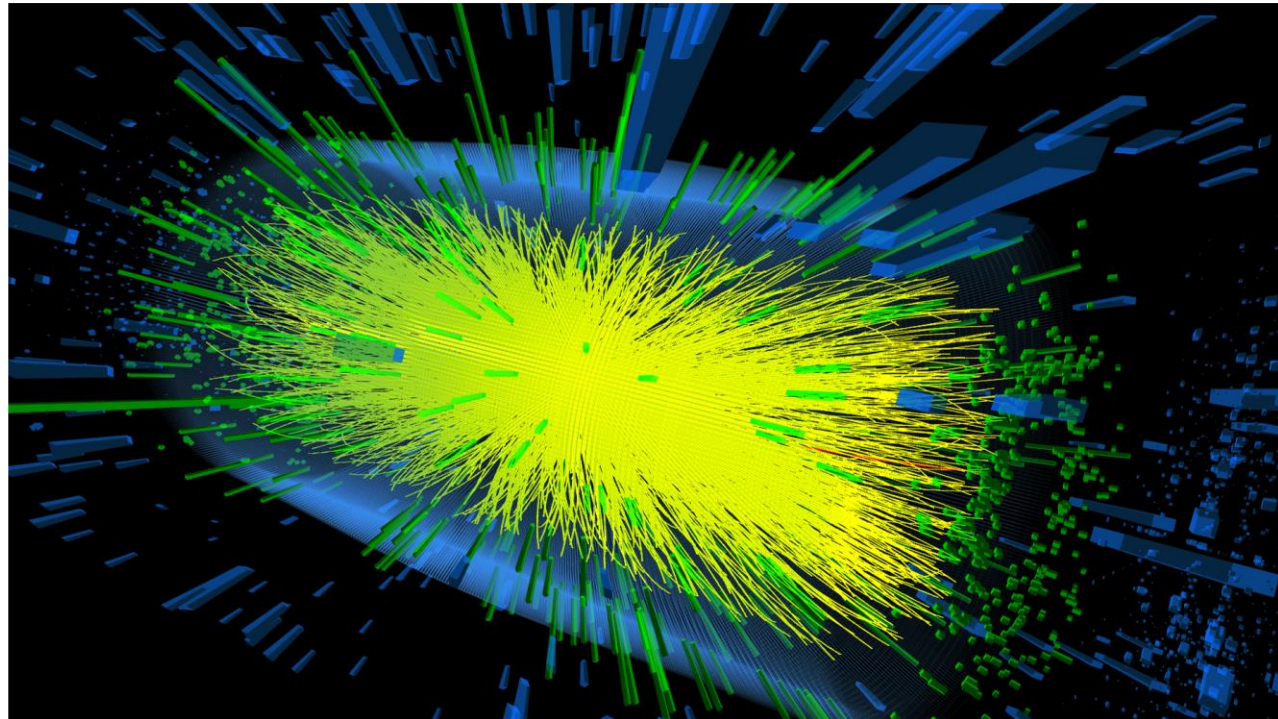


The alignment challenge in complex high resolution trackers

KIT Colloquium

Rainer Mankel (DESY)

Karlsruhe, 12 May 2022



Outline

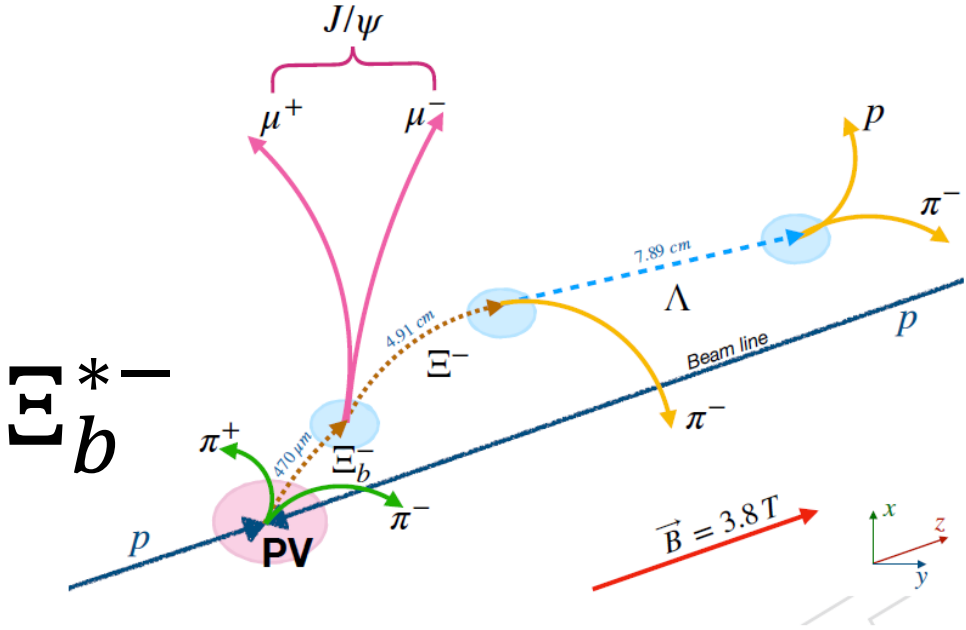
- Motivation
- Alignment challenges in complex tracking systems
- Methodology & solutions
- Practical experience with alignment in a huge tracking system
 - inputs & workflows
 - evolution with time
 - systematic effects & their impact on physics performance

Includes already results from our upcoming new paper:

Strategies and performance of the CMS silicon tracker alignment during LHC Run 2, arxiv:2111.08757
(will appear soon in Nucl. Instr. Meth. A)

Motivation: tracking in the LHC era

- How about discovering a new beauty-strange baryon...
 - possibly an excitation of Ξ_b^- , quark content (bsd)
 - produced at the interaction point \rightarrow expect complex decay cascade

