The effective track to new physics

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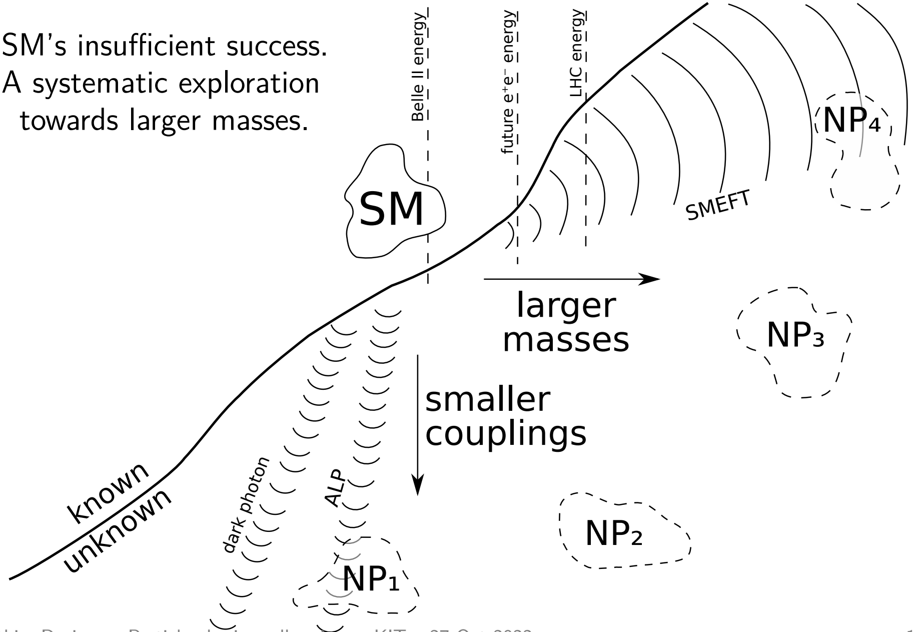
(CERN)

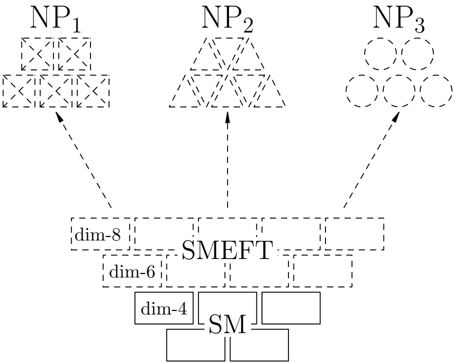
Particle physics colloquium – KIT – 27 Oct 2022

The particle physics landscape

SM's insufficient success.
A systematic exploration
towards larger masses.

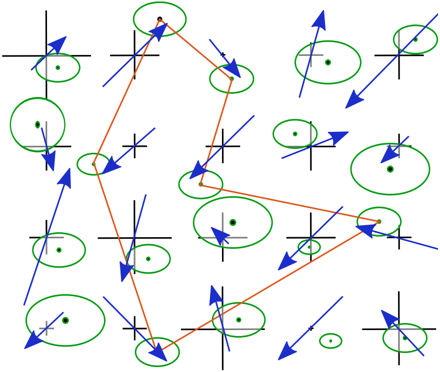


Taking the SM to higher dimensions



- using established bricks (fields and symmetries)
- extension organised by relevance (dimension)
- including all deformations (theory space coverage)

Isolating patterns of new physics



array of sensitive observables

- precise SM-EFT predictions
- precise measurements

→ correlate deviations

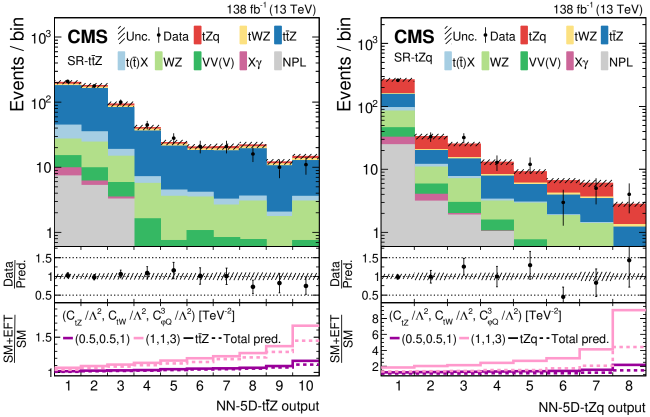
SMEFT challenges

1. improved sensitivity (exploit powerful and complementary obs.)
2. more global picture (combine sectors)
3. precise data interpretation (include quantum corrections)
4. new-physics implications (map to models)
5. framework understanding (leverage amplitude techniques)

1. Improved sensitivity

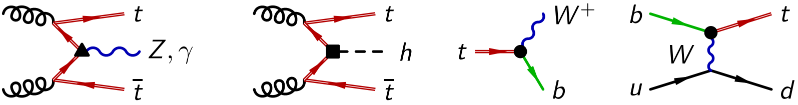
Neural networks for SMEFT

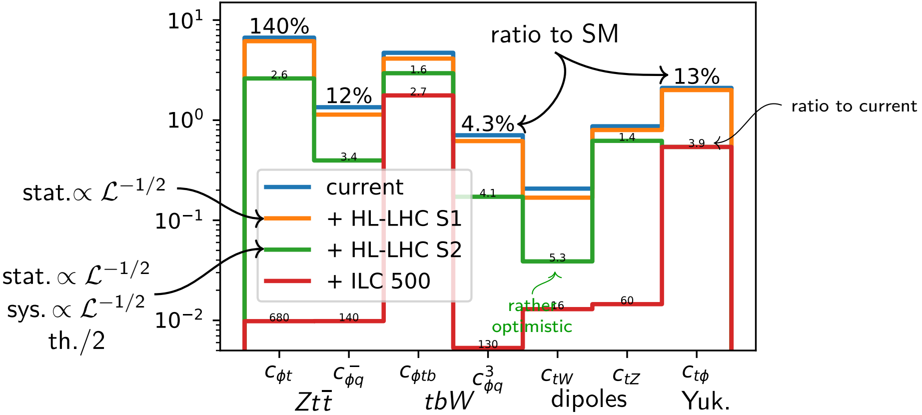
- $tZ + X$ process in the three-lepton signal region
- first discriminate $t\bar{t}Z$, tZj signals and backgrounds
- then SM vs. $(c_{tZ}, c_{tW}, c^3_{\phi q})$ from reweighted sample



Top electroweak interactions

[GD, Irles, Miralles, Peñuelas, Pöschl, Perellò, Vos ’19]
see also: [GD, Gutiérrez, Mantani, Miralles, Miralles, Moreno, Poncelet, Vryonidou, Vos ’22]





Ratios to cancel systematics

[Schulze, Soreq '16]
[Mangano, Plehn, Reimitz, Schell, Shao '15]

$t\bar{t}V/t\bar{t}$

NLO scale: 20% → 3%

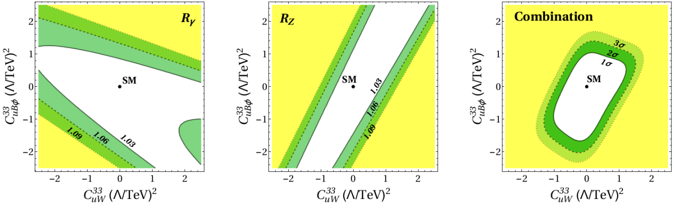
$$\mathcal{R}_\gamma^{\rm SM}\times 10^{-3} = \begin{cases} 11.4^{-0.7\%}_{+0.7\%} & \text{at LO,} \\ 12.6^{+3.1\%}_{-1.8\%} & \text{at NLO QCD.} \end{cases}$$

$$\mathcal{R}_Z^{\rm SM}\times 10^{-4} = \begin{cases} 2.27^{-1.7\%}_{+2.0\%} & \text{at LO,} \\ 1.99^{-1.9\%}_{+2.8\%} & \text{at NLO QCD.} \end{cases}$$

PDF: 10% → 1%

$$\mathcal{R}_\gamma^{\rm LO}\times 10^{-3} = \begin{cases} 11.5 & \text{with NNPDF3.0 [70]} \\ 11.4 & \text{with CTEQ6L1 [72],} \\ 11.5 & \text{with MSTW08 [73],} \end{cases}$$

$$\mathcal{R}_Z^{\rm LO}\times 10^{-4} = \begin{cases} 2.29 & \text{with NNPDF3.0,} \\ 2.27 & \text{with CTEQ6L1,} \\ 2.27 & \text{with MSTW08.} \end{cases}$$



$t\bar{t}H/t\bar{t}Z$

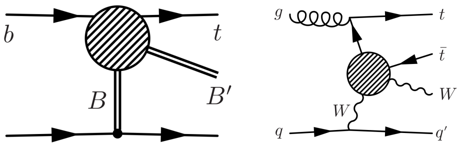
NLO scale: 10% → 3%, PDF: 3% → 0.5%

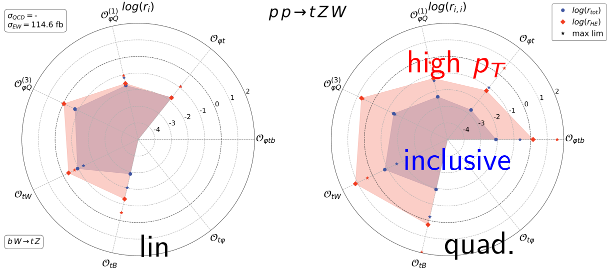
	$\sigma(t\bar{t}H)$[pb]	$\sigma(t\bar{t}Z)$[pb]	$\frac{\sigma(t\bar{t}H)}{\sigma(t\bar{t}Z)}$
13 TeV	$0.475^{+5.79\%+3.33\%}_{-9.04\%-3.08\%}$	$0.785^{+9.81\%+3.27\%}_{-11.2\%-3.12\%}$	$0.606^{+2.45\%+0.525\%}_{-3.66\%-0.319\%}$
100 TeV	$33.9^{+7.06\%+2.17\%}_{-8.29\%-2.18\%}$	$57.9^{+8.93\%+2.24\%}_{-9.46\%-2.43\%}$	$0.585^{+1.29\%+0.314\%}_{-2.02\%-0.147\%}$

Rare processes and high energies

[Dror, Farina, Salvioni, Serra '15]
[Maltoni, Mantani, Mimasu '19]
[El Faham, Maltoni, Mimasu, Zaro '21]

deviations from SM couplings lead to energy-growing effects



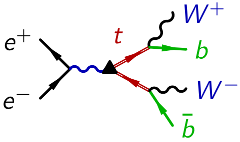


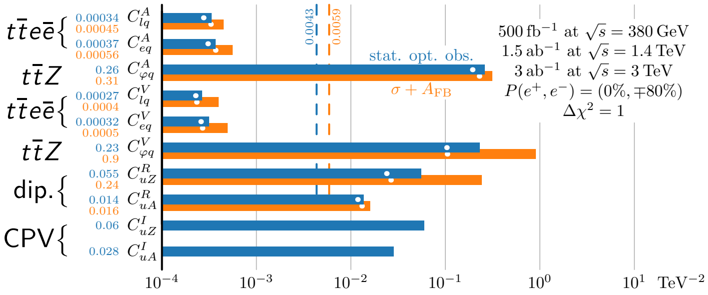
Top EW couplings at lepton colliders

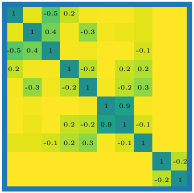
[GD, Perelló, Vos, Zhang '18]
[CLICdp '18]

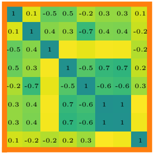
Statistically optimal observables:

- exploiting differential distributions
- covering multidimensional parameter spaces
- enhancing linear terms and EFT validity





