

The Failure of Naturalness

Tevong You

Contents

- Historical introduction
- The naturalness problem
- Cosmological solutions
- Colliders as general-purpose particle observatories
- Radical BSM outcomes is still a possibility

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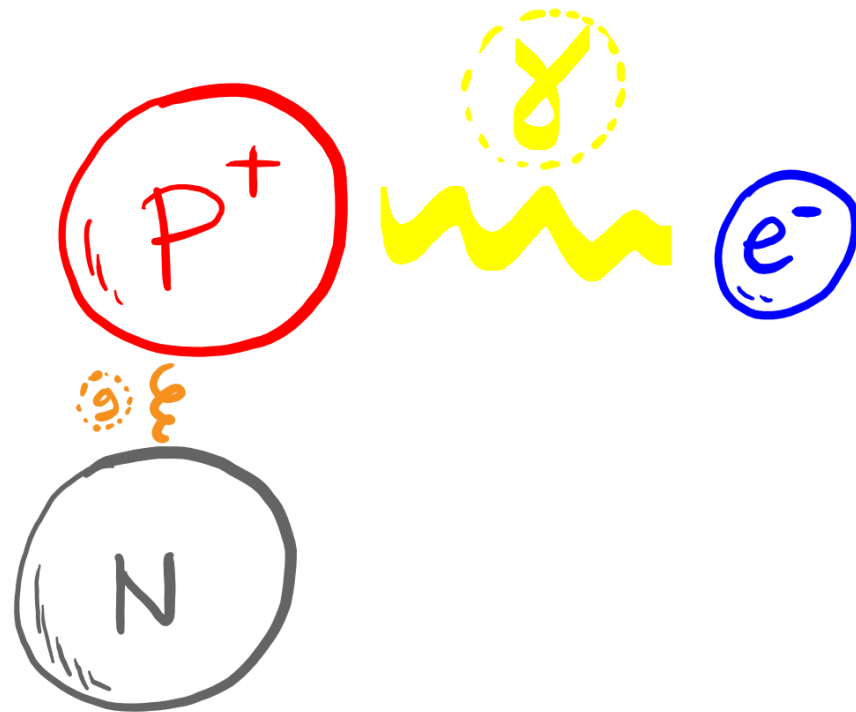
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Introduction

- **3 lessons** from history

Introduction

- *1930s: everything* is made of **protons**, **neutrons**, and **electrons**

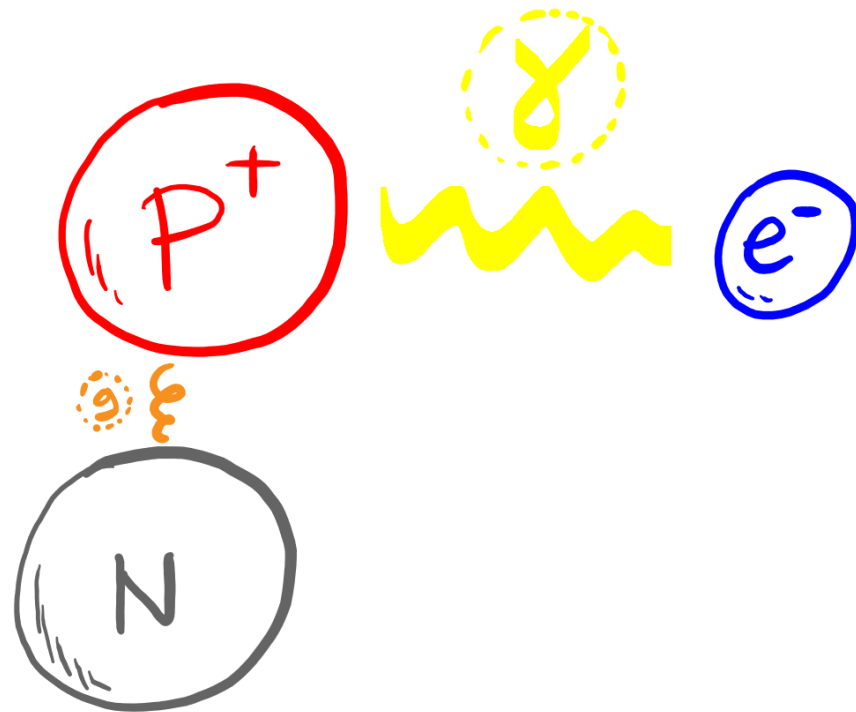


Minimal, economical theory!
However...

- Held together by **electromagnetism** and the **strong force**

Introduction

- *1930s: everything* is made of **protons**, **neutrons**, and **electrons**



"If we consider protons and neutrons as elementary particles, we would have three kinds of elementary particles [p,n,e].... This number may seem large but, from that point of view, two is already a large number."

Paul Dirac 1933 Solvay Conference

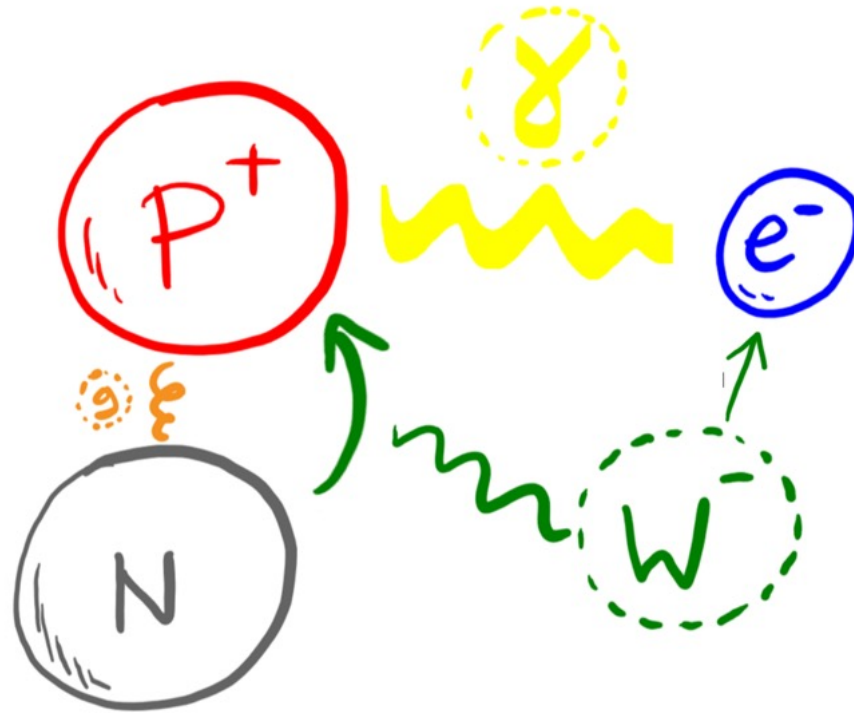
From D. Tong slide

Lesson 1: Beauty in fundamental physics is not an economy of particle multiplicities, it's an *economy of theoretical principles*

- Held together by **electromagnetism** and the **strong force**

Introduction

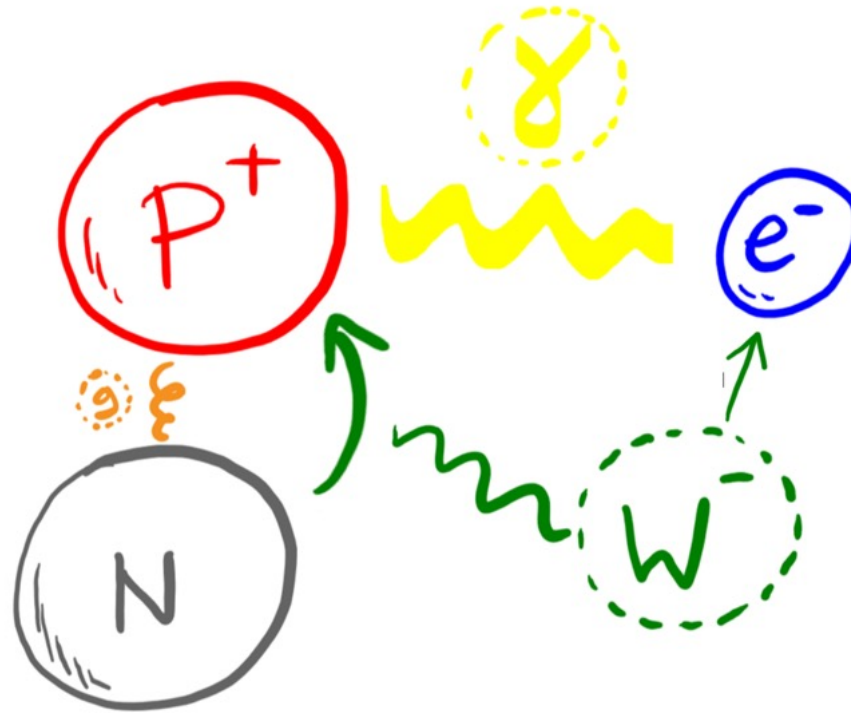
- **Weak force** explains *radioactivity*



- **Neutron** can change into **proton**, emitting **electron**

Introduction

- **Weak force** explains *radioactivity*

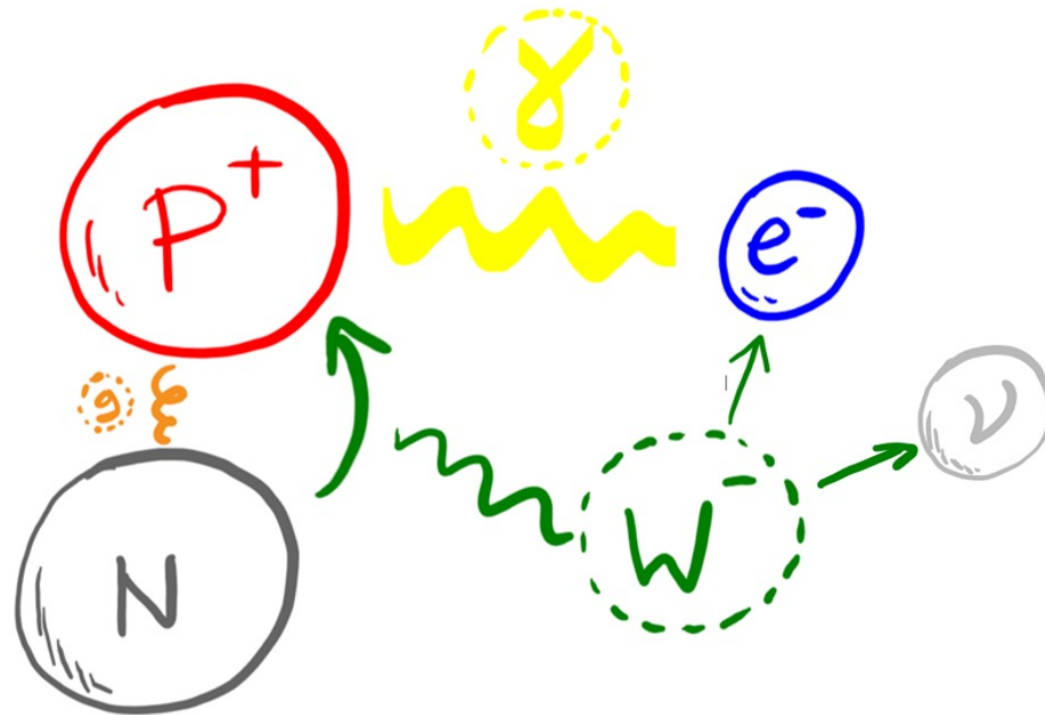


Missing energy? Pauli postulates “a desperate remedy”

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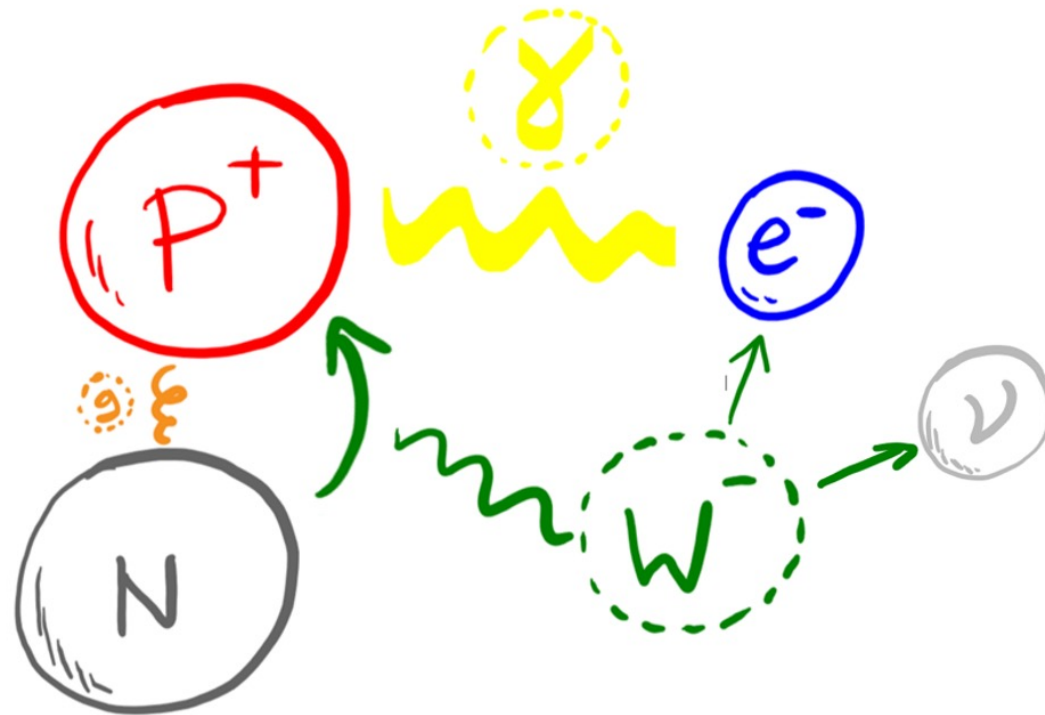


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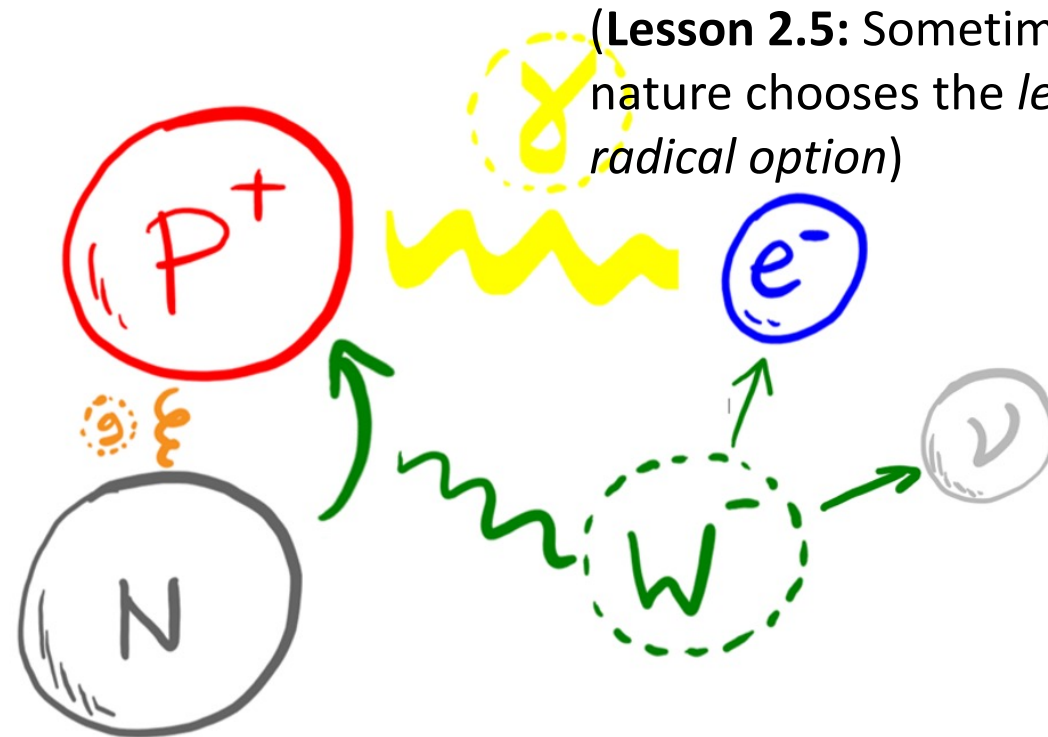
Missing energy? Pauli postulates “a desperate remedy”

Lesson 2: *perceived* prospects of experimental confirmation is *not a useful scientific criteria* for establishing **what nature actually does**

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- **Weak force** explains *radioactivity*



Missing energy? Pauli postulates “*a desperate remedy*”

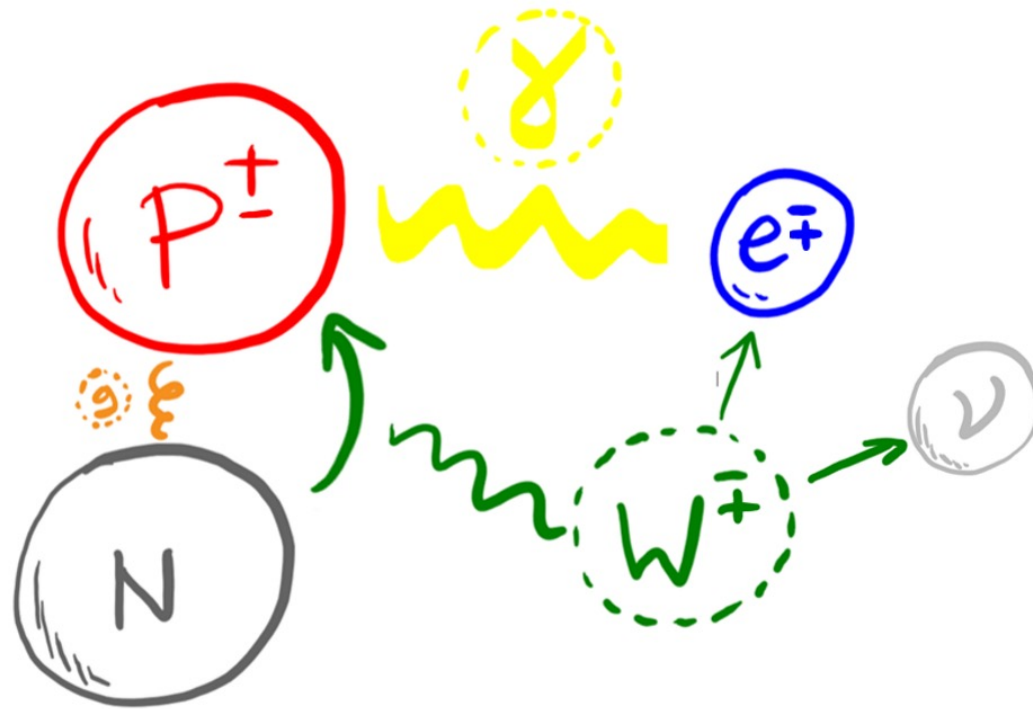
(Bohr suggests fundamental violation of energy conservation principle)

Lesson 2: *perceived* prospects of experimental confirmation is *not a useful scientific criteria* for establishing **what nature actually does**

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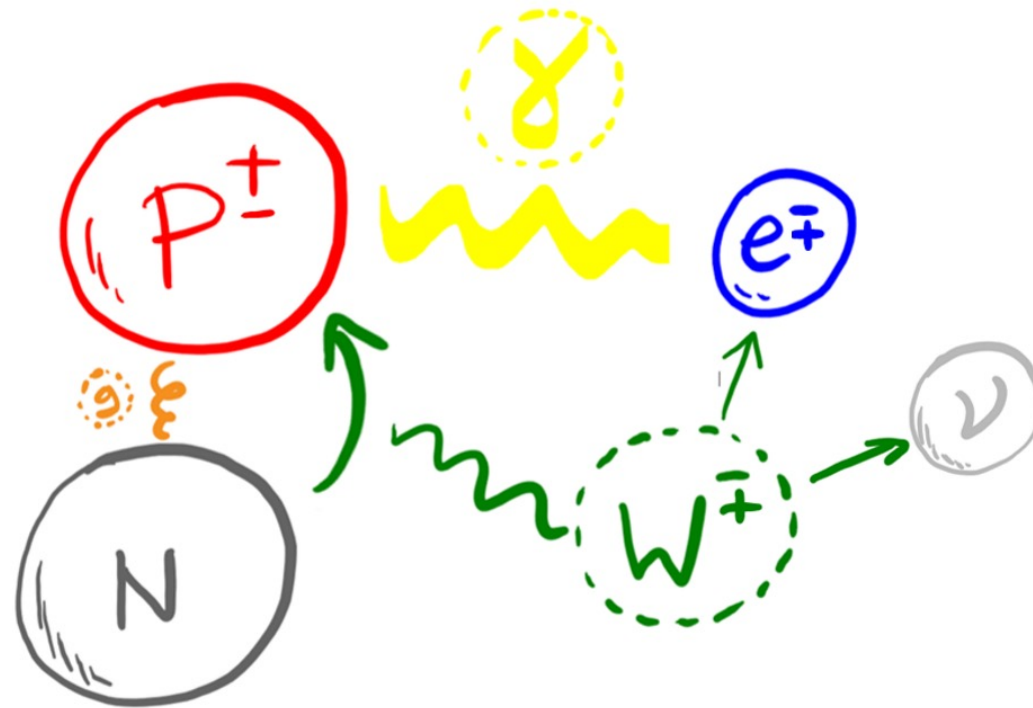
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- *Every particle has an oppositely charged antiparticle partner*

Introduction

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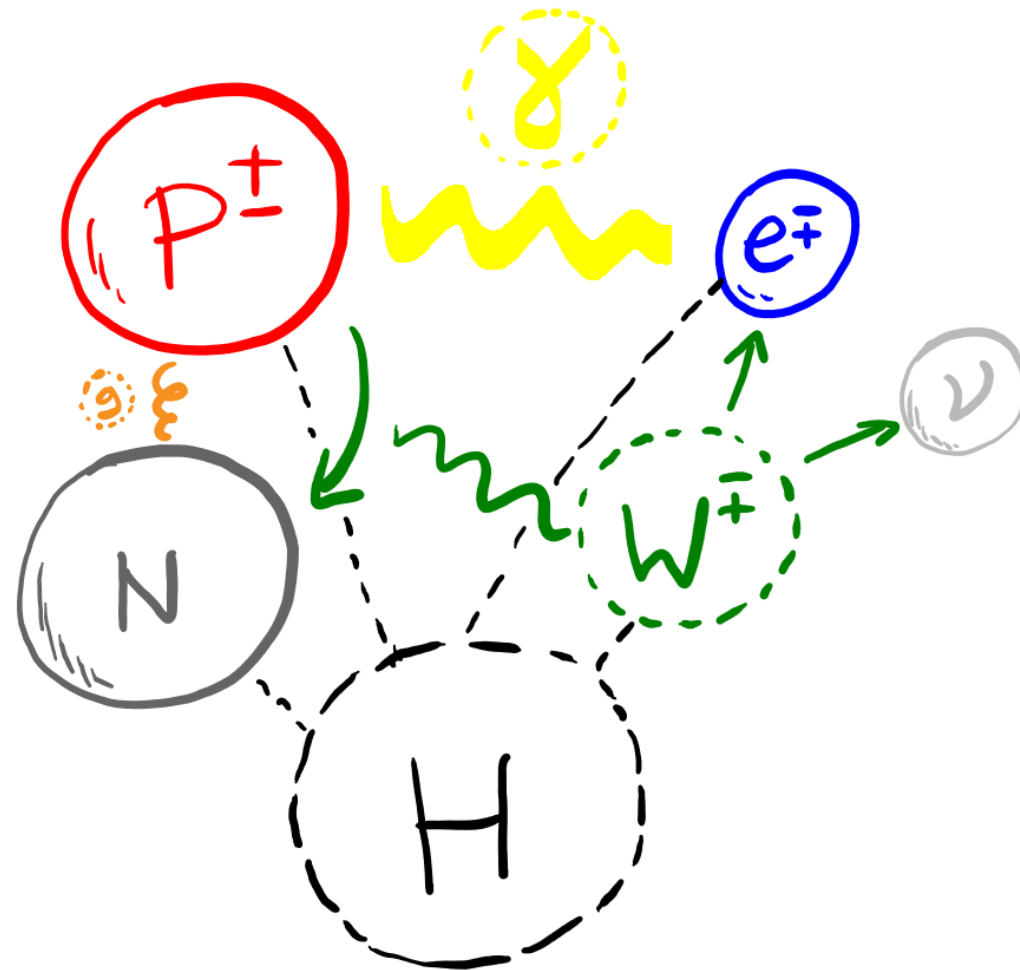


c.f. Lesson 1: antiparticles double the particle spectrum. Nevertheless, the theory is much tighter, less arbitrary, and more elegant