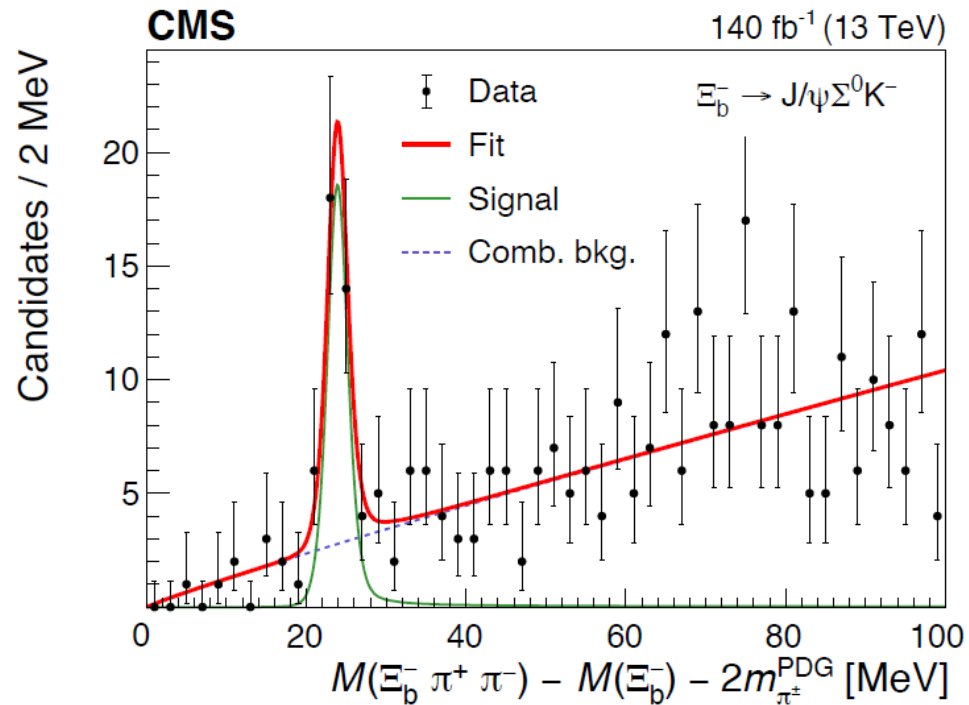
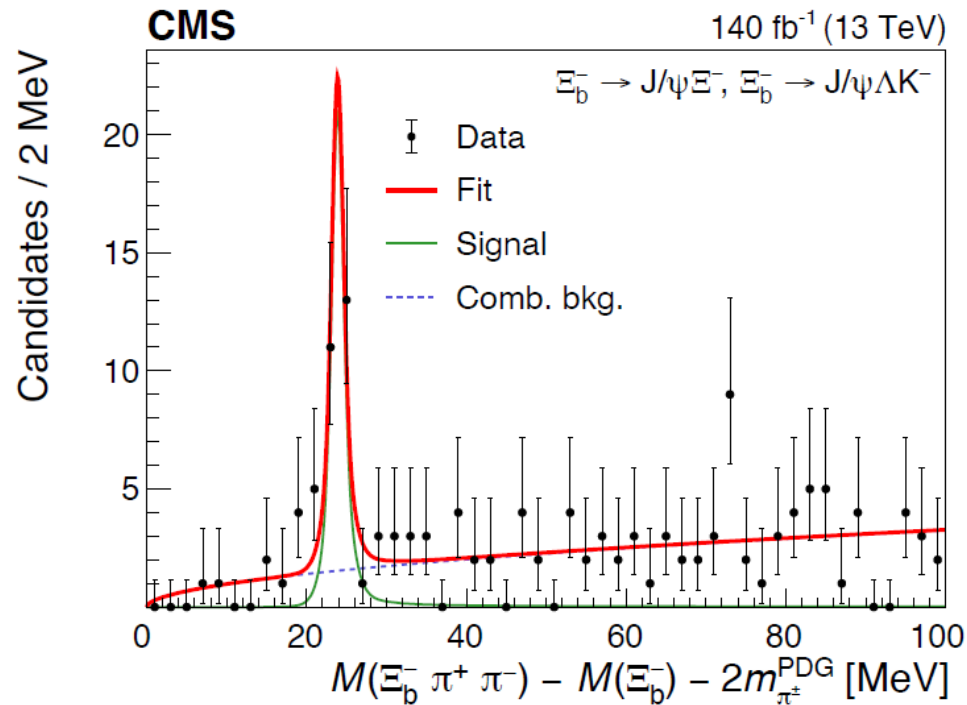


- Reminiscent of bubble chamber physics in the 60's
- But can we do this in presence of a pileup of 60 and more inelastic interactions in the detector for each event...?

Observation of a new excited beauty strange baryon

Phys. Rev. Lett. 126, 252003 (June 2021)

- First observation of $\Xi_b(6100)^-$
 - orbital excitation of $\Xi_b^-, J^P = 3/2^-$



6.2 – 6.7 σ

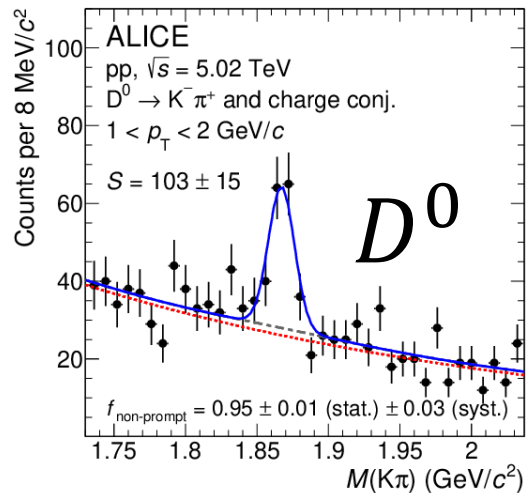
- Very low background due to lifetime signature. Excellent mass resolution
 - precision tracking at the LHC

