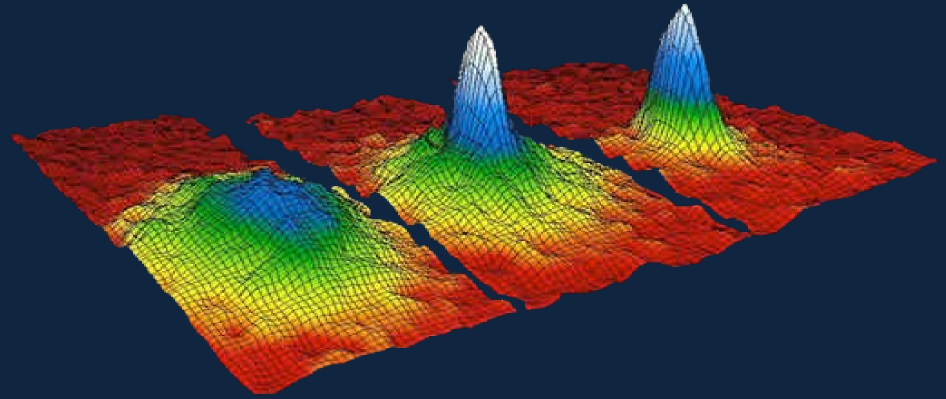
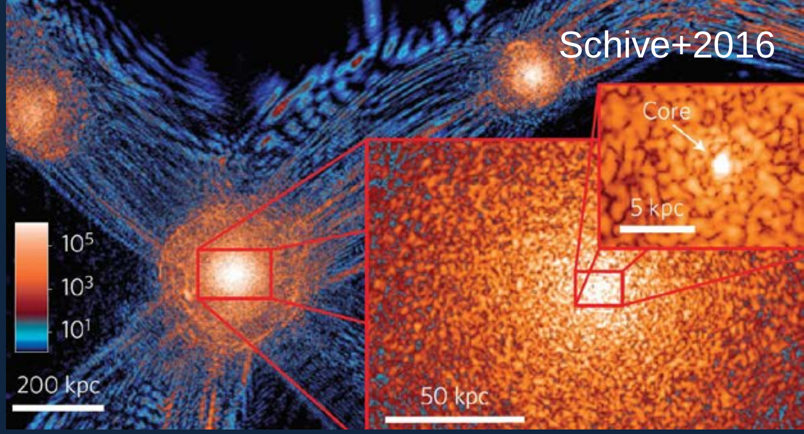
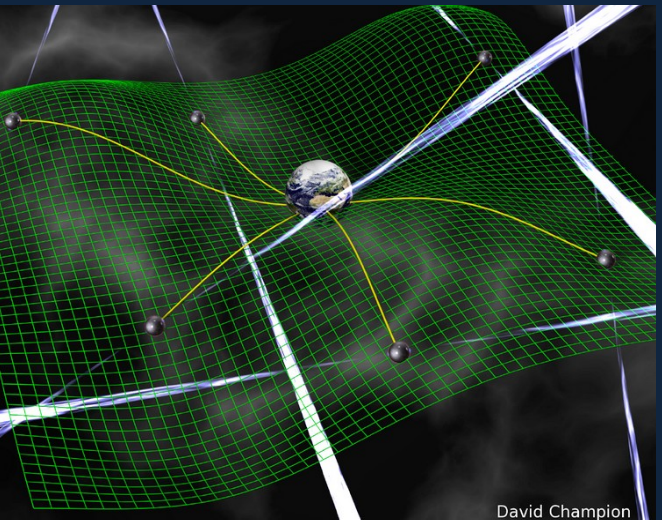




Cosmology in a Universe with Complex Scalar-Field Dark Matter

PAUL R. SHAPIRO[1], BOHUA LI[1,2] ,TANJA RINDLER-DALLER[3]

[1] The University of Texas at Austin [2] Tsinghua University, Beijing [3] University of Vienna



2017, PRD 96, 063505 (ARXIV: 1611.07961)

2014, PRD 89, 083536 (ARXIV: 1310.6061)

DARK MATTER 2018, UCLA, Los Angeles 2/22/2018

OUTLINE

- Dark Matter Candidate: Complex Scalar Field Dark Matter (SFDM, or BEC SFDM)
- ΛSFDM Cosmology: Observational Constraints
- Stochastic Gravitational-Wave Background (SGWB) from Inflation: Amplification in ΛSFDM
- Prediction of Detectability of the Inflationary SGWB by Advanced LIGO/Virgo

Complex Scalar Field Dark Matter (SFDM), aka Bose-Einstein Condensed Cold Dark Matter (BEC-CDM)

- Alternative to WIMP CDM
- Ultralight bosons (m $\gtrsim 10^{-22}$ eV/c^2)
- **Complex** scalar field: $\psi = |\psi| e^{i\theta}$
- Global U(1) symmetry $\Leftrightarrow$ conserved particle number

$$\rho_{SFDM,0} = n_{SFDM,0} mc^2 = \Omega_{DM} \rho_{crit,0}$$

- Particles created with low entropy per particle $\Longrightarrow$ BEC
- Add repulsive self-interaction: $V_{SI} = \lambda |\psi|^4 / 2$
- Small-scale structure suppressed for L < L_{SFDM}:

$$L_{SFDM} = \max \{ \lambda_{deBroglie}, l_{SI} \}$$

λ_{deB} $\longleftrightarrow$ quantum pressure

l_{SI} $\longleftrightarrow$ self-interaction

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➔ complex SFDM is *asymmetric* dark matter

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SFDM has 3 phases

Einstein + Klein-Gordon $\Rightarrow$

$$(p/\rho)_{SFDM} = w(t)$$

(1) Late: w=0
(Non-relativistic matter)

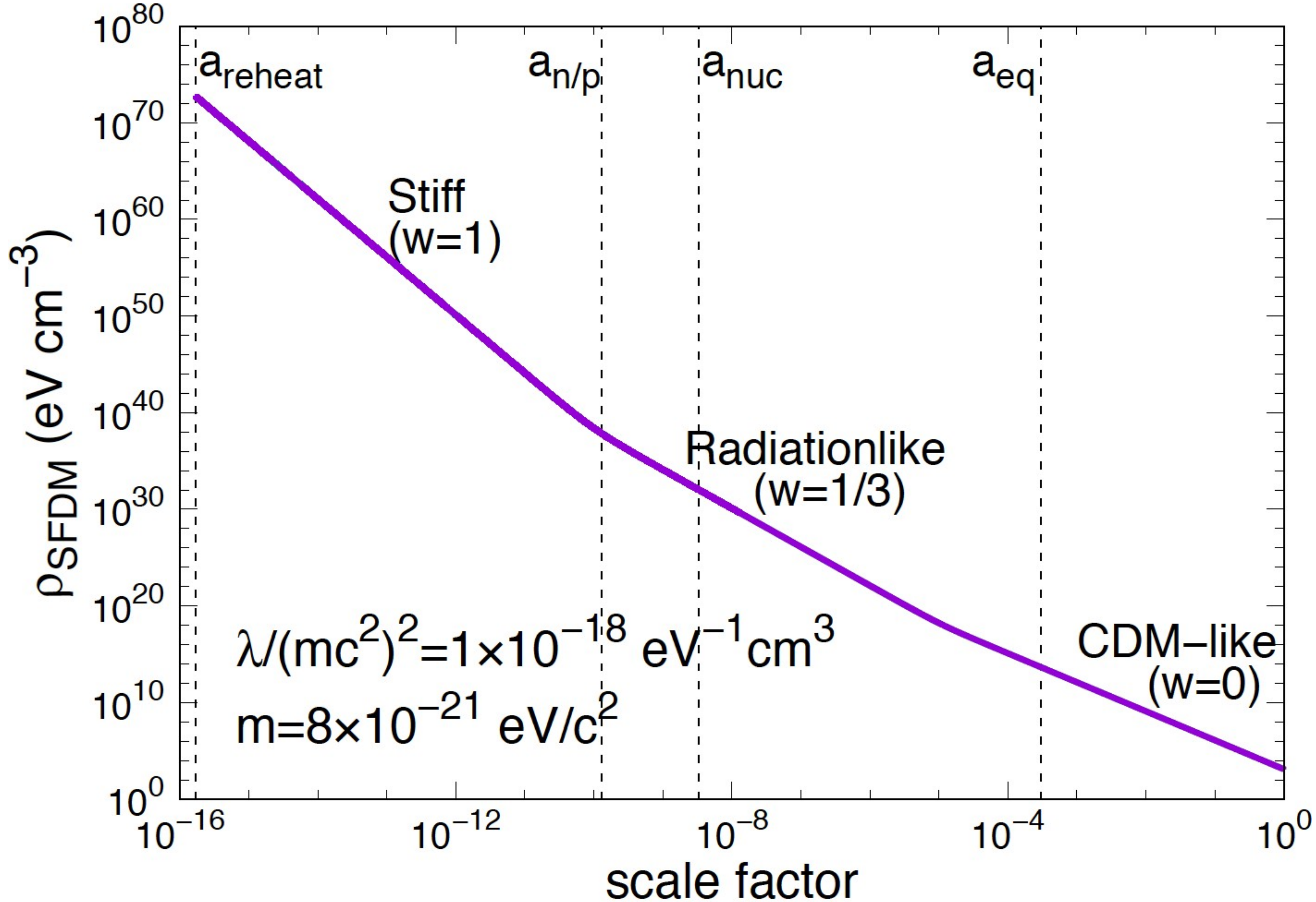
(2) Intermediate: w=1/3
(Radiationlike)

(3) Early: w=1
(Stiff)

(1) + (2) $\Rightarrow$ Just like ΛCDM

but **(3)** $\Rightarrow$ $\Omega_{SFDM} \to 1$ as $a \to 0$

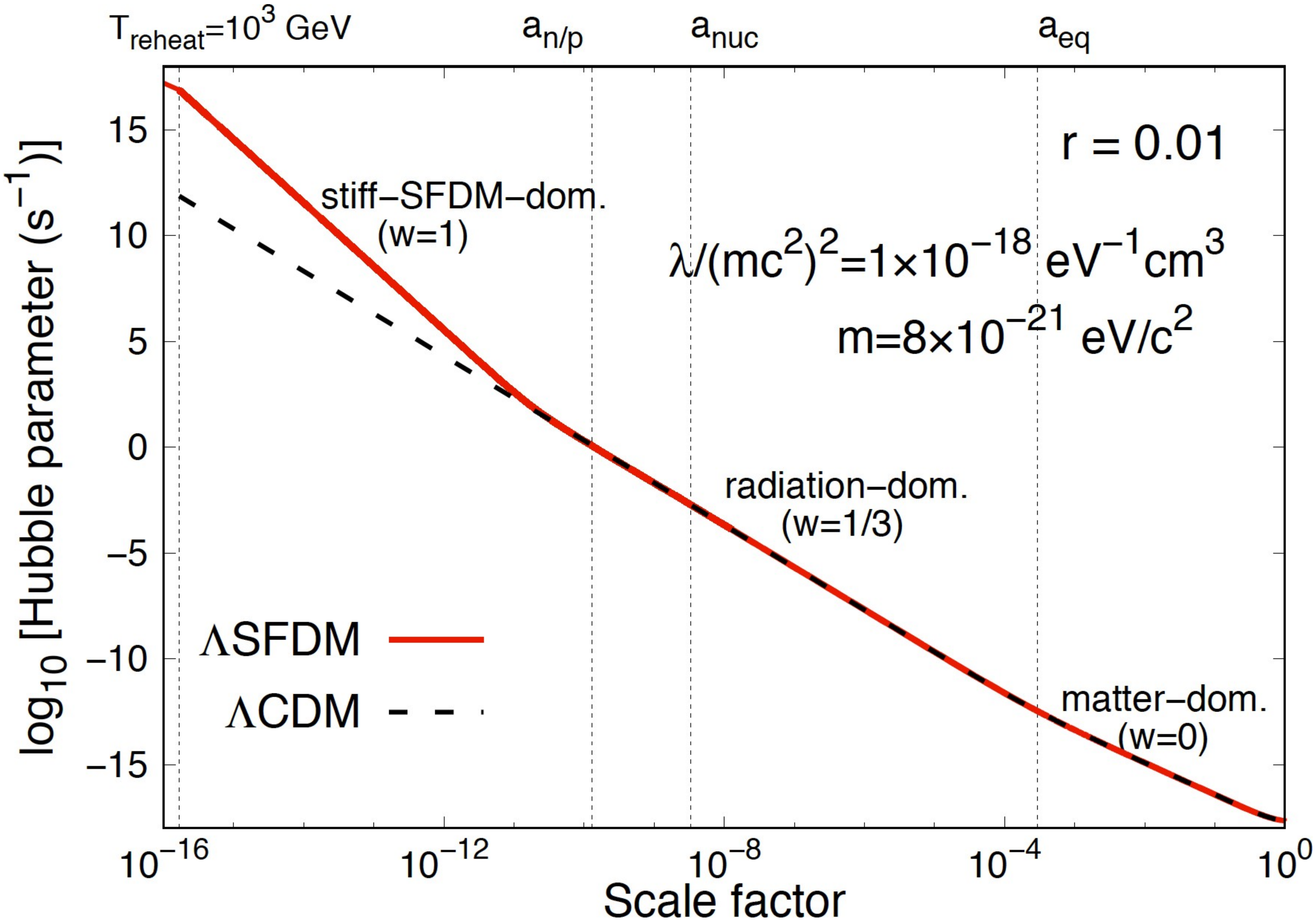
$\Rightarrow$ Stiff-SFDM-dominated early Universe