- *Every particle has an oppositely charged antiparticle partner*

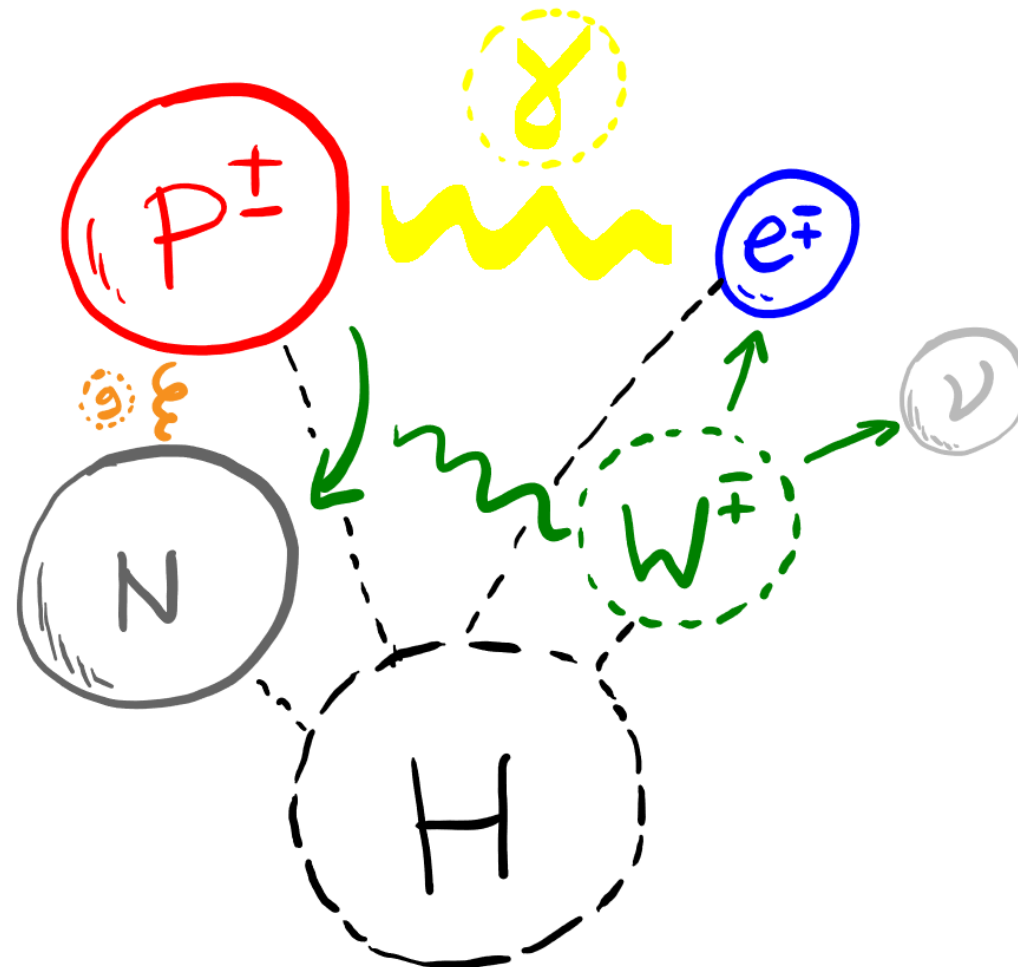
Introduction

- *Higgs(+Brout+Englert)*: **particle masses** require a new **scalar boson H**



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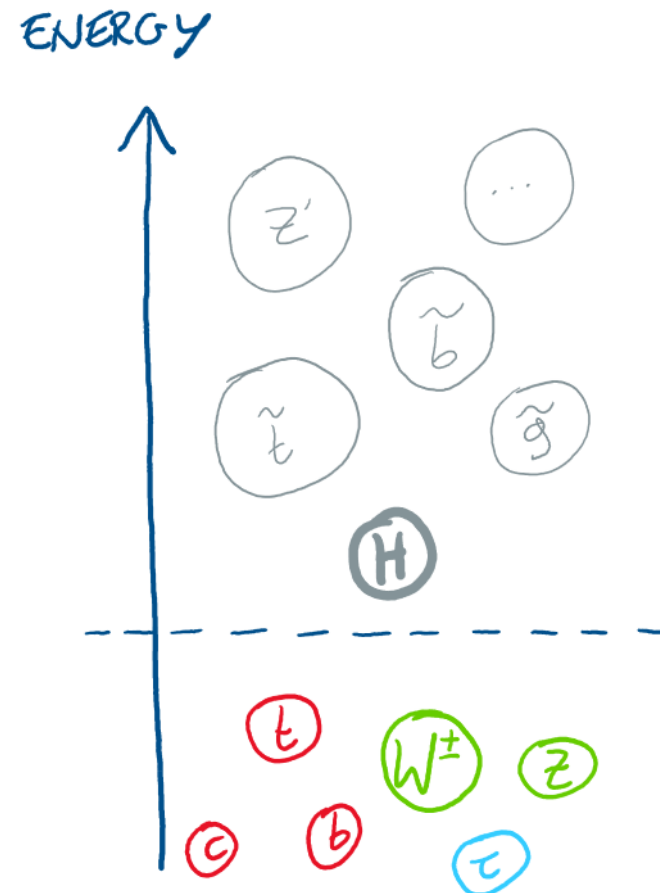
Lesson 3: Keep an open mind. Ideas initially dismissed as **unrealistic** (e.g. non-abelian gauge theories and spontaneous symmetry breaking, because they predicted **unobserved massless bosons**) can click together suddenly and make sense

Introduction

- 1930-40s: Success of QED. **QFT** emerges as the *new fundamental description of Nature*.
- 1960s: QFT is **unfashionable**, non-Abelian theory dismissed as an **unrealistic generalisation** of local symmetry-based forces. Widely believed a **radically new framework** will be required *e.g. to understand the strong force*.
- 1970s: **QFT triumphs** following Yang-Mills+Higgs+asymptotic freedom+renormalisation. Nature is **radically conservative**, *but more unified than ever*.
- 1980s: Success of SM. QFT understood as **most general EFT consistent with symmetry**. *Higgs* (and cosmological constant) *violates symmetry expectation*.
- **Tremendous progress** since, *despite lack of BSM*

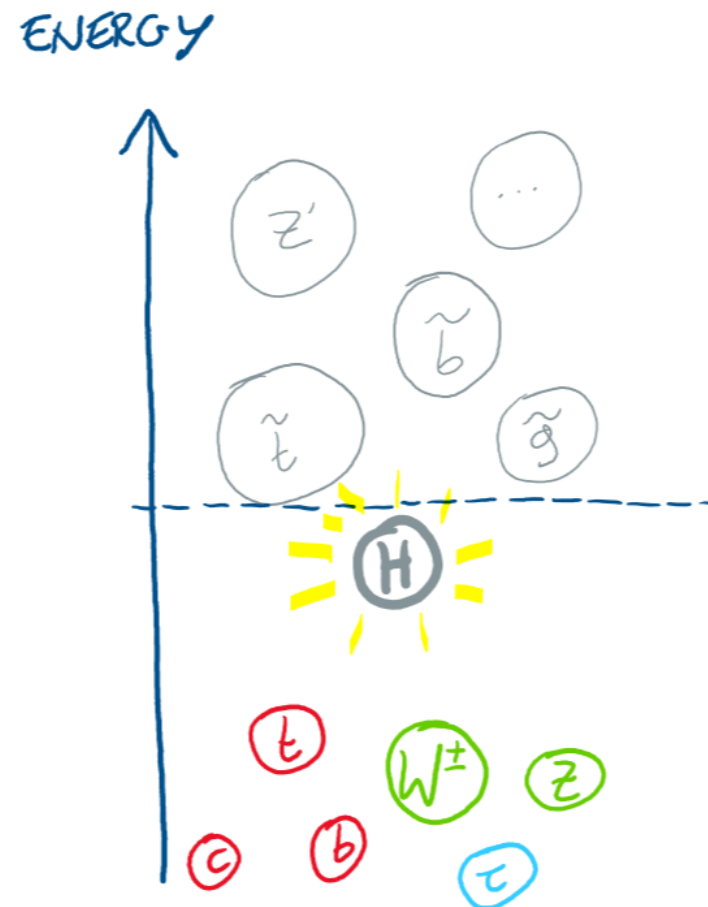
Motivation

- Until now, there had been a **clear roadmap**



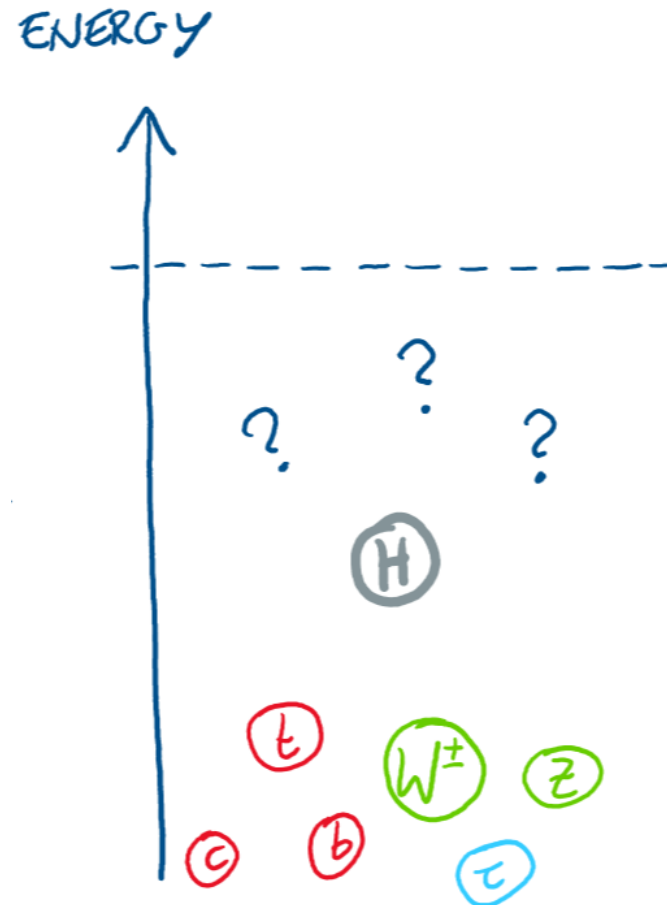
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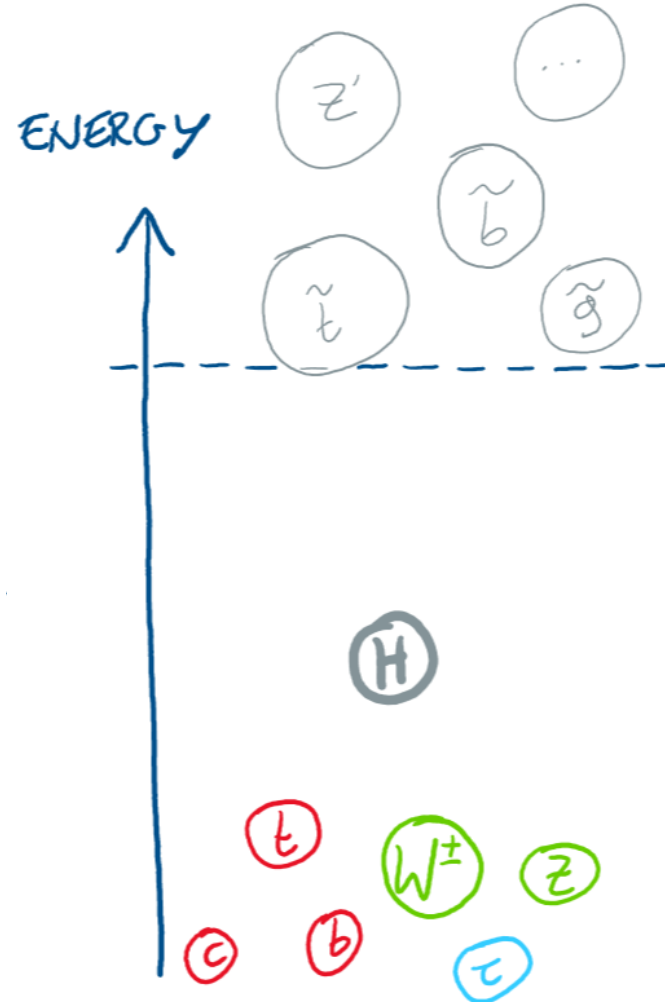
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Conventional
symmetry-based
solutions *have not*
shown up!

Motivation

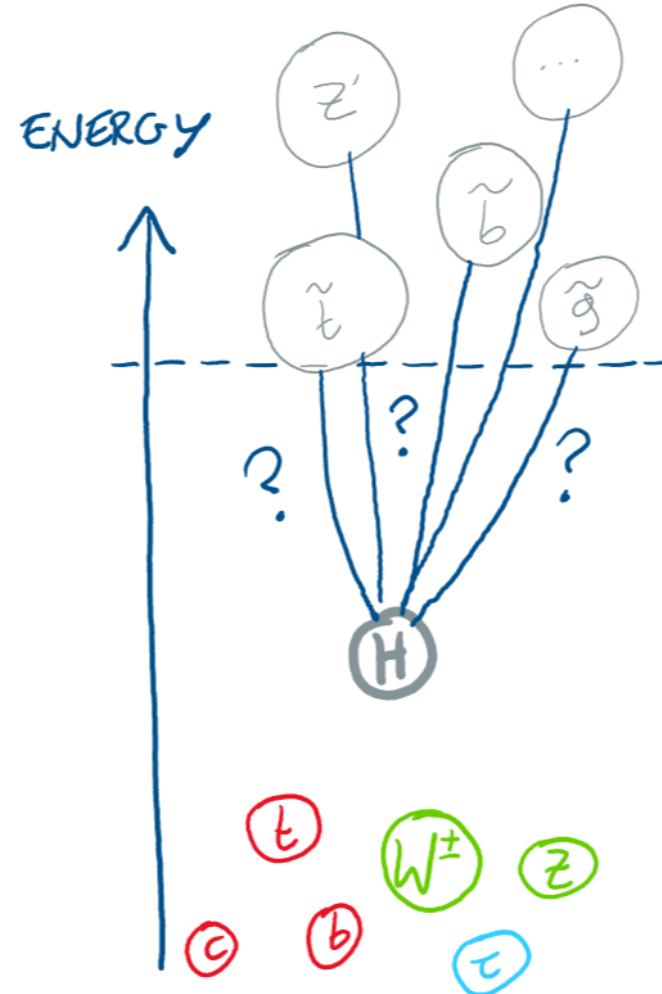
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Maybe **just around the corner...**

Motivation

- Until now, there had been a **clear roadmap**



...but the larger the separation of scales, the more **fine-tuned** the underlying theory is!

The Higgs boson's hierarchy problem is a **profound mystery**, that is **even more perplexing** in the absence of new physics at the LHC.

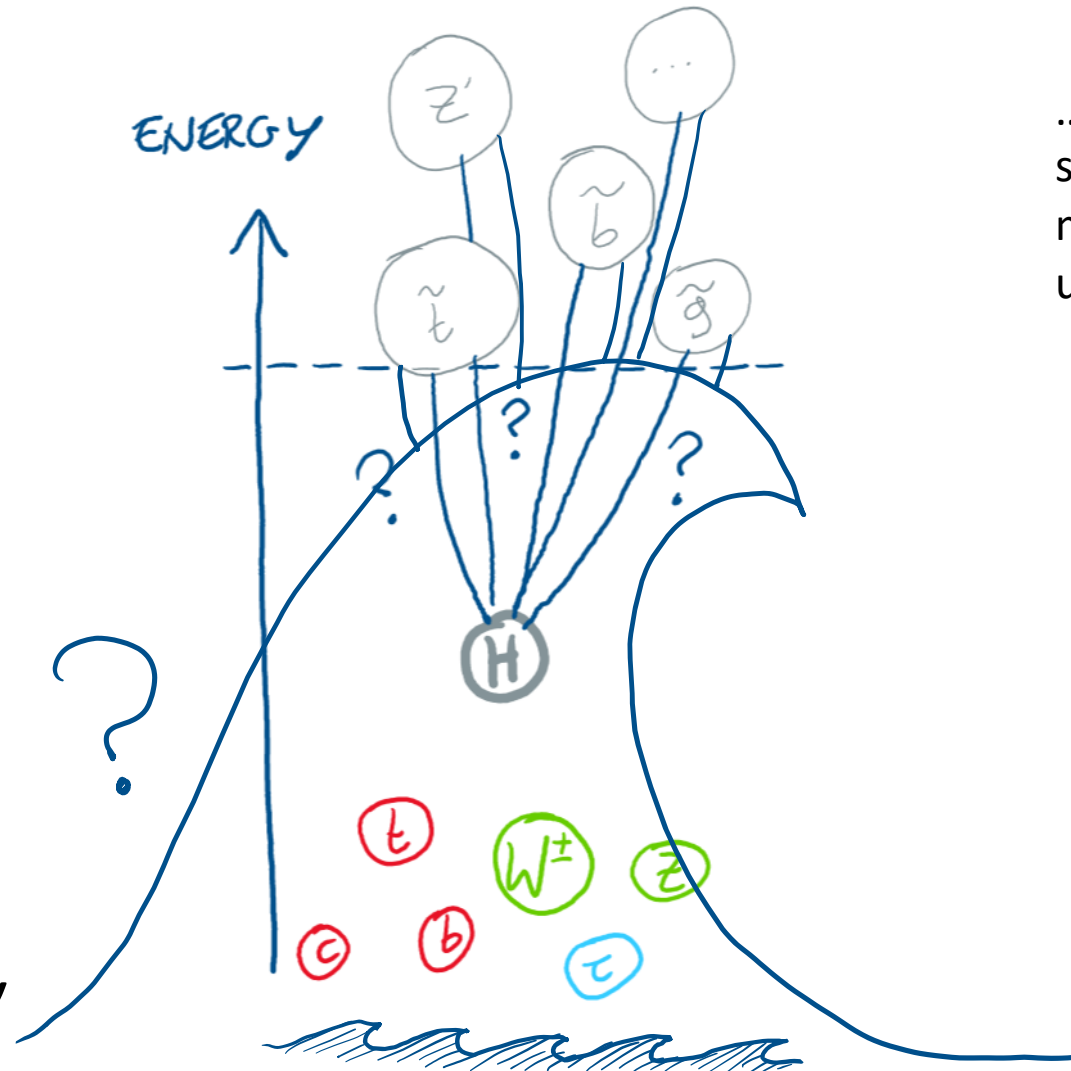
Our **Michelson-Morley moment?**

Motivation

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...but the larger the separation of scales, the more **fine-tuned** the underlying theory is

Vacuum energy is also *peculiarly tiny*



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Our **Michelson-Morley moment?**

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Effective Field Theory

$$\mathcal{L} = \Lambda^4 + \Lambda^2 \mathcal{O}^{(2)} + m \mathcal{O}^{(3)} + \mathcal{O}^{(4)} + \frac{1}{\Lambda} \mathcal{O}^{(5)} + \frac{1}{\Lambda^2} \mathcal{O}^{(6)} + \frac{1}{\Lambda^3} \mathcal{O}^{(7)} + \frac{1}{\Lambda^4} \mathcal{O}^{(8)} + \dots$$

1960s point of view: renormalisability of a *finite* number of parameters is essential

Modern point of view: our QFTs are really EFTs - include *all* operators allowed by symmetries

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Effective Field Theory

Suppressed!

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Effective Field Theory

Naturalness?

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1960s point of view: renormalisability of a *finite* number of parameters is essential

Modern point of view: our QFTs are really EFTs - include *all* operators allowed by symmetries

Naturalness

Take aesthetic problems seriously.

Example 1

$$F = m_{inertia}a \qquad F \propto \frac{q_1 q_2}{r^2}$$

Inertial mass and charge have nothing to do with each other, and yet for gravity we arbitrarily set by hand

$$q = m_{inertia}$$

Solution to this equivalence problem took centuries: Newtonian gravity → GR

Naturalness

Take fine-tuning problems seriously.

e.g. 2205.05708 N. Craig - Snowmass review,
1307.7879 G. Giudice - Naturalness after LHC

Example 2

$$(m_e c^2)_{obs} = (m_e c^2)_{bare} + \Delta E_{\text{Coulomb}} \quad \Delta E_{\text{Coulomb}} = \frac{1}{4\pi\epsilon_0} \frac{e^2}{r_e}$$

Avoiding cancellation between “bare” mass and divergent self-energy in classical electrodynamics requires new physics around

$$e^2/(4\pi\epsilon_0 m_e c^2) = 2.8 \times 10^{-13} \text{ cm}$$

Indeed, the positron and quantum-mechanics appears just before!

$$\Delta E = \Delta E_{\text{Coulomb}} + \Delta E_{\text{pair}} = \frac{3\alpha}{4\pi} m_e c^2 \log \frac{\hbar}{m_e c r_e}$$

Naturalness

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Example 3

Divergence in pion mass: $m_{\pi^\pm}^2 - m_{\pi^0}^2 = \frac{3\alpha}{4\pi} \Lambda^2$

Experimental value is $m_{\pi^\pm}^2 - m_{\pi^0}^2 \sim (35.5 \text{ MeV})^2$

Expect new physics at $\Lambda \sim 850 \text{ MeV}$ to avoid fine-tuned cancellation.

ρ meson appears at 775 MeV!

Naturalness

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Example 4

Divergence in Kaons mass difference in a theory with only up, down, strange:

$$m_{K_L^0} - m_{K_S^0} \simeq \frac{1}{16\pi^2} m_K f_K^2 G_F^2 \sin^2 \theta_C \cos^2 \theta_C \times \Lambda^2 ;$$

Avoiding fine-tuned cancellation requires $\Lambda < 3 \text{ GeV}$.

Gaillard & Lee in 1974 predicted the charm quark mass!

Naturalness

Take fine-tuning problems seriously.

e.g. 2205.05708 N. Craig - Snowmass review,
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Higgs?

Higgs also has a quadratically divergent contribution to its mass

$$\Delta m_H^2 = \frac{\Lambda^2}{16\pi^2} \left(-6y_t^2 + \frac{9}{4}g^2 + \frac{3}{4}g'^2 + 6\lambda \right)$$

Avoiding fine-tuned cancellation requires $\Lambda < O(100)$ GeV??

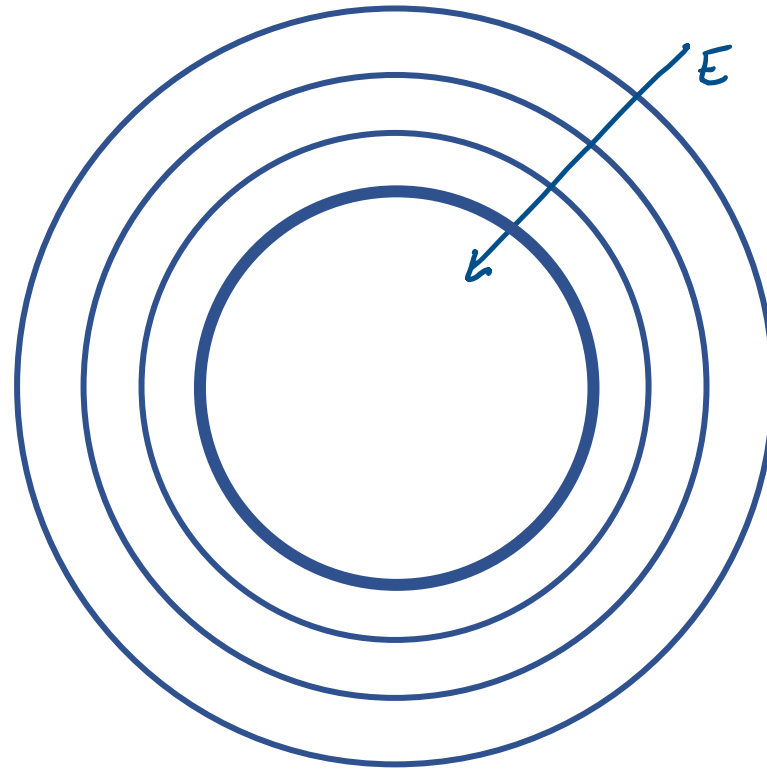
As Λ is pushed to the TeV scale by null results, tuning is around 10% - 1%.

Note: in the SM the Higgs mass is a parameter to be measured, not calculated. What the quadratic divergence represents (independently of the choice of renormalisation scheme) is the fine-tuning in an underlying theory in which we expect the Higgs mass to be calculable.