Why precision tracking matters

Tracking is more important than ever

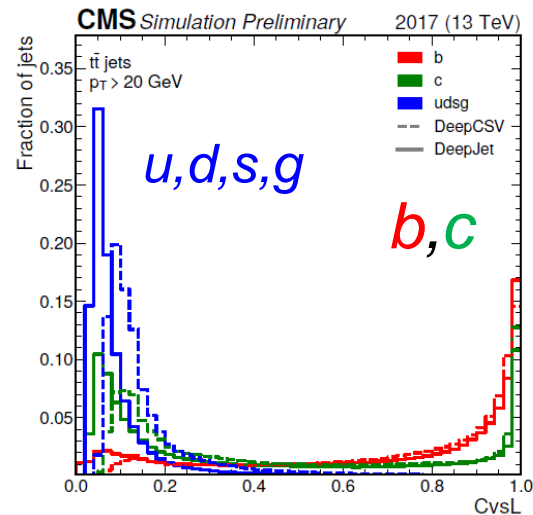
Secondary vertices

Example: properties of charm and beauty hadrons



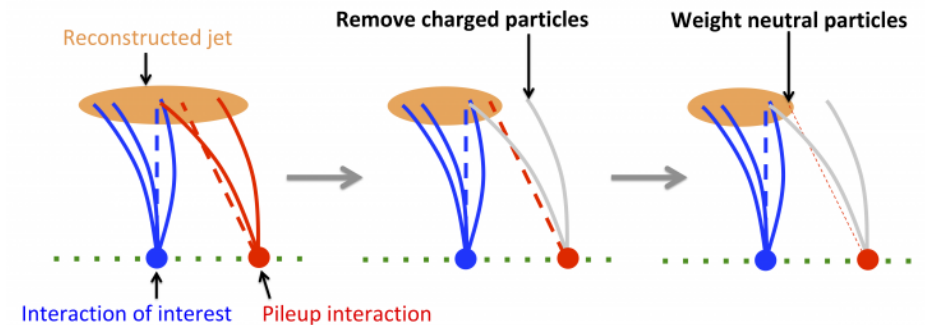
Flavor tagging of jets

Example: $H \rightarrow bb$, $H \rightarrow cc$



Improved jet reconstruction & pileup mitigation

Key technology for HL-LHC



- Precision tracking and alignment are key drivers of physics performance

Why is alignment important?

- Intrinsic coordinate resolution:
 - $\sigma_{hit} \sim 9 \mu\text{m}$ (pixel), $\sigma_{hit} \sim 20\text{-}60 \mu\text{m}$ (strip)
- The effective coordinate resolution emerges from combination of **intrinsic resolution** and **alignment**

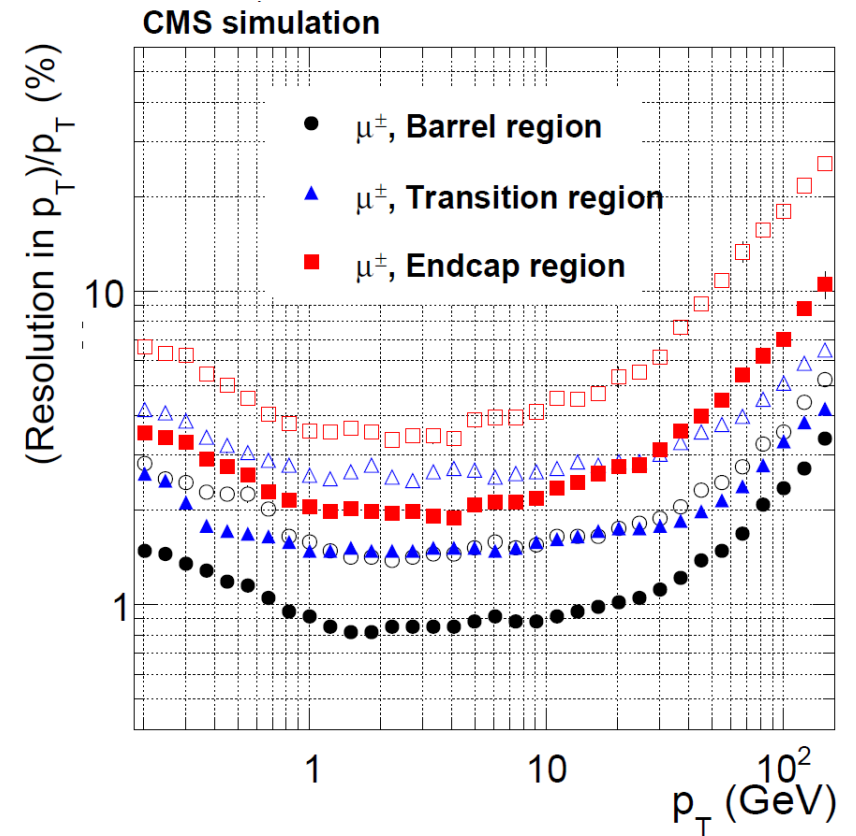
$$\sigma_{meas} \sim \sqrt{\sigma_{hit}^2 + \sigma_{alignment}^2}$$

- In a simplified model, the relative momentum resolution is the combined effect of **coordinate resolution** and **multiple scattering**

$$\frac{\delta p_T}{p_T} = C_1 \cdot p_T \oplus C_2 \quad \text{where } C_1 \propto \sigma_{meas}$$