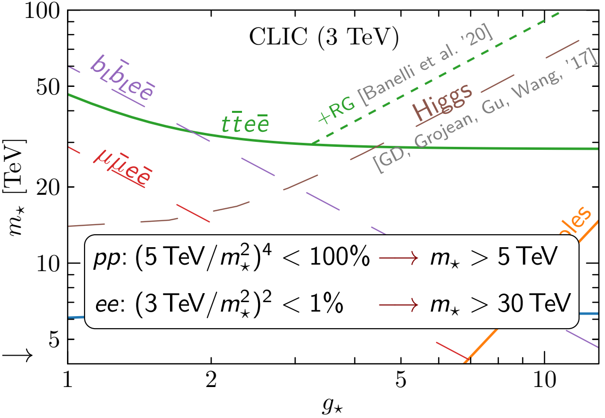




[GD, Matsedonskyi '18]

Impact on compositeness scenarios

- 1σ sensitivities
- fully composite t_R ($\epsilon_t = 1$ in $y_t = \epsilon_t \epsilon_q g_\star$)
- up to $\pm\mathcal{O}(1)$ factors



HL-LHC $\longrightarrow$
[Farina et al. '18]

Complementarity between bottom/top/Higgs measurements

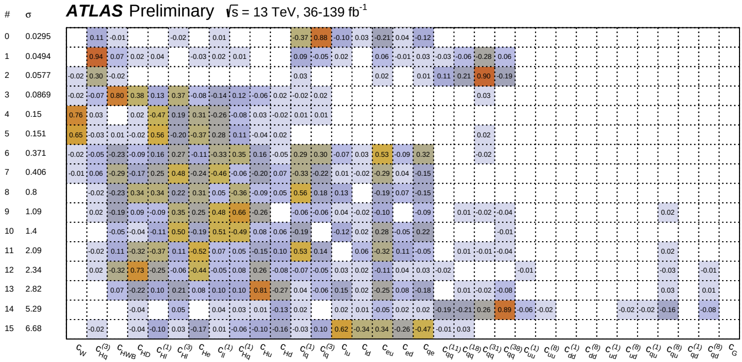
Going beyond HL-LHC reach

2. More global picture

Fitting with degeneracies

[ATLAS '21]
see also Higgs combination: [ATLAS '20]

- diagonalise the Gaussian covariance in SMEFT space
- keep constrained eigenvectors, group operators by 'type'
- $WW+WZ+4\ell+Zjj$ differential measurements



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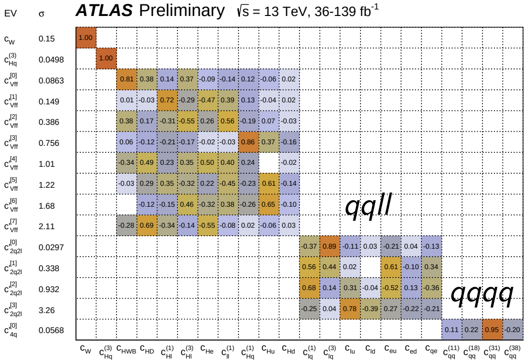
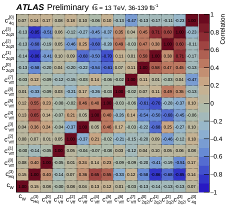
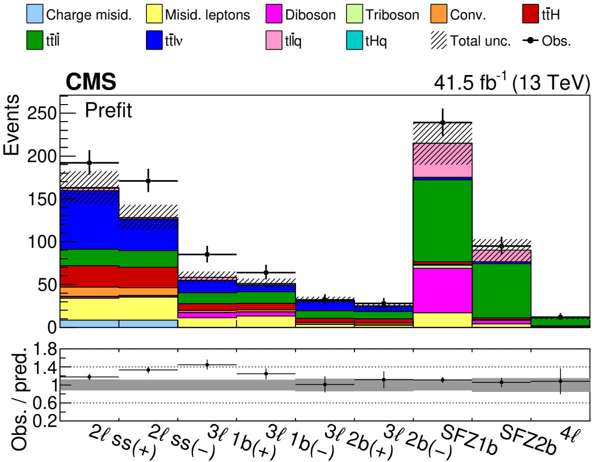




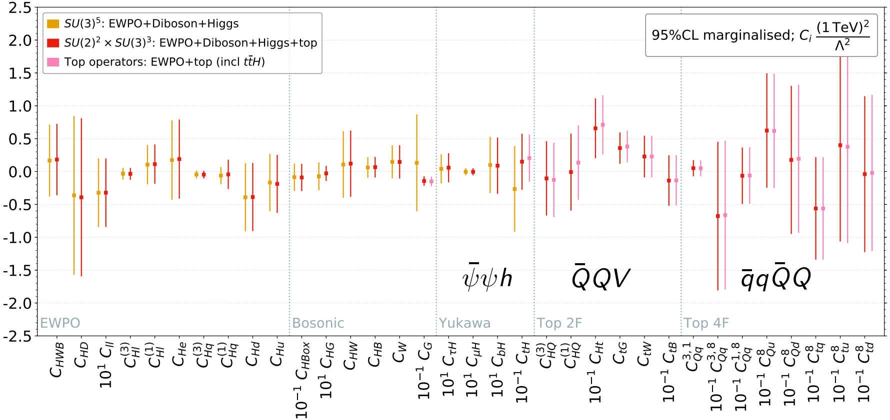
Abolishing the signal-background distinction

- leptons+b's final state, spit into 35 signal regions
- contains tth, ttZ, ttW, tZq, tHq, diboson, etc.
- 16 top operator contributions to all of them



EW+Higgs+top

[Ellis, Madigan, Mimasu, Sanz, You '20]
see also: [Ethier, Magni, Maltoni, Mantani, et al. '21]



- tension in various observables but no large deviation consistent overall

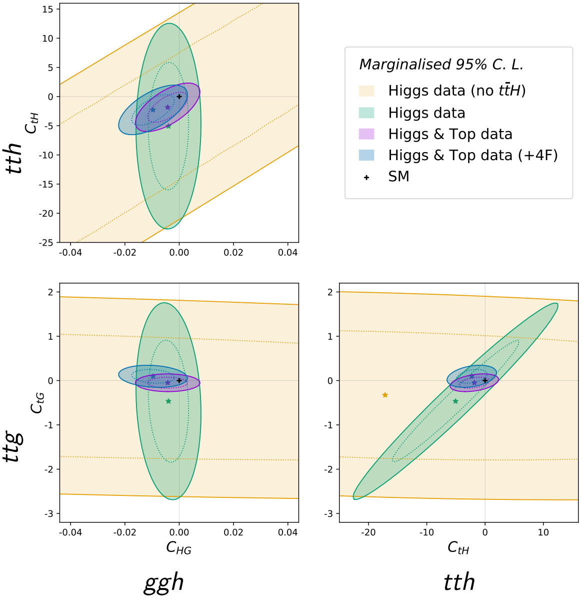
e.g.

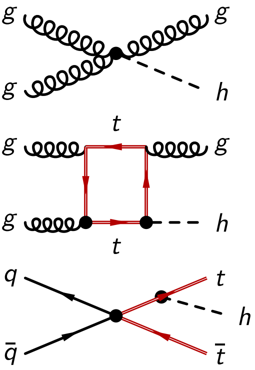
obs.	ttW	m_{tt}&y_{tt}	$p_T^{\text{t-chan}}$
χ^2/ndf	2	1.5	5

- largest correlations btw EW and Higgs, no significant degradation from top

EW+Higgs+top

[Ellis, Madigan, Mimasu, Sanz, You '20]





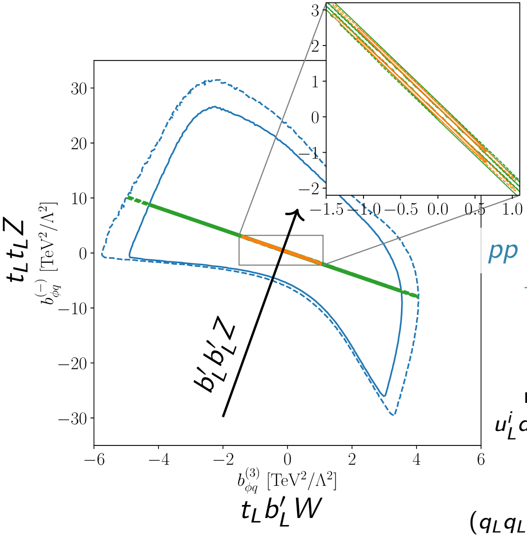
- Higgs/top complementarity among C_{HG}, C_{tH}, C_{tG}
- in subset of 9 operators $C_{H\Box}$, C_{HG}, C_{HW}, C_{HB}, C_{tG}, C_{tH}, C_{bH}, $C_{\tau H}$, $C_{\mu H}$
- robust against $\bar{q}q\bar{t}t$ op.

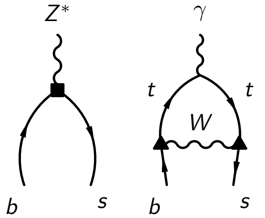
Top+bottom

[Bruggisser et al. '21]
[see also Brod et al. '14]

$B_s \to \mu^+\mu^-$: $b'_L b'_L Z$ current
$B \to X_s\gamma$: $t_L b'_L W$ current

with $b'_L \equiv V_{td} d_L + V_{ts} s_L + V_{tb} b_L$





$pp \to ttZ, tZ, tW, tj$
$+\ B_s \to \mu^+\mu^-$
$+\ B \to X_s\gamma$
(no $Z \to bb$)

marginalized over
$u_L^i d_L^i W / u_L^i u_L^i Z / d_L^i d_L^i Z$,
$\Delta\chi^2 = 2.3$ & 6

($q_L q_L q_L q_L$ op. also studied)

Top+bottom

[Bißmann et al. '20]

Operators [8]

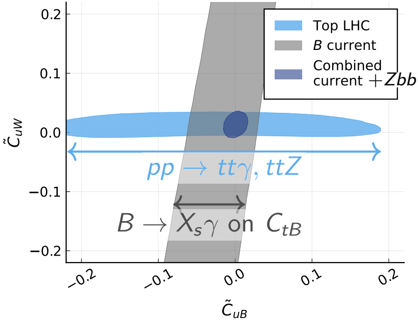
- top dipoles [3]
- top currents [3]
- $b'_L b'_L \ell\ell$ [2]

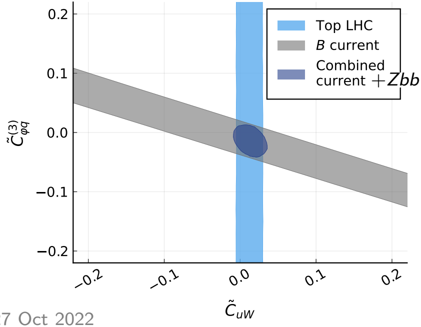
Constraints

- $t\bar{t}$, $t\bar{t}\gamma$, $t\bar{t}Z$ rates
- W helicity fractions
- $Z \to b\bar{b}$ (at tree level)
- $b \to s\gamma$, $b \to s\ell\ell$ (`flavio`+`wilson`)
- B_s mixing, $b \to s\nu\bar{\nu}$
- + future $e^+e^- \to t\,\bar{t}$ $(\sigma, A_{\mathrm{FB}})$

Improvements from b

- mostly on C_{uB}, $C^3_{\varphi q}$ $(b \to s\gamma)$
- not much in $C_{\varphi u}$
- none in C_{tW}, C_{tG}





