AXION DARK MATTER CLUMPS

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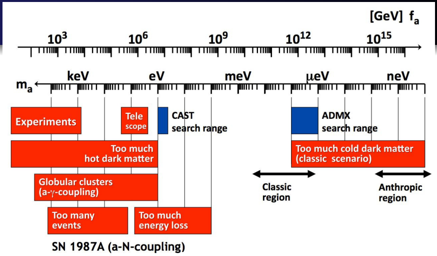
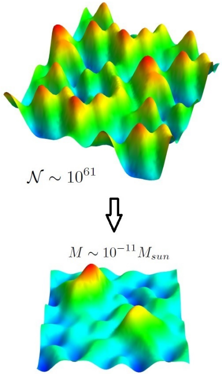


- Large scale structure and CMB observations are very well fit by cold dark matter.
- A favorite candidate: Axion.
- The axion is the pseudo-Goldstone boson associated with a spontaneously broken symmetry $U(1)_{PQ}$ (Peccei and Quinn, 1977).

- The axion is a field that acquires a mass in the early universe, after the QCD phase transition, and can then begin to act as a form of cold dark matter (Preskill et al. 1983; Abbott and Sikivie 1983).
- If the PQ phase transition happens after inflation, then the field remains inhomogeneous from one Hubble patch to the next as suggested by causality.
- Large fluctuations already present in the axion field after the QCD phase transition can lead to the formation of a kind of Bose-Einstein condensate.



(Guth, M.H., Prescod-Weinstein, 2015)

- Axions are described in field theory by a real scalar field $\phi(x)$:

$$\mathcal{L} = \sqrt{-g}\left[\frac{1}{2} g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi - V(\phi)\right]\ , \text{ with } V(\phi) = \Lambda^4[1-\cos(\phi/f_a)]\,. \qquad (1)$$

- Here $\Lambda \sim 0.1$ GeV, f_a is the PQ symmetry breaking scale, and $m = \Lambda^2/f_a$. We shall often take $m = 10^{-5}$ eV with $f_a = 6x10^{11}$ GeV.
- The non-relativistic field theory approximation for axions is often very well justified. It is useful to express the real field $\phi(x)$ as

$$\phi(\mathbf{x},t) = \frac{1}{\sqrt{2m}}\left[e^{-imt}\psi(\mathbf{x},t) + e^{imt}\psi^*(\mathbf{x},t)\right]\ . \qquad (2)$$

- We want to replace this expression into the axion Lagrangian density:
  - Drop all terms proportional to a power of e^{-imt} or e^{imt}.
  - Take $|\dot{\psi}|/m \ll |\psi|$ in the kinetic term of the Lagrangian density.
  - Use the weak field Newtonian metric $g_{00} = 1 + 2\phi_N(\psi^*,\psi)$.
- We obtain $\mathcal{L}_{nr} = \frac{i}{2}\left(\dot{\psi}\psi^* - \psi\dot{\psi}^*\right) - \frac{\nabla\psi^*\cdot\nabla\psi}{2m} - V_{nr}(\psi,\psi^*) - m\,\psi^*\psi\,\phi_N(\psi^*,\psi)\,,$ where the non-relativistic effective potential is $V_{nr}(\psi,\psi^*) = -\frac{\psi^{*2}\psi^2}{16\,f_a^2}\,.$

- Now, we treat ψ and ψ^* as independent fields and calculate the total Hamiltonian as: $H = H_{kin} + H_{int} + H_{grav}$.

$$H_{kin} \equiv \int d^3x \, \frac{1}{2m} \nabla\psi^* \cdot \nabla\psi \,, \quad H_{int} \equiv \int d^3x V_{nr}(\psi, \psi^*) \,, \tag{3}$$

$$H_{grav} \equiv -\frac{Gm^2}{2} \int d^3x \int d^3x' \, \frac{\psi^*(\mathbf{x})\psi^*(\mathbf{x}')\psi(\mathbf{x})\psi(\mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|} \,. \tag{4}$$

- The (non-relativistic) full equation of motion is

$$i\,\dot{\psi} = -\frac{\nabla^2\psi}{2m} - Gm^2\,\psi \int d^3x' \, \frac{\psi^*(\mathbf{x}')\psi(\mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|} + \frac{\partial}{\partial\psi^*} V_{nr}(\psi, \psi^*) \tag{5}$$