- **Need to keep $\sigma_{alignment} \ll \sigma_{hit}$**

JINST 9 (2014) P10009

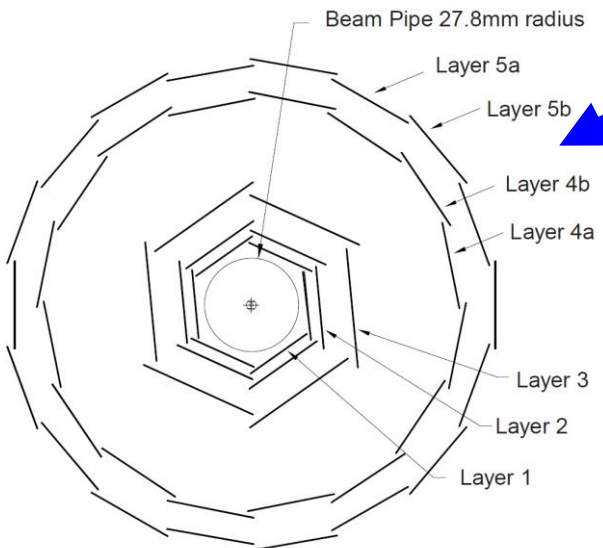


Complexity evolution of silicon trackers

A very arbitrary selection

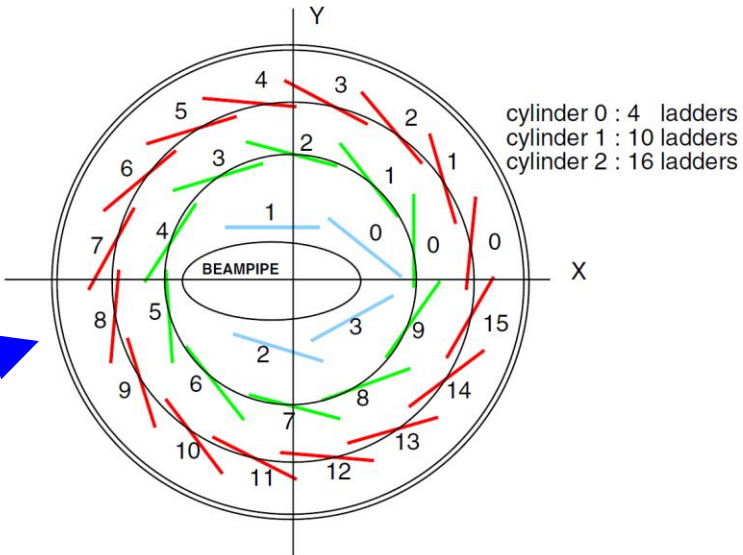
BaBar SVT

- 340 wafers



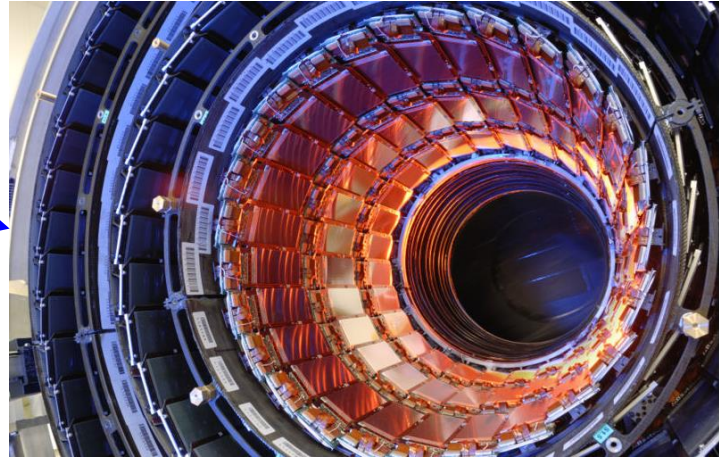
ZEUS MVD

- 712 single-sided strip detectors



CMS tracker

- 15,148 strip modules, 1,852 pixel modules (Phase 1)
→ ~17,000 modules



The CMS all-silicon tracker

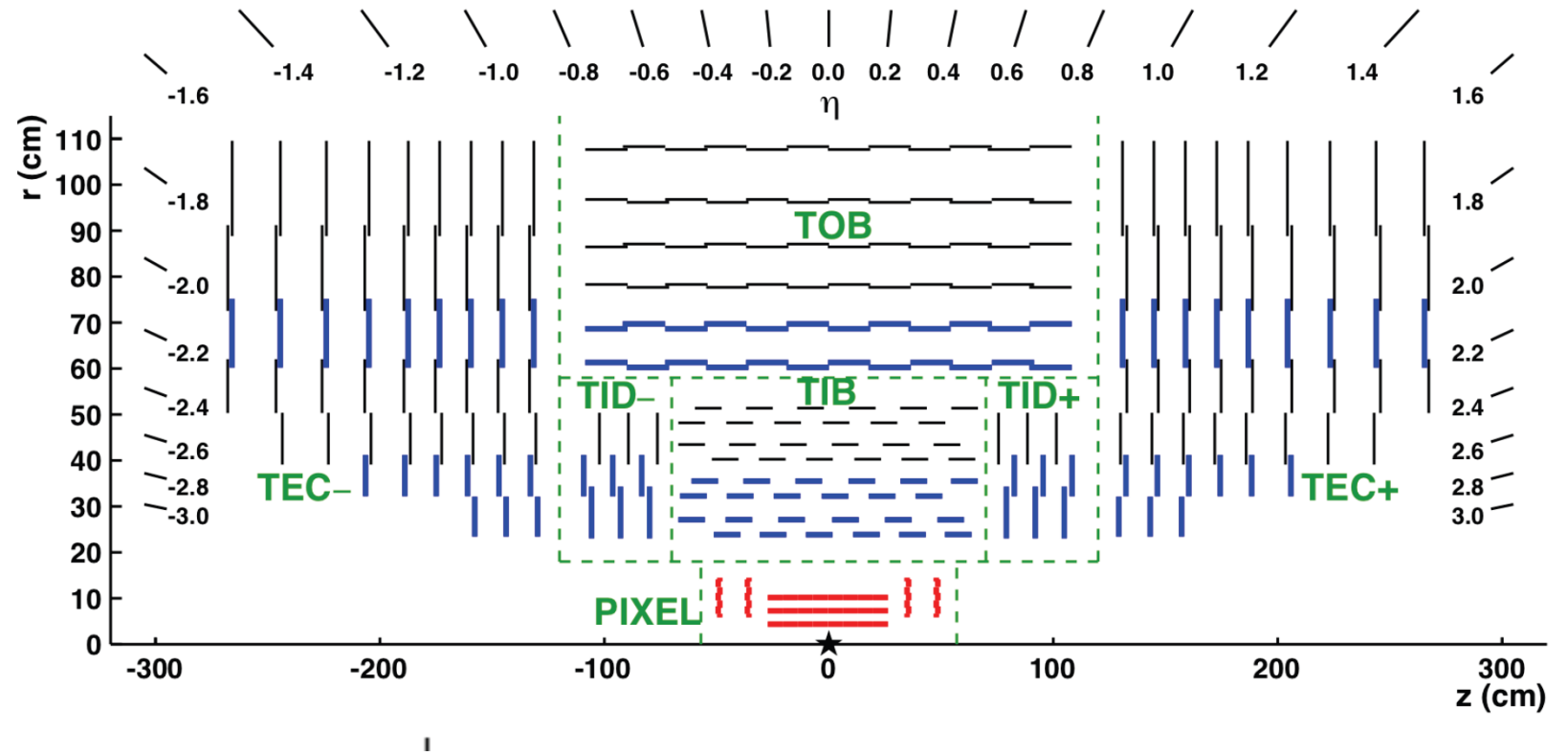
The largest silicon tracker ever built

- Si-Pixel Detector ("Phase 1 upgrade" in 2017)

- 66 M (124 M) pixels
- $100 \times 150 \mu\text{m}^2$
- 3 (4) barrel layers
- 2x 2 (2x 3) endcap wheels
- $4.7 < r < 10.2 \text{ cm}$
($2.9 < r < 16 \text{ cm}$)