Naturalness is still a fundamental problem

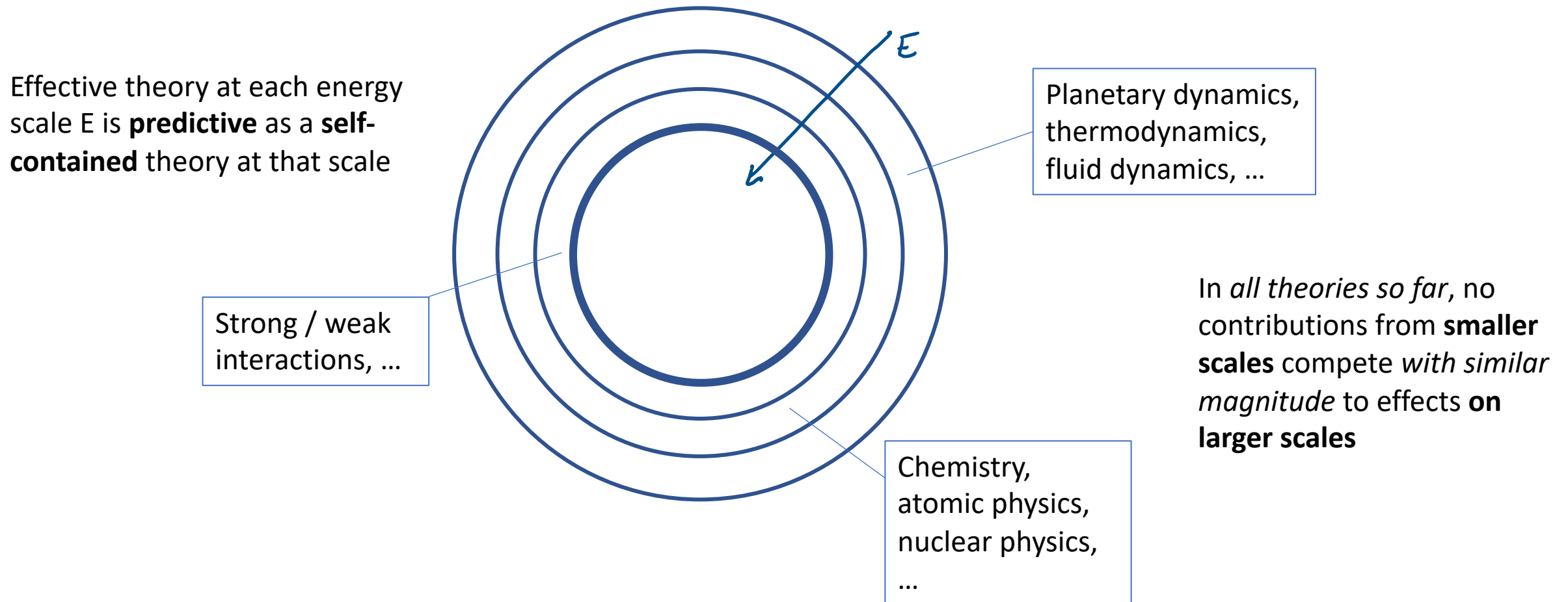
- *Why is unnatural fine-tuning such a big deal?*

Effective theory at each energy scale E is **predictive** as a **self-contained** theory at that scale



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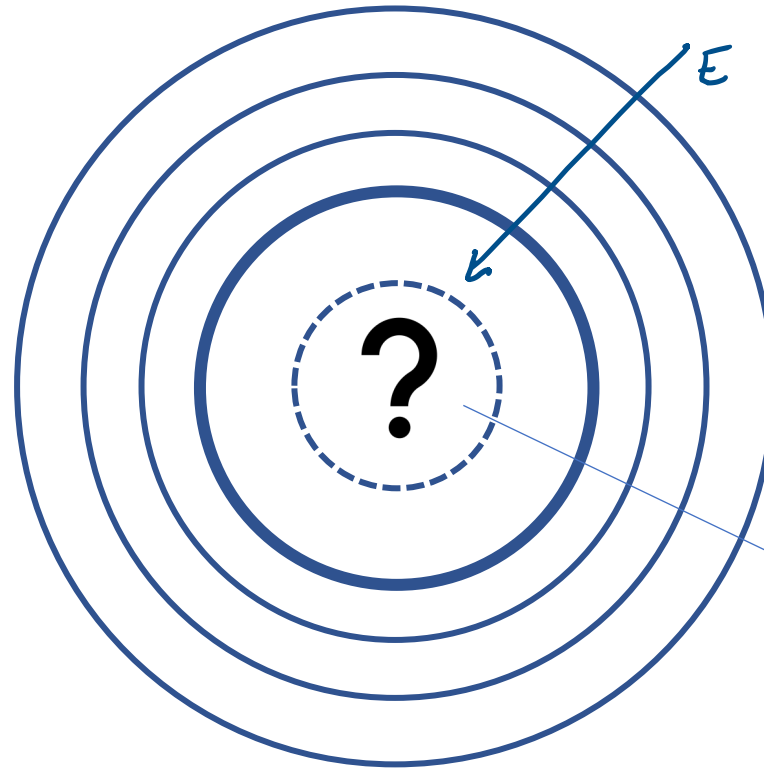
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- Indicates *an unprecedented breakdown* of the **effective theory** structure of nature

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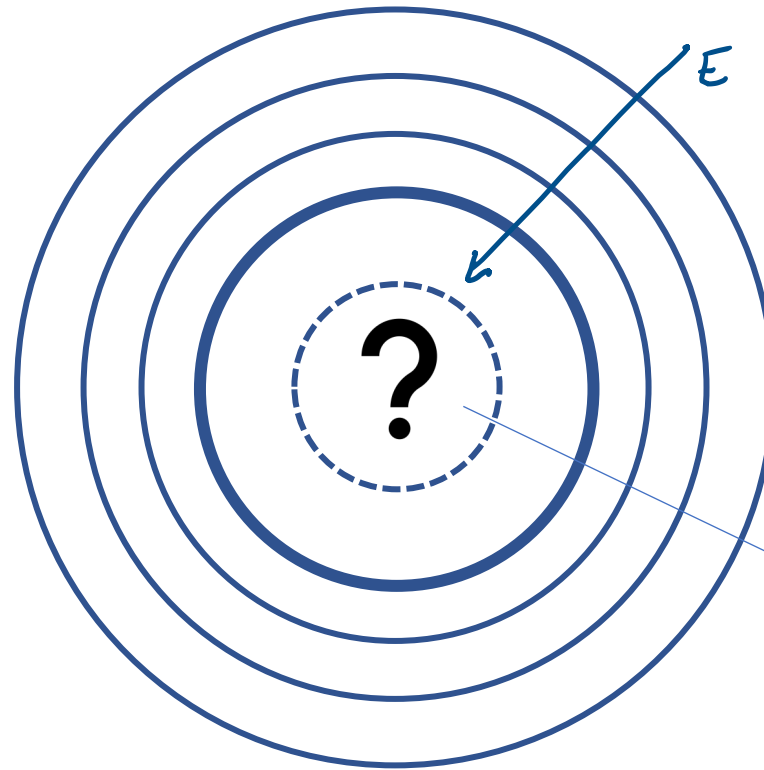


Unnatural Higgs means the next layer *is no longer predictive* without including contributions *from much smaller scales*

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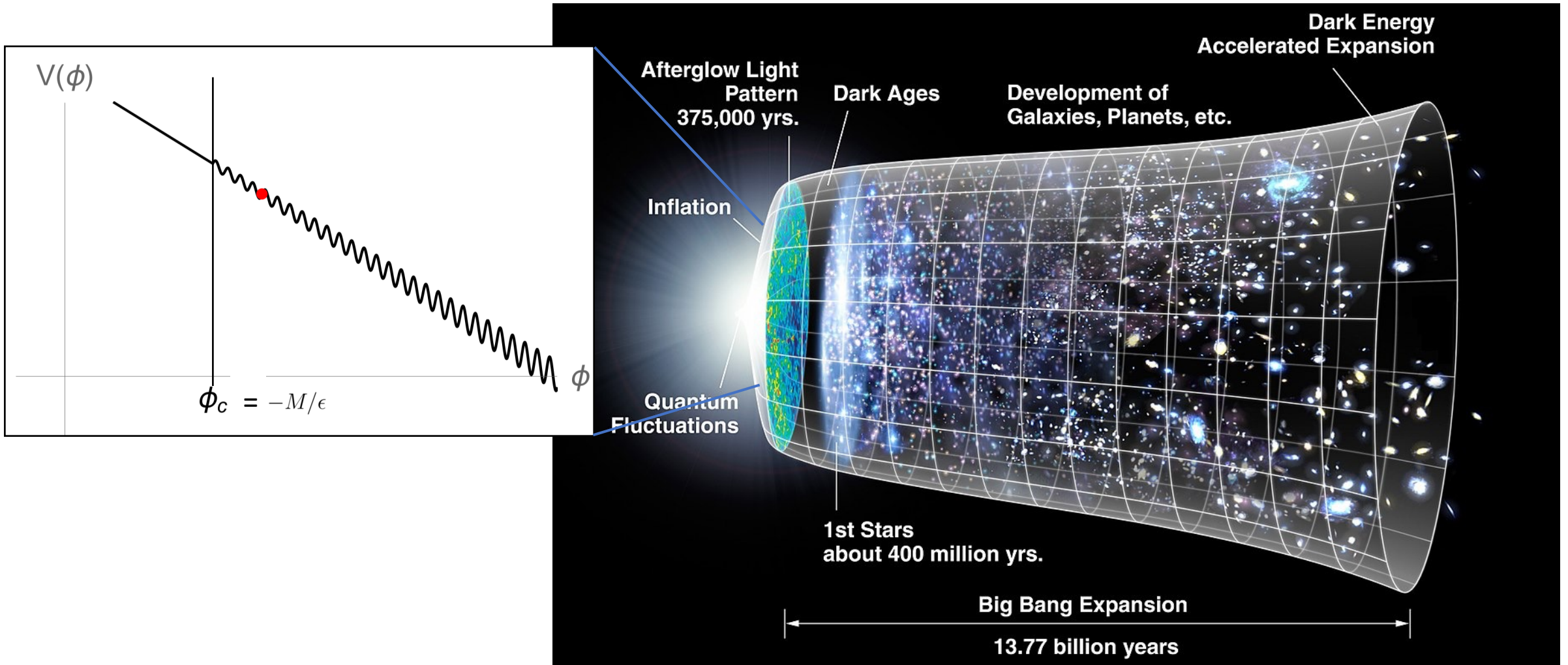
- Are we missing a **fundamentally new** “*post-naturalness*” principle? c.f. null results in search for aether

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Cosmological solution to naturalness?

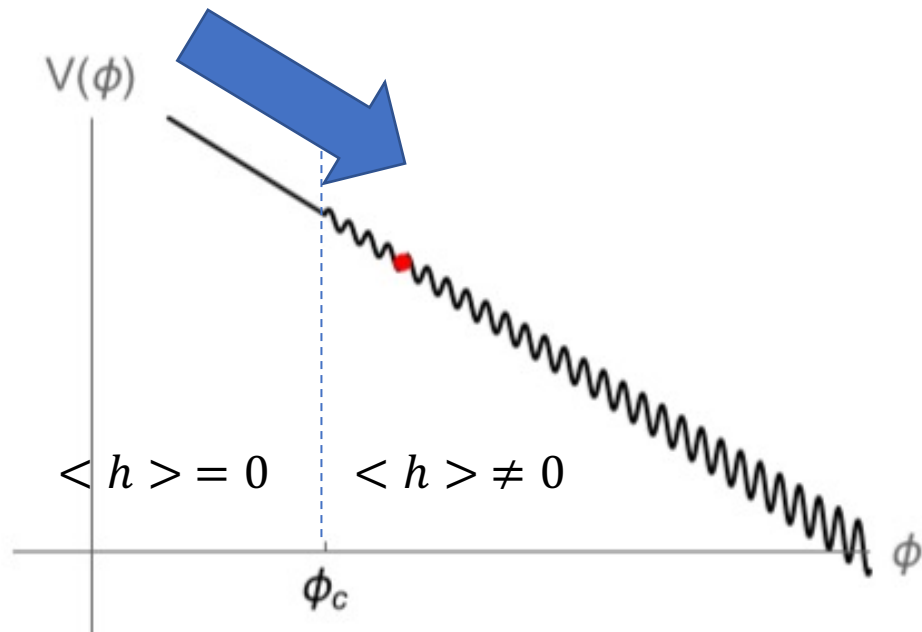
- **Cosmological evolution** could play a role



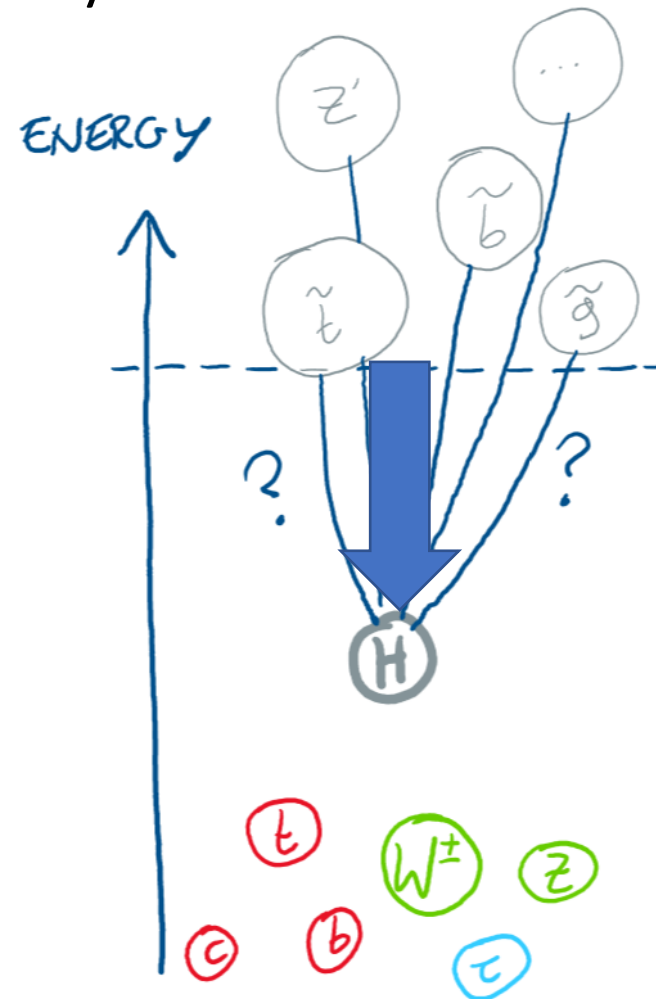
Cosmological relaxation of the weak scale

- **Axions** could solve a variety of fundamental problems
- **Relaxion** scanning the Higgs mass in the early universe

A dynamical solution to the hierarchy problem



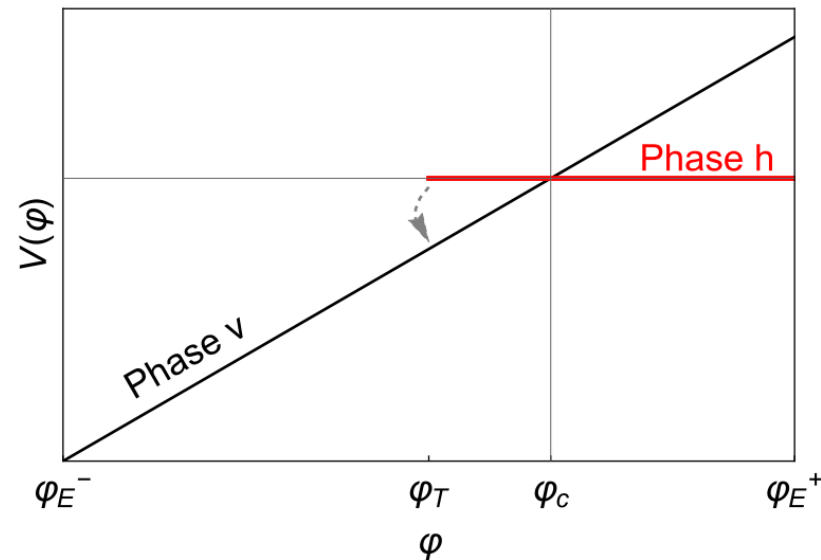
Graham, Kaplan, Rajendran '15



Cosmological Self-Organised Criticality

(See also J. Khoury et al
1907.07693, 1912.06706,
2003.12594)

- **Self-Organised Localisation (SOL)** Giudice, McCullough, TY (2105.08617)
- Can relate **Higgs mass** to vacuum instability scale (*requires e.g. VL fermions*)
- Potential solution to the vacuum energy *Cosmological Constant (CC) problem*



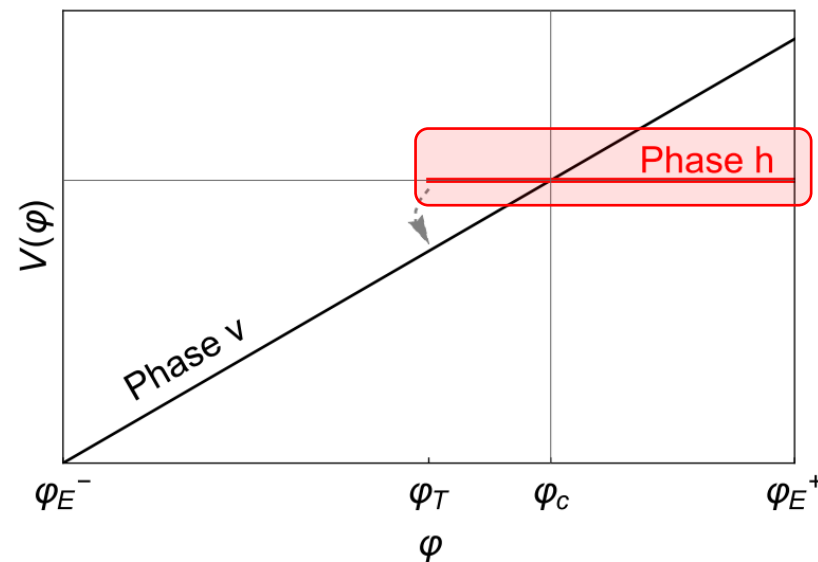
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Phase v: *visible* vacuum with broken supersymmetry but *SOL localises at critical point with **vanishing CC***

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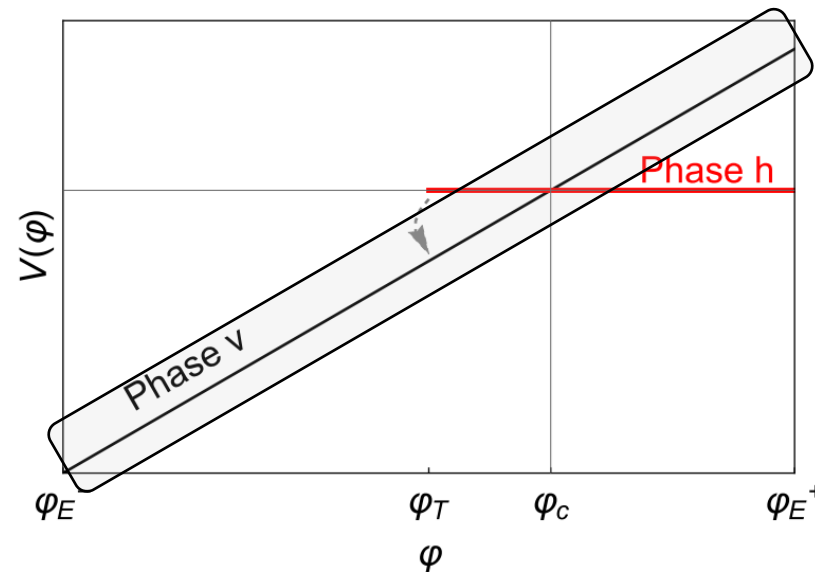
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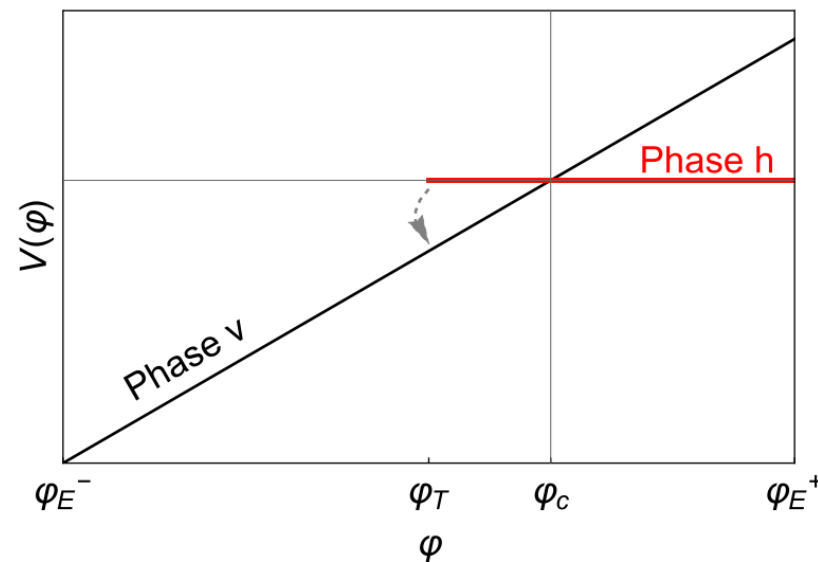
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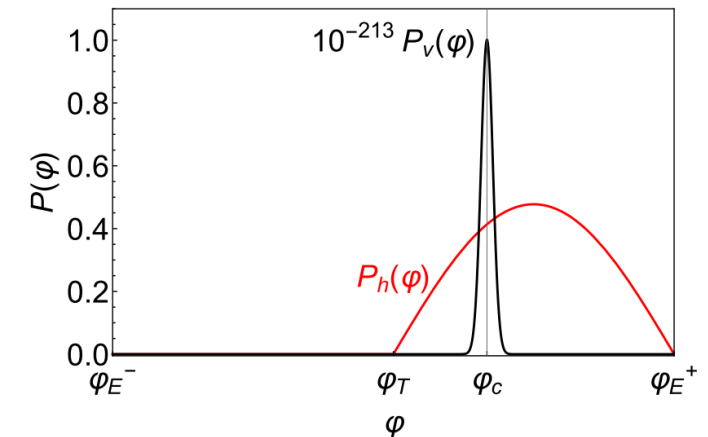
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Light boson **localises** itself at **critical point**:



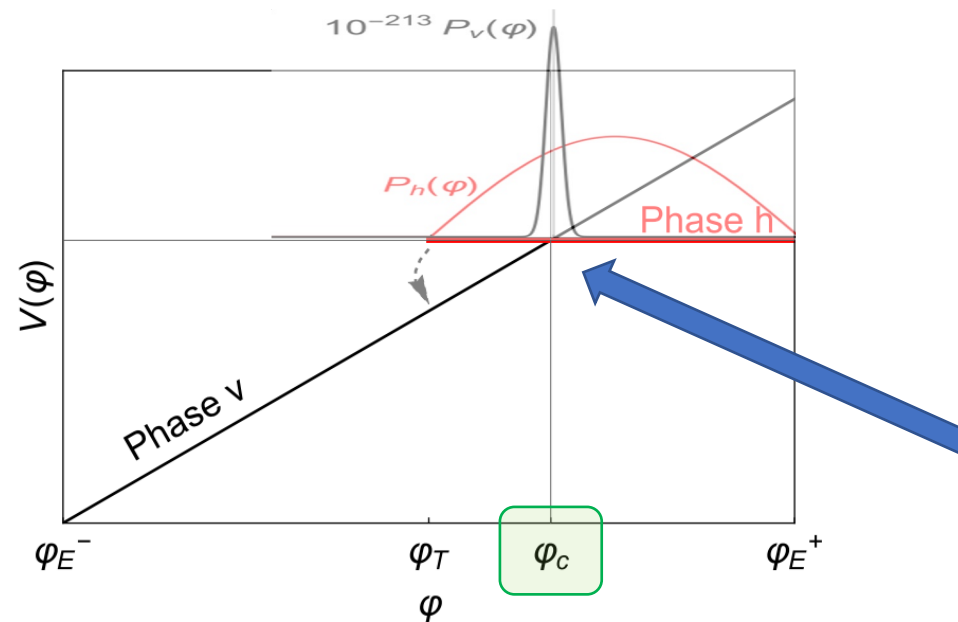
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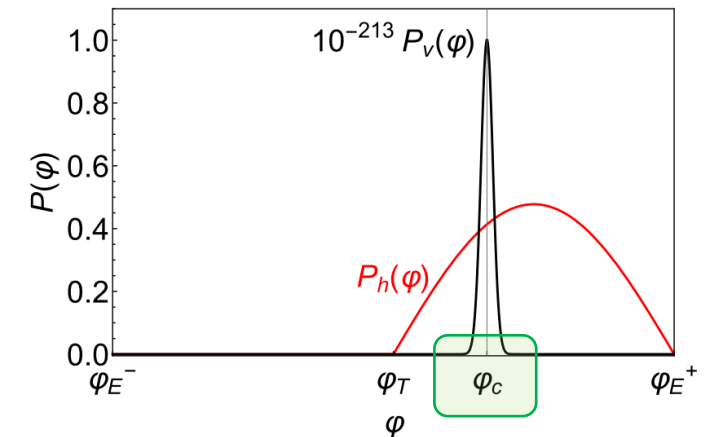
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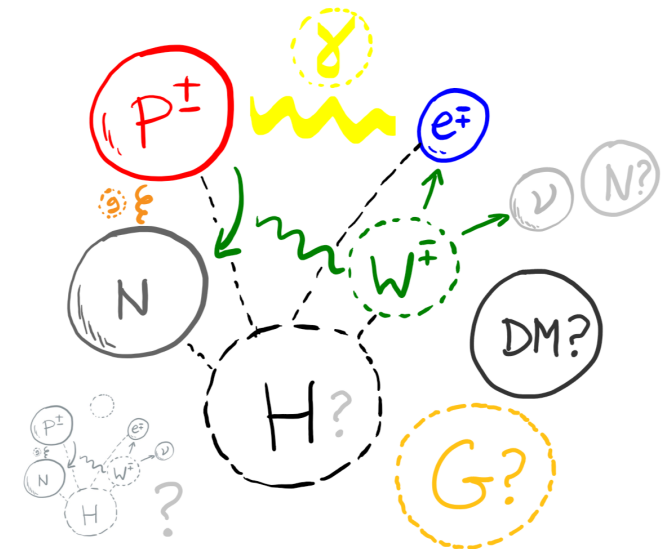
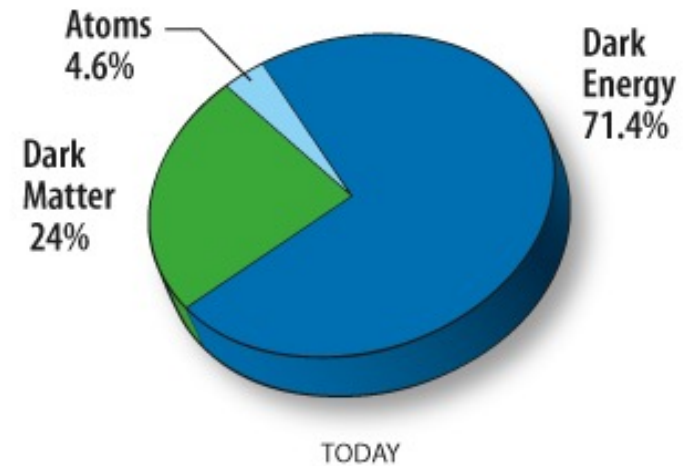
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Naturalness aside, many more open questions