- The local number density of particles, $n(\mathbf{x})$, and local mass density, $\rho(\mathbf{x})$, are given by the usual expressions:

$$n(\mathbf{x}) = \psi^*(\mathbf{x})\psi(\mathbf{x}) \,, \tag{6}$$

$$\rho(\mathbf{x}) = m\,\psi^*(\mathbf{x})\psi(\mathbf{x}) \,. \tag{7}$$

- The ground is guaranteed to be spherically symmetric to avoid additional energy from angular momentum $\rightarrow \psi_g(r,t) = \Psi(r)\, e^{-i\,\mu\, t}$.
- The time independent field equation for a spherically symmetric eigenstate is

$$\mu\,\Psi = -\frac{1}{2m}\left(\Psi'' + \frac{2}{r}\Psi'\right) - 4\pi G m^2 \Psi \int_0^\infty dr'\, r'^2\, \frac{\Psi(r')^2}{r_>} + \frac{1}{2}\frac{\partial}{\partial \Psi} V_{nr}(\Psi)\,. \quad (8)$$

- Far field region:
    - $\Psi \rightarrow 0$ as $r \rightarrow \infty$.
    - At large distances we can ignore the self-interactions.
    - In the gravitational term we can replace $r_> \rightarrow r$ in the far region.
- Hence

$$\mu\,\Psi \approx -\frac{1}{2m}\left(\Psi'' + \frac{2}{r}\Psi'\right) - \frac{G m^2 N}{r}\Psi \quad \text{(far region)}\,. \quad (9)$$

- Identical to the structure of the time independent Schrödinger equation for the hydrogen atom $(G m^2 N \rightarrow e^2)$. The ground state solution is:

$$\Psi(r) = \text{Poly}_n(r) e^{-G m^3 N\, r/n} \quad \text{(far region)}\,. \quad (10)$$

- Near field region:
    - Corrections from self-interactions become important and the structure of the gravitational term is altered.
    - There are no known full analytical solutions.

- A simple choice (decay length scale R acts as a variational parameter):

$$\Psi_R(r) = \sqrt{\frac{N}{\pi R^3}}\, e^{-r/R} \,. \qquad (11)$$

- The total number of particles, $N = \int d^3x\, n(\mathbf{x})$, is ensured by the prefactor of Eq. (11) and is assumed to be fixed as we perform our variation.

- The exponential ansatz has the disadvantage that it cannot be correct for small r because we need that $\Psi' \to 0$ as $r \to 0$.

- Better options are:

$$\Psi(r) = \sqrt{\frac{3N}{\pi^3 R^3}}\operatorname{sech}(r/R)\,, \qquad (12)$$

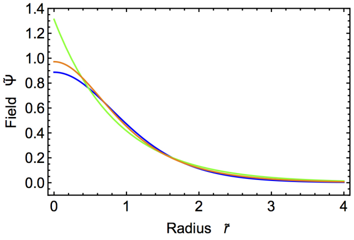
$$\Psi(r) = \sqrt{\frac{N}{7\pi R^3}}(1 + r/R)e^{-r/R}\,. \qquad (13)$$

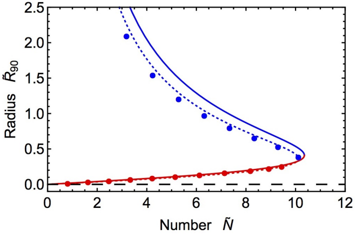
- Inserting any localized ansatz of a single variational parameter R into the Total Hamiltonian and using $\tilde{R} \equiv m f_a \sqrt{G}\, R$, $\tilde{N} \equiv \frac{m^2\sqrt{G}}{f_a} N$, $\tilde{H} \equiv \frac{m}{f_a^3\sqrt{G}} H$, we have

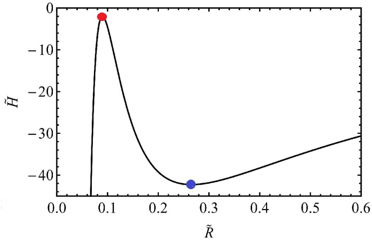
$$\tilde{H}(\tilde{R}) = a\frac{\tilde{N}}{\tilde{R}^2} - b\frac{\tilde{N}^2}{\tilde{R}} - c\frac{\tilde{N}^2}{\tilde{R}^3}\,. \qquad (14)$$

- Extremizing the Hamiltonian $\tilde{H}$ with respect to $\tilde{R}$, we obtain the condition for stationary solutions.
- For any values of (a, b, c) there is a stable branch for large $\tilde{R}$ and an unstable branch for low $\tilde{R}$, given by $\tilde{R} = \left(a \pm \sqrt{a^2 - 3bc\tilde{N}^2}\right)/(b\tilde{N})$.