ΛSFDM: the Universe has 6 eras

Inflation ➔ **Reheating** ➔ **Stiff-SFDM-dom.** ➔ **Radiation-dom.** ➔ **Matter-dom.** ➔ **Λ-dom.**

(w= -1) **(w=0)** **(w=1)** **(w=1/3)** **(w=0)** **(w= -1)**



ΛSFDM Model Parameters

- SFDM particle parameters: m, $\lambda/(mc^2)^2$

$$\lambda/(mc^2)^2 = 1\times10^{-18}\,\mathrm{eV}^{-1}\mathrm{cm}^3 \quad \Longrightarrow \quad l_{SI} \approx 0.8kpc$$

$$\mathcal{L} = \frac{\hbar^2}{2m} g^{\mu\nu}\partial_\mu\psi^*\partial_\nu\psi - \frac{1}{2}mc^2|\psi|^2 - \frac{\lambda}{2}|\psi|^4,$$

- Global U(1) symmetry ⟹ Charge (particle number density) conservation

$$Q \equiv n - \bar{n} = \rho_{SFDM,0}/(mc^2) \qquad (\bar{n} = 0)$$

- BEC ⟹ Classical field description

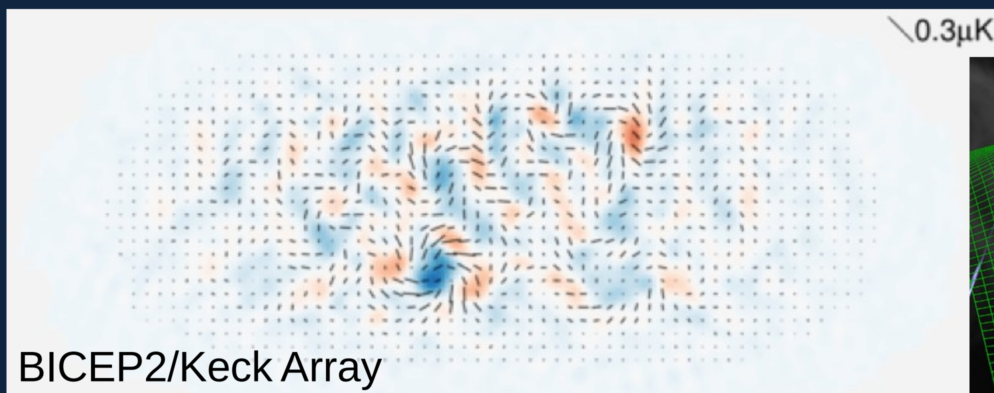
- Tensor-to-scalar ratio: r $= A_T/A_S$

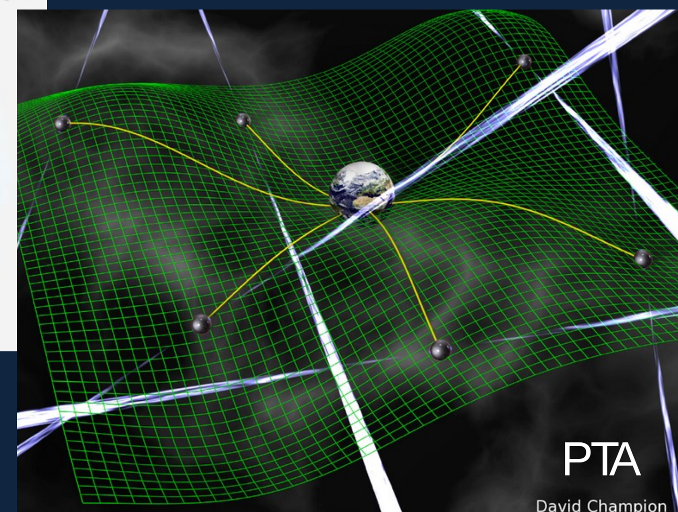
$$H_{\inf} = \frac{\pi M_{\mathrm{pl}}}{\hbar}\sqrt{rA_s}$$

inflationary paradigm

- Reheat temperature: T_{reheat}

Stochastic Gravitational-Wave Background from *Inflation*





Single-field slow-roll inflation

- $r > 0.001$
- Consistency relation $n_t = -r/8$

Subhorizon inflationary SGWB energy density spectrum:

$$\Omega_{GW}(k,a) = \frac{\Delta^2_{h,init}(k)}{12}\left(\frac{kc}{aH}\right)^2 T_h(k,a), \qquad \Delta^2_{h,init}(k) = A_T(k/k_*)^{n_t}$$

Holistic Evolution of the ΛSFDM Universe

- Friedmann equation

$$H^2(t) \equiv \left(\frac{\mathrm{d}a/\mathrm{d}t}{a}\right)^2 = \begin{cases} H_{\mathrm{inf}}^2, & a < a_{\mathrm{inf}}, \\ H_{\mathrm{inf}}^2\left(\frac{a_{\mathrm{inf}}}{a(t)}\right)^3, & a_{\mathrm{inf}} < a < a_{\mathrm{reheat}}, \\ \frac{8\pi G}{3c^2}\left[\rho_r(t) + \rho_b(t) + \rho_\Lambda(t) + \rho_{\mathrm{SFDM}}(t) + \rho_{\mathrm{GW}}(t)\right], & a > a_{\mathrm{reheat}}, \end{cases}$$

SGWB contribution to the expansion history *self-consistently* included

$$\Omega_{\mathrm{GW}}(k,a) \equiv \frac{\mathrm{d}\Omega_{\mathrm{GW}}(a)}{\mathrm{d}\ln k} = \frac{1}{\rho_{\mathrm{crit}}(a)}\frac{\mathrm{d}\rho_{\mathrm{GW}}(a)}{\mathrm{d}\ln k}$$
$$= \frac{\Delta_h^2(k,a)c^2}{24a^2H^2(a)}\left(\left|\frac{h_k'(a(\tau))}{h_k(a(\tau))}\right|^2 + k^2\right)$$

conformal time: $d\tau \equiv dt\,/\,a(t)$

- Klein-Gordon Equation

$$\frac{\hbar^2}{2mc^2}\ddot{\psi} + 3\frac{\hbar^2}{2mc^2}\frac{\dot{a}}{a}\dot{\psi} + \frac{1}{2}mc^2\psi + \lambda|\psi|^2\psi = 0,$$

ρ_{GW} (t): Tensor Mode Perturbations in the ΛSFDM Universe

Tensor mode equation of motion in Fourier space:

$$h_k''(\tau) + 2\frac{a'(\tau)}{a(\tau)}h_k'(\tau) + k^2 h_k(\tau) = 0$$

GW spectrum vs. k at scale factor a(t):

$$\Omega_{\rm GW}(k,a) \equiv \frac{{\rm d}\Omega_{\rm GW}(a)}{{\rm d}\ln k} = \frac{1}{\rho_{\rm crit}(a)}\frac{{\rm d}\rho_{\rm GW}(a)}{{\rm d}\ln k} = \frac{\Delta_h^2(k,a)c^2}{24a^2H^2(a)}\left(\left|\frac{h_k'(a(\tau))}{h_k(a(\tau))}\right|^2 + k^2\right)$$

conformal time: $d\tau \equiv dt\,/\,a(t)$

- In subhorizon limit, different modes contribute to ρ_{GW} (t) according to the expansion phase during which they re-entered the horizon, how many e-foldings elapse in each phase since horizon crossing, and the initial power spectrum: $\Delta_{h,\rm init}^2(k) \simeq k^0$

w = 0 (reheating era) ←→ $\Omega_{\rm GW}^{\rm m}(k,\tau) \simeq \frac{\Delta_{h,\rm init}^2(k)}{24}\cdot\frac{9}{4}\frac{1}{(k\tau)^2},$ Red tilt

w = 1 (stiff-SFDM-dominated) era ←→ $\Omega_{\rm GW}^{\rm stiff}(k,\tau) \simeq \frac{\Delta_{h,\rm init}^2(k)}{24}\cdot\frac{8}{\pi}k\tau,$ Blue tilt

w = 1/3 (radiation-dominated era) ←→ $\Omega_{\rm GW}^{\rm rad}(k,\tau) \simeq \frac{\Delta_{h,\rm init}^2(k)}{24}.$