- What is the **origin of the Higgs?**
- What is the **origin of matter?**
- What is the **origin of flavour?**
- What is the **origin of dark matter and dark energy?**
- What is the **origin of neutrino mass?**
- What is the **origin of the Standard Model?**



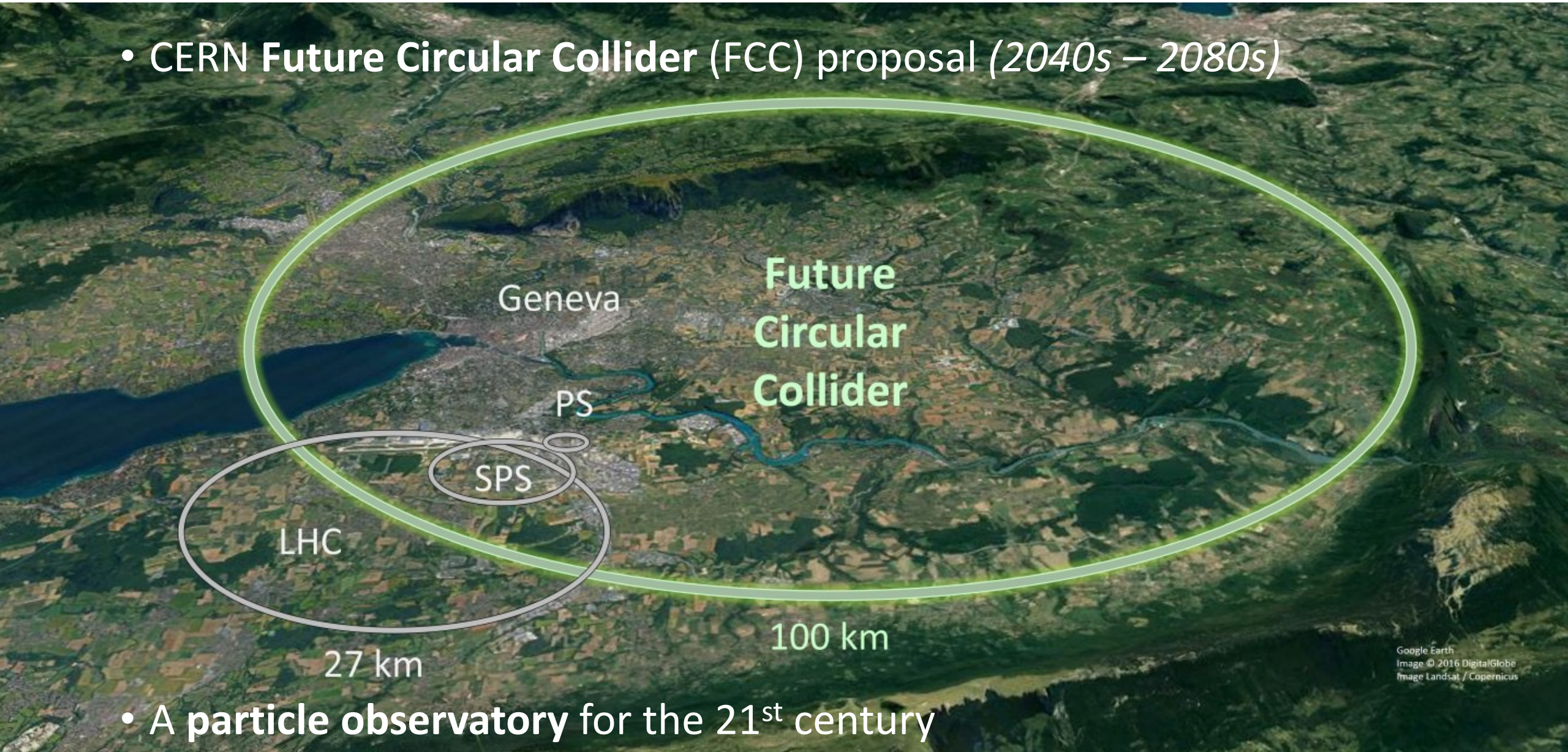


- Telescopes are observatories of the very large
- Colliders observe the very small
- We need *all eyes open on all scales* in our universe



FCC-ee \rightarrow FCC-hh

- CERN Future Circular Collider (FCC) proposal (2040s – 2080s)



- A **particle observatory** for the 21st century

“Discovery prospects” → “Exploring origins”

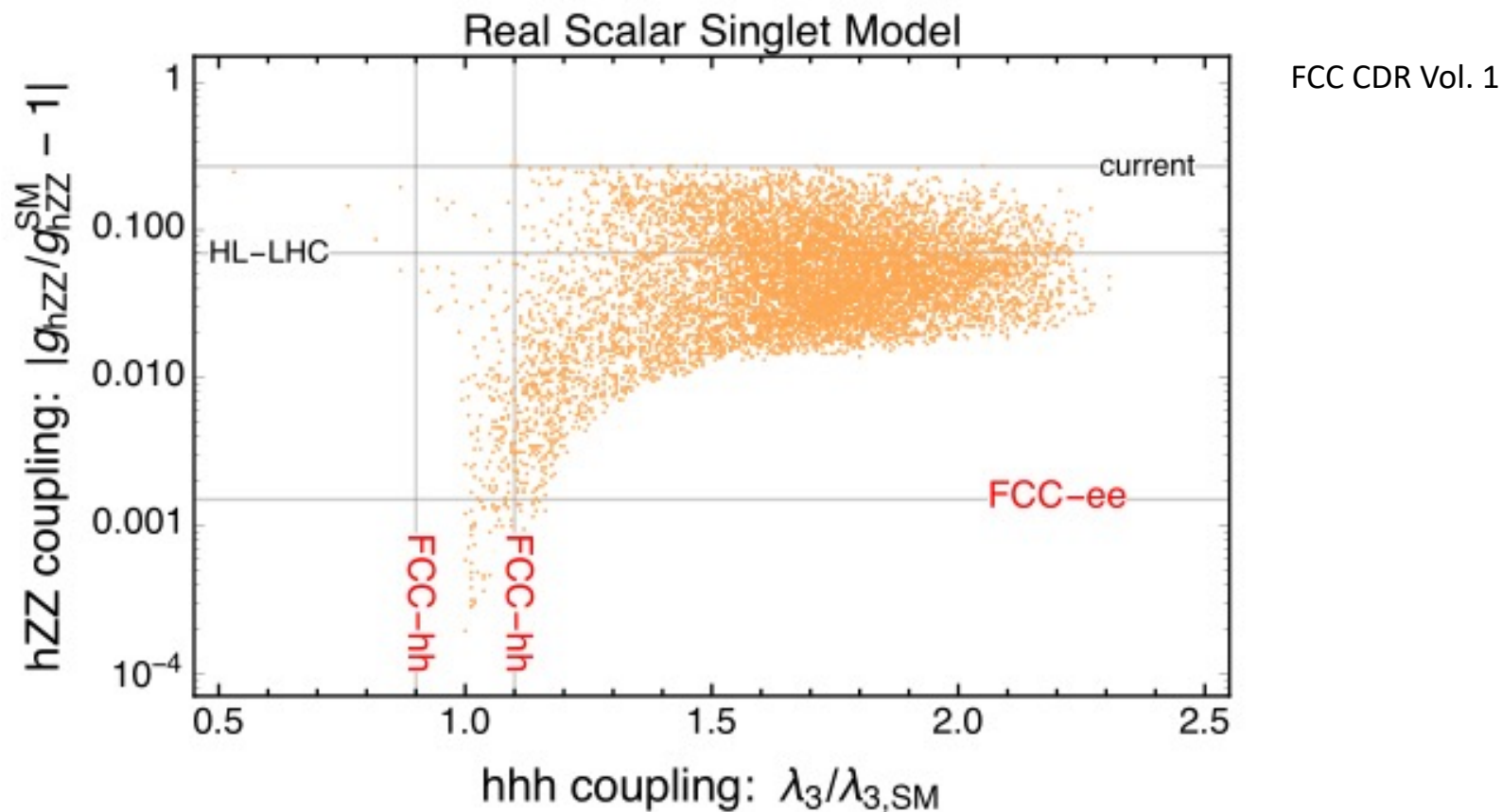
- What is the **purpose** of a **next-generation particle observatory**?

To explore the fundamental origins of our universe and its laws

- **Exploring**, not searching
 - “*Exploring the origins of our universe*” is a more accurate **mission statement**, unlike e.g. “*searching for supersymmetry and dark matter*”
 - “*Exploring the origin of the Higgs*” simpler to convey than **naturalness**
- “*Discovery stories*” risks putting the focus on *promising* to **find new physics**
- “*Exploring origins*” puts the focus on **open BSM questions** to be answered
 - Emphasises colliders as a **general-purpose particle observatory** with a *wide-ranging physics programme*

Origin of matter

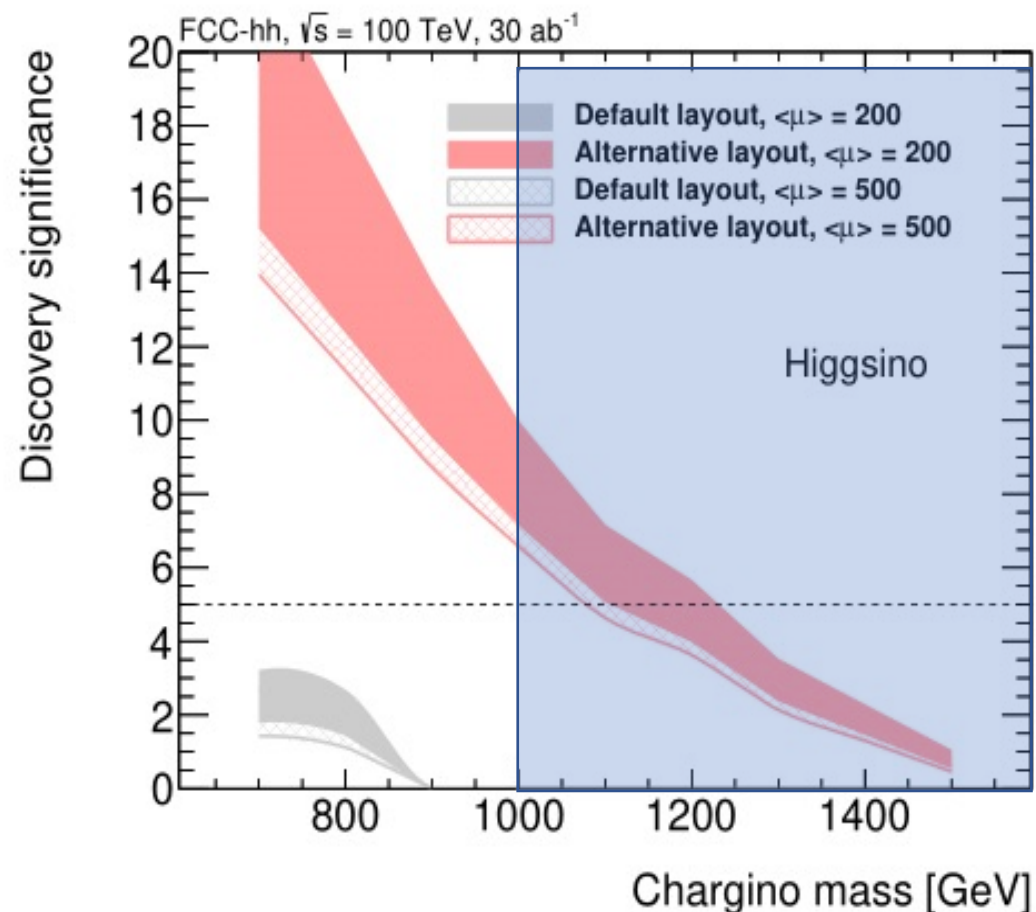
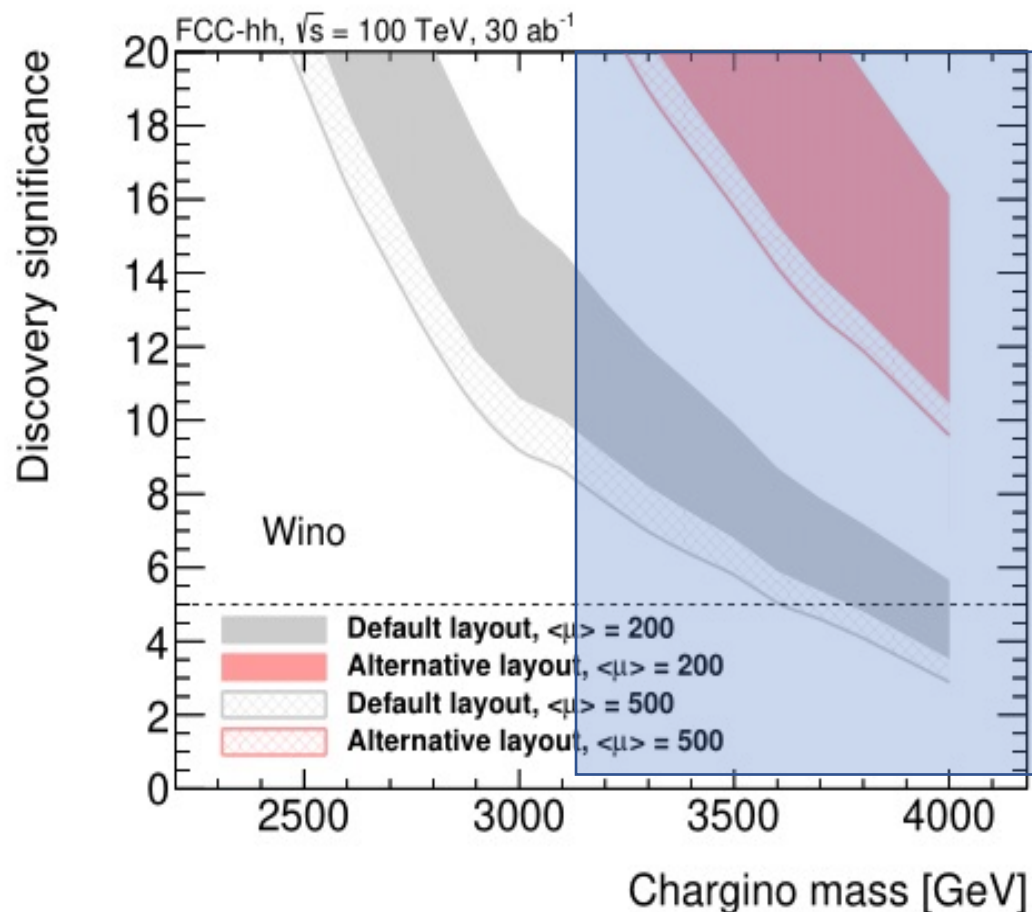
- Nature of the **electroweak phase transition**: *first or second order?*



- *Potential corroboration* with **gravitational wave signal** at LISA

Origin of dark matter

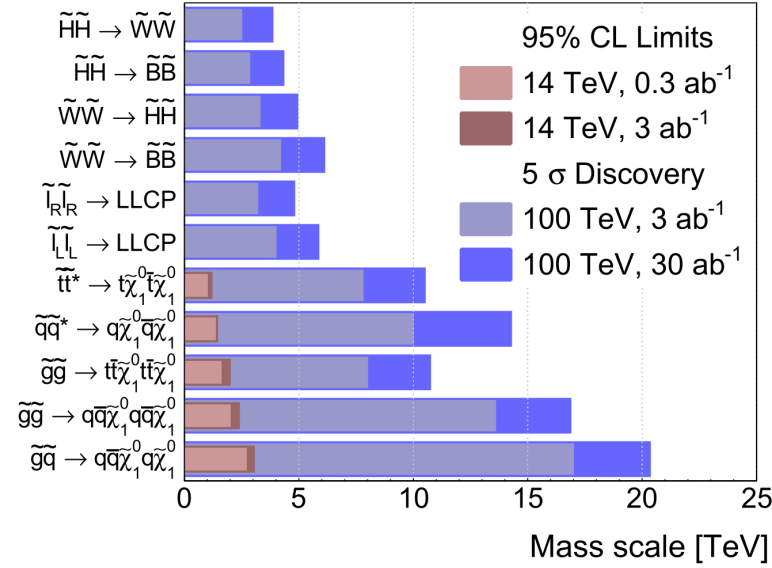
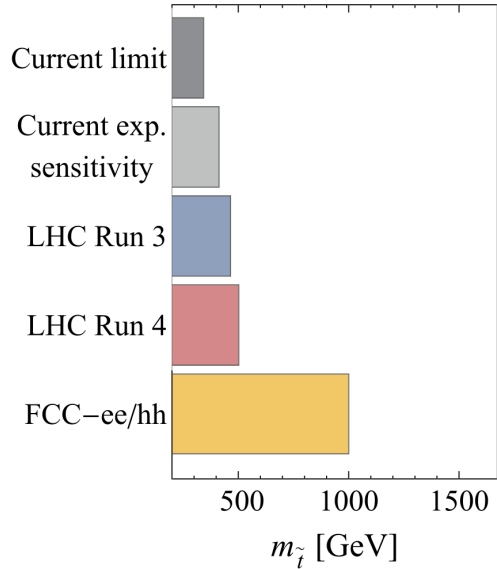
- Coverage of *entire* **doublet** and **triplet thermal WIMP** mass range



Origin of the Higgs

FCC CDR Vol. 1

Note: naturalness aside, still motivation in exploring origin of Higgs in models from which it emerges, where its mass is *calculable*

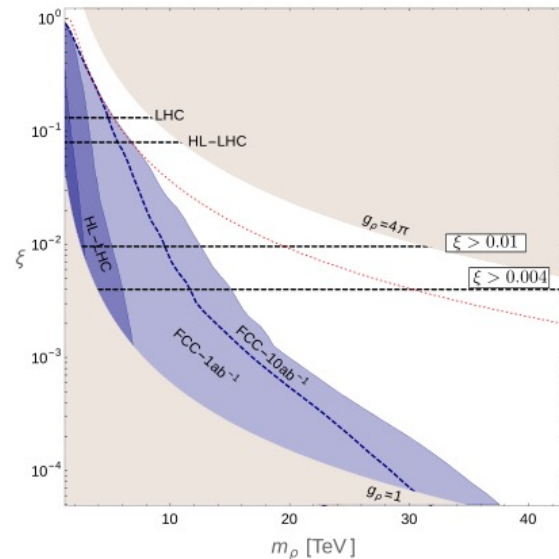
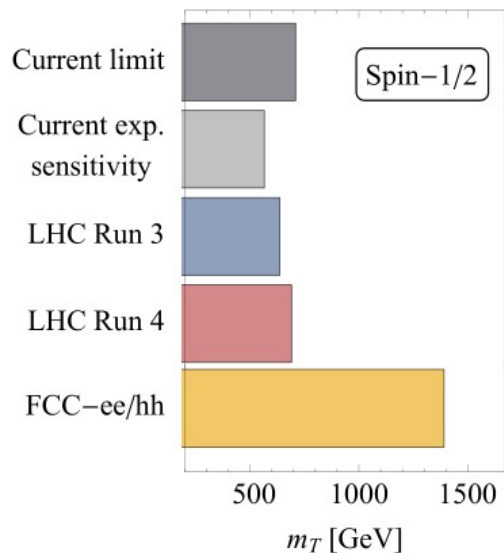


• Supersymmetry

- Massless spins 0, $\frac{1}{2}$, 1, $\frac{3}{2}$, 2 *only*
- Spin $\frac{3}{2}$ *must* be supersymmetric
- (Ir)relevant for solving **naturalness**?

• Composite Higgs / extra dimensions

- Is the Higgs **elementary** or **composite**?
- Are there *accessible* extra dimensions?



Contents

- Historical introduction
- The naturalness problem
- Cosmological solutions
- Colliders as general-purpose particle observatories
- **Radical BSM outcomes is still a possibility**

Potential BSM outcomes for naturalness at FCC

- **Radically conservative:** naturalness restored just around the corner
 - Natural supersymmetry
 - Composite Higgs/extra dimensions
- **Creatively conservative**
 - Twin Higgs
 - Stealth supersymmetry
- **Post-naturalness BSM**
 - Split supersymmetry
 - Vector-like fermions only
 - Lowered vacuum instability scale
 - Weak-scale new physics for cosmological dynamics
- **Radically new?**
 - Hard to imagine what form this might take, by definition
 - How might this show up?

Potential BSM outcomes for naturalness at FCC

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“Radically conservative” historical precedent

- 1930-40s: Success of QED. **QFT** emerges as the *new fundamental description of Nature*.
- 1960s: QFT is **unfashionable**, non-Abelian theory dismissed as an **unrealistic generalisation** of local symmetry-based forces. Widely believed a **radically new framework** will be required *e.g. to understand the strong force*.
- 1970s: **QFT triumphs** following Yang-Mills+Higgs+asymptotic freedom+renormalisation. Nature is **radically conservative**, *but more unified than ever*.
- 1980s: Success of SM. QFT understood as **most general EFT consistent with symmetry**. Higgs and cosmological constant *violate this symmetry principle*.

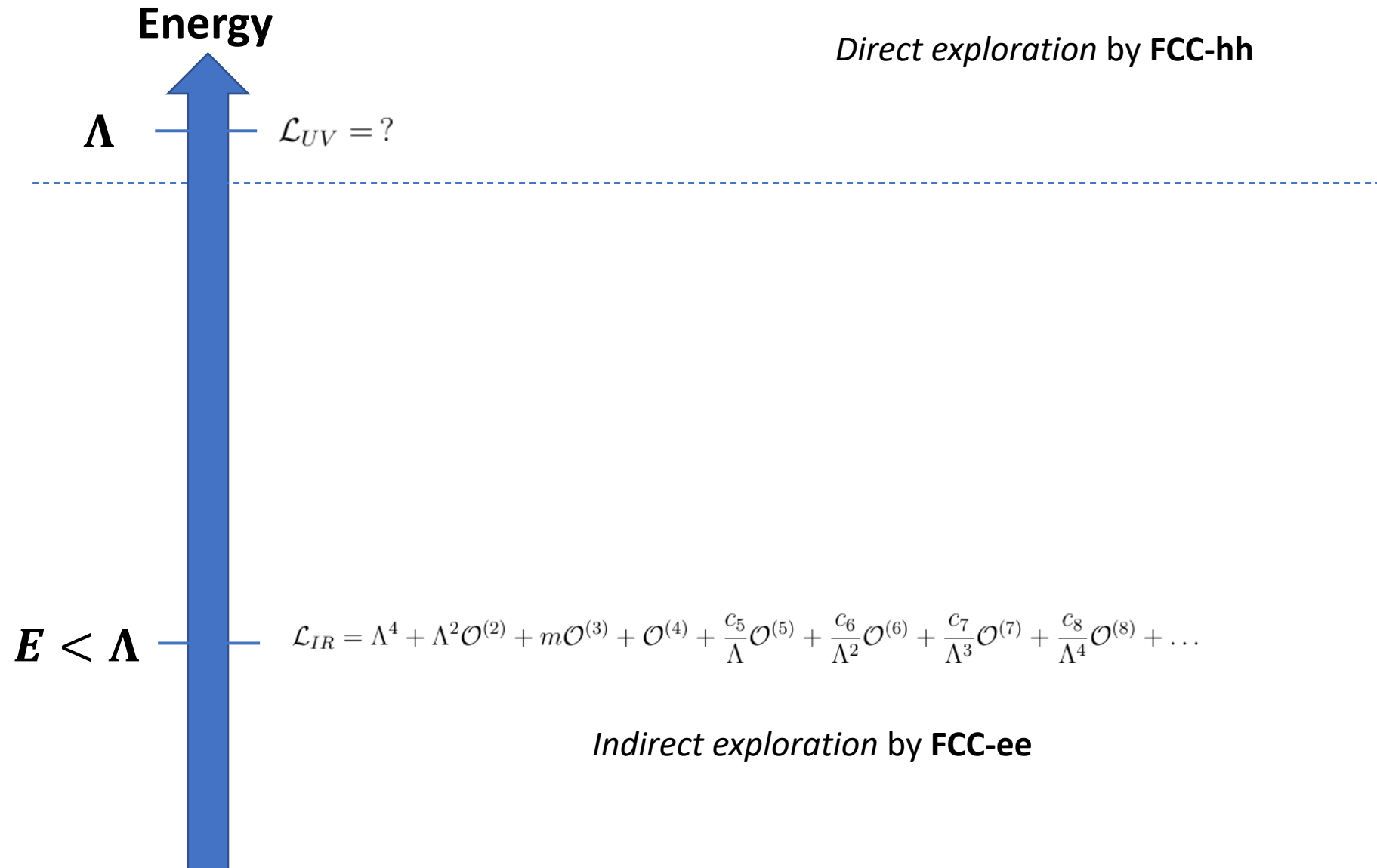
“Radically conservative” naturalness solution at FCC?

