Inflation and the Structure of the Higgs Vacuum

Based on work with Gian Giudice and Tevong You.

Dec 14th 2022

KCETA Colloquium

Matthew McCullough

ESA/Hubble and Harald Süpfle


Criticality
ESA/Hubble and Harald Süpfle

Naturalness

Suppose you're travelling to a conference (at sea level). You've rented a room and turn up on time to find the room freezing (almost).

The air temperature is 0°C, to more decimal places than you can measure.

Naturalness

Suppose you're travelling to a conference (at sea level). You've rented a room and turn up on time to find the room freezing (almost).

You then notice that it stays fixed at 0°C, regardless of the outside air temperature.

Naturalness

To you, as an observer, the temperature is a fundamental parameter of this room.

There’s an obvious explanation for the fixed temperature: A good thermostat and good insulation.

Naturalness

To you, as an observer, the temperature is a fundamental parameter of this room.

But the fact that it is fixed to be on the edge of a phase transition is just downright odd...

We do prepare analogous situations in the laboratory.

Consider superconductors...

Ginzburg-Landau

The G-L Theory of superconductivity involves a complex scalar field and the photon (magnetic vector potential)

$$\Phi \qquad\qquad A$$

The Free energy for this theory is

$$F = |(\nabla + 2ieA)\Phi|^2$$

$$+m^2(T)|\Phi|^2 + \lambda|\Phi|^4 + \ldots$$

Where the mass depends on the temperature.

Ginzburg-Landau

$$F = |(\nabla + 2ieA)\Phi|^2$$

$$+m^2(T)|\Phi|^2 + \lambda|\Phi|^4 + \ldots$$

At high temperatures the mass-squared is positive:

Just a hot metal.

Ginzburg-Landau

$$F = |(\nabla + 2ieA)\Phi|^2$$

$$+m^2(T)|\Phi|^2 + \lambda|\Phi|^4 + \ldots$$

At the critical temperature the mass-squared vanishes:

Strange theory with massless fluctuations. Two phases coexisting.

Ginzburg-Landau

$$F = |(\nabla + 2ieA)\Phi|^2$$

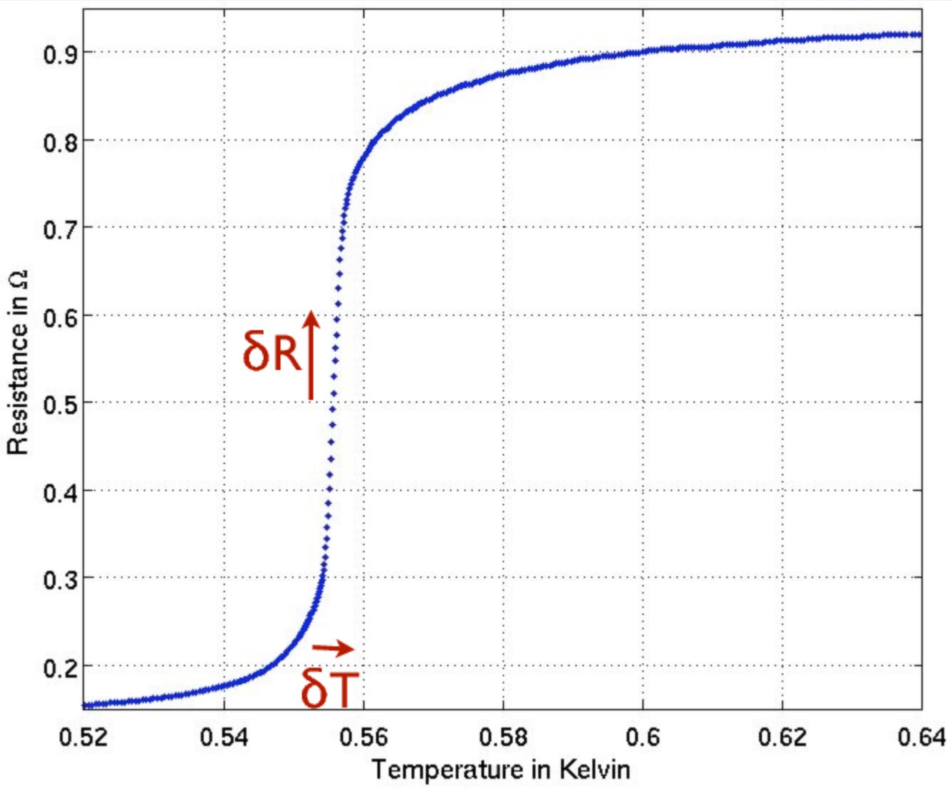
$$+m^2(T)|\Phi|^2 + \lambda|\Phi|^4 + \ldots$$

Below the critical temperature the mass-squared is negative:

Photon has become massive: $m_A \sim e\langle\Phi\rangle$

TES

In a transition edge sensor the temperature is fine-tuned, through a feedback loop, to sit at precisely the critical point…

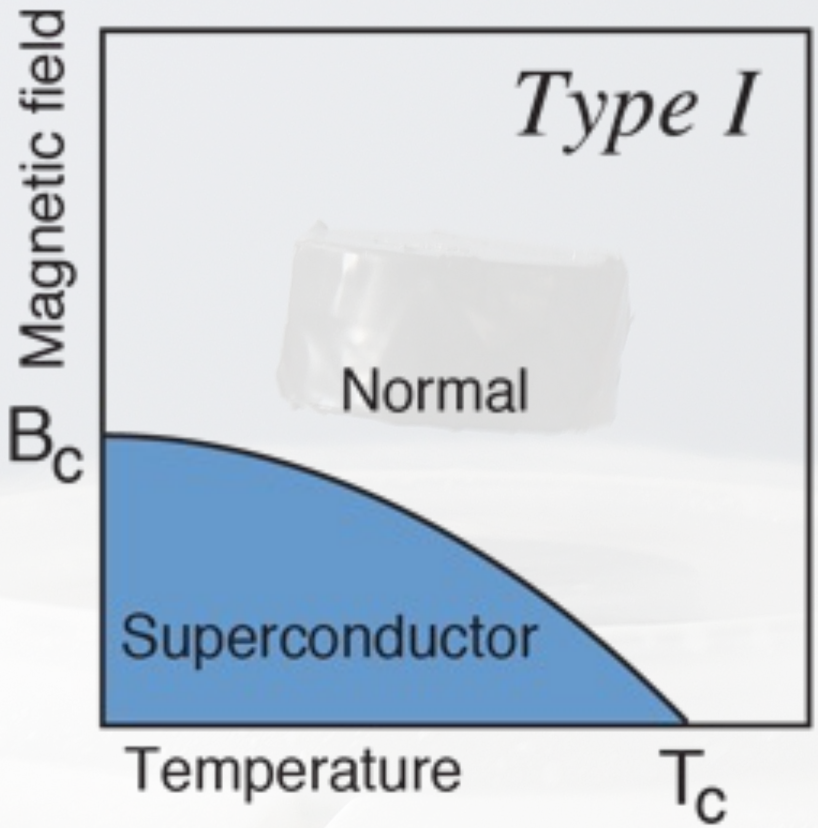


Taken from 1309.5383

A small fluctuation gives a big change!

Other Background Parameters

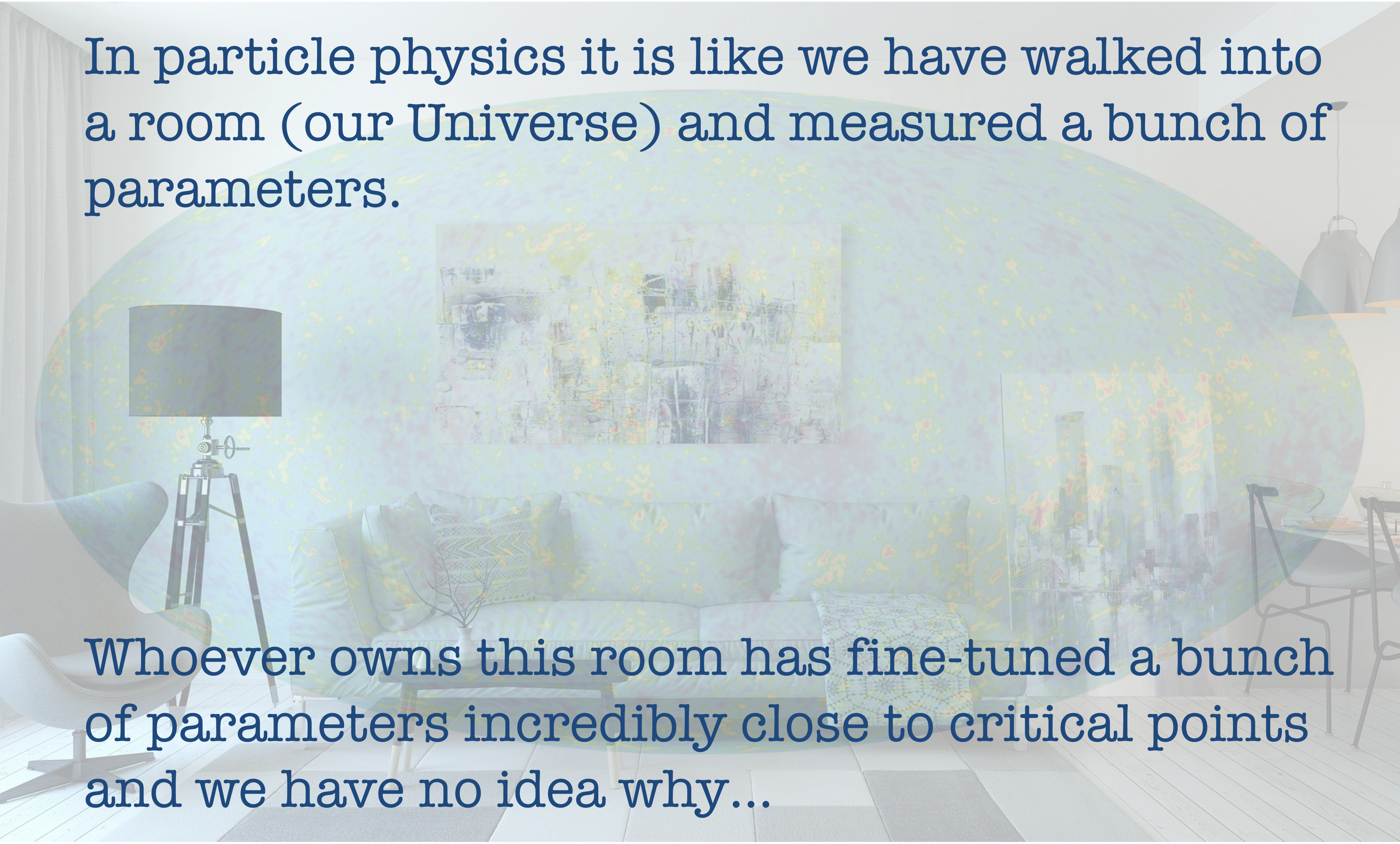
The phase doesn't only depend on the temperature, but other background parameters such as an external magnetic field.



Taken from Hyperphysics

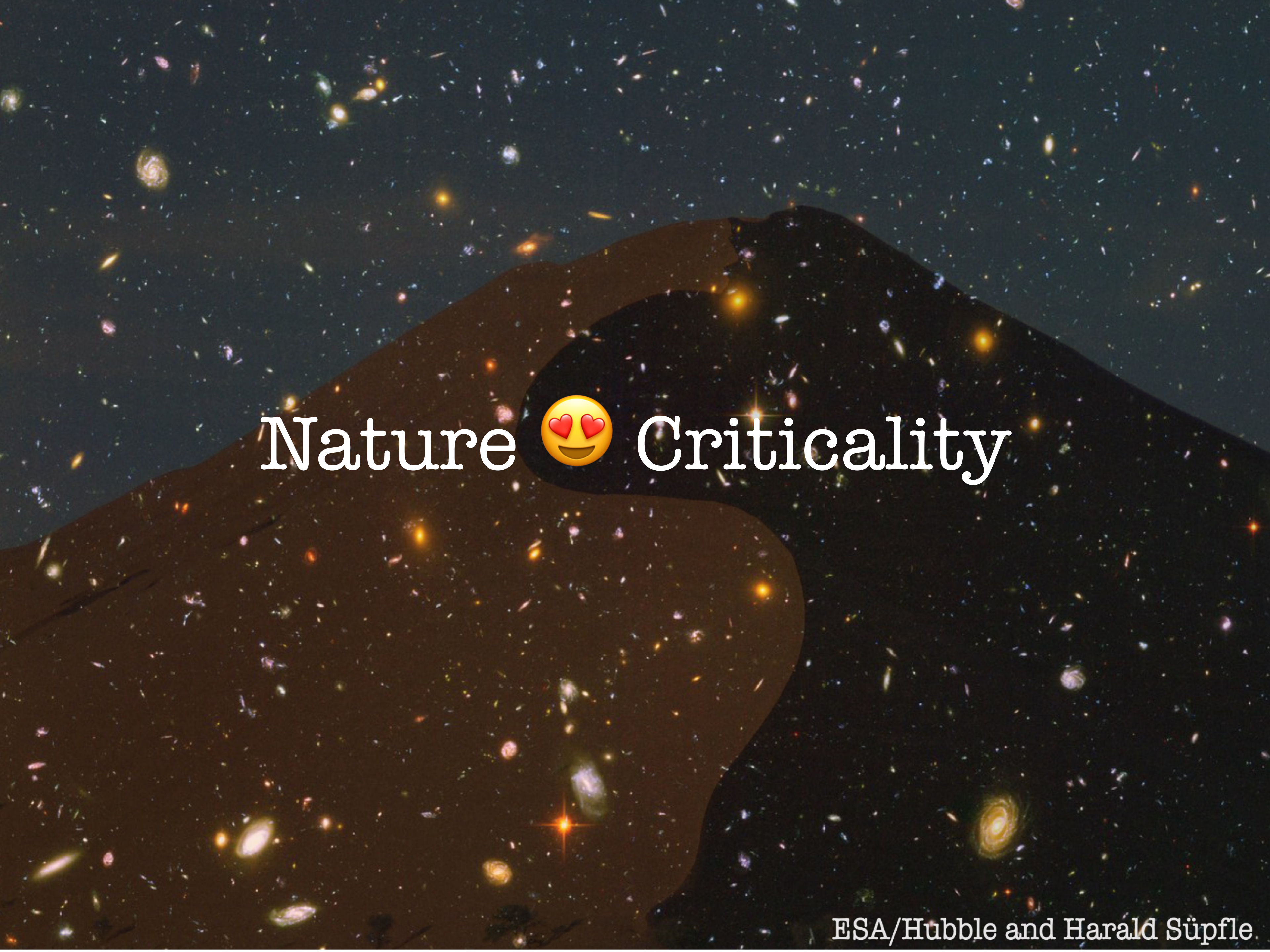
For a range of temperatures could tune B to sit arbitrarily close to the critical point.

Naturalness



In particle physics it is like we have walked into a room (our Universe) and measured a bunch of parameters.

Whoever owns this room has fine-tuned a bunch of parameters incredibly close to critical points and we have no idea why…


Nature 😍 Criticality
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Puzzle 1

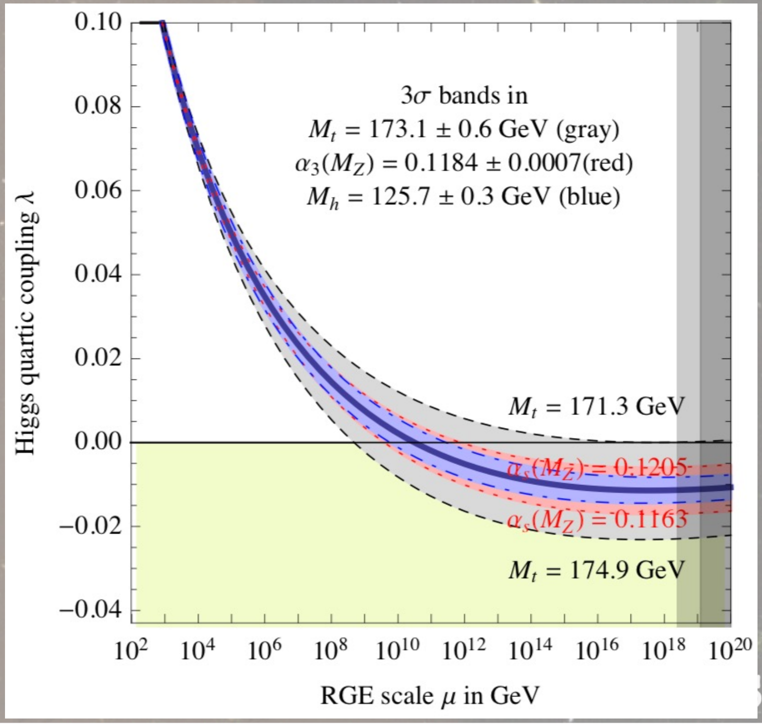
ESA/Hubble and Harald Süpfle

A Metastable Universe?