ρ_{GW} (t): Tensor Mode Perturbations in the ΛSFDM Universe

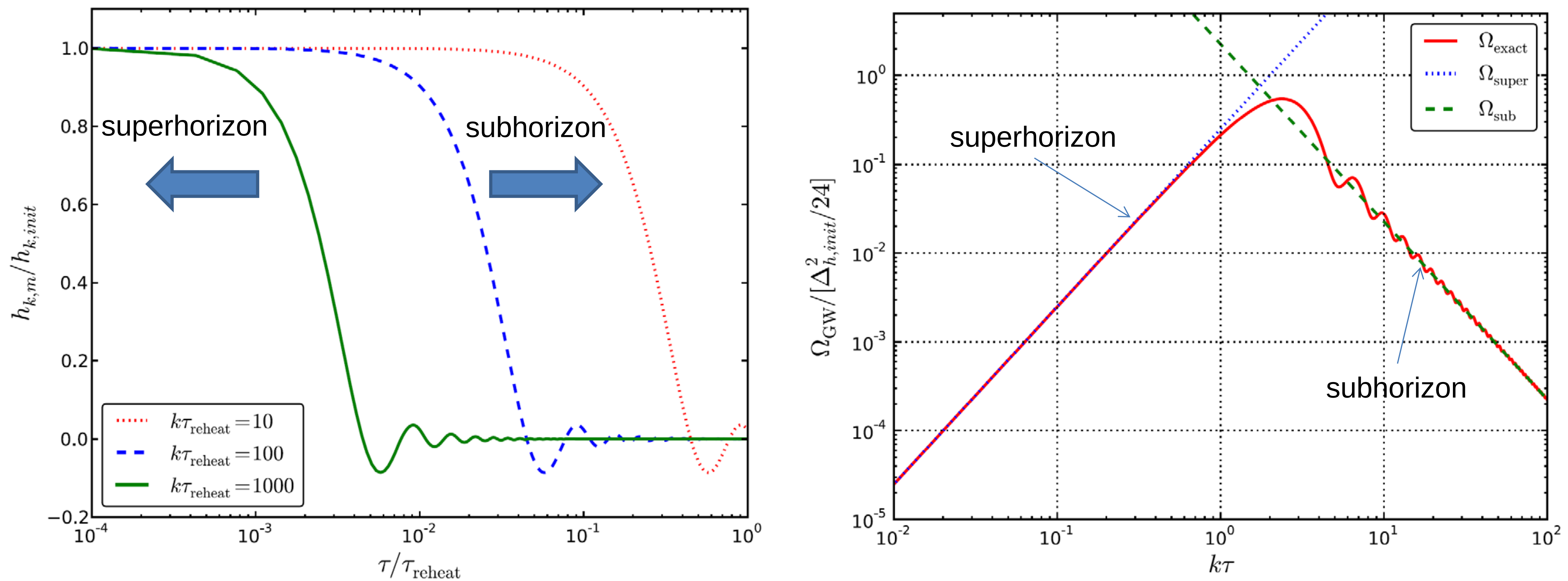
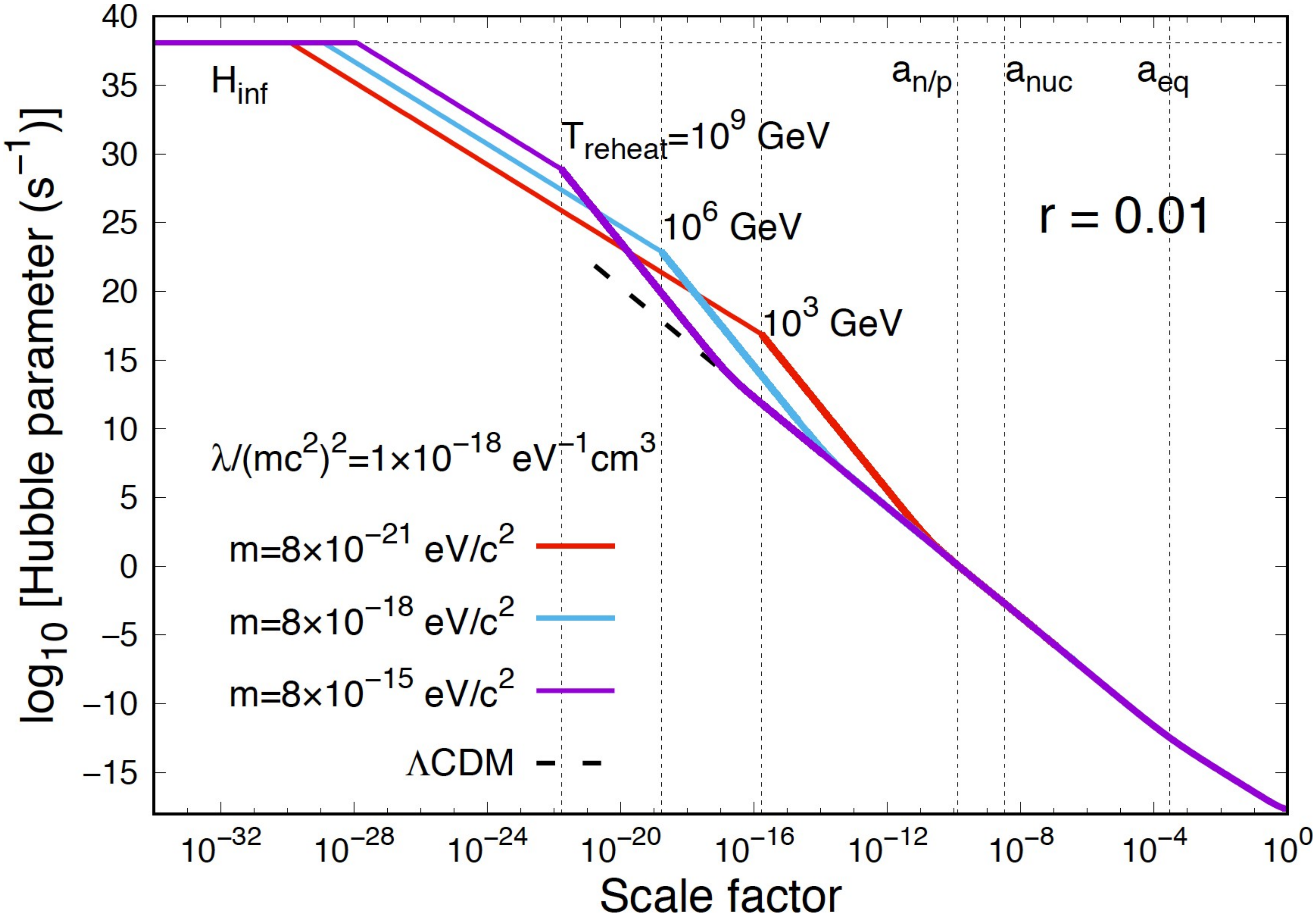


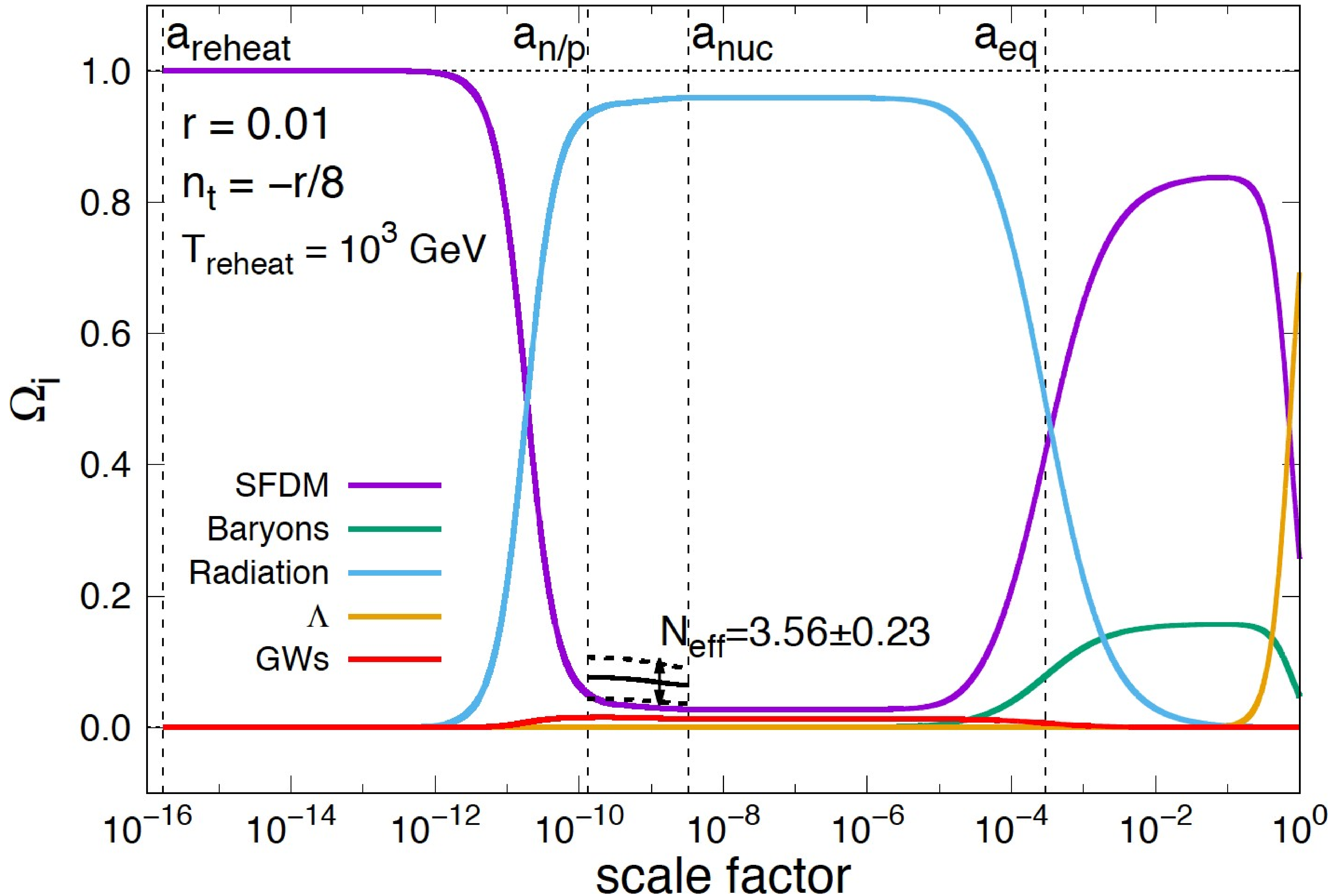
FIG. 13. Left: Tensor perturbations for different k modes, as they reenter the horizon during reheating (with $w = 0$) at different times. At $\tau/\tau_{\rm reheat} = 1$, the reheating era gives rise to the stiff era. The tensor modes (strains) are normalized over their initial amplitude $h_{k,\rm init}$ for each k. Right: The exact solution for $\Omega_{\rm GW}(k,\tau)$ as a function of $k\tau$ (solid curve), as well as the respective asymptotic expressions (with the superhorizon and subhorizon as dotted and dashed lines, respectively), for a reheating era with $w = 0$. $\Omega_{\rm GW}$ is normalized over $\Delta^2_{h,\rm init}/24$.

Example: Tensor modes of different *k* that re-enter horizon during the reheating era : w = 0

Holistic Evolution of the ΛSFDM Universe



Holistic Evolution of the ΛSFDM Universe



Cosmological Constraints on the SFDM Particle Parameters

- Matter-radiation equality: z_{eq}

$$1 + z_{\text{eq}} \equiv \frac{1}{a_{\text{eq}}} = \frac{\Omega_b h^2 + \Omega_c h^2}{\Omega_r h^2 + \Omega_{\text{GW}} h^2},$$

- Effective number of neutrino species at BBN: N_{eff}

$$\frac{\Delta N_{\text{eff,BBN}}(a)}{N_{\text{eff,standard}}} = \frac{\Omega_{\text{SFDM}}(a) + \Omega_{\text{GW}}(a)}{\Omega_\nu(a)},$$

Bohua Li, Tanja Rindler-Daller, Paul R. Shapiro 2014, PRD, 89, 083536
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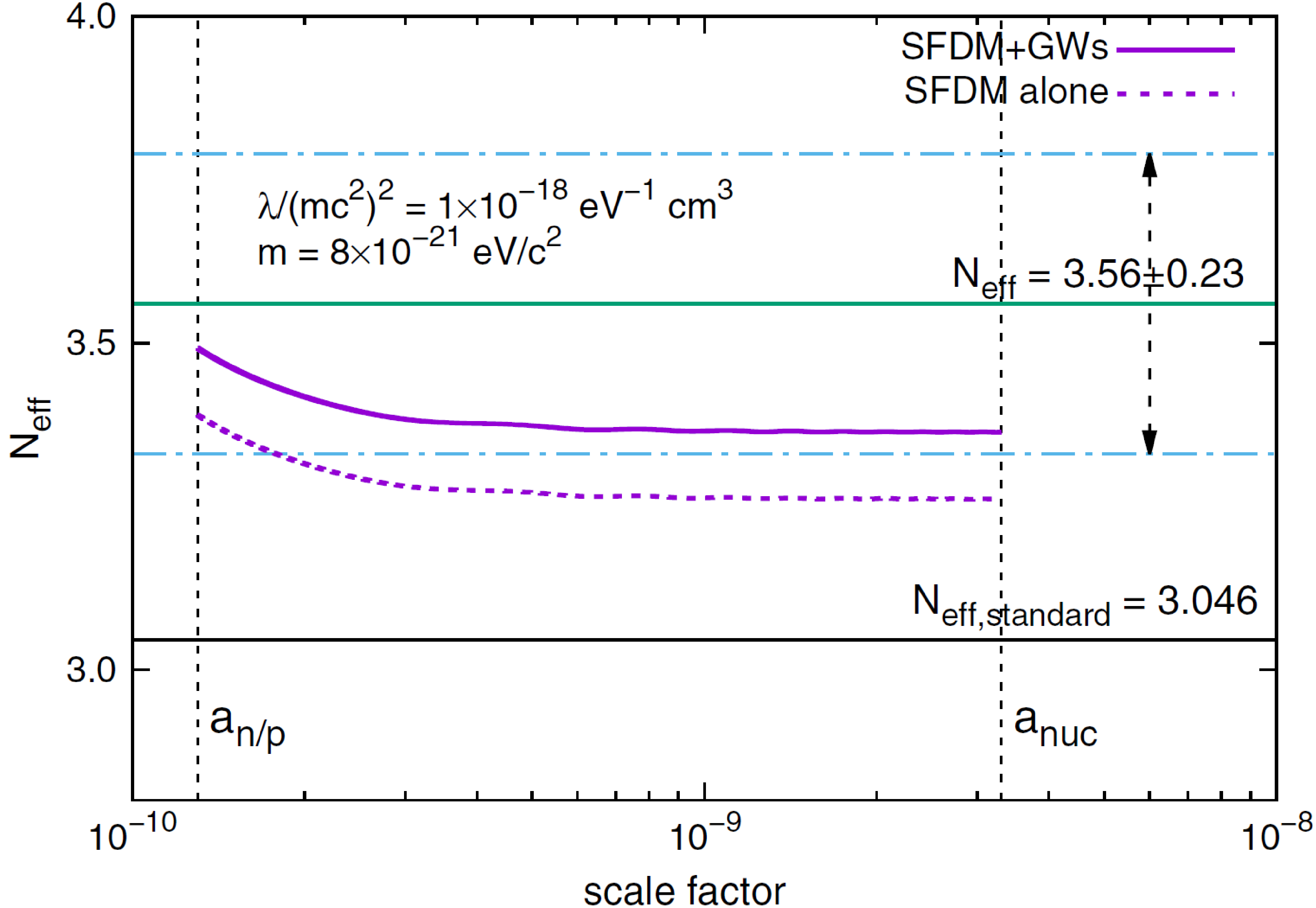