- 1980-2020s: Success of SM, established as the *fundamental description of Nature up to TeV scale*.
- 2040s: QFT is **unfashionable**, supersymmetry theory dismissed as an **unrealistic generalisation** of symmetry principles. Widely believed a **radically new framework** will be required *e.g. to understand naturalness*.
- 2060s: **QFT triumphs** following Yang-Mills+Higgs+asymptotic freedom+renormalisation+**supersymmetry**. Nature is **radically conservative**, *but more unified than ever*.
- 2080s: Success of MSSM

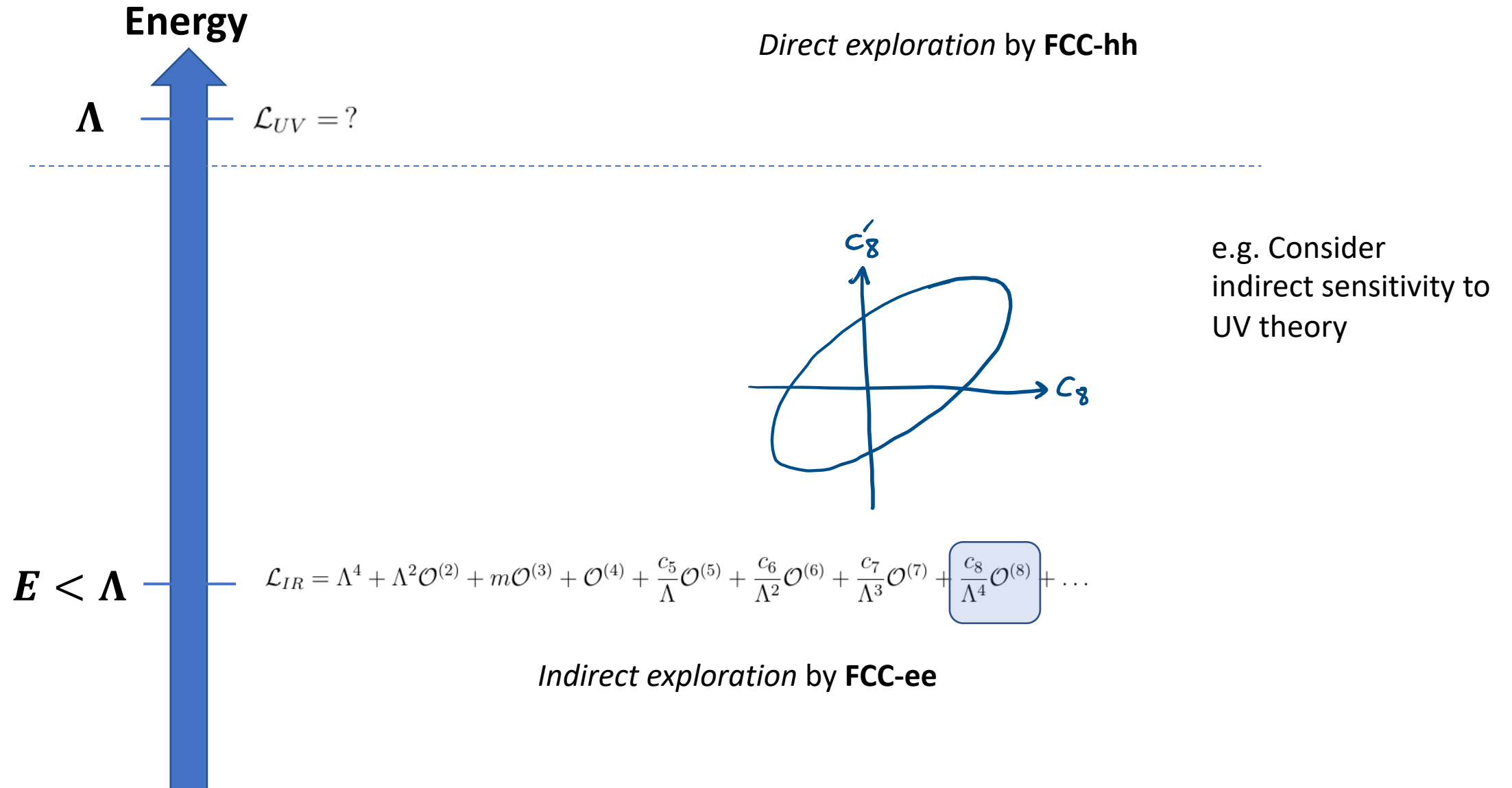
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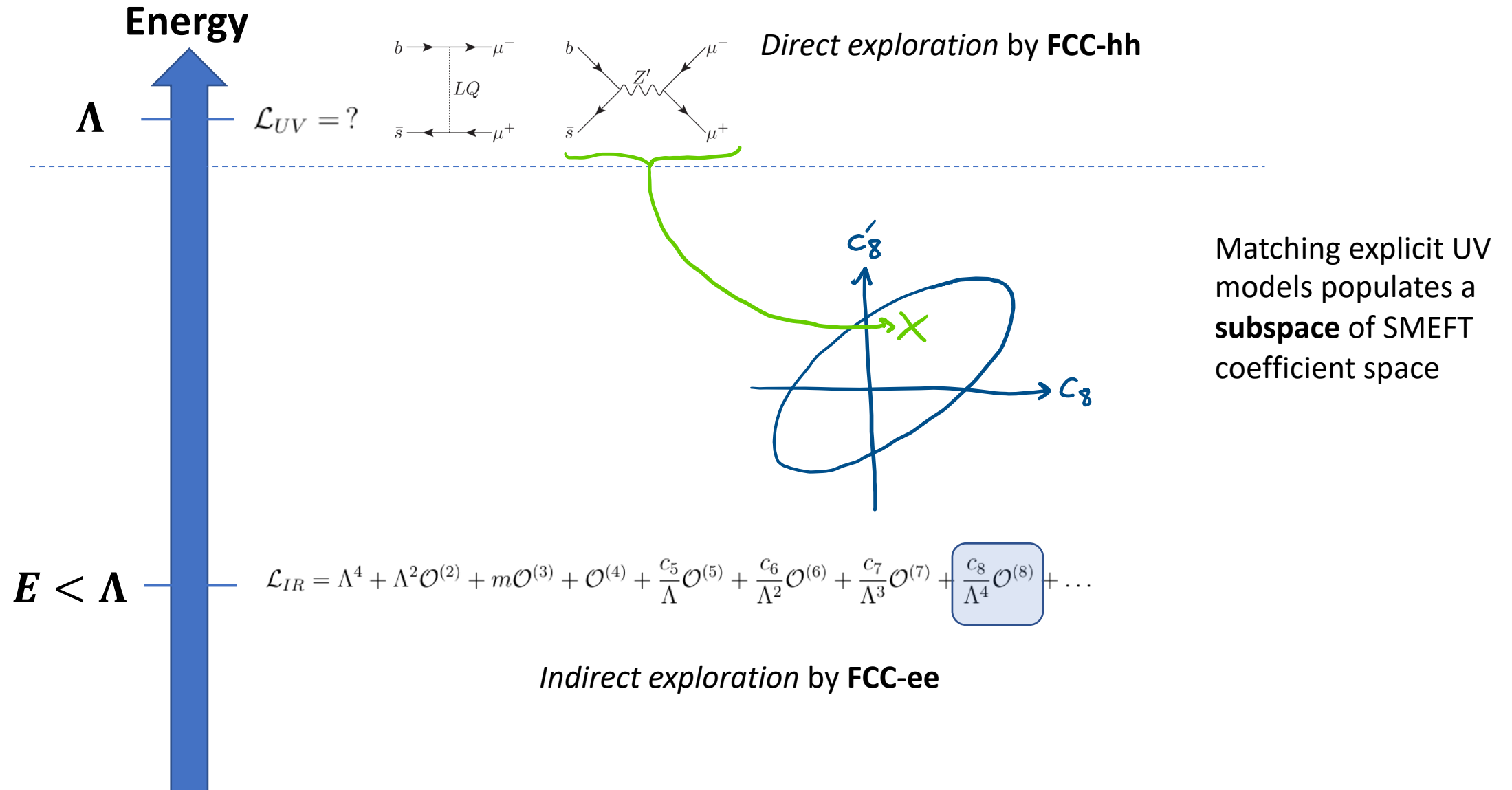
Radically new BSM?



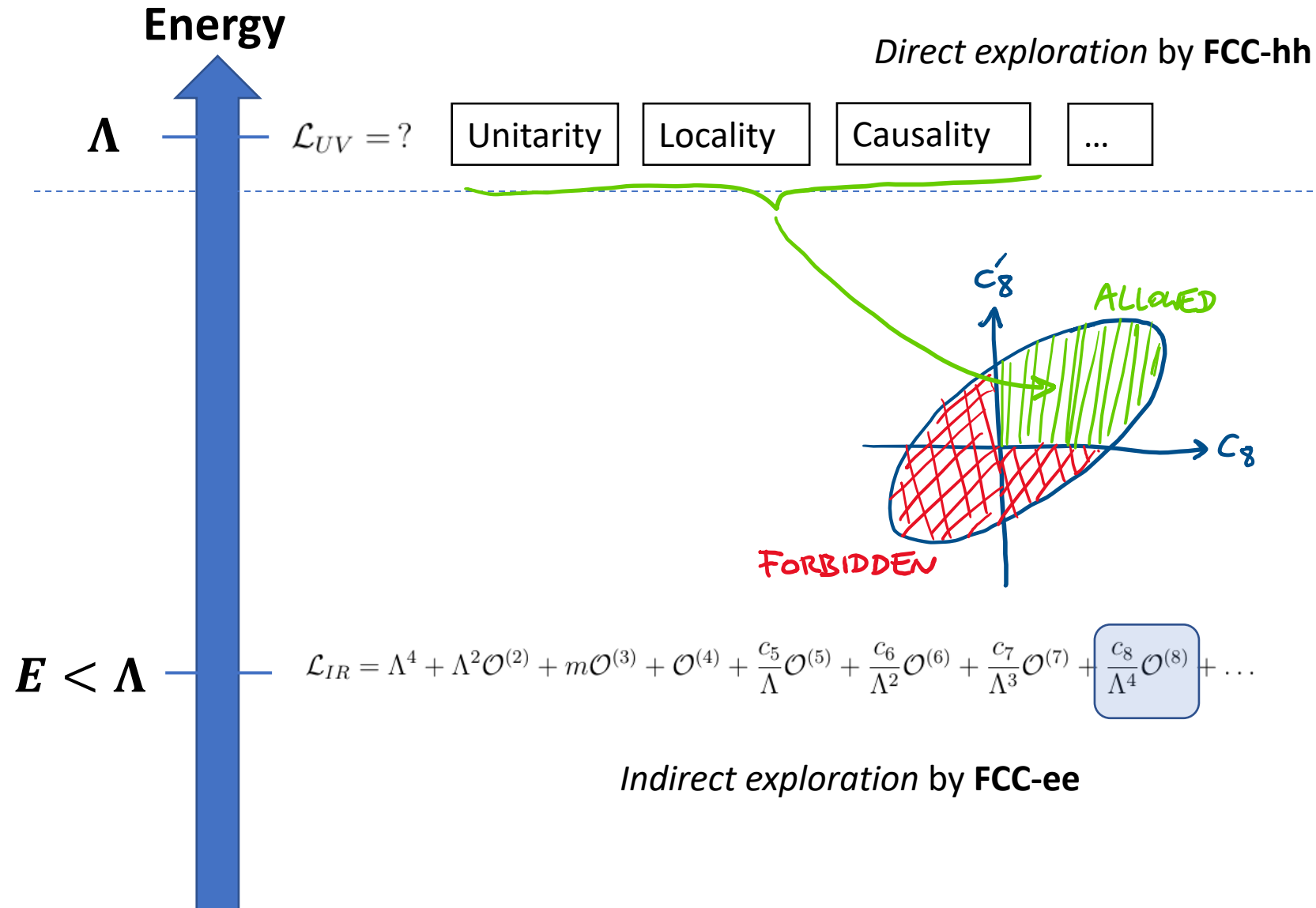
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Radically new BSM?



Radically new BSM?

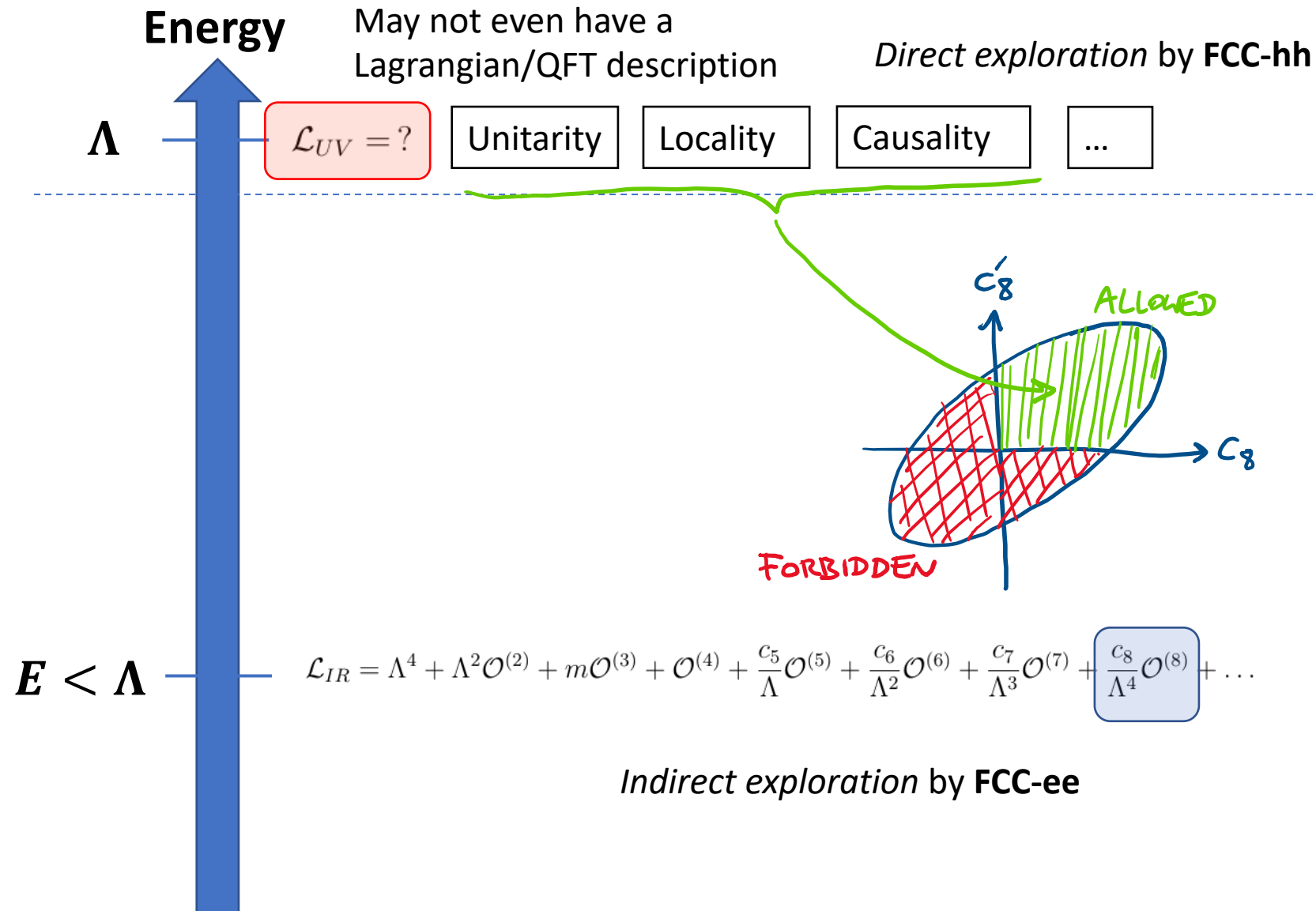


Direct exploration by FCC-hh

Positivity bounds forbid **negative signs** of SMEFT coefficients assuming only general fundamental principles in the UV

Measuring the “*wrong*” sign experimentally would have **truly revolutionary** consequences for the underlying theory!

Radically new BSM?



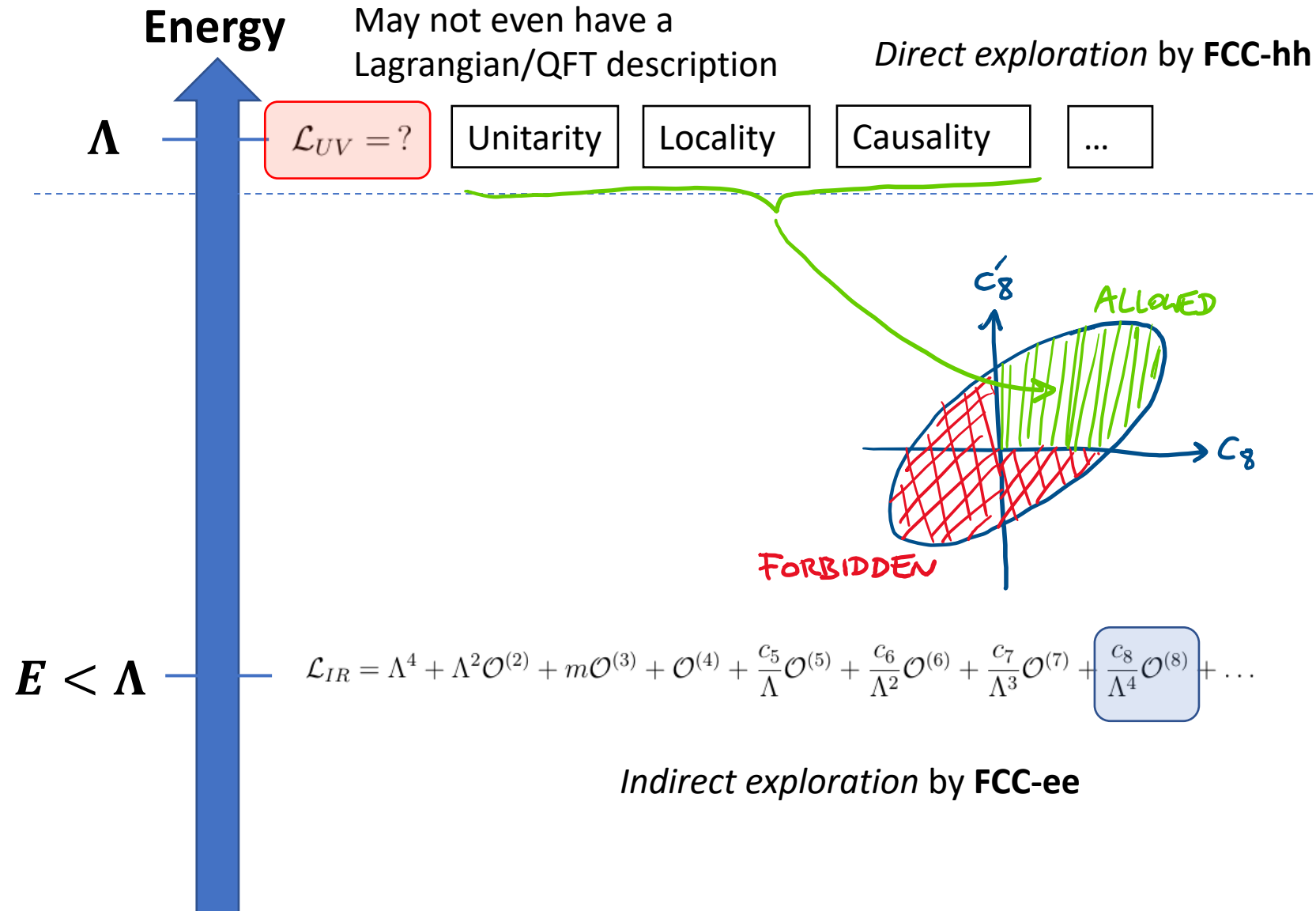
Positivity bounds forbid **negative signs** of SMEFT coefficients assuming only general fundamental principles in the UV

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Radically new BSM?

Positivity may also be related to the electroweak hierarchy problem

2308.06226 Davighi, Melville, Mimasu, TY



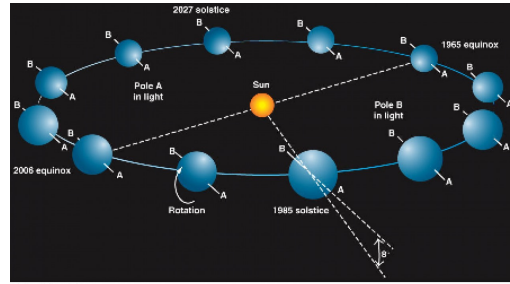
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Radically new BSM?

- Sometimes an anomaly in **indirect precision** measurement = *something missing*

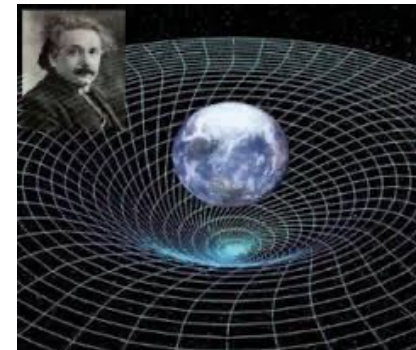
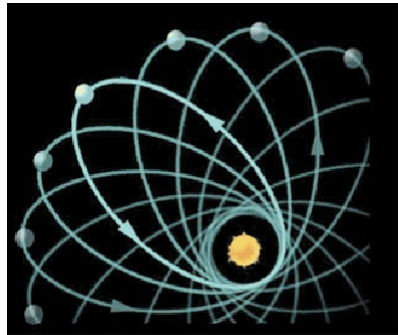
Anomaly in orbit of Uranus



Discovery of Neptune

- Sometimes its implications are *far more radical*

Anomaly in orbit of Mercury



Explained by General Relativity

Conclusion

- *“What would be the use of such extreme refinement in the science of measurement? [...] The more important fundamental laws and facts of physical science have all been discovered, and these are so firmly established that the possibility of their ever being supplanted in consequence of new discoveries is exceedingly remote. [...]”*

–A. Michelson 1903

Conclusion

- *“What would be the use of such extreme refinement in the science of measurement? **Very briefly and in general terms the answer would be that in this direction the greater part of all future discovery must lie.** The more important fundamental laws and facts of physical science have all been discovered, and these are so firmly established that the possibility of their ever being supplanted in consequence of new discoveries is exceedingly remote. **Nevertheless, it has been found that there are apparent exceptions to most of these laws, and this is particularly true when the observations are pushed to a limit, i.e., whenever the circumstances of experiment are such that extreme cases can be examined.**”*

–A. Michelson 1903

Conclusion

- 1900: Almost all data agree spectacularly with the fundamental framework of the time, *no reason to doubt its universal applicability or completeness.*
- 1920s: A combination of **precision measurements** (Mercury), **aesthetic arguments** (relativity) supported by **null experimental results** (Michelson-Morley), and **theoretical inconsistencies** (Rayleigh-Jeans UV catastrophe) lead to an overhaul of the fundamental picture at **smaller scales** and **higher energies** after *pushing the frontiers of technology and theory into new regimes.*

Conclusion

- 2020: Almost all data agree spectacularly with the fundamental framework of the time, *no reason to doubt its universal applicability or completeness.*
- 2050s: A combination of **precision measurements** (MW, Hubble), **aesthetic arguments** (naturalness) supported by **null experimental results** (LHC), and **theoretical inconsistencies** (black hole information paradox) lead to an overhaul of the fundamental picture at **smaller scales** and **higher energies** after *pushing the frontiers of technology and theory into new regimes.*

Backup

Is it too expensive?

- No, not relative to other taxpayer-funded big projects
- Olympic games costs \$10-20 billion to a single nation for a summer's entertainment
- FCC-ee+hh costs \$20 billion shared between dozens of countries over decades for improving our fundamental knowledge of the universe
- Astrophysics missions are billion-dollar proposals, e.g. Dragonfly Titan. FCC-ee's vast physics case is easily > 10 astrophysics instruments.