3. Precise data interpretation

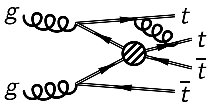
SMEFT at the loop level

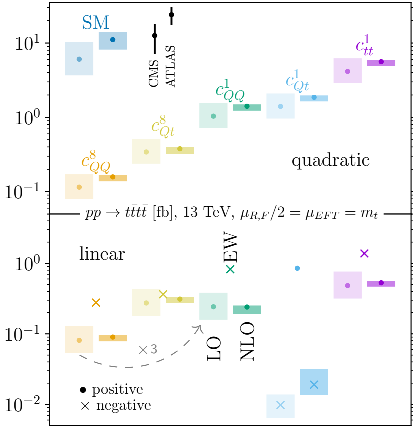
- $pp \to jj$ ($q\bar{q}q\bar{q}$) [Gao, Li, Wang, Zhu, Yuan '11]
- $pp \to t\bar{t}$ ($q\bar{q}t\bar{t}$) [Shao, Li, Wang, Gao, Zhang, Zhu '11]
- $pp \to VV$ [Dixon, Kunszt, Signer '99] [Melia, Nason, Röntsch, Zanderighi '11] [Baglio, Dawson, Lewis '17, '18, '19] [Chiesa, Denner, Lang '18]
- EWPO (top) [Zhang, Greiner, Willenbrock '12]
- top decays [Zhang '14] [Boughezal, Chen, Petriello, Wiegand '19]
- top FCNCs UFO [Degrande, Maltoni, Wang, Zhang '14] [GD, Maltoni, Zhang '14]
- $pp \to t\bar{t}$ (chromo-dipole) [Franzosi, Zhang '15]
- $h \to \gamma\gamma$, VV, γZ [Hartmann, Trott '15] [Ghezzi, Gomez-Ambrosio, Passarino, Uccirati '15] [Dawson, Giardino '18] [Dedes, Paraskevas, Rosiek, Suxho, Trifyllis '18] [Dawson, Giardino '18] [Dedes, Suxho, Trifyllis '19]
- $h \to f\bar{f}$ [Gauld, Pecjak, Scott '15, '16] [Cullen, Pecjak, Scott '19, '20]
- $pp \to tj$ [Zhang '16] [de Beurs, Laenen, Vreeswijk, Vryonidou '18]
- $pp \to t\bar{t}Z$, $gg \to ZH$ [Röntsch, Markus Schulze '14] [Bylund, Maltoni, Vryonidou, Zhang '16]
- $pp \to t\bar{t}H$, $gg \to Hj, HH$ [Maltoni, Vryonidou, Zhang '16]
- $pp \to HV$ [Degrande, Fuks, Mawatari, Mimasu, Sanz '16] [Alioli, Dekens, Girard, Mereghetti '18]
- Z, W poles [Hartmann, Shepherd, Trott '16] [Dawson, Ismail, Giardino '18, '18, '19]
- $pp \to h$ [Grazzini, Ilnicka, Spira, Wiesemann '16] [Deutschmann, Duhr, Maltoni, Vryonidou '17]
- $pp \to tjZ, tjh$ [Degrande, Maltoni, Mimasu, Vryonidou, Zhang '18]
- $pp \to$ jets (triple gluon) UFO [Hirshi, Maltoni, Tsinikos, Vryonidou '18]
- Higgs self-coupling [McCullough '13] [Gorbahn, Haisch '16] [Degrassi et al. '16, '17] [Bizon et al. '16] [Kribs et al. '16] [Maltoni, Pagani, Shivaji, Zhao '17] [Di Vita, GD, Grojean, Gu, Liu, Panico, Riembau, Vantalon '17]
- EW Higgs & WW (top) [Vryonidou, Zhang '18] [GD, Gu, Vryonidou, Zhang '18] [Boselli, Hunter, Mitov '18]
- EW $pp \to t\bar{t}$ (ttZ,tth) [Martini, Schulze '19] [Martini, Pan, Schulze, Xiao '21]
- all QCD and four-quarks UFO [Degrande, GD, Maltoni, Mimasu, Vryonidou, Zhang '20]
- EW $pp \to \ell^+\ell^-$ [Dawson, Giardino '21, '22]
- EW $QQQQ$ in $gg \to h$, $h \to bb$, $pp \to tth$ [Alasfar, de Blas, Gröber '22]
- NNLO $pp \to Zh \to \ell^+\ell^- b\bar{b}$ [Haisch, Scott, Wiesemann, Zanderighi, Zanoli '22]
- NNLO VBF [Asteriadis, Caola, Melnikov, Röntsch '22]

[Degrande, GD, Maltoni, Mimasu, Vryonidou, Zhang '20]

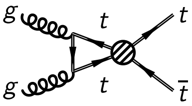
SMEFT at one loop: automation

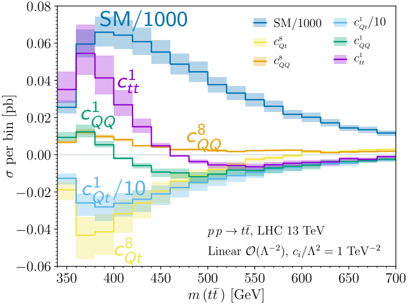
Better accuracy and uncertainties





New sensitivities



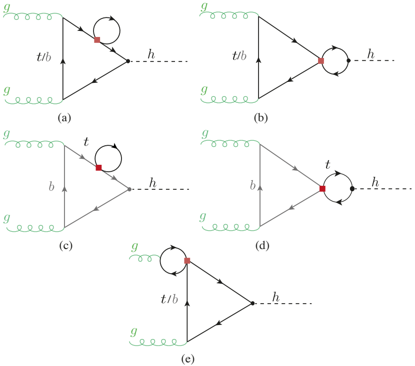


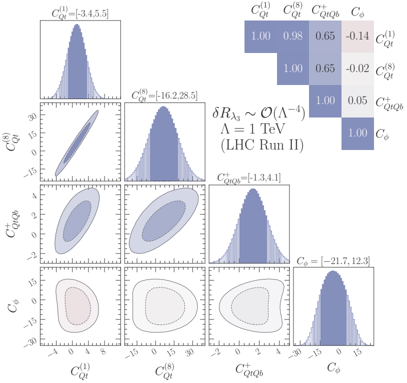
technicalities:

- anomaly cancellation
  [Bonnefoy et al. '20] [Feruglio '20]
- evanescent operators

$t\bar{t}Q\bar{Q}$ in Higgs processes

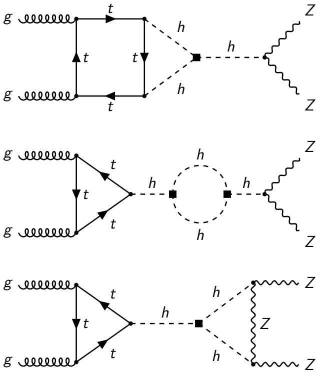
- sensitivity in $gg \to h$, $h \to \gamma\gamma$, $pp \to t\bar{t}h$ comparable to $pp \to t\bar{t}t\bar{t}$ and $t\bar{t}b\bar{b}$
- spoils the loop sensitivity to the Higgs self-coupling

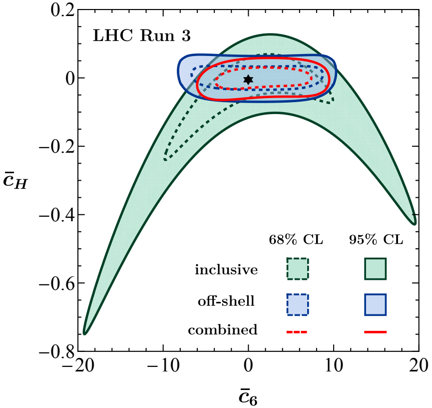




Self-coupling in off-shell $gg \to 4\ell$

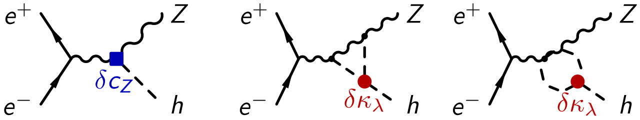
- extra discriminating power in differential distributions
- leveraged with matrix-element based observable

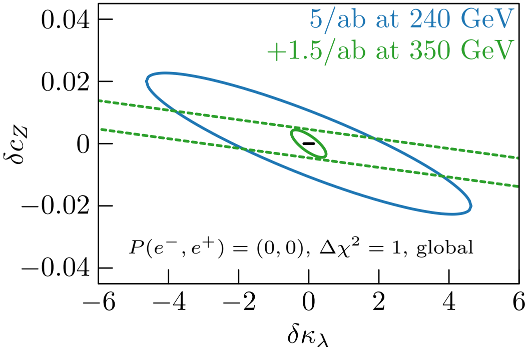




Self-coupling loops at lepton colliders

[Di Vita, GD, Grojean, Gu, Liu, et al. '17]
[McCullough '13]
[Gorbahn, Haisch '16]
[Degrassi et al. '16]
[Bizon et al. '16]
[Degrassi et al. '17]
[Kribs et al. '17]
[Maltoni et al. '17]
[Maltoni et al. '18]
[Gorbahn, Haisch '19]
[Degrassi, Vitti '19]
[Degrassi et al. '21]
[Haisch, Koole '21]



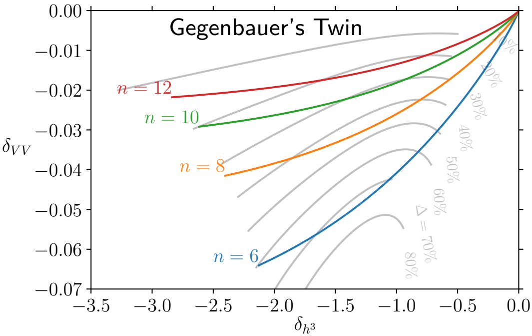


Correlations with single-Higgs couplings require two $\sqrt{s}$.

Models with large δ_{h^3}/δ_{VV} ?

[GD, McCullough, Salvioni '21, '22, '22]
see also: [Di Luzio, Gröber, Spannowsky '17]
[Gupta, Rzehak, Wells '13] [Falkowski, Rattazzi '19] [etc.]
[Logan, Rentala '15] [Chala, Krause, Nardini '18]

Gegenbauer potentials $G_n^{(N-1)/2}(\cos\frac{h}{f})$ are radiatively stable for pseudo-Nambu-Goldstone bosons of $\mathrm{SO}(N+1) \to \mathrm{SO}(N)$.



Naturally features $\mathcal{O}(1\%)$ Higgs deviations,

but yields $\mathcal{O}(100\%)$ self-coupling modifications.

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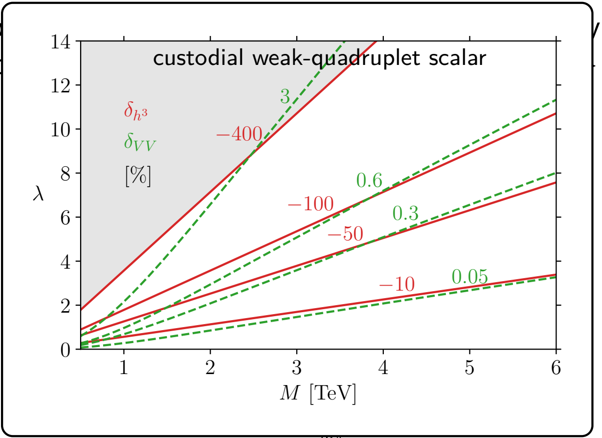
Gegenb … y stable
for pseud … SO(N).



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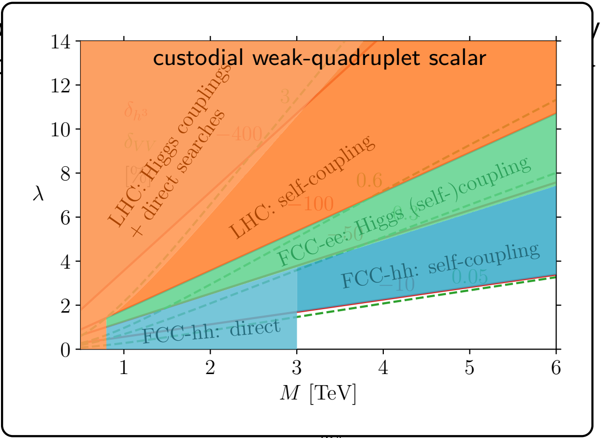
Gegenb… stable
for pseud… SO(N).