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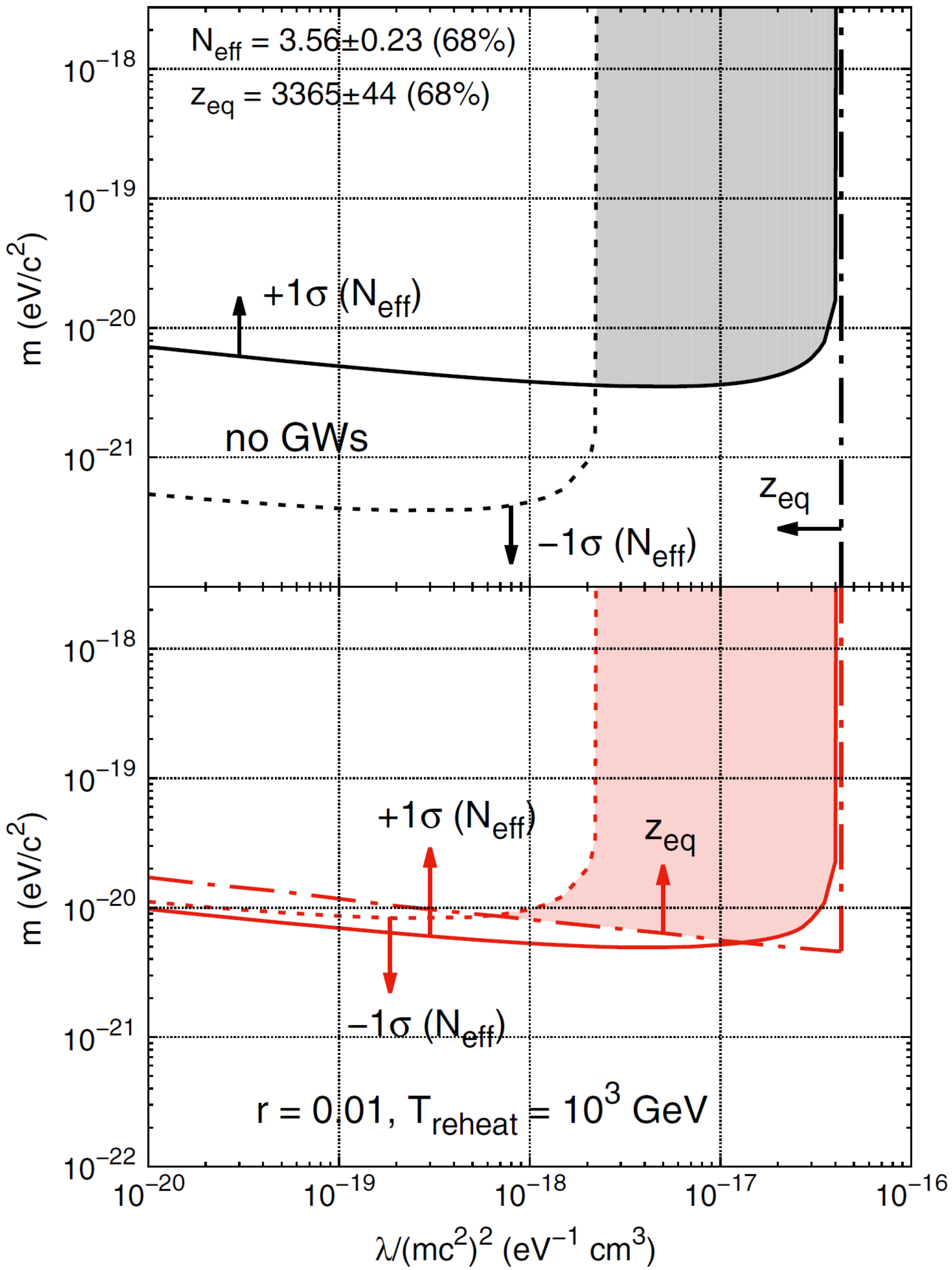

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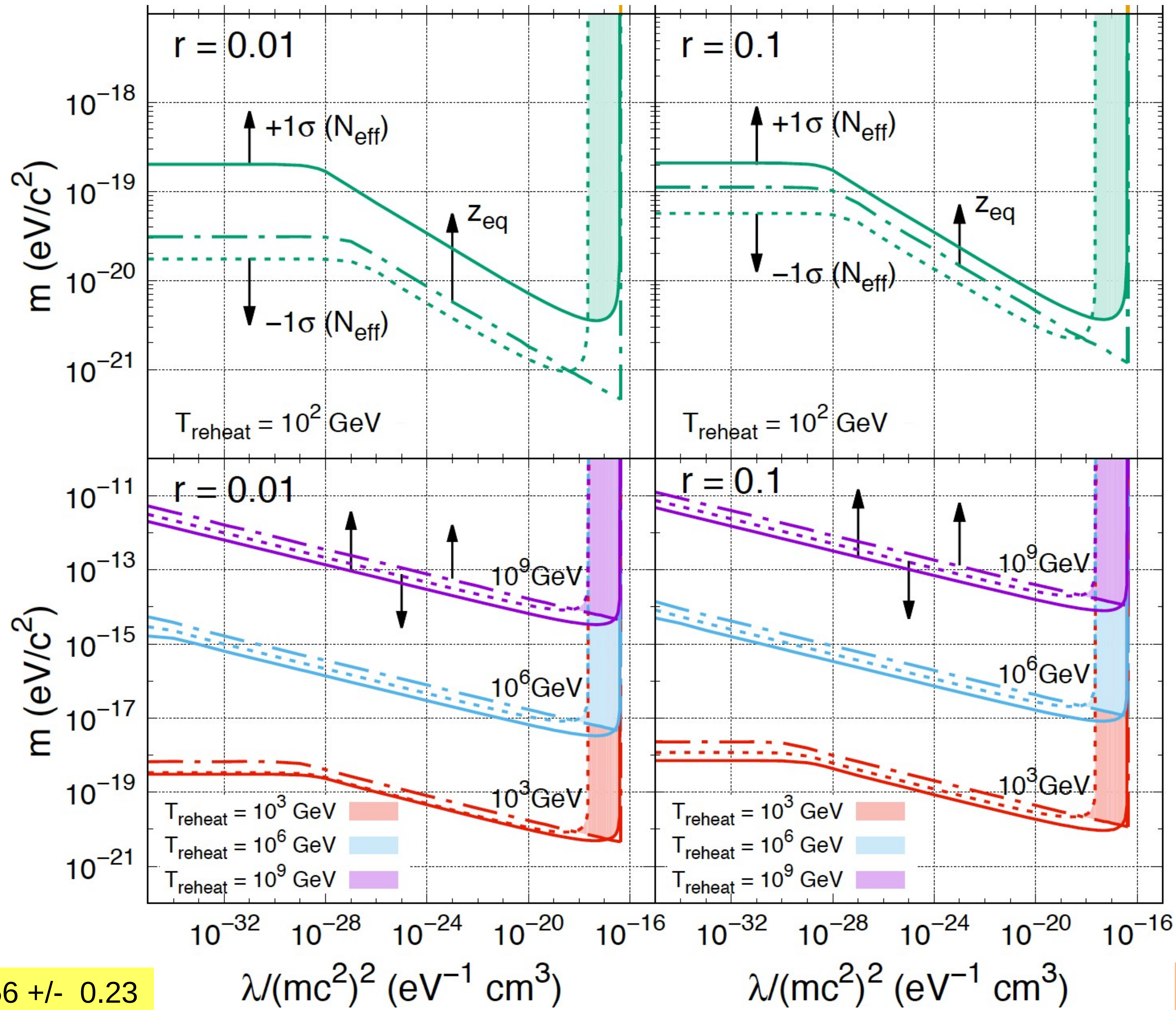
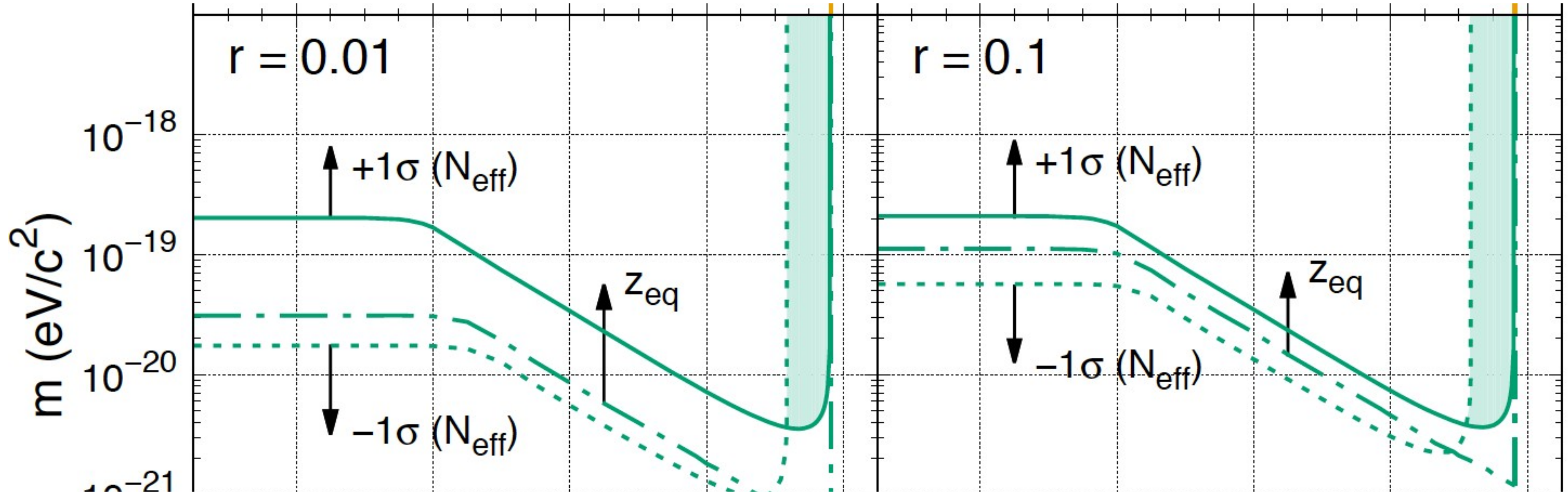


Neff, BBN = 3.56 +/- 0.23 (68% C.L.)

