Renormalons in QCD

(Towards higher precision)

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☆Plan of Talk

1. Introduction to renormalons

2. Renormalon cancellation in Heavy Quarkonium

3. Renormalon cancellation in general observables

   Dual space approach in OPE

4. Summary

Pert. QCD

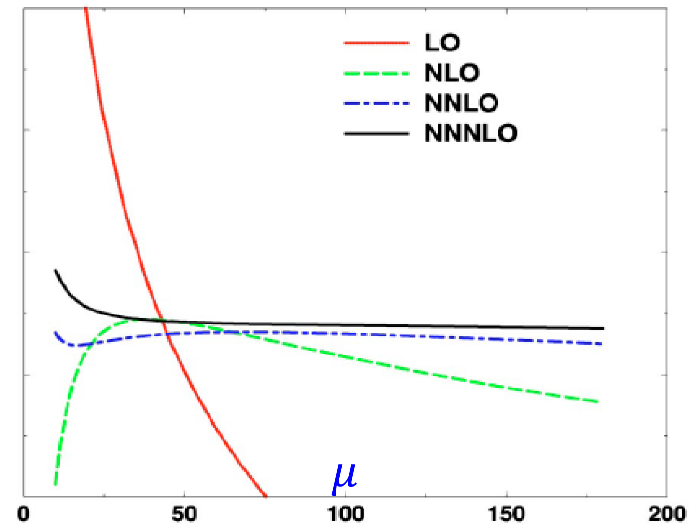
renormalization scale

$$\mathcal{L}_{QCD}(\alpha_s, m_i; \mu)$$

Theory of quarks and gluons

Same input parameters as full QCD.
Systematic: has its own way of estimating errors.
(Dependence on μ is used to estimate errors.)

Differs from a model

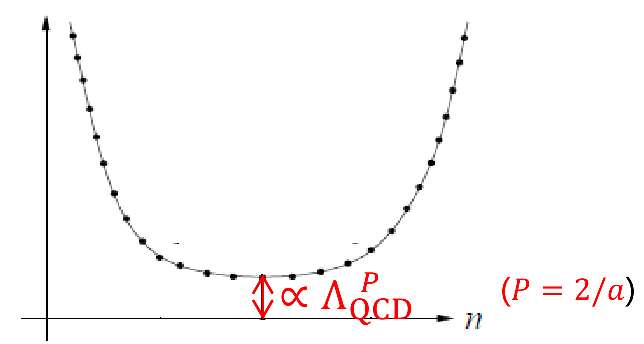


Renormalon uncertainty

’t Hooft (See review by Beneke)

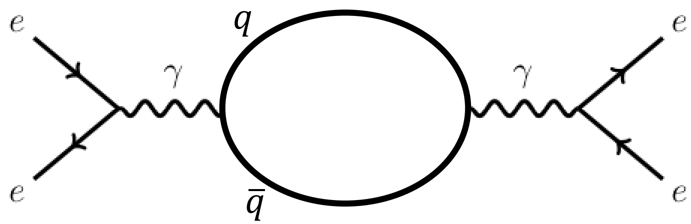
$$A = \sum_n c_n \alpha_s^n \quad ; \quad c_n \sim n!\, a^n$$

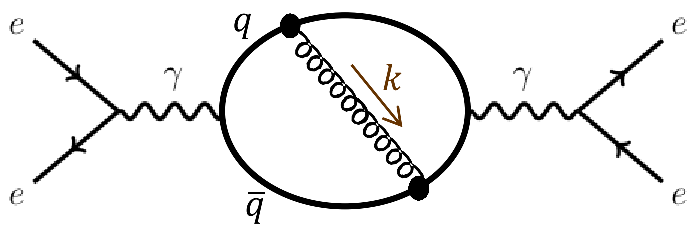
$c_n\, \alpha_s^n \sim n!\, a^n\, \alpha_s^n$



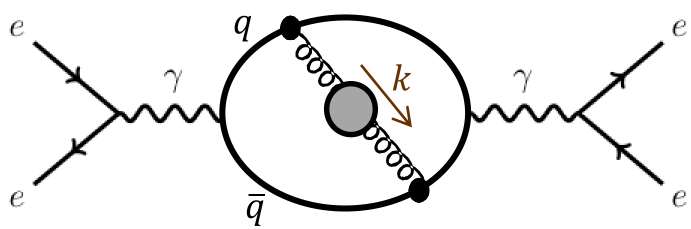
$\Lambda_{\mathrm{QCD}} \sim 300$ MeV

$\sigma(e^+e^- \rightarrow hadrons; E)$

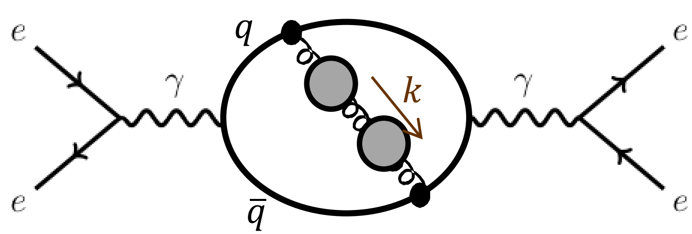




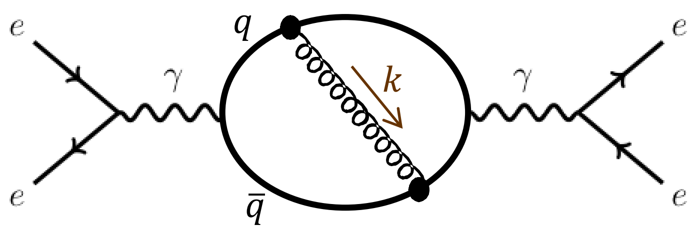
$\alpha_s(\mu)$



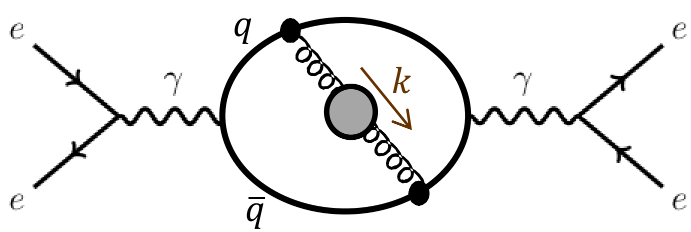
$\alpha_s(\mu) \times b_0\ \alpha_s(\mu) \log(\frac{\mu}{k})$



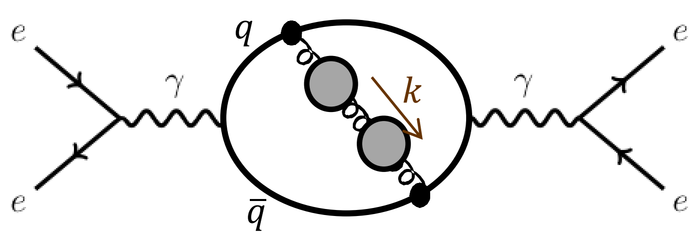
$\alpha_s(\mu) \times b_0^2 \alpha_s^2\ (\mu) \log^2(\frac{\mu}{k})$



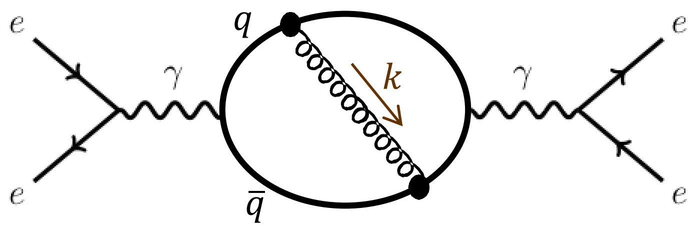
$\alpha_s(\mu)$



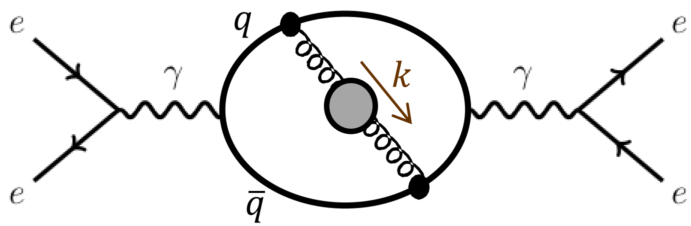
$\alpha_s(\mu) \times b_0\ \alpha_s(\mu) \log(\frac{\mu}{k})$



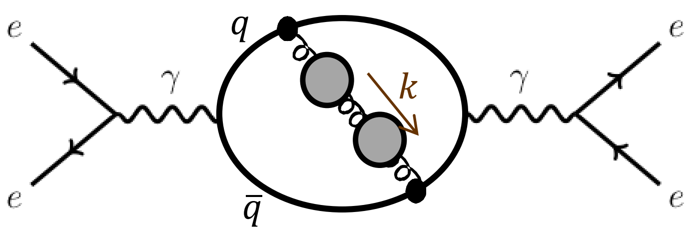
$\alpha_s(\mu) \times b_0^2 \alpha_s^2\ (\mu) \log^2(\frac{\mu}{k})$