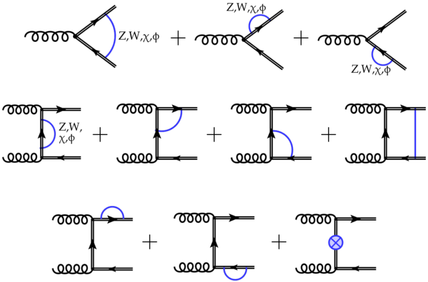


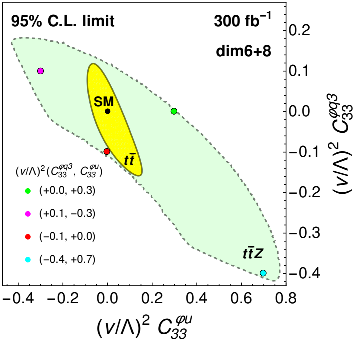
Naturally features $\mathcal{O}(1\%)$ Higgs deviations,

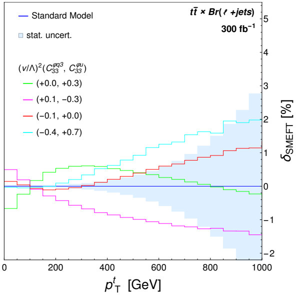
but yields $\mathcal{O}(100\%)$ self-coupling modifications.

Top EW interactions in $pp \to t\bar{t}$

[Martini, Schulze '19]
see also tth: [Kühn, Scharf, Uwer '13]
and CPV tth: [Martini, Pan, Schulze, Xiao '21]





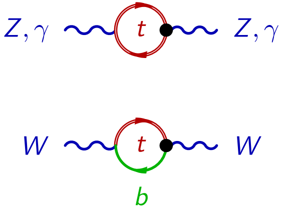


- linear + square in $C^1_{\varphi q}$, $C^3_{\varphi q}$, $C_{\varphi u}$
- assuming $C^1_{\varphi q} + C^3_{\varphi q} = 0$
  from $Z \to b\bar{b}$ constraint
- using $\Delta\varphi_{\ell\ell}$ in $t\bar{t}(Z \to \ell\ell)$ NLO QCD
  and $p_T(t)$ in $t\bar{t}$ NLO QCD+EW
- sys. from state-of-the-art scale unc.:
  correlated flat ±15% in $t\bar{t}Z$, and ±5% in $t\bar{t}$

Top EW interactions at lepton colliders

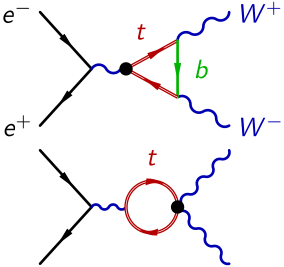
EWPO

[Zhang, Greiner, Willenbrock '12]



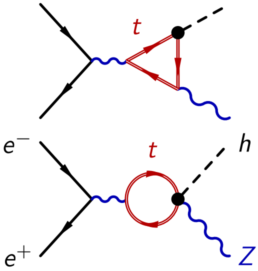
Diboson

[GD, Gu, Vrionidou, Zhang '18]



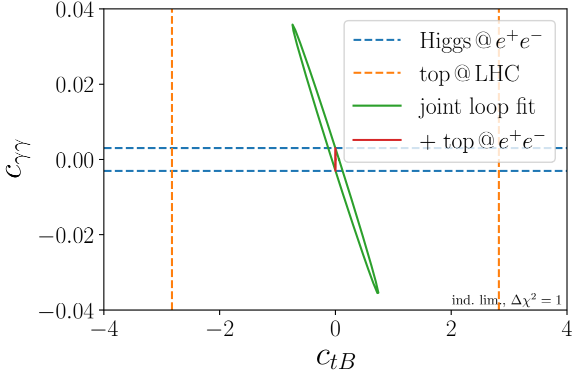
Higgs

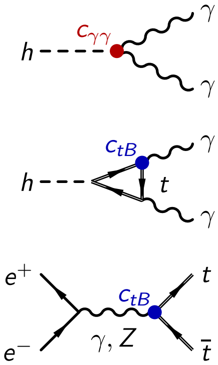
[Vrionidou, Zhang, '18]
[see also Boselli et al '18]



Top-Higgs interplay at lepton colliders

[GD, Gu, Vryonidou, Zhang '18]
see also: [Jung, Lee, Perelló, Tian, Vos '20]





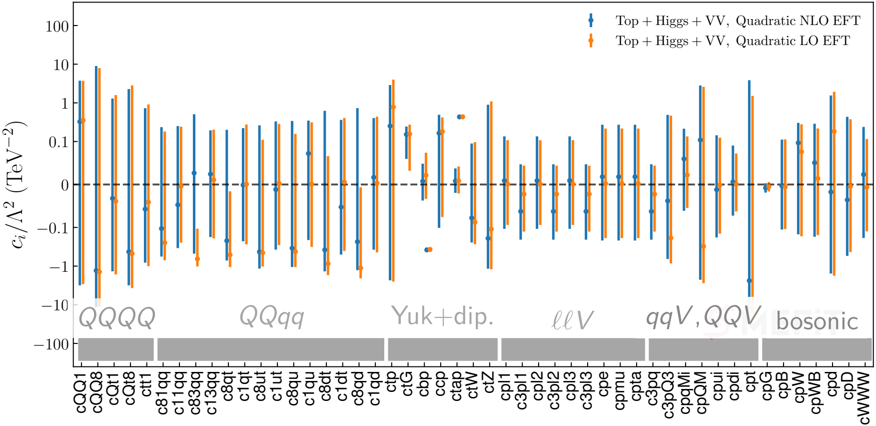
Higgs@e^+e^- helps improving top coupling precision.

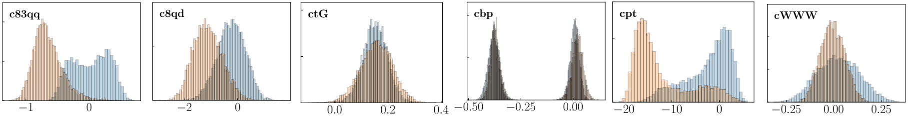
Higgs precision is however contaminated by top uncertainties.

Top@e^+e^- is needed to achieve the full potential of Higgs@e^+e^-.

NLO in diboson+Higgs+top

NLO SMEFT in $t\bar{t}$, single top, $gg \to h$, hV, tth, $h \to bb$, diboson





5. Framework understanding

Going on-shell

construct amplitudes directly and recursively

bypass unphysical fields, operators, Lagrangians

avoid gauge and field-redefinition redundancies

e.g. graviton Feynman rules [De Witt '67]

3 pt.

$$\frac{\delta^3 S}{\delta\varphi_{\mu\nu}\delta\varphi_{\sigma'\tau'}\delta\varphi_{\rho''\lambda''}} \rightarrow \mathrm{Sym}\Big[-\tfrac{1}{4}P_3(p\cdot p'\eta^{\mu\nu}\eta^{\sigma\tau}\eta^{\rho\lambda})-\tfrac{1}{4}P_6(p^\sigma p^\tau\eta^{\mu\nu}\eta^{\rho\lambda})+\tfrac{1}{4}P_3(p\cdot p'\eta^{\mu\sigma}\eta^{\nu\tau}\eta^{\rho\lambda})+\tfrac{1}{2}P_6(p\cdot p'\eta^{\mu\nu}\eta^{\sigma\rho}\eta^{\tau\lambda})+P_3(p^\sigma p^\lambda\eta^{\mu\nu}\eta^{\tau\rho})$$
$$-\tfrac{1}{2}P_3(p^\tau p'^\mu\eta^{\nu\sigma}\eta^{\rho\lambda})+\tfrac{1}{2}P_3(p^\rho p'^\lambda\eta^{\mu\sigma}\eta^{\nu\tau})+\tfrac{1}{2}P_6(p^\rho p^\lambda\eta^{\mu\sigma}\eta^{\nu\tau})+P_6(p^\sigma p'^\lambda\eta^{\tau\mu}\eta^{\nu\rho})+P_3(p^\sigma p'^\mu\eta^{\tau\rho}\eta^{\lambda\nu})$$
$$-P_3(p\cdot p'\eta^{\nu\sigma}\eta^{\tau\rho}\eta^{\lambda\mu})\Big],$$

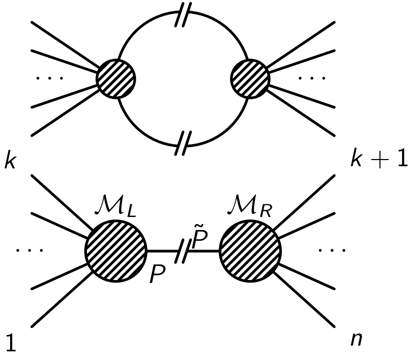
171 terms vs. $\left([12]^3/[23][31]\right)^2$

4 pt.

$$\frac{\delta^4 S}{\delta\varphi_{\mu\nu}\delta\varphi_{\sigma'\tau'}\delta\varphi_{\rho''\lambda''}\delta\varphi_{\iota'''\kappa'''}} \rightarrow \mathrm{Sym}\Big[-\tfrac{1}{8}P_6(p\cdot p'\eta^{\mu\nu}\eta^{\sigma\tau}\eta^{\rho\lambda}\eta^{\iota\kappa})-\tfrac{1}{8}P_{12}(p^\sigma p^\tau\eta^{\mu\nu}\eta^{\rho\lambda}\eta^{\iota\kappa})-\tfrac{1}{4}P_6(p^\sigma p'^\mu\eta^{\nu\tau}\eta^{\rho\lambda}\eta^{\iota\kappa})+\tfrac{1}{8}P_6(p\cdot p'\eta^{\mu\sigma}\eta^{\nu\tau}\eta^{\rho\lambda}\eta^{\iota\kappa})$$
$$+\tfrac{1}{4}P_6(p\cdot p'\eta^{\mu\nu}\eta^{\sigma\tau}\eta^{\rho\iota}\eta^{\lambda\kappa})+\tfrac{1}{4}P_{12}(p^\sigma p^\tau\eta^{\mu\nu}\eta^{\rho\iota}\eta^{\lambda\kappa})+\tfrac{1}{2}P_6(p^\sigma p'^\mu\eta^{\nu\tau}\eta^{\rho\iota}\eta^{\lambda\kappa})-\tfrac{1}{4}P_6(p\cdot p'\eta^{\mu\sigma}\eta^{\nu\tau}\eta^{\rho\iota}\eta^{\lambda\kappa})$$
$$+\tfrac{1}{4}P_{24}(p\cdot p'\eta^{\mu\nu}\eta^{\sigma\rho}\eta^{\tau\lambda}\eta^{\iota\kappa})+\tfrac{1}{4}P_{24}(p^\sigma p^\tau\eta^{\mu\rho}\eta^{\nu\lambda}\eta^{\iota\kappa})+\tfrac{1}{4}P_{12}(p^\rho p'^\lambda\eta^{\mu\sigma}\eta^{\nu\tau}\eta^{\iota\kappa})+\tfrac{1}{2}P_{24}(p^\sigma p'^\rho\eta^{\tau\mu}\eta^{\nu\lambda}\eta^{\iota\kappa})$$
$$-\tfrac{1}{2}P_{12}(p\cdot p'\eta^{\nu\sigma}\eta^{\tau\rho}\eta^{\lambda\mu}\eta^{\iota\kappa})-\tfrac{1}{2}P_{12}(p^\sigma p'^\mu\eta^{\tau\rho}\eta^{\lambda\nu}\eta^{\iota\kappa})+\tfrac{1}{2}P_{12}(p^\sigma p^\rho\eta^{\tau\lambda}\eta^{\mu\nu}\eta^{\iota\kappa})-\tfrac{1}{2}P_{24}(p\cdot p'\eta^{\mu\nu}\eta^{\tau\rho}\eta^{\lambda\iota}\eta^{\kappa\sigma})$$
$$-P_{12}(p^\sigma p^\tau\eta^{\nu\rho}\eta^{\lambda\iota}\eta^{\kappa\mu})-P_{12}(p^\rho p'^\lambda\eta^{\nu\iota}\eta^{\kappa\sigma}\eta^{\tau\mu})-P_{24}(p_\sigma p'^\rho\eta^{\tau\iota}\eta^{\kappa\mu}\eta^{\nu\lambda})-P_{12}(p^\rho p'^\iota\eta^{\lambda\sigma}\eta^{\tau\mu}\eta^{\nu\kappa})$$
$$+P_6(p\cdot p'\eta^{\nu\rho}\eta^{\lambda\sigma}\eta^{\tau\iota}\eta^{\kappa\mu})-P_{12}(p^\sigma p^\rho\eta^{\mu\nu}\eta^{\tau\iota}\eta^{\kappa\lambda})-\tfrac{1}{2}P_{12}(p\cdot p'\eta^{\mu\rho}\eta^{\nu\lambda}\eta^{\sigma\iota}\eta^{\tau\kappa})-P_{12}(p^\sigma p^\rho\eta^{\tau\lambda}\eta^{\mu\iota}\eta^{\nu\kappa})$$
$$-P_6(p^\rho p'^\iota\eta^{\lambda\kappa}\eta^{\mu\sigma}\eta^{\nu\tau})-P_{24}(p^\sigma p'^\rho\eta^{\tau\mu}\eta^{\nu\iota}\eta^{\kappa\lambda})-P_{12}(p^\sigma p'^\mu\eta^{\tau\rho}\eta^{\lambda\iota}\eta^{\kappa\nu})+2P_6(p\cdot p'\eta^{\nu\sigma}\eta^{\tau\rho}\eta^{\lambda\iota}\eta^{\kappa\mu})\Big].$$