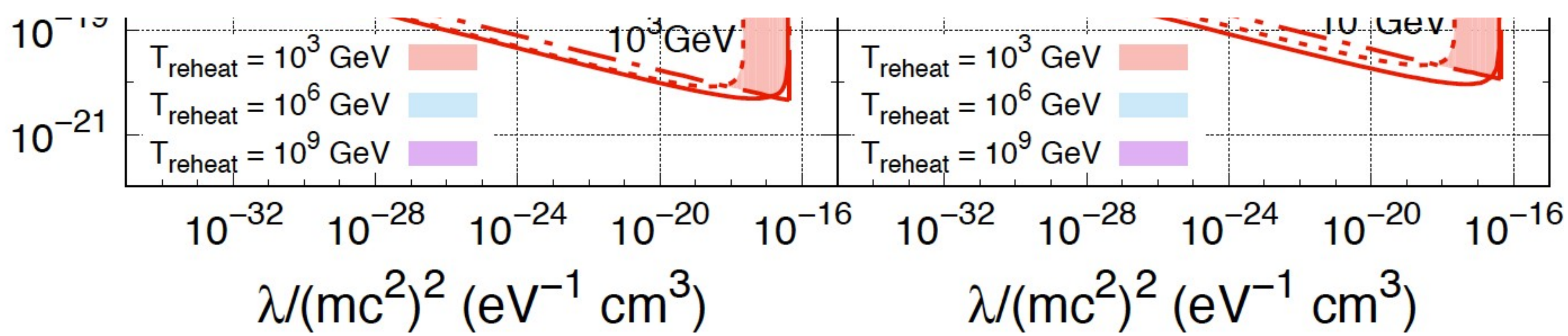
Zeq = 3365 +/- 44 (68% C.L.)

Cosmological Constraints on the SFDM Particle Parameters



$$2.3\times10^{-18}\ \mathrm{eV}^{-1}\ \mathrm{cm}^3 \leq \frac{\lambda}{(\mathrm{mc}^2)^2} \leq 4.1\times10^{-17}\ \mathrm{eV}^{-1}\ \mathrm{cm}^3,$$

$$m_{\min} \simeq (5\times10^{-21}\ \mathrm{eV}/c^2)\times\begin{cases}\dfrac{T_{\mathrm{reheat}}}{10^3\ \mathrm{GeV}}\sqrt{\dfrac{r}{0.01}}, & T_{\mathrm{reheat}} \gtrsim 10^3\ \mathrm{GeV},\\[2ex] 1, & T_{\mathrm{reheat}} < 10^3\ \mathrm{GeV}.\end{cases}$$



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$$\frac{\Delta N_{\rm eff,BBN}(a)}{N_{\rm eff,standard}} = \frac{\Omega_{\rm SFDM}(a) + \Omega_{\rm GW}(a)}{\Omega_\nu(a)},$$

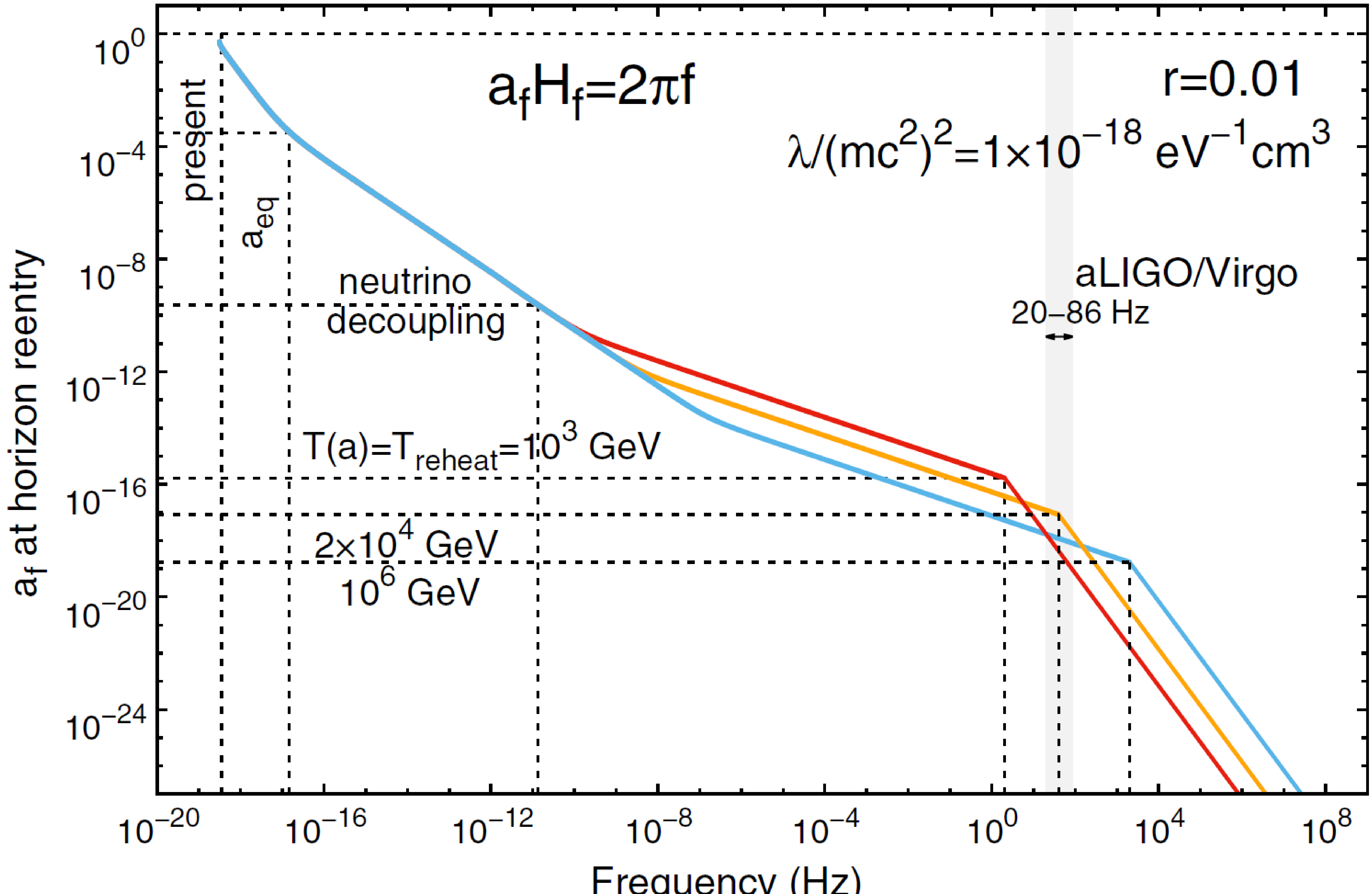
- SGWB measured by laser interferometers:

$\Omega_{GW}(f)$ at a=1

Stiff-SFDM-dominated era amplifies SGWB from inflation

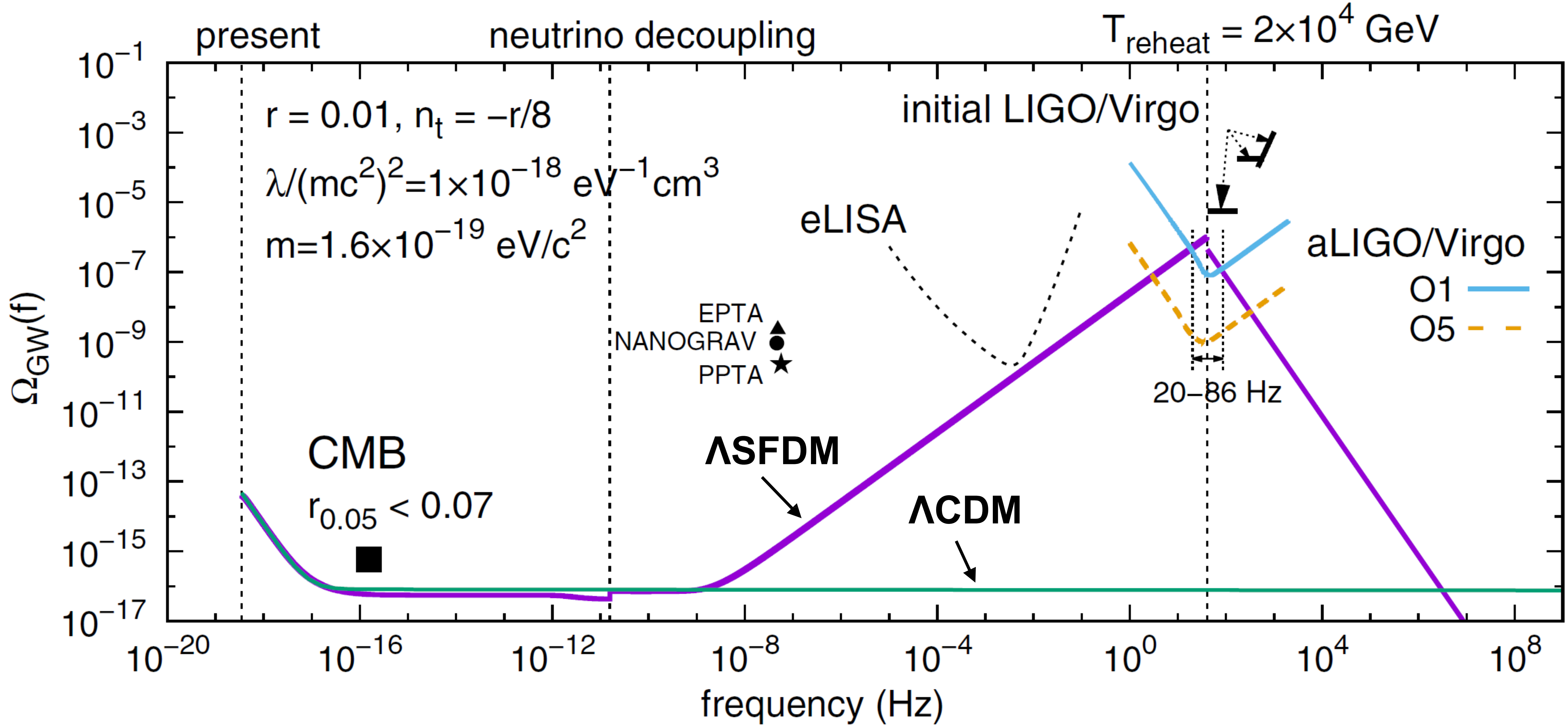
- LIGO can detect ΛSFDM-amplified inflationary SGWB for a range of SFDM parameters **(m, λ)** that satisfy cosmological constraints, for values of tensor-to-scalar ratio **r** currently allowed by CMB expmnt and a large range of reheat temperatures $\mathbf{T_{reheat}}$.
- For given **r** and $\mathbf{\lambda/(mc^2)^2}$, the marginally-allowed model for each $\mathbf{T_{reheat}}$ has the smallest m that satisfies cosmological constraints and maximizes the present energy density of the SGWB for that $\mathbf{T_{reheat}}$
- SGWB is then maximally ***detectable*** for $\mathbf{T_{reheat}}$ values for which modes that re-enter horizon when reheating ends have frequencies today inside LIGO sensitive band.
- GW experiments can already place a new kind of cosmological constraint on SFDM!

scale factor at horizon re-entry for modes of frequency f today



Stiff-SFDM-dominated era amplifies SGWB from inflation

Example 1 prediction for aLIGO/Virgo



ΛSFDM predicts 2-parameter broken power-law spectrum at high frequencies:

$$\Omega_{GW}(f) = \Omega_{GW,peak} \begin{cases} f / f_{peak}, & f \leq f_{peak} \\ (9\pi/64)(f/f_{peak})^{-2}, & f > f_{peak} \end{cases}$$