$\alpha_s(\mu)$

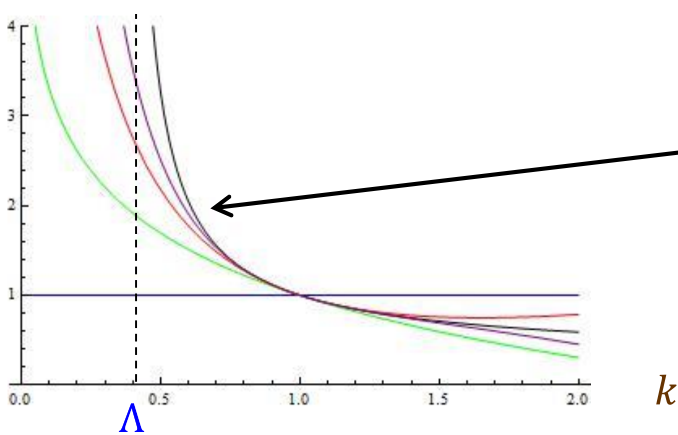


$\alpha_s(\mu) \times b_0\, \alpha_s(\mu) \log(\frac{\mu}{k})$

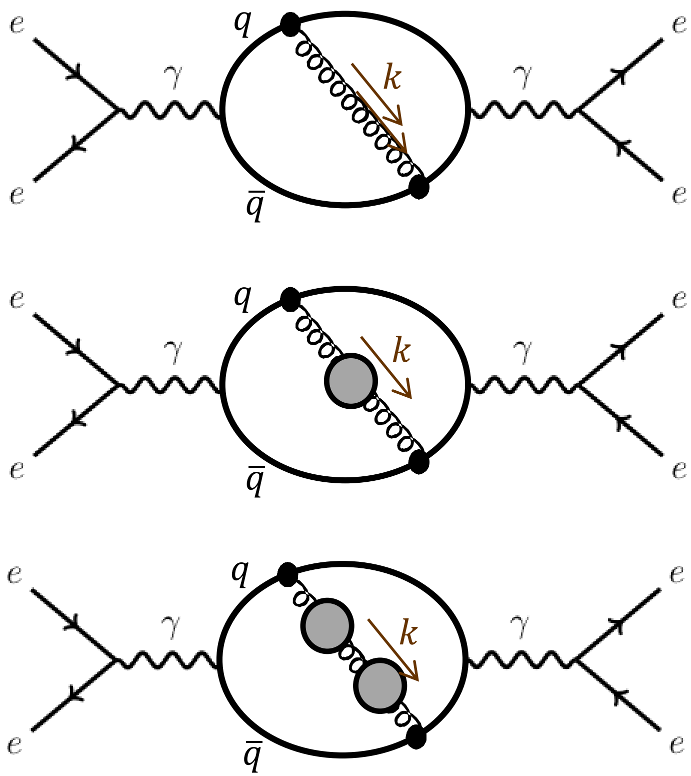


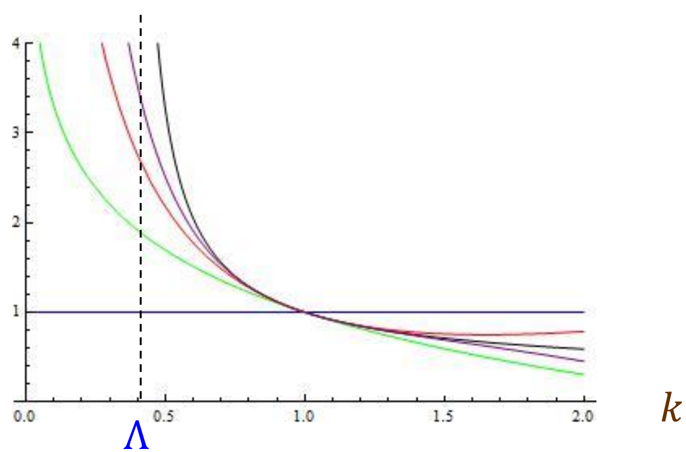
$\alpha_s(\mu) \times b_0^2 \alpha_s^2\,(\mu) \log^2(\frac{\mu}{k})$

⇓ Infinite sum



$$\alpha_s(k) = \frac{\alpha_s(\mu)}{1 - b_0 \alpha_s(\mu) \log(\frac{\mu}{k})} = \frac{1}{b_0 \log(\frac{k}{\Lambda})}$$

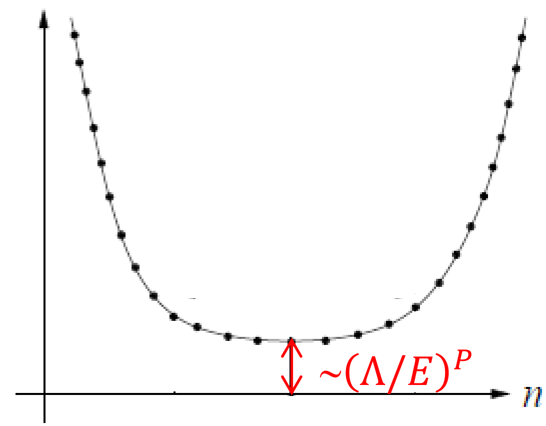




Consequence

Renormalon uncertainty

$c_n(E/\mu)\,\alpha_s^{\,n}(\mu)$

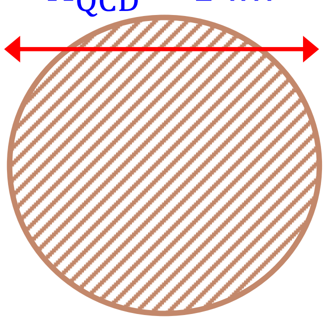


Asymptotic series
Limited accuracy

Renormalon cancellation in heavy quarkonium

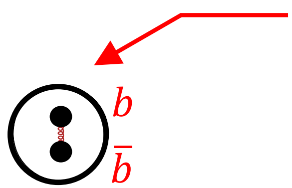
Motivation

$\Lambda_{\rm QCD}^{-1} \sim 1$ fm



ordinary hadrons

$p, n, \pi, K, B, D, \ldots$



Gluons

$\lambda_g \ll \Lambda_{\rm QCD}^{-1} \sim 1$ fm

in binding dynamics

Bottomonium

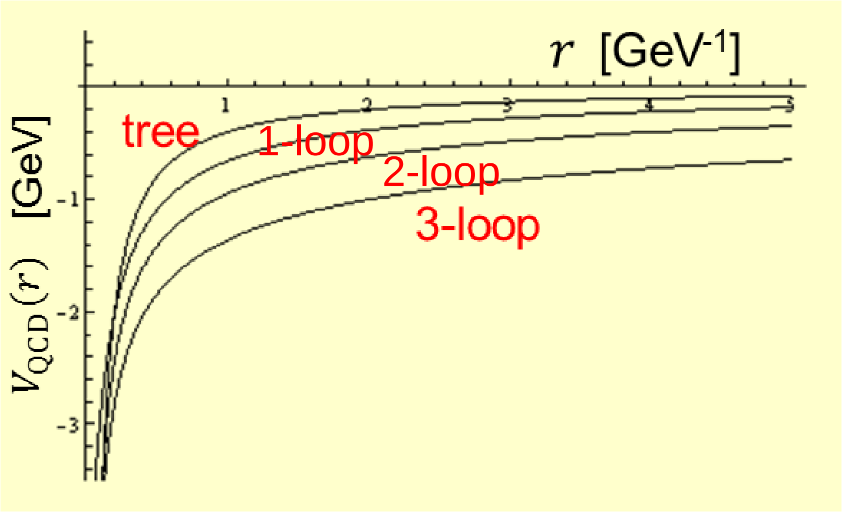
⇧

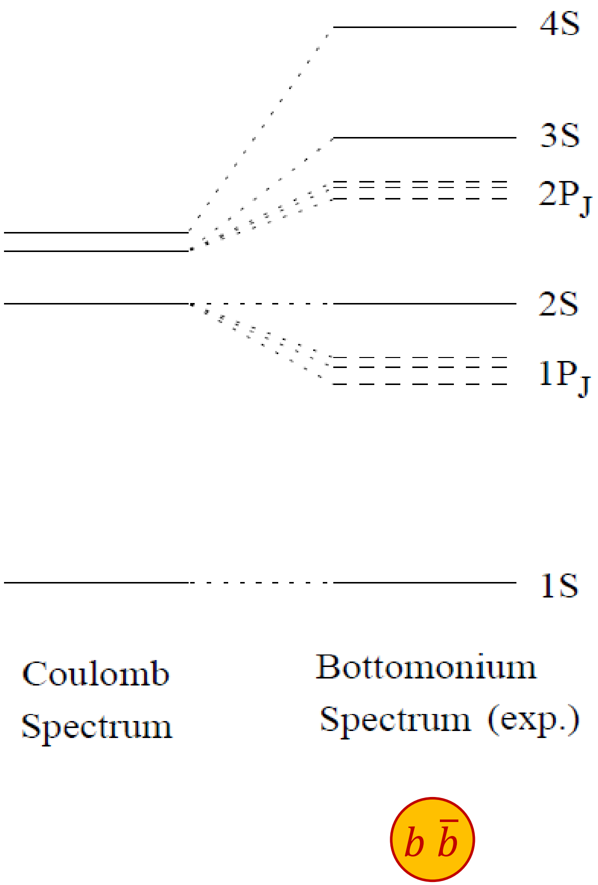
Unique existing hadrons, for which pert. QCD alone can calculate various properties.

Convergence of

perturbative QCD potential $V_{\mathrm{QCD}}(r)$

used to be very bad !

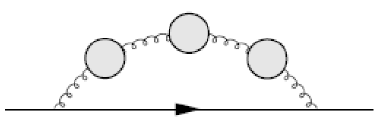




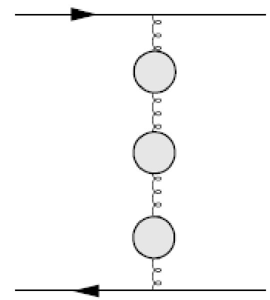
Accuracy of perturbative prediction for $2m_{\mathrm{pole}} + V_{\mathrm{QCD}}(r)$ improved dramatically around year 1998,

Pineda
Hoang,Smith,Stelzer,Willenbrock
Beneke

if we re-express the quark pole mass (m_{pole}) by the $\overline{\mathrm{MS}}$ mass ($m_{\overline{\mathrm{MS}}}$).

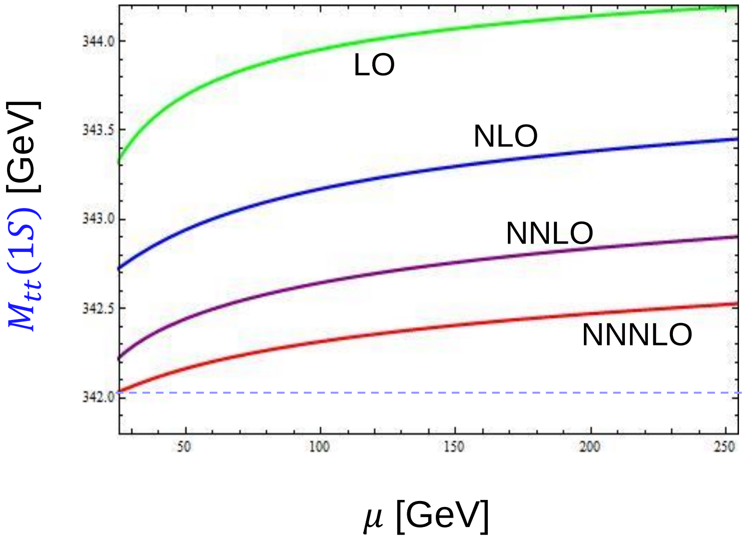


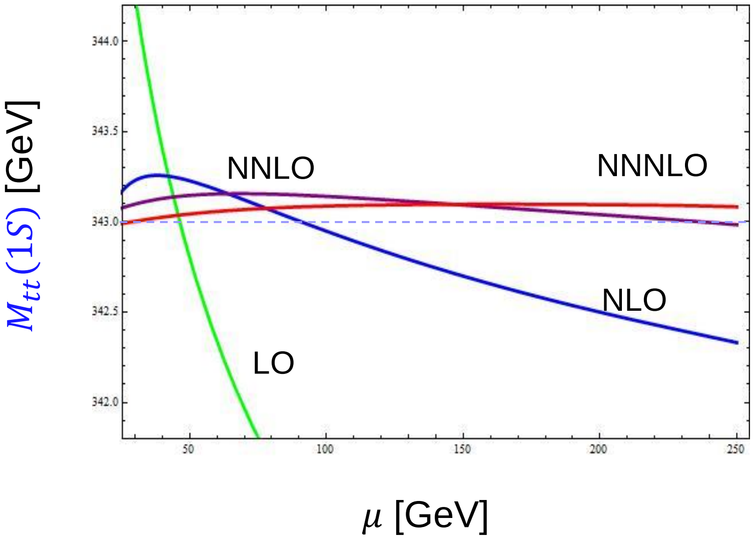
$\times 2$



IR renormalons cancel

μ dependence and convergence of $M_{Q\bar{Q}}(1S)$





- $\Upsilon(1S):\quad M_{\Upsilon(1S)} = 9.94 - 0.10 - 0.15 - 0.20 - 0.26$ GeV (Pole mass)