Astro/cosmo captures the public imagination

- So does particle physics: the Higgs boson has become a household name
- Don't underestimate the public – they are fascinated by big fundamental ideas, not just pretty pictures

Is it worth it?

- See talk

When do we stop?

- When we lose our spirit of exploration and curiosity
- When we don't learn anything or gains become marginal
- Far from being marginal, the gains are huge
- We just washed ashore upon *terra incognita* and have barely left the beach
- LHC enters threshold of TeV-scale physics that FCC can explore fully

What about climate change?

- 90% of CERN's energy is from non-warming sources
- All activity contributes to climate change. This question implies particle physics is not an activity worth continuing.
- Expanding our fundamental knowledge of the smallest scales is as important as many other human endeavours we would not want cancelled completely
- Of course, we should make particle physics as efficient as possible
- Particle physics is also part of the solution, by shaping society positively

I won't be alive to see it

- Ensuring particle physics thrives for the rest of the century is more important
- These ambitious multi-generational projects are the cathedrals of our era

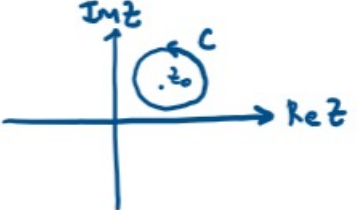
Why not skip FCC-ee and do FCC-hh first?

- We can't – technology and cost won't be feasible on that timescale
- FCC-ee is just as exciting and worth doing in its own right

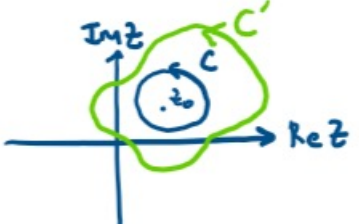
EFT Positivity Bounds

Contour integral isolates coefficient of simple pole

$f(z) = d_n (z-z_0)^n + \dots + a_1 (z-z_0) + a_0 + \frac{b_1}{z-z_0} + \frac{b_2}{(z-z_0)^2} + \dots$



$\Rightarrow \oint_C \frac{dz}{2\pi i} f(z) = b_1$

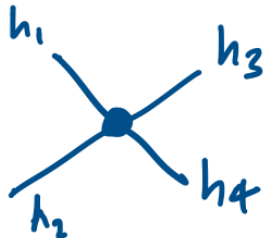


$\Rightarrow \oint_{C'} \frac{dz}{2\pi i} f(z) = b_1$

Analyticity allows contour deformation

EFT Positivity Bounds

Analytically continue 2-to-2 scattering amplitude $A(s)$ to complex s


$$= \hat{A}_{h_1 h_2 h_3 h_4}(s, t) = \sum_{i, j=0} C_{i, j} \frac{s^i t^j}{\Lambda^{2i+2j}}$$

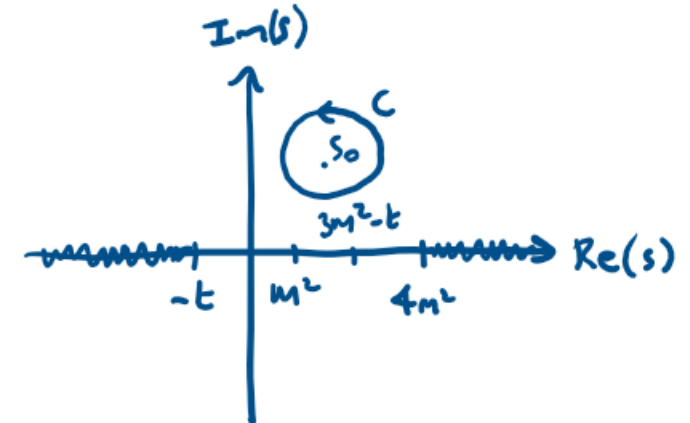
e.g. $2_{\text{dim}=8} = \frac{\bar{c}_8}{\Lambda^4} (\Psi \partial \Psi)^2 \Rightarrow \hat{A} \sim \frac{\bar{c}_8 s^2}{\Lambda^4}$

Higher-dimensional operators contribute to amplitude at different powers of s

EFT Positivity Bounds

Contour integral isolates higher-dimensional operator contributions for choice of N

$$\partial_s^{(2N)} \hat{A}_{h_1 h_2 h_3 h_4}(s_0, t) = \oint_C \frac{ds}{2\pi i} \frac{\hat{A}_{h_1 h_2 h_3 h_4}(s, t)}{(s - s_0)^{2N+1}}$$

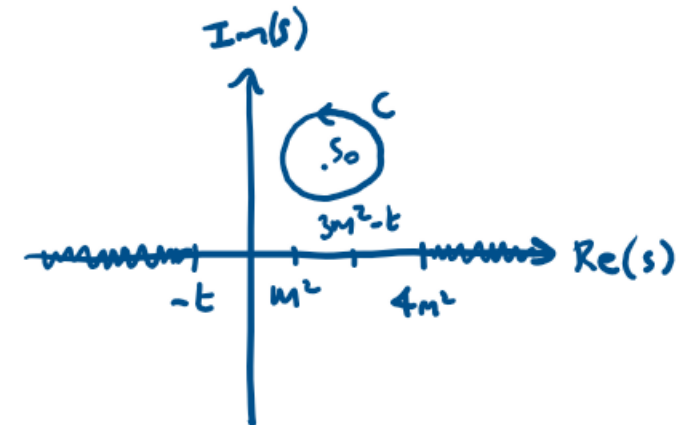


Analyticity (causality) of 2-to-2 scattering amplitude allows contour deformation sensitive to UV at large s

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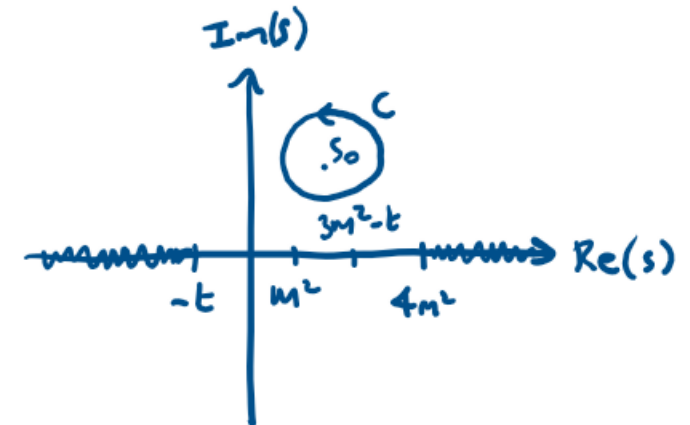


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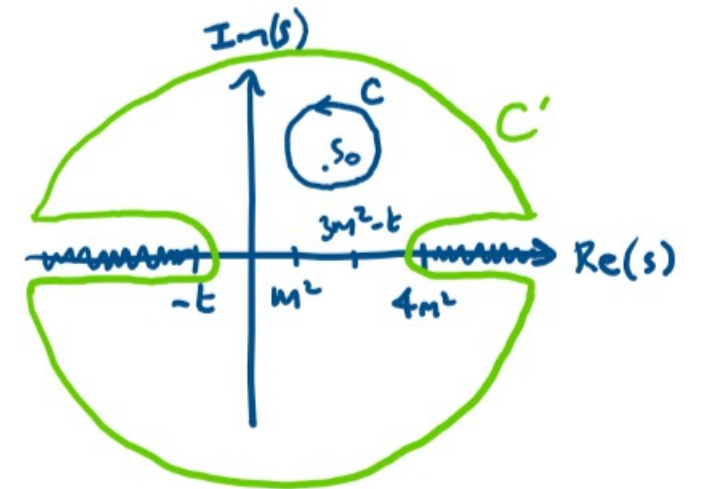


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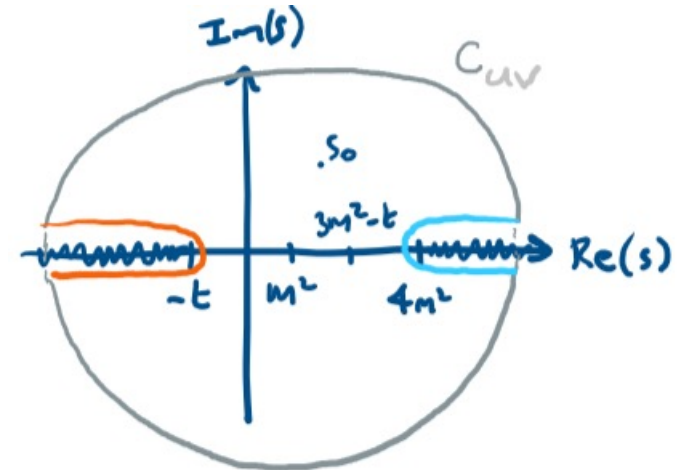


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 &= \oint_{C'} \frac{ds}{2\pi i} \frac{\hat{A}_{h_1 h_2 h_3 h_4}(s, t)}{(s - s_0)^{2N+1}} \\
 &= \underbrace{I_{uv}}_{\text{locality}} + \underbrace{I_s + I_u}_{\text{unitarity}} > 0 ?
 \end{aligned}$$



Analyticity (causality) of 2-to-2 scattering amplitude allows contour deformation sensitive to UV at large s

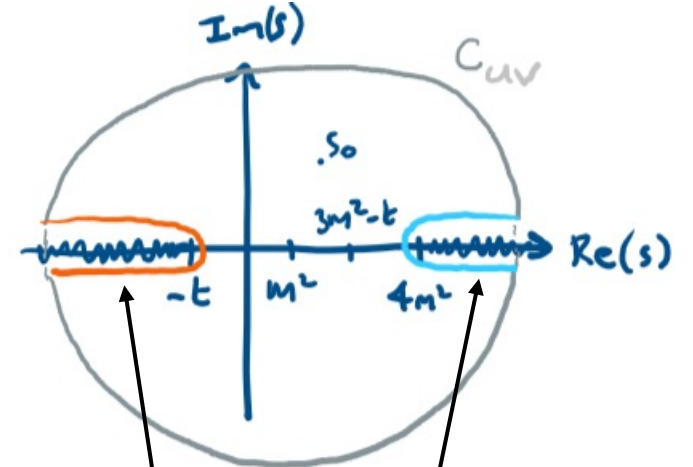
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$$= \underbrace{I_{uv}}_{\text{locality}} + \underbrace{I_s + I_u}_{\text{unitarity}} > 0 ?$$



$$\lim_{|s| \rightarrow \infty} \hat{A}(s) < s^2$$

$$\mathbb{1} = S^\dagger S = (\mathbb{1} + iT)^\dagger (\mathbb{1} + iT) \Rightarrow i(T^\dagger - T) = |T|^2 \geq 0$$

Analyticity (causality) of 2-to-2 scattering amplitude allows contour deformation sensitive to UV at large s

EFT Positivity Bounds

e.g. 2203.06805 Snowmass review

Contour integral isolates higher-dimensional operator contributions for choice of N

$$\Rightarrow \partial_s^{(2N)} \hat{A}_{h_1 h_2 h_3 h_4}(s_0, t) > 0$$

Positivity mandated by unitarity, locality, causality (and Lorentz invariance) of UV

EFT Positivity Bounds

e.g. Contour integral isolates dimension-8 operator contributions for $N = 1$

$$\mathcal{L}_{\text{EFT}}[v] = \bar{c}_8 \frac{\mathcal{O}_8}{\Lambda^4}$$

$$\bar{c}_8 \sim \partial_s^{(2N)} \hat{A}_{h_1 h_2 h_3 h_4}(s_0, t) > 0$$

Positivity mandated by unitarity, locality, causality (and Lorentz invariance) of UV

Potential Positivity Bounds

Scalar potentials with a stable vev can contribute to positivity bounds

2308.06226 Davighi, Melville, Mimasu, TY

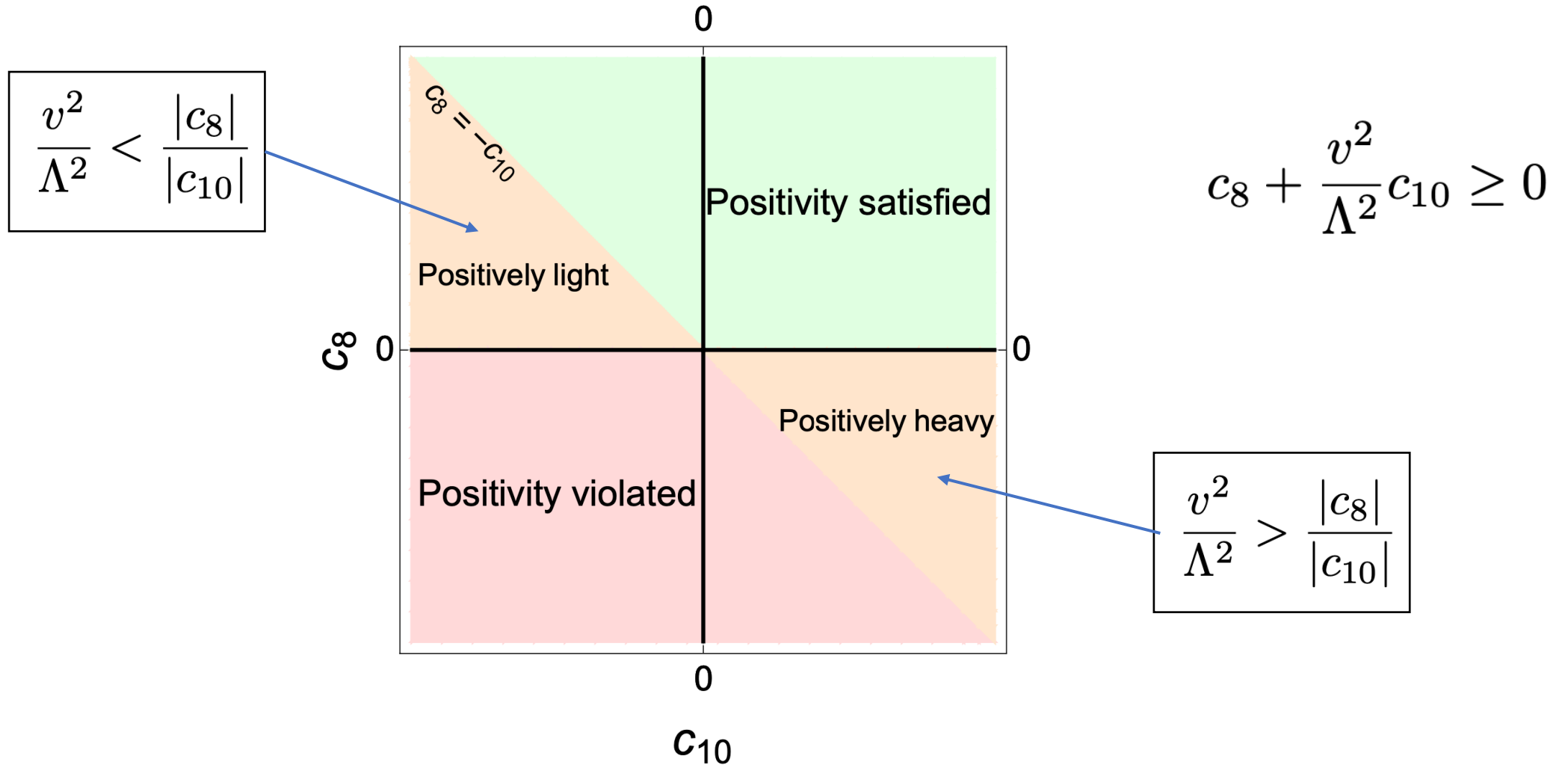
$$\mathcal{L}_{\text{EFT}}[H] = c_8 \frac{\mathcal{O}_8}{\Lambda^4} + c_{10} \frac{|H|^2 \mathcal{O}_8}{\Lambda^6}$$

$$c_8 + \frac{v^2}{\Lambda^2} c_{10} \geq 0$$

Positivity mandated by unitarity, locality, causality (and Lorentz invariance) of UV

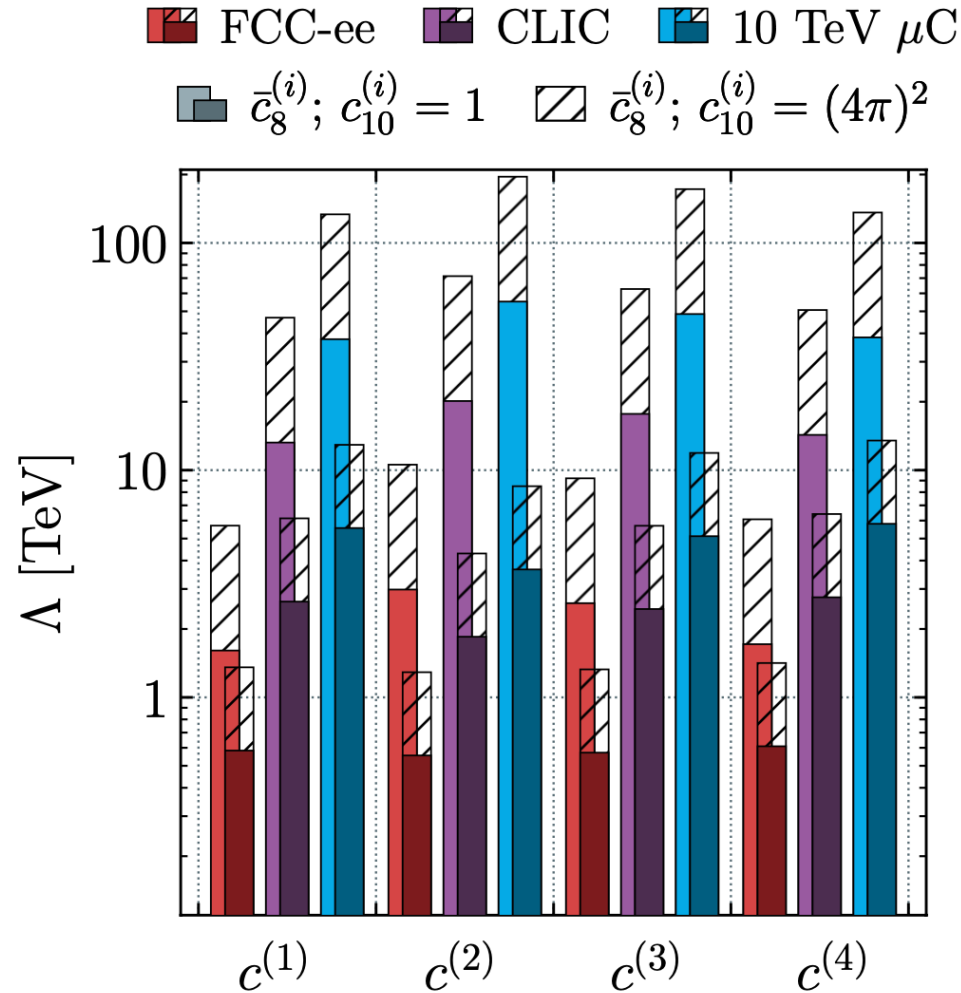
Positively light Higgs

A unitary, local, and causal UV theory that lives in $|c_8| \ll |c_{10}|$ EFT parameter space *necessarily* has restricted vev v



Positively light Higgs

This scenario could in principle be established experimentally for a little hierarchy up to O(10) TeV



$$\mathcal{L}_{\text{EFT}}[H] = c_8 \frac{\mathcal{O}_8}{\Lambda^4} + c_{10} \frac{|H|^2 \mathcal{O}_8}{\Lambda^6}$$

$$\mathcal{O}_8^{(1)} = \partial^\nu (\bar{e}_i \gamma^\mu e_i) \partial_\nu (\bar{e}_i \gamma_\mu e_i) ,$$

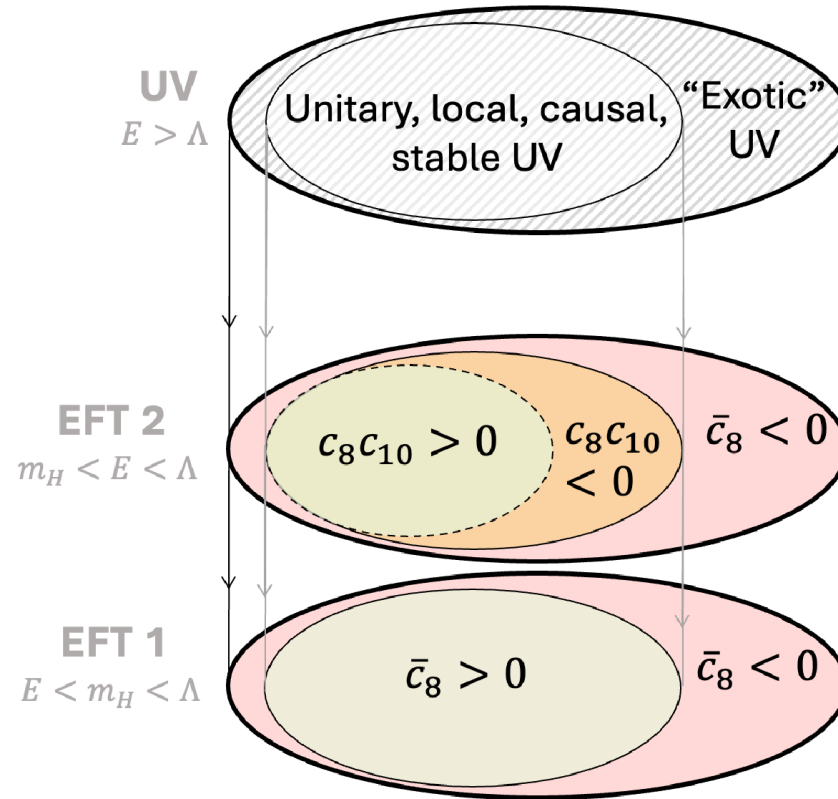
$$\mathcal{O}_8^{(2)} = \partial^\nu (\bar{e}_i \gamma^\mu e_i) \partial_\nu (\bar{L}_i \gamma_\mu L_i) ,$$

$$\mathcal{O}_8^{(3)} = D^\nu (\bar{e}_i L_i) D_\nu (\bar{L}_i e_i) ,$$

$$\mathcal{O}_8^{(4)} = \partial^\nu (\bar{L}_i \gamma^\mu L_i) \partial_\nu (\bar{L}_i \gamma_\mu L_i) ,$$

Conclusion

There exists a region of EFT parameter space where positivity is conditional upon a scalar vev hierarchy



Connects an *a priori* unrelated IR observable to a restricted Higgs vev through general UV assumptions

(*c.f.* Fifth force and Weak Gravity Conjecture = light Higgs) [1407.7865 Cheung & Remmen]

Conclusion

Everything about the SM Higgs potential coefficients are highly non-generic:

Dimension-0 operator (cosmological constant) balanced between implosion and explosion

Dimension-4 operator (Higgs quartic) on boundary of vacuum stability and instability

$$\mathcal{L} = \Lambda^4 + \Lambda^2 \mathcal{O}^{(2)} + m \mathcal{O}^{(3)} + \mathcal{O}^{(4)} + \frac{1}{\Lambda} \mathcal{O}^{(5)} + \frac{1}{\Lambda^2} \mathcal{O}^{(6)} + \frac{1}{\Lambda^3} \mathcal{O}^{(7)} + \frac{1}{\Lambda^4} \mathcal{O}^{(8)} + \dots$$

Dimension-2 operator (Higgs mass) tuned between unbroken and phase phases

Higher-dimensional operator coefficients may also place us on the edge of positive and non-positive theory space!