- Si-Strip Detector

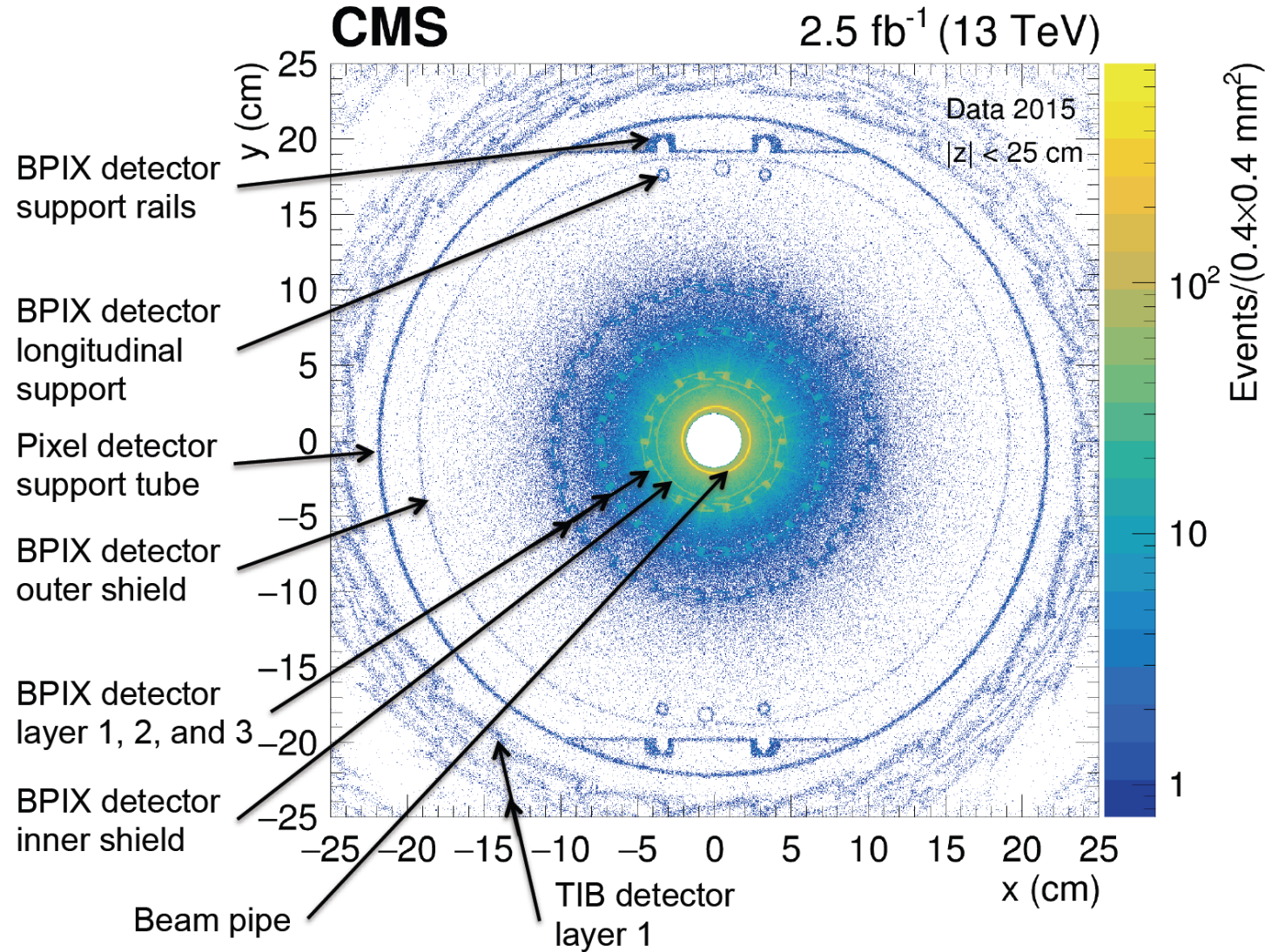
- 10 M strips in 10 layers
- $> 200 \text{ m}^2$ of silicon
- $20 < r < 116 \text{ cm}$
- 80—184 μm pitch



An “X-ray view” of the tracker in operation (2015 data)

Hadrography

- Based on reconstructed vertices from nuclear interactions in the material
- ➔ Detailed map of both sensitive and “dead” material



The LHC: a new level of challenge for detector alignment

- In the beginning, we were entering new territory in terms of tracker complexity.
Even in 2008, it was not entirely clear if/how the problem could be managed
- Very clearly, major methodological developments were necessary
- ➔ A series of three LHC alignment workshops, with experts also from previous experiments, were organized to address these problems

LHC Detector Alignment Workshop 2009

<https://indico.cern.ch/event/50502/>

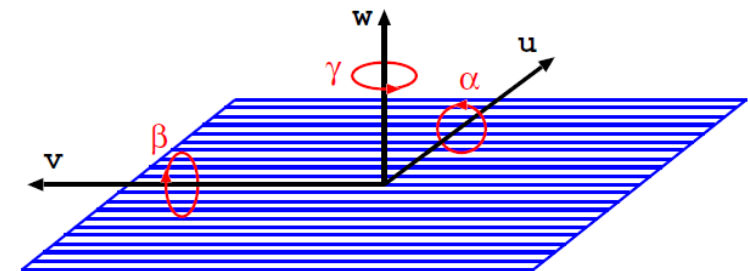
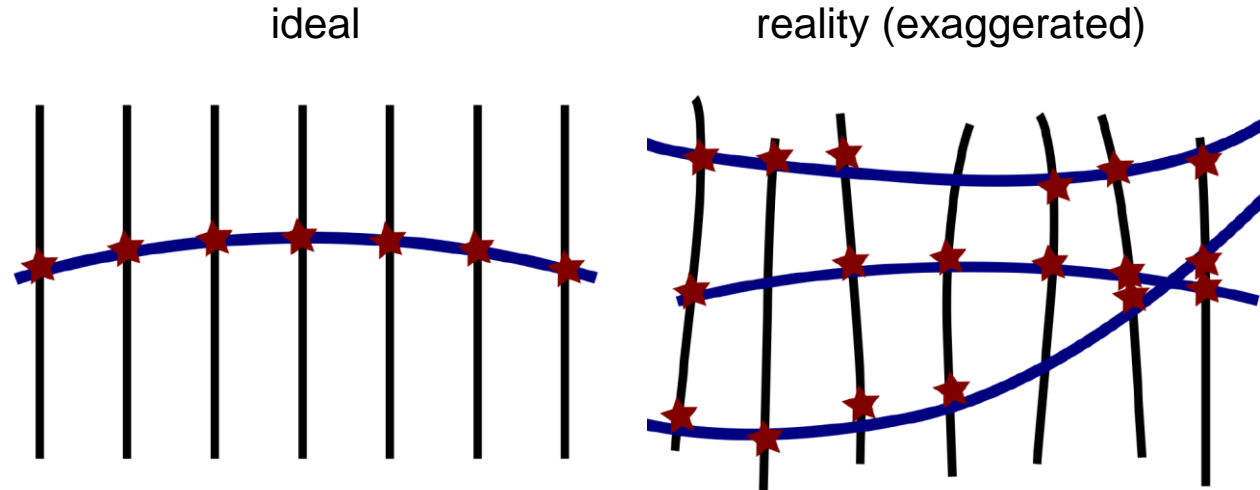