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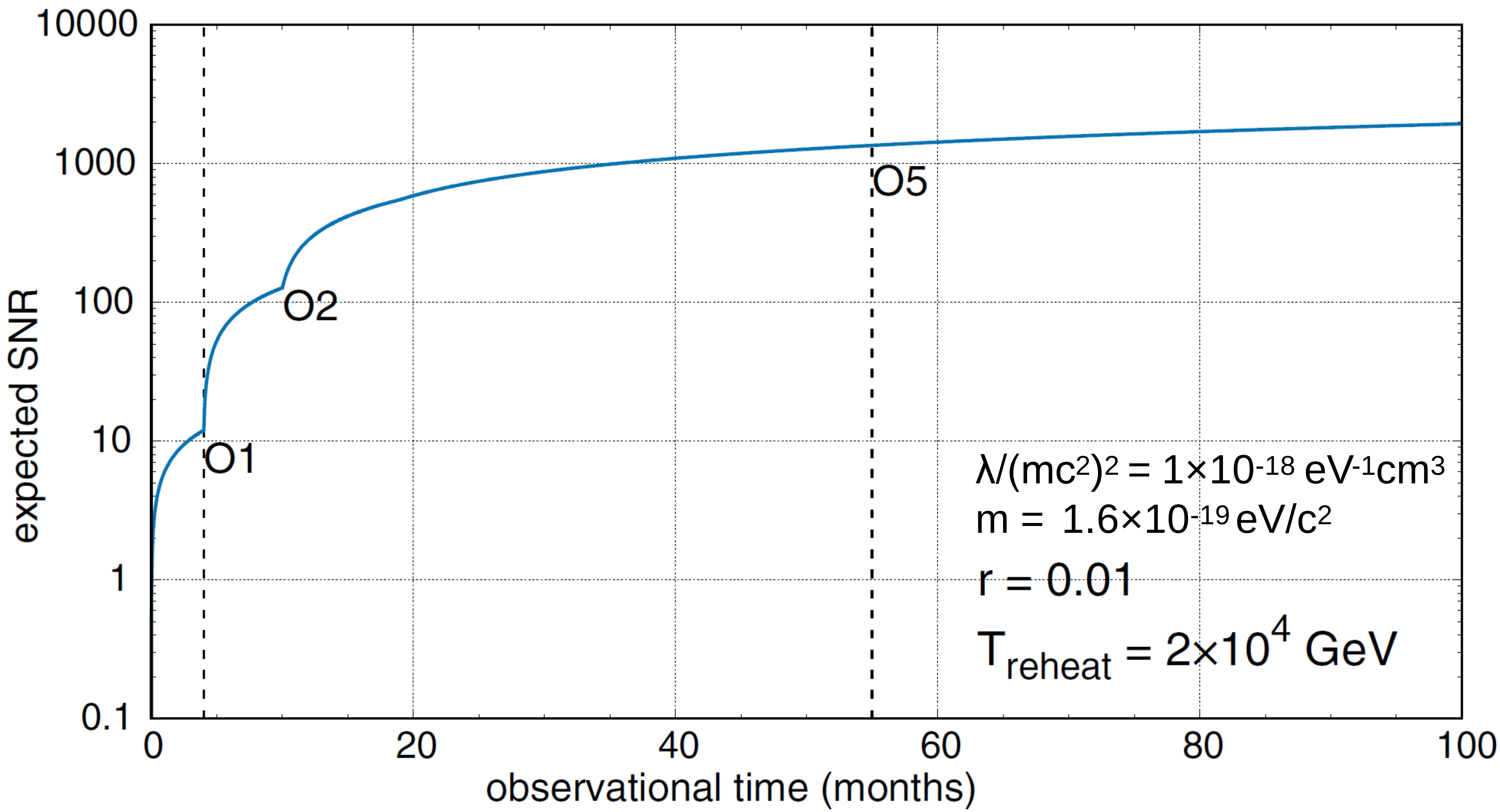

Spoiler alert! Upper limit from O1 data excludes this example case at 95% CL (1612.02029)

Stiff-SFDM-dominated era amplifies SGWB from inflation

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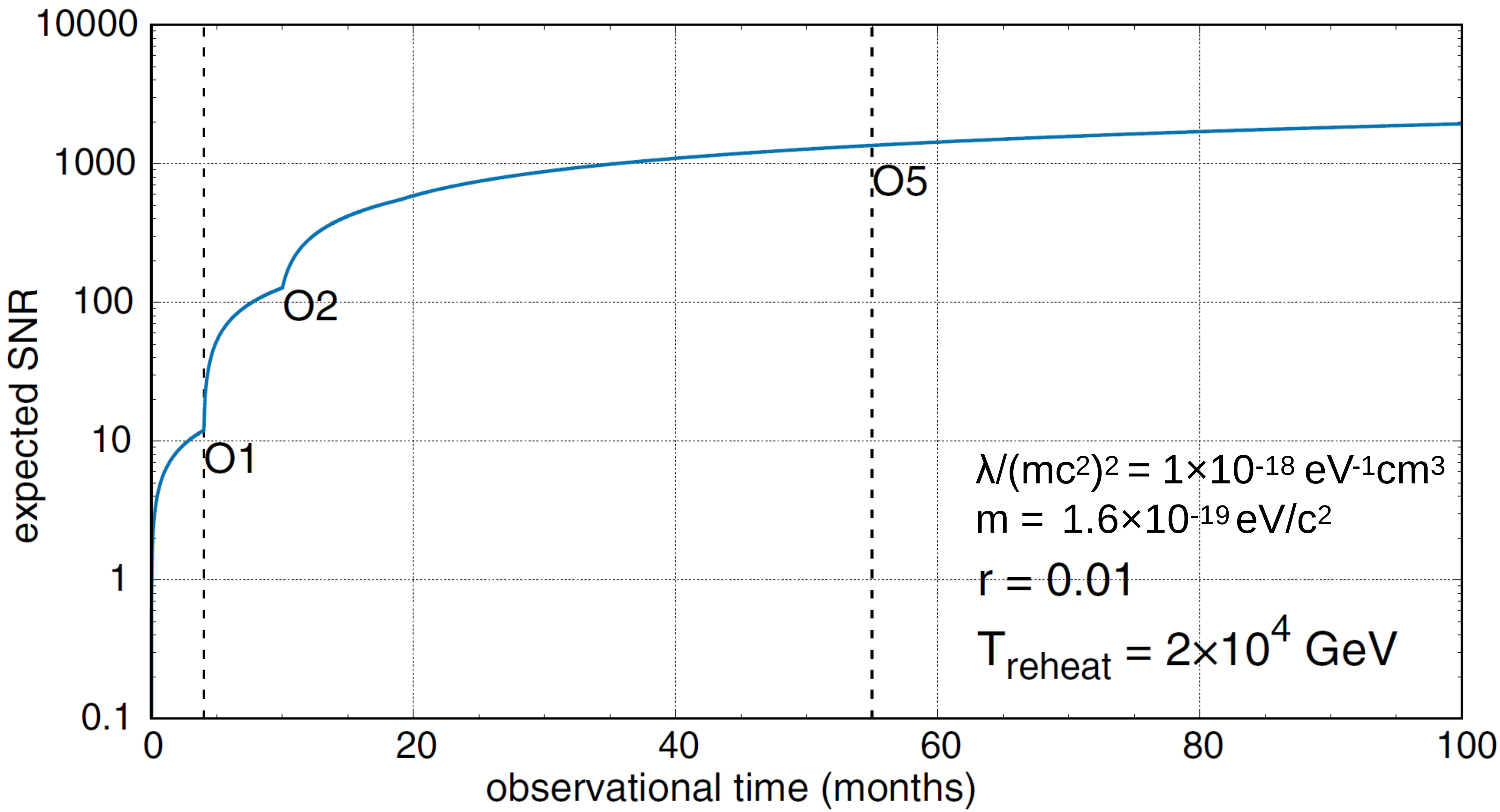

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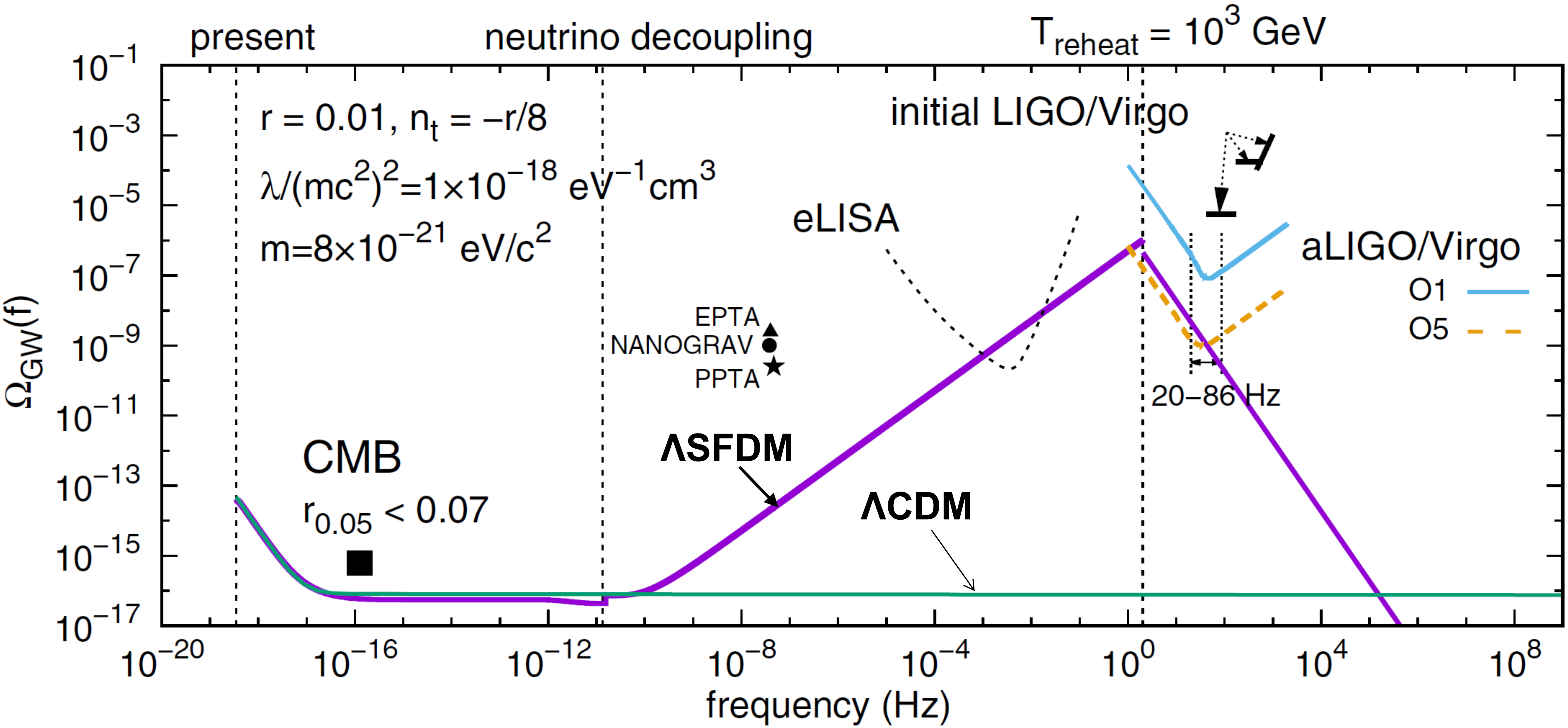
$\Longrightarrow$ The Age of Dark Matter Search by GW Detection has Begun!

