CLASSICAL BLACK HOLE SCATTERING FROM A WORLDLINE QFT

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Based on joint work with

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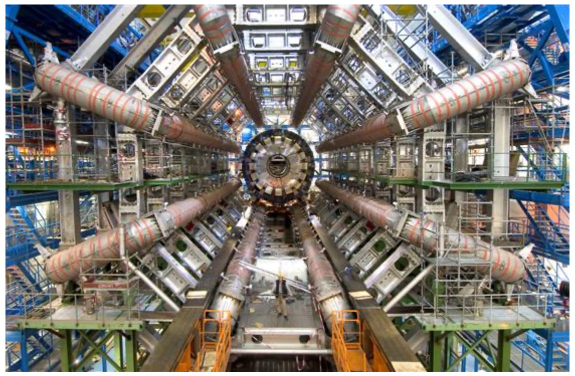
2010:02865, *JHEP* 02 (2021) 048
2101.12688, *PRL* 126 (2021) 20
2106.10256, *PRL* 128 (2022) 1
2109.04465, *JHEP 01 (2022) 027*
* 2201.07778, *PRL* 128 *(2022) 14*
2207.00569

RTG 2575: Rethinking Quantum Field Theory

KIT Particle Physics Colloquium, 07/22

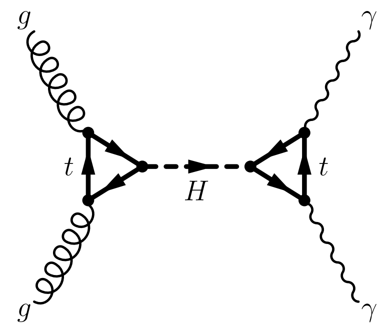
Particle Physics: Paradigmatic experiment is Scattering in Colliders

Theory: Relativistic Quantum Field Theory

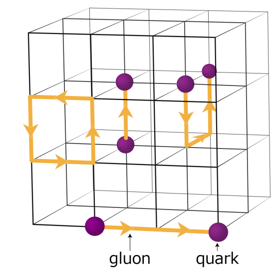


$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i\bar{\psi} \not{D} \psi + \psi_i Y_{ij} \psi_j \, \phi + |D_\mu \phi|^2 - \lambda |\phi|^4 - m^2 \, |\phi|^2$$

path integral quantization

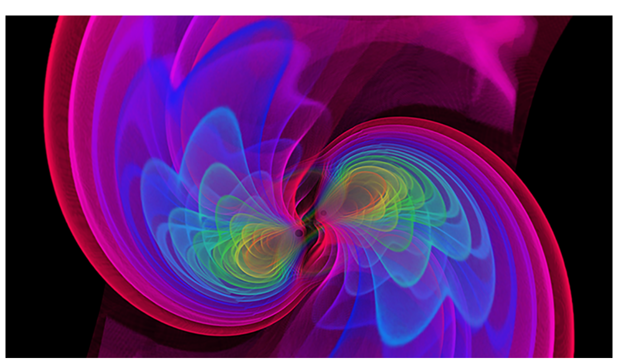


Perturbative QFT: S-matrix



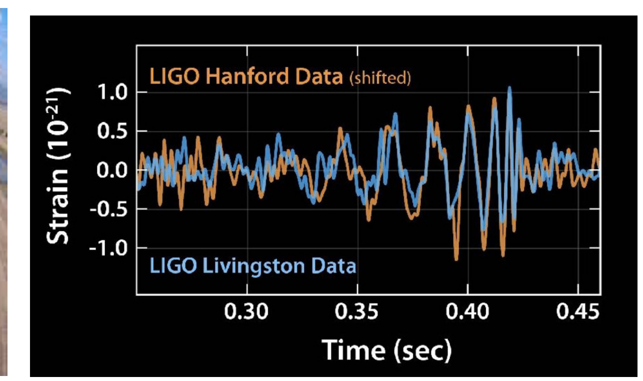
Lattice Field Thy: Bound system

Gravity: Gravitational wave emission in Black Hole and Neutron Star encounters now routinely measured in LIGO-Virgo-Karga GW detectors



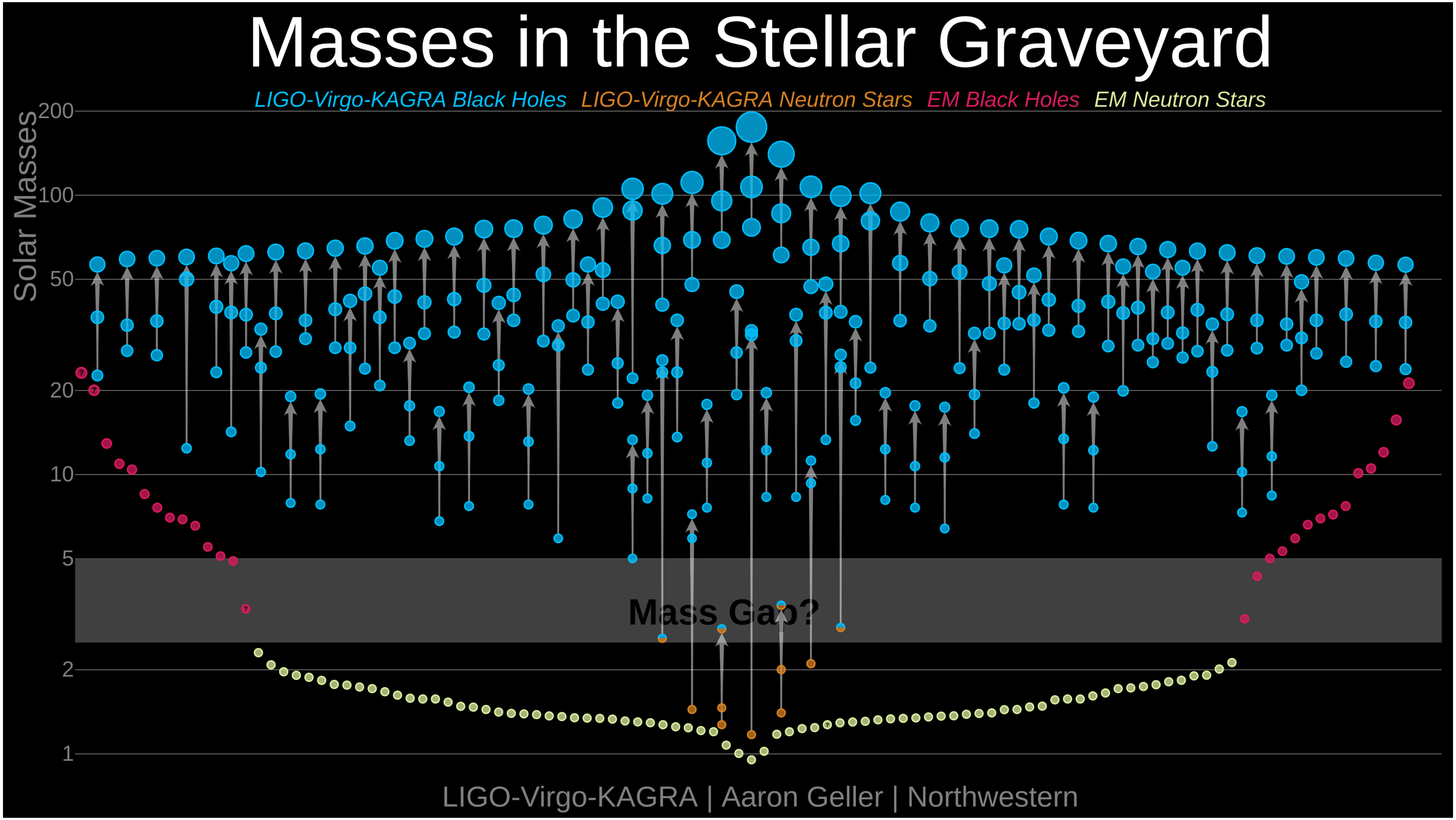
Classical radiative field theory

$$\mathcal{L} = \frac{1}{16\pi G} \sqrt{-g} \, R + \mathcal{L}_{\text{Matter}}$$



Theory: Needs perturbative Solution of classical gravitational two-body problem: Apply perturbative QFT techniques!

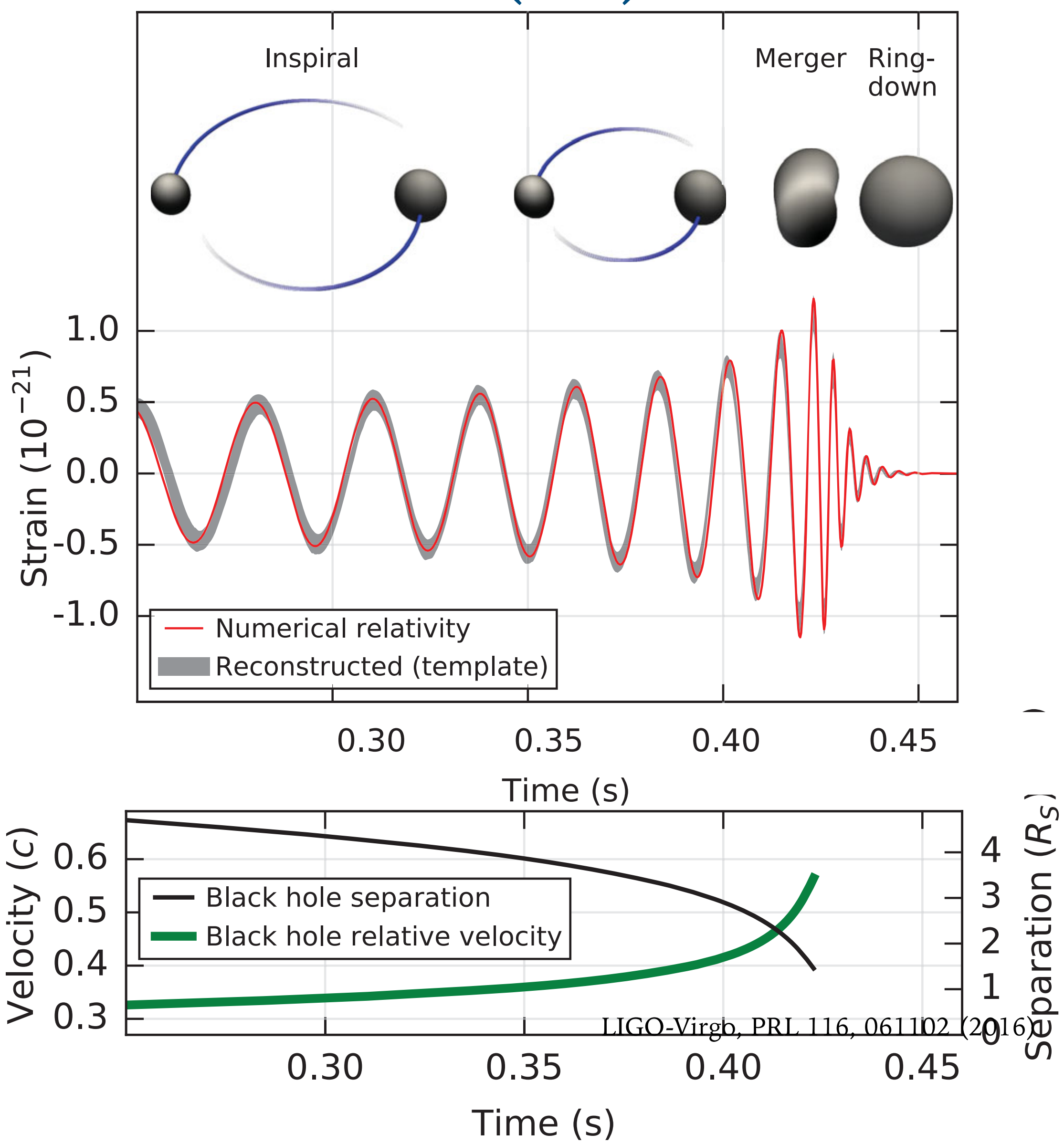
GRAVITATIONAL WAVES: A NEW OBSERVATIONAL ERA



Following GW150914: To date 90 binary mergers detected by LIGO-Virgo-Karga Collaboration