2850 terms vs. $[12]^4\langle 34\rangle^4/stu$

Recursive amplitude construction



- loops cut into trees
  - + rational terms
- trees cut into trees
  - (e.g. with recursion relations)
  - + contact terms

$$\mathcal{M}^{\text{tree}}(1, ...k, ...n) = \sum_{\text{channels}} \frac{\mathcal{M}_L^{\text{tree}}(1, ...k, P)\, \mathcal{M}_R^{\text{tree}}(\tilde{P}, k+1, ...n)}{P^2 - m^2}$$

$$+ \mathcal{M}^{\text{contact}}(1, ...k, ...n)$$

On-shell SMEFT

a. operator enumeration

[Shadmi, Weiss '18], [Ma, Shu, Xiao '19], [Falkowski '19], [GD, Machado '19], [Li, Ren, et al. '20, '20]

b. non-renormalisation, non-interference, anomalous dim.

[Cheung, Shen '15], [Azatov et al. '16], [Bern et al. '19, '20], [Jiang et al. '20], [Elias Miró et al. '20, '21], [Baratella et al. '20, '20, '21], [Accettulli Huber, De Angelis '21], [Delle Rose et al. '22], [Baratella '22], [Machado, Renner, Sutherland '22]

c. massive amplitude construction

[Arkani-Hamed, Huang, Huang '17],
dim$\leq$4: [Christensen, Field '18], [Bachu, Yelleshpur '19], [Liu, Yin '22],
SMEFT: [Aoude, Machado '19], [GD, Kitahara, Shadmi, Weiss '19], [GD et al. '20], [Balkin et al. '21], [Dong, Ma, Shu, Zheng '21, '22], [De Angelis '22]

...

+ d. EFT double copy

a. Operator enumeration

Helicity spinors

[Mangano, Parke '91]
[Dreiner, Haber, Martin '08]
[Helvang, Huang '13]
[Dixon '13]
[Schwartz '14]
[Cheung '17]

As brackets

$$u_{i^+} = \begin{pmatrix} 0 \\ i] \end{pmatrix}, \qquad u_{i^-} = \begin{pmatrix} i\rangle \\ 0 \end{pmatrix} \qquad \text{for particle } i$$

Rewritting momenta (and polarizations vectors)

$$p_i^\mu \sigma_\mu = i\rangle[i \qquad \left(\varepsilon^\mu_{i^+}\sigma_\mu = \frac{\zeta\rangle[i}{\sqrt{2}\langle\zeta i\rangle}, \qquad \varepsilon^\mu_{i^-}\sigma_\mu = \frac{i\rangle[\zeta}{\sqrt{2}\langle i\zeta\rangle} \right)$$

Trivializing $p_i^2 = \langle ii\rangle[ii]/2 = 0$

$$\langle ii\rangle = \epsilon_{\alpha\beta}\; i\rangle^\alpha i\rangle^\beta = 0, \qquad [ii] = \epsilon^{\dot\alpha\dot\beta}\; i]_{\dot\alpha} i]_{\dot\beta} = 0$$

Little-group covariance

Little-group transformations leave p_i invariant

Little group includes U(1) for massless p_i

Spinors $i], i\rangle$ pick up phases proportional $\pm 1/2$

The amplitude picks up a phase proportional to h_i

Massless three-point contact terms

fully determined by little-group covariance

$$\mathcal{M}(1^{h_1}, 2^{h_2}, 3^{h_3}) = g \begin{cases} [12]^{h_1+h_2-h_3} \quad [23]^{h_2+h_3-h_1} \quad [31]^{h_3+h_1-h_2} & \text{for } h_1+h_2+h_3 > 0 \\ \langle 12\rangle^{-h_1-h_2+h_3} \langle 23\rangle^{-h_2-h_3+h_1} \langle 31\rangle^{-h_3-h_1+h_2} & \text{for } h_1+h_2+h_3 < 0 \end{cases}$$

f^+f^+s $[12]$

v^+v^+s $[12]^2$

$f^+f^-v^+$ $[13]^2/[12]$

$v^+v^+v^-$ $[12]^3/[23][31]$

$t^+t^+t^-$ $\left([12]^3/[23][31]\right)^2$

$$[g] = 1 - |h|$$

h: $\sum h_i$

Massless higher-point contact terms

Multiple independent structures for given helicities

non-vanishing Lorentz invariants (s_{ij}, ϵ_{ijkl})

solving
· little-group constraints
· momentum conservation
· Schouten identities e.g. $[12][34] - [13][24] + [14][23] = 0$

Construction

· *harmonics* and Young tableaux [Henning, Melia '19] [Li, Ren, et al. '20, '20]

· *twistors* trivializing momentum conservation [Falkowski '19]

· systematic algorithm and explicit construction [GD, Machado '19] [see also Accettulli Huber, De Angelis '21]

Massless applications

- SM+graviton operators up to dim-8:

$$
\begin{array}{rr}
t^+t^+t^+t^+: & [12]^4[34]^4+[13]^4[24]^4+[14]^4[23]^4 \\
t^+t^+v^+v^+: & [12]^4[34]^2,\,[12]^2[13][14][24][23] \\
t^+v^+f^+f^-: & [12]^2[13][124\rangle \\
t^+f^+f^+f^+f^+: & [12][13][14][15] \\
\cdots & \cdots
\end{array}
\qquad \times f(s_{ij},\epsilon_{ijkl})
$$

also from Hilbert series: [Ruhdorfer et al. '19]

- minimal dim. of operators contributing to any helicity amp.:

$$
\dim\{\text{operator}\} \geq n - \sum_i \max(0, \text{ceil}\{|h_i|-1\}) + \sum_i |h_i| + 2\max\begin{bmatrix} \{\sum_{h_i>0} 2h_i\} \mod 2 \\ 2\max_{h_i>0}\{|h_i|\} - \sum_{h_i>0}|h_i| \\ 2\max_{h_i<0}\{|h_i|\} - \sum_{h_i<0}|h_i| \end{bmatrix}
$$

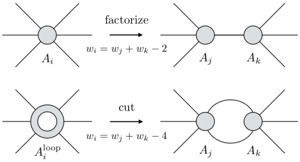
b. Non-renormalisation, non-interference, anomalous dimensions

Non-renormalisation

[Cheung, Shen '15]

vanishing tree helicity amp. $\Rightarrow$ vanishing one-loop divergences

define (anti)holomorphic weights $\overleftrightarrow{w} \equiv n \mp h$
renormalisable trees: $\overleftrightarrow{w}^{\text{tree}}_{\text{reno}} \geq 4$ for $n \geq 4$
(except for e.g. Yukawa amps)
from cut: $\overleftrightarrow{w}^{\text{loop}}_{\text{EFT}} = \overleftrightarrow{w}^{\text{tree}}_{\text{EFT}} + \overleftrightarrow{w}^{\text{tree}}_{\text{reno}} - 4$
so $\overleftrightarrow{w}^{\text{loop}}_{\text{EFT}} \geq \overleftrightarrow{w}^{\text{tree}}_{\text{EFT}}$



	$(w,\bar{w})$	F^3 $(0,6)$	$F^2\phi^2$ $(2,6)$	$F\psi^2\phi$ $(2,6)$	ψ^4 $(2,6)$	$\psi^2\phi^3$ $(4,6)$	$\bar{F}^3$ $(6,0)$	$\bar{F}^2\phi^2$ $(6,2)$	$\bar{F}\bar{\psi}^2\phi$ $(6,2)$	$\bar{\psi}^4$ $(6,2)$	$\bar{\psi}^2\phi^3$ $(6,4)$	$\bar{\psi}^2\psi^2$ $(4,4)$	$\bar{\psi}\psi\phi^2 D$ $(4,4)$	$\phi^4 D^2$ $(4,4)$	ϕ^6 $(6,6)$
F^3	$(0,6)$			×	×	×			×	×	×	×	×	×	×
$F^2\phi^2$	$(2,6)$				×	×				×	×	×			×
$F\psi^2\phi$	$(2,6)$									×				×	×
ψ^4	$(2,6)$	×	×			×	×	×	×	×	×	y^2		×	×
$\psi^2\phi^3$	$(4,6)$	×*									y^2				×
$\bar{F}^3$	$(6,0)$			×	×	×			×	×	×	×	×	×	×
$\bar{F}^2\phi^2$	$(6,2)$				×	×				×	×	×			×
$\bar{F}\bar{\psi}^2\phi$	$(6,2)$				×									×	×
$\bar{\psi}^4$	$(6,2)$	×	×	×	×	×	×	×			×	$\bar{y}^2$		×	×
$\bar{\psi}^2\phi^3$	$(6,4)$					$\bar{y}^2$	×*								×
$\bar{\psi}^2\psi^2$	$(4,4)$		×		$\bar{y}^2$	×		×		y^2	×			×	×
$\bar{\psi}\psi\phi^2 D$	$(4,4)$														×
$\phi^4 D^2$	$(4,4)$				×					×		×			×
ϕ^6	$(6,6)$	×*		×	×		×*		×	×		×			

+ angular momentum selection rules

[Jiang, Shu, Xiao, Zheng '20]

Non-interference

[Azatov, Contino, Machado, Riva '16]

massless tree four-point amplitudes involving transverse bosons
do not overlap in helicity at dim-4 and dim-6

A_4	$\lvert h(A_4^{\rm SM})\rvert$	$\lvert h(A_4^{\rm BSM})\rvert$
$VVVV$	0	4,2
$VV\phi\phi$	0	2
$VV\psi\psi$	0	2
$V\psi\psi\phi$	0	2
$\psi\psi\psi\psi$	2,0	2,0
$\psi\psi\phi\phi$	0	0
$\phi\phi\phi\phi$	0	0

interference mass- or loop- suppressed, recovered in the
azimuthal angle of decay products or through extra radiation

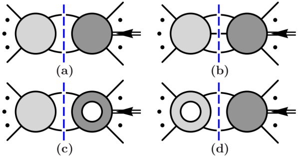
Non-renormalisation at $L > 1$

[Bern, Parra-Martinez, Sawyer '19, '20]

$$\text{length}(\mathcal{O}_i) < \text{length}(\mathcal{O}_j) - L$$

only maximal cut, between tree amplitudes, at minimal L order

	F^3	ϕ^2F^2	$F\phi\psi^2$	$D^2\phi^4$	$D\phi^2\psi^2$	ψ^4	$\phi^3\psi^2$	ϕ^6
F^3		$\times_1$	(2)	$\times_2$	$\times_2$	$\times_2$	$\times_3$	$\times_3$
ϕ^2F^2							(2)	$\times_2$
$F\phi\psi^2$							$\times_1$	$\times_3$
$D^2\phi^4$							$\times_1$	$\times_2$
$D\phi^2\psi^2$							$\times_1$	(3)
ψ^4							(2)	(4)
$\phi^3\psi^2$								(2)
ϕ^6								



Anomalous dimensions from cuts

[Caron-Huot, Wilhelm '16]