Stiff-SFDM-dominated era amplifies SGWB from inflation

Example 2 prediction for aLIGO/Virgo



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Stiff-SFDM-dominated era amplifies SGWB from inflation

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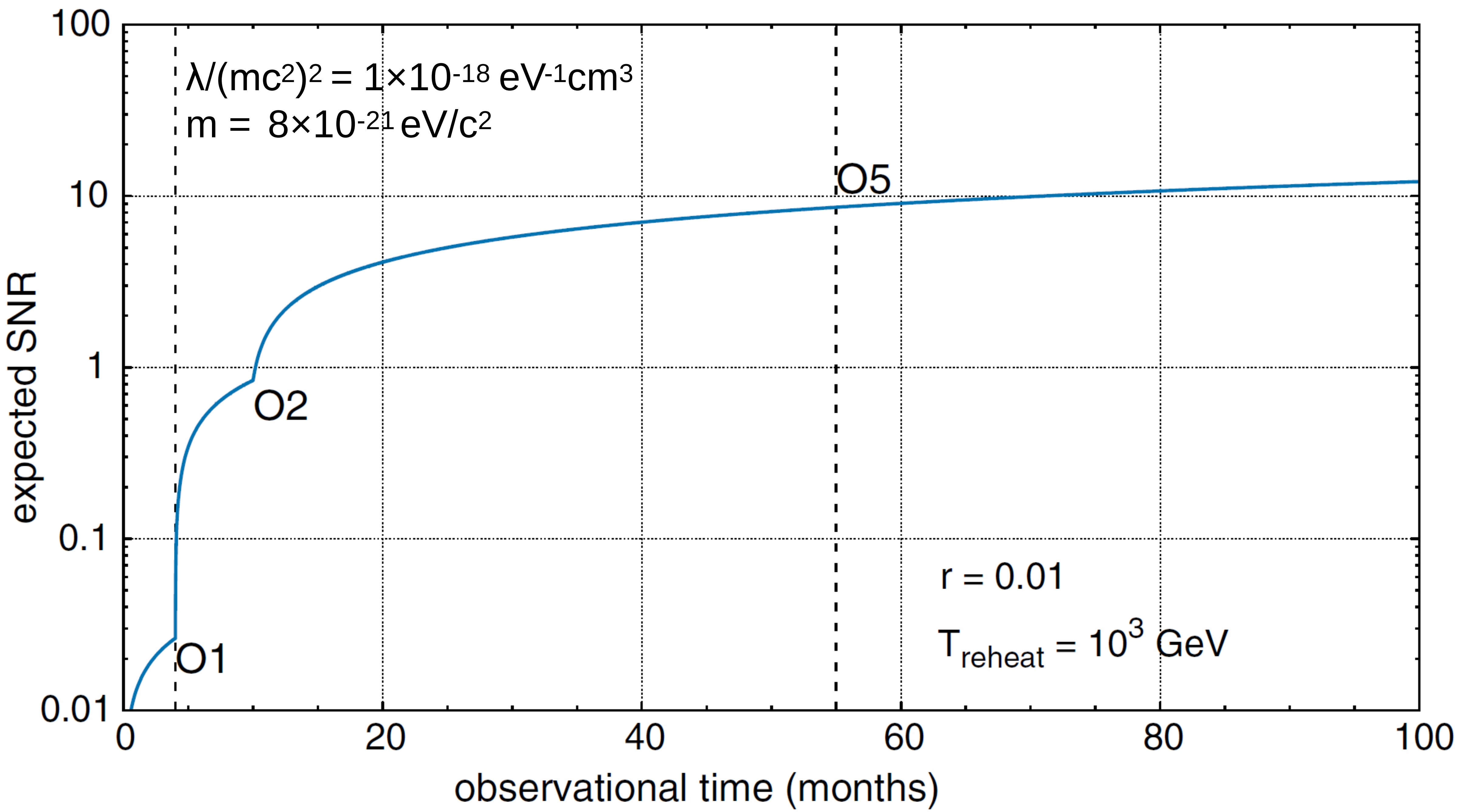
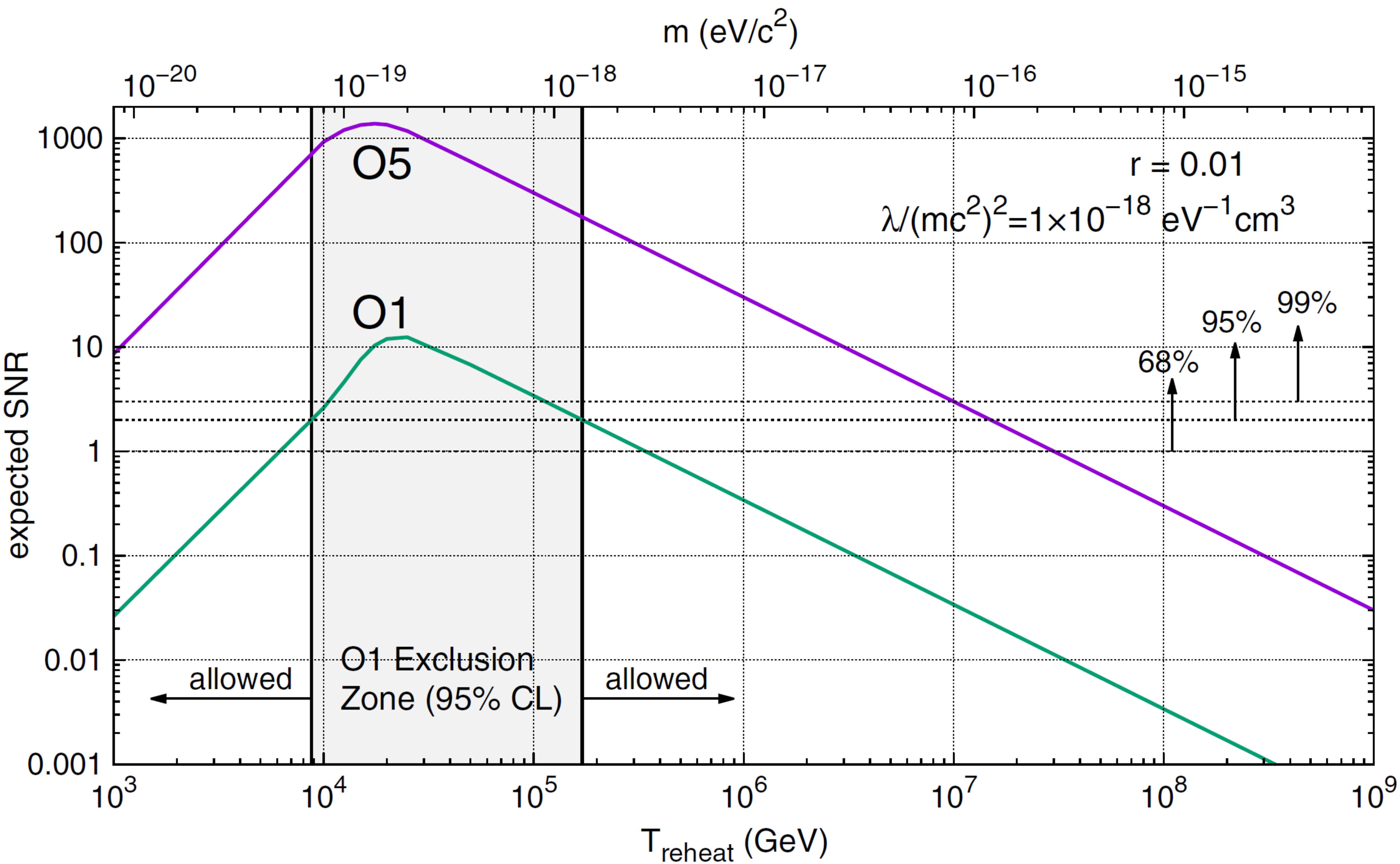


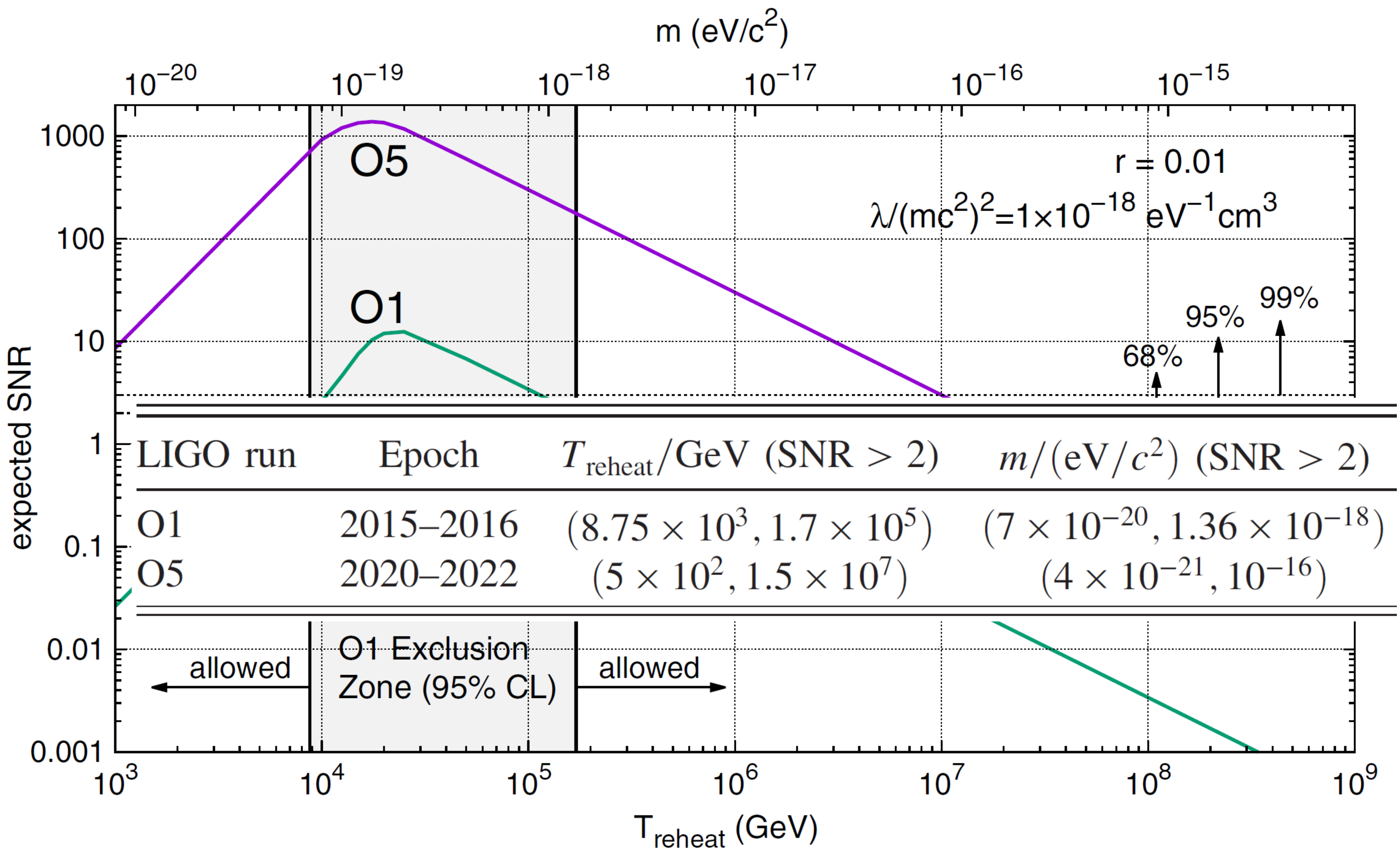
This example case is NOT excluded by O1 data!