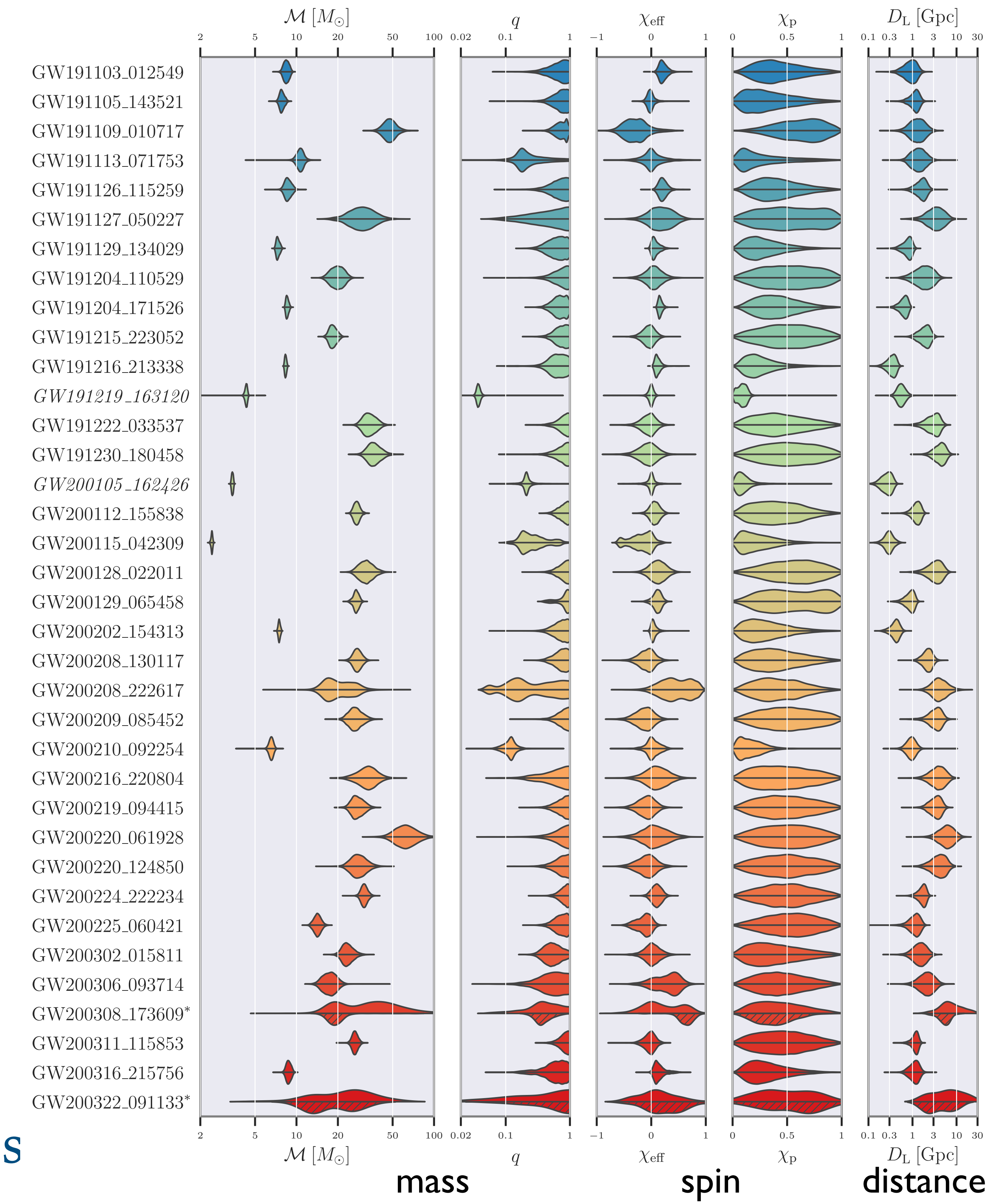
GRAVITATIONAL WAVES: A NEW OBSERVATIONAL ERA

Binary mergers of black holes (BHs) and neutron stars (NS)



Measurement of binary parameters

Masses, Spins, Distance



LVG collaboration arXiv:2111.03606

PHYSICS CASES

AEI

- 3rd generation of GW observatories (Einstein Telescope; Advanced LIGO, LISA) to start in 2030's.
- Highly increased sensitivity expected: Need for high precision theory predictions

Astrophysics:

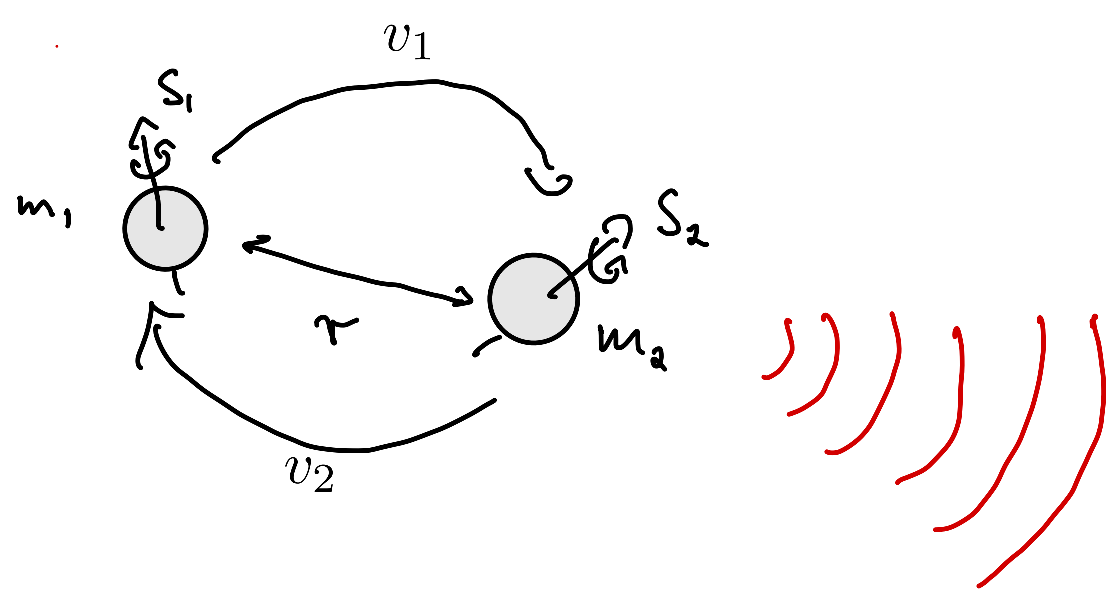
- Black hole formation & evolution
- Neutron star properties: Equation of state, strong interacting matter
- Multi-messenger astronomy
- New astrophysical sources of GW

Fundamental physics:

- Precision tests of (strong field) GR
- New physics signals? Modifications of GR, Higher curvature terms, Dark Matter…

THE GENERAL RELATIVISTIC 2-BODY PROBLEM

As in Newtonian case has either **bound** or **unbound** orbits.



Inspiral of 2 BHs or NSs:

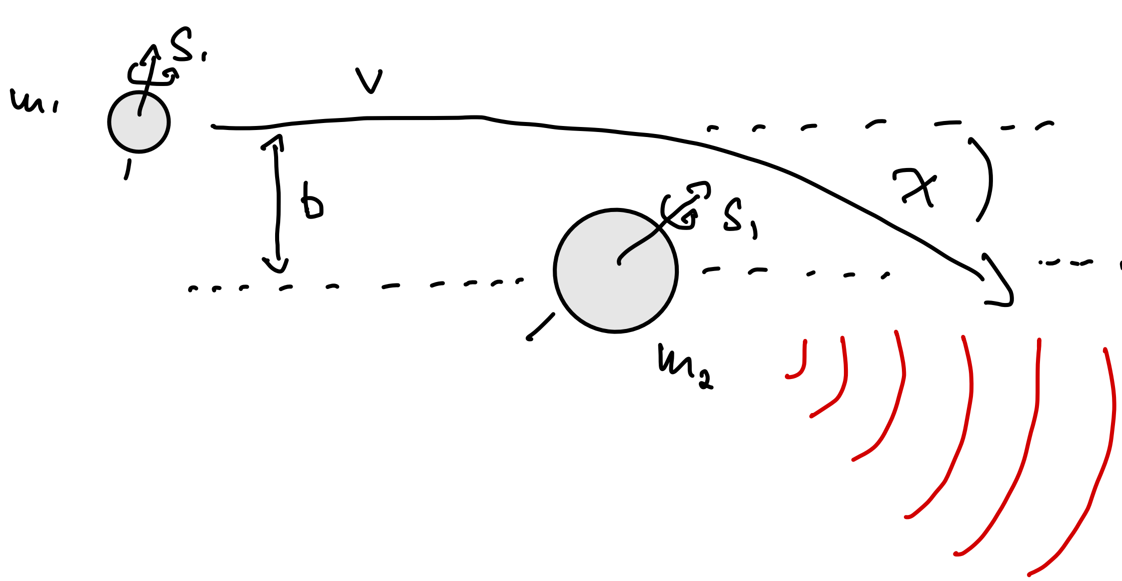
Virial-thm: $\frac{GM}{r} \sim v^2$

post-Newtonian (PN) expansion:

Weak field expansion: $g_{\mu\nu} = \eta_{\mu\nu} + \kappa\, h_{\mu\nu}$

$\kappa = \sqrt{32\pi G}$

Newton's constant



Scattering of 2 BHs or NSs:

Weak field (G), exact in v

post-Newtonian (PM) expansion

THE POST NEWTONIAN EXPANSION

Effective (conservative) action known up to 5PN order:

1PN: [Newton (1687)]

$$S=\sum_i \int dt\Big[-m_i+\frac{1}{c^2}\Big(\frac{m_i\mathbf{v}_i^2}{2}+\sum_{j\neq i}\frac{Gm_im_j}{2r_{ij}}\Big)$$

$$+\frac{1}{c^4}\Big(\frac{m_i\mathbf{v}_i^4}{8}+\sum_{j\neq i}\frac{Gm_im_j}{4r_{ij}}\left(6\,\mathbf{v}_i^2-(\mathbf{n}_{ij}\cdot\mathbf{v}_i)(\mathbf{n}_{ij}\cdot\mathbf{v}_j)\;-7\,\mathbf{v}_i\cdot\mathbf{v}_j\right)-\sum_{j\neq i}\sum_{k\neq i}\frac{G^2m_im_jm_k}{2r_{ij}r_{ik}}\Big)\Big]$$

2PN: [Einstein,Infeld, Hofmann(1938)]

$$+\quad\sum_i\int\frac{\mathrm{d}t}{c^6}\Big\{\frac{m_i\mathbf{v}_i^6}{16}+\sum_{j\neq i}\frac{Gm_im_j}{16r_{ij}}\Big[3(\mathbf{n}_{ij}\cdot\mathbf{v}_i)^2(\mathbf{n}_{ij}\cdot\mathbf{v}_j)^2-6\,\mathbf{n}_{ij}\cdot\mathbf{v}_i\,\mathbf{n}_{ij}\cdot\mathbf{v}_j\,\mathbf{v}_{ij}^2-2\,(\mathbf{n}_{ij}\cdot\mathbf{v}_j)^2\,\mathbf{v}_i^2$$

$$+3\,\mathbf{v}_i^2\,\mathbf{v}_j^2+2\,(\mathbf{v}_i\cdot\mathbf{v}_j)^2-20\,\mathbf{v}_i^2\,\mathbf{v}_i\cdot\mathbf{v}_j+14\,\mathbf{v}_i^4\Big]+\sum_{j\neq i}\frac{G^2m_im_j^2}{2r_{ij}^2}\left[33\,(\mathbf{n}_{ij}\cdot\mathbf{v}_{ij})^2-17\mathbf{v}_{ij}^2\right]$$

$$+\sum_{j\neq i}\sum_{k\neq i}\frac{G^2m_im_jm_k}{8}\Big[\frac{1}{r_{ij}r_{ik}}\left(4(\mathbf{n}_{ij}\cdot\mathbf{v}_j)^2+18\,\mathbf{v}_i^2-16\,\mathbf{v}_j^2-32\,\mathbf{v}_i\cdot\mathbf{v}_j+32\,\mathbf{v}_j\cdot\mathbf{v}_k\right)$$

$$+\frac{1}{r_{ij}^2}\left(14\,\mathbf{n}_{ik}\cdot\mathbf{v}_k\,\mathbf{n}_{ij}\cdot\mathbf{v}_k-12\,\mathbf{n}_{ij}\cdot\mathbf{v}_i\,\mathbf{n}_{ik}\cdot\mathbf{v}_k+\mathbf{n}_{ij}\cdot\mathbf{n}_{ik}\,(\mathbf{n}_{ik}\cdot\mathbf{v}_k)^2-\mathbf{n}_{ij}\cdot\mathbf{n}_{ik}\,\mathbf{v}_k^2\right)\Big]$$

3PN: [Ohta,Okamura,Hiida, Kimura (1974)]

$$+\sum_{j\neq i}\sum_{k\neq i,j}G^2m_im_jm_k\Big[\frac{2(\mathbf{n}_{ij}-\mathbf{n}_{jk})\cdot\mathbf{v}_{ij}}{(r_{ij}+r_{ik}+r_{jk})^2}\left(4\,(\mathbf{n}_{ij}+\mathbf{n}_{ik})\cdot\mathbf{v}_{ij}+(\mathbf{n}_{ik}+\mathbf{n}_{jk})\cdot\mathbf{v}_{ik}\right)$$

$$+\frac{9\,(\mathbf{n}_{ij}\cdot\mathbf{v}_{ij})^2-9\,\mathbf{v}_{ij}^2+2\,(\mathbf{n}_{ij}\cdot\mathbf{v}_{ik})^2-2\,\mathbf{v}_{ik}^2}{r_{ij}\,(r_{ij}+r_{ik}+r_{jk})}\Big]\Big\}+G^3\times[\text{static term}]\,,\;+\dots$$

4PN: [Damour, Jaranowski,Schaefer (2016); Blanchet,Bohe,Faye (2015)]

5PN: [Bini,Damour,Geralico (2019); Foffa (2017);Porto, Rothstein, Sturani (2019)]

Partial results at 6PN…

POST-NEWTONIAN VS POST-MINKOWSKIAN EXPANSIONS

Conservative non-spinning 2-body dynamics:

		0PN	1PN	2PN	3PN	4PN	5PN		Integration complexity
0PM [Einstein]	1	v^2	v^4	v^6	v^8	v^{10}	v^{12}	…	
1PM [Westpfahl]		G/r [Newton]	$G\ v^2/r$ [EIH]	$G\ v^4/r$	$G\ v^6/r$	$G\ v^8/r$	$G\ v^{10}/r$	…	~ tree-level
2PM [many]			$G^2\ 1/r^2$	$G^2\ v^2/r^2$	$G^2\ v^4/r^2$	$G^2\ v^6/r^2$	$G^2\ v^8/r^2$	…	~ 1-loop
3PM				$G^3\ 1/r^3$	$G^3\ v^2/r^3$	$G^3\ v^6/r^3$	$G^3\ v^8/r^3$	…	~ 2-loop