Alignment basics

- For track-based alignment, we use many millions of tracks and study how they match to the hits in the detector modules

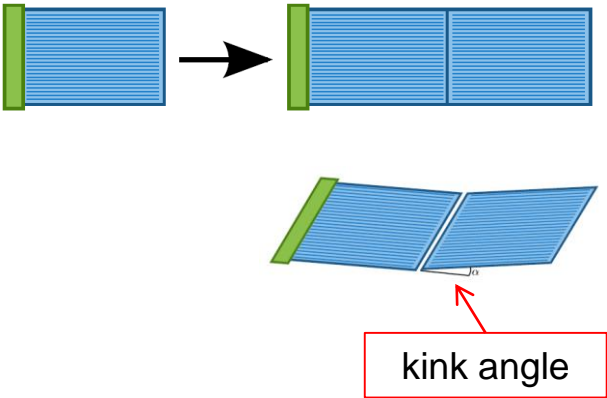
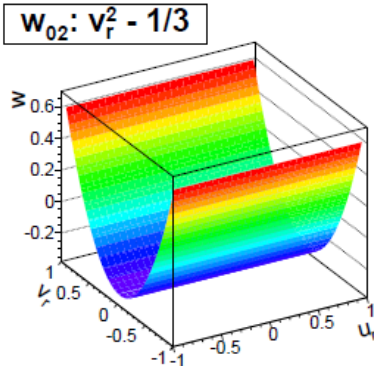
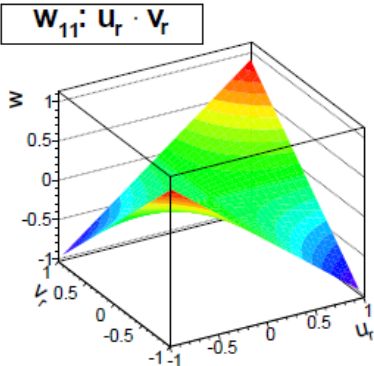
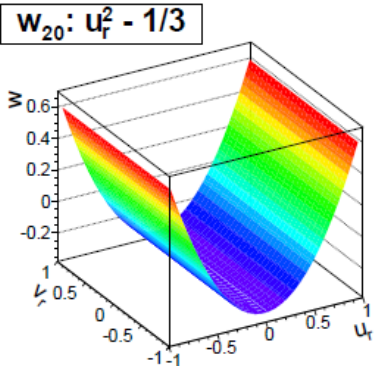
- distance between track and hit: “residual”

- We introduce **corrections** to the module geometry (alignment parameters) such that they match well with the tracks
- Typically, there are three translational and three rotational alignment parameters per module (assuming planar shape)
 - corrections assumed to be relatively small
- But in practice, things are less simple...



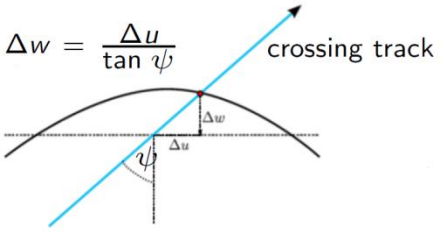
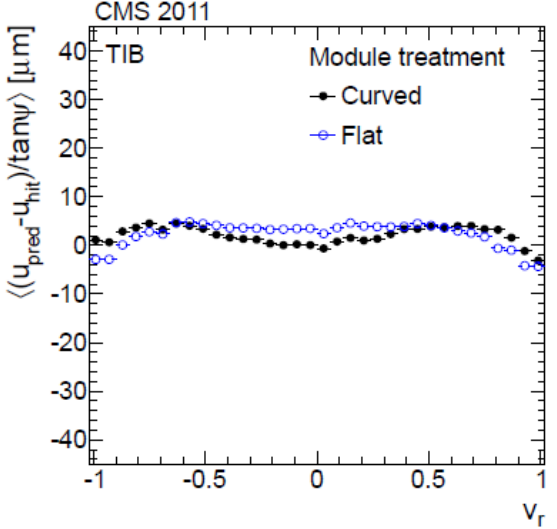
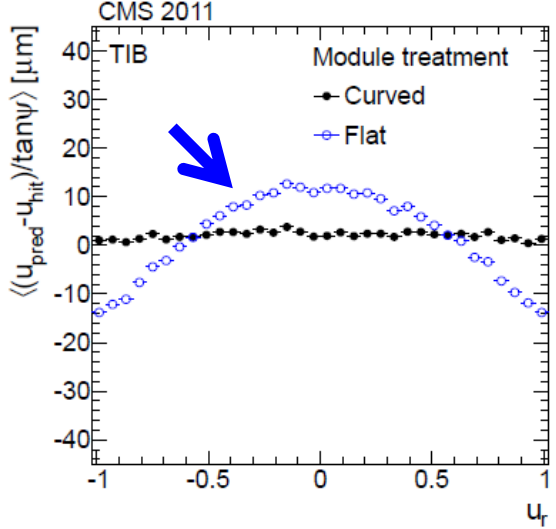
Sensor shape parameters

- In real life, sensors are not planar
 - without correction, coordinate measurement of non-perpendicular tracks is biased
- Introduce three additional **curvature parameters** per sensor
- In addition, "**kink angles**" and offsets are introduced between daisy-chained sensors in TOB modules
- Increases the number of alignment parameters 80,000 → 200,000

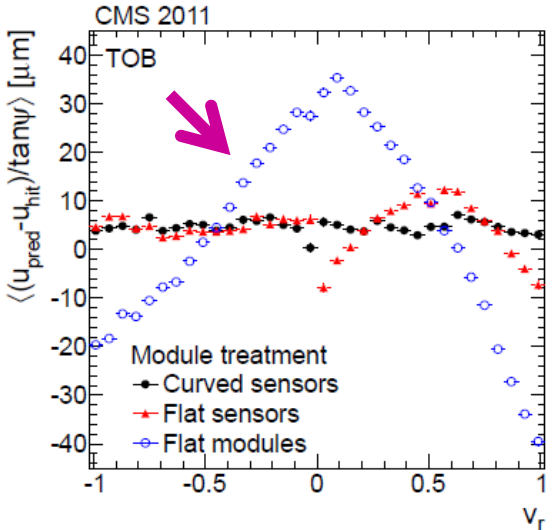
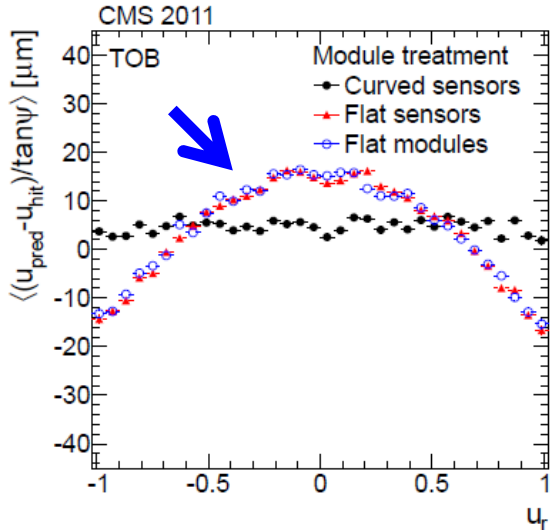