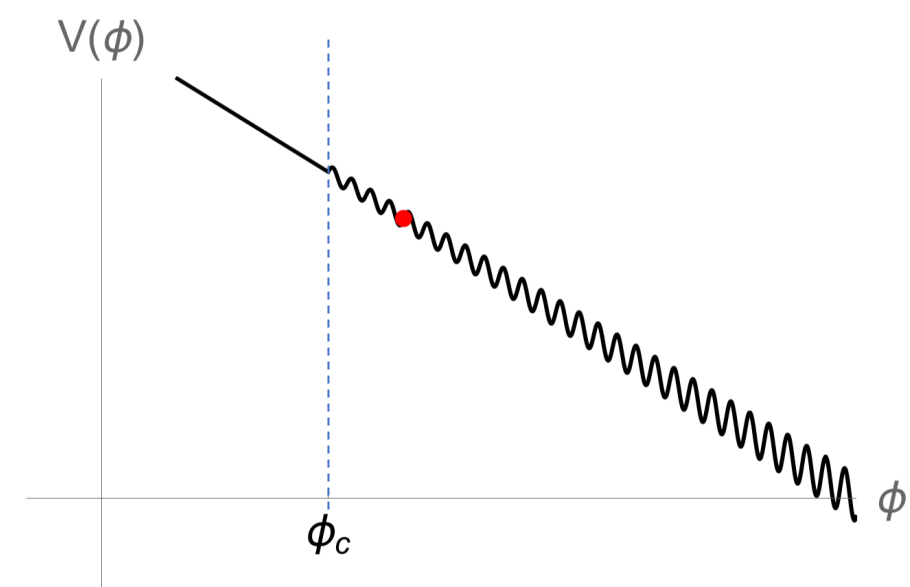
Cosmological relaxation

P. W. Graham, D. E. Kaplan and S. Rajendran,
[arXiv:1504.07551]

- Assume Higgs mass is naturally large at cut-off M

$$\mathcal{L} \supset (M^2 + \epsilon M \phi) |h|^2 + \epsilon M^3 \phi + \dots + \Lambda_p^{4-n} v^n \cos\left(\frac{\phi}{f_p}\right)$$

- Higgs quadratic term scanned by axion-like field ϕ during inflation
- ϕ protected by shift symmetry, explicitly broken by small parameter ϵ
- Backreaction when $\langle h \rangle \sim v$ stops ϕ evolution at small electroweak scale v



$$\epsilon M^3 \simeq \frac{\Lambda_p^{4-n} v^n}{f_p}$$

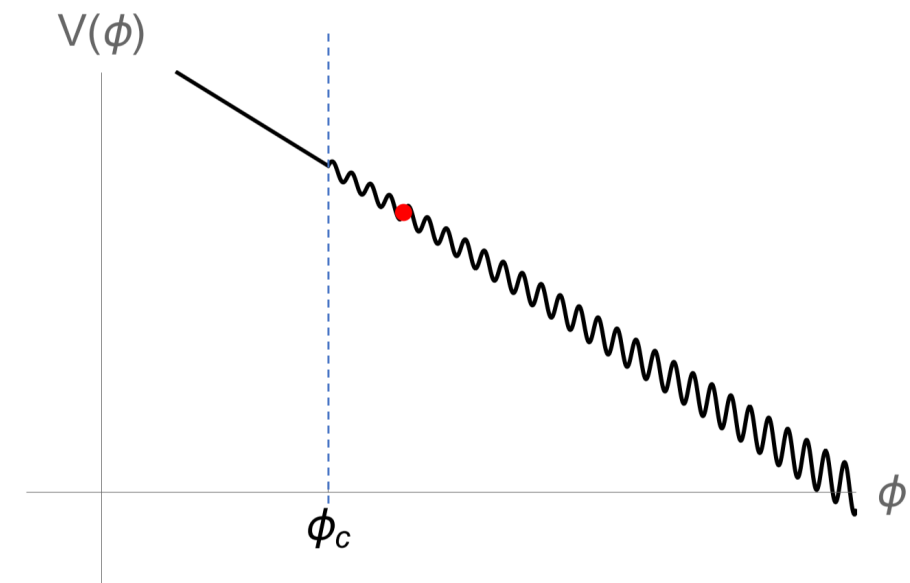
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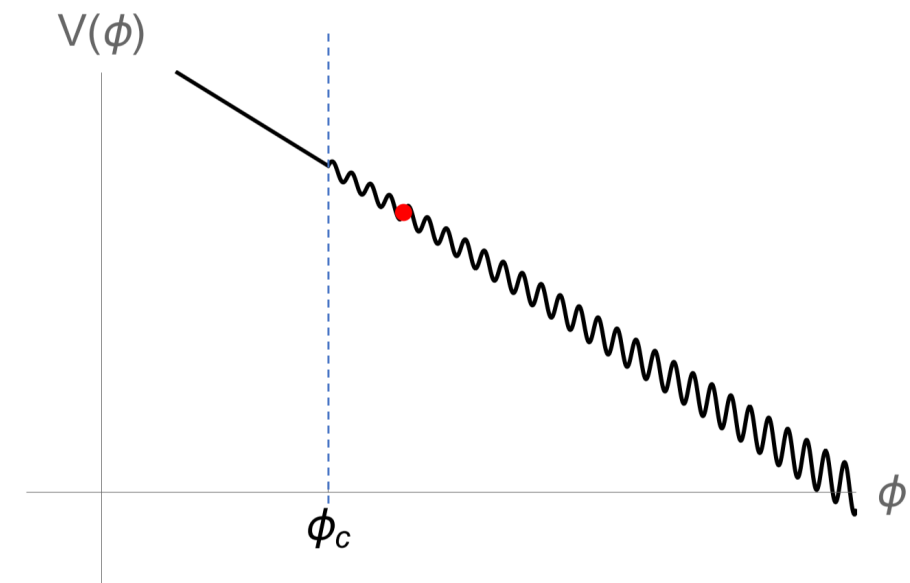
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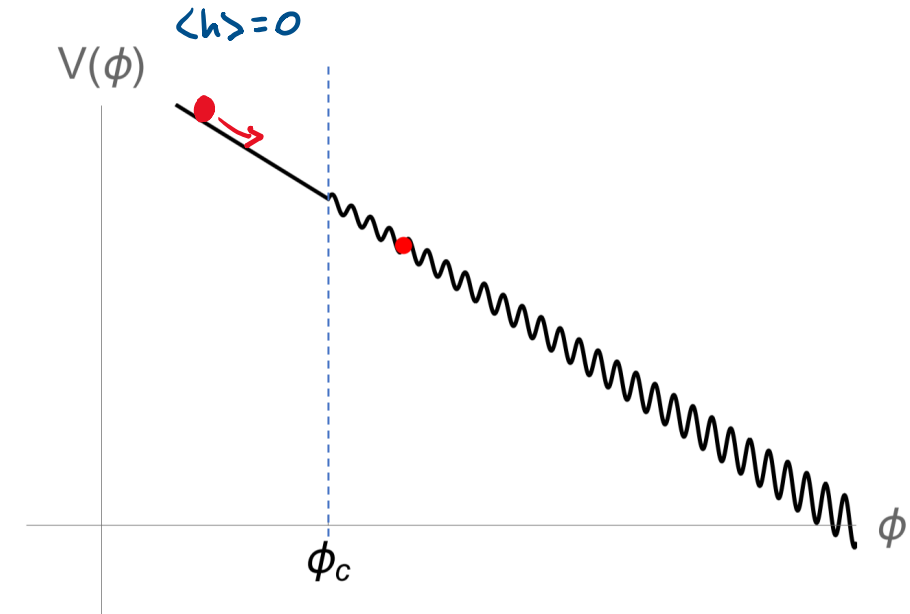
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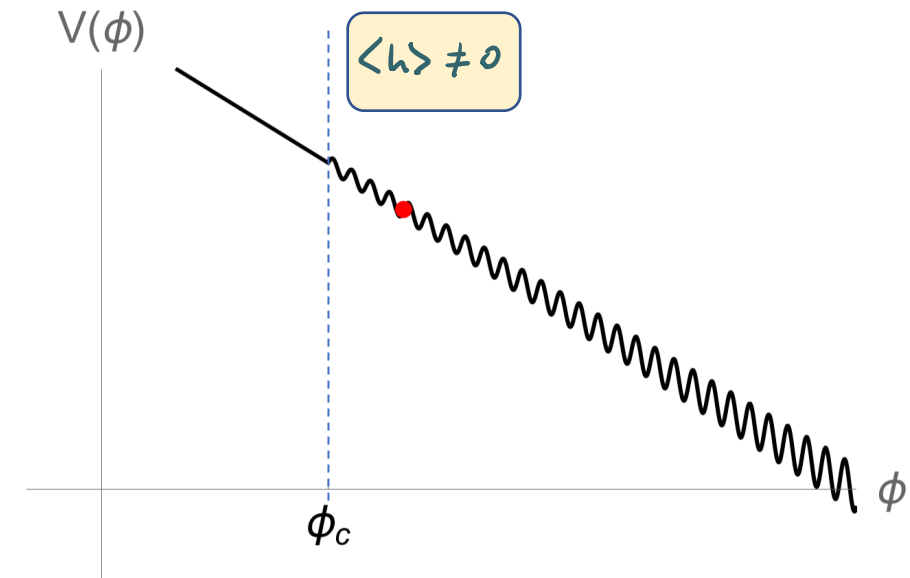
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Cosmological relaxatio

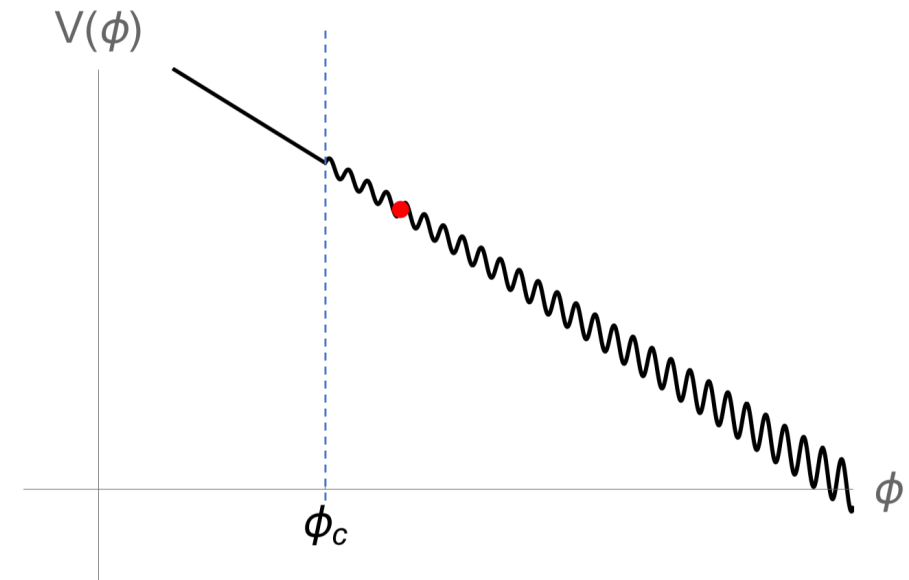
Constraints: $H < v$, classical rolling vs quantum, inflaton energy density dominates relaxion, etc.

Very small ϵ and natural scanning range lead to super-planckian field excursions, exponential e-foldings...

- Assume Higgs mass is naturally l

$$\mathcal{L} \supset (M^2 + \epsilon M \phi) |h|^2 + \epsilon M^3 \phi + \dots + \Lambda_p^{4-n} v^n \cos\left(\frac{\phi}{f_p}\right)$$

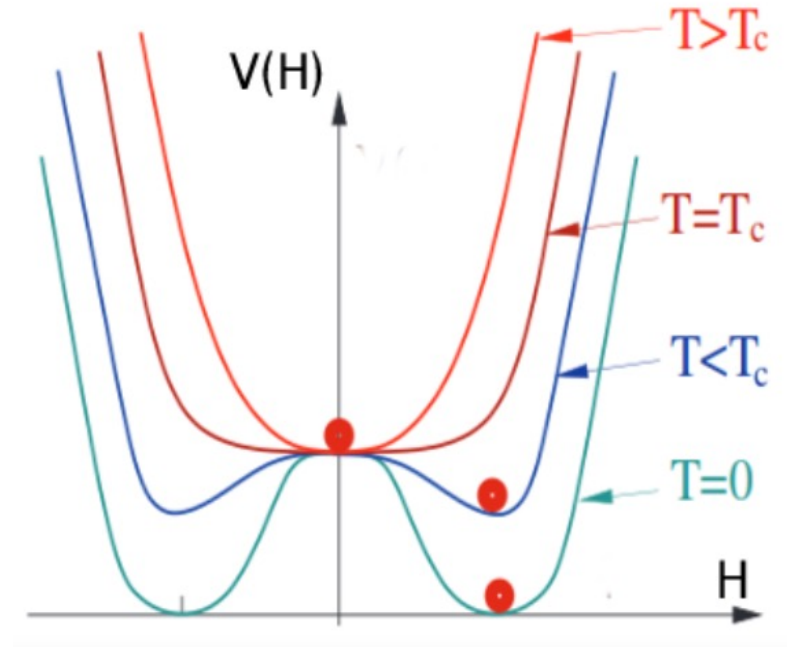
- Higgs quadratic term scanned by axion-like field ϕ during inflation
- ϕ protected by shift symmetry, explicitly broken by small parameter ϵ
- Backreaction when $\langle h \rangle \sim v$ stops ϕ evolution at small electroweak scale v



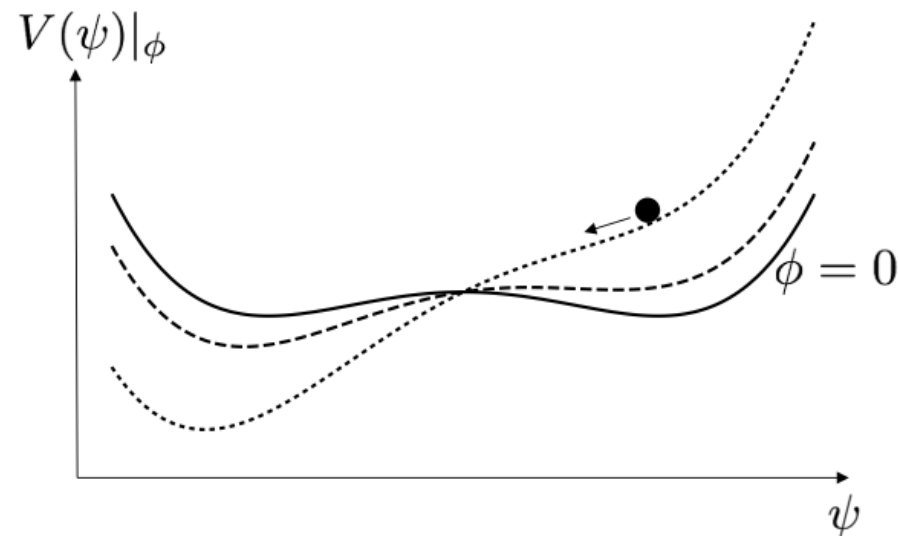
$$\epsilon M^3 \simeq \frac{\Lambda_p^{4-n} v^n}{f_p}$$

Phase Transitions (PT)

- **Classical PT:** varying background temperature
- **Quantum PT:** varying background field



$$V = \frac{\lambda}{4} (\psi^2 - \rho^2)^2 + \kappa\phi\psi$$



Fokker-Planck Volume (FPV) equation

- **Langevin equation:** classical slow-roll + Hubble quantum fluctuations

$$\phi(t + \Delta t) = \phi(t) - \frac{V'}{3H} \Delta t + \eta_{\Delta t}(t)$$

- Volume-averaged Langevin trajectories: **FPV for volume distribution** $P(\phi, t)$

$$\frac{\partial}{\partial \phi} \left[\frac{\hbar}{8\pi^2} \frac{\partial(H^3 P)}{\partial \phi} + \frac{V' P}{3H} \right] + 3HP = \frac{\partial P}{\partial t}$$

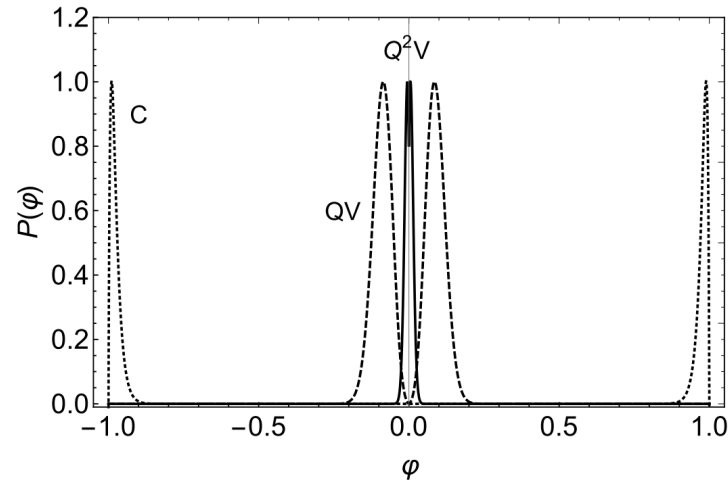
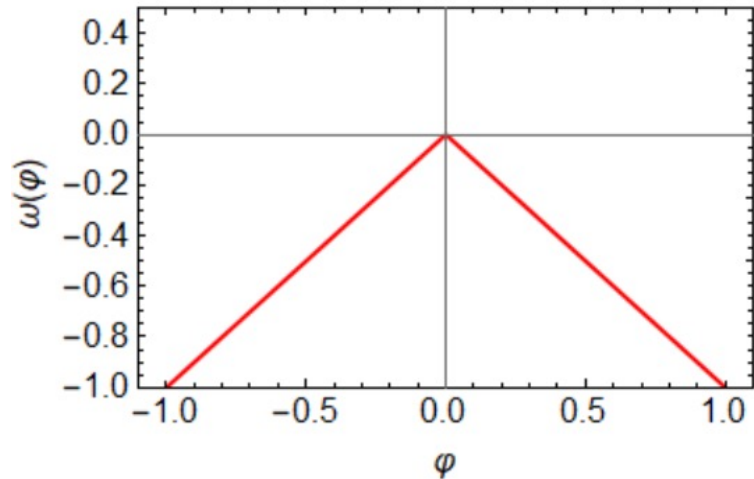
$$H(\phi) = \sqrt{\frac{V(\phi)}{3M_p^2}}$$

Quantum
diffusion term

Classical drift
term

Volume term

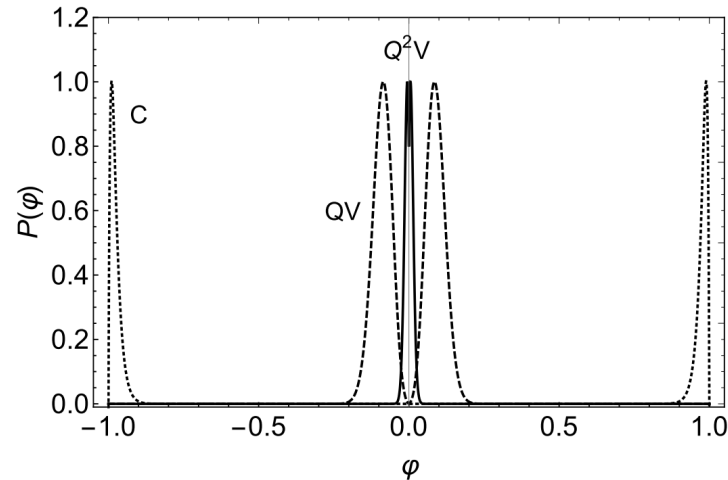
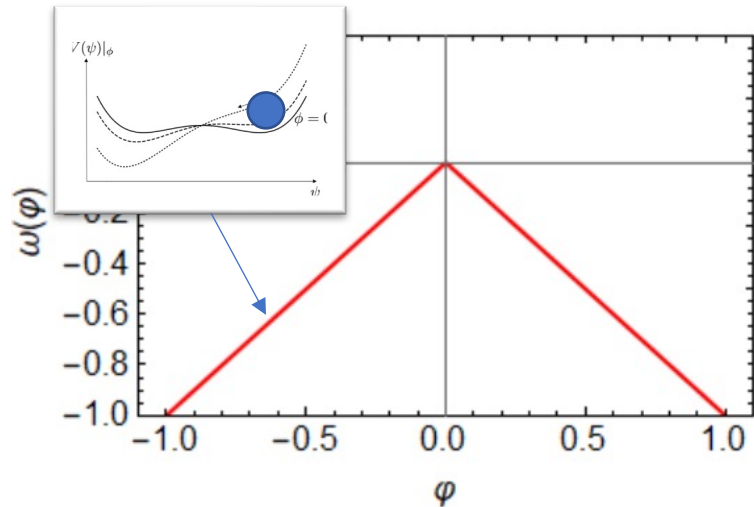
Junction conditions at phase transitions



- ϕ triggers 1st order **quantum phase transition** at ϕ_c
- **Discontinuity** in V' leads to discontinuous P'
- Requiring **continuity of FPV** across the critical point gives a **junction condition** to satisfy

$$\lim_{\epsilon \rightarrow 0} \int_{\phi_c - \epsilon}^{\phi_c + \epsilon} d\phi \frac{\partial}{\partial \phi} \left[\frac{V'P}{3H} + \frac{\hbar}{8\pi^2} \frac{\partial}{\partial \phi} (H^3 P) \right] = 0 \quad \Rightarrow \quad \frac{\Delta P'}{P(\phi_c)} = -\frac{2\Delta\omega'}{\alpha}$$

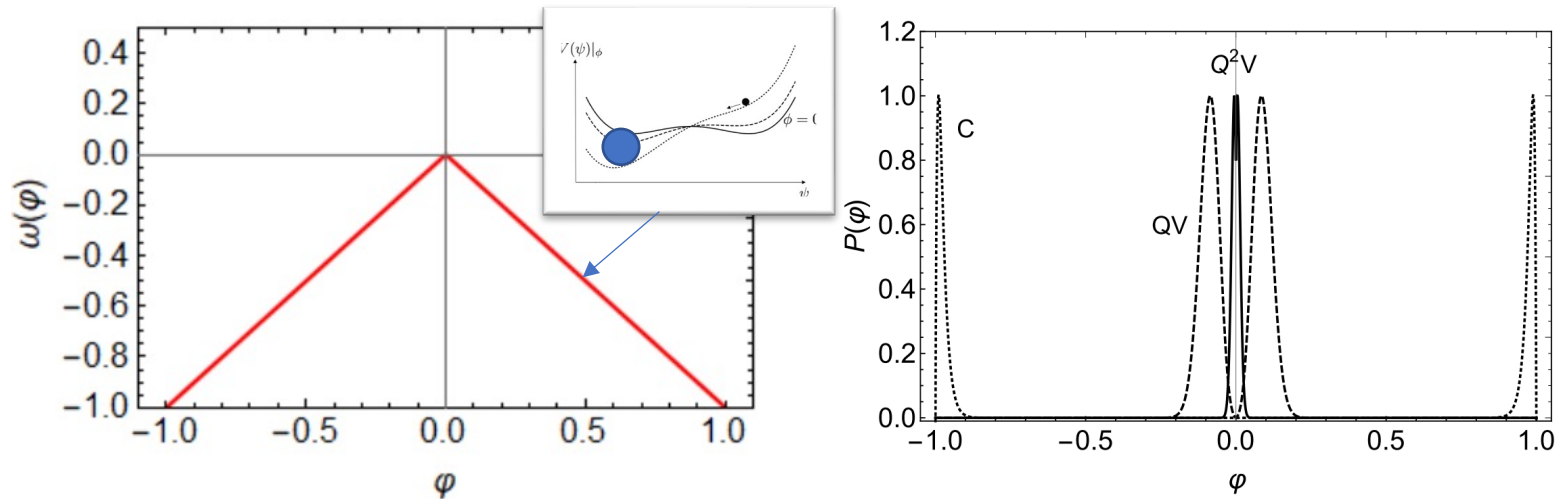
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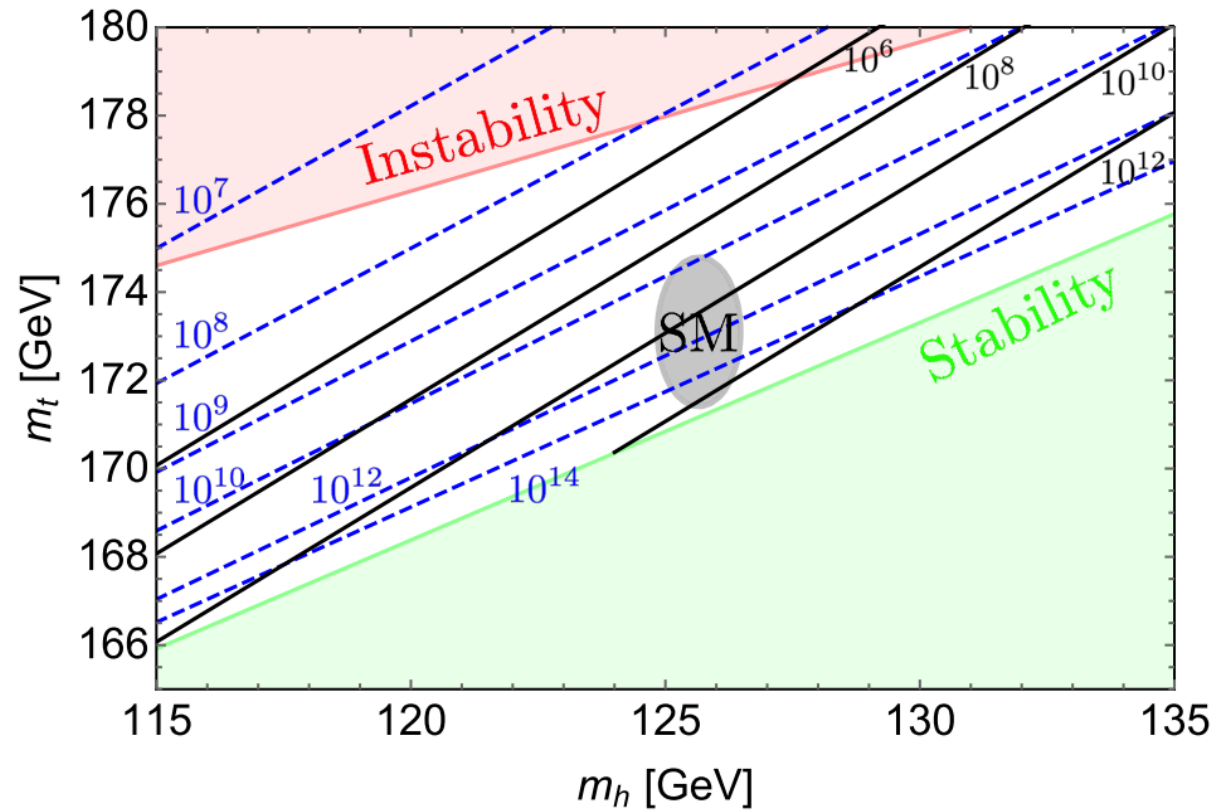
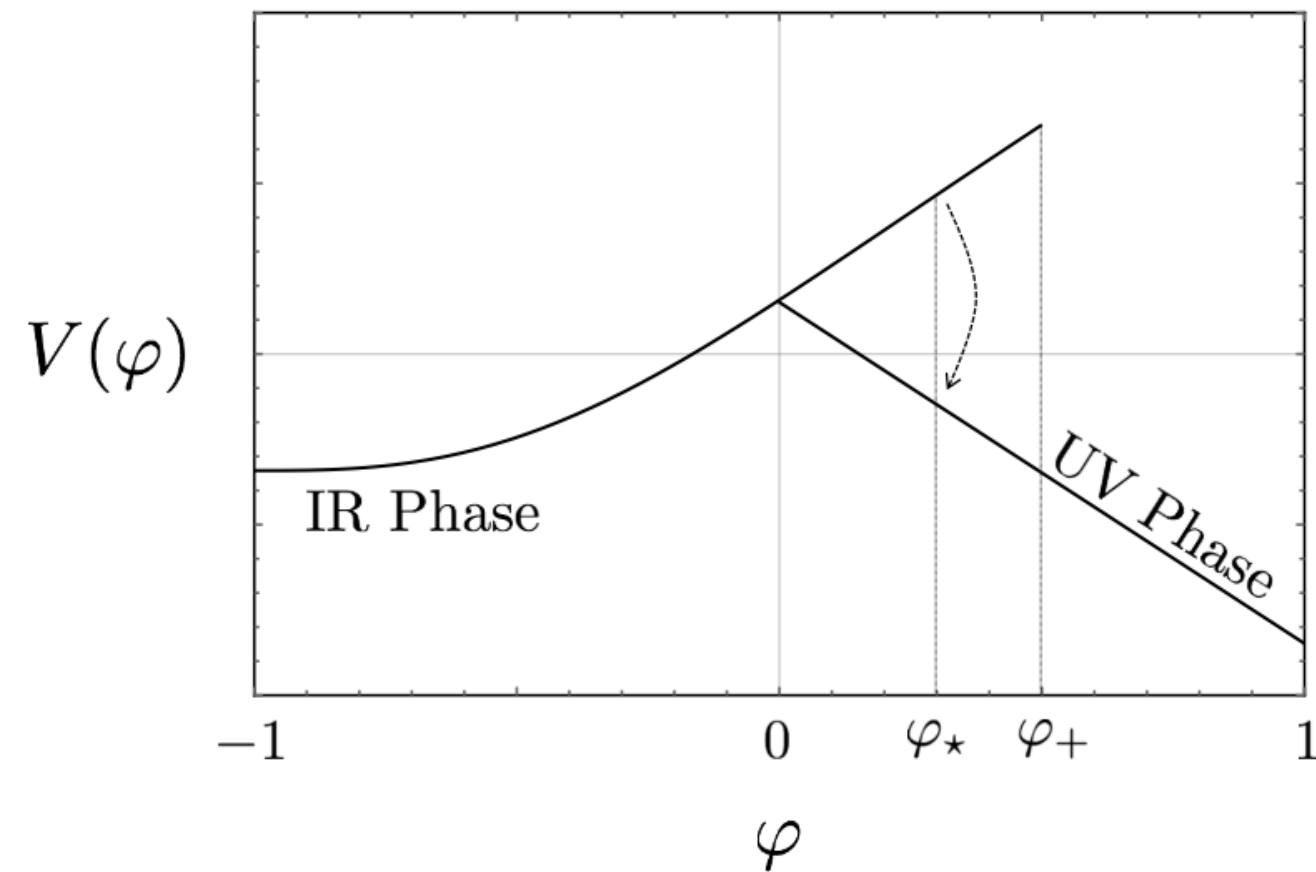
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Higgs metastability