[Bern,Cheung,Roiban,Shen, Solon,Zeng][Kälin, Liu, Porto][Di Vecchia, Heissenberg, Russo,Veneziano]
[Bjerrum-Bohr,Vanhove,Damgaard][Brandhuber,Chen,Travaglini,Wen][Jakobsen,Mogull,JP,Sauer]

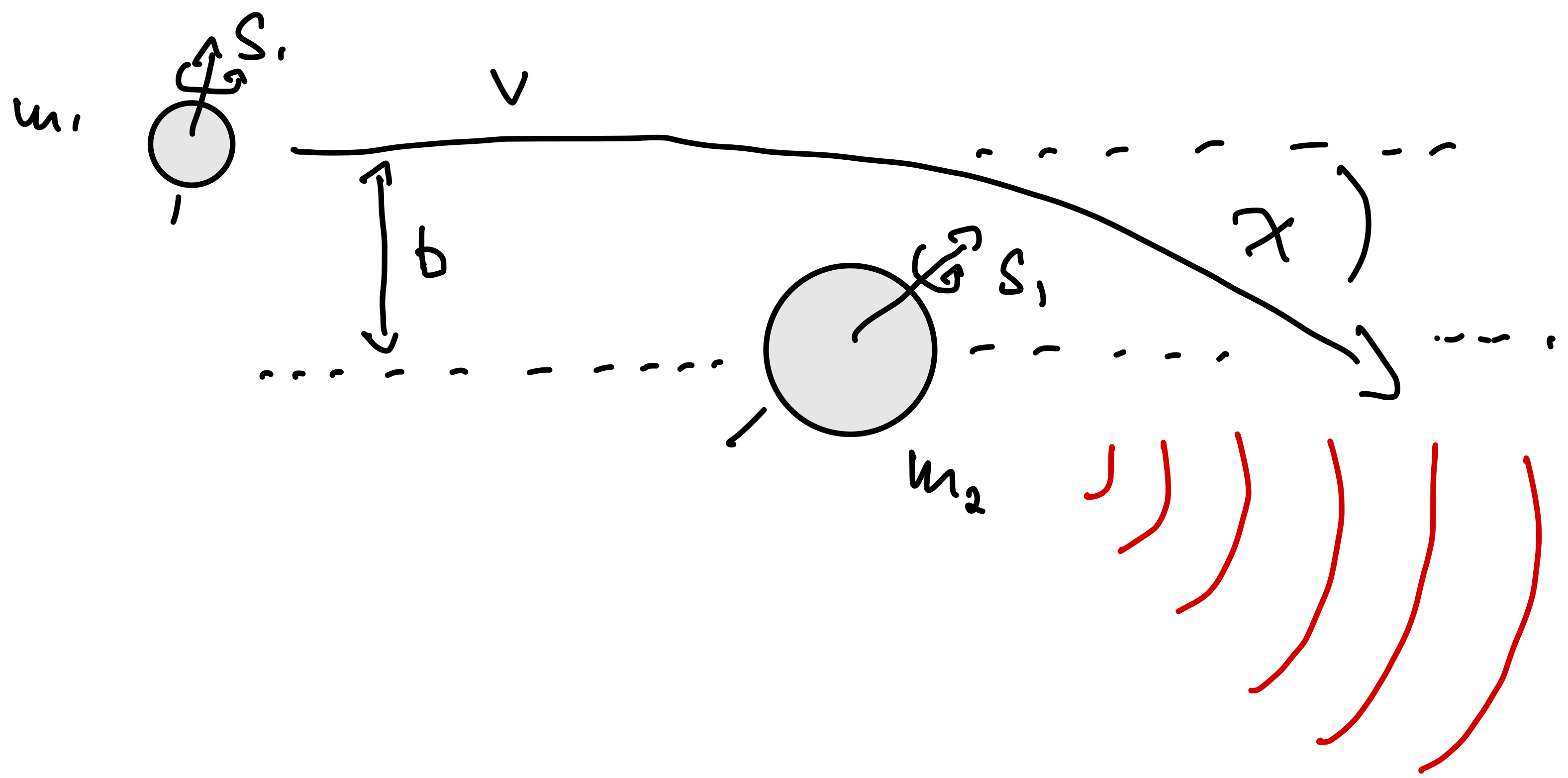
4PM	PM state-of-the-art		→		$G^4\ 1/r^4$	$G^4\ v^2/r^4$	$G^4\ v^6/r^4$	…	~ 3-loop

[Bern,Parra-Martinez,Roiban,Ruf,Shen,Solon,Zeng][Dlapa,Källin,Liu,Porto]

....						:	↑ :	

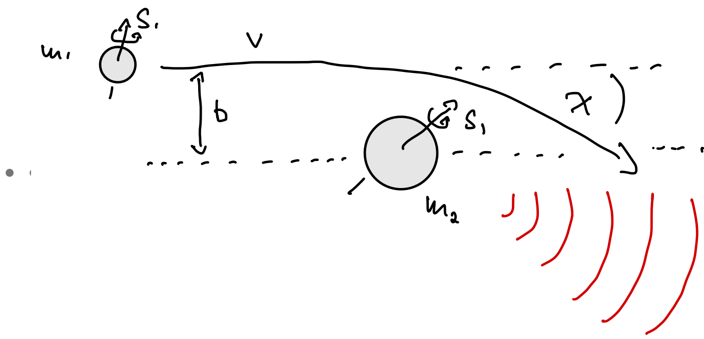
PN state-of-the-art

THE POST-MINKOWSKIAN EXPANSION



$$f_{\mu\nu} = \sum_{n=1}^{\infty} G^n \, f_{\mu\nu}^{(n)}$$

THE GENERAL REALTIVISTIC TWO BODY PROBLEM IN PM: TRADITIONAL APPROACH



Point-particle approximation for BHs (or NSs)

$$S = -\sum_{i=1}^{2}\int d\tau_i\sqrt{g_{\mu\nu}\dot{x}_i^\mu(\tau_i)\dot{x}_i^\nu(\tau_i)} + \frac{1}{16\pi G}\int d^4x\sqrt{-g}R + S_{\text{g.f.}}$$

Point particle — Bulk gravity & gauge

1) Equations of motion:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu} = \frac{\kappa^2}{8}T_{\mu\nu} \qquad \ddot{x}_i^\mu + \Gamma^\mu{}_{\nu\rho}\dot{x}_i^\nu\dot{x}_i^\rho = 0$$

Einstein's eqs. — Geodesic

2) Solve iteratively in $\kappa = \sqrt{32\pi G}$

$$g_{\mu\nu} = \eta_{\mu\nu} + \sum_{n=1}^{\infty}\kappa^n h_{\mu\nu}^{(n)}(x) \qquad x_i^\mu(\tau) = b_i^\mu + v_i^\mu\,\tau + \sum_{n=1}^{\infty}\kappa^n z_i^{\mu\,(n)}(\tau)$$

emitted radiation — straight line: „in“ state — deflections

3) Construct observables

Far field waveform:

$$\lim_{r\to\infty} h_{\mu\nu} = \frac{f_{\mu\nu}(t-r,\theta,\varphi)}{r} + \mathcal{O}(\frac{1}{r^2})$$

„Impulse“ (change in momentum):

$$\Delta p_i^\mu = m_i\dot{x}_i^\mu\Big|_{\tau=-\infty}^{\tau=+\infty} = \int d\tau\,\ddot{x}_i^\mu(\tau)$$

USE OF QUANTUM FIELD THEORY TECHNIQUES FOR CLASSICAL 2-BODY PROBLEM

1) Effective world-line field theory:

[Källin,Porto,Dlapa][Mougiakos,Riva,Vernizzi]

Construct effective action: $$e^{\frac{i}{\hbar}S_{\rm eff}[x_i]} = \int [Dh_{\mu\nu}] e^{\frac{i}{\hbar}(S_{pp}+S_G)}$$

Solve e.oms for $x_i(\tau)$: $$\frac{\delta S_{\rm eff}[x_i]}{\delta x_i} = 0$$

2) Scattering amplitudes:

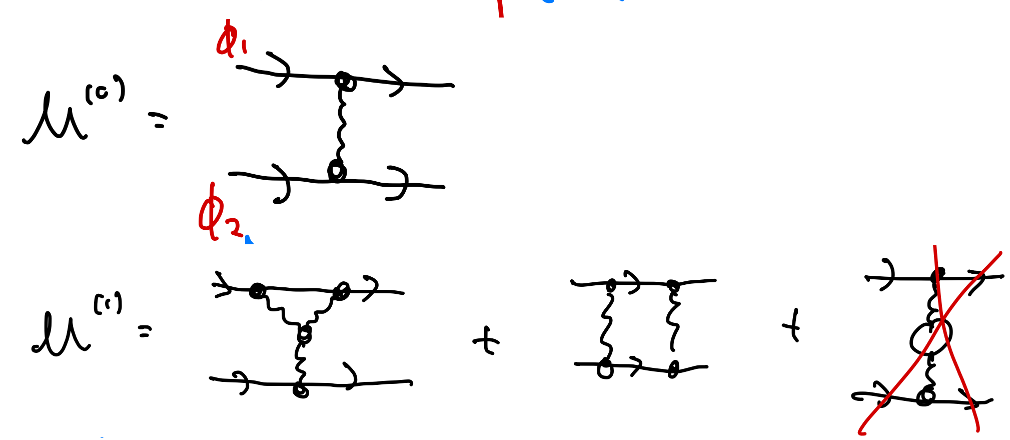
[Bern,Cheung,Roiban,Solon,Parra-Martinex,Ruf,Zeng][Bjerrum-Bohr,Damgaard,Vanhove,Cristo
[DiVecchia,Heissenberg,Russo,Venneziano][Kosower,Maybee,O'Connell,Vines]

Scalar fields as avatars of BHs & NSs:

$$\mathcal{M} = \underbrace{\kappa^2 \mathcal{M}^{(0)}}_{1PM} + \underbrace{\kappa^4 \mathcal{M}^{(1)}}_{2PM} + \dots$$

+ Modern on-shell techniques:

- Non-trivial classical limit
- Opaque relation to observables



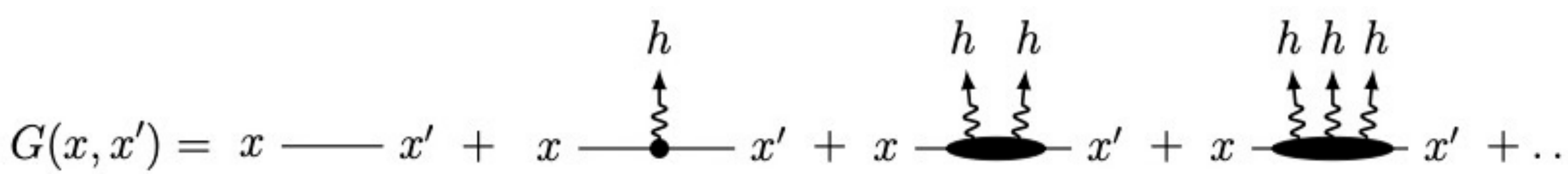
3) World line quantum field theory: Best of 1) & 2)

[Jakobsen,Mogull,JP,Steinhoff]

Philosophy: Focus on observables (here one-point functions @ tree-level

Use 1) but also path integrate over $x_i(\tau)$!

WORLDLINE QUANTUM FIELD THEORY



USING QFT TECHNIQUES TO SOLVE CLASSICAL FIELD EQUATIONS

CONSIDER SCALAR FIELD THY AS PROXY:

$$S[\phi; Q] = \frac{1}{2}\int d^4x \left[(\partial_\mu \phi)^2 + m^2 \phi^2 \right] + S_{int}[\phi; Q]$$

Q: PHYSICAL SOURCE OR BACKGROUND

GOAL: (PERTURBATIVE) SOLUTION OF E.O.M.:

$$\left.\frac{\delta S[\phi, Q]}{\delta \phi}\right|_{\phi = \phi_{class}(x)} \stackrel{!}{=} 0$$

QFT: GENERATING FUNCTIONAL

$$e^{\frac{i}{\hbar} W[J]} = \int [D\phi] \exp\left\{ \frac{i}{\hbar} S[\phi; Q] + \frac{i}{\hbar}\int d^4x \, J(x)\, \phi(x) \right\}$$

ONE-POINT FUNCTION

$$\langle \hat{\phi}_H(x) \rangle_{IN-OUT} = \left.\frac{\delta W[J]}{\delta J(x)}\right|_{J=0}$$

EFFECTIVE ACTION:

(LEGENDRE-TRANSFORM)

$$S_{eff}[\phi] = \frac{i}{\hbar}\int d^4x \, J(x)\, \phi(x) - W[J]$$

ONE-POINT FUNCTION & E.O.M.

① EFFECTIVE E.O.M. ARE SOLVED BY ONE-POINT FUNCTION '

$$\left.\frac{\delta S_{eff}[\phi]}{\delta \phi(x)}\right|_{\phi(x) = \langle \hat{\phi}_H(x) \rangle} = 0$$

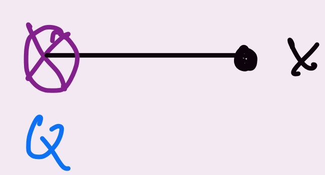
② TREE-LEVEL $\Rightarrow$ CLASSICAL ACTION : $S_{eff}[\phi] = S[\phi; Q] + \mathcal{O}(\hbar)$

$\Rightarrow$ TREE-LEVEL (FEYNMAN-DIAGRAMATIC) EVALUATION OF $\langle \hat{\phi}_H \rangle$ YIELDS SOLUTION TO CLASSICAL E.O.M.