Relate dilatation operator to S-matrix phase

$$e^{-i\pi D}F^* = SF^* \qquad \text{form-factor } F \equiv \langle p_1, \dots p_n|\mathcal{O}(q)|0\rangle$$

momentum influx (q)

So at one-loop,

$$(\gamma_{ij} - \gamma_{ij}^{\text{IR}} + \beta_g \partial_g)^{(1)} \langle p1, \dots p_n|\mathcal{O}_i|0\rangle^{(0)} = -\frac{1}{\pi}\langle p_1, \dots p_n|\mathcal{M}\otimes\mathcal{O}_j|0\rangle^{(0)}$$

- γ_{ij}^{IR}: absent at first non-vanishing order
- $\beta_g \partial_g$: absent for 'minimal' form factors
- $\mathcal{M}\otimes$: two-particle phase-space integral

c. Massive amplitude construction

Massive spin spinors

[Arkani-Hamed, Huang, Huang '17]

Two massless for one massive

$$p^i_\mu \sigma^\mu_{\alpha\dot\alpha} \;=\; q^i\rangle[q^i + k^i\rangle[k^i \;=\; i^J\rangle[i_J \qquad \text{with } k_i^2 = 0 = q_i^2,\;\; J = 1,2 \quad 2k^i\cdot q^i = m_i^2$$

Little group is now SO(3) $\sim$ SU(2)

Spin s from $2s$ symmetrized spin 1/2

Bolded spinors with implicit symmetrization

e.g. $\langle 1^I 3^J\rangle\langle 2^K 3^{J'}\rangle + (J \leftrightarrow J') \to \langle \mathbf{13}\rangle\langle\mathbf{23}\rangle$

Spin quantisation axis unspecified / little-group covariance

ffs $[\mathbf{12}],\ \langle\mathbf{12}\rangle$

vvs $\langle\mathbf{12}\rangle^2,\ \langle\mathbf{12}\rangle[\mathbf{12}],\ [\mathbf{12}]^2$

ssv $[\mathbf{3}(\mathbf{1}-\mathbf{2})\mathbf{3}\rangle \equiv [\mathbf{3}(p_\mathbf{1}-p_\mathbf{2})\mathbf{3}\rangle$

ffv $\langle\mathbf{13}\rangle\langle\mathbf{23}\rangle,\ \langle\mathbf{13}\rangle[\mathbf{23}],\ [\mathbf{13}]\langle\mathbf{23}\rangle,\ [\mathbf{13}][\mathbf{23}]$

...

Massive three-points

Counting from angular momentum

number of irreps in the spin addition: [Costa, Penedones, Poland, Rychkov '11]

$$(2s_1+1)(2s_2+1)-p(p+1) \quad \text{with} \quad \begin{cases} p \equiv \max\{0, s_1+s_2-s_3\} \\ s_1 \le s_2 \le s_3 \end{cases}$$

Construction by correcting a massless-like ansatz

[GD, Kitahara, Machado, Shadmi, Weiss '20]

$$(\mathbf{12})^{s_1+s_2-\tilde{s}_3}\,(\mathbf{23})^{-s_1+s_2+\tilde{s}_3}\,(\mathbf{13})^{s_1-s_2+\tilde{s}_3}\,[\mathbf{3}(\mathbf{1}-\mathbf{2})\mathbf{3}\rangle^{s_3-\tilde{s}_3}$$

with

$$\left|\begin{array}{l} (\boldsymbol{ij})^k \equiv \text{ any } \langle \boldsymbol{ij}\rangle^{k-l}[\boldsymbol{ij}]^l \quad \text{for } l=0,\dots,k \\ s_1 \le s_2 \le s_3 \\ \tilde{s}_3 \equiv s_3 - \max\{0, s_3-s_2-s_1\} \end{array}\right.$$

removing occurrences of $\epsilon(\boldsymbol{\varepsilon}_1, \boldsymbol{\varepsilon}_2, \boldsymbol{\varepsilon}_3, \boldsymbol{p_1}+\boldsymbol{p_2}+\boldsymbol{p_3})$

$$\begin{aligned} & m_1\langle\mathbf{12}\rangle\langle\mathbf{13}\rangle[\mathbf{23}] + m_2\langle\mathbf{12}\rangle[\mathbf{13}]\langle\mathbf{23}\rangle + m_3[\mathbf{12}]\langle\mathbf{13}\rangle\langle\mathbf{23}\rangle \\ = {} & m_1[\mathbf{12}]\,[\mathbf{13}]\langle\mathbf{23}\rangle + m_2[\mathbf{12}]\langle\mathbf{13}\rangle[\mathbf{23}] + m_3\langle\mathbf{12}\rangle[\mathbf{13}]\,[\mathbf{23}] \end{aligned}$$