- The maximum value of $\tilde{N}$ is given by $\tilde{N} < \tilde{N}_{max} = a/\sqrt{3bc}$.
- We have $\tilde{N}_{max} \approx (10.36, 10.12, 10.15)$ for exponential, sech, and exponential $\times$ linear ansatz, respectively.





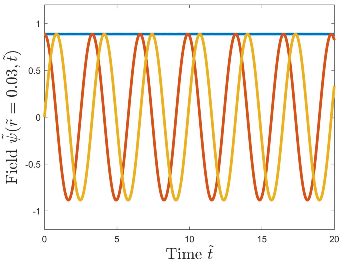


- We would like to solve the full equation of motion for the axion field, Eq. (5), within the spherically symmetric ansatz:

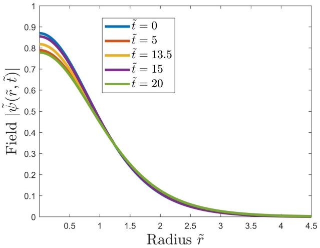
$$i\frac{\partial\tilde{\psi}}{\partial\tilde{t}} = -\frac{1}{2\tilde{r}}\frac{\partial^2}{\partial\tilde{r}^2}\left(\tilde{r}\tilde{\psi}\right) + \tilde{\phi}_N\tilde{\psi} - \frac{1}{8}|\tilde{\psi}|^2\tilde{\psi} \quad \text{and} \quad \frac{1}{\tilde{r}}\frac{\partial^2}{\partial\tilde{r}^2}\left(\tilde{r}\tilde{\phi}_N\right) = 4\pi|\tilde{\psi}|^2 \,. \qquad (15)$$

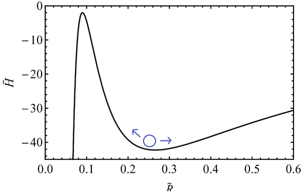
- Here $\tilde{\psi}(\tilde{r},\tilde{t})$ and $\tilde{\phi}_N(\tilde{r},\tilde{t})$ are the axion field and the newtonian potential, respectively, and $\tilde{r}$ and $\tilde{t}$ are the radial and time coordinates, respectively, all in dimensionless variables.

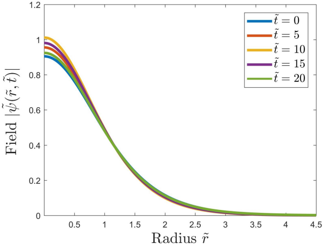
- The time evolution of a clump that lives exactly on the stable branch solution is the expected.



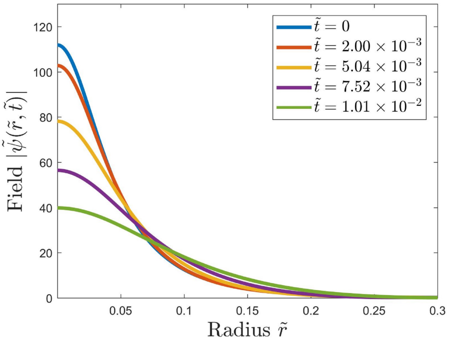
- We perturb the stable and unstable solutions: $\tilde{\psi}_{initial}(\tilde{r}) = (1 + \epsilon)Re(\Psi(\tilde{r}))$



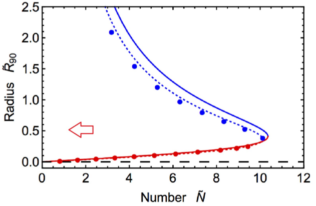


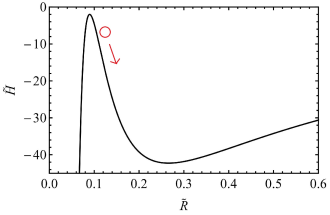


-/+ 2% perturbation (Stable Branch)



- 2% perturbation (Unstable Branch)





- Using $\tilde{f}_a \equiv f_a/(6 \times 10^{11}\ \text{GeV})$ and $\tilde{m} \equiv m/(10^{-5}\ \text{eV})$, we have for the stable branch

$$N_{max} \approx 8 \times 10^{59}\,(\tilde{m}^{-2}\tilde{f}_a)\,, \qquad (16)$$

$$M_{max} \approx 1.4 \times 10^{19}\,\text{kg}\,(\tilde{m}^{-1}\tilde{f}_a)\,, \qquad (17)$$

$$R_{90,min} \approx 130\,\text{km}\,(\tilde{m}^{-1}\tilde{f}_a^{-1})\,, \qquad (18)$$