CAUSALITY:

EXAMPLE: $S[\phi] = \frac{1}{2}\int d^4x \left[(\partial_\mu \phi)^2 + m^2 \phi(x)^2 + Q(x)\,\phi(x) \right]$

IN-IN ONE POINT FUNCTION:

$\langle \hat{\phi}_H(x) \rangle_{\text{IN-OUT}} =$ 

$= \int d^4y\, G_{\text{FEYN}}(x-y)\, Q(y)$

SOLVES E.O.M BUT WE WANT RETARDED PROPAGATOR!

IN-OUT FORMALISM: STANDARD PATH INTEGRAL

[Galley,Tiglio] [Jordan]

INTERACTION PICTURE:

$$|\psi(t)\rangle = \mathcal{U}(t,-\infty)\,|\psi\rangle$$

STATE EVOLVES WITH $\hat{H}_{INT}$,
OPERATORS WITH $\hat{H}_0$

$$\mathcal{U}(T,T') = \mathcal{T}\exp\left[\frac{i}{\hbar}\int_T^{T'} dt \int d^3x\; \hat{H}_{INT}[\phi_I(\vec{x},t)]\right]$$

HEISENBERG PICTURE: OPERATORS INERT, STATE EVOLVE WITH $\hat{H}_0 + \hat{H}_{INT}$

RELATION:

$$\hat{\phi}_I(t,\vec{x}) = \mathcal{U}(t,-\infty)\,\hat{\phi}_H(t,\vec{x})\,\mathcal{U}(-\infty,t)$$

PATH INTEGRAL REPRESENTATION

$|0\rangle$: GROUNDSTATE AT $T=-\infty$

$$\langle 0|\,\mathcal{U}_J(\infty,-\infty)\,|0\rangle = \int[D\phi]\exp\left\{\frac{i}{\hbar}S[\phi;Q] + \frac{i}{\hbar}\int d^4x\, J(x)\,\phi(x)\right\} = e^{\frac{i}{\hbar}W[J]}$$

$$\langle\hat{\phi}_H(t,\vec{x})\rangle_{IN\text{-}OUT} = \left.\frac{\delta W[J]}{\delta J(t,\vec{x})}\right|_{J=0} = \langle 0|\,\mathcal{U}(\infty,t)\,\hat{\phi}_I(t,\vec{x})\,\mathcal{U}(t,-\infty)\,|0\rangle$$

$$= \langle 0|\,\mathcal{U}(\infty,-\infty)\;\hat{\phi}_H(t,\vec{x})\;|0\rangle = {}_{OUT}\langle 0|\hat{\phi}_H(x)|0\rangle_{IN}$$

IN-IN (SCHWINGER-KELDYSH) FORMALISM

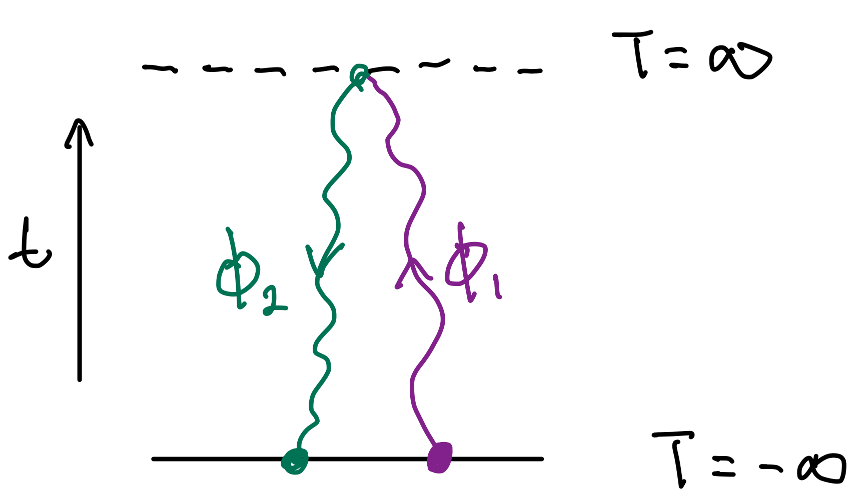
[Galley,Tiglio] [Jordan]

IN-OUT (STANDARD) FORMALISM YIELDS $\langle \hat{\phi}_H(x) \rangle_{IN-OUT} = {}_{OUT}\langle 0 | \hat{\phi}_H(x) | 0 \rangle_{IN}$ BUT WANT

$$\langle \hat{\phi}_H(x) \rangle_{IN-IN} := {}_{IN}\langle 0 | \hat{\phi}_H(x) | 0 \rangle_{IN} = \langle 0 | \hat{\mathcal{U}}(-\infty, t)\, \hat{\phi}_I(t, \vec{x})\, \hat{\mathcal{U}}(t, -\infty) | 0 \rangle$$

NEED TWO TIME EVOLUTION OPERATORS $\Rightarrow$ DOUBLE FIELDS IN PATH-INTEGRAL

$$e^{\frac{i}{\hbar} W[J_1, J_2]} = \langle 0 | \hat{U}_{J_2}(-\infty, \infty)\, \hat{U}_{J_1}(\infty, -\infty) | 0 \rangle$$
$$= \int \mathcal{D}\phi_1 \mathcal{D}\phi_2 \exp\left\{ \frac{i}{\hbar} \left(S[\phi_1] - S[\phi_2] + \int d^4x\, J_1(x)\phi_1(x) - J_2(x)\phi_2(x) \right) \right\}$$



BOUNDARY CONDITIONS:

$$\phi_1(T=\infty, \vec{x}) = \phi_2(T=\infty, \vec{x})$$
$$\phi_1(T=-\infty, \vec{x}) = \phi_2(T=-\infty, \vec{x}) = 0$$

$$\Rightarrow \boxed{\langle \Phi_H(x) \rangle_{IN-IN} = \left. \frac{\delta W[J_1, J_2]}{\delta J_1(x)} \right|_{J_i = 0}}$$

KELDYSH BASIS

$$\phi_+ = \frac{1}{2}(\phi_1 + \phi_2) \qquad \phi_- = \phi_1 - \phi_2$$

THIS YIELDS (SAME FOR $J_\pm$)

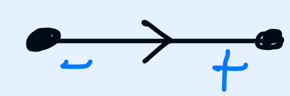
$$e^{\frac{i}{\hbar}W[J_+,J_-]} = \int \mathcal{D}\phi_+ \mathcal{D}\phi_- \exp\left\{\frac{i}{\hbar}\left(S[\phi_+ + \tfrac{1}{2}\phi_-] - S[\phi_+ - \tfrac{1}{2}\phi_-] + \int d^4x\,(J_+\phi_- + J_-\phi_+)\right)\right\}$$

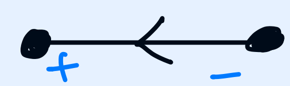
PROPAGATOR MATRIX FROM FREE PART:

$$\Rightarrow D^{ab}(x,y) = \begin{array}{c|cc} & + & - \\ \hline + & 0 & D_{adv}(x,y) \\ - & D_{ret}(x,y) & \frac{i}{2}D_H(x,y) \end{array}$$

RETARDED PROPAGATOR

$\langle\{\phi(x),\phi(y)\}\rangle$ IRRELEVANT @ TREE-LEVEL

$D_{ret}(k) =$  $= \dfrac{i}{(k^0 + i\varepsilon)^2 - \vec{k}^2}$

$D_{adv}(k) =$  $= \dfrac{-i}{(k^0 - i\varepsilon)^2 - \vec{k}^2}$

VERTICES FROM

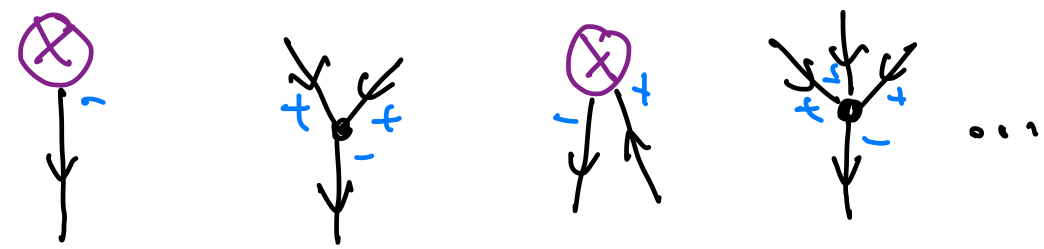
$$S_{INT}[\phi_+ + \tfrac{1}{2}\phi_-] - S_{INT}[\phi_+ - \tfrac{1}{2}\phi_-] = \phi_-\left(\frac{\delta S_{INT}(\phi)}{\delta\phi}\right)_{\phi\to\phi_+} + \mathcal{O}(\phi_-^3)$$

$\Rightarrow$ $\boxed{\text{ONLY ODD NUMBER OF } \phi_- \text{ LEGS}}$

ONE-POINT FUNCTIONS @ TREE-LEVEL

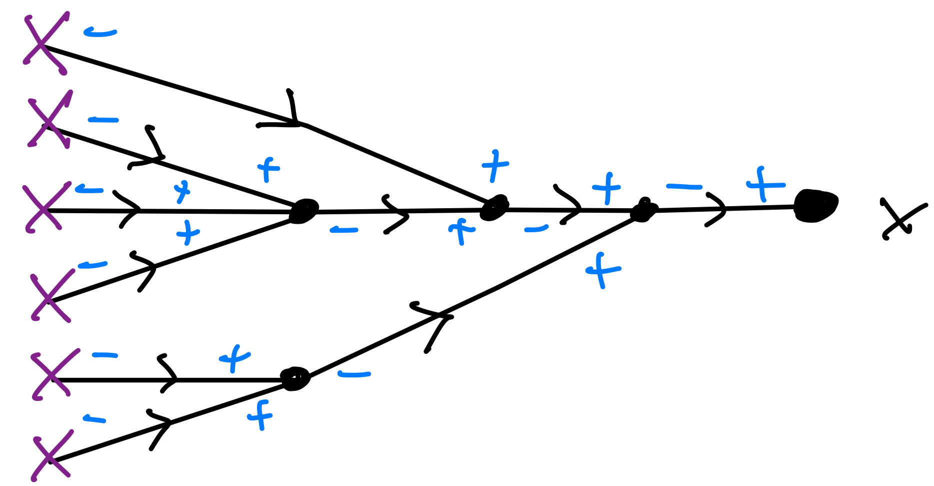
$$S_{\text{INT}}[\phi; Q] \xrightarrow[\text{IN-IN}]{} \phi_- \left(\frac{\delta S_{\text{INT}}[\phi, Q]}{\delta \phi} \right)_{\phi \to \phi_+} + O(\phi_-^3)$$

<u>VERTICES:</u>



ONE-POINT FCT. $\Rightarrow$

$$\langle \hat{\phi}_H(x) \rangle_{\text{IN-IN}} =$$



ONLY RETARDED PROPAGATORS CONTRIBUTE

WORLDLINE EFFECTIVE FIELD THEORY

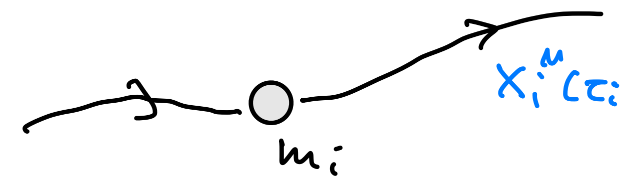
[Goldberger,Rothstein] [Porto,Källin] [Foffa,Sturani]

- MODEL BHs/NSs AS POINT PARTICLES:

$$S_p = -\sum_{i=1}^{2} m_i \int_{-\infty}^{\infty} d\tau_i \sqrt{g_{\mu\nu}\, \dot{x}_i^\mu\, \dot{x}_i^\nu}$$

BETTER: INTRODUCE EINBEIN $e(\tau)$:

$$S_p = -\frac{m}{2} \int d\tau \left(e^{-1} g_{\mu\nu}\, \dot{x}^\mu \dot{x}^\nu + e \right)$$



ALGEBRAIC E.O.M. YIELD $e^2 = g_{\mu\nu}\, \dot{x}^\mu \dot{x}^\nu \Rightarrow$ PROPER TIME GAUGE $e = 1 \Leftrightarrow \dot{x}^2 = 1.$

- ## INCLUSION OF FINITE SIZE/TIDAL EFFECTS

$$S_p = -\frac{m}{2} \int d\tau \left(g_{\mu\nu}\, \dot{x}^\mu \dot{x}^\nu + c_1 R\, \dot{x}^2 + c_2 R_{\mu\nu}\, \dot{x}^\mu \dot{x}^\nu + \right.$$
$$\left. + c_{E^2} \left(R_{\mu\alpha\nu\beta}\, \dot{x}^\alpha \dot{x}^\alpha \right)^2 + c_{B^2} \left(R^*_{\mu\alpha\nu\beta}\, \dot{x}^\alpha \dot{x}^\alpha \right)^2 + \dots \right.$$

- ## COUPLE TO GRAVITY

$$S_G = \frac{2}{\kappa^2} \int d^4x \sqrt{-g}\, R + S_{g.f.}$$

WEAK GRAVITATIONAL FIELD

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa \cdot h_{\mu\nu}$$

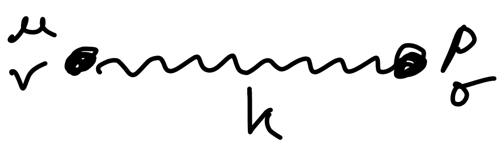
WORLDLINE QFT: FLUCTUATE GRAVITON & WORLDLINE

OBJECTIVE: FOCUS ON OBSERVABLES!