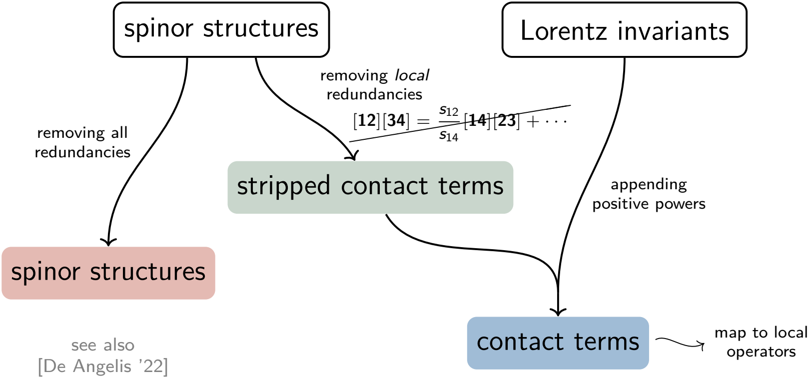
Massive three-points

s_1	s_2	s_3	$n^{3\text{-pt}}$	n_{rel}	spinor structures
0	0	0	1		constant
0	0	1	1		$[\mathbf{3}(\mathbf{1}-\mathbf{2})\mathbf{3}\rangle$
0	0	2	1		$[\mathbf{3}(\mathbf{1}-\mathbf{2})\mathbf{3}\rangle^2$
0	0	3	1		$[\mathbf{3}(\mathbf{1}-\mathbf{2})\mathbf{3}\rangle^3$
0	1/2	1/2	2		$([\mathbf{23}],\langle\mathbf{23}\rangle)$
0	1/2	3/2	2		$[\mathbf{3}(\mathbf{1}-\mathbf{2})\mathbf{3}\rangle\otimes([\mathbf{23}],\langle\mathbf{23}\rangle)$
0	1/2	5/2	2		$[\mathbf{3}(\mathbf{1}-\mathbf{2})\mathbf{3}\rangle^2\otimes([\mathbf{23}],\langle\mathbf{23}\rangle)$
0	1	1	3		$([\mathbf{23}]^2,\langle\mathbf{23}\rangle[\mathbf{23}],\langle\mathbf{23}\rangle^2)$
0	1	2	3		$[\mathbf{3}(\mathbf{1}-\mathbf{2})\mathbf{3}\rangle\otimes([\mathbf{23}]^2,\langle\mathbf{23}\rangle[\mathbf{23}],\langle\mathbf{23}\rangle^2)$
0	1	3	3		$[\mathbf{3}(\mathbf{1}-\mathbf{2})\mathbf{3}\rangle^2\otimes([\mathbf{23}]^2,\langle\mathbf{23}\rangle[\mathbf{23}],\langle\mathbf{23}\rangle^2)$
0	3/2	3/2	4		$([\mathbf{23}]^3,\langle\mathbf{23}\rangle[\mathbf{23}]^2,\langle\mathbf{23}\rangle^2[\mathbf{23}],\langle\mathbf{23}\rangle^3)$
0	3/2	5/2	4		$[\mathbf{3}(\mathbf{1}-\mathbf{2})\mathbf{3}\rangle\otimes([\mathbf{23}]^3,\langle\mathbf{23}\rangle[\mathbf{23}]^2,\langle\mathbf{23}\rangle^2[\mathbf{23}],\langle\mathbf{23}\rangle^3)$
0	2	2	5		$([\mathbf{23}]^4,\langle\mathbf{23}\rangle[\mathbf{23}]^3,\langle\mathbf{23}\rangle^2[\mathbf{23}]^2,\langle\mathbf{23}\rangle^3[\mathbf{23}],\langle\mathbf{23}\rangle^4)$
0	2	3	5		$[\mathbf{3}(\mathbf{1}-\mathbf{2})\mathbf{3}\rangle\otimes([\mathbf{23}]^4,\langle\mathbf{23}\rangle[\mathbf{23}]^3,\langle\mathbf{23}\rangle^2[\mathbf{23}]^2,\langle\mathbf{23}\rangle^3[\mathbf{23}],\langle\mathbf{23}\rangle^4)$
0	5/2	5/2	6		$([\mathbf{23}]^5,\langle\mathbf{23}\rangle[\mathbf{23}]^4,\langle\mathbf{23}\rangle^2[\mathbf{23}]^3,\langle\mathbf{23}\rangle^3[\mathbf{23}]^2,\langle\mathbf{23}\rangle^4[\mathbf{23}],\langle\mathbf{23}\rangle^5)$
0	3	3	7		$([\mathbf{23}]^6,\langle\mathbf{23}\rangle[\mathbf{23}]^5,\langle\mathbf{23}\rangle^2[\mathbf{23}]^4,\langle\mathbf{23}\rangle^3[\mathbf{23}]^3,\langle\mathbf{23}\rangle^4[\mathbf{23}]^2,\langle\mathbf{23}\rangle^5[\mathbf{23}],\langle\mathbf{23}\rangle^6)$
1/2	1/2	1	4		$([\mathbf{23}],\langle\mathbf{23}\rangle)\otimes([\mathbf{13}],\langle\mathbf{13}\rangle)$
1/2	1/2	2	4		$[\mathbf{3}(\mathbf{1}-\mathbf{2})\mathbf{3}\rangle\otimes([\mathbf{23}],\langle\mathbf{23}\rangle)\otimes([\mathbf{13}],\langle\mathbf{13}\rangle)$
1/2	1/2	3	4		$[\mathbf{3}(\mathbf{1}-\mathbf{2})\mathbf{3}\rangle^2\otimes([\mathbf{23}],\langle\mathbf{23}\rangle)\otimes([\mathbf{13}],\langle\mathbf{13}\rangle)$
1/2	1	3/2	6		$([\mathbf{23}]^2,\langle\mathbf{23}\rangle[\mathbf{23}],\langle\mathbf{23}\rangle^2)\otimes([\mathbf{13}],\langle\mathbf{13}\rangle)$
1/2	1	5/2	6		$[\mathbf{3}(\mathbf{1}-\mathbf{2})\mathbf{3}\rangle\otimes([\mathbf{23}]^2,\langle\mathbf{23}\rangle[\mathbf{23}],\langle\mathbf{23}\rangle^2)\otimes([\mathbf{13}],\langle\mathbf{13}\rangle)$
1/2	3/2	2	8		$([\mathbf{23}]^3,\langle\mathbf{23}\rangle[\mathbf{23}]^2,\langle\mathbf{23}\rangle^2[\mathbf{23}],\langle\mathbf{23}\rangle^3)\otimes([\mathbf{13}],\langle\mathbf{13}\rangle)$
1/2	3/2	3	8		$[\mathbf{3}(\mathbf{1}-\mathbf{2})\mathbf{3}\rangle\otimes([\mathbf{23}]^3,\langle\mathbf{23}\rangle[\mathbf{23}]^2,\langle\mathbf{23}\rangle^2[\mathbf{23}],\langle\mathbf{23}\rangle^3)\otimes([\mathbf{13}],\langle\mathbf{13}\rangle)$
1/2	2	5/2	10		$([\mathbf{23}]^4,\langle\mathbf{23}\rangle[\mathbf{23}]^3,\langle\mathbf{23}\rangle^2[\mathbf{23}]^2,\langle\mathbf{23}\rangle^3[\mathbf{23}],\langle\mathbf{23}\rangle^4)\otimes([\mathbf{13}],\langle\mathbf{13}\rangle)$
1/2	5/2	3	12		$([\mathbf{23}]^5,\langle\mathbf{23}\rangle[\mathbf{23}]^4,\langle\mathbf{23}\rangle^2[\mathbf{23}]^3,\langle\mathbf{23}\rangle^3[\mathbf{23}]^2,\langle\mathbf{23}\rangle^4[\mathbf{23}],\langle\mathbf{23}\rangle^5)\otimes([\mathbf{13}],\langle\mathbf{13}\rangle)$
1	1	1	7	1	$([\mathbf{12}],\langle\mathbf{12}\rangle)\otimes([\mathbf{23}],\langle\mathbf{23}\rangle)\otimes([\mathbf{13}],\langle\mathbf{13}\rangle)$
1	1	2	9		$([\mathbf{23}]^2,\langle\mathbf{23}\rangle[\mathbf{23}],\langle\mathbf{23}\rangle^2)\otimes([\mathbf{13}]^2,\langle\mathbf{13}\rangle[\mathbf{13}],\langle\mathbf{13}\rangle^2)$
1	1	3	9		$[\mathbf{3}(\mathbf{1}-\mathbf{2})\mathbf{3}\rangle\otimes([\mathbf{23}]^2,\langle\mathbf{23}\rangle[\mathbf{23}],\langle\mathbf{23}\rangle^2)\otimes([\mathbf{13}]^2,\langle\mathbf{13}\rangle[\mathbf{13}],\langle\mathbf{13}\rangle^2)$
1	3/2	3/2	10	2	$([\mathbf{12}],\langle\mathbf{12}\rangle)\otimes([\mathbf{23}]^2,\langle\mathbf{23}\rangle[\mathbf{23}],\langle\mathbf{23}\rangle^2)\otimes([\mathbf{13}],\langle\mathbf{13}\rangle)$
1	3/2	5/2	12		$([\mathbf{23}]^3,\langle\mathbf{23}\rangle[\mathbf{23}]^2,\langle\mathbf{23}\rangle^2[\mathbf{23}],\langle\mathbf{23}\rangle^3)\otimes([\mathbf{13}]^2,\langle\mathbf{13}\rangle[\mathbf{13}],\langle\mathbf{13}\rangle^2)$
1	2	2	13	3	$([\mathbf{12}],\langle\mathbf{12}\rangle)\otimes([\mathbf{23}]^3,\langle\mathbf{23}\rangle[\mathbf{23}]^2,\langle\mathbf{23}\rangle^2[\mathbf{23}],\langle\mathbf{23}\rangle^3)\otimes([\mathbf{13}],\langle\mathbf{13}\rangle)$
1	2	3	15		$([\mathbf{23}]^4,\langle\mathbf{23}\rangle[\mathbf{23}]^3,\langle\mathbf{23}\rangle^2[\mathbf{23}]^2,\langle\mathbf{23}\rangle^3[\mathbf{23}],\langle\mathbf{23}\rangle^4)\otimes([\mathbf{13}]^2,\langle\mathbf{13}\rangle[\mathbf{13}],\langle\mathbf{13}\rangle^2)$
1	5/2	5/2	16	4	$([\mathbf{12}],\langle\mathbf{12}\rangle)\otimes([\mathbf{23}]^4,\langle\mathbf{23}\rangle[\mathbf{23}]^3,\langle\mathbf{23}\rangle^2[\mathbf{23}]^2,\langle\mathbf{23}\rangle^3[\mathbf{23}],\langle\mathbf{23}\rangle^4)\otimes([\mathbf{13}],\langle\mathbf{13}\rangle)$
1	3	3	19	5	$([\mathbf{12}],\langle\mathbf{12}\rangle)\otimes([\mathbf{23}]^5,\langle\mathbf{23}\rangle[\mathbf{23}]^4,\langle\mathbf{23}\rangle^2[\mathbf{23}]^3,\langle\mathbf{23}\rangle^3[\mathbf{23}]^2,\langle\mathbf{23}\rangle^4[\mathbf{23}],\langle\mathbf{23}\rangle^5)\otimes([\mathbf{13}],\langle\mathbf{13}\rangle)$
3/2	3/2	2	14	4	$([\mathbf{12}],\langle\mathbf{12}\rangle)\otimes([\mathbf{23}]^2,\langle\mathbf{23}\rangle[\mathbf{23}],\langle\mathbf{23}\rangle^2)\otimes([\mathbf{13}]^2,\langle\mathbf{13}\rangle[\mathbf{13}],\langle\mathbf{13}\rangle^2)$
3/2	3/2	3	16		$([\mathbf{23}]^3,\langle\mathbf{23}\rangle[\mathbf{23}]^2,\langle\mathbf{23}\rangle^2[\mathbf{23}],\langle\mathbf{23}\rangle^3)\otimes([\mathbf{13}]^3,\langle\mathbf{13}\rangle[\mathbf{13}]^2,\langle\mathbf{13}\rangle^2[\mathbf{13}],\langle\mathbf{13}\rangle^3)$
3/2	2	5/2	18	6	$([\mathbf{12}],\langle\mathbf{12}\rangle)\otimes([\mathbf{23}]^3,\langle\mathbf{23}\rangle[\mathbf{23}]^2,\langle\mathbf{23}\rangle^2[\mathbf{23}],\langle\mathbf{23}\rangle^3)\otimes([\mathbf{13}]^2,\langle\mathbf{13}\rangle[\mathbf{13}],\langle\mathbf{13}\rangle^2)$
3/2	5/2	3	22	8	$([\mathbf{12}],\langle\mathbf{12}\rangle)\otimes([\mathbf{23}]^4,\langle\mathbf{23}\rangle[\mathbf{23}]^3,\langle\mathbf{23}\rangle^2[\mathbf{23}]^2,\langle\mathbf{23}\rangle^3[\mathbf{23}],\langle\mathbf{23}\rangle^4)\otimes([\mathbf{13}]^2,\langle\mathbf{13}\rangle[\mathbf{13}],\langle\mathbf{13}\rangle^2)$
2	2	2	19	8	$([\mathbf{12}]^2,\langle\mathbf{12}\rangle[\mathbf{12}],\langle\mathbf{12}\rangle^2)\otimes([\mathbf{23}]^2,\langle\mathbf{23}\rangle[\mathbf{23}],\langle\mathbf{23}\rangle^2)\otimes([\mathbf{13}]^2,\langle\mathbf{13}\rangle[\mathbf{13}],\langle\mathbf{13}\rangle^2)$
2	2	3	23	9	$([\mathbf{12}],\langle\mathbf{12}\rangle)\otimes([\mathbf{23}]^3,\langle\mathbf{23}\rangle[\mathbf{23}]^2,\langle\mathbf{23}\rangle^2[\mathbf{23}],\langle\mathbf{23}\rangle^3)\otimes([\mathbf{13}]^3,\langle\mathbf{13}\rangle[\mathbf{13}]^2,\langle\mathbf{13}\rangle^2[\mathbf{13}],\langle\mathbf{13}\rangle^3)$
2	5/2	5/2	24	12	$([\mathbf{12}]^2,\langle\mathbf{12}\rangle[\mathbf{12}],\langle\mathbf{12}\rangle^2)\otimes([\mathbf{23}]^3,\langle\mathbf{23}\rangle[\mathbf{23}]^2,\langle\mathbf{23}\rangle^2[\mathbf{23}],\langle\mathbf{23}\rangle^3)\otimes([\mathbf{13}]^2,\langle\mathbf{13}\rangle[\mathbf{13}],\langle\mathbf{13}\rangle^2)$
2	3	3	29	16	$([\mathbf{12}]^2,\langle\mathbf{12}\rangle[\mathbf{12}],\langle\mathbf{12}\rangle^2)\otimes([\mathbf{23}]^4,\langle\mathbf{23}\rangle[\mathbf{23}]^3,\langle\mathbf{23}\rangle^2[\mathbf{23}]^2,\langle\mathbf{23}\rangle^3[\mathbf{23}],\langle\mathbf{23}\rangle^4)\otimes([\mathbf{13}]^2,\langle\mathbf{13}\rangle[\mathbf{13}],\langle\mathbf{13}\rangle^2)$
5/2	5/2	3	30	18	$([\mathbf{12}]^2,\langle\mathbf{12}\rangle[\mathbf{12}],\langle\mathbf{12}\rangle^2)\otimes([\mathbf{23}]^3,\langle\mathbf{23}\rangle[\mathbf{23}]^2,\langle\mathbf{23}\rangle^2[\mathbf{23}],\langle\mathbf{23}\rangle^3)\otimes([\mathbf{13}]^3,\langle\mathbf{13}\rangle[\mathbf{13}]^2,\langle\mathbf{13}\rangle^2[\mathbf{13}],\langle\mathbf{13}\rangle^3)$
3	3	3	37	27	$([\mathbf{12}]^3,\langle\mathbf{12}\rangle[\mathbf{12}]^2,\langle\mathbf{12}\rangle^2[\mathbf{12}],\langle\mathbf{12}\rangle^3)\otimes([\mathbf{23}]^3,\langle\mathbf{23}\rangle[\mathbf{23}]^2,\langle\mathbf{23}\rangle^2[\mathbf{23}],\langle\mathbf{23}\rangle^3)\otimes([\mathbf{13}]^3,\langle\mathbf{13}\rangle[\mathbf{13}]^2,\langle\mathbf{13}\rangle^2[\mathbf{13}],\langle\mathbf{13}\rangle^3)$

Massive higher-points

[GD, Kitahara, Machado, Shadmi, Weiss '20]

e.g. $W^+W^+W^-W^-$: $\dfrac{[\mathbf{13}][\mathbf{24}]\langle\mathbf{13}\rangle\langle\mathbf{24}\rangle - [\mathbf{14}][\mathbf{23}]\langle\mathbf{14}\rangle\langle\mathbf{23}\rangle}{m_1 m_2 m_3 m_4}$ $(\tilde{s}_{13} - \tilde{s}_{14} - \tilde{s}_{23} + \tilde{s}_{24})$



see also
[De Angelis '22]
[Dong, Ma, Shu, Zheng '22]

and stripped contact term enumeration
from Hilbert series *secondaries*
[Chang, Chen, Liu, Luty, to appear]

→ 4-points & spins≤1

Massive high

[GD, Kit... hado, Shadmi, Weiss '20]