- The ground state is well described by the weak field gravitational approximation :

$$\frac{R}{R_S} > \frac{R_{min}}{2GM_{max}} \approx 4 \times 10^{12}\tilde{f}_a^{-2}\,. \qquad (20)$$

- The typical number of axions in inhomogeneous patches in the early universe is (Guth et al. 2015):

$$N_\xi \sim \frac{T_{eq}M_{pl}^3}{T_{QCD}^3\,m} \sim 10^{61}\,\tilde{m}^{-1}. \qquad (19)$$

- So there is no possibility for black hole formation of these low density objects when $f_a \ll M_{pl}$.
- Strong field effects can emerge if one were to move away from the traditional QCD axion and investigate extremely high values of f_a (Helfer et al. 2017).

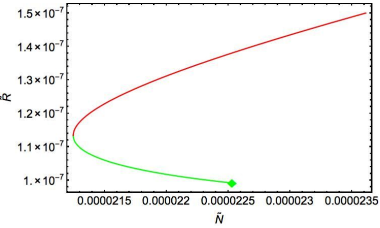
- We need $\omega \approx m$ to trust non-relativistic approximations ($\phi << f_a$).
- For the blue stable branch this condition is always satisfied. For the red unstable branch this condition is broken when $\tilde{N} \lesssim \mathcal{O}(10^{-5})$.
- We ignore the gravitational corrections and take an approximate periodic clump solution as

$$\phi(r,t) = \Phi(r)\cos(\omega\, t)\,. \tag{21}$$

- We insert this into the relativistic Hamiltonian (ignoring gravity) and average over a period of oscillation $T = 2\pi/\omega$ as

$$\langle H \rangle = \frac{1}{T}\int_0^T dt\, H. \tag{22}$$

- To specify the condition for ω, we take the time average of equation of motion and integrate over space.
- We use an exponential ansatz for the radial profile: $\Phi(r) = 2\pi\,\varepsilon\, f_a\, e^{-r/R}$, where $0 < \varepsilon < 1 \to |\phi| < 2\pi\, f_a$.
- We extremize $\langle H \rangle$ using $\langle N \rangle = \int d^3x\, \omega \langle \phi^2 \rangle$.



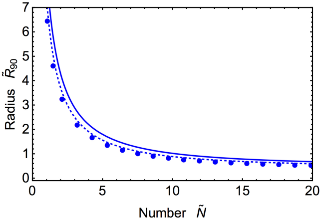
- Consider generic light scalar dark matter candidate that may be described by a *repulsive* $+\lambda_r\phi^4$ interaction.
- In the non-relativistic regime, $V(\phi)=\frac{1}{2}m^2\phi^2+\frac{\lambda_r}{4!}\phi^4$ leads to exactly the same set of equations as we described earlier, but now

$$V_{nr}(\psi, psi^*)=\lambda_r\frac{\psi^{*2}\psi^2}{16m^2}\,. \tag{23}$$

- We again pass to the dimensionless variables as before ($f_a\to m/\sqrt{\lambda_r}$). For any localized clump ansatz of a single length scale $\tilde{R}$, we have

$$\tilde{H}(\tilde{R})\approx a\frac{\tilde{N}}{\tilde{R}^2}-b\frac{\tilde{N}^2}{\tilde{R}}+c\frac{\tilde{N}^2}{\tilde{R}^3}\,. \tag{24}$$

- Unlike the previous case of attractive interactions, here there is only one branch of extrema, which is stable, and given by $\tilde{R}=\frac{a+\sqrt{a^2+3bc\tilde{N}^2}}{n\tilde{N}}$



Conclusion

- Mapping out the basic solutions of the axion-gravity-self-interacting system, we find that:
  - For sufficiently spatially large clumps, gravity dominates, and the system is stable.
  - For sufficiently spatially small clumps, self-interaction dominates, and the system is unstable.
  - For extremely small clumps, the full cosine potential and relativistic corrections become important, and a new (narrow) axiton-branch emerges.
- The typical number of axions in a clump is comparable to the typical number of axions in one coherence length in the early universe (scenario in which the PQ phase transition occurs after inflation).
- We also examined more generic scalar dark matter, allowing for repulsive self-interactions, which has only a stable clump solution branch that extends to arbitrarily large particle number and its rather compact.