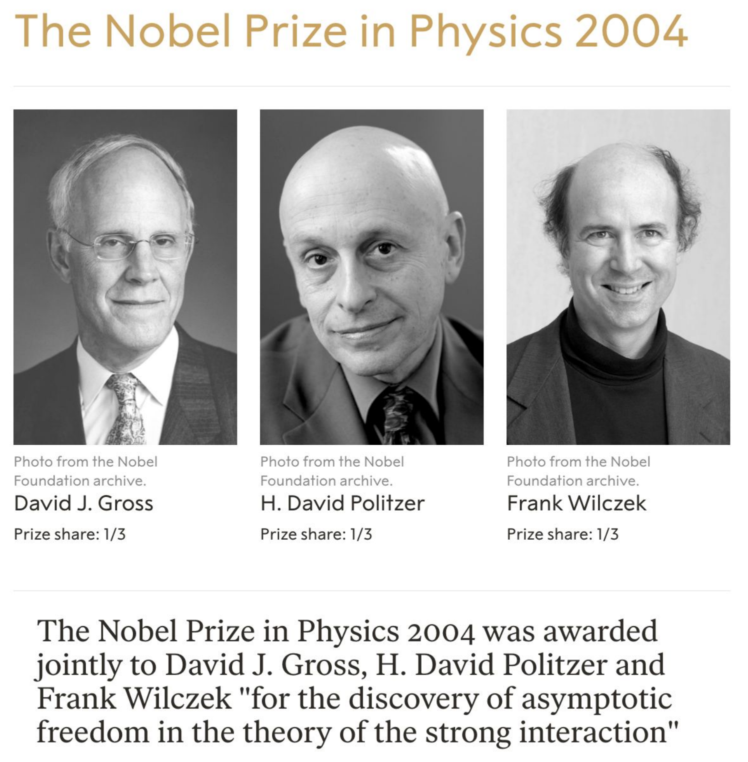
---

[*] Work supported in part by the National Science Foundation under Grant Number GP40397X.

We are delighted that many of the people from this period are here.

Nobel Prize in 2004





The Nobel Prize in Physics 2004

Photo from the Nobel Foundation archive.
David J. Gross
Prize share: 1/3

Photo from the Nobel Foundation archive.
H. David Politzer
Prize share: 1/3

Photo from the Nobel Foundation archive.
Frank Wilczek
Prize share: 1/3

The Nobel Prize in Physics 2004 was awarded jointly to David J. Gross, H. David Politzer and Frank Wilczek "for the discovery of asymptotic freedom in the theory of the strong interaction"

Conference venues

Monday, Tuesday: Luskin Centennial AB
Tuesday Bhaumik Lecture: Luskin Centennial CD

Wednesday, Thursday, Friday: Luskin Legacy room

Conference events

Monday

6:00 - 7:30 pm: conference reception

7:30 - 9:30 pm: conference banquet

dinner talk by Prof. Helen Quinn

Conference events

Tuesday

5:00 - 6:00 pm: reception

6:00 - 7:30 pm: Bhaumik public lecture by Prof. David Gross

UCLA Mani L. Bhaumik Institute for Theoretical Physics

Location: Luskin Conference Center
Centennial Ballroom AB
Date: Tuesday, September 12th, 2023
Time: 6PM

Speaker: Professor David Gross

Fifty Years of Quantum Chromodynamics (The Theory of The Strong Nuclear Force)

Quantum Chromodynamics is fifty years old this year. I shall discuss the past, present and future of this remarkable theory.

Prof. David J. Gross has been at the forefront of theoretical physics for almost 60 years. He is the Chancellor's Chair professor of theoretical physics and the former director of the Kavli Institute for Theoretical Physics at the University of California, Santa Barbara. He received his Ph.D. in 1966 from the University of California, Berkeley. Before joining the Kavli Institute, he was the Thomas Jones professor of mathematical physics at Princeton University. Gross was awarded the 2004 Nobel Prize in Physics, along with H. David Politzer and Frank Wilczek, "for the discovery of asymptotic freedom in the theory of the strong interaction." His other awards include the Sakurai Prize, a MacArthur fellowship, the Dirac Medal, the Oskar Klein Medal, the Harvey Prize, the High Energy and Particle Physics Prize of the European Physical Society, and the Grande Médaille d'Or of the French Academy of Sciences. He is a member of the National Academy of Sciences, the American Academy of Arts and Sciences, the American Philosophical Society, the Indian Academy of Sciences and the Chinese Academy of Sciences. In 2020, he became Past President of the American Physical Society.

Talk logistics

Send slides to **qcd50@googlegroups.com** well in advance

Talk: 50 minutes = 40’ + 10’

The conference is hybrid with zoom component

All talks are recorded

Lunch break

Lunch: UCLA Ackerman Union (food court) is right next to Luskin conference (across street, less than 5 minutes walk)

Dinner: many restaurants in Westwood (within walking distance)

Looking forward to
an exciting and productive conference!

UCLA **Mani L. Bhaumik**
Institute for Theoretical Physics