The Higgs potential depends on the Higgs itself, due to quantum mechanics:

$$V_H = M^2(H)|H|^2 + \lambda(H)|H|^4$$

The Higgs quartic interaction effectively turns negative at large field values:



Taken from 1205.6497

A/Hubble and Harald Süpfle

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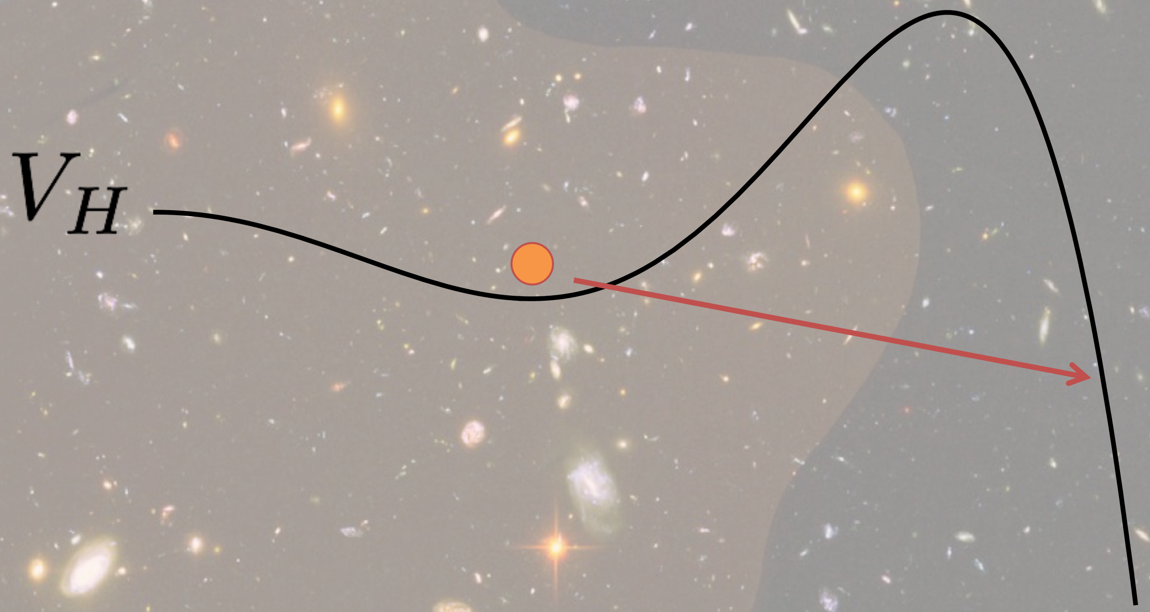
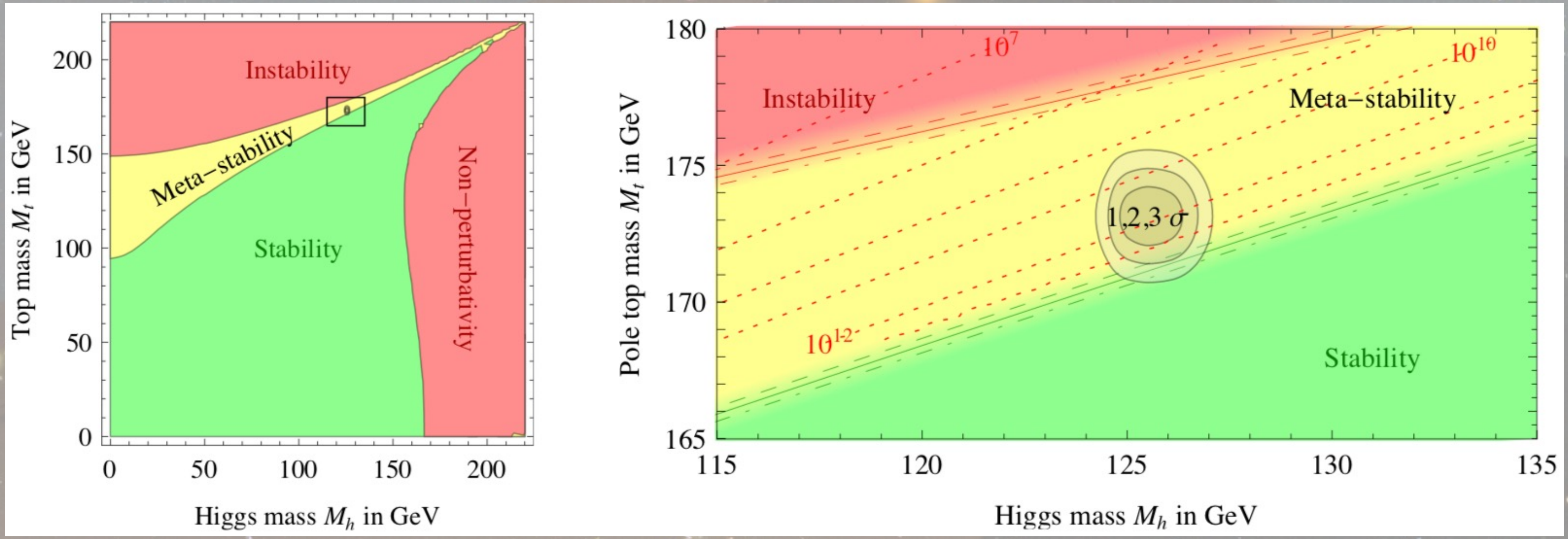


A Metastable Universe?

This means that the vacuum we are in, as in the Mexican hat pictures, is just local, but there is a deeper one out at large field values.



The fundamental parameters we have measured in our "room" imply that nature is delicately balanced at a critical point where two Higgs phases may coexist.

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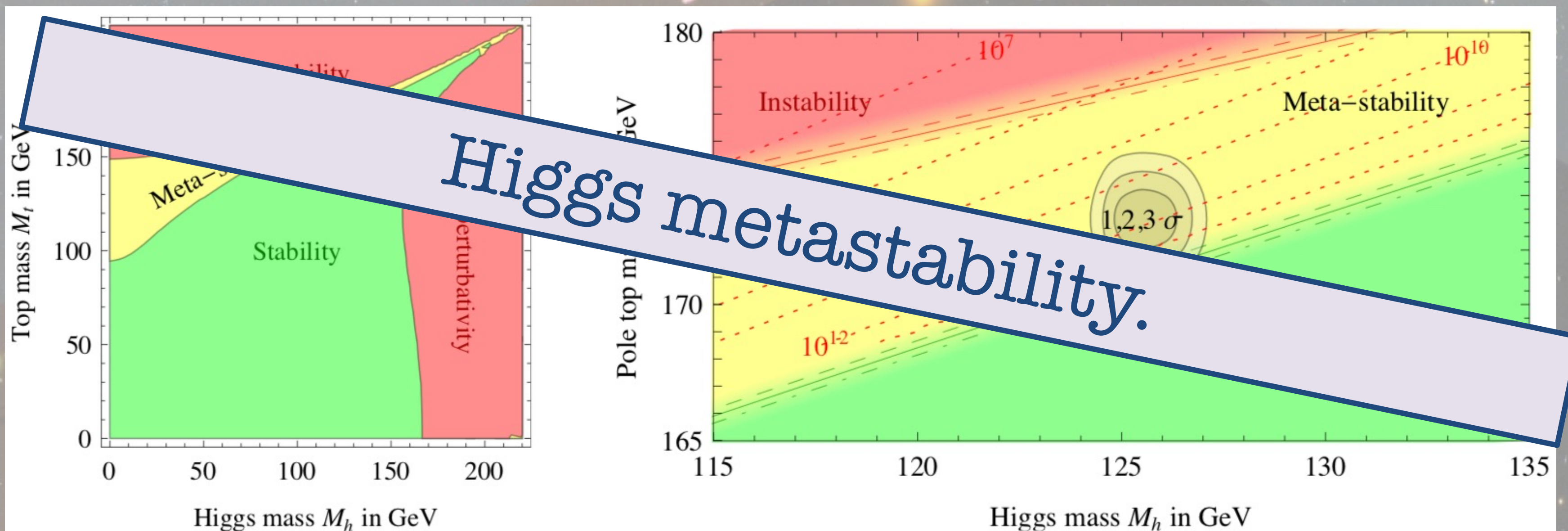

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Puzzle 2

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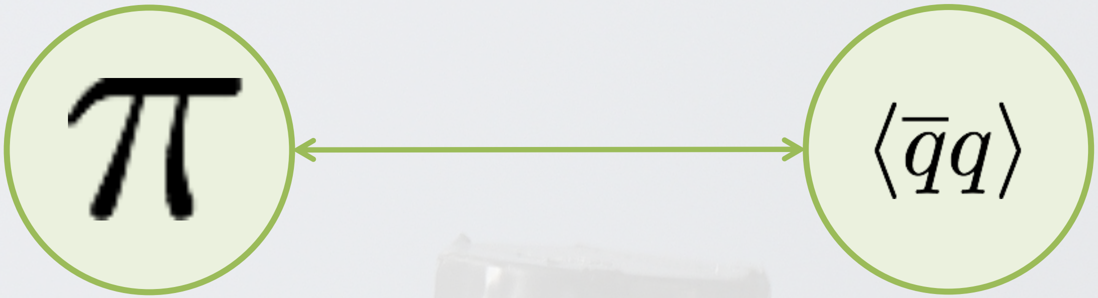
Critical Higgs

Consider pions:



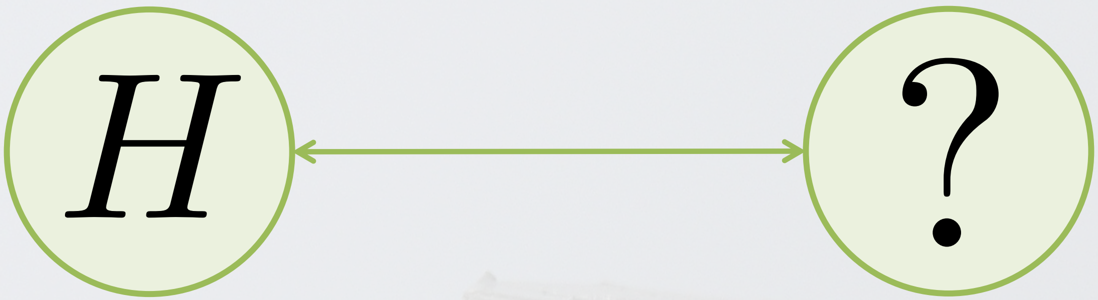
The order parameter for the condensate and the pion mass are both calculable in terms of microscopic theory

$$f_\pi \sim \frac{\Lambda_{\mathrm{QCD}}}{g_\star} \qquad\qquad m_\pi^2 \sim m_q \Lambda_{\mathrm{QCD}}$$

and both follow typical symmetries + scales.

Critical Higgs

What about the Higgs?

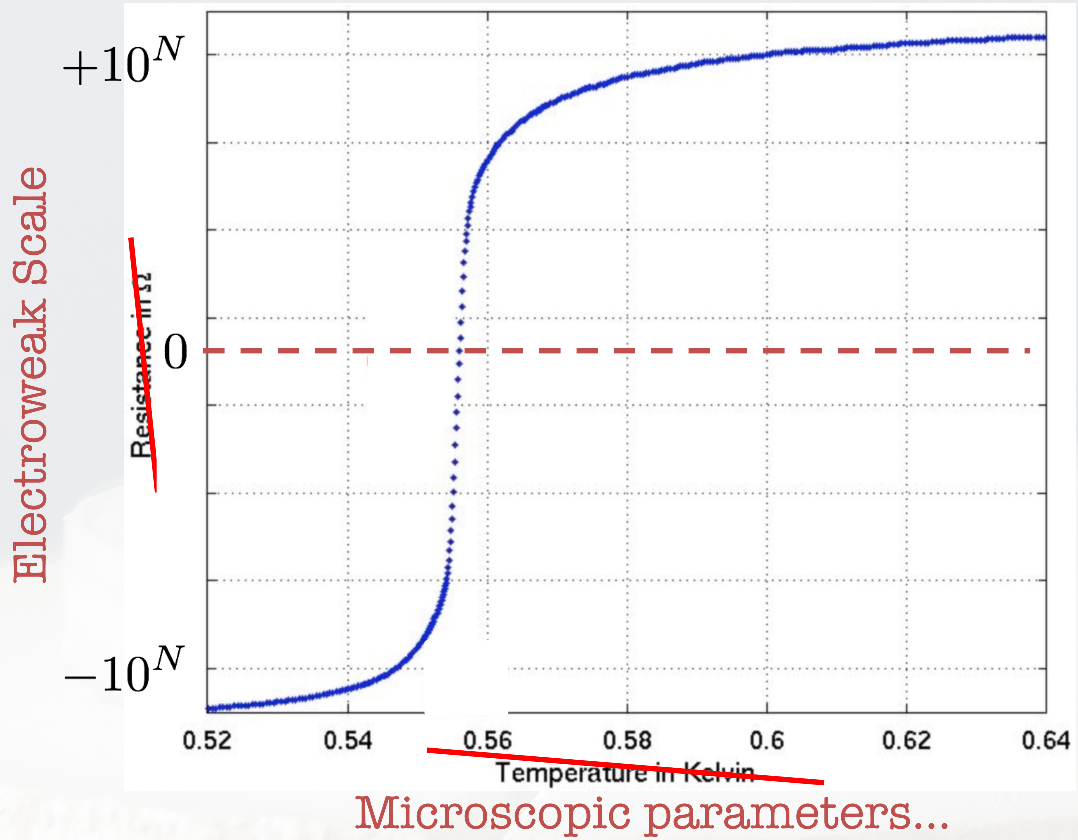


If there is some scale at which the electroweak scale (order parameter) and Higgs mass become <u>calculable</u> in terms of the microscopic theory then the LHC is telling us that:

$$v \ll \frac{\Lambda}{g_\star} \qquad\qquad m_h^2 \ll \hbar\lambda^2\Lambda^2$$

Critical Higgs

Essentially, it seems like the Universe is just like a Transition Edge Sensor:



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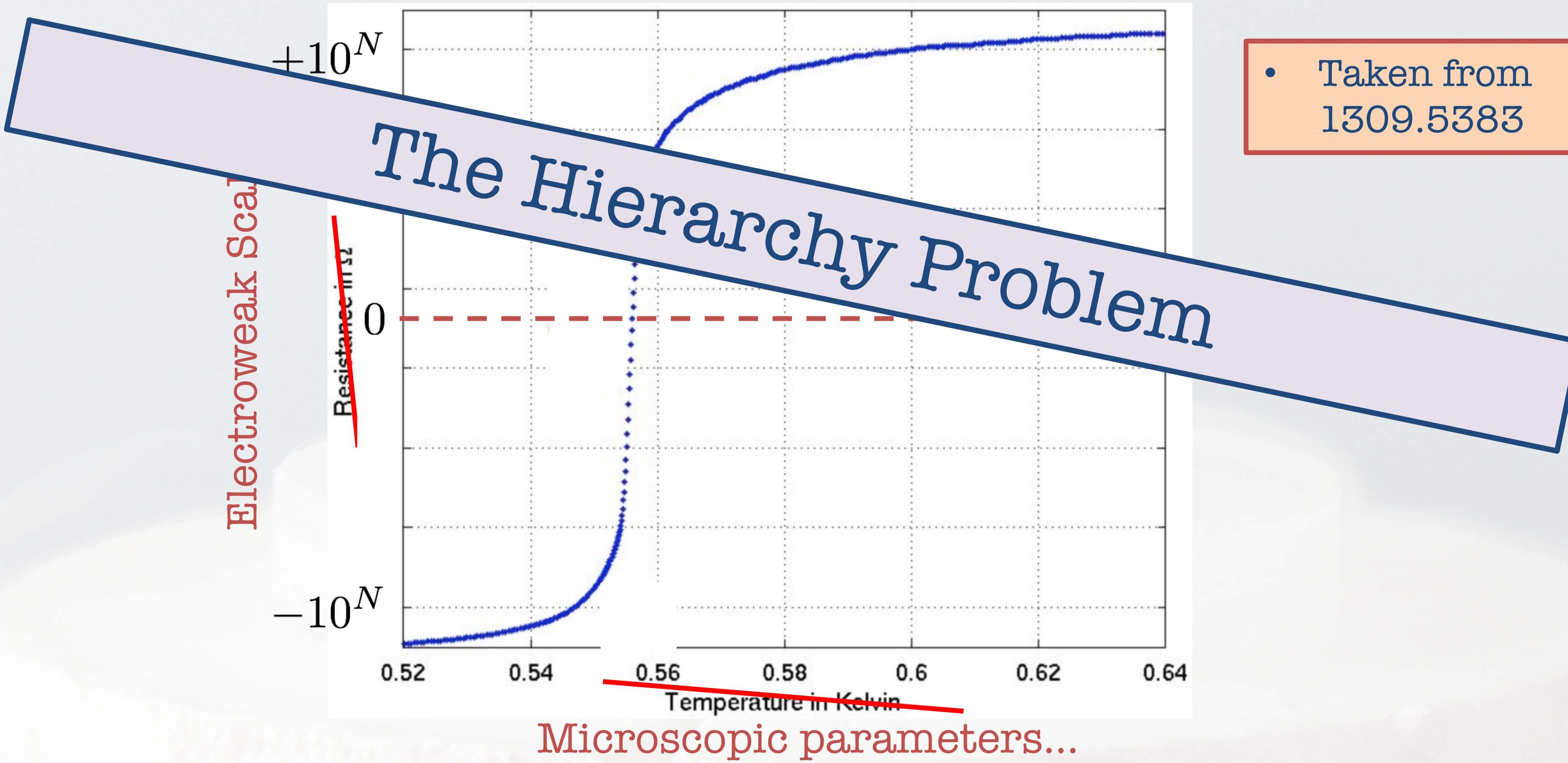

- Taken from 1309.5383

Traditionally these questions are considered independently. A different explanation for each.

But what if this common thread, criticality, is telling us about a broader general phenomenon?

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Inflation
28
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General relativity has solutions which support exponential spatial expansion: The greater the vacuum energy (scalar potential) the faster the inflation.

Finite Hubble parameter physics has many connections with finite-temperature physics at the Gibbons-Hawking temperature.