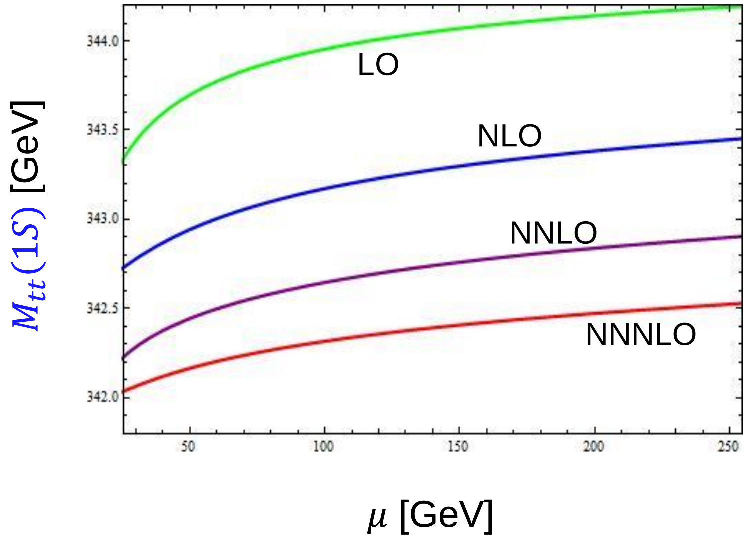
  $= 8.43 + 0.72 + 0.25 + 0.07 - 0.02$ GeV ($\overline{\text{MS}}$ mass)

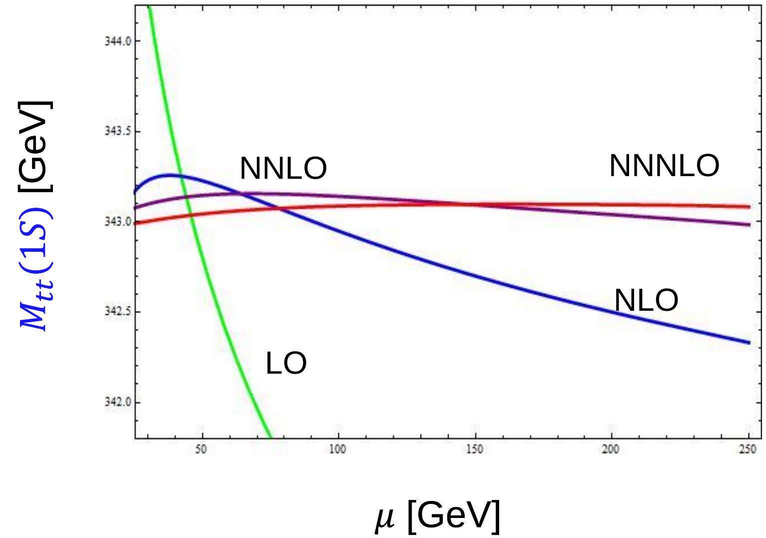
- $\Upsilon(2S):\quad M_{\Upsilon(2S)} = 9.94 - 0.06 - 0.11 - 0.22 - 0.41$ GeV (Pole mass)

  $= 8.43 + 1.17 + 0.26 + 0.10 - 0.04$ GeV ($\overline{\text{MS}}$ mass)

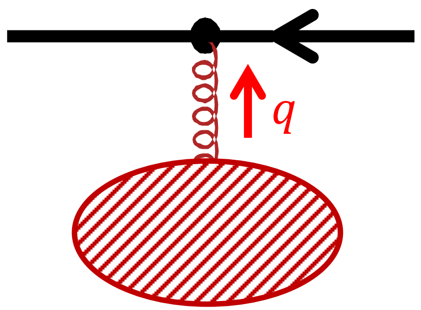
Bottomonium spectrum: Brambilla, YS, Vairo; Kiyo, YS

μ dependence and convergence of $M_{Q\bar{Q}}(1S)$





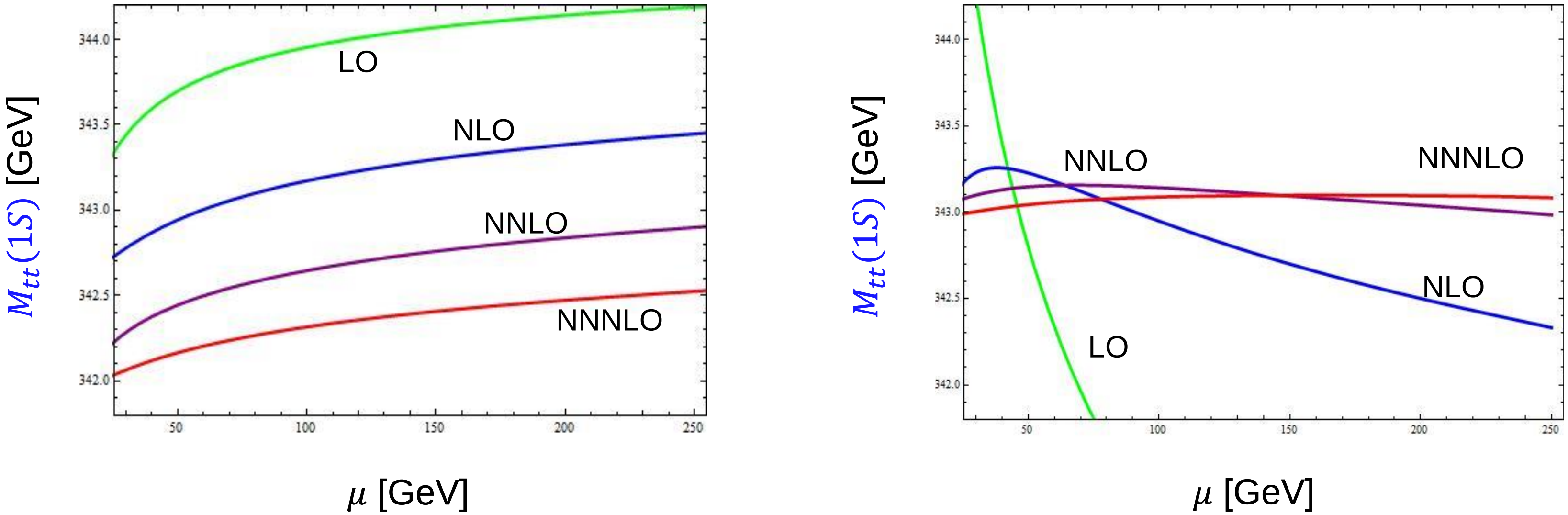
General feature of QCD beyond large β_0 or leading-log approx.



$A_\mu(q)\, j^\mu(-q)$ $\qquad$ $j^\mu(x) = \delta^{\mu 0}\delta^3(\vec{x} - \vec{r}/2)$

Couples to total charge as $q \to 0$.

μ dependence and convergence of $M_{Q\bar{Q}}(1S)$



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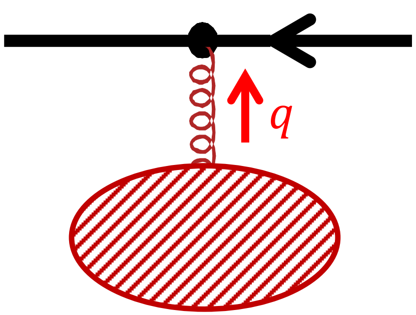

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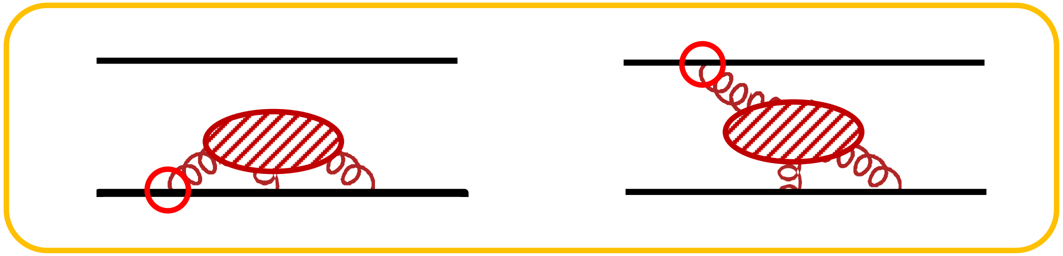
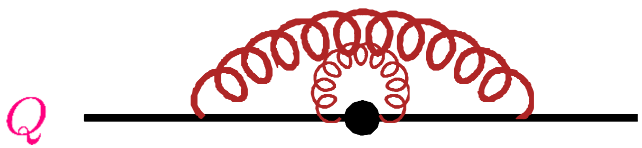


Pole mass m_{pole}

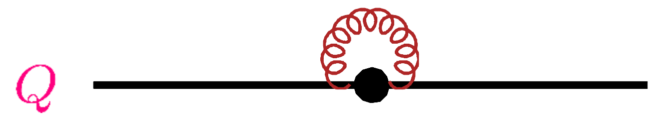
$0 < \lambda_g < \infty$



m_{bare}

$\overline{\text{MS}}$ mass $\overline{m} \equiv m_{\overline{\text{MS}}}(m_{\overline{\text{MS}}})$

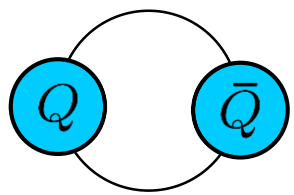
$0 < \lambda_g < 1/\overline{m}$



m_{bare}

Computation of spectrum of Heavy Quarkonium

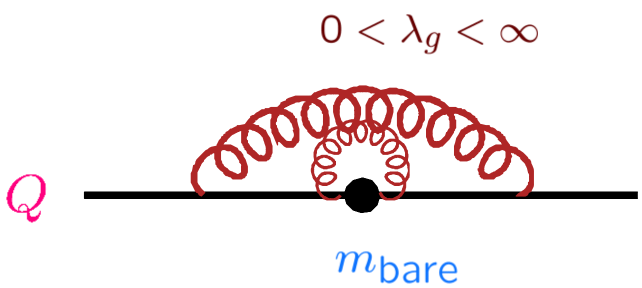
had followed calculation of QED bound states



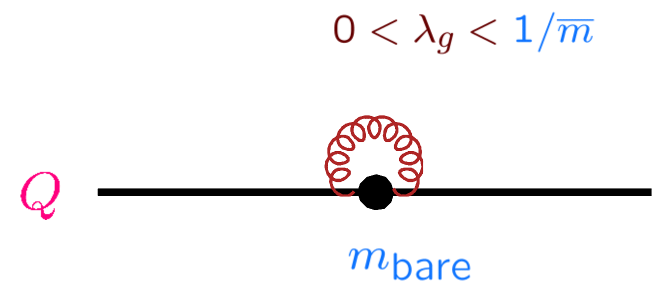
Free Q and $\bar{Q}$:

$$E_{\text{tot}} = 2m_{\text{pole}} - E_{\text{bin}}$$

Pole mass m_{pole}

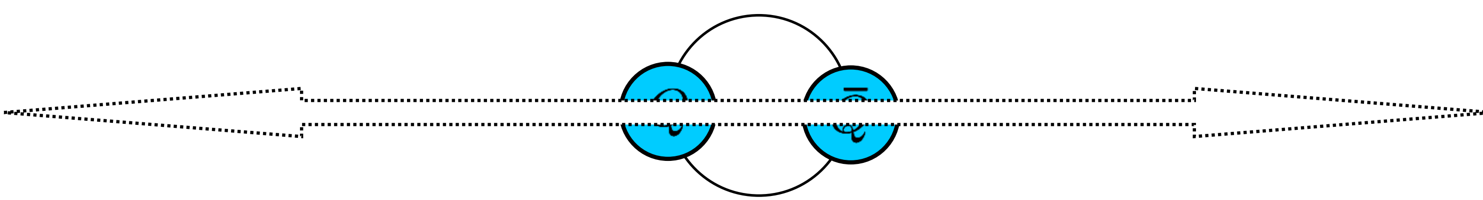


$\overline{\text{MS}}$ mass $\overline{m} \equiv m_{\overline{\text{MS}}}(m_{\overline{\text{MS}}})$