(strongly exaggerated)

Sensor shape parameters (cont'd)



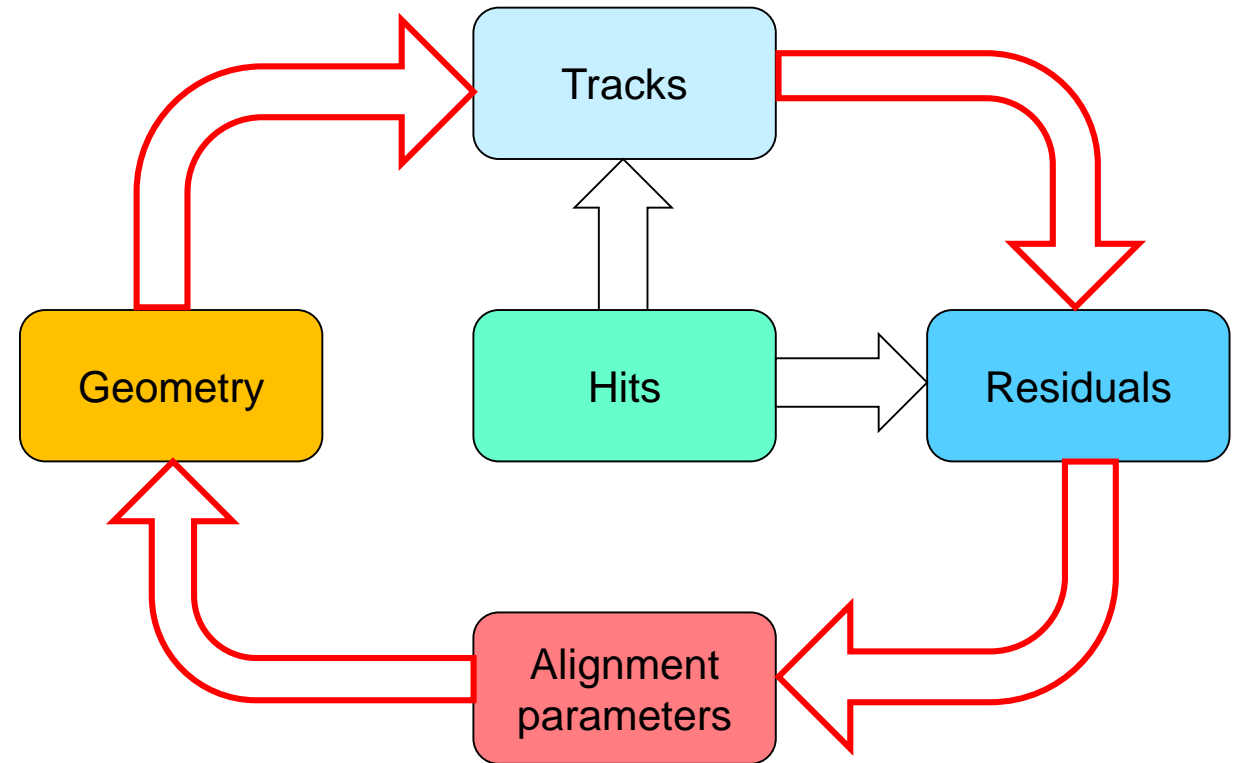
- Curvature in TIB and TOB modules (in direction transverse to strips)



- Kink between sensors in TOB modules (in direction parallel to strips)