[Jakobsen,Mogull,JP,Steinhoff]

$$S = -2m_{Pl}^2 \int d^4x \sqrt{-g} R - \sum_i \frac{m_i}{2} \int d\tau_i \, g_{\mu\nu} \dot{x}_i^\mu \dot{x}_i^\nu \quad \left.\begin{array}{l} g_{\mu\nu}(x) = \eta_{\mu\nu} + \kappa h_{\mu\nu}(x) \\ x_i^\mu(\tau_i) = b_i^\mu + \tau_i v_i^\mu + \underbrace{z_i^\mu(\tau_i)}_{\text{QUANTUM FIELDS}} \end{array}\right.$$

Graviton propagator in de Donder gauge



$$= i \frac{P_{\mu\nu;\rho\sigma}}{(k^0 + i\epsilon)^2 - \vec{k}^2}$$

$$P_{\mu\nu;\rho\sigma} = \eta_{\mu(\rho}\eta_{\sigma)\nu} - \frac{1}{2}\eta_{\mu\nu}\eta_{\rho\sigma}$$

Worldline fluctuation propagator:

$$= -\frac{i}{m} \frac{\eta^{\mu\nu}}{(\omega + i\epsilon)^2}$$

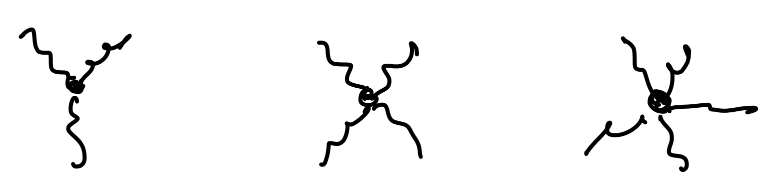
N.B.: $i\epsilon$ prescription is crucial here!

For classical physics want retarded prop.

$\Rightarrow$ IN-IN FORMALISM

[Schwinger,Keldysh]

Graviton interactions:

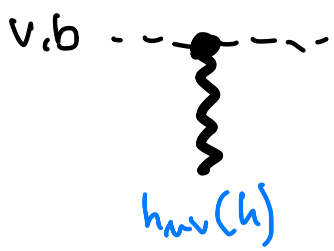


$+ \ldots$

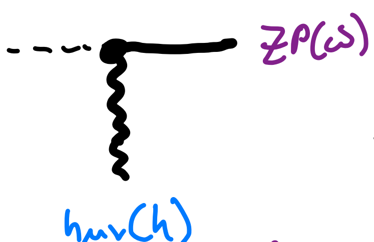
Worldline Interactions

EMERGE FROM $h_{\mu\nu}[X(\tau)]\,\dot{X}^{\mu}(\tau)\,\dot{X}^{\nu}(\tau)$ WITH $X_i^{\mu}(\tau_i) = \underbrace{b_i^{\mu} + \tau_i v_i^{\mu}}_{\text{“}Q\text{”}} + \underbrace{z_i^{\mu}(\tau_i)}_{\text{“}\phi\text{”}}$

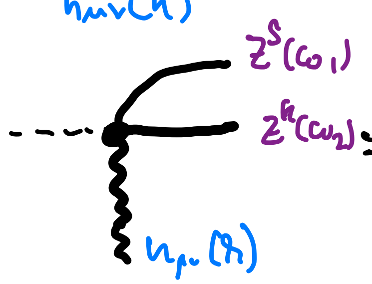
IN MOMENTUM SPACE



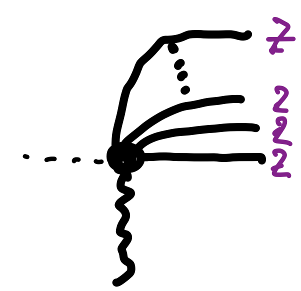
$$= -im\,\kappa\, e^{ik\cdot b}\,\delta(k\cdot v)\,v^{\mu}v^{\nu}$$



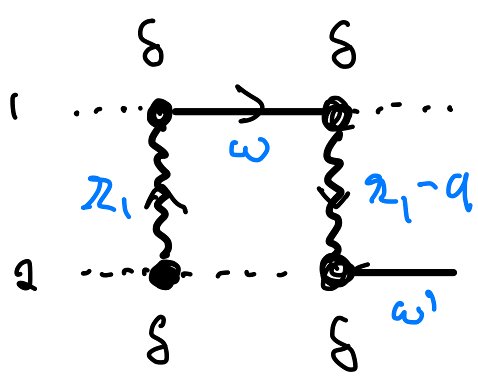
$$= m\,\kappa\, e^{ik\cdot b}\,\delta(k\cdot v+\omega)\left(2\omega\, v^{(\mu}\delta^{\nu)}_{\rho} + v^{\mu}v^{\nu}k_{\rho}\right)$$



$= \;\ldots$ and higher !



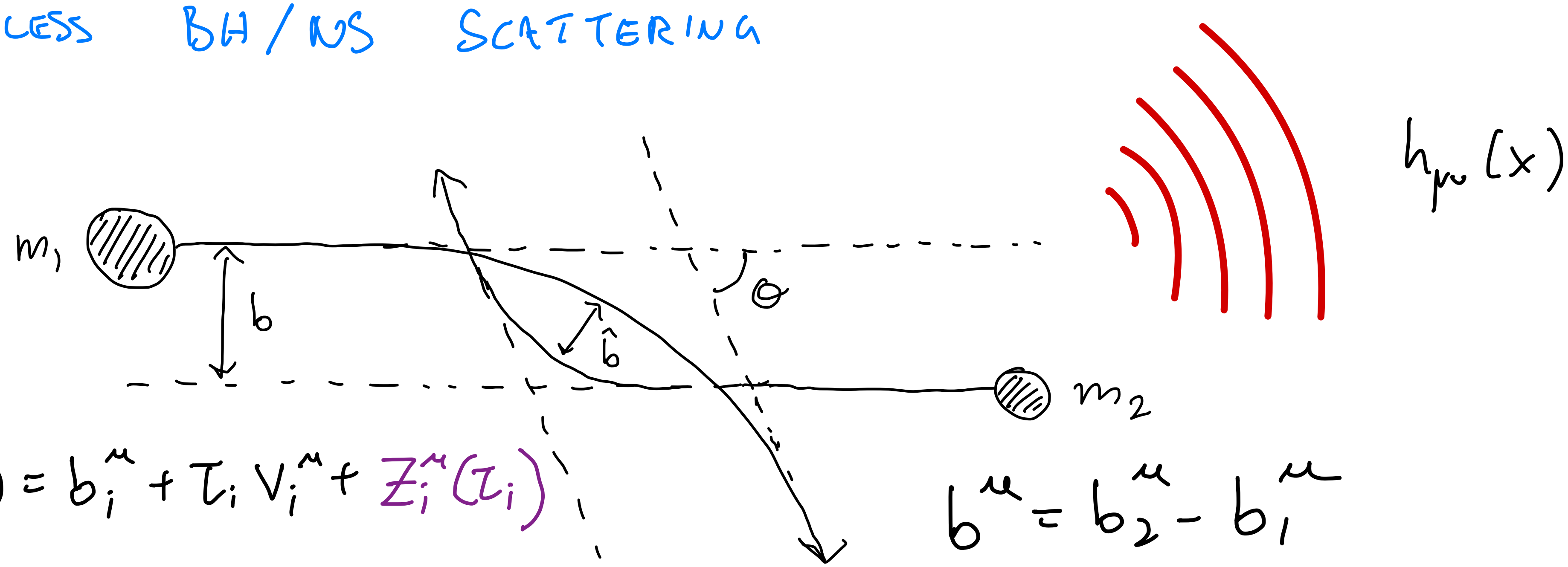
TREE LEVEL WQFT GRAPHS YIELD LOOP-LEVEL FEYNMAN INTEGRALS



$$\hat{=} \int d^4\ell_1 \int d\omega\; \delta^4(\ldots)\ldots = \delta(q\cdot v_1)\,\delta(q\cdot v_2) \underbrace{\int d^4\ell_1\, \delta(\ell_1\cdot v_1)\ldots}_{\text{1-LOOP}}$$

WQFT OBSERVABLES: ONE-POINT FUNCTIONS

SPIN-LESS BH / NS SCATTERING



$$X_i^\mu(\tau_i) = b_i^\mu + \tau_i v_i^\mu + Z_i^\mu(\tau_i)$$

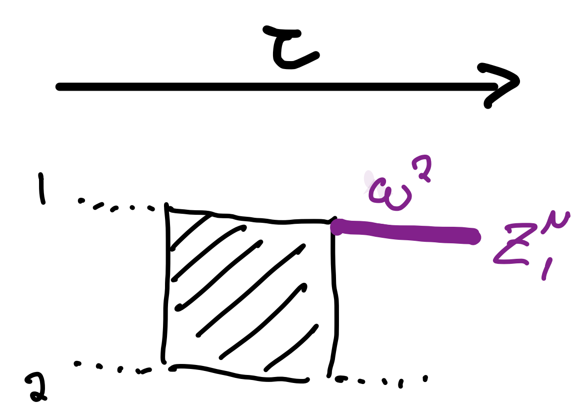
$$b^\mu = b_2^\mu - b_1^\mu$$

① IMPULSE: CHANGE OF MOMENTUM

$$\Delta p_i^\mu = m_i \dot{X}(\tau)\Big|_{\tau=-\infty}^{\tau=\infty} = m_i \int_{-\infty}^{\infty} d\tau \, \langle \ddot{X}_i^\mu(\tau) \rangle_{WQFT} = m_i \int_{-\infty}^{\infty} d\tau \, \frac{d^2}{d\tau^2} \langle Z_i^\mu(\tau) \rangle_{WQFT}$$

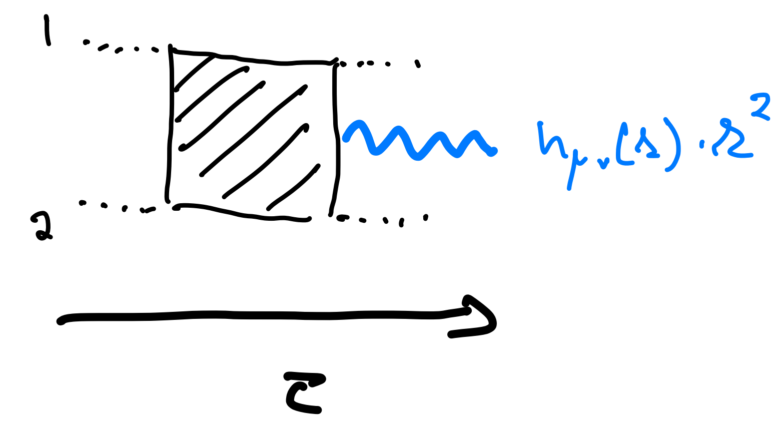
F.T.

$$= -m_i \, \omega^2 \langle Z_i^\mu(\omega) \rangle_{WQFT} \Big|_{\omega=0}$$



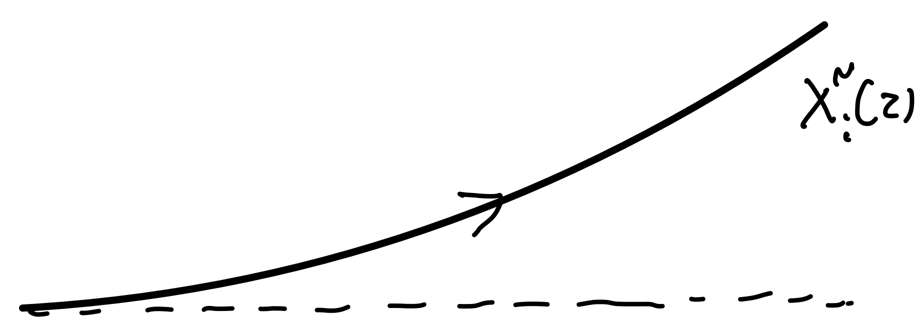
② EMITTED WAVEFORM: "BREMSSTRAHLUNG"

$$\tau^{T.T.}_{\mu\nu} = \kappa^2 \left\langle h_{\mu\nu}(k) \right\rangle_{WQFT}$$

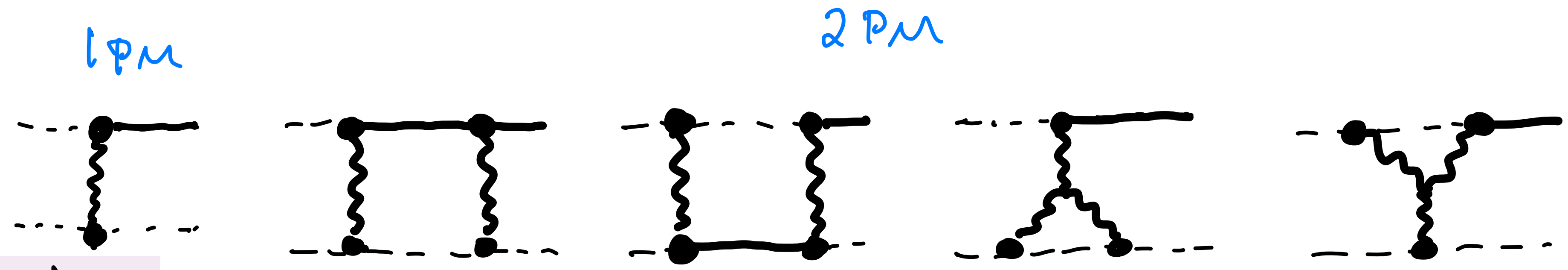


③ TRAJECTORY:

$$x^\mu_i(\tau) = b^\mu_i + v^\mu_i \tau + \int d\omega \, e^{i\omega\cdot\tau} \left\langle z^\mu_i(\omega) \right\rangle_{WQFT}$$