Computation of spectrum of Heavy Quarkonium

had followed calculation of QED bound states



Free Q and $\bar{Q}$: } not well-defined

$E_{\text{tot}} = 2m_{\text{pole}} - E_{\text{bin}}$

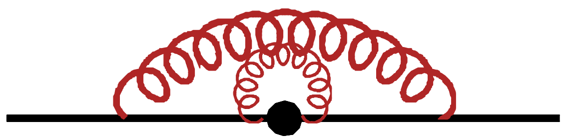
Poorly convergent perturbative series

Pole mass m_{pole}

$\overline{\text{MS}}$ mass $\overline{m} \equiv m_{\overline{\text{MS}}}(m_{\overline{\text{MS}}})$

$0 < \lambda_g < \infty$

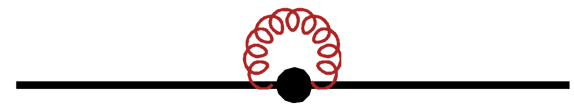
Q



m_{bare}

$0 < \lambda_g < 1/\overline{m}$

Q



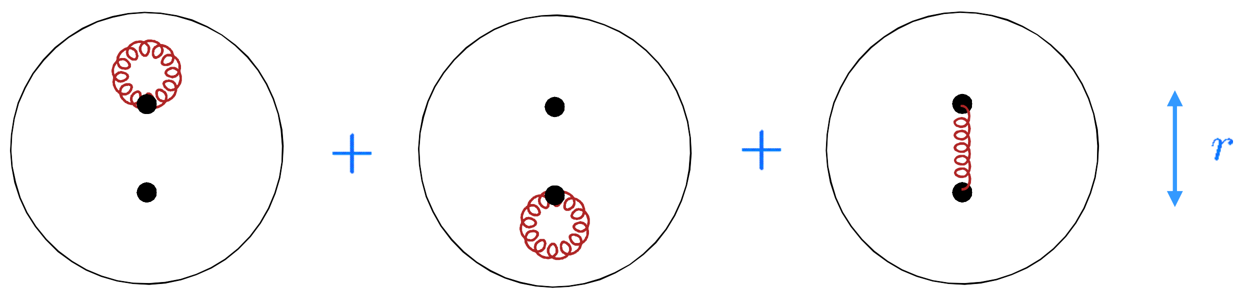
m_{bare}

Computation of spectrum of Heavy Quarkonium

Using $\overline{\text{MS}}$ mass $\quad 2m_{\text{pole}} = 2\overline{m}\,(1 + c_1\,\alpha_S + c_2\,\alpha_S^2 + c_3\,\alpha_S^3 + \cdots)$

Chetyrkin, Steinhauser; Melnikov, Ritbergen; Marquard, Smirnov, Smirnov, Steinhauser, Wellmann

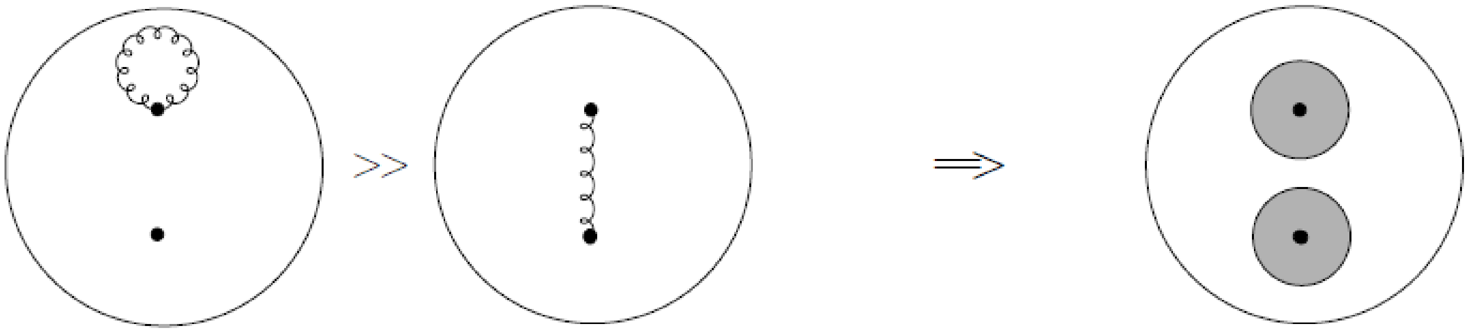
$E_{\text{tot}} =$

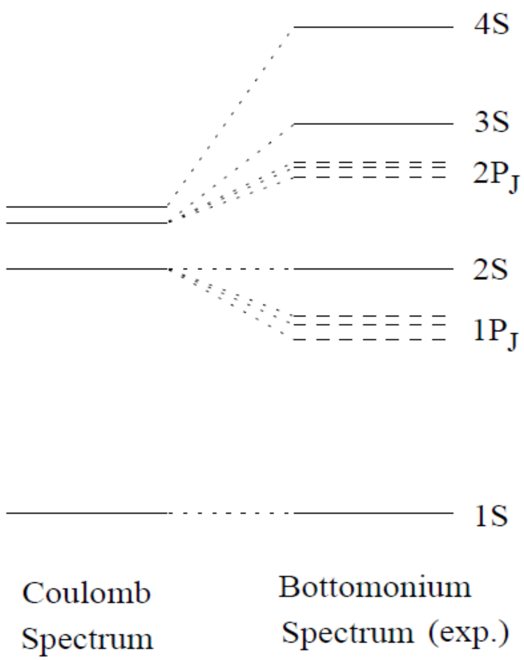


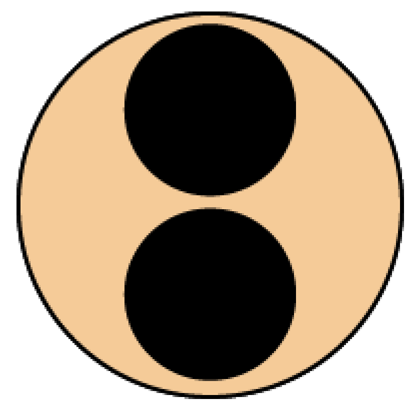
IR gluons $\lambda_g \gg r$ decouple ⇨ much more convergent series

Rapid growth of masses of excited states originates from rapid growth of self-energies of Q & $\bar{Q}$ due to IR gluons.

Brambilla, Y.S., Vairo







$$E_X \approx 2m_b^{\overline{MS}}(\mu) + \int_0^{\mu} dq\, f_X(q)\alpha_s(q)$$

Brambilla,YS,Vairo
Recksiegel,YS

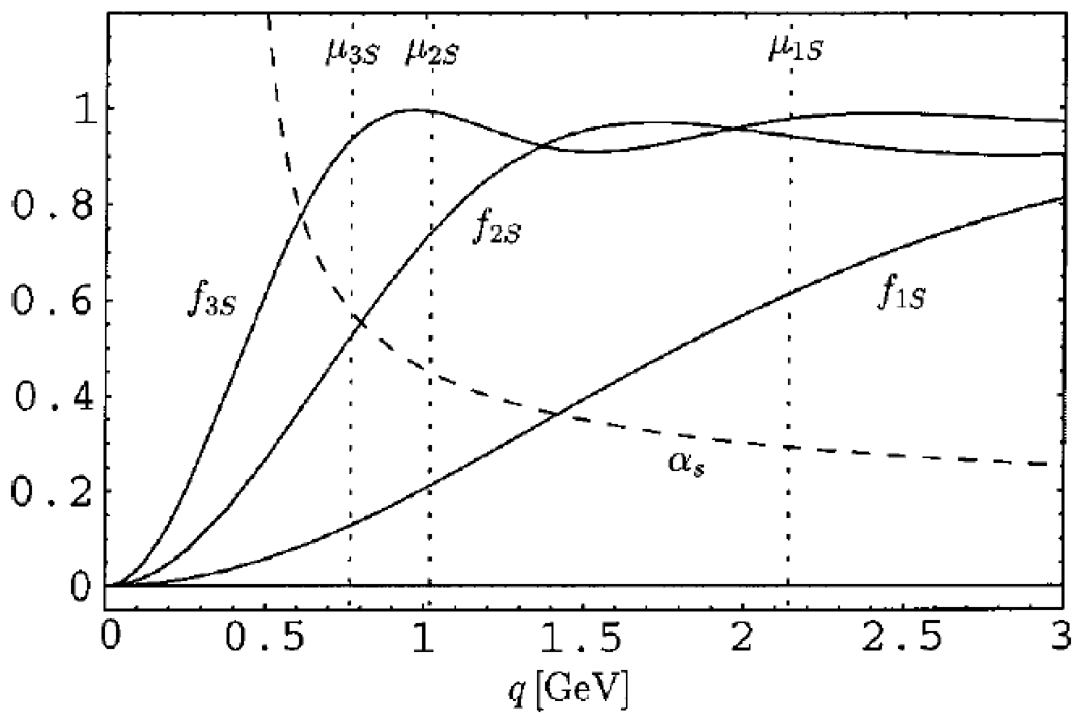
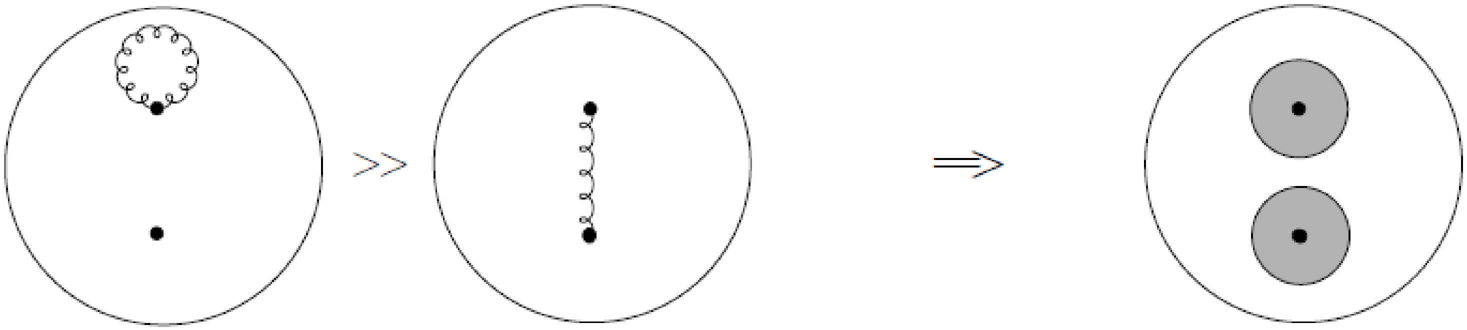
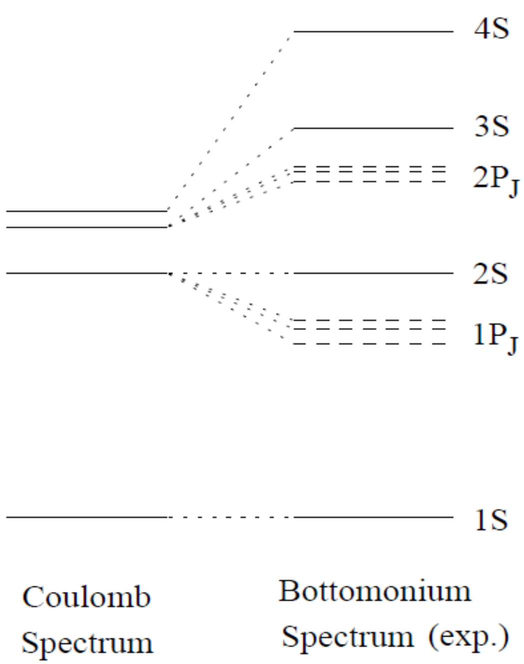