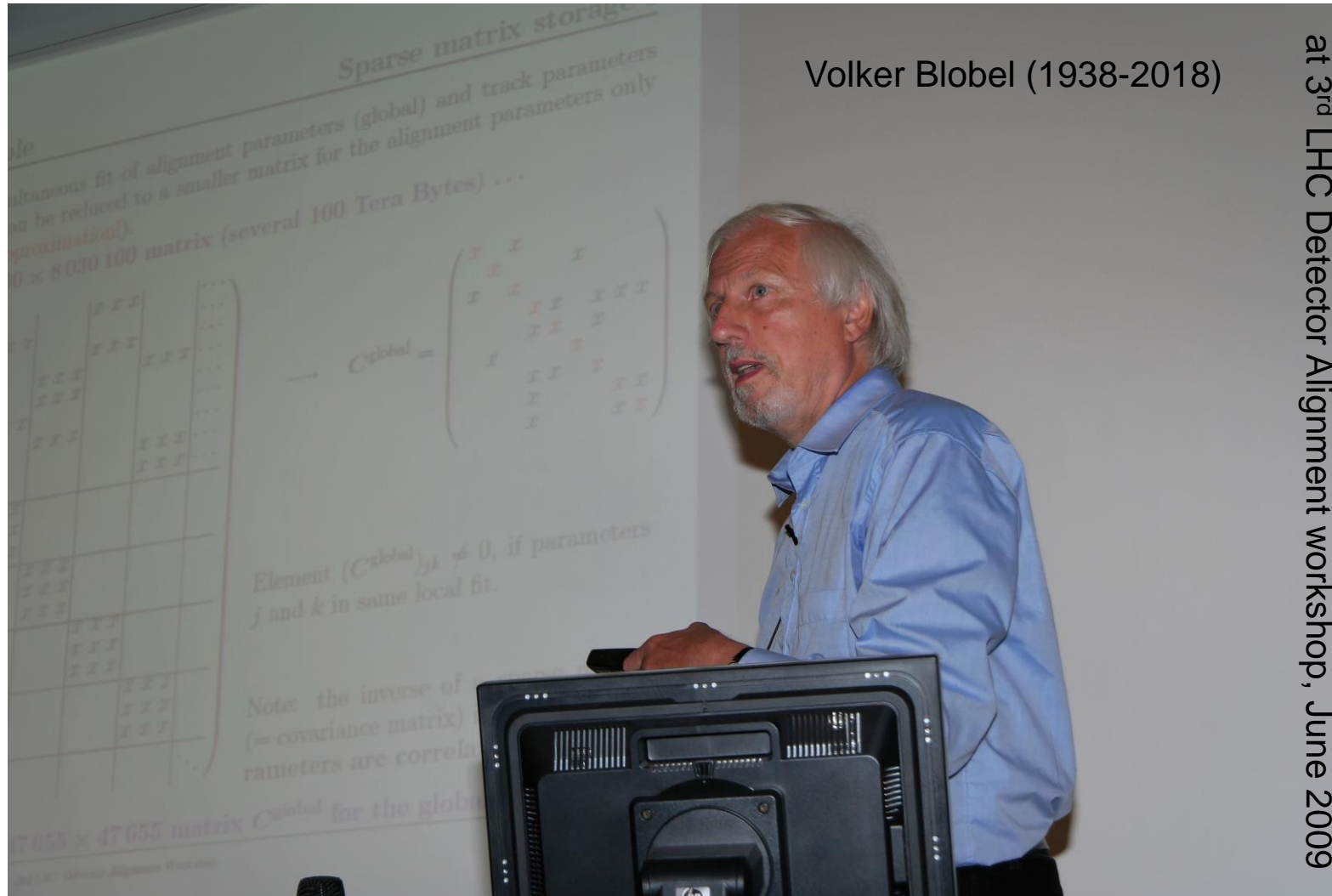
Alignment with residuals

- Straight-forward approach:
 - for each alignable object, evaluate track-hit residuals for all tracks, and compute alignment corrections by means of a least-squares fit
 - this leads to an updated geometry
- The problem:
 - also **tracks will change** when updating geometry
 - need to iterate (this procedure is actually applied in various experiments. in CMS: “HipPy” algorithm)
 - but convergence not guaranteed!
 - in a fit, correlations are important, and no good to ignore them
- The rigorous solution:
 - **simultaneous fit** of all tracks and all alignment parameters



The Millepede idea

A rigorous solution that is computationally manageable



Volker Blobel (1938-2018)

at 3rd LHC Detector Alignment workshop, June 2009

The Millepede idea (cont'd)

- Blobel's example: 1,596,489 tracks (@ 5 parameters); 47,655 alignment parameters
 - >8 M free parameters to be determined → equation system characterized by 8M x 8M matrix (several 100 TB!)

$$C^{\text{total}} = \begin{pmatrix} x & & & & & & & & & & \dots \\ & x & & & & & & & & & \dots \\ & & x & & & & & & & & \dots \\ & & & x & & & & & & & \dots \\ & & & & x & & & & & & \dots \\ & & & & & x & & & & & \dots \\ & & & & & & x & & & & \dots \\ & & & & & & & x & & & \dots \\ & & & & & & & & x & & \dots \\ & & & & & & & & & x & \dots \\ \hline x & & & & & & & & & & \dots \\ x & & & & & & & & & & \dots \\ x & & & & & & & & & & \dots \\ \hline & x & & & & & & & & & \dots \\ & & x & & & & & & & & \dots \\ & & & x & & & & & & & \dots \\ \hline x & & & & & & & & & & \dots \\ x & & & & & & & & & & \dots \\ x & & & & & & & & & & \dots \\ \hline & x & & & & & & & & & \dots \\ & & x & & & & & & & & \dots \\ & & & x & & & & & & & \dots \\ \hline \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix} \quad \rightarrow \quad C^{\text{global}} = \begin{pmatrix} x & & & & & & & & & & \dots \\ & x & & & & & & & & & \dots \\ & & x & & & & & & & & \dots \\ & & & x & & & & & & & \dots \\ & & & & x & & & & & & \dots \\ & & & & & x & & & & & \dots \\ & & & & & & x & & & & \dots \\ & & & & & & & x & & & \dots \\ & & & & & & & & x & & \dots \\ & & & & & & & & & x & \dots \end{pmatrix}$$

- With a smart transformation, using Schur complements, this problem can be reduced to one with a **much smaller matrix** for the alignment parameters only
 - 47,655 x 47,655
 - no approximation involved
 - this is a sparse (!) matrix

<https://indico.cern.ch/event/50502/contributions/1183071/attachments/964111/1368903/cernali.pdf>

The Millepede program

- Millepede (I):
 - since 1998 used in H1 for vertex detector and central jet chamber
 - since 2000 downloadable from the web... adopted by many experiments, still used today
 - used for up to 4,800 alignment parameters
- With LHC on the horizon it became clear that this program could not meet the highest demands
 - for example, CMS:
 - 17,000 modules → ~100,000 alignment parameters in straight-forward implementation
 - number of matrix elements → exceeds largest possible 4 byte integer
 - numerical methods for solving in Millepede I not adequate
 - today's CMS alignment campaigns even exceed 200,000 parameters
- **Development of Millepede-II** → cutting-edge solving of massive linear problems

Millepede-II: computational/numerical technology

- Simply speaking, track-based alignment can be described as solving a huge linear equation system:

$$\begin{array}{ccc}
 \begin{pmatrix} \square & \cdots & \square \\ \vdots & \ddots & \vdots \\ \square & \cdots & \square \end{pmatrix} & \underbrace{\begin{pmatrix} \Delta p_1 \\ \vdots \\ \Delta p_n \end{pmatrix}}_{\text{alignment parameters}} & = & \begin{pmatrix} \square \\ \vdots \\ \square \end{pmatrix} \\
 C' & & & b'
 \end{array}$$

In CMS, C' is typically a matrix with 50,000 - 200,000 rows and columns

- Straight-forward solution (= inversion of the matrix C') only possible for “small” number of parameters

Method	Computing time	Solution type	Error calculation
Inversion (Gauss–Jordan)	$\sim n^3$	Exact	Yes
Cholesky decomposition	$\sim n^3$	Exact	Skipped (for speed)
MINRES [24, 25]	$\sim n^2 \times n_{it}$	Approximate	No

- Very good turnaround thanks to exploitation of matrix sparsity, multithreading, and dedicated large-memory machines

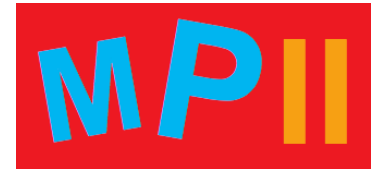
Millepede-II: further information



- Millepede-II is maintained & further developed by Claus Kleinwort (DESY)