Broader ΛSFDM parameter range to be tested



Marginally allowed ΛSFDM models for $\lambda/(mc^2)^2 = 1\times10^{-18}\,eV^{-1}cm^3$

Broader ΛSFDM parameter range to be tested

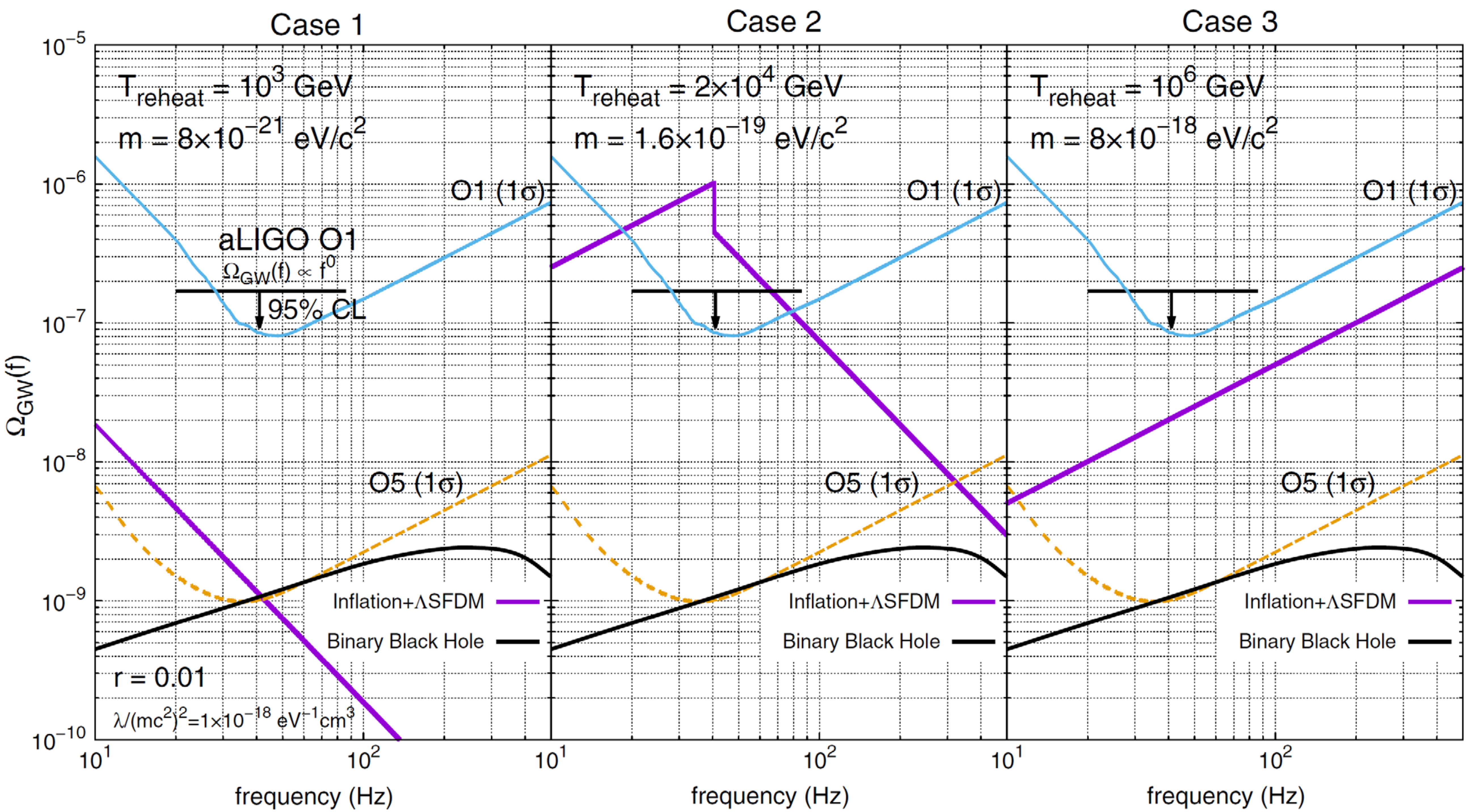


LIGO run	Epoch	$T_{\rm reheat}/\mathrm{GeV}$ (SNR > 2)	$m/(\mathrm{eV}/c^2)$ (SNR > 2)
O1	2015–2016	$(8.75\times10^3, 1.7\times10^5)$	$(7\times10^{-20}, 1.36\times10^{-18})$
O5	2020–2022	$(5\times10^2, 1.5\times10^7)$	$(4\times10^{-21}, 10^{-16})$

Marginally allowed ΛSFDM models for $\lambda/(mc^2)^2 = 1\times10^{-18}\,\mathrm{eV}^{-1}\mathrm{cm}^3$

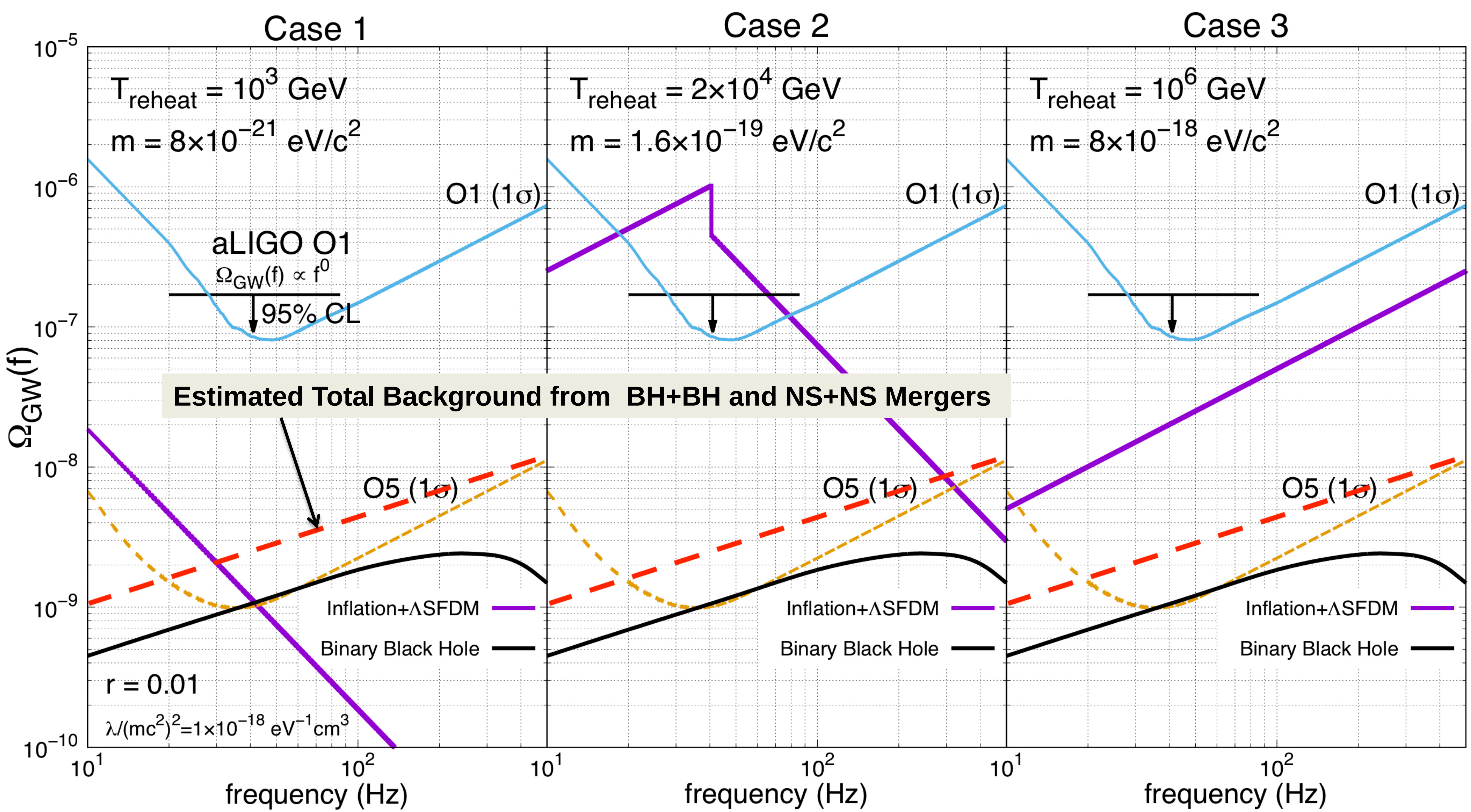
Stiff-SFDM-dominated era amplifies SGWB from inflation

SGWB's from (SFDM + Inflation) vs. (Unresolved Binary Black Hole Mergers)



Stiff-SFDM-dominated era amplifies SGWB from inflation

SGWB's from (SFDM + Inflation) vs. (Unresolved BH + BH and NS + NS Binary Mergers)



Summary

(A) Complex SFDM has stiff and radiation-like relativistic phases

➔ Increases the expansion rate of the early universe

(B) Stiff-SFDM-dominated era amplifies SGWB from inflation

➔ Subhorizon tensor modes contribute a radiation-like energy density to the background universe
➔ Further increases the expansion rate during the radiation-dominated era

(A) + (B) ➔ Cosmological Constraints on SFDM Particle Parameters

- Observational constraints on N_{eff} and z_{eq} ➔ constraints on allowed range of (m, λ)

$$2.3\times10^{-18}\ \mathrm{eV}^{-1}\ \mathrm{cm}^3 \leq \frac{\lambda}{(\mathrm{mc}^2)^2} \leq 4.1\times10^{-17}\ \mathrm{eV}^{-1}\ \mathrm{cm}^3,$$

$$m_{\min} \simeq (5\times10^{-21}\ \mathrm{eV}/c^2) \times \begin{cases} \dfrac{T_{\mathrm{reheat}}}{10^3\ \mathrm{GeV}}\sqrt{\dfrac{r}{0.01}}, & T_{\mathrm{reheat}} \gtrsim 10^3\ \mathrm{GeV}, \\ 1, & T_{\mathrm{reheat}} < 10^3\ \mathrm{GeV}. \end{cases}$$

Summary (cont.)

Stiff-SFDM-dominated era amplifies SGWB from inflation ➔ GWs detectable!

- **ΛSFDM predicts 2-parameter broken power-law SGWB spectrum**

$$\Omega_{GW}(f) = \Omega_{GW,peak} \begin{cases} f / f_{peak}, & f \le f_{peak} \\ (9\pi/64)(f/f_{peak})^{-2}, & f > f_{peak} \end{cases}$$

Expected SNR depends on the position of f_{peak} relative to LIGO band

- **LIGO can detect ΛSFDM-amplified inflationary SGWB for a range of SFDM (m, λ) that satisfy cosmological constraints, for tensor-to-scalar ratio r values currently allowed by CMB and a large range of reheat temperatures T_{reheat}.**
- **For given r and $\lambda/(mc^2)^2$, the marginally-allowed model for each T_{reheat} has the smallest m that satisfies cosmological contraints and maximizes the present energy density of the SGWB for that T_{reheat}.**
- **SGWB is then maximally *detectable* if T_{reheat} s.t. modes that re-enter horizon when reheating ends have frequencies inside LIGO sensitive band.**

➡ e.g. **For marginally-allowed models with r=0.01 and $\lambda/(mc^2)^2 = 1\times10^{-18}$ $eV^{-1}cm^3$, $8.75\times10^3 < T_{reheat}$ (GeV) $< 1.7\times10^5$ is excluded at 95 CL by LIGO O1 data.**

➡ **But for the same illustrative family, 3σ detection by O5 data (in 2022) is possible for $600 < T_{reheat}$ (GeV) $< 10^7$ GeV.**

➡ **GW experiments can already place a new kind of cosmological constraint on**

- **SGWB (inflation + ΛSFDM) can exceed that from unresolved BH and NS mergers!**

Summary (cont.)

Q: What happens to ΛSFDM if $N_{eff, BBN} = N_{eff, standard} = 3.046$ is someday favored by abundance measurements?

A: Upper limit on $\Delta N_{eff, BBN}$ remains, but lower limit is relaxed

➔ allows $\lambda \rightarrow 0$ limit (i.e. SFDM non-self-interacting), since SFDM then has no radiation-like ($w = 1/3$) intermediate phase

➔ SFDM transitions directly from stiff ($w = 1$) to matter-like ($w = 0$)

➔ But stiff phase must still end before BBN ➔ $m > m_{min}$

$$m_{\min} \simeq (5 \times 10^{-21}\ \mathrm{eV}/c^2) \times \begin{cases} \dfrac{T_{\mathrm{reheat}}}{10^3\ \mathrm{GeV}} \sqrt{\dfrac{r}{0.01}}, & T_{\mathrm{reheat}} \gtrsim 10^3\ \mathrm{GeV}, \\ 1, & T_{\mathrm{reheat}} < 10^3\ \mathrm{GeV}. \end{cases}$$

➔ AND even if $\lambda \rightarrow 0$, ΛSFDM stiff phase amplifies the inflationary SGWB enough to be detectable!!