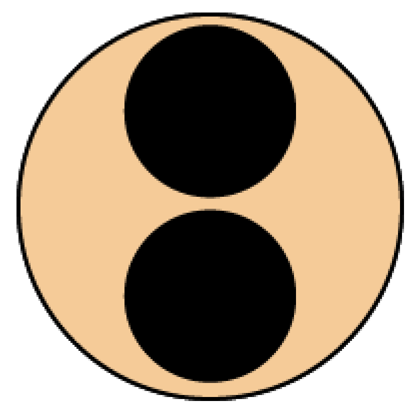
FIG. 5. Support functions for the S states. The solid curves show the support functions as defined in Eq. (19); for comparison of the relevant scales, $\alpha_s^{(4)}(\mu)$ is also plotted (dashed curve). Since the analysis that we advocate in this work does not attribute scales to the individual states, the scales indicated by the dotted lines are taken from [3], Table II.

Rapid growth of masses of excited states originates from rapid growth of self-energies of Q & $\overline{Q}$ due to IR gluons.

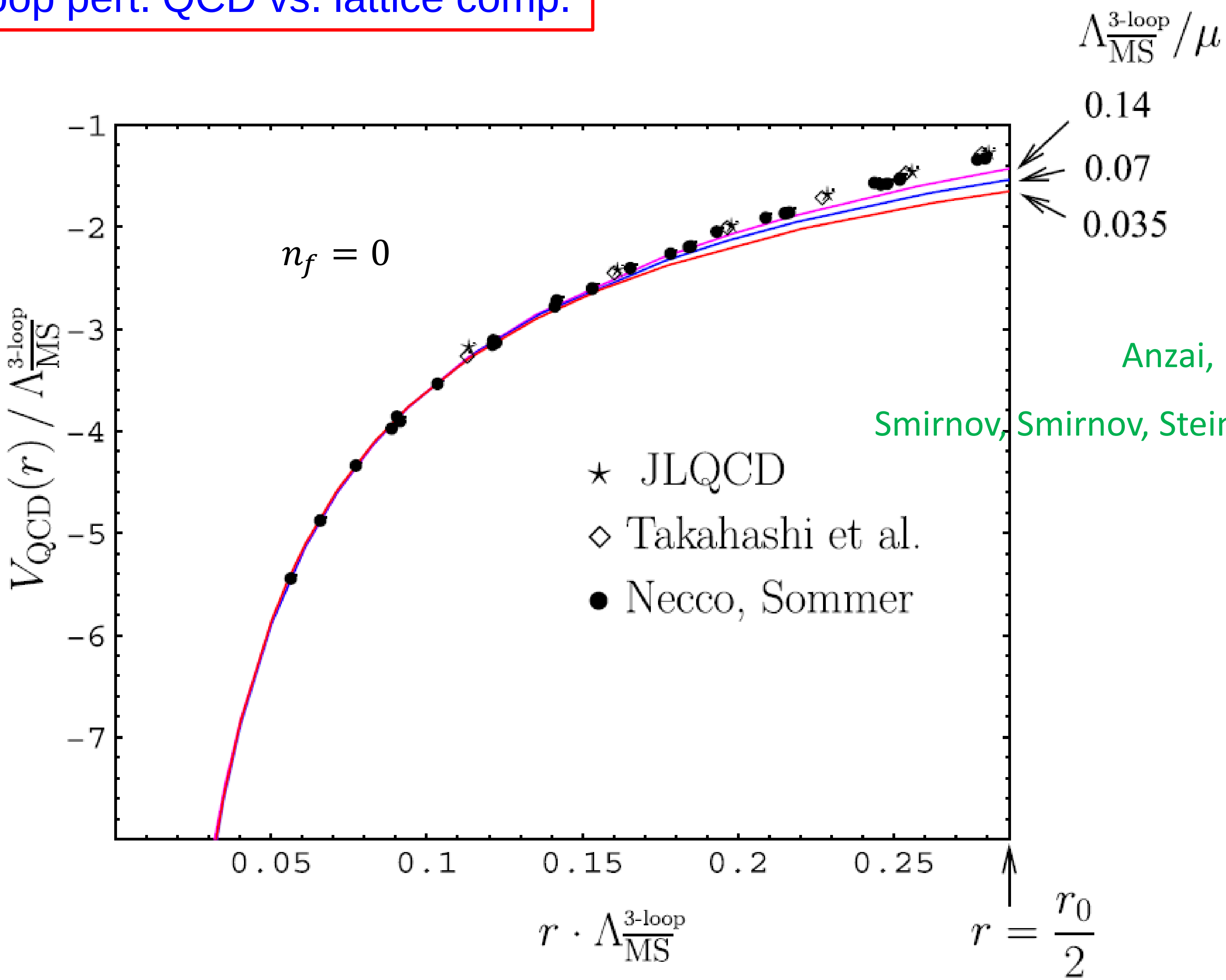
Brambilla, Y.S., Vairo







3-loop pert. QCD vs. lattice comp.



Anzai, Kiyo, YS

Smirnov, Smirnov, Steinhauser

What do we learn?

- Renormalon ambiguity can be absorbed into a (non-perturbative) parameter ($= m_{\mathrm{pole}}$).
- Remaining part (UV dominant) is more convergent, leading to more accurate theoretical prediction.

More General Framework

Solution to renormalon problem = OPE (Operator Product Expansion)

Müller
Parisi

For $Q \gg \Lambda$,

Λ^4/Q^4

$$A(Q)_{\mathrm{OPE}} = C_{\mathbf{1}}(Q)\,\langle \mathbf{1} \rangle + C_{G^2}(Q)\,\frac{\langle G^a_{\mu\nu} G^a_{\mu\nu} \rangle}{Q^4} + C_{G^3}(Q)\,\frac{\langle G^3 \rangle}{Q^6} + \cdots$$

Λ^6/Q^6

How to separate renormalons from Wilson coefficients $C_i(Q)$'s **?**

⇨ Dual Space Approach

For every observable, ∃ dual space where renormalons are suppressed or vanish.
(single scale)

Hayashi, YS, Takaura
Hayashi, Mishima, YS, Takaura

Q-space ⟷ τ-space

dual transf.

$A(Q)_{\rm OPE}$ ⟷ $\tilde{A}(Q)_{\rm OPE}$

$$A(Q) \sim \int_0^\infty d\tau\ e^{-\tau/Q}\ \tilde{A}(\tau)$$

$$A(Q)_{\rm OPE} \sim \underbrace{\int d\tau\ e^{-\tau/Q}\,\tilde{A}(\tau)_{\rm OPE}}_{\tau \gg \Lambda} + \underbrace{\int d\tau \left\{1 - \frac{\tau}{Q} + \frac{1}{2!}\left(\frac{\tau}{Q}\right)^2 - \cdots\right\}\tilde{A}(\tau)}_{\tau \lesssim \Lambda \ll Q}$$

$\sim C_i(Q)$ $\qquad\qquad$ $\sim \langle O_i\rangle / Q^{n_i}$

Renormalons from: $\tau \gtrsim \Lambda$ ⟷ $\tau \lesssim \Lambda$

separable $\qquad$ cancel $\qquad$ *separable*

Non-linear σ model

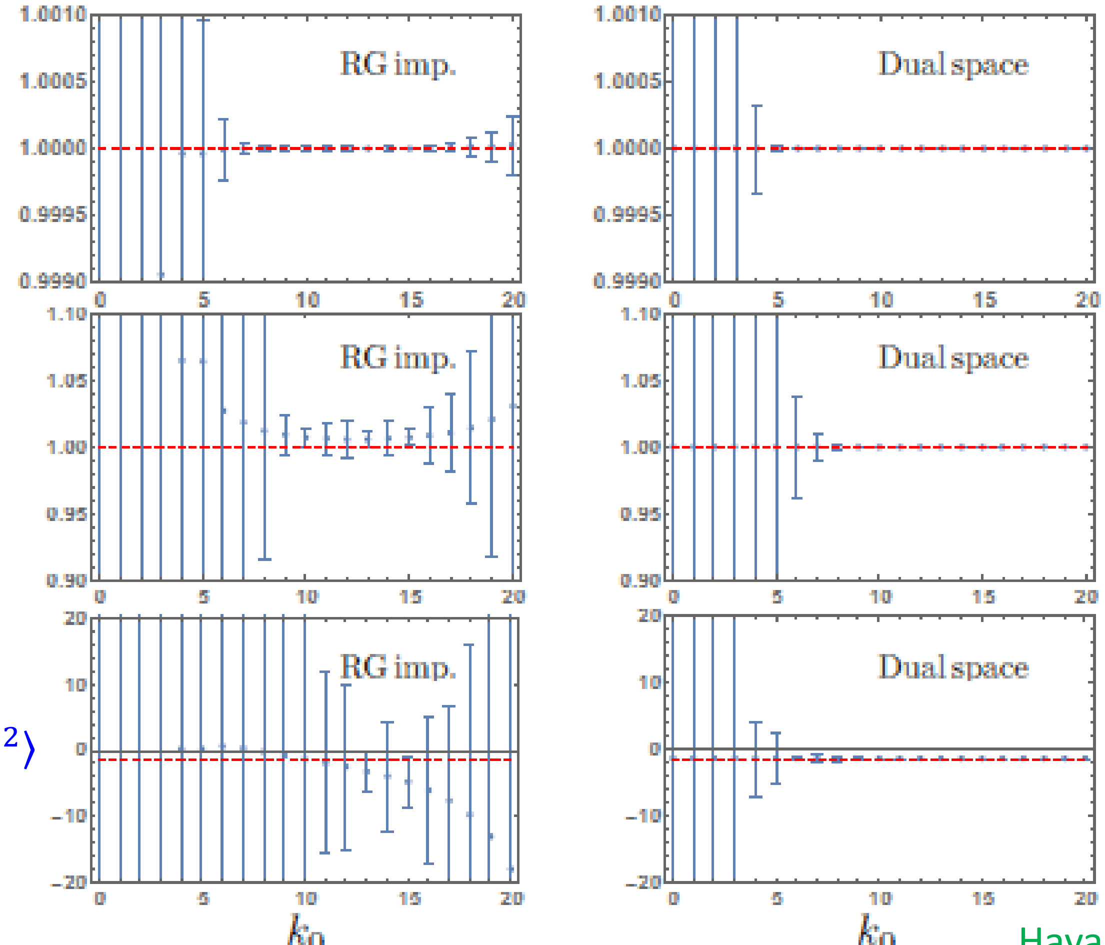
Simulation: Extracting non-pert. parameters by a fit to exact values (experimental data)

$$A(Q)_{\mathrm{OPE}} = C_{\mathbf{1}}(Q)\,\langle \mathbf{1} \rangle + C_{\alpha}(Q)\,\frac{\langle \alpha \rangle}{Q^2} + C_{\alpha^2}(Q)\,\frac{\langle \alpha \rangle^2}{Q^4} + C_{\delta\alpha^2}(Q)\,\frac{\langle (\delta\alpha)^2 \rangle}{Q^4} + \cdots$$