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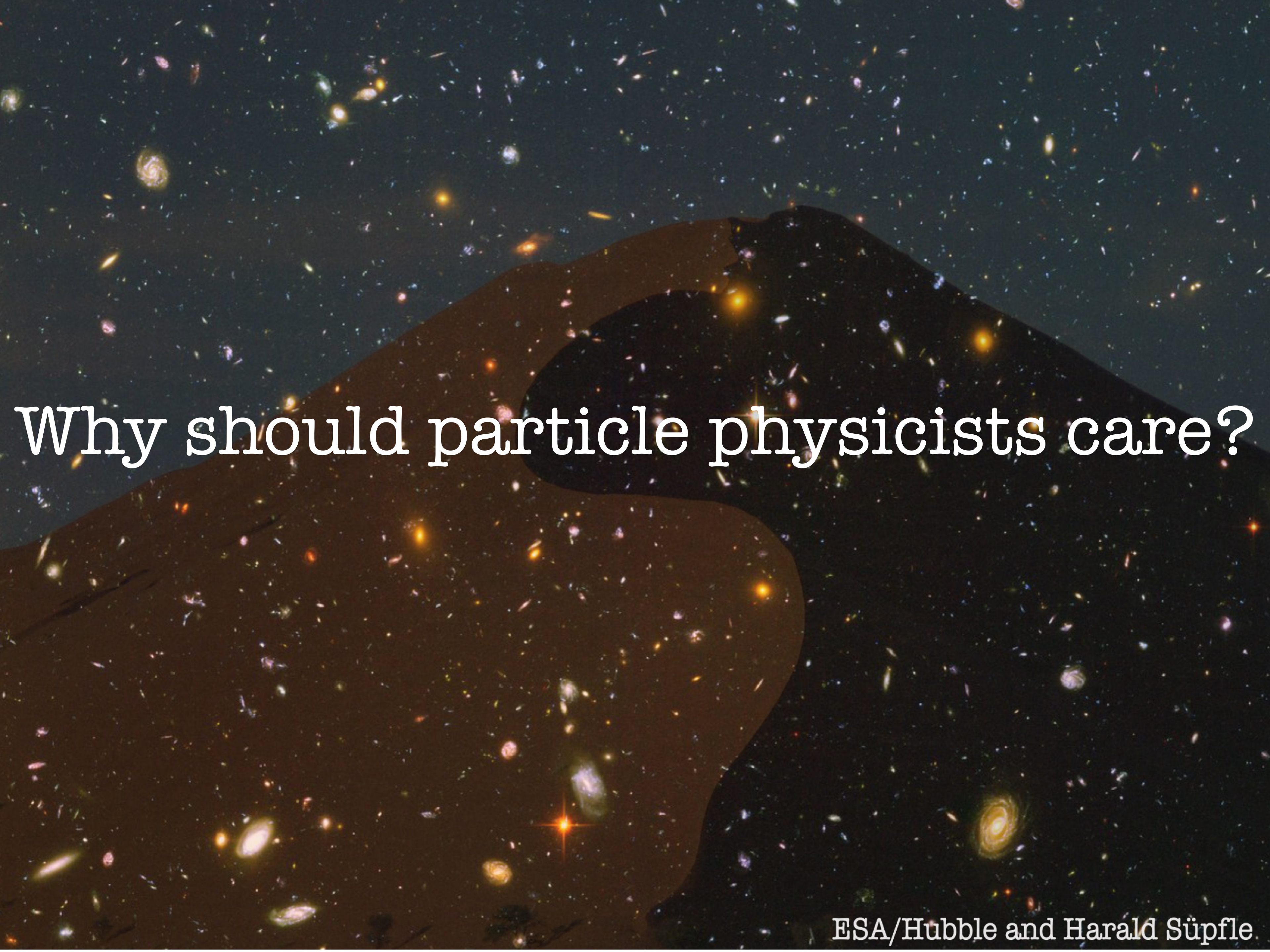
Theory of Inflation

The stochastic approach to inflation developed by Starobinsky and others has been a useful guide.

Quantum calculations have confirmed many aspects of this approach as a leading order picture, in the pre-eternal regime.

But eternal regime seems typical...


Why should particle physicicists care?
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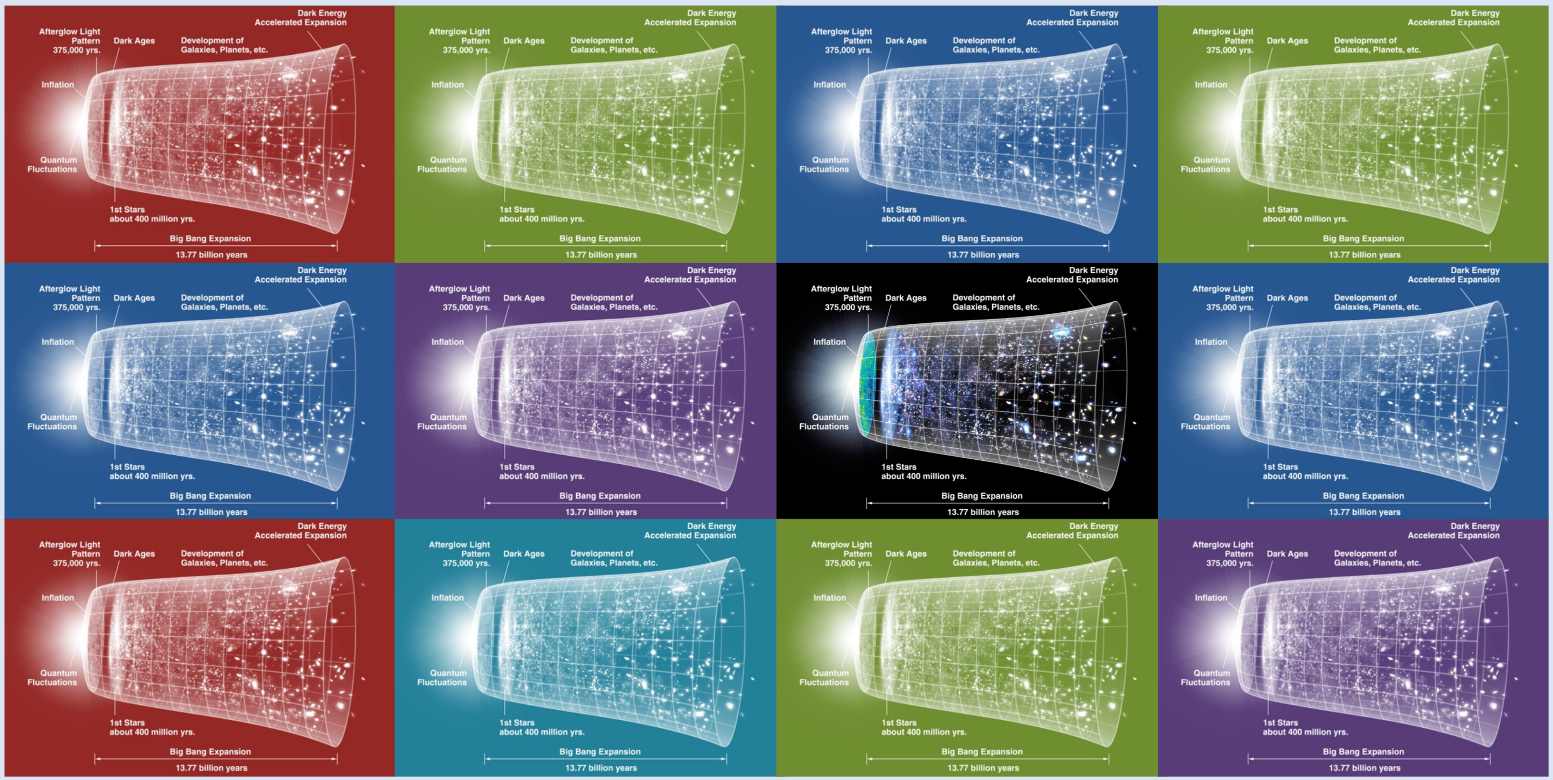
Inflation

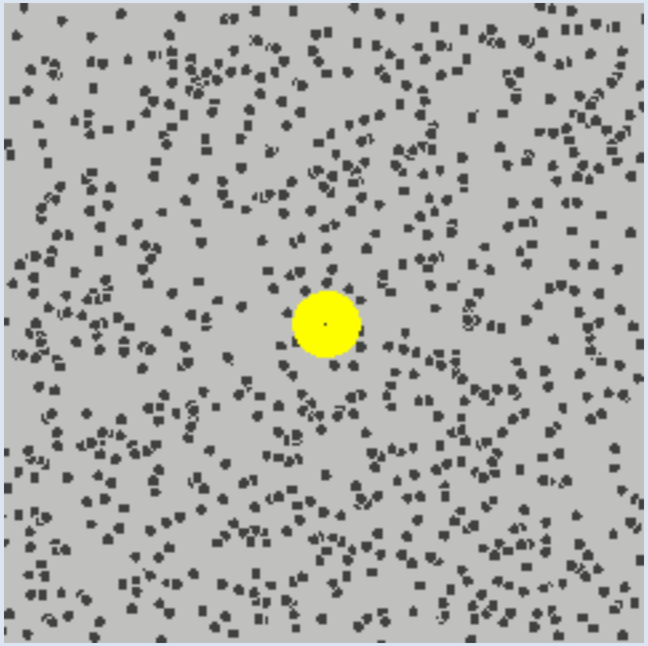
Eternal inflation leads to a multiverse of different Universes. We are but one…



In each one, different parameters, forces…

Inflation

Suppose you have a box of gas and you measure the velocity of one atom, once.



Is that value of velocity likely, or unlikely?

Inflation

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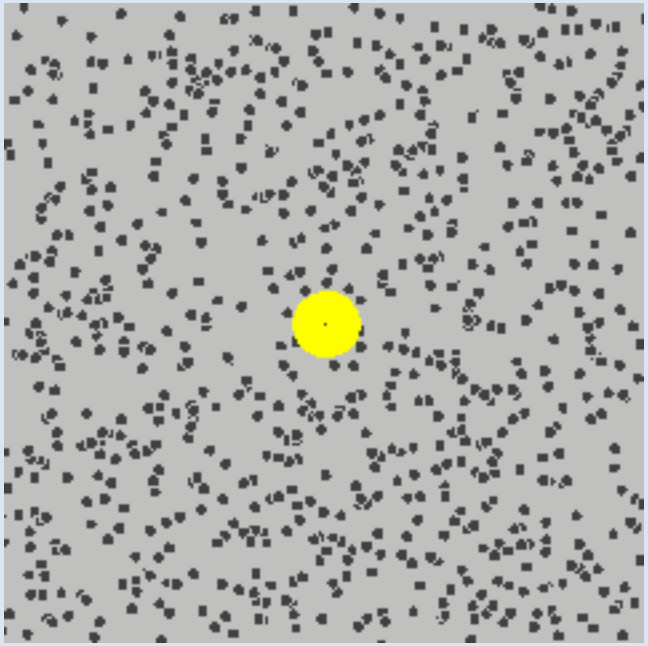
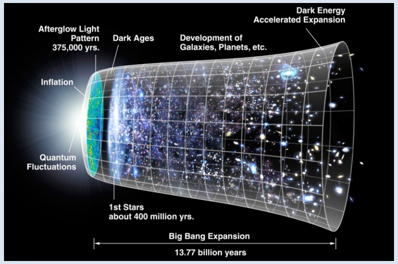


If we know the properties of the statistical ensemble at equilibrium, we have context.

Inflation

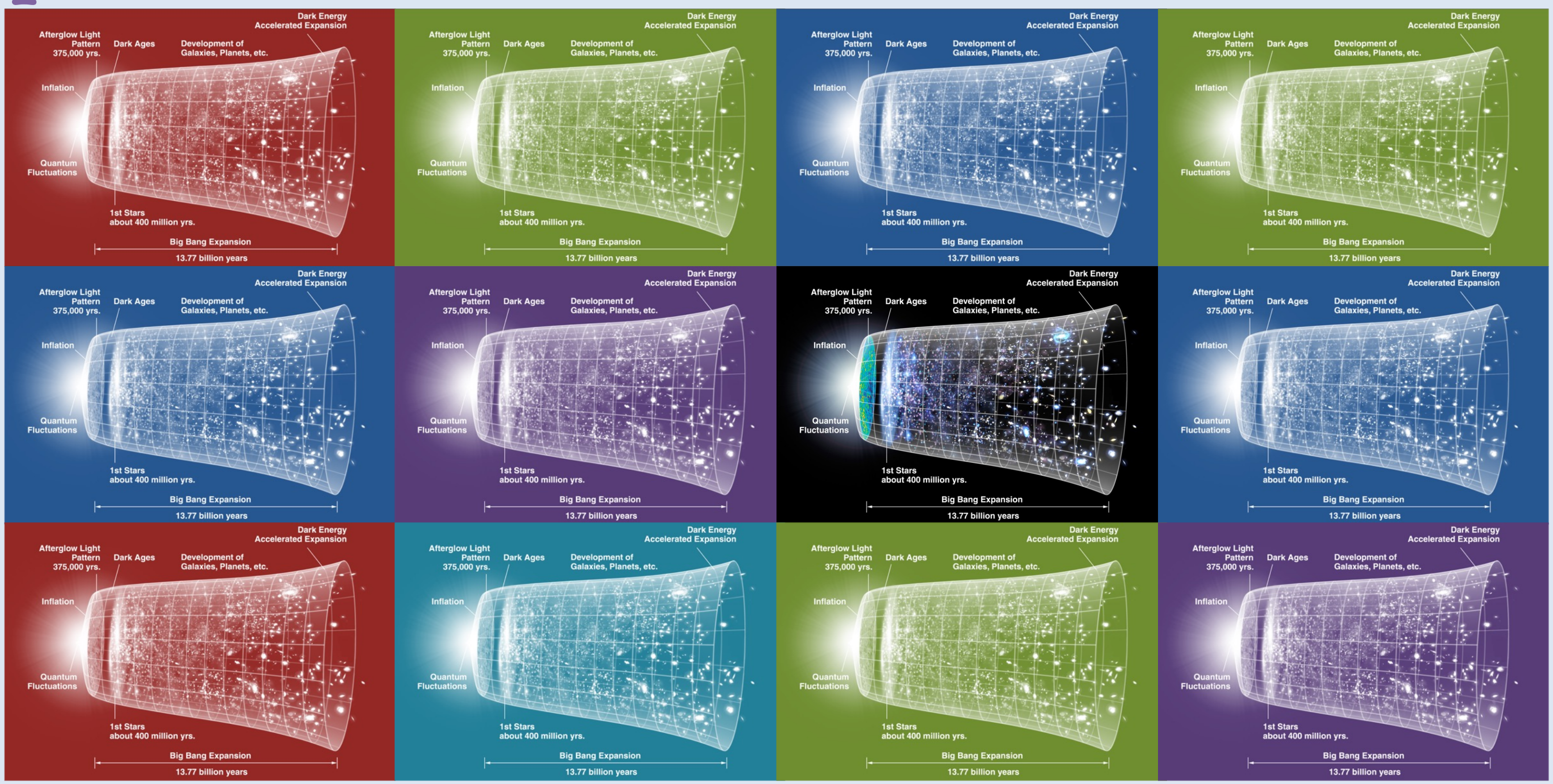
If there is a multiverse in which parameters, forces, etc are scanned then by measuring SM parameters...



how can we know if they are likely, unlikely, tuned, etc? Anthropics...?

Inflation

Instead, we need to know the macroscopic properties of the statistical ensemble



to assign context to parameters.


Towards quasi-statistics...
ESA/Hubble and Harald Süpfle

Random-ish Walks

The stochastic approach offers possibility of estimating scalar field distributions.

$$\frac{\partial}{\partial\phi}\left[\frac{\hbar}{8\pi^2}\frac{\partial(H^3\,P_{\rm FP})}{\partial\phi}+\frac{V'\,P_{\rm FP}}{3H}\right]=\frac{\partial P_{\rm FP}}{\partial t}$$

"Quantum" Gaussian Noise

EOM Drift

Time Evolution

Light scalar fields follow a Langevin-like trajectory. Average of trajectories described by a Fokker-Planck equation.

Turning the Volume Up

We're interested in the volume distribution

$$\frac{\partial}{\partial\phi}\left[\frac{\hbar}{8\pi^2}\frac{\partial(H^3\,P)}{\partial\phi}+\frac{V'\,P}{3H}\right]+3H\,P=\frac{\partial P}{\partial t}$$

"Quantum" Gaussian Noise — EOM Drift — Volume expansion — Time Evolution

Light scalar fields follow a Langevin-like trajectory. Volume average of trajectories described by a Fokker-Planck-like equation.

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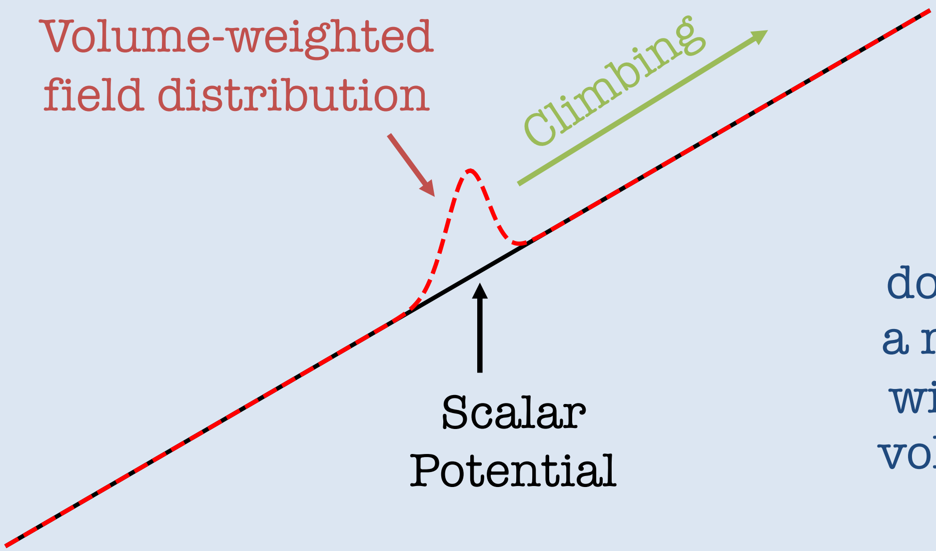
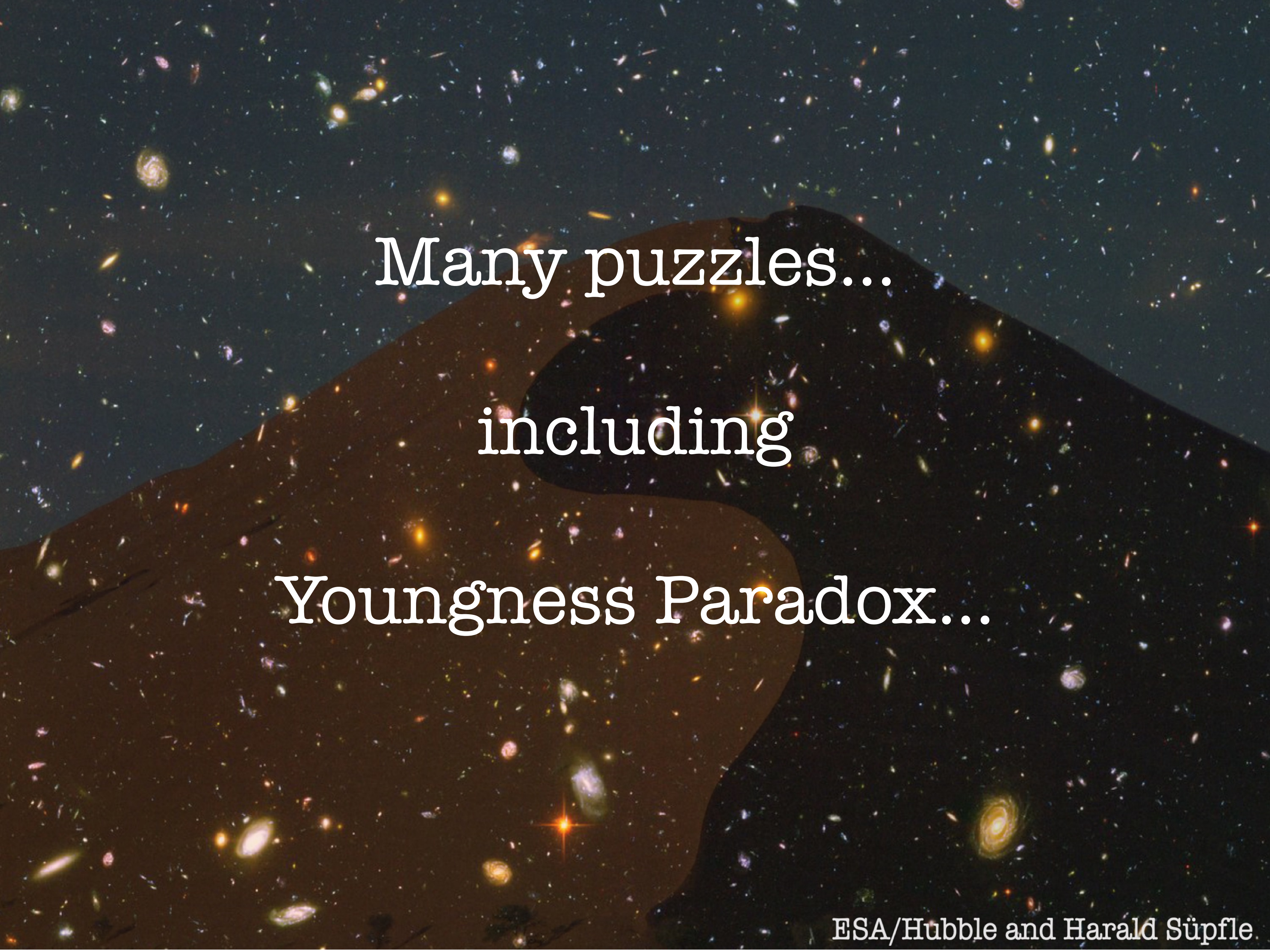


Even if a scalar field wants to roll down the scalar potential, on average, a rare upward fluctuation is rewarded with exponential growth. As a result, volume-weighted field distribution can climb up a potential!