$$V(\varphi, h) = \frac{M^4}{g_*^2} \omega(\varphi) + \frac{\lambda(\varphi, h)}{4} (h^2 - v^2)^2$$

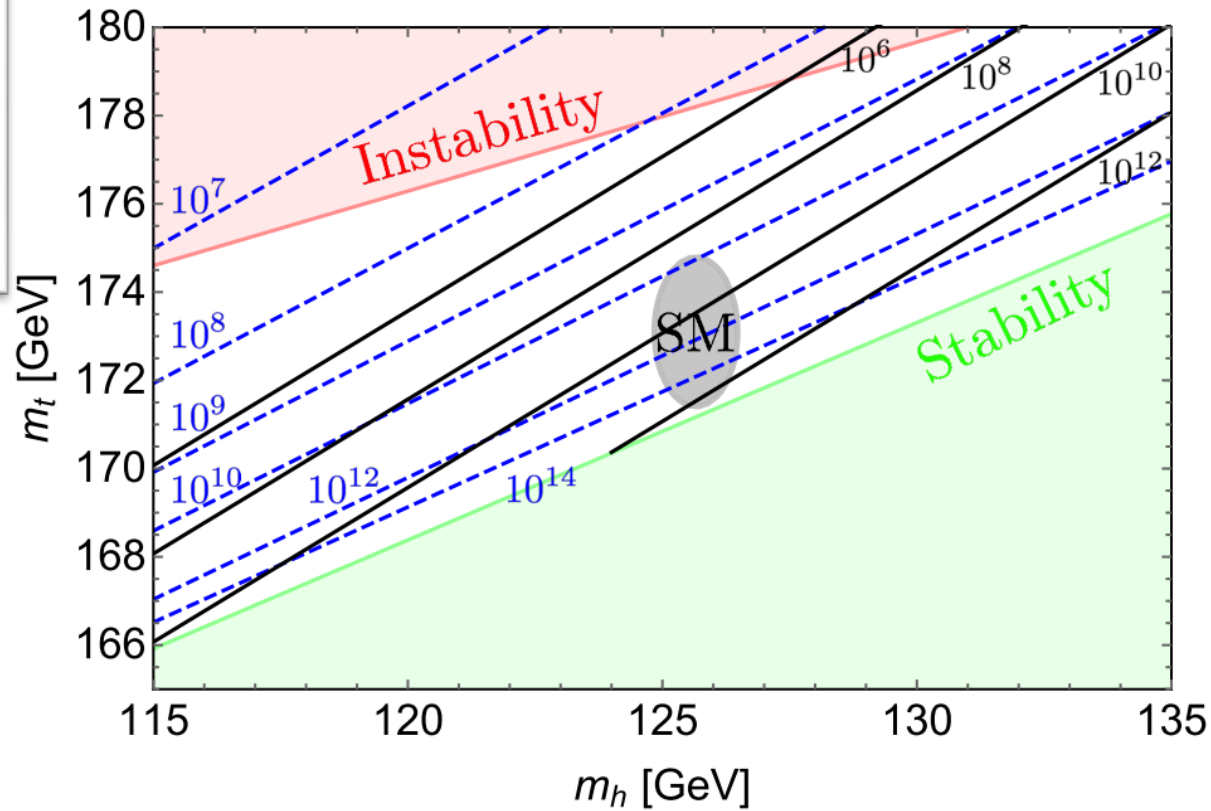
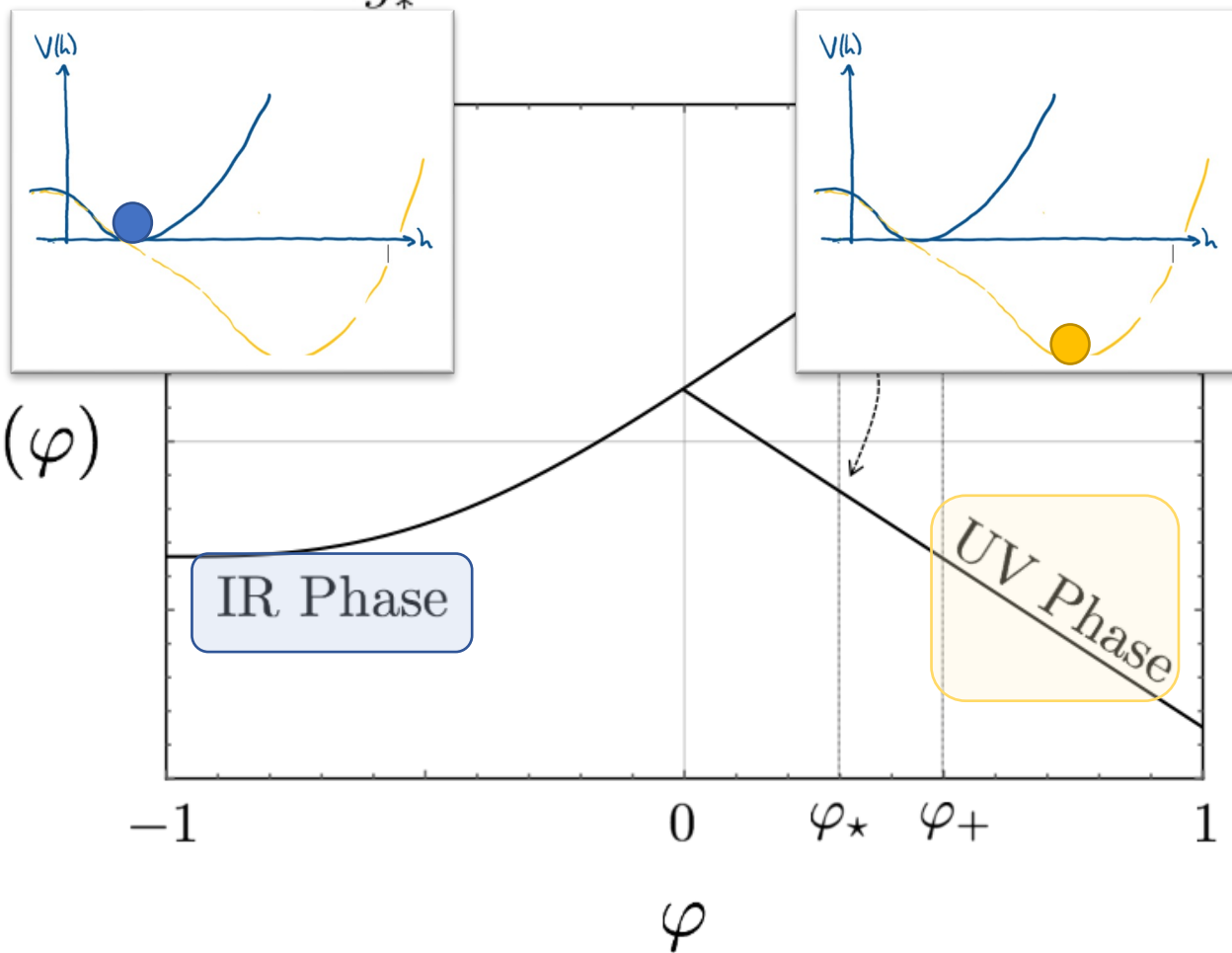
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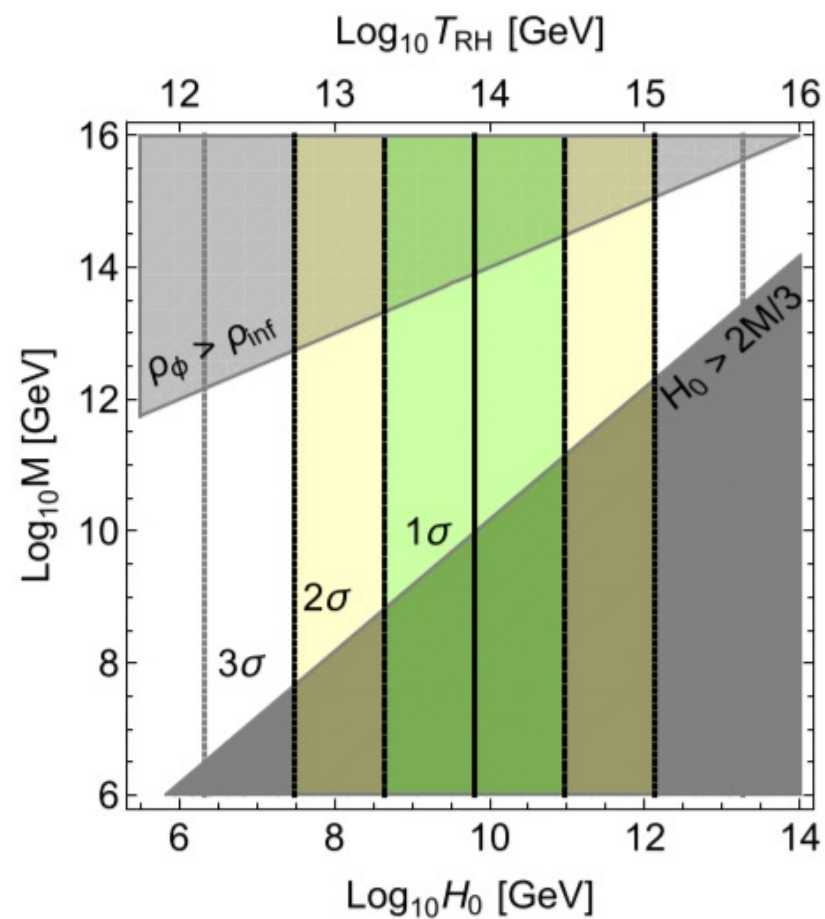
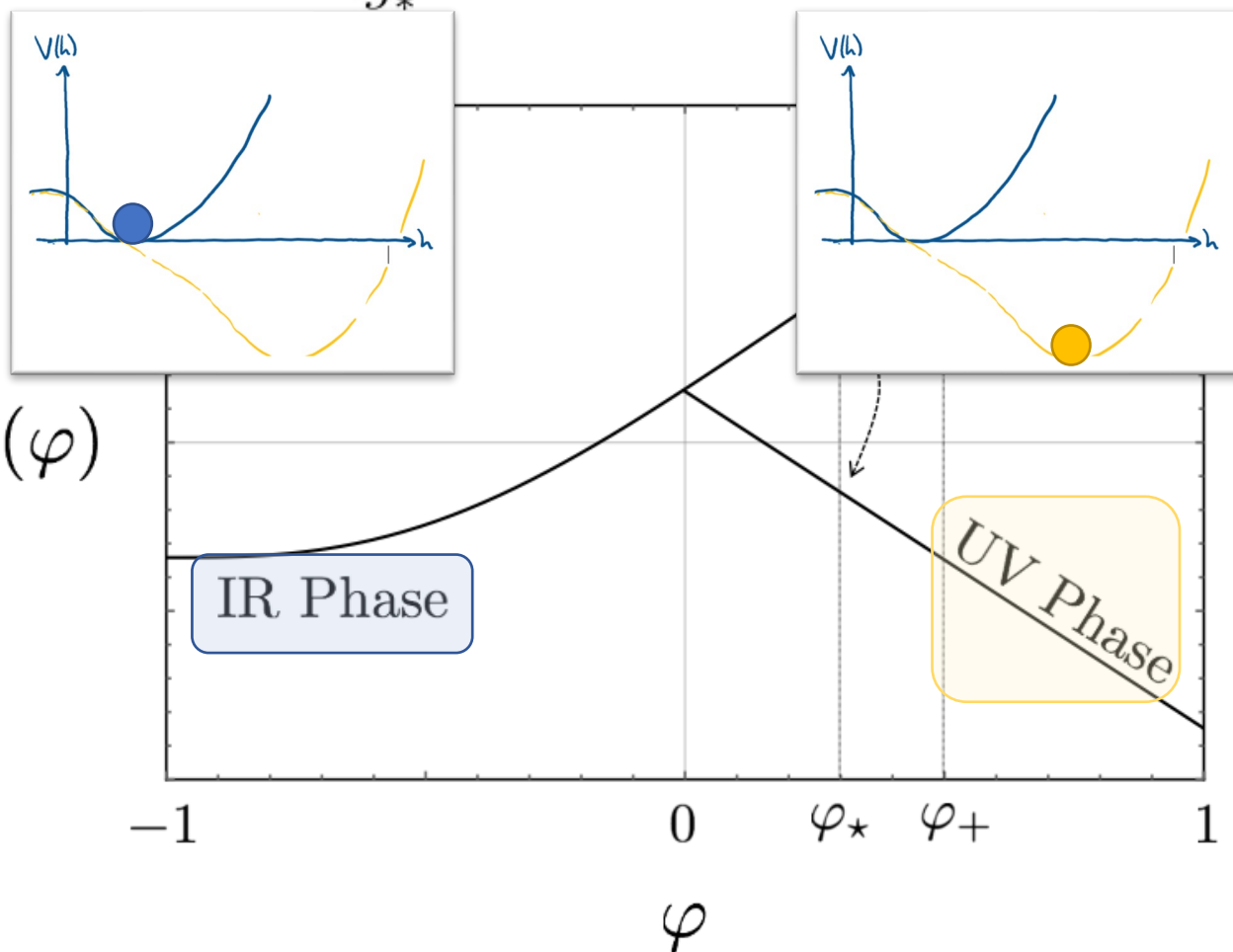
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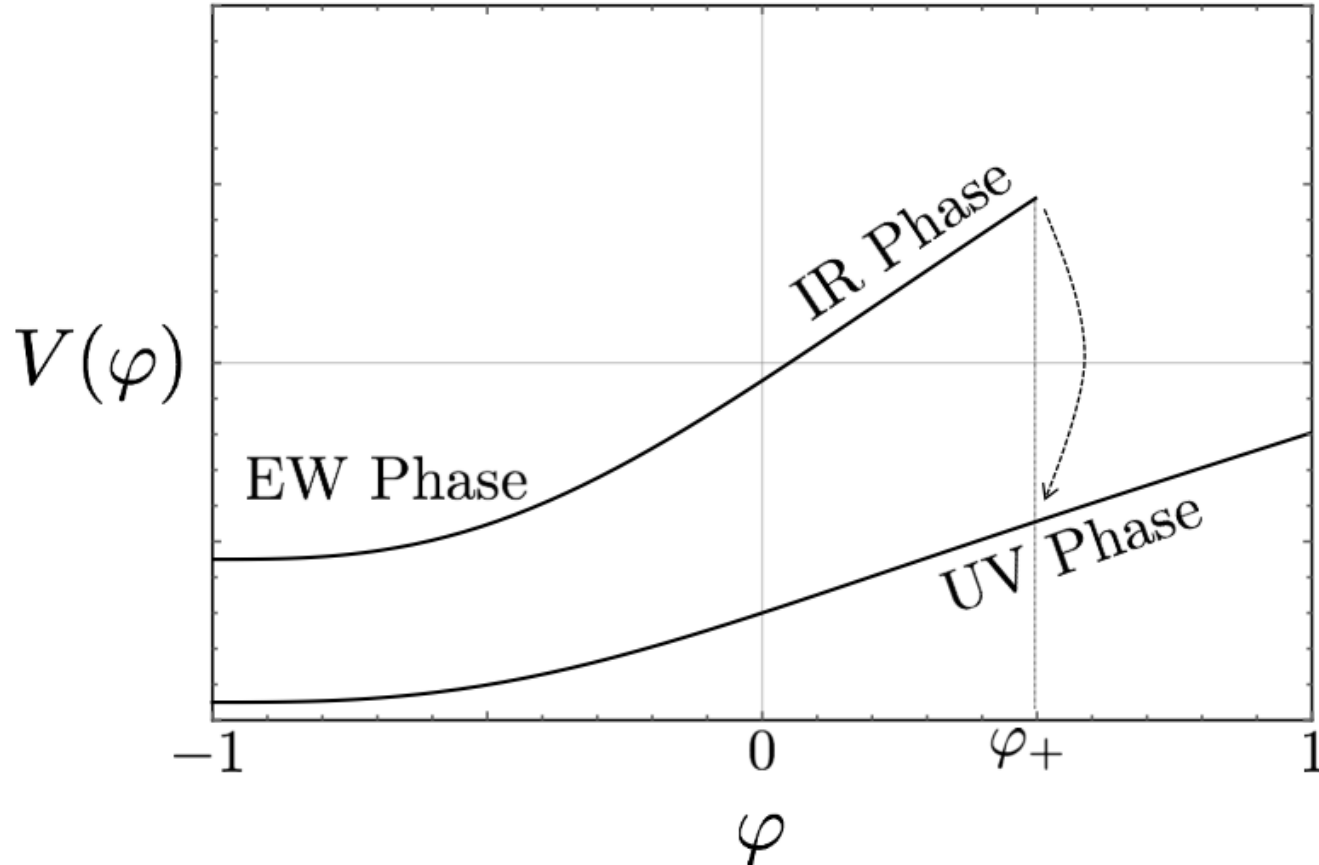


Higgs mass naturalness

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- Use **broken IR to broken UV** phase transition

$$\varphi_+ = \frac{-\beta_I e^{-\frac{3}{2}} \Lambda_I^2}{M^2} \quad \longrightarrow \quad v = e^{-\frac{3}{4}} \Lambda_I$$

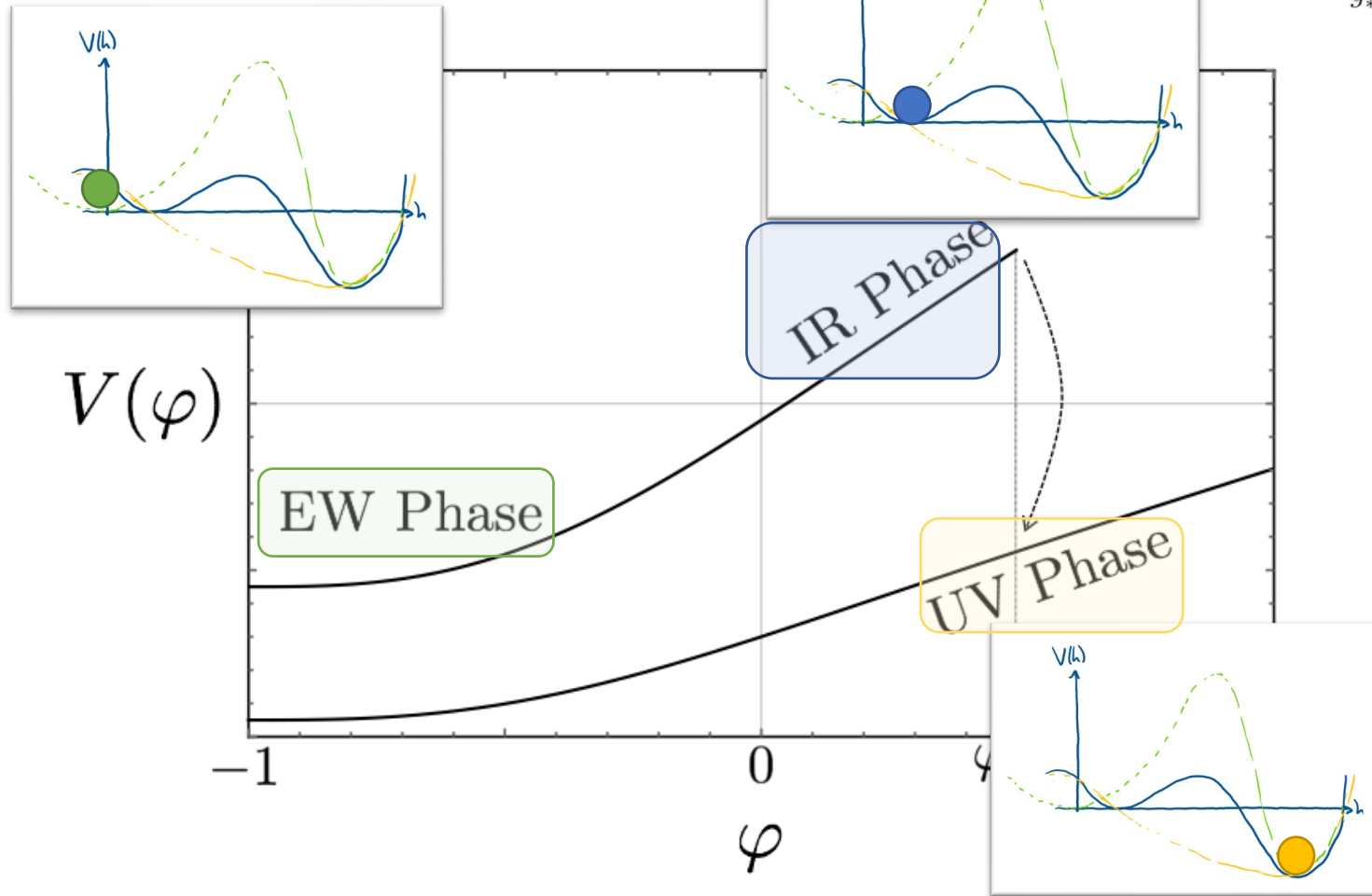
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