Scale unit Λ

$\langle \alpha \rangle$

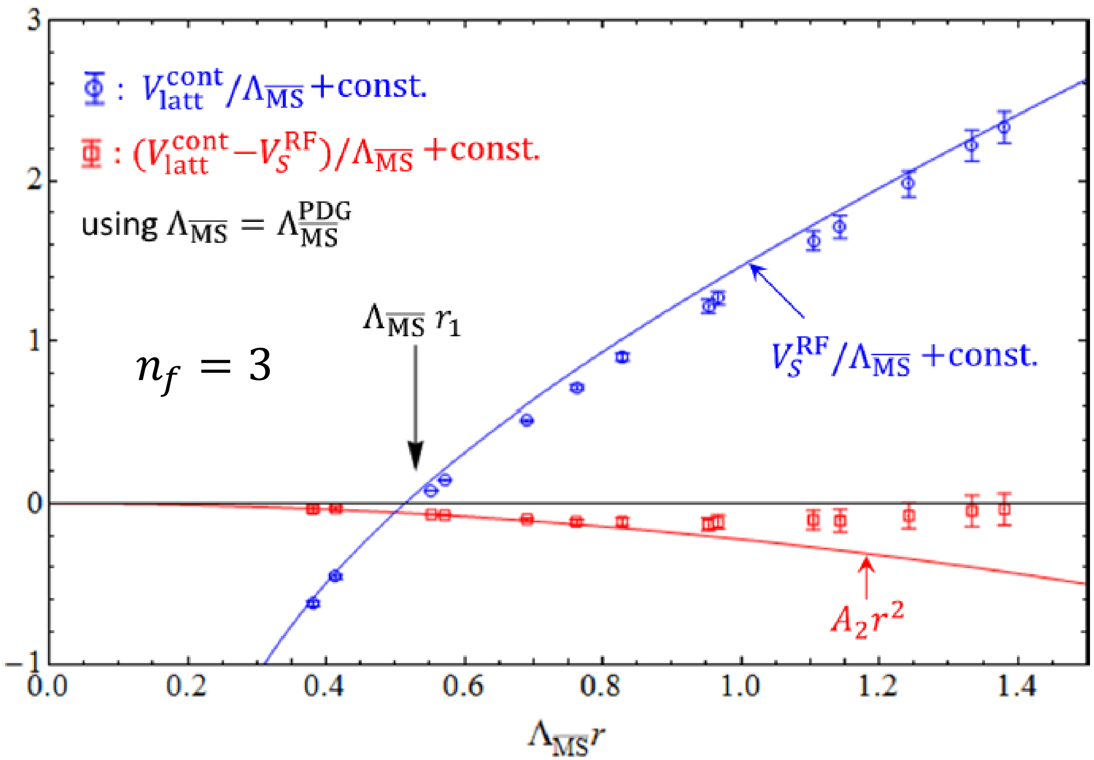
$\langle (\delta\alpha)^2 \rangle$



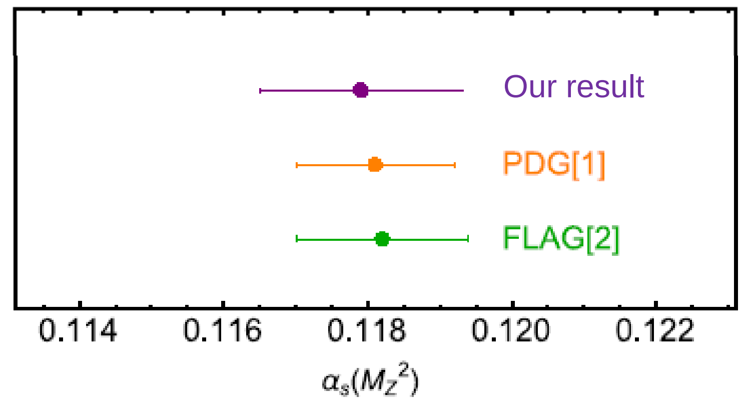
Hayashi, Mishima, YS, Takaura

OPE of QCD potential vs. lattice results and α_s determination

Takaura, Kaneko, Kiyo, YS



$$\alpha_s(M_Z) = 0.1179^{+0.0015}_{-0.0014}$$



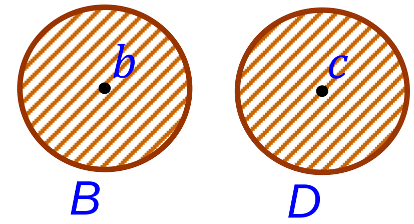
$V_{\rm QCD}(r)$ [JLQCD] consistent with OPE at $r\,\Lambda_{\overline{MS}} \lesssim 0.8$

First time to subtract NLO renormalon

$$V_{\rm QCD}(r) = V_S^{RF}(r) + V_{IR}^{RF}(r)$$

NNNLL fit fn: $A_0 + A_2 r^2$

Future Prospects



Hayashi, Mishima, YS, Takaura

(Hayashi, Ph.D. Thesis)

- Apply renormalon separation to B, D physics

  Non-pert. matrix elements, $|V_{cb}|, m_c, m_b$

- Heavy quarkonium obs.

- Other obs: Adler fn., τ-decay, *R*-ratio, …

- [Challenge] m_t from top decay at LHC

Summary

High precision QCD predictions by separating renormalons

1. Cancellation of $O(\Lambda_{\rm QCD})$ renormalons by re-expressing $m_{\rm pole}$ by $m_{\overline{\rm MS}}$

   $2m_{\rm pole} + V_{\rm QCD}(r)$ in heavy quarkonium system

   spectroscopy, production cross sections, level transitions, decays,…

   m_c, m_b, m_t

2. How to separate renormalons (general framework) in OPE:

   Dual transf. by one-param. integral

   Practical applications: α_s determination from $V_{\rm QCD}(r)$;

   $|V_{cb}|, m_c, m_b$ from B, D; other obs.

タイトル　**量子色力学QCDの定量的理解への挑戦**

Q: Quality（品質）
C: Cost（価格）
D: Delivery（納期）

あおい技研　＞　業務改善コラム　＞　QCD　＞　QCDとは？初心者向けに4つのポイントで重要な理由や関係性を解説！

QCDとは？初心者向けに4つのポイントで重要な理由や関係性を解説！

2022年9月20日　QCD　生産管理　製造業

QCDとは、Quality（品質）・Cost（コスト）・Delivery（納期）の頭文字をとった用語です。QCDとは、製造業における生産管理やマネジメントの現場で、よく使われている専門用語です。特に製造業では欠かせない要素であり、それぞれをバランス良く満たすことが、企業の発展や利益向上に大きく関わります。

しかしQCDが製造業において、具体的にどのような意味を持っているのか、少し理解しづらい部分もあります。今回は、QCDの基本を初心者向けに4つのポイントで解説したいと思います。QCDが製造業で重要な理由やそれぞれの関係性も紹介するため、理解を深めたい人はご参考にしてみてください。

https://aoigk.co.jp/column/what-is-qcd/


c
t
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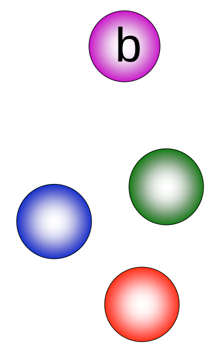

b

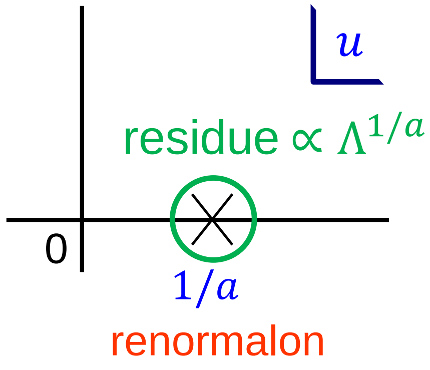
More recent & on-going works

Renormalon uncertainty

$$\sum_n c_n \alpha_s^n \quad ; \quad c_n \sim n!\, a^n$$

⇩ Borel transf.

$$\sum_n \frac{c_n}{n!} u^n \sim \frac{1}{1 - a\,u}$$



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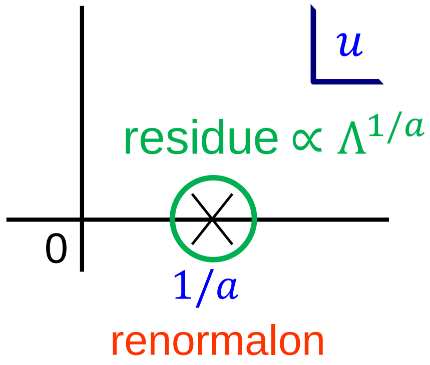
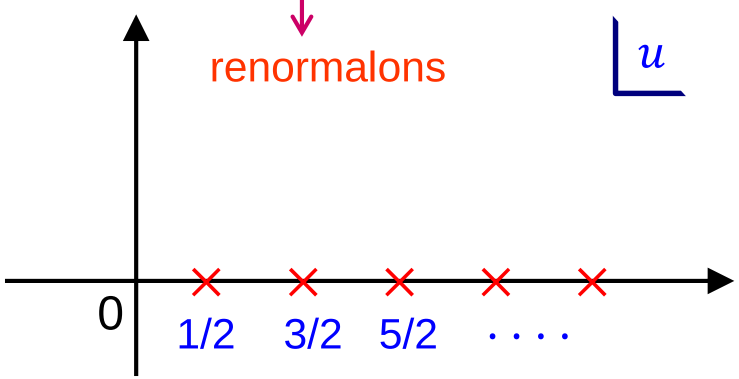


$$V_{\rm QCD}(r) = \int \frac{d^3\vec{q}}{(2\pi)^3} \, e^{-i\vec{q}\cdot\vec{r}} \, \underline{\tilde{V}_{\rm QCD}(q)}$$

No renormalon !