OBSERVABLES @ NLO

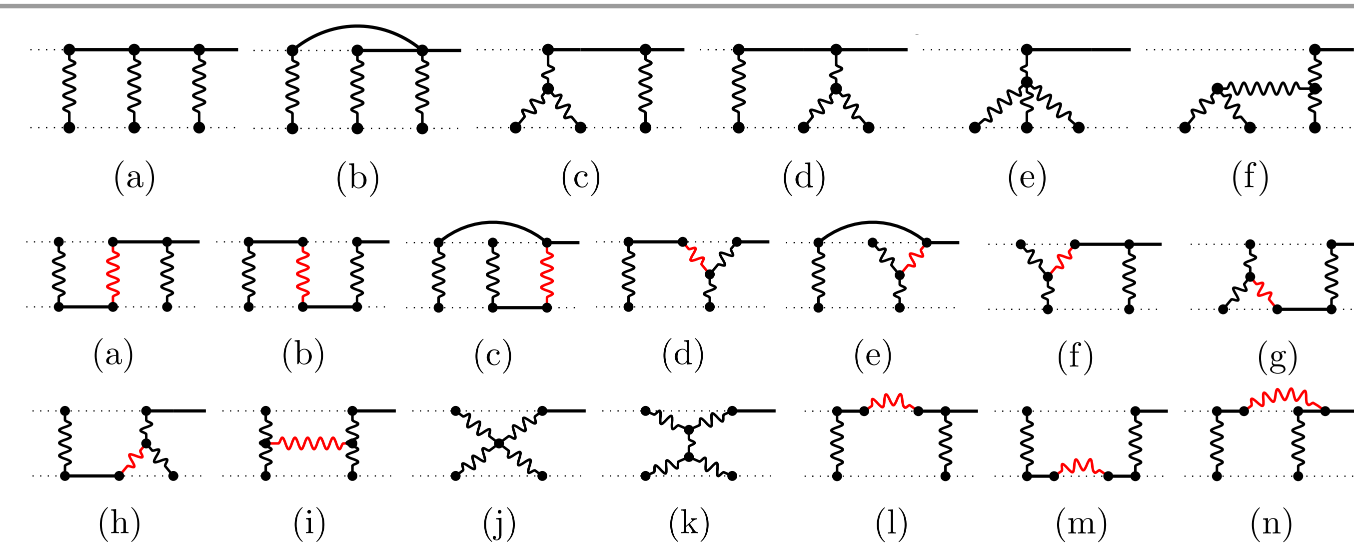


DEFLECTION

$$\Delta p_1^{\mu} = -m_1\, \omega^2 \langle z_1^{\mu}(\omega)\rangle_{WQFT}\Big|_{\omega=0}$$

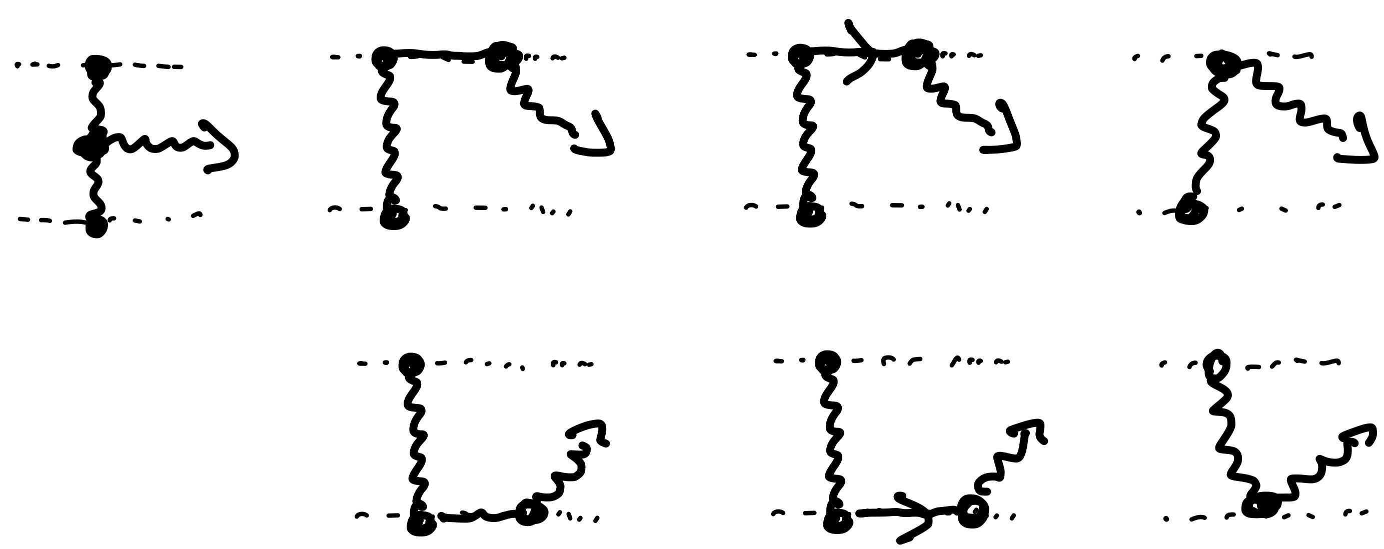
3PM

appearance of radiation reaction effects: i0 prescription crucial



BREMSSTRAHLUNG

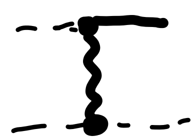
$$-i\kappa^2 \langle h^{\mu\nu}(k)\rangle_{WQFT}$$



Deflections

$$\Delta p_i^\mu = -m_i \omega^2 \langle Z_i^\mu(\omega)\rangle_{\mathrm{WQFT}}\Big|_{\omega=0}$$

Graphs with single outgoing worldline excitation Z_i^μ

1PM: 

outgoing line, $\omega=0$

2PM: 

Δp_1^μ

WE ONLY COMPUTE TREE-LEVEL GRAPHS

$(\hbar=0)$

Integration gives (w/o spin)

$$\Delta p_1^\mu = \frac{G m_1 m_2 b^\mu}{b^2}\left(\frac{2(2\gamma^2-1)}{\sqrt{\gamma^2-1}} + \frac{3\pi}{4}\frac{(5\gamma^2-1)}{\sqrt{\gamma^2-1}}\frac{G(m_1+m_2)}{b}\right) + O(G^3) \qquad \gamma = v_1\cdot v_2$$

1PM [Mogull,JP,Steinhoff]

2PM [Jakobsen,Mogull,JP,Steinhoff]

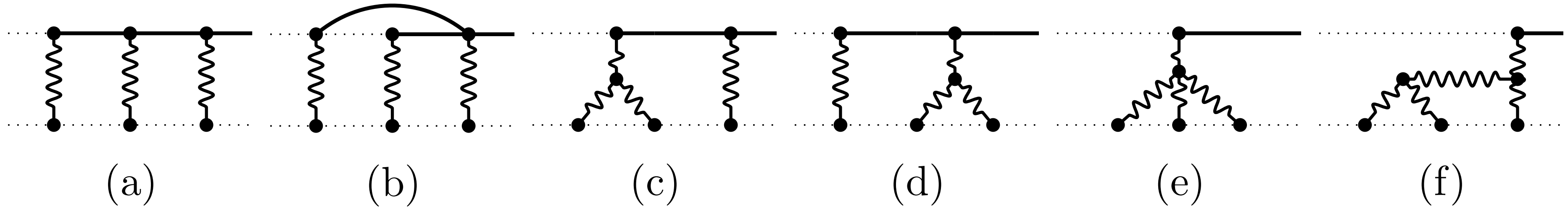
3PM [Jakobsen,Mogull]

AGREES WITH WEFT & AMPLITUDE APPROACHES [Källin,Porto][Bern et al][Brandhuber et al][Bjerrum-Bohr er al]

MOMENTUM DEFLECTION (IMPULSE) @ 3PM ORDER:

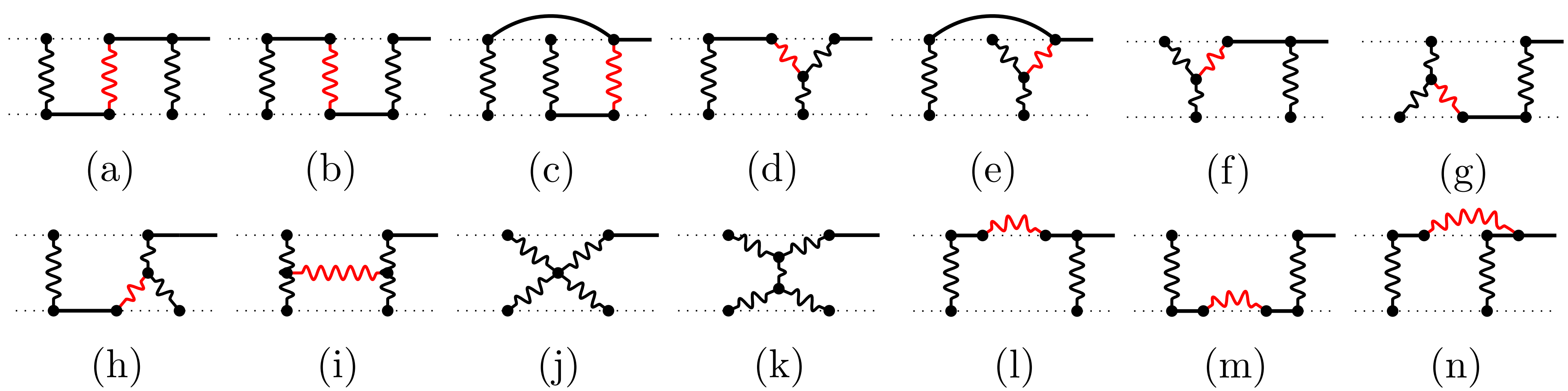
[Jakobsen,Mogull,JP,Sauer]

1) Test body diagrams (geodesic motion in Schwarzschild background):



$\sim G^3 m_1\, m_2^3$

2) Comparable mass diagrams (i0 prescription relevant for red propagators):



$\sim G^3 m_1^2\, m_2^2$

A 2-loop computation!

Integral family (with retarded propagators!)

$$I_{n_1 n_2 n_3 n_4 n_5 n_6 n_7} :=$$

$$\int d^d\ell_1\, d^d\ell_2 \frac{\delta(\ell_1\cdot v_2)\,\delta(\ell_2\cdot v_1)}{\underbrace{(\ell_i\cdot v_1 \pm i\epsilon)^{n_1}(\ell_j\cdot v_2 \pm i\epsilon)^{n_2}}_{\text{active worldline prop.}}\underbrace{\left((\ell_1+\ell_2-q)^2 + i\epsilon\, \mathrm{sgn}(\ell_1^0+\ell_2^0-q^0)\right)}_{\text{active graviton propagators.}}{}^{n_3}(\ell_1^2)^{n_4}(\ell_2^2)^{n_5}((\ell_1-q)^2)^{n_6}((\ell_2-q)^2)^{n_7}}$$

RESULT IMPULSE @ 3PM ORDER:

[Jakobsen,Mogull,JP,Sauer]

$$\boxed{\Delta p_1^\mu = p_\infty \sin\theta \frac{b^\mu}{|b|} + (\cos\theta - 1)\frac{m_1 m_2}{E^2}[(\gamma m_1 + m_2)v_1^\mu - (\gamma m_2 + m_1)v_2^\mu] - v_2\cdot P_{\rm rad}\, w_2^\mu}$$

Scattering angle:

$$\gamma = v_1\cdot v_2 \qquad w_1^\mu = \frac{\gamma v_2^\mu - v_1^\mu}{\gamma^2-1}$$

$$\frac{\theta}{\Gamma} = \underbrace{\frac{GM}{|b|}\frac{2(2\gamma^2-1)}{\gamma^2-1}}_{\text{1PM}} + \underbrace{\left(\frac{GM}{|b|}\right)^2\frac{3\pi(5\gamma^2-1)}{4(\gamma^2-1)}}_{\text{2PM}} + \underbrace{\left(\frac{GM}{|b|}\right)^3\left(2\frac{64\gamma^6-120\gamma^4+60\gamma^2-5}{3(\gamma^2-1)^3}\Gamma^2 - \frac{8\nu\gamma(14\gamma^2+25)}{3(\gamma^2-1)} - 8\nu\frac{(4\gamma^4-12\gamma^2-3)}{(\gamma^2-1)}\frac{\operatorname{arccosh}\gamma}{\sqrt{\gamma^2-1}}\right)}_{\text{3PM conservative}}$$

$$+\underbrace{\left(\frac{GM}{|b|}\right)^3\frac{4\nu(2\gamma^2-1)^2}{(\gamma^2-1)^{3/2}}\left(-\frac{8}{3}+\frac{1}{v^2}+\frac{(3v^2-1)}{v^3}\operatorname{arccosh}\gamma\right)}_{\text{3PM radiation-reaction}}$$

$$\Gamma = E/M = \sqrt{1+2\nu(\gamma-1)} \qquad \nu = \frac{m_1 m_2}{M^2}$$

Radiated 4-momentum:

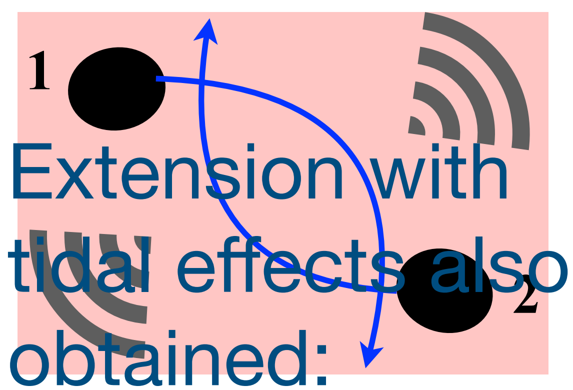
$$P_{\rm rad}^\mu = -\Delta p_1^\mu - \Delta p_2^\mu$$

$$P_{\rm rad}^\mu = \frac{G^3 m_1^2 m_2^2 \pi}{|b|^3}\frac{v_1^\mu + v_2^\mu}{\gamma+1}\left[e_1 + e_2\log\left(\frac{\gamma+1}{2}\right) + e_3\frac{\operatorname{arccosh}\gamma}{\sqrt{\gamma^2-1}}\right]$$

$$e_1 = \frac{210\gamma^6 - 552\gamma^5 + 339\gamma^4 - 912\gamma^3 + 3148\gamma^2 - 3336\gamma + 1151}{48(\gamma^2-1)^{3/2}}$$

$$e_2 = -\frac{35\gamma^4 + 60\gamma^3 - 150\gamma^2 + 76\gamma - 5}{8\sqrt{\gamma^2-1}},$$

$$e_3 = \frac{\gamma(2\gamma^2-3)(35\gamma^4-30\gamma^2+11)}{16(\gamma^2-1)^{3/2}}.$$



Extension with tidal effects also obtained:

$$C_{E^2}\left(R_{\mu\alpha\nu\beta}\,\dot{x}^\alpha\,\dot{x}^\alpha\right)^2$$

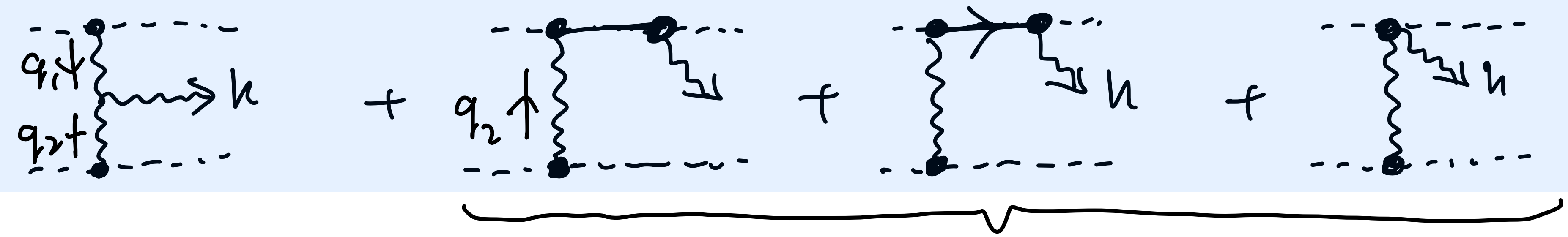
$$+\; C_{B^2}\left(R^*_{\mu\alpha\nu\beta}\,\dot{x}^\alpha\,\dot{x}^\alpha\right)^2$$

FAR FIELD WAVEFORM @ NLO

[Jakobsen,Mogull,JP,Steinhoff]

Sum on diagrams with an outgoing graviton. Integrate on internal lines:

$$\langle h_{\mu\nu}(k)\rangle =$$



$+ (1\leftrightarrow 2)$

We obtain the time-domain waveform for large $|\underline{x}|=r$. This requires integrating on the outgoing energy:

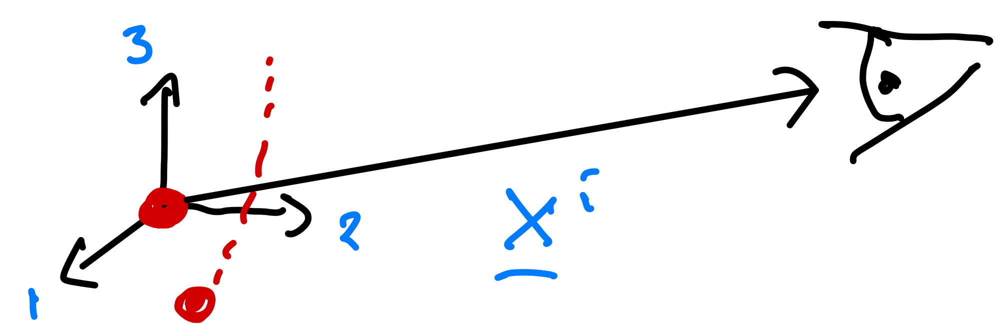
$$\frac{f_{+,\times}(u,\hat{\underline{x}})}{r} = \frac{4G}{r}\int_{\Omega} e^{-ik\cdot x}\,\varepsilon^{\mu\nu}_{+,\times}\langle h_{\mu\nu}(k=\Omega p)\rangle \quad \Big\}\ p^\mu=(1,\hat{\underline{x}})$$

where $k^\mu=\Omega p^\mu$, $p^\mu=(1,\hat{\underline{x}})$ points to the observer.

$k\cdot x = \Omega p\cdot x = \Omega(t-r) = \Omega u$
$u = t-r =$ retarded time

Two components: $+/\times$ polarizations.

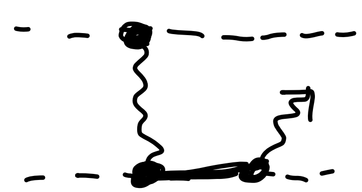
$$f_{+,\times}(m_1, m_2, u, \underbrace{\Theta, \phi}_{\hat{\underline{x}}}, \underbrace{v}_{\gamma}, |b|)$$

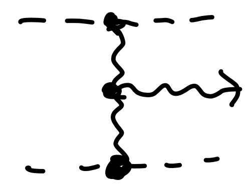


Integrated waveform

first computed by [Kovacs,Thorne `75] in 4 long papers!

$$\frac{f^{(2)}}{m_1 m_2} = 4\pi \int_q e^{i q\cdot \tilde{b}} \left(\underbrace{\frac{N_\mu q^\mu}{q^2 (q\cdot \hat{e}_1 - i\varepsilon)}} + \underbrace{\frac{M_{\mu\nu} q^\mu q^\nu}{q^2 (q^2 + q\cdot L \cdot q)}} \right)$$





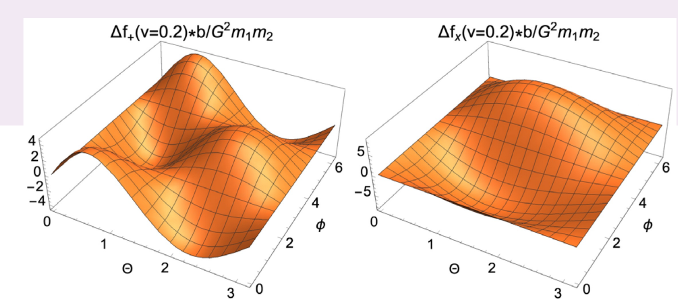
Performing these integrals yields time-domain waveform:

VIDEO

$$\boxed{\frac{f^{(2)}}{m_1 m_2} = \frac{\hat{e}_1\cdot N}{\sqrt{b^2+\tau^2}} - \frac{b\cdot N}{b^2}\left(1 + \frac{\tau}{\sqrt{b^2+\tau^2}}\right) + \frac{2M^{ij}}{\Delta(G)}\left[\frac{(G_0+\alpha G_1)A^{ij} - (G_1+\alpha G_2)B^{ij}}{\sqrt{G(\alpha)}}\right]_{\alpha=0}^{\alpha=1}}$$

Wave memory

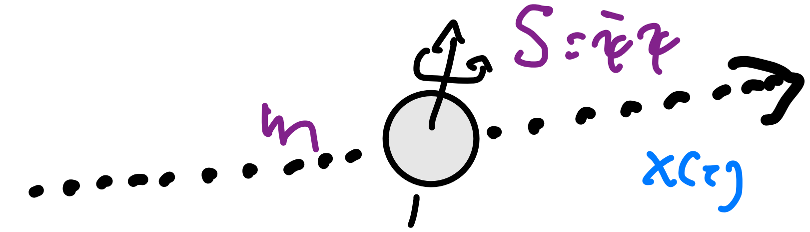
$$\frac{f^{(2)}(u=+\infty) - f^{(2)}(u=-\infty)}{m_1 m_2} = \frac{4(2\gamma^2-1)\, \varepsilon\cdot v_1 \left(2\, b\cdot\varepsilon\, \rho\cdot v_1 - b\cdot\rho\, \varepsilon\cdot v_1\right)}{b^2 \sqrt{\gamma^2-1}\, (\rho\cdot v_1)^2} + 1 \leftrightarrow 2$$



$\gamma = v_1\cdot v_2$ $\quad b = b_2 - b_1$ $\quad \rho = (1, \hat{x})$ $\quad \varepsilon$: polarization

SUSY in the Sky with Gravitons

PUTTING SPIN ON THE WORLD-LINE



- Traditional approach:

Spin tensor $S_i^{\mu\nu}(\tau)$ & co-moving frame $\Lambda_i^{A\,\mu}(\tau)$

Eoms: $\frac{Dp^\nu}{D\tau} + \frac{1}{2} S^{\mu\rho} R_{\mu\rho\nu\kappa}\dot{x}^\kappa = 0 \qquad \frac{DS^{\mu\nu}}{D\tau} + 2\dot{x}^{[\mu}\, p^{\nu]} = 0$ [Matthisson-Papapetrou-Dixon]

Freedom of imposing a
Spin-Supplementary Condition: $p_\mu\, S^{\mu\nu} = 0$

$\Leftrightarrow Q_\alpha\, \psi_\alpha = 0$
Susy = SSC:

- Our approach: Spinning super-particle [Howe,Penati,Pernici,Townsend]

- Worldline fields: $x^\mu(\tau)$; $\psi^a_\alpha(\tau)$ $\alpha = 1, \ldots, N$

In flat space-time

- Supercharge: $Q_\alpha = p \cdot \psi_\alpha$

$\{\psi^a_\alpha, \psi^b_\beta\} = i\,\delta_{\alpha\beta}\,\eta^{ab} \Longrightarrow$ $\{Q_\alpha, Q_\beta\}_{P.B.} = -2i\,\delta_{\alpha\beta}\, H$

Hamiltonian: $H = \frac{1}{2} p^2$ R-Charge: $R_{\alpha\beta} = \psi_\alpha \cdot \psi_\beta$

Describes free spin N/2 particle. Spin: $S^{\mu\nu} = \psi^\mu_\alpha \psi^\nu_\alpha$

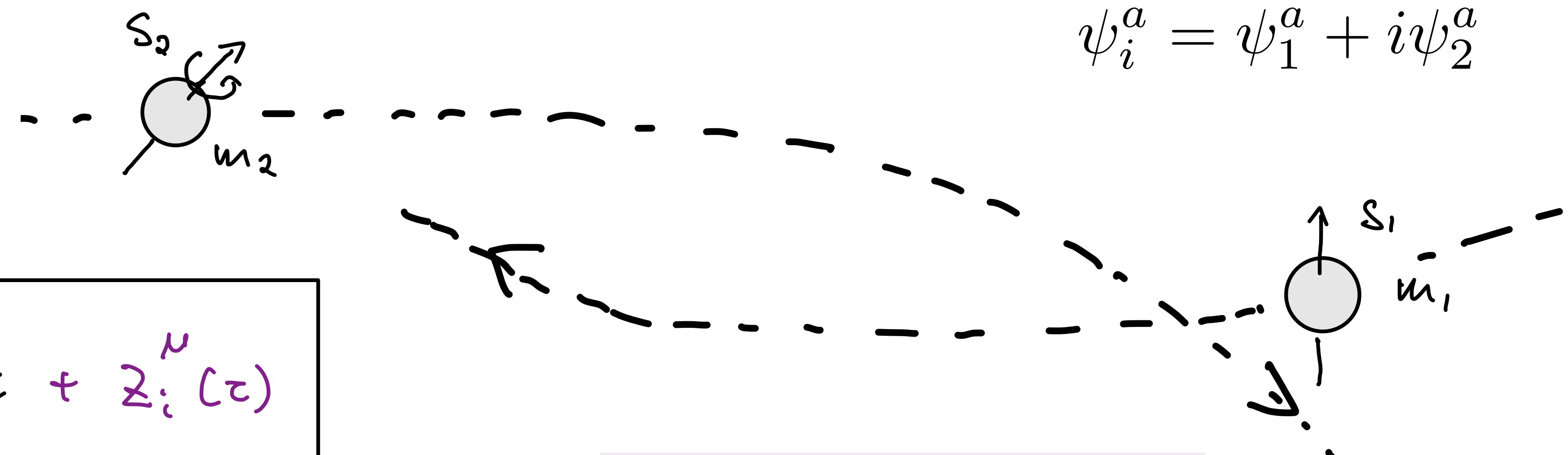
N=2 SUPERPARTICLE IN CURVED SPACE = KERR-BLACK HOLE

In curved space-time SUSY only preserved up to N=2 (= spin 1 particle):

$$S_{\text{SWQFT}} = \int d\tau \sum_{i=1}^{2} \Big[-\frac{m}{2} g_{\mu\nu} \dot{x}_i^\mu \dot{x}_i^\nu + i \bar{\psi}_{i,a} \frac{D\psi_i^a}{D\tau} + \frac{1}{2m} R_{abcd}\, \bar{\psi}_i^a \psi_i^b \bar{\psi}_i^c \psi_i^d$$
$$+ \frac{C_E}{2m} R_{a\mu b\nu} \dot{x}_i^\mu \dot{x}_i^\nu\, \bar{\psi}_i^a \psi_i^b\, \bar{\psi}_i \cdot \psi_i \Big] + \frac{2}{\kappa^2} \int d^4x \sqrt{-g}\, R + S_{\text{G.F.}}$$

Scattering scenario:

$$\psi_i^a = \psi_1^a + i\psi_2^a$$



$$\boxed{\begin{aligned} x_i^\mu(\tau) &= b_i^\mu + v_i^\mu \tau + z_i^\mu(\tau) \\ \psi_i^a(\tau) &= \underline{\Psi}_i^a + \psi_i'^a(\tau) \end{aligned}}$$

$\Rightarrow$

$$S_i^{ab} = -2i\, \bar{\Psi}_i^{[a} \Psi_i^{b]}$$

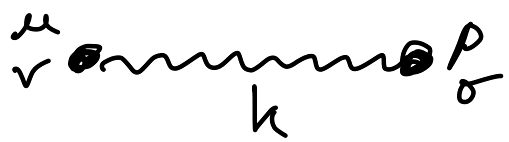
Initial spins of BHs/NSs

Integrate out $z_i^\mu, \psi_i'^a, \bar{\psi}_i'^a$ perturbatively!