e.g. $W^+W^+W^-W$ ... $\tilde{s}_{23} + \tilde{s}_{24})$

spins	$n_{\rm SCT}$	n_s	hel. cat.	spinor structures	$n_{\rm perm}$	$\min\{d_{\rm op}\}$
$ssss$	1	1	(0000)	constant	1	4
$vsss$	$4\to3$	3	(0000)	$[\mathbf{121}\rangle, [\mathbf{131}\rangle$	1	5
			(+000)	$[\mathbf{1231}] \to [\mathbf{1231}] - \langle\mathbf{1231}\rangle$	$\not{2}\to1$	7
$ffss$	4	4	(++00)	$[\mathbf{12}]$	2	5
			(+−00)	$[\mathbf{132}\rangle$	2	6
$vvss$	$10\to9$	9	(0000)	$[\mathbf{12}]\langle\mathbf{12}\rangle, [\mathbf{131}\rangle[\mathbf{232}\rangle$	1	4,6
			(+000)	$[\mathbf{12}][\mathbf{132}\rangle$	4	6
			(++00)	$[\mathbf{12}]^2$	2	6
			(+−00)	$[\mathbf{132}\rangle^2 \to [\mathbf{132}\rangle^2 - \langle\mathbf{132}]^2$	$\not{2}\to1$	8
$ffvs$	$14\to12$	12	(++00)	$[\mathbf{12}]\{[\mathbf{313}\rangle, [\mathbf{323}\rangle\}$	2	6
			(+−00)	$[\mathbf{13}]\langle\mathbf{23}\rangle$	2	5
			(+++0)	$[\mathbf{13}][\mathbf{23}]$	2	6
			(++−0)	$[\mathbf{12}]\langle\mathbf{3123}\rangle \to \varnothing$	$\not{2}\to0$	8
			(+−+0)	$[\mathbf{13}][\mathbf{312}\rangle$	4	7
$ffff$	18	16	(++++)	$[\mathbf{12}][\mathbf{34}], [\mathbf{13}][\mathbf{24}]$	2	6
			(++−−)	$[\mathbf{12}]\langle\mathbf{34}\rangle$	6	6
			(+++−)	$[\mathbf{12}][\mathbf{324}\rangle$	8	7
$vvvs$	$35\to27$	27	(0000)	$[\mathbf{12}][\mathbf{343}\rangle\langle\mathbf{12}\rangle, [\mathbf{13}][\mathbf{242}\rangle\langle\mathbf{13}\rangle, [\mathbf{23}][\mathbf{141}\rangle\langle\mathbf{23}\rangle$	1	5
			(+000)	$[\mathbf{12}][\mathbf{13}]\langle\mathbf{23}\rangle$	6	5
			(++00)	$[\mathbf{12}]^2\{[\mathbf{313}\rangle, [\mathbf{323}\rangle\}$	6	7
			(+−00)	$[\mathbf{13}][\mathbf{132}\rangle\langle\mathbf{23}\rangle$	$6\to4$	7
			(+++0)	$[\mathbf{12}][\mathbf{13}][\mathbf{23}]$	2	7
			(++−0)	$[\mathbf{12}]^2\langle\mathbf{3123}\rangle \to \varnothing$	$\not{6}\to0$	9
$vvff$	$46\to38$	36	(00++)	$\langle\mathbf{12}\rangle \times \{[\mathbf{12}][\mathbf{34}], [\mathbf{13}][\mathbf{24}]\}$	2	5
			(00+−)	$\langle\mathbf{14}\rangle\langle\mathbf{231}][\mathbf{23}], \langle\mathbf{24}\rangle\langle\mathbf{132}][\mathbf{13}]$	2	6
			(0−++)	$\langle\mathbf{12}\rangle[\mathbf{34}]\langle\mathbf{241}] \to \langle\mathbf{12}\rangle[\mathbf{34}](\langle\mathbf{241}]/m_1 - \langle\mathbf{142}]/m_2)$	$\not{4}\to2$	7
			(0+++)	$\langle\mathbf{132}] \times \{[\mathbf{12}][\mathbf{34}], [\mathbf{13}][\mathbf{24}]\}$	4	7
			(0++−)	$\langle\mathbf{14}\rangle[\mathbf{12}][\mathbf{23}]$	8	6
			(+++−)	$[\mathbf{12}]^2[\mathbf{314}\rangle$	4	8
			(++++)	$[\mathbf{12}] \times \{[\mathbf{12}][\mathbf{34}], [\mathbf{13}][\mathbf{24}]\}$	2	7
			(−+++)	$\langle\mathbf{1231}\rangle[\mathbf{23}][\mathbf{24}] \to \varnothing$	$\not{4}\to0$	9
			(++−−)	$[\mathbf{12}]^2\langle\mathbf{34}\rangle$	2	7
			(+−−+)	$[\mathbf{14}][\mathbf{132}\rangle\langle\mathbf{23}\rangle \to [\mathbf{14}][\mathbf{132}\rangle\langle\mathbf{23}\rangle - [\mathbf{24}][\mathbf{231}\rangle\langle\mathbf{13}\rangle$	$\not{4}\to2$	8
$vvvv$	$116\to85$	81	(0000)	$\{[\mathbf{12}][\mathbf{34}], [\mathbf{13}][\mathbf{24}]\} \times \{\langle\mathbf{12}\rangle\langle\mathbf{34}\rangle, \langle\mathbf{13}\rangle\langle\mathbf{24}\rangle\}$	1	4
			(+000)	$\{[\mathbf{12}][\mathbf{34}], [\mathbf{13}][\mathbf{24}]\} \times [\mathbf{142}\rangle\langle\mathbf{34}\rangle \to \cdots$	$\not{8}\to6$	6
			(++00)	$\{[\mathbf{12}][\mathbf{34}], [\mathbf{13}][\mathbf{24}]\} \times [\mathbf{12}]\langle\mathbf{34}\rangle$	12	6
			(+−00)	$[\mathbf{13}][\mathbf{14}]\langle\mathbf{23}\rangle\langle\mathbf{24}\rangle$	12	6
			(+++0)	$\{[\mathbf{12}][\mathbf{34}], [\mathbf{13}][\mathbf{24}]\} \times [\mathbf{23}][\mathbf{134}\rangle$	8	8
			(++−0)	$[\mathbf{12}]^2\langle\mathbf{34}\rangle\langle\mathbf{324}] \to [\mathbf{12}]^2\langle\mathbf{34}\rangle(\langle\mathbf{324}]/m_4 - \langle\mathbf{423}]/m_3) \to \cdots$	$\not{24}\to\not{12}\to5$	8
			(++++)	$[\mathbf{12}]^2[\mathbf{34}]^2, [\mathbf{12}][\mathbf{13}][\mathbf{24}][\mathbf{34}], [\mathbf{13}]^2[\mathbf{24}]^2$	2	8
			(+++−)	$[\mathbf{12}][\mathbf{13}][\mathbf{23}]\langle\mathbf{4124}\rangle \to \varnothing$	$\not{8}\to0$	10
			(++−−)	$[\mathbf{12}]^2\langle\mathbf{34}\rangle^2$	6	8

invariants

removing all redundancies

appending positive powers

spinor

ms → map to local operators

pins≤1

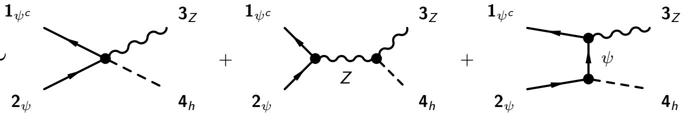
see also
[De Angelis '22]
[Dong, Ma,

and stripped cont
from Hilbert
[Chang, Chen,

EW symmetry from perturbative unitarity

[GD, Kitahara, Shadmi, Weiss '19]

$\psi^c \psi\, Z\, h \sim$



contact terms — factorisable s, t, u channels

$$\begin{matrix}[\mathbf{13}][\mathbf{23}]\\ [\mathbf{13}]\langle\mathbf{23}\rangle\\ [\mathbf{123}\rangle\langle\mathbf{23}\rangle\\ \cdots\end{matrix} \times f(s_{ij})$$

$$\begin{matrix}[\mathbf{1P}][\mathbf{2P}]\\ [\mathbf{1P}]\langle\mathbf{2P}\rangle\\ \langle\mathbf{1P}\rangle[\mathbf{2P}]\\ \langle\mathbf{1P}\rangle\langle\mathbf{2P}\rangle\end{matrix} \otimes \begin{matrix}[\mathbf{P3}]^2\\ [\mathbf{P3}]\langle\mathbf{P3}\rangle\\ \langle\mathbf{P3}\rangle^2\end{matrix} \qquad \begin{matrix}[\mathbf{13}][\mathbf{P3}]\\ [\mathbf{13}]\langle\mathbf{P3}\rangle\\ \langle\mathbf{13}\rangle[\mathbf{P3}]\\ \langle\mathbf{13}\rangle\langle\mathbf{P3}\rangle\end{matrix} \otimes \begin{matrix}[\mathbf{P2}]\\ \langle\mathbf{P2}\rangle\end{matrix}$$

$$\xrightarrow[\text{energy}]{\text{high}} \begin{cases} \dfrac{[12]}{m_Z}\left(c^{\text{left}}_{\psi^c\psi Z} - c^{\text{right}}_{\psi^c\psi Z}\right)\left(c^{\text{right}}_{\psi^c\psi h} - c_{ZZh}\dfrac{m_\psi}{2m_Z}\right) \\ \dfrac{\langle 12\rangle}{m_Z}\left(c^{\text{left}}_{\psi\psi Z} - c^{\text{right}}_{\psi^c\psi Z}\right)\left(c^{\text{left}}_{\psi^c\psi h} - c_{ZZh}\dfrac{m_\psi}{2m_Z}\right) \end{cases}$$

as for the SM in the '70

[Llewellyn-Smith '73]
[Joglekar '73]
[Conwall et al. '73, '74]

Massive → massless

high-energy limit / unbolding

· choice for the decomposition $p^\mu = (E, p\,\hat{n}) = k^\mu + q^\mu$:

$$k^\mu = \frac{E+p}{2}(1, +\hat{n}), \qquad q^\mu = \frac{E-p}{2}(1, -\hat{n})$$

$\rightarrow$ spin quantization axis $k^\mu - q^\mu \sim (1, \hat{n}) \rightsquigarrow$ helicity

$\rightarrow$ $k], k\rangle \sim \sqrt{E}$ and $q], q\rangle \sim m/\sqrt{E}$

· massless limit: *un-bolding* $+\mathcal{O}(m)$

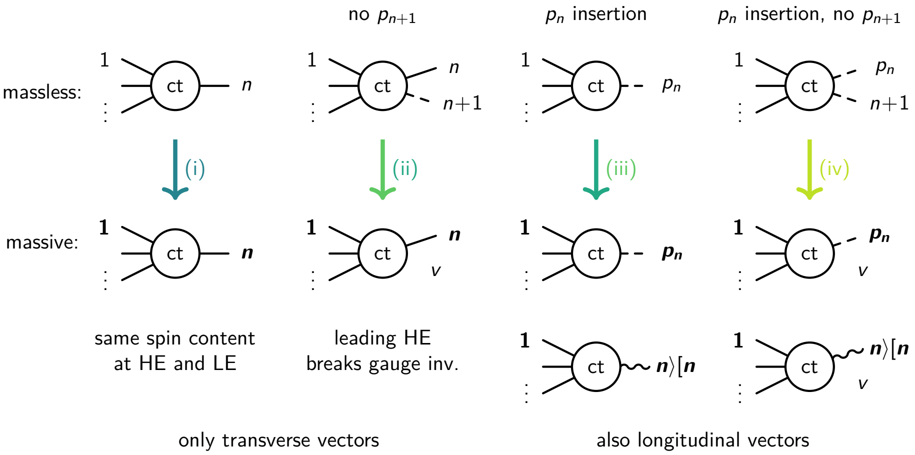
e.g. $\langle\mathbf{13}\rangle\langle\mathbf{23}\rangle \rightarrow \langle 13\rangle\langle 23\rangle + \mathcal{O}(m)$

$\langle\mathbf{13}\rangle[\mathbf{23}] \rightarrow \cancel{\langle 1 p_3 2]} + \mathcal{O}(m)$

Massive ← massless

[Balkin, GD, Kitahara, Shadmi, Weiss ’21]

little-group covariantisation from the leading term / bolding



d. EFT double copy

EFT double copy

[Bonnefoy, GD, Grojean, Machado, Roosmale-Nepveu '21]

×	BAS	NLSM	YM
$\widetilde{\text{BAS}}$	BAS	NLSM	YM
$\widetilde{\text{NLSM}}$		sGal	BI
$\widetilde{\text{YM}}$			GR

EFTs allowed as inputs?

EFTs obtained as outputs?

Colour-Kinematics (CK)

$c^{\text{adj}} \cdot P \cdot n^{\text{adj}} \longrightarrow \tilde{n}^{\text{adj}} \cdot P \cdot n^{\text{adj}}$

composition rules for $c^{\text{adj}}_{\text{hd}}(\text{col}, \text{kin})$

[Carrasco, Rodina, Yin, Zekioglu '19, '21]

Kawai-Lewellen-Tye (KLT)

$c^{\text{tr}} \cdot A \longrightarrow \tilde{A} \otimes A$

bootstrap equations for $\otimes_{\text{hd}}$, A_{hd}

[Chi, Elvang, Herderschee, Jones, Paranjape '21]

Numerator seeds

simpler building blocks for $c^{\text{adj}}_{\text{hd}}(\text{col}, \text{kin})$

relation to $\otimes_{\text{hd}}$, A_{hd}

The effective track to new physics

SMEFT could identify small correlated deviations
in an array of observables.

A global approach preserves the systematic theory-space coverage
and attacks new-physics from all fronts.

Precise EFT predictions yield new sensitivities
and sharpen potential new-physics patterns.

Model interpretations locate landmarks
in the explored territory.

Amplitude techniques contribute to
improving the theory understanding of EFTs.