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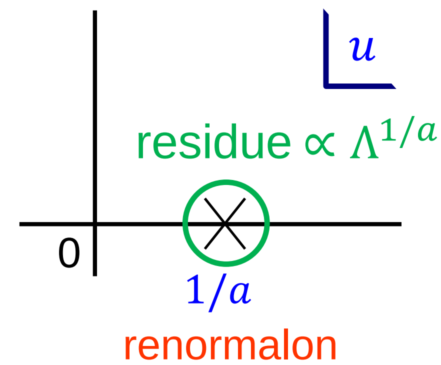
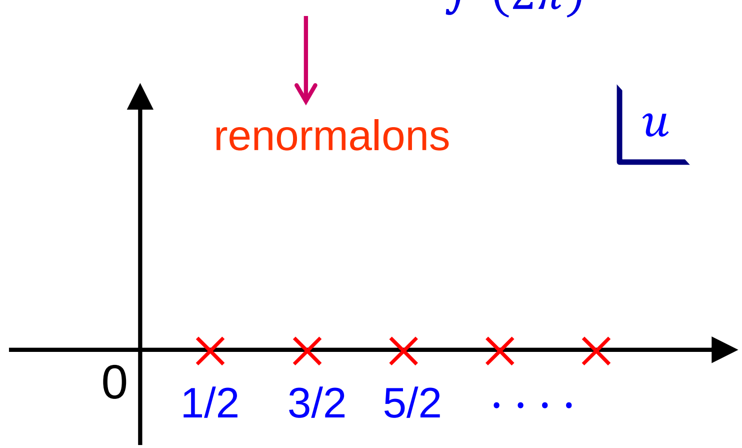


$$V_{\mathrm{QCD}}(r) = \int \frac{d^3\vec{q}}{(2\pi)^3} \, e^{-i\vec{q}\cdot\vec{r}} \, \underline{\tilde{V}_{\mathrm{QCD}}(q)}$$



No renormalon !

$$\because \quad \delta\tilde{V}_{\mathrm{QCD}}(q) = \int d^3\vec{r} \; e^{i\vec{q}\cdot\vec{r}} \, \delta V_{\mathrm{QCD}}(r)$$

$$\propto \int d^3\vec{r} \; e^{i\vec{q}\cdot\vec{r}} \, (r\Lambda_{\mathrm{QCD}})^{2P}/r$$

$$\propto \; \Gamma(2P+1)\cos(\pi P)$$

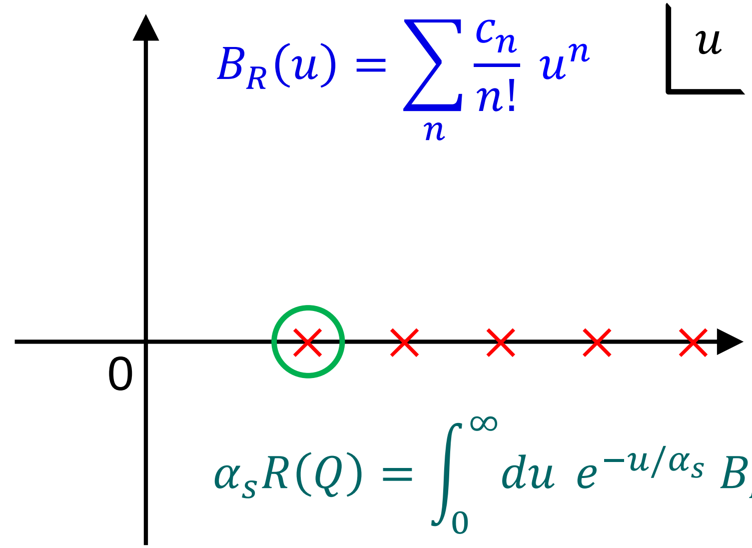
zero at $P = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \ldots$

Takaura

Renormalon uncertainty

't Hooft

$$B_R(u) = \sum_n \frac{c_n}{n!} u^n$$

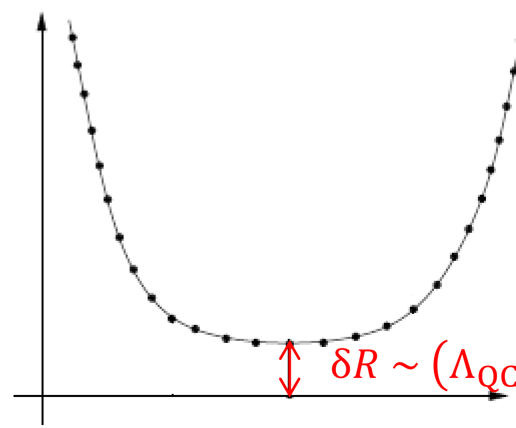


$$\alpha_s R(Q) = \int_0^\infty du\ e^{-u/\alpha_s} B_R(u)$$

Later it was shown that renormalon uncertainties can be absorbed into non-pert. matrix elements in OPE.

Mueller

$c_n\, \alpha_s^n(\mu)$



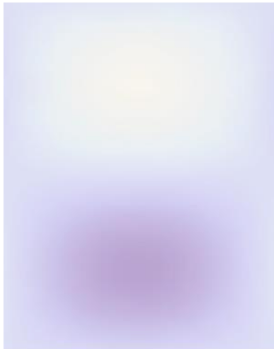
$$\delta R \sim \left(\Lambda_{\rm QCD}/Q\right)^{P+1}$$

$$\alpha_s(q) = \frac{\alpha_s(\mu)}{1 - b_0 \alpha_s(\mu) \log(\mu/q)}$$

$$R(Q) \propto \int_0^Q dq\, q^P \alpha_s(q) = \sum_n c_n\, \alpha_s^n(\mu)$$

with $c_n \sim n!$

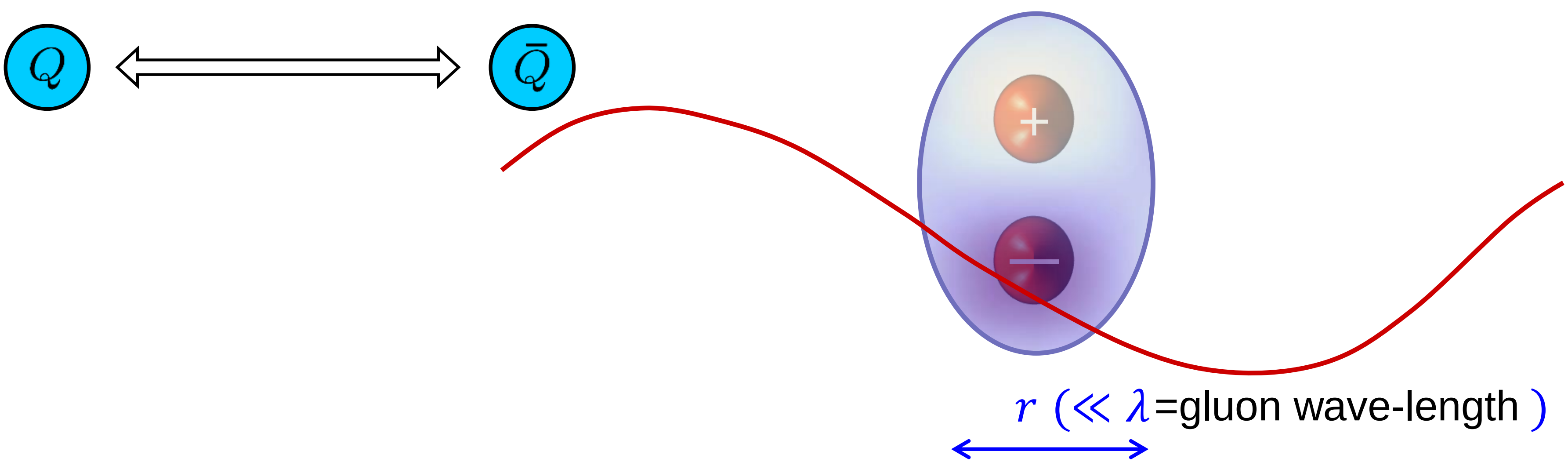
Asymptotic series ⇨ Limited accuracy



$$\delta\tilde{V}_{\rm QCD}(q) = \int d^3\vec{r}\ e^{i\vec{q}\cdot\vec{r}}\ \delta V_{\rm QCD}(r)$$
$$\propto \int d^3\vec{r}\ e^{i\vec{q}\cdot\vec{r}}\ (r\Lambda_{\rm QCD})^{2P}$$
$$\propto\ \Gamma(2P+1)\cos(\pi P)$$

$\delta V_{\rm QCD}(r) = N(u_*)\, r^{-1}(r\Lambda_{\rm QCD})^{2u_*}$ by renormalon at $u = u_*$

$$\delta\tilde{V}_{\rm QCD}(q) = N(u_*)\, q^{-2}(\Lambda_{\rm QCD}/q)^{2u_*}\ \Gamma(2u_*+1)\cos(\pi u_*)$$



Pert. QCD

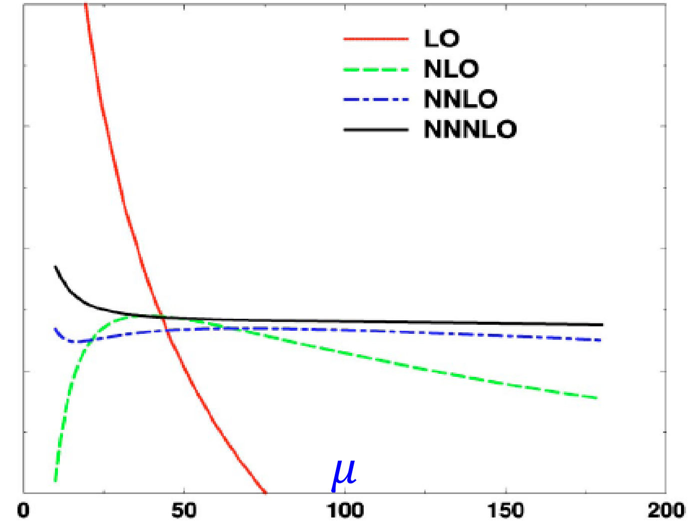
renormalization scale

$$\mathcal{L}_{QCD}(\alpha_s, m_i; \mu)$$

Theory of quarks and gluons

Same input parameters as full QCD.
Systematic: has its own way of estimating errors.
(Dependence on μ is used to estimate errors.)