[Jakobsen,Mogull,JP,Steinhoff]

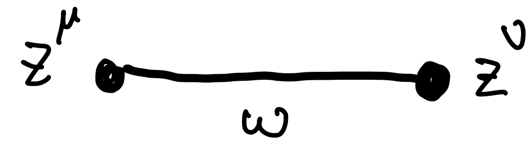
Spinning WQFT Feynman rules

Graviton propagator

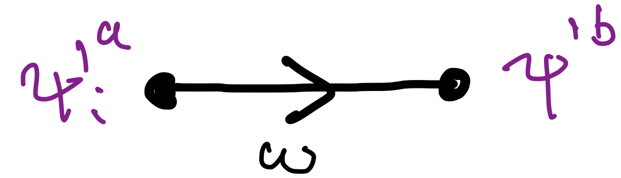


$$= i \frac{P_{\mu\nu;\rho\sigma}}{(k^0 + i\varepsilon)^2 - \vec{k}^2}$$

Worldline fluctuation propagator:

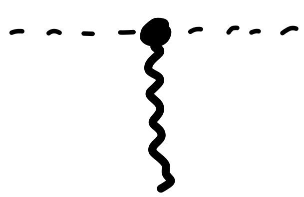


$$= -\frac{i}{m} \frac{\eta^{\mu\nu}}{(\omega + i\varepsilon)^2}$$

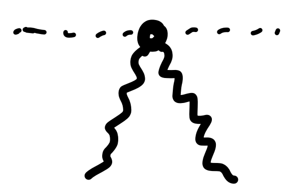


$$= i \frac{\eta^{ab}}{\omega + i\varepsilon}$$

Worldline interactions

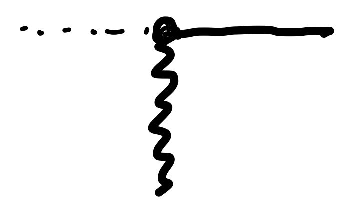


$\sim v^2 + Sv + S^2$

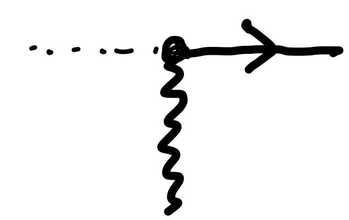


$\sim S^2$

...

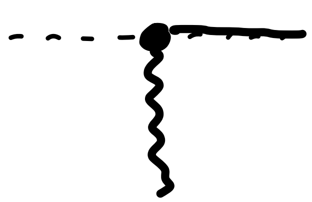


$\sim v^2 + Sv + S^2$

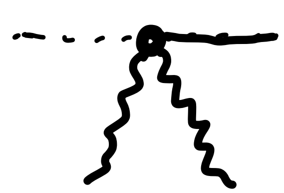


$\sim v + S$

...

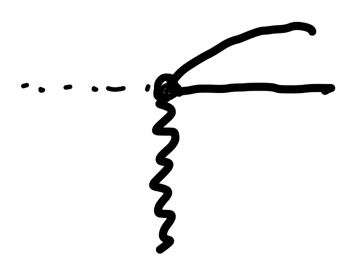


$\sim v^2 + Sv + v^2$

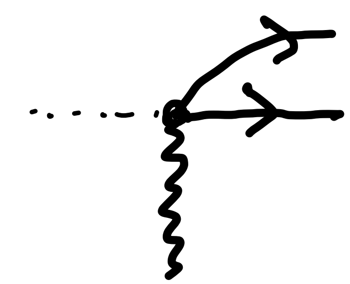


$\sim S^2$

...



$\sim v^2 + Sv + v^2$



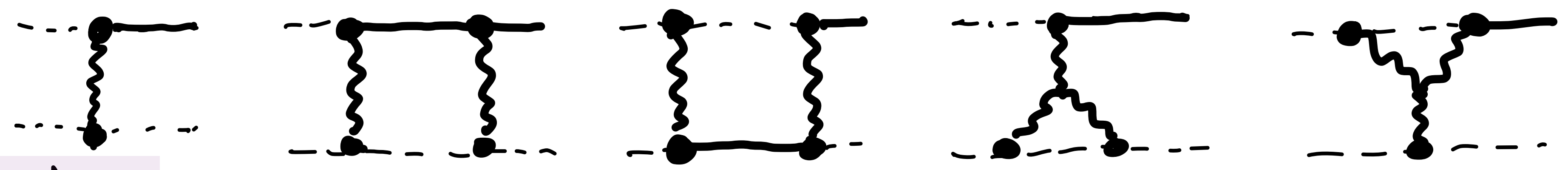
$\sim v + S$

...

OBSERVABLES @ NLO

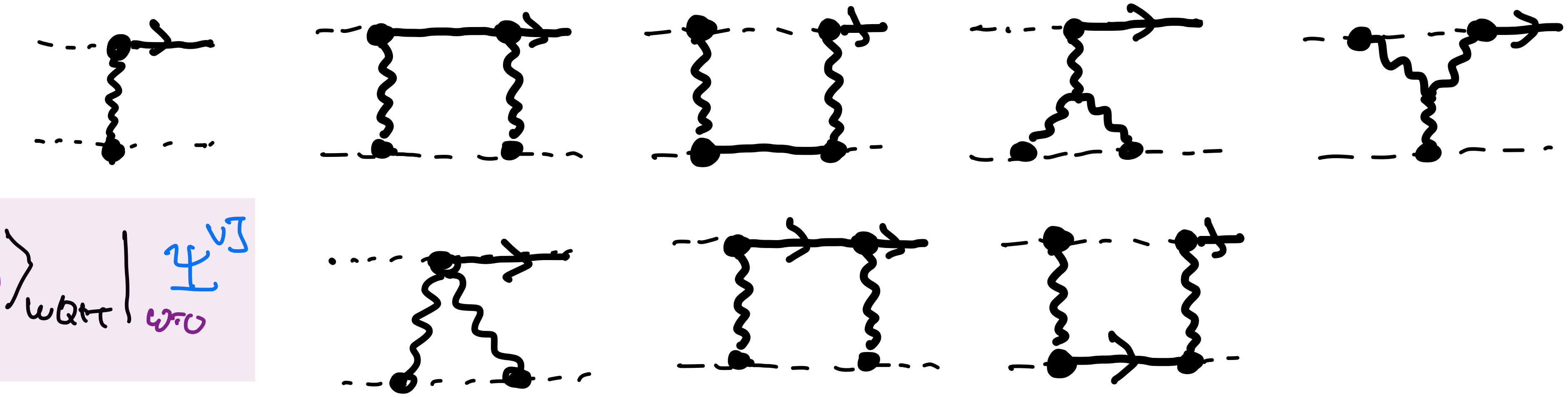
1PM 2PM

- **DEFLECTION**



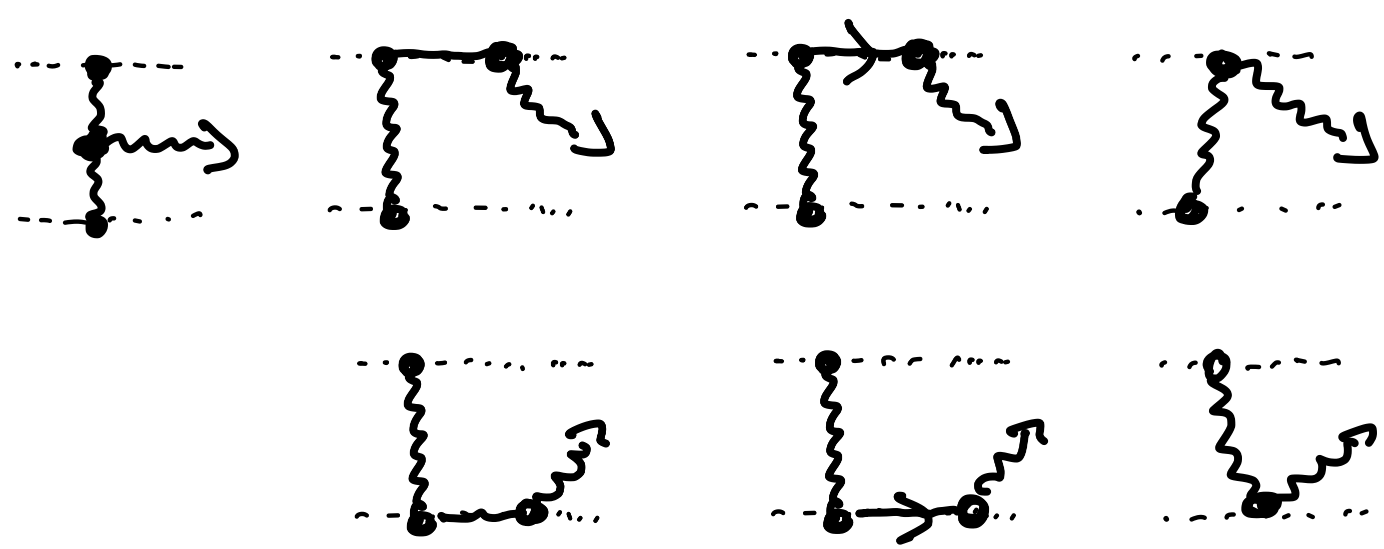
$$\Delta p_1^{\mu} = -m_1\,\omega^2 \left\langle z_1^{\mu}(\omega)\right\rangle_{\rm WQFT}\Big|_{\omega=0}$$

- **SPIN KICK**



$$\Delta S_1^{\mu\nu} = -2i\omega \left\langle \bar{\psi}^{[\mu}(\omega)\right\rangle_{\rm WQFT}\Big|_{\omega=0} \Psi^{\nu]}$$

- **BREMSSTRAHLUNG**



$$-i\kappa^2 \left\langle h^{\mu\nu}(k)\right\rangle_{\rm WQFT}$$

POST-MINKOWSKIAN SCATTERING PRECISSION RACE

WQFT [us]

WEFT Worldline effective theory [Källin,Porto,Dlapa,Cho,Liu,..] [Riva,Vernizzi,Mougiakakos..]

Amps Scattering amplitudes [Bern,Roiban,Shen,Parra-Martinez,Ruf,..] [Di Vecchia,Veneziano,Heissenberg,Russo] [Solon,Cheung,..][Huang,..][Guevera,Ochirov,Vines,…] [Bjerrum-Bohr,Damgaard,Vanhove,..][Johansson,Pichini,…] [Kosower,O'Connell,Maybee,Cristofoli,Gonzo…]

HEFT Heavy BH effective theory [Aoude,Haddad,Helset] [Brandhuber,Travaglini,Chen]

	deflection & spin kick				waveform			
	plain	**spin²**	**spin>2**	**tidal**	**plain**	**spin²**	**tidal**	Integration complexity
1PM	WQFT WEFT Amps HEFT	WQFT WEFT Amps HEFT	Amps HEFT	X	trivial	trivial	trivial	~ tree-level
2PM	WQFT WEFT Amps HEFT	WQFT WEFT HEFT	Amps	WQFT WEFT Amps	WQFT WEFT (Amps)	WQFT WEFT	WQFT WEFT	~ 1-loop
3PM w/o r-r	WQFT WEFT Amps HEFT	WQFT (Amps)		WQFT WEFT				~ 2-loop
3PM r-r	WQFT WEFT Amps HEFT	WQFT (WEFT)		WQFT WEFT				~ 2-loop
4PM w/o r-r	WEFT Amps							~ 3-loop

r-r: Radiation-reaction

(…) : partial results

SUMMARY

- WQFT HIGHLY EFFICIENT FOR CLASSICAL SCATTERING:
  - FOCUS ON OBSERVABLES BY "QUANTIZING" WORLDLINE D.O.F.
  - ONLY COMPUTE TREE-DIAGRAMS (NO "SUPER-CLASSICAL" CONTRIBUTIONS)
  - ALL PROPAGATORS RETARDED: NO "SPECIAL" TREATMENTS OF CONSERVATIVE & RADIATION-REACTED CONTRIBUTIONS
- SPIN CARRIED BY GRASSMANN VECTORS ON THE WORLDLINE (à la STRING THEORY)

OUTLOOK

- RELATION TO SELF-FORCE APPROACH?
- BOUND ORBITS ?
- HIGHER ORDERS IN SPIN ?
- OBSERVABLES @ 4PM ?

THANK YOU !