Many puzzles...
including
Youngness Paradox...
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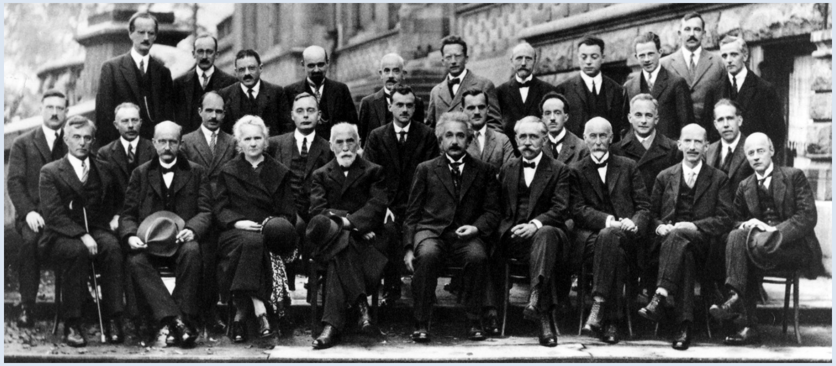
Personal View

Past breakthroughs were made by venturing into incomplete frameworks, often even involving unregulated infinities. QM, QFT...



Given the success of symmetry-based approaches in taking us beyond the SM, perhaps we need to spend more time in uncharted territory?

Criticality and Eternal Inflation…

Our Setup

Suppose the background parameters are controlled by some scalar field. In general language:

$$V = 3H_0^2 M_P^2 + g_\epsilon^2 f^4 \omega(\varphi) \ , \qquad \omega(\varphi) = \sum_{n=1}^{\infty} \frac{c_n}{n!}\varphi^n \ , \qquad \varphi \equiv \frac{\phi}{f} \ , \qquad \omega(0) = 0$$

When scalar potential is a small perturbation across the field range can expand perturbatively to find:

$$\frac{\alpha}{2}\frac{\partial^2 P}{\partial \varphi^2} + \frac{\partial\left(\omega' P\right)}{\partial \varphi} + \beta \omega P = \frac{\partial P}{\partial T}$$

Where:

$$\alpha = \frac{3\hbar H_0^4}{4\pi^2 g_\epsilon^2 f^4} \ , \qquad \beta = \frac{3\xi f^2}{2M_P^2} \ , \qquad T = \frac{t}{t_R} \ , \qquad t_R = \frac{3H_0}{g_\epsilon^2 f^2}$$

Quantum — Range — Clock — Timescale

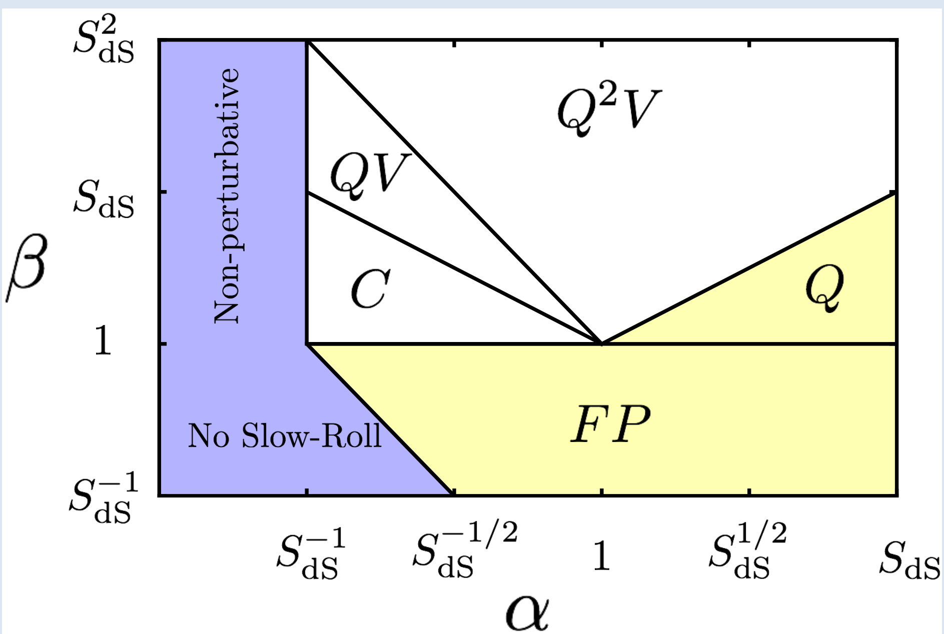
In Stasis

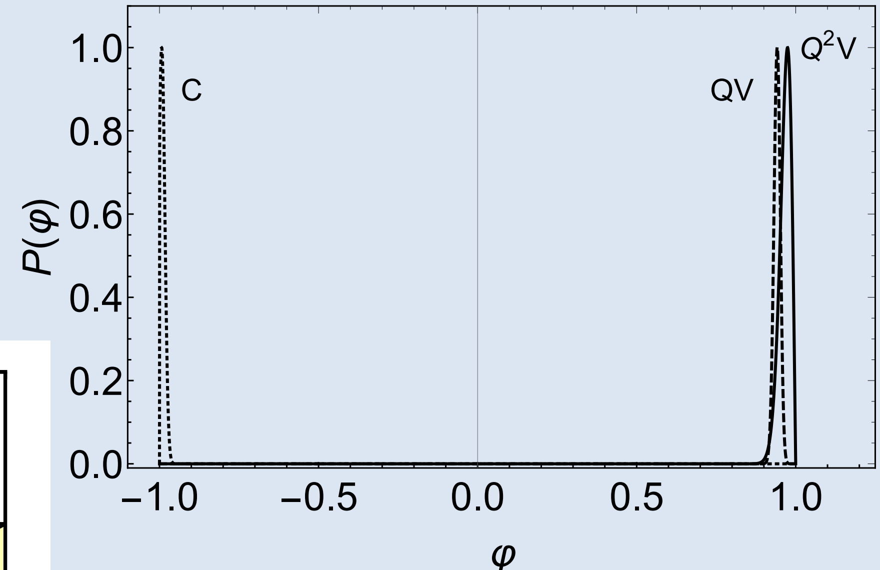
On a linear slope we identify three distinct parameter regimes:

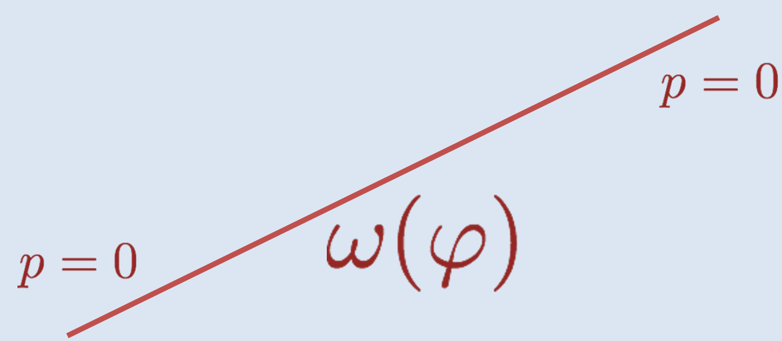
$$C : \alpha\beta < 1$$

$$QV : \alpha\beta > 1$$

$$Q^2V : \alpha^2\beta > 1$$







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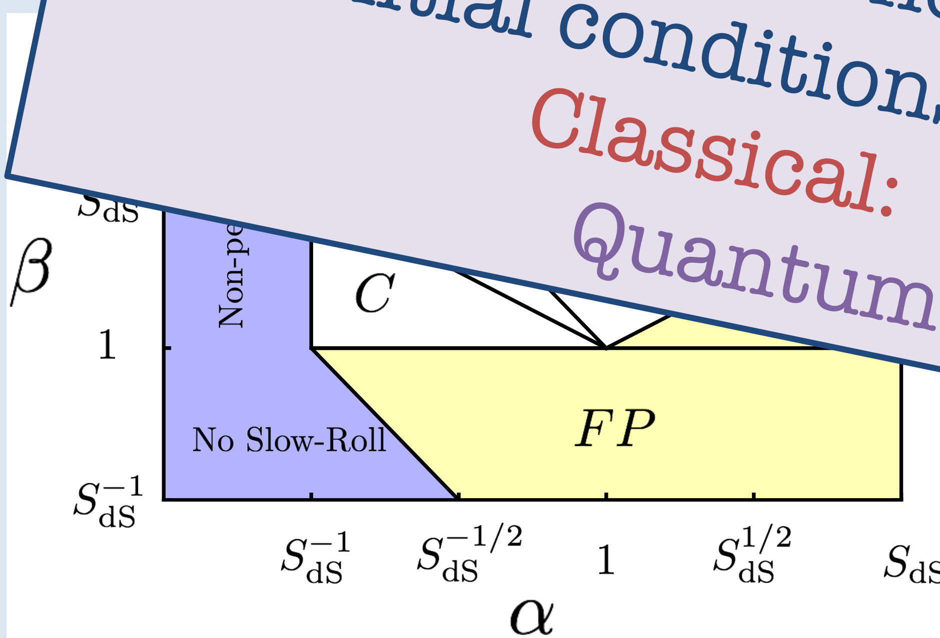
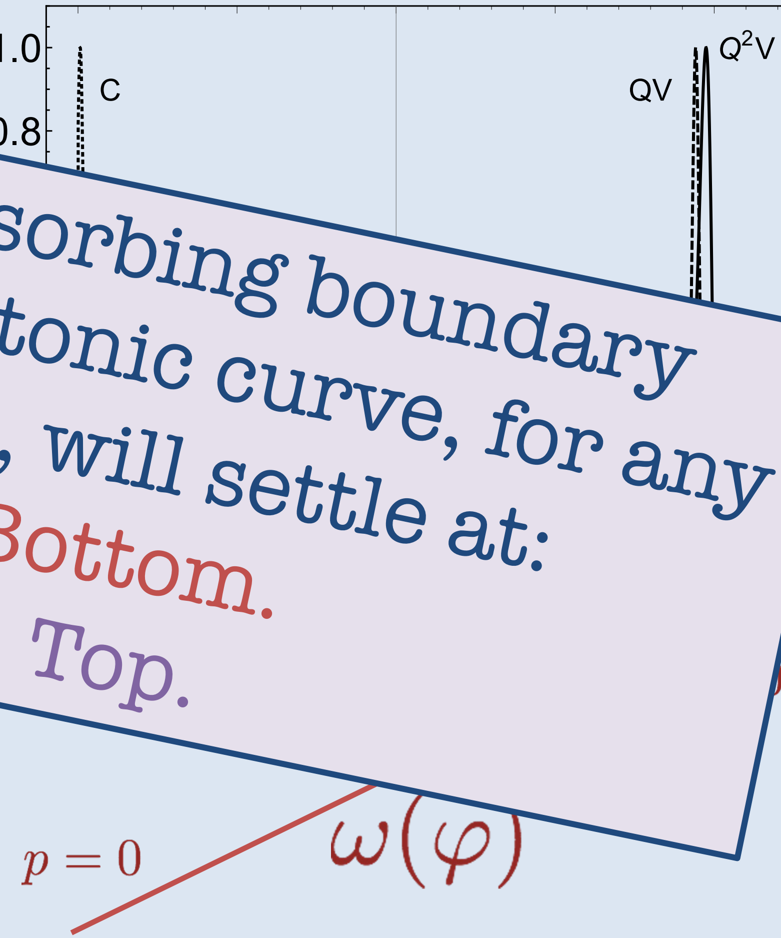




Punchline: For absorbing boundary conditions on a monotonic curve, for any initial conditions, will settle at:

Classical: Bottom.

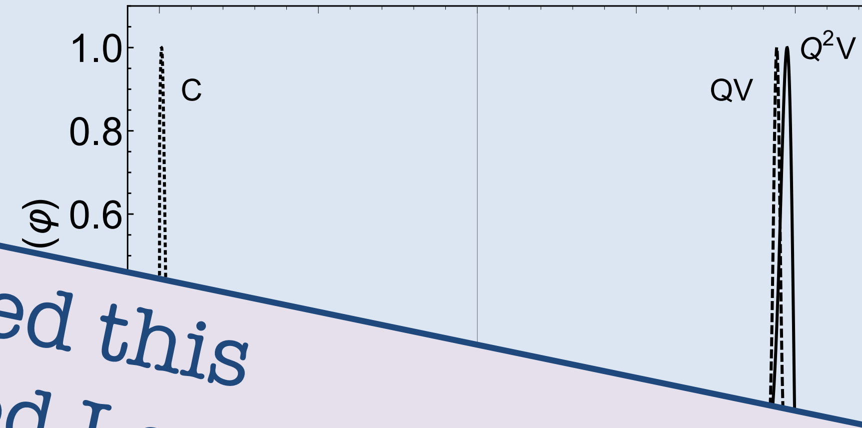
Quantum: Top.

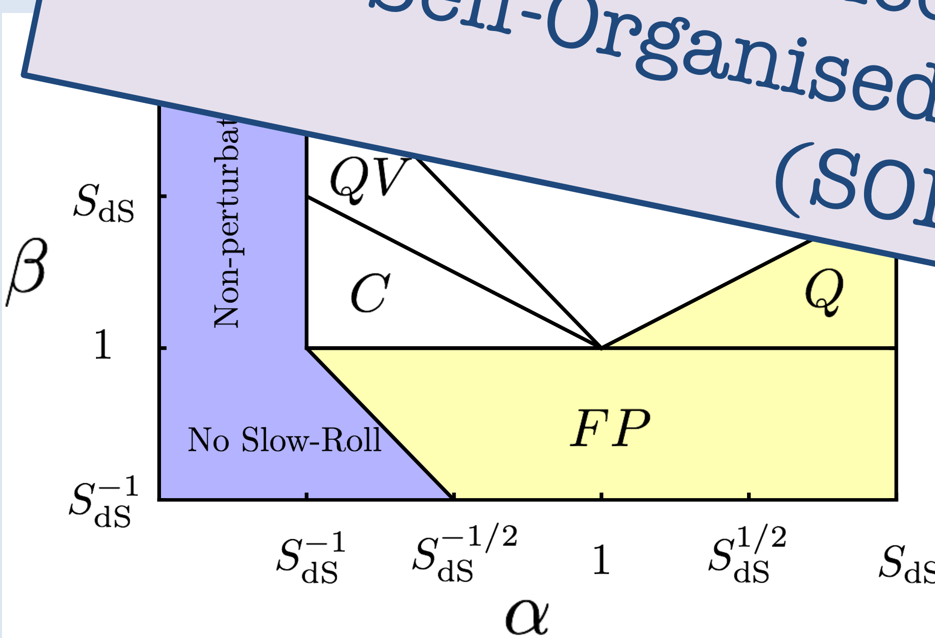
In Stasis

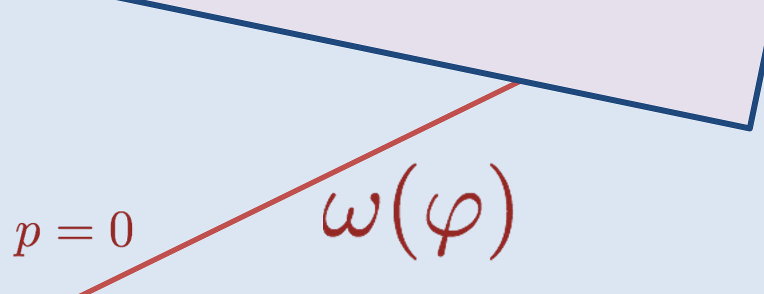
On a linear slope we identify three distinct parameter regimes:

$$C : \alpha\beta < 1$$

$$\beta > 1$$







Called this “Self-Organised Localisation” (SOL)

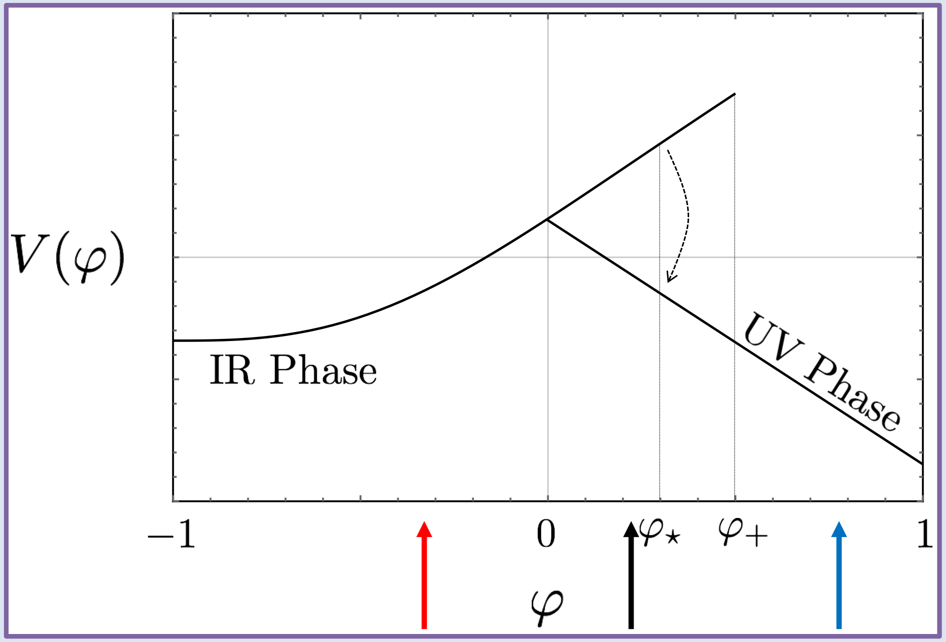
Putting this to use...

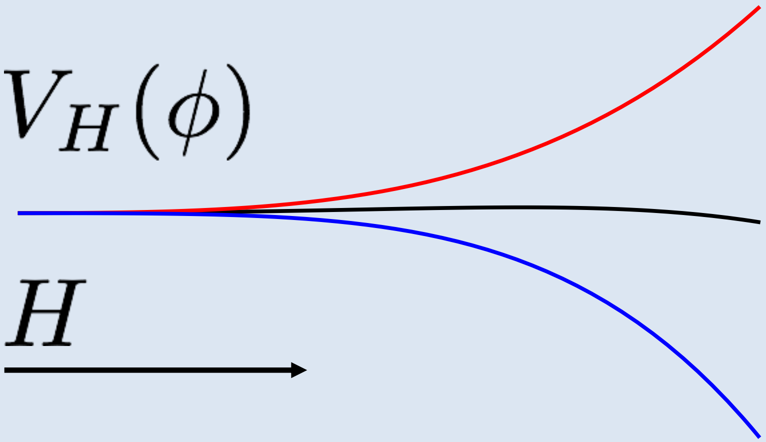
Application: SM Quartic

Suppose the scalar is scanning the SM parameters. In particular, the Higgs quartic, consistent with EFT

$$V(\varphi, h) = \frac{M^4}{g_*^2}\,\omega(\varphi) + \frac{\lambda(\varphi, h)}{4}\left(h^2 - v^2\right)^2$$

Scalar potential is:





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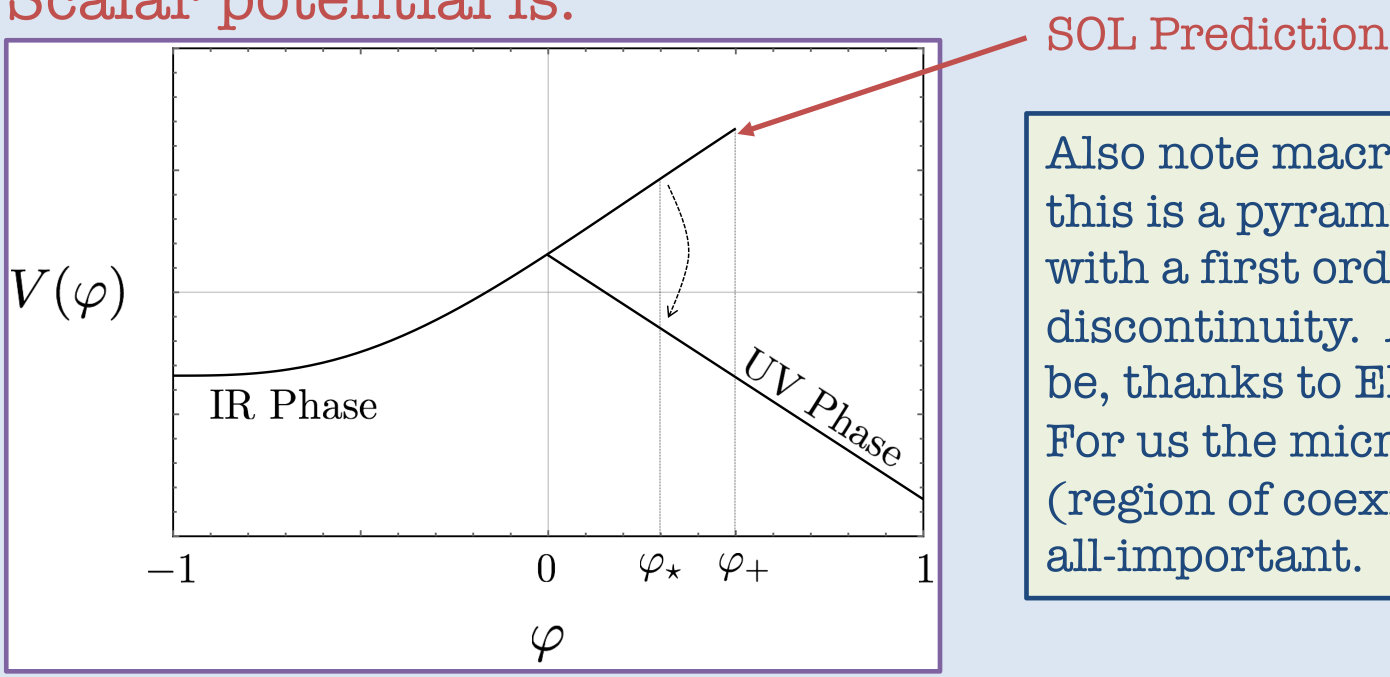
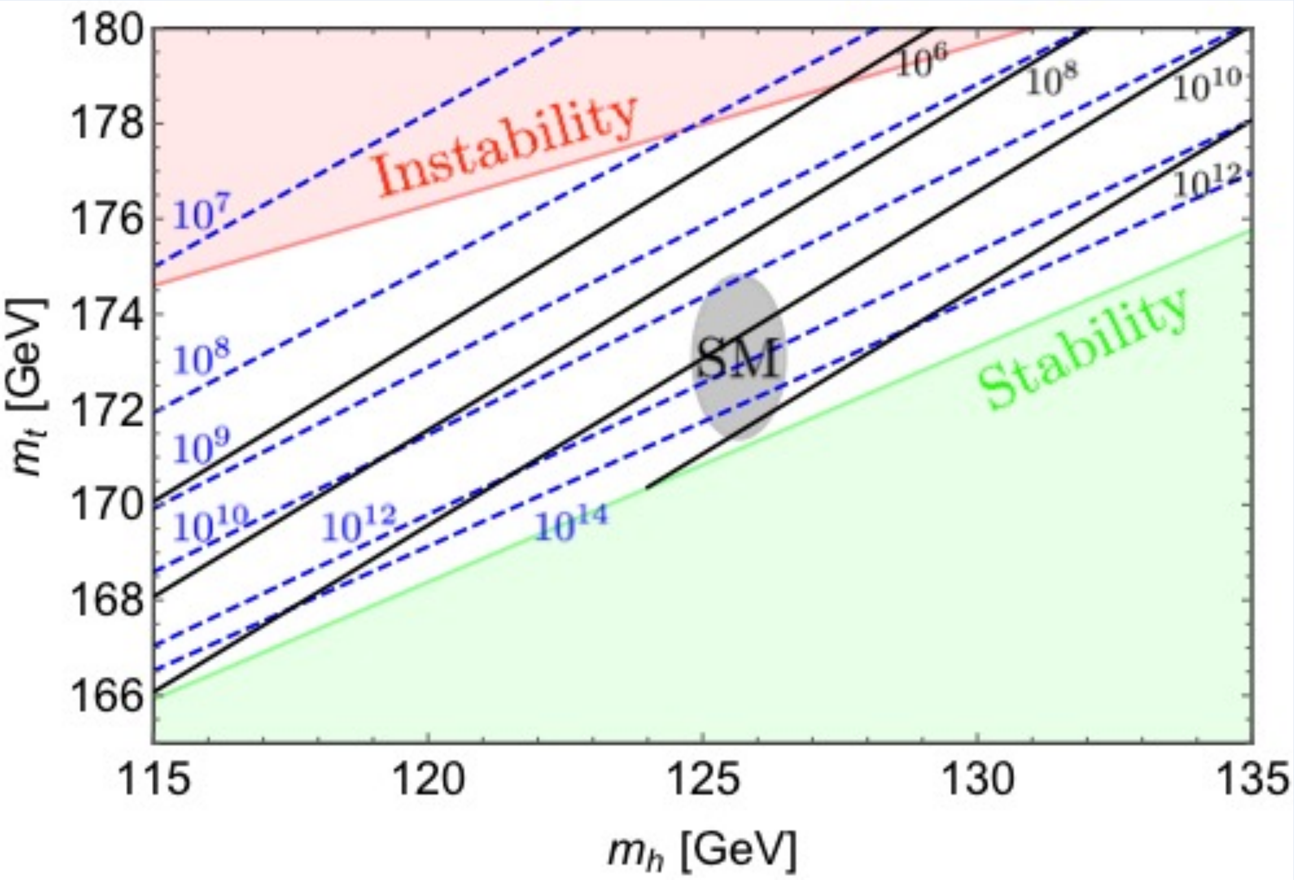


Also note macroscopically this is a pyramid potential with a first order discontinuity. As it should be, thanks to Ehrenfest. For us the microscopics (region of coexistence) is all-important.

Application: SM Quartic

Prediction is metastability region, since top of potential:



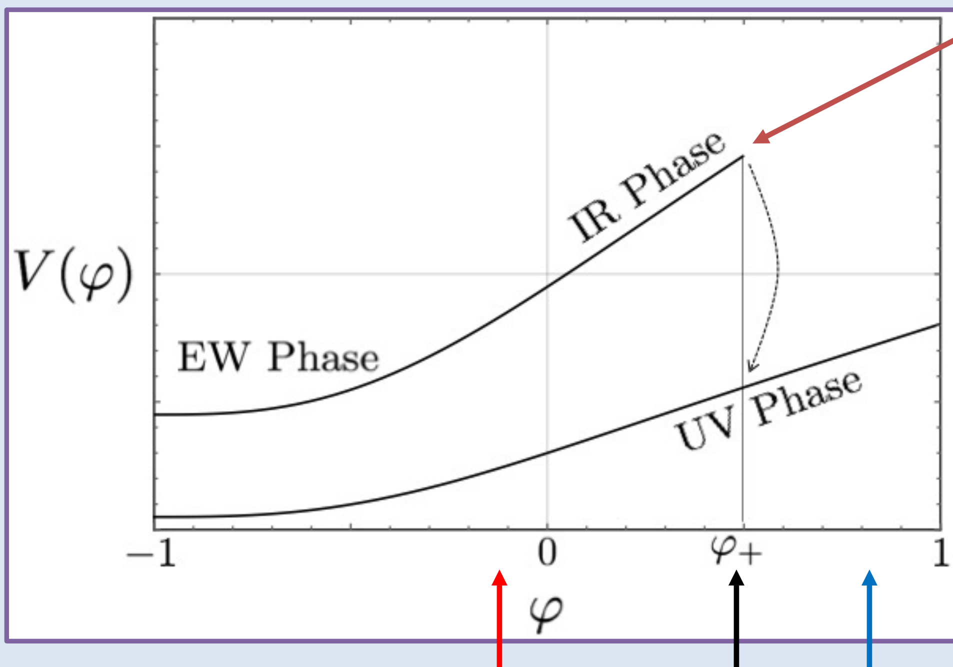
Blue is instability scale, black fixed Hubble contours.

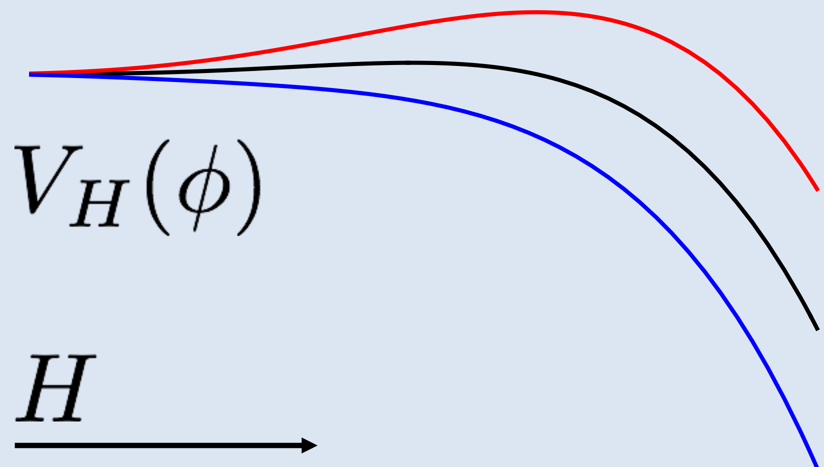
Application: SM Naturalness

Consider the same Higgs instability question, but with a field-dependent mass bilinear:

$$V(\varphi, h) = \frac{M^4}{g_*^2}\,\omega(\varphi) - \frac{\varphi M^2 h^2}{2} + \frac{\lambda(h)\,h^4}{4}$$

Scalar potential is:

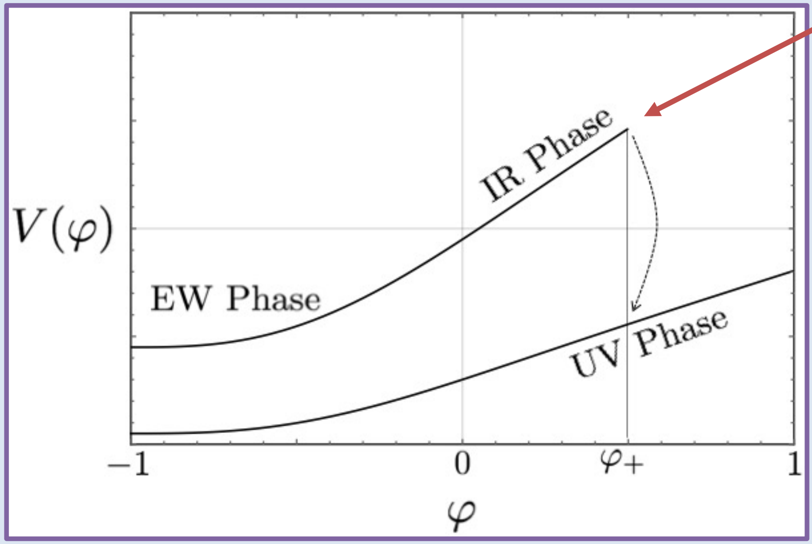




Application: SM Naturalness

Within the SM alone the SOL prediction is the SM instability scale. This is remarkable: Quartic running generates an exponential scale separation between cutoff and instability scale.

SOL Prediction



But the instability scale is still way above the weak scale…

$$\Lambda_{\rm Inst} \sim 10^{10}\ {\rm GeV}$$

Summary

Whatever your thoughts on eternal inflation.

Whatever your thoughts on eternal inflation.

Whatever your thoughts on eternal inflation.

Self-Organised Localisation could offer answers as to why our Universe is so determinedly critical.

O.T.' R.U. ATOME?. $\iint$ Sphar. dS was done in the most general form in 1867. I have now bagged ξ & η from T & T' and done the numerical value of $\iint (Y_i^{(s)})^2 dS$ in 4 lines, thus verifying T+T''s value of $\iint (\mathfrak{D}_i^{(s)})^2 dS$

Your plan seems indep^t of T + T' or of me. Publish!

I am busy supplying the physical necessities of scientific life at Dens 11 Scroope Terrace, Cambridge. Prooves have got as far as grooves, corrugated plates, gratings and rings. If you have time for criticism then

$$\iint (Y_i^{(s)})^2 dS = \frac{8\pi a^2}{2i+1} \frac{\underline{|i+s} \; \underline{|i-s}}{2^{2s} \; \underline{|i} \; \underline{|i}}$$

except when $s=0$ when $\iint (Q_i)^2 dS = \frac{4\pi a^2}{2i+1}$

Hence
$$\int_{-1}^{+1} (\mathfrak{D}_i^{(s)})^2 d\mu = \frac{2}{2i+1} \frac{2^{2s} \underline{|i-s} \; \underline{|s} \; \underline{|s}}{\underline{|i+s}}$$ without exception

see $\frac{dp}{dq}$

The Good

We're interested in the volume distribution

$$\ldots \partial(H^3 P) \ldots + \frac{V' P}{\partial H \ldots}\Big] + 3HP = \frac{\partial P}{\partial t}$$

We're interested in the equilibrium solutions, where time-dependence is trivial $\dot{P} \propto P$, since these are the final states, independent of initial conditions.

Scalar field follo... trajectory. Volume average... described by a Fokker-Planck-like equation.

The Good

We're interested in the volume distribution

$$\cdots \frac{\partial(H^3\,P)}{\cdots} + \frac{V'\,P}{\cdots}\Bigg] + 3HP = \frac{\partial P}{\partial t}$$

Boundary conditions are just as much a part of the theory as the equation itself. I won't always explicitly state them, but ask at any point about sensitivity to BCs.

Scalar field follo… trajectory. Volume average… described by a Fokker-Planck-like equation.

The Good?

In our applications to reach steady state we have to wait for

$$N > S_{\mathrm{dS}} = \frac{8\pi^2 M_P^2}{\hbar\, H^2}$$

e-foldings.

This necessarily implies eternal inflation. (See e.g. Arkani-Hamed, Dubovsky, Senatore, Villadoro).

The Bad

We have to choose a clock:

$$dt_\xi / dt = (H/H_0)^{1-\xi}$$

Because Hubble depends on the scalar field, the clock can involve field dependence. For example, evolution w.r.t. proper time is not the same as scale factor.

$$\frac{\partial}{\partial\phi}\left[\frac{\hbar}{8\pi^2}\frac{\partial(H^{2+\xi}\,P)}{\partial\phi} + \frac{V'\,P}{3H^{2-\xi}}\right] + 3H^{\xi}P = H_0^{\xi-1}\,\frac{\partial P}{\partial t_\xi}$$

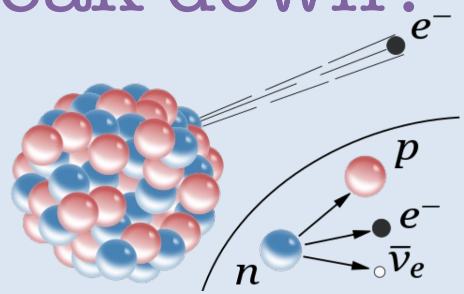
Worse

Quantum corrections accumulate to large effect on classical system, essentially because field climbs the potential. Does semi-classics break down?

Similar to the page time for BHs. For BHs it seems semi-classics remains valid, but new field configurations may be important.

Worse

Quantum corrections accumulate to large effect on classical system, essentially because field climbs the potential. Does semi-classics break down?



Take a large quantity of radioactive material that beta-decays. EFT valid for each decay, yet after long enough system is completely changed.

Even Worse

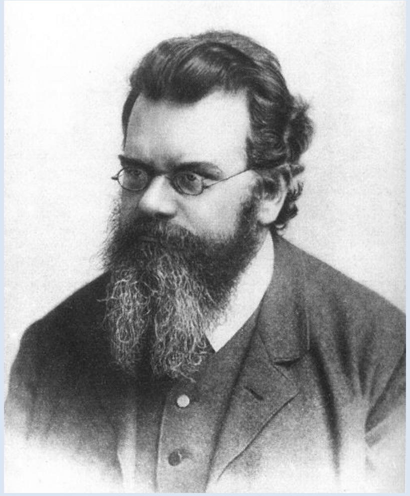
In inflation reheating occurs when the inflaton passes the "reheating surface". Can perform statistics by studying this surface.

$$\frac{\infty}{\infty}$$

But in eternal inflation reheating surface is infinite. Must be regulated somehow. Results depend on measure...

The Worst

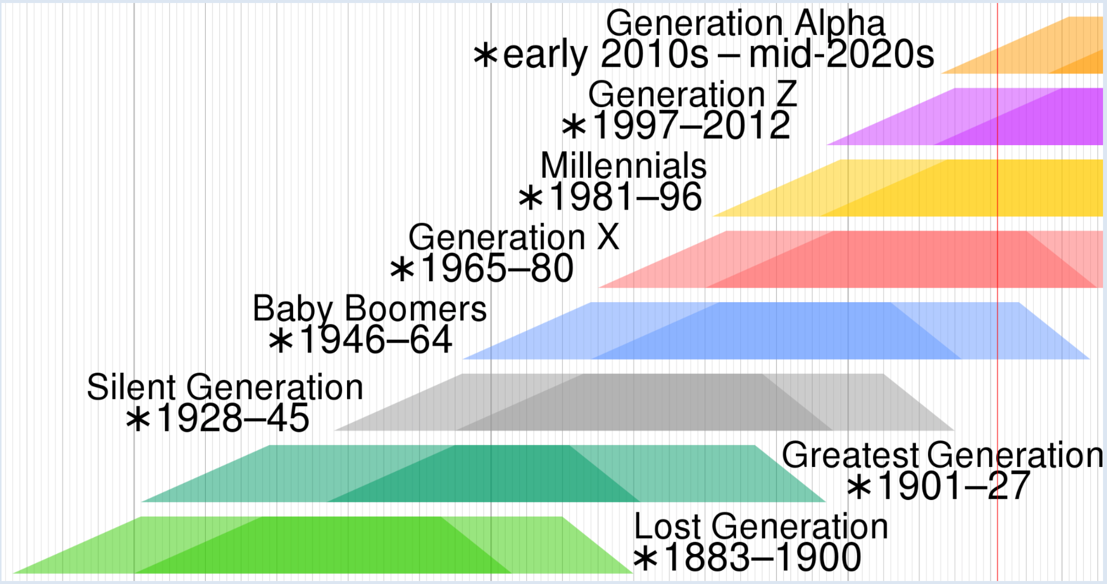
Since we don't commit to a specific inflationary model, we take proper time as our time-slicing measure.



Issues such as Boltzmann brains arise in this case. May not apply to our applications though, as all our Universes are unstable...

The Worst

The youngness paradox is much more severe. Emphasised to us by Andrei Linde.



Proper Time Prediction

Our Universe

Universe should be much younger and hotter if proper time cutoff naively extrapolated to our time.

The Ugly

Our scenario (and most of inflationary parameter space) is in The Swampland.
De Sitter conjecture is:

$$|\nabla V| > c\frac{V}{M_P}$$

Slow-roll parameter is:

$$\epsilon = \frac{M_P^2\, V'^2}{2\, V^2}$$

Clear tension...

The Ugly

Our scenario (and most of inflationary parameter space) is in The Swampland.

D... conjecture is:

V ...

Also, in all our models, the field range is super-Planckian, violating distance conjecture too.

Slow-roll parame...

$$\epsilon = \frac{M_P^2 \, V'^2}{2 \, V^2}$$

Clear tension...

In Stasis

In stasis the solution is an eigenstate of time. Subject to BCs, field distribution is a solution of:

$$\frac{\alpha}{2}p'' + \omega' p' + (\omega'' + \beta\omega - \lambda)\, p = 0$$

Reminder:

$$\alpha = \frac{3\hbar H_0^4}{4\pi^2 g_\epsilon^2 f^4}\,, \qquad \beta = \frac{3\xi f^2}{2M_P^2}\,, \qquad T = \frac{t}{t_R}\,, \qquad t_R = \frac{3H_0}{g_\epsilon^2 f^2}$$

Quantum | Range | Clock | Timescale

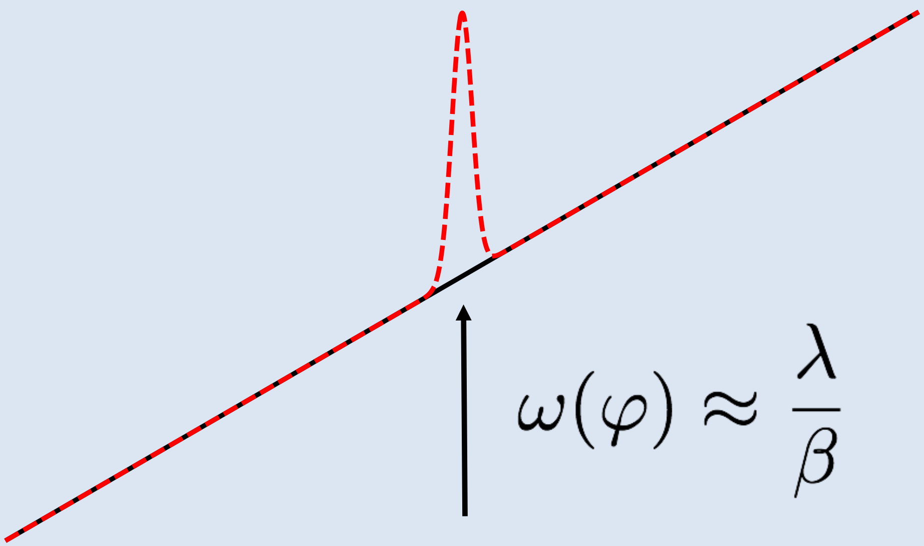
A peak, if it exists, will have position determined by the inflationary rate.

In Stasis

In stasis the solution is an eigenstate of time. Subject to BCs, field distribution is a solution of:

$$\frac{\alpha}{2}\,p'' + \omega' p' + (\omega'' + \beta\omega - \lambda)\,p = 0$$

A peak, if it exists, will have position determined by the inflationary rate (eigenvalue):



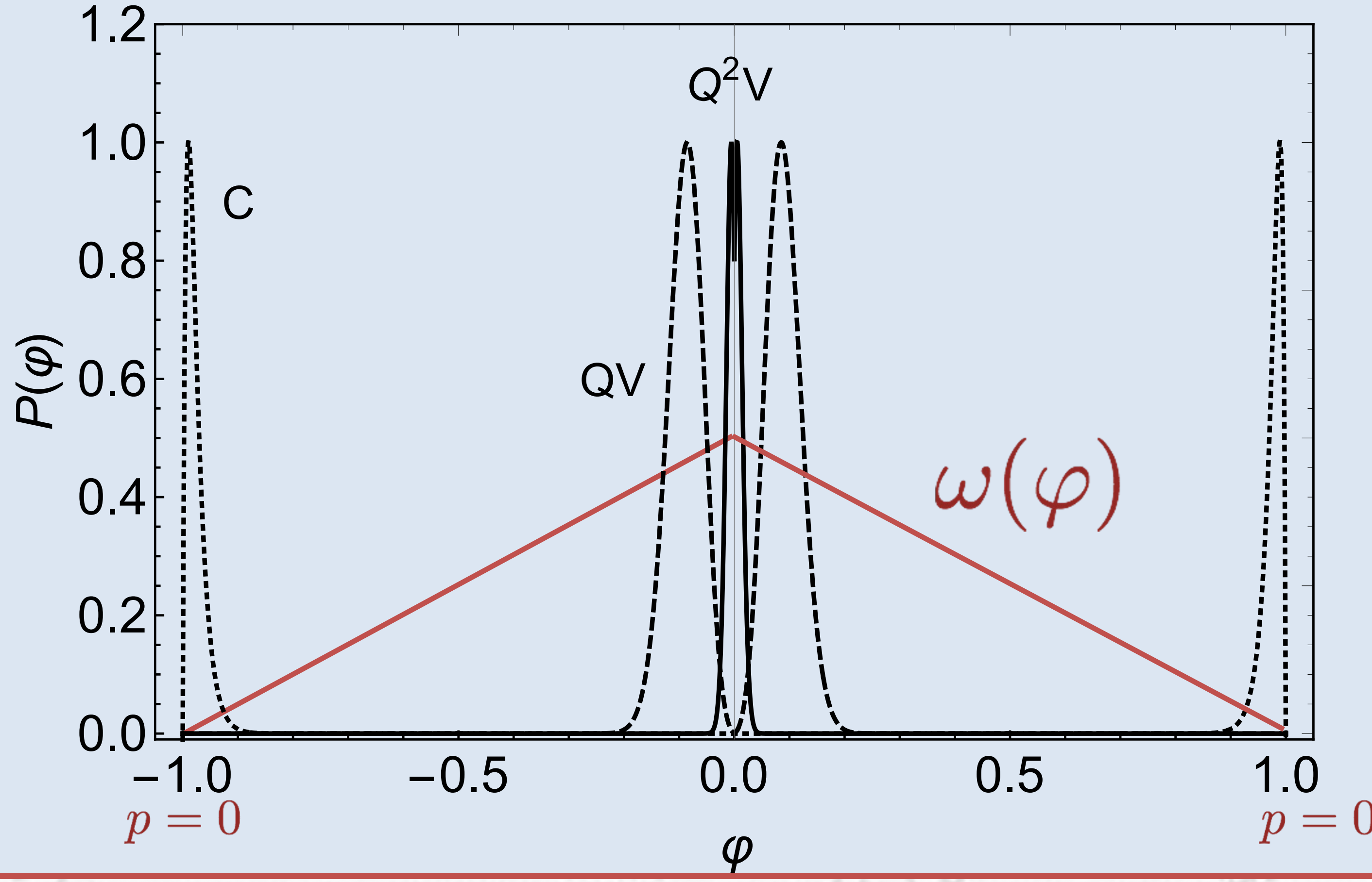
$$\omega(\varphi) \approx \frac{\lambda}{\beta}$$

This is very intuitive. If the field is localised at some position, the vacuum energy in slow roll is just the height it is localised at.

The vacuum energy is the inflationary rate.

In Stasis

On a pyramid we identify the same three distinct parameter regimes:



In Stasis

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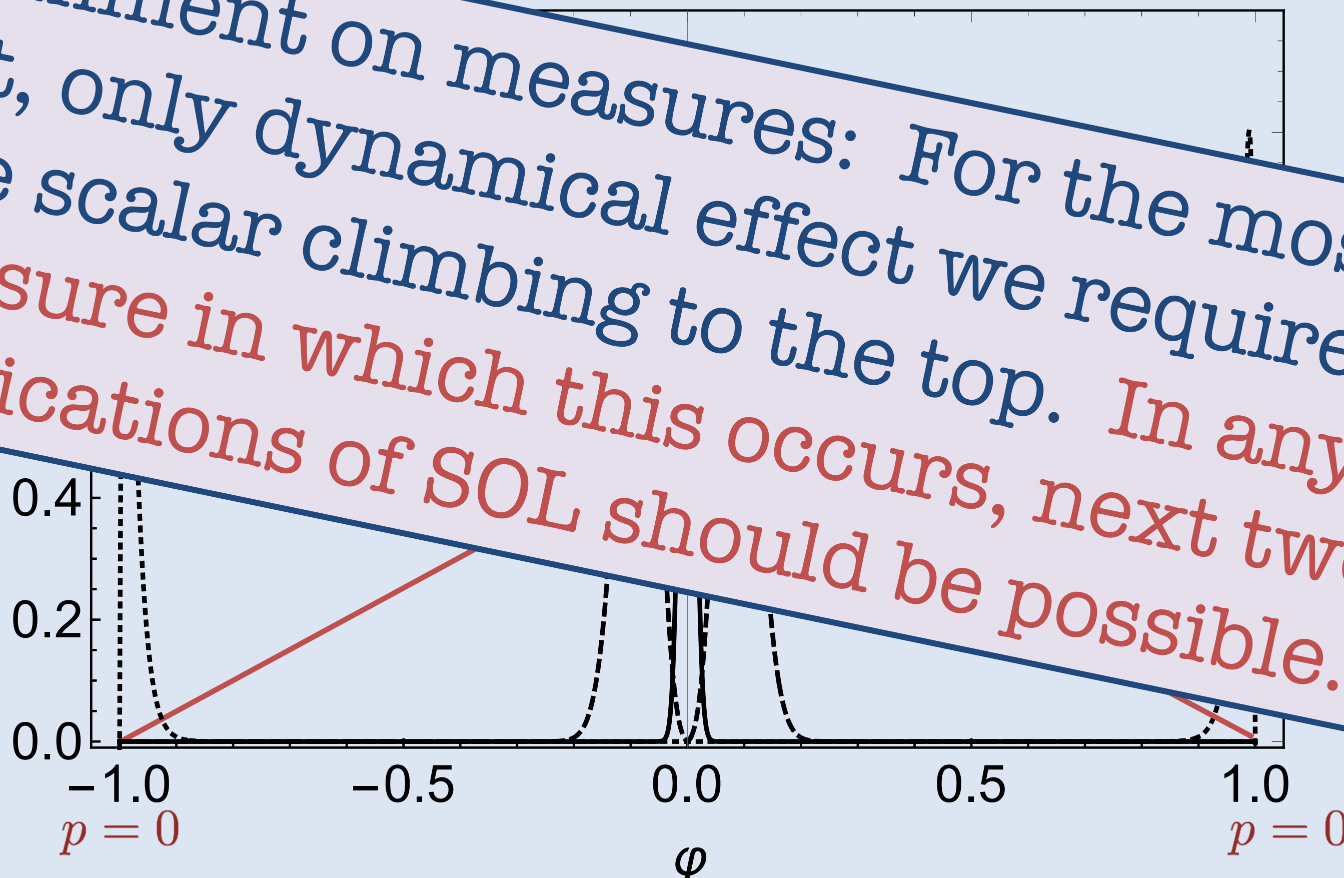


Comment on measures: For the most part, only dynamical effect we require is the scalar climbing to the top. In any measure in which this occurs, next two applications of SOL should be possible.

How long until stasis?

If we are going to reach a relatively stationary state, in order to be independent of boundary conditions we have to wait a time

$$\alpha\beta t_S \quad , \quad \alpha\beta < 1$$

in the classical regime, where t_S is the entropy-bound timescale, and

$$\sqrt{\alpha\beta} t_S \quad , \quad \alpha\beta > 1$$

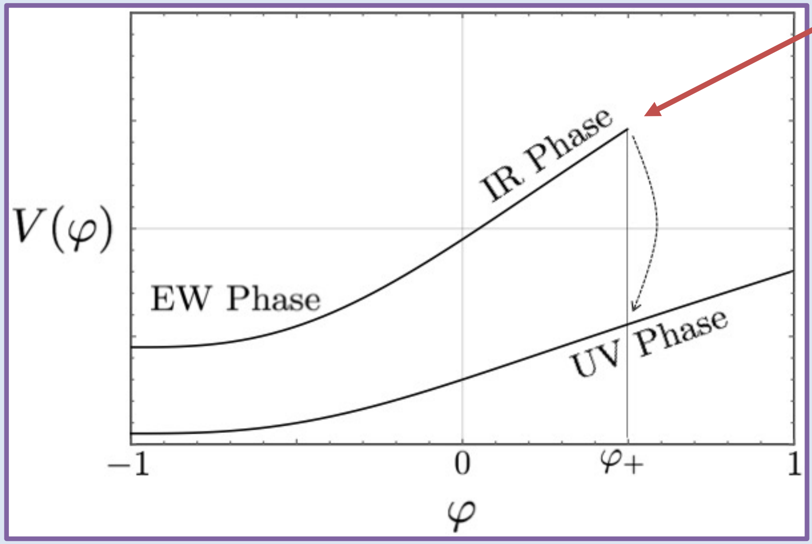
in the quantum regime.

Sherpas are eternal...

Application: SM Naturalness

Within the SM alone the SOL prediction is the SM instability scale. This is remarkable: Quartic running generates an exponential scale separation between cutoff and instability scale.

SOL Prediction

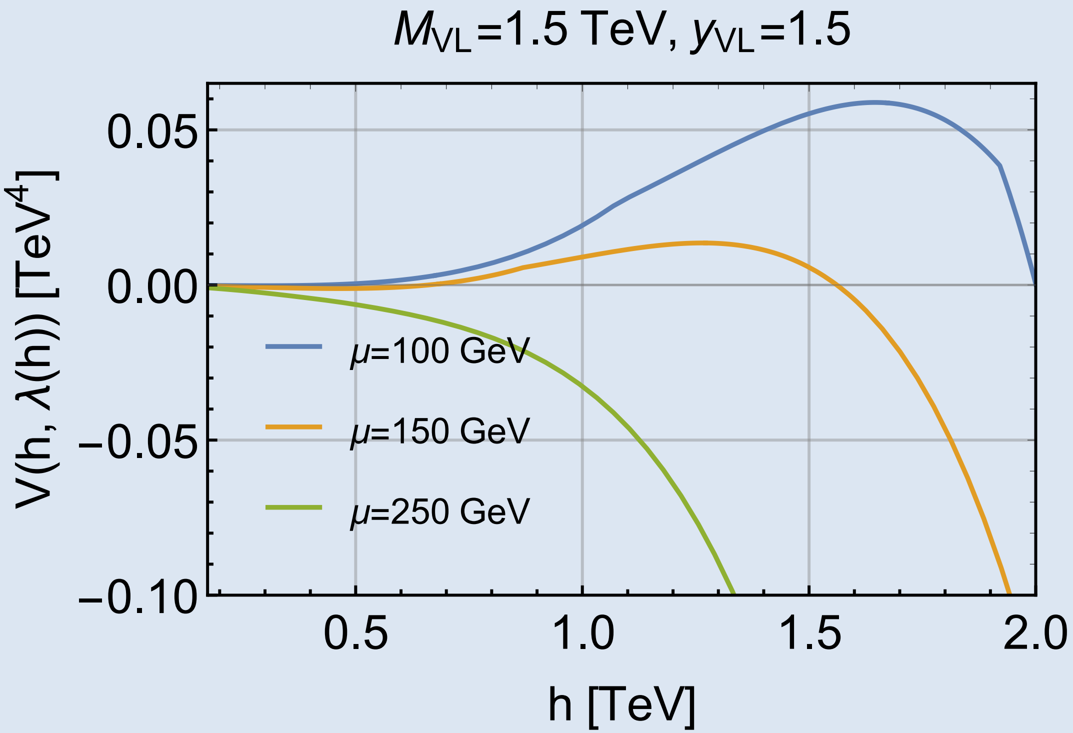


But the instability scale is still way above the weak scale…

$$\Lambda_{\rm Inst} \sim 10^{10}\ \text{GeV}$$

Junk Food

If we add (naturally) light vector-like-fermions coupled to the Higgs the instability scale can be brought down significantly, just about consistent with SM stability.



Yet another motivation for new states near the TeV scale.

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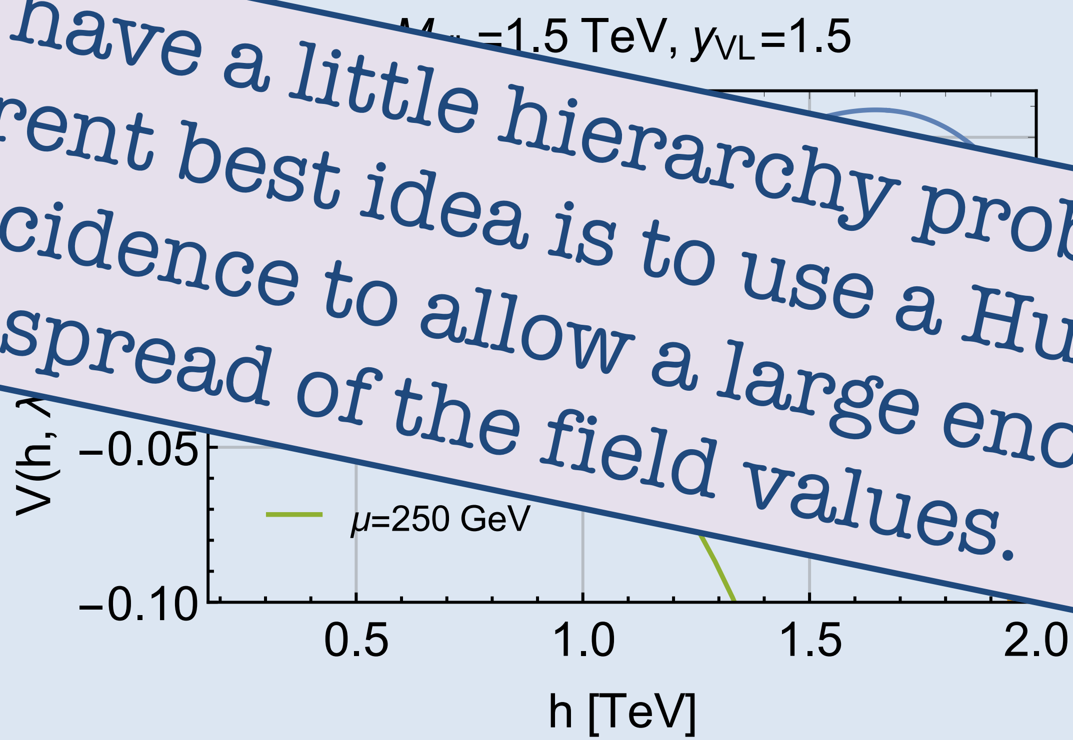


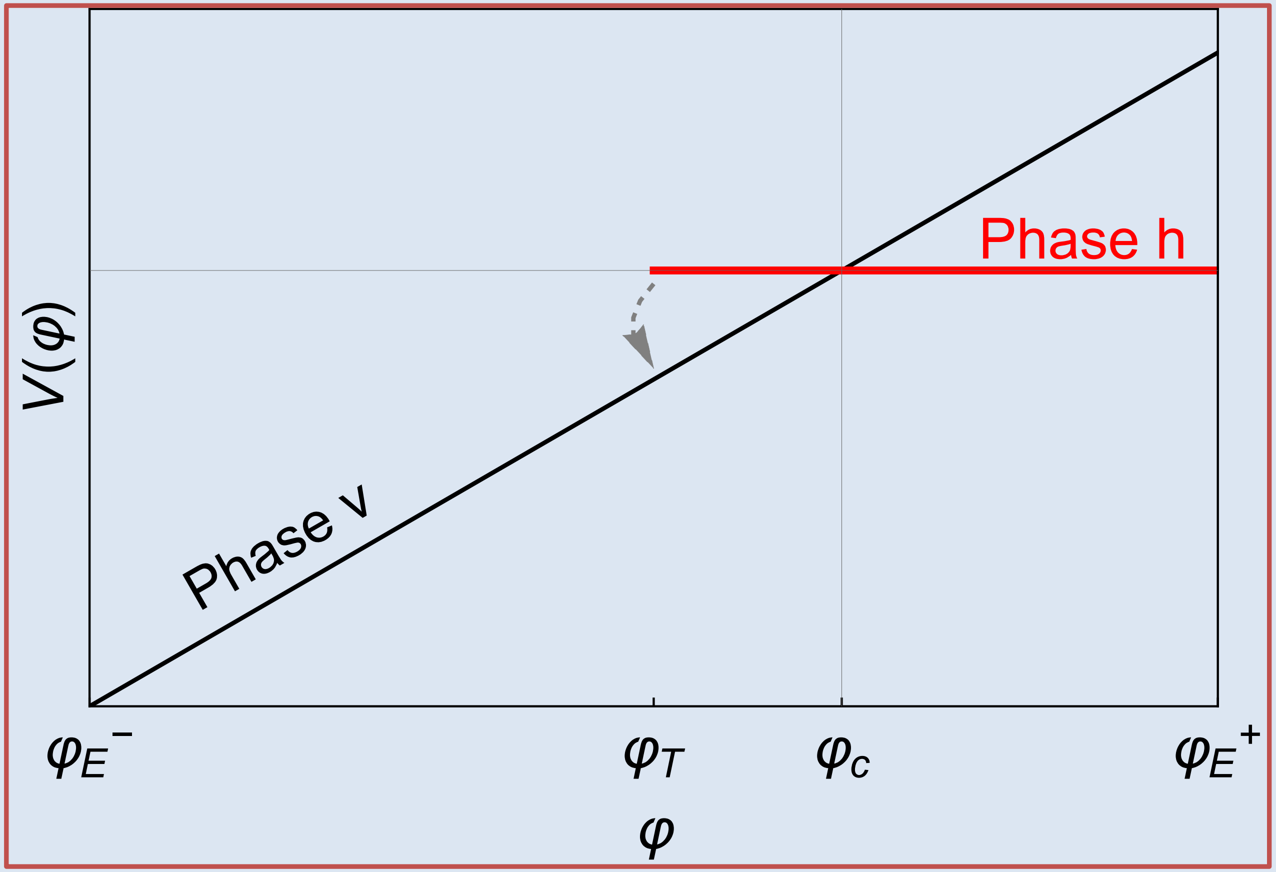
Still have a little hierarchy problem… Current best idea is to use a Hubble-coincidence to allow a large enough spread of the field values.

Yet another motivation for new states near the TeV scale.

From quite speculative, to spectacularly speculative...

Waterfalls

Consider the following "Waterfall" scalar potential:



What does the field distribution look like in steady state?

Waterfalls

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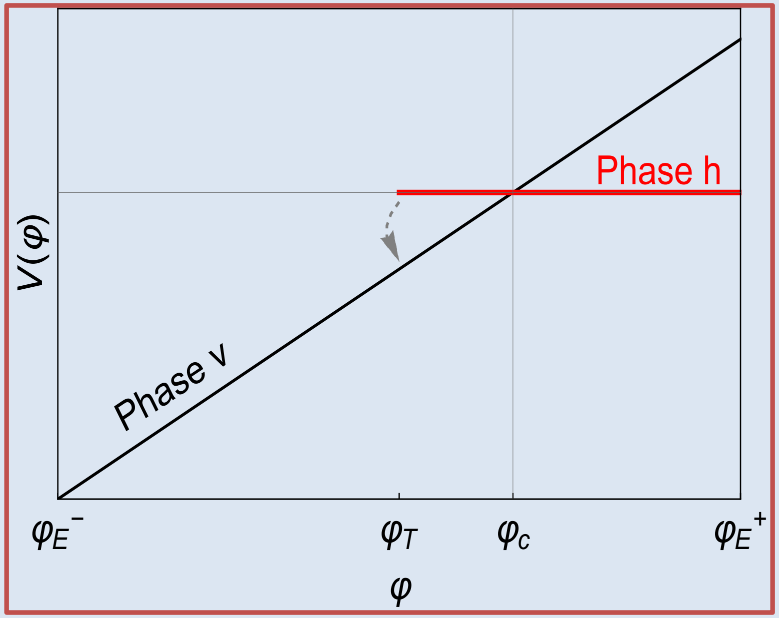

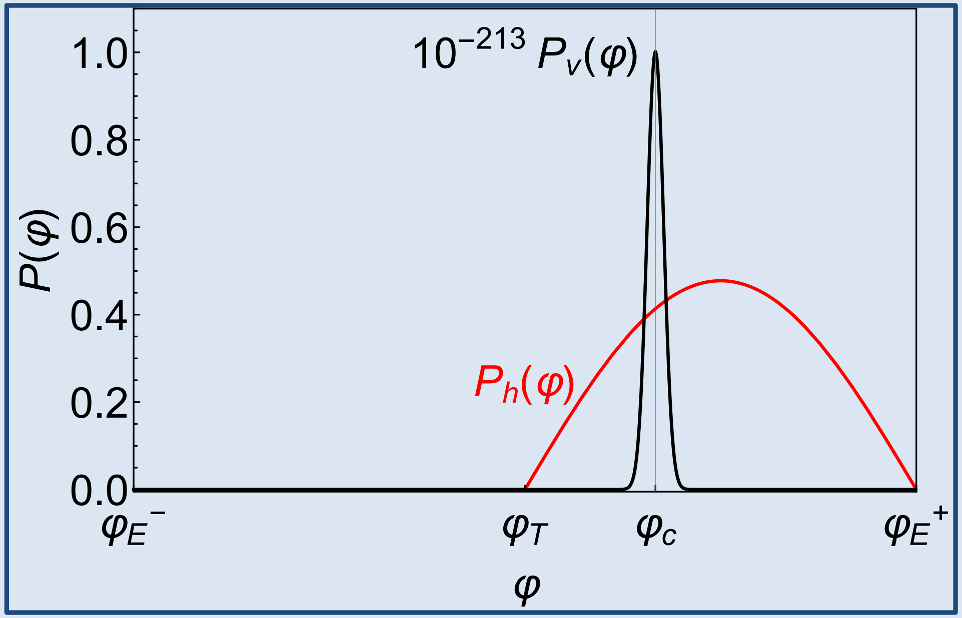

In steady state the solution on the "v" branch is localised at the point where it crosses the same height at the "h" branch, even though tunneling far away.

Waterfalls

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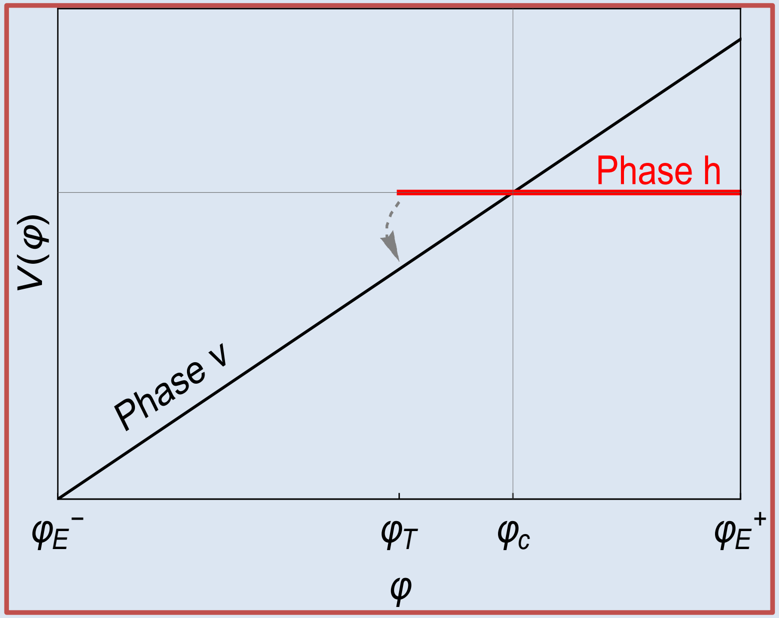

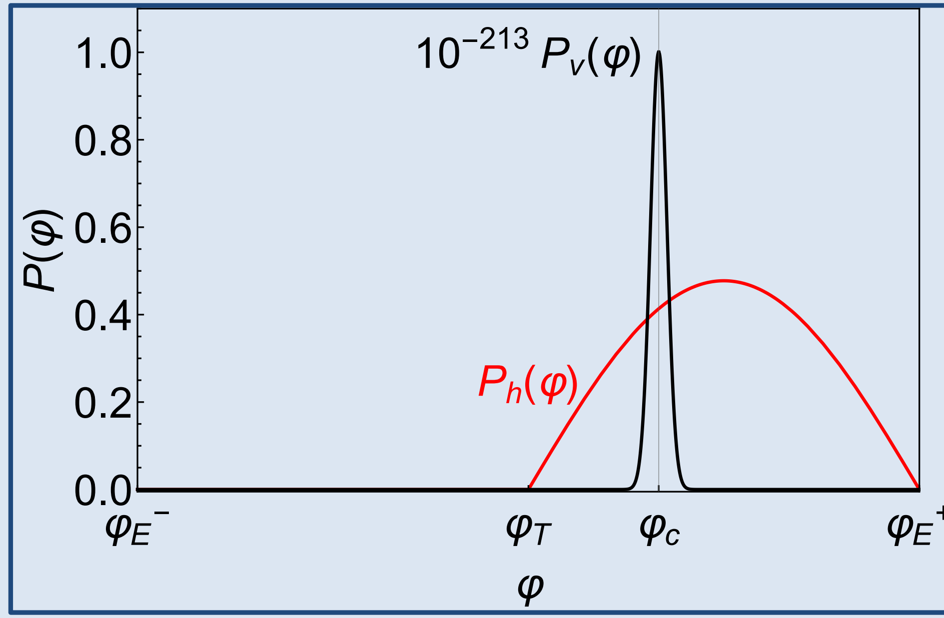

Reason for this is simple. To have non-zero solutions on both branches in steady state, they must both, on average, inflate at the same rate = same Hubble.

Waterfalls

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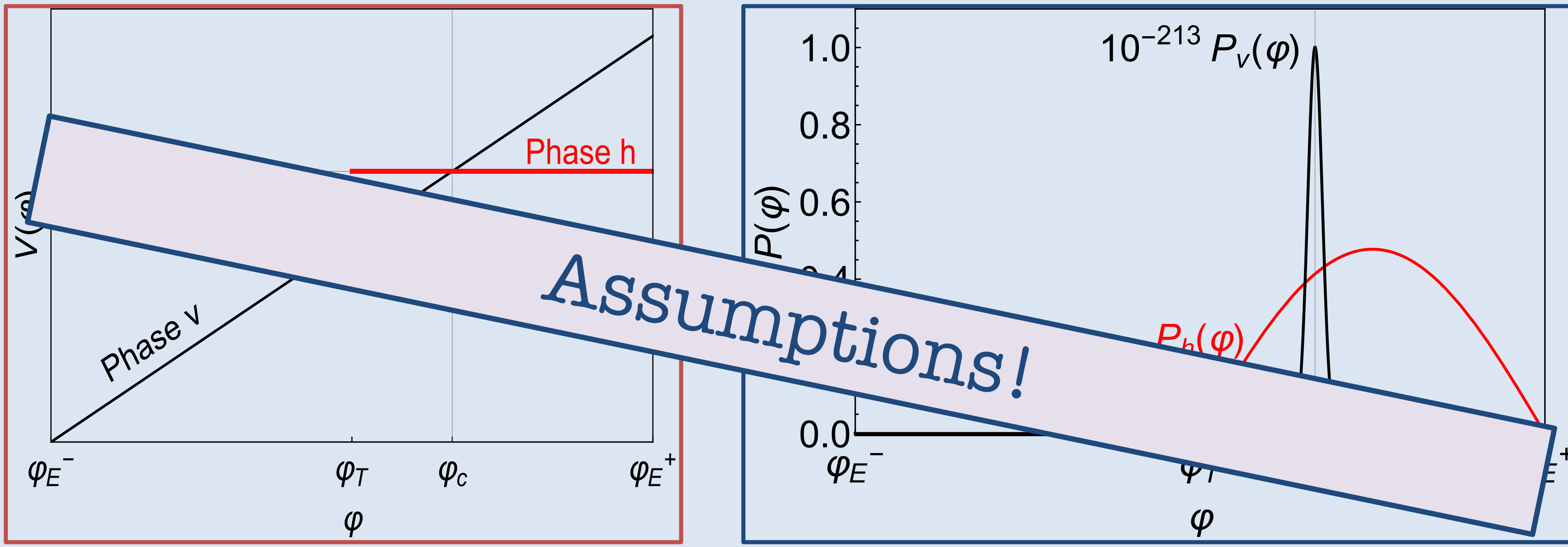

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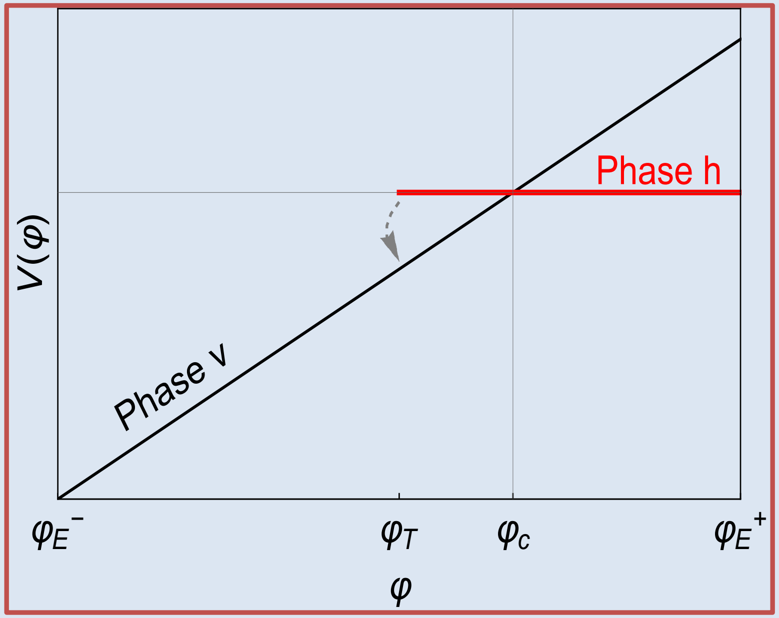

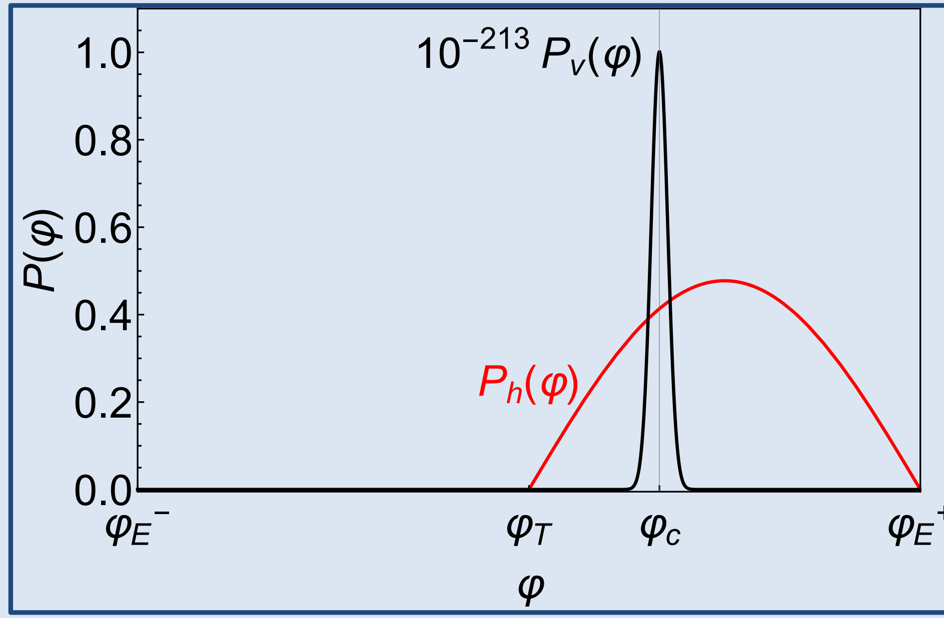

- "v" has non-zero flux injected at the top.
- "h" branch also has flux.
- (No significant sensitivity to these BCs though.)

Waterfalls

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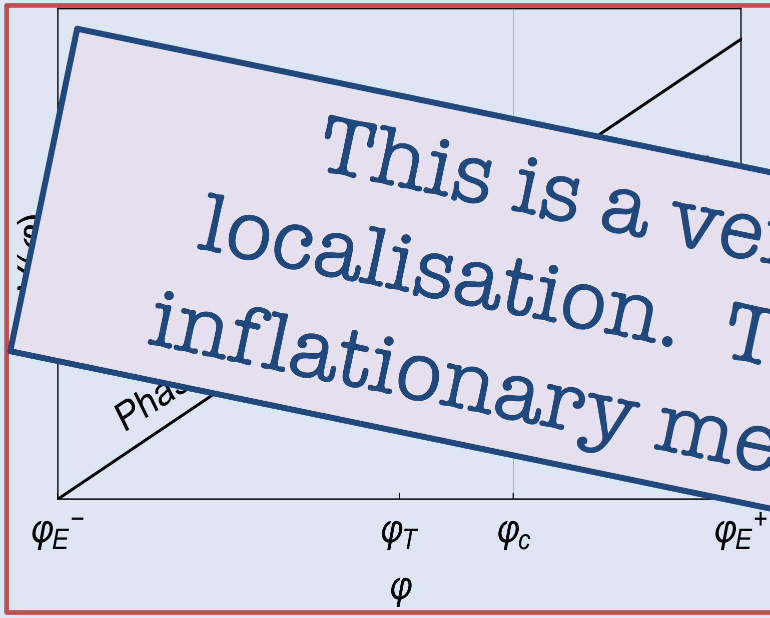
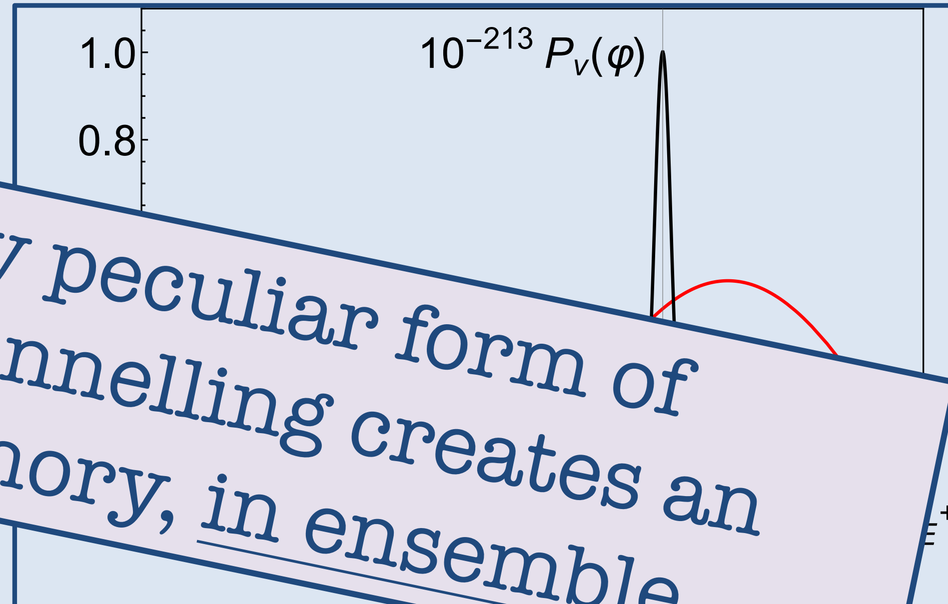




This is a very peculiar form of localisation. Tunnelling creates an inflationary memory, <u>in ensemble</u>....

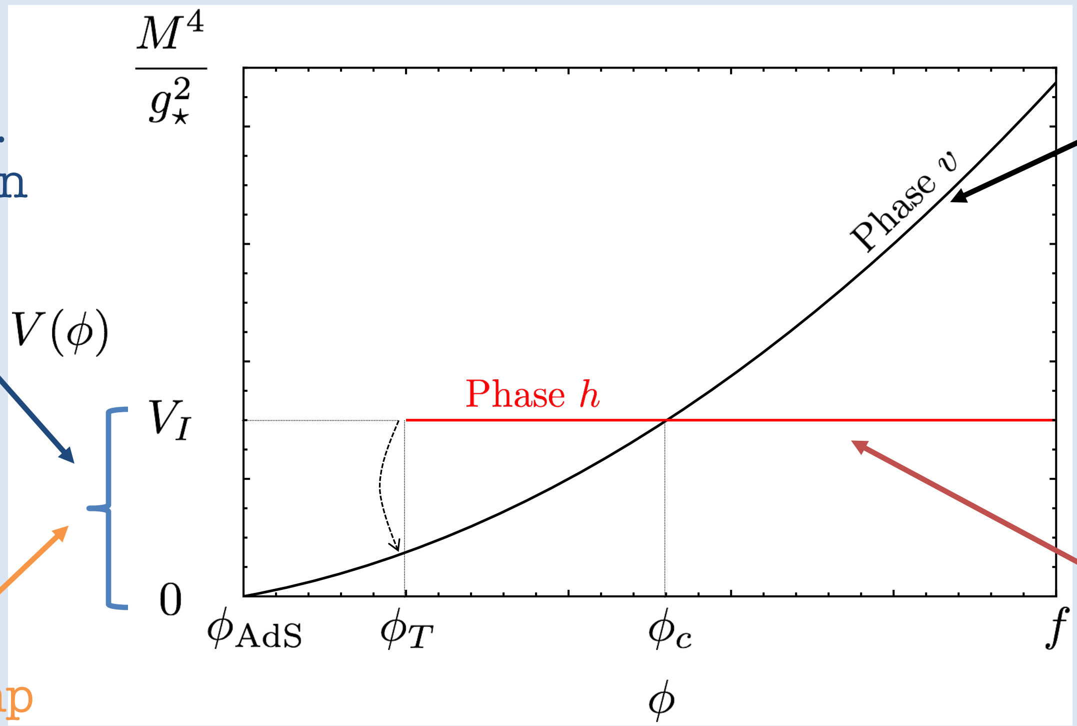
- "v" has non-zero flux injected at the top.
- "h" branch also has flux.
- (No significant sensitivity to these BCs though.)

Application...

Waterfalls

Consider the following "Waterfall" scalar potential in a SUSY setup:

Inflationary sector secluded. this contribution vanishes after reheating.



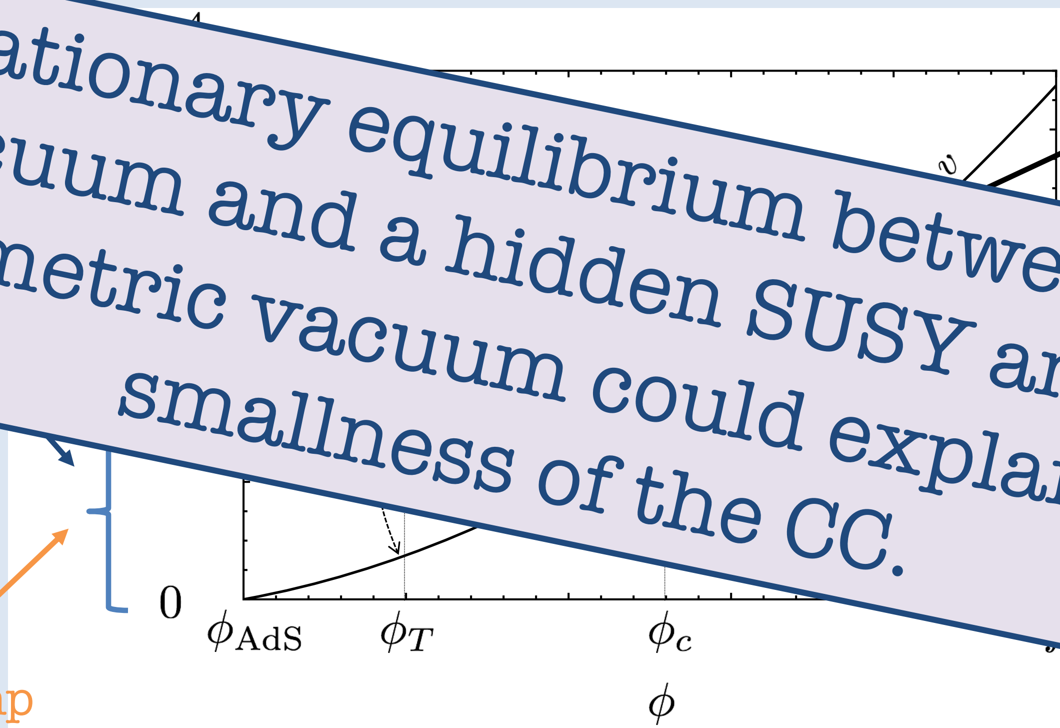
SUSY and R-symmetry broken, large symmetry breaking.

SUSY vacuum, scalar a pNGB with small explicit breaking. No R-breaking.

For C regime max reheat temp is 25 MeV or so.

Waterfalls

Consider the following "Waterfall" scalar potential in a SUSY setup:



SUSY and R-symmetry broken, large symmetry

For C regime max reheat temp is 25 MeV or so.

explicit breaking. No R-breaking.

Inflationary equilibrium between our vacuum and a hidden SUSY and R-symmetric vacuum could explain the smallness of the CC.

Life after SOL...

Post-Reheating Dynamics

For these models on their own, the scalar will continue rolling after inflation. Many potential effects, but limits on the evolution of dark energy imply:

$$\alpha^2 \beta > \left(\frac{\hbar\, H_0^4}{M_P H_{\rm now} \Lambda^2} \right)^2 = \left(\frac{H_0}{2 \times 10^{-3}\ {\rm eV}} \right)^8$$

Unless the Hubble scale is extremely low during inflation, this constraint forces us towards the Q regime for most applications.

Alternatively, could have some post-inflationary trapping etc, but we opt for simplicity.

Experimental Predictions

Details differ in each implementation, however a common feature is a very very light scalar field which would still be rolling down its scalar potential.

Predicts that dark energy is not a exactly a constant and evidence for $w=-1$ being violated would provide support for SOL.

Inflation

The theory of inflation has been remarkably successful at solving puzzles:

- Horizon
- Flatness
- …

And at making detailed predictions for our Hubble patch:

- Structures from gravitational collapse of quantum mechanical fluctuations
- Almost scale-invariant spectrum…

In general inflation doing pretty well…

The Good

The stochastic approach offers possibility of ... macroscopic parameters.

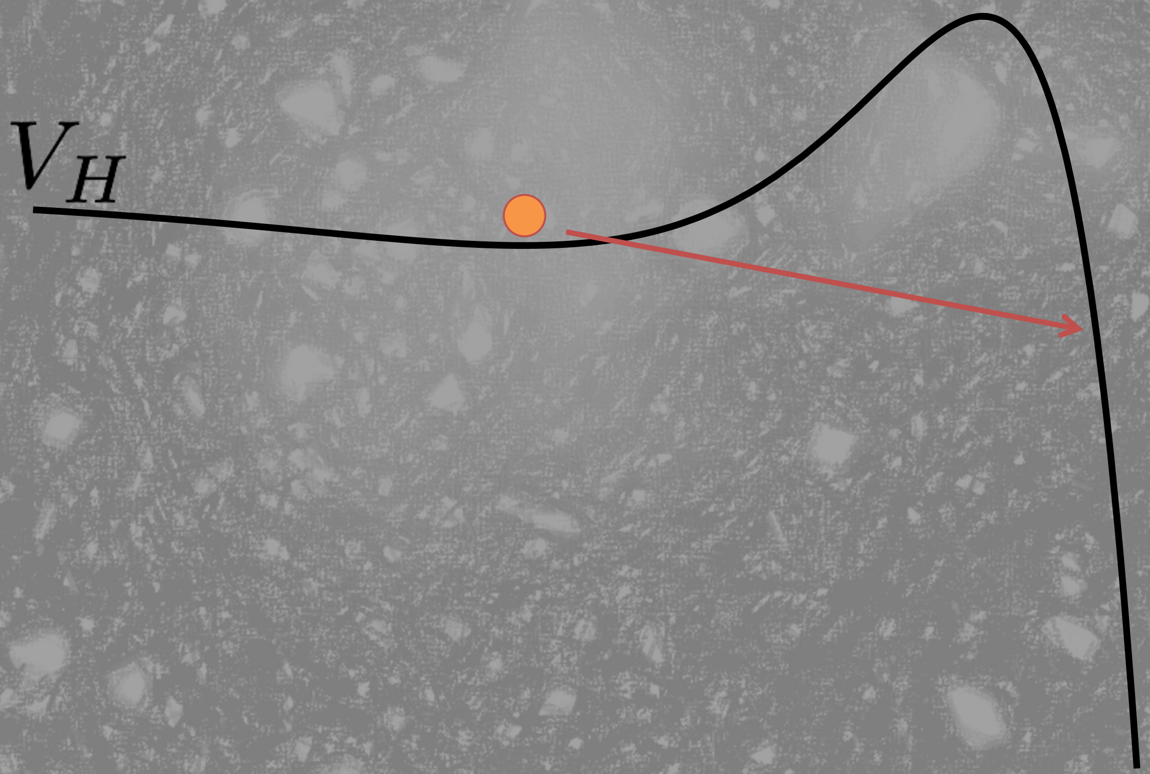
Scalar field ... trajectory. Average of traje... described by a Fokker-Planck equation.

The connection with statistical mechanics is not just coincidence...

"In statistical mechanics, the Fokker–Planck equation is a partial differential equation that describes the time evolution of the probability density function of the velocity of a particle under the influence of drag forces and random forces, as in Brownian motion." Wikipedia

A Metastable Universe?

This means that the vacuum we are in, as in the Mexican hat pictures, is just local, but there is a deeper one out at large field values.



The Elephant in the Room

Ginzburg-Landau is just a phenomenological model, with no explanation of parameters. The macroscopic parameters follow from the detailed microscopic BCS theory and there are no big surprises.

The order parameter at generic temperatures is of the typical scale associated with underlying microscopic parameters. Criticality means fine-tuning against the fundamental scale.

A Stable Universe?

It is as if there is some additional piece in the potential for the entire Universe that knew in advance, before the electroweak phase transition had even happened, to precisely cancel the Higgs/QCD... contribution

$$V = V_H + V_0$$

where

$$V \ll V_H$$

The Universe is delicately and calmly balanced between two violent phases. Why?

ESA/Hubble and Harald Süpfle