Differs from a model



Predictable observables

testable hypothesis

(i) Inclusive observables (hadronic inclusive) $\cdots$ insensitive to hadronization

- R-ratio: $R(E) \equiv \dfrac{\sigma(e^+e^- \to hadrons; E)}{\sigma(e^+e^- \to \mu^+\mu^-; E)} = \sum_q 3Q_q^2 \left[1 + \sum_{n=1}^{\infty} c_n(E/\mu)\, \alpha_s^n(\mu)\right]$
- Inclusive decay widths
- Distributions of non-colored particles, $\ell, \gamma, W, H, \cdots$

(ii) Observables of heavy quarkonium states (the only individual hadronic states)

- spectrum, leptonic decay width, transition rates

QCD potential

$$V_{\rm QCD}(r) = V_S(r)\cdot\langle \mathbf{1}\rangle + \text{const.} + \delta E_{US}(r) + \cdots$$

$$\delta E_{US}(r) = g^2 \int_0^\infty dt\, e^{-it\Delta V} \left\langle \vec{r}\cdot\vec{E}^a(t)\,\varphi_{ab}(t)\,\vec{r}\cdot\vec{E}^b(0)\right\rangle$$

Expand in r: $V_C(r) + C_0^V\cdot\Lambda_{\rm QCD} + C_1^V\cdot\Lambda_{\rm QCD}^2 r + C_2^V\cdot\Lambda_{\rm QCD}^3 r^2 + \cdots$

YS, Takaura

renormalon

renormalon

$$= V_S^{RF}(r)\cdot\langle \mathbf{1}\rangle + \text{const.} + \delta E_{US}^{RF}(r) + \cdots$$

$A_2 r^2$ or $A_2 r^2(1 + c\log r)$

fitting param.

compare

$V_{\rm latt}(r)$

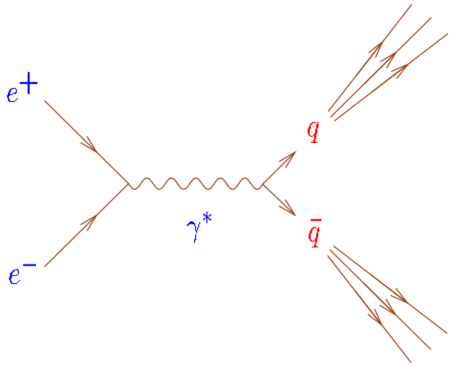
Theoretical predictions

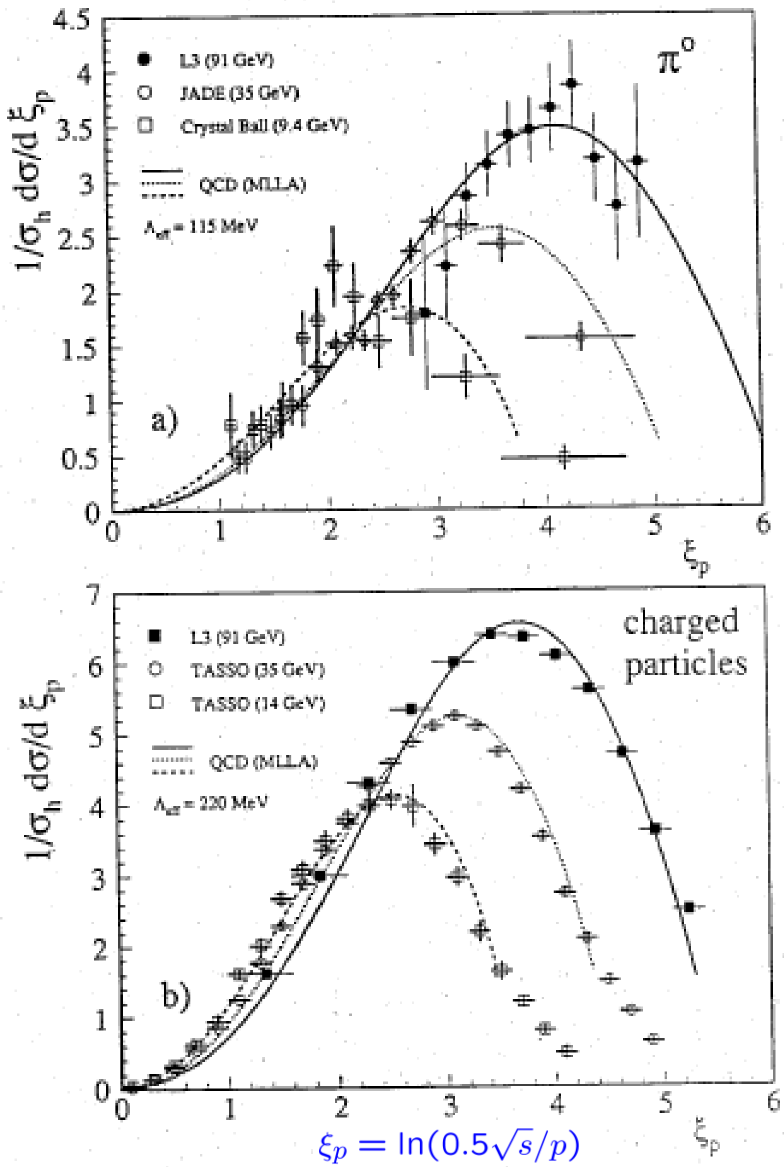
Possible to predict (semi-)analytically:

- Mean multiplicity
- Multiplicity distribution
- Patterns of energy and multiplicity flow
- Inclusive energy spectrum
- Correlations between partons

⋮

without recourse to hadronization scheme.





☆ Predictions are very restrictive, with few adjustable parameters.

$$E_X \approx 2m_b^{\overline{MS}}(\mu) + \int_0^{\mu} dq\, f_X(q)\alpha_s(q)$$

Brambilla,YS,Vairo
Recksiegel,YS

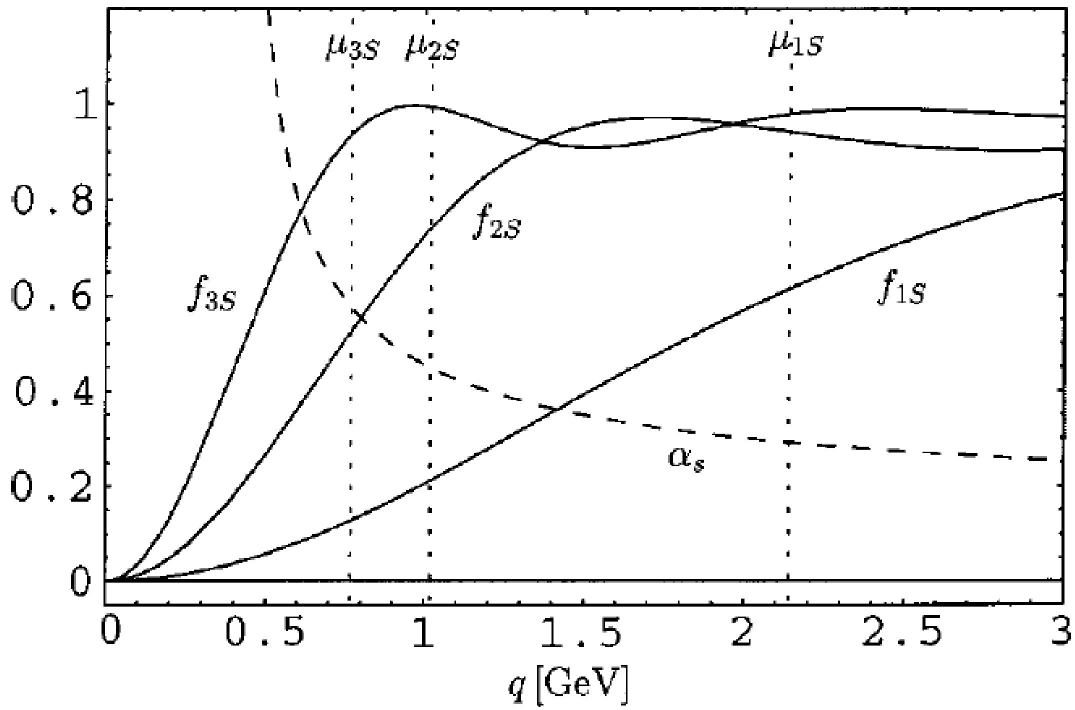
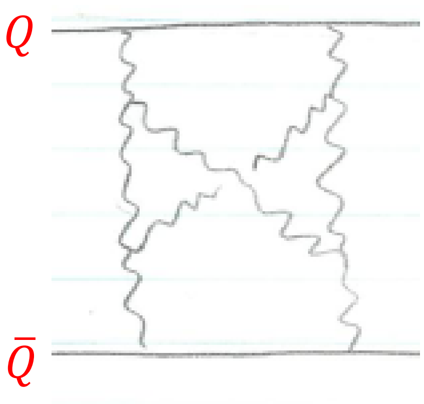
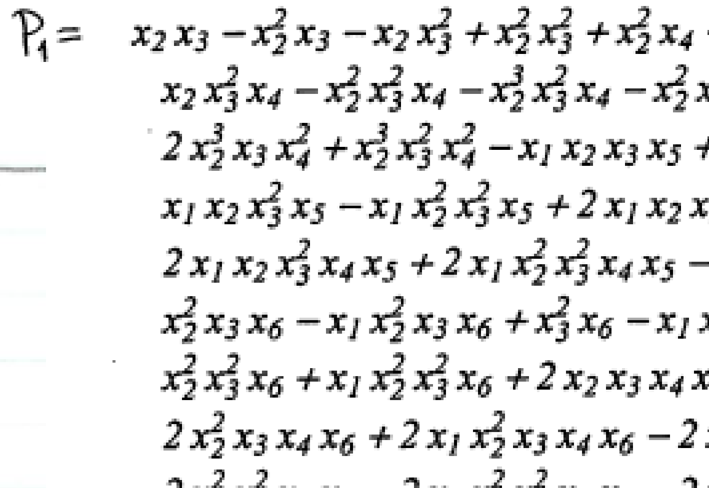
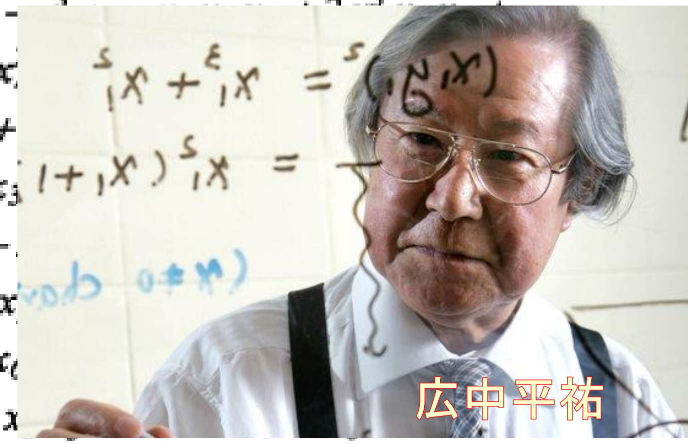
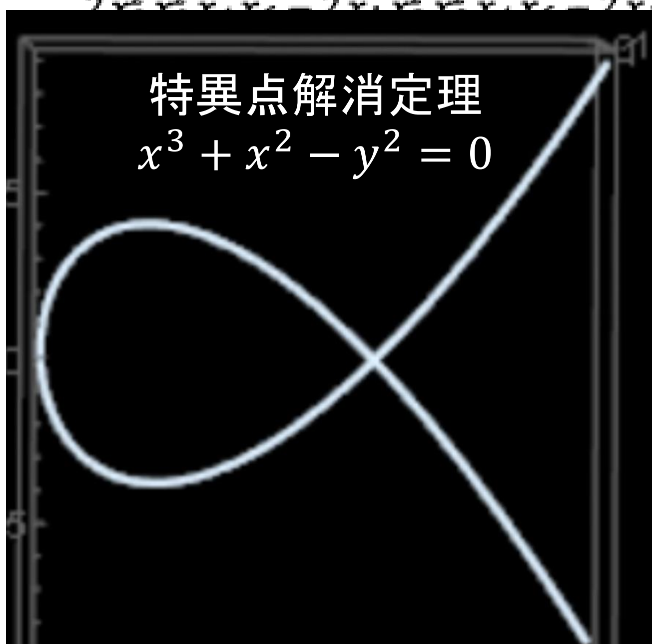


FIG. 5. Support functions for the S states. The solid curves show the support functions as defined in Eq. (19); for comparison of the relevant scales, $\alpha_s^{(4)}(\mu)$ is also plotted (dashed curve). Since the analysis that we advocate in this work does not attribute scales to the individual states, the scales indicated by the dotted lines are taken from [3], Table II.









$$\int_0^1 dx_1 \cdots dx_7 \frac{P_3}{P_1 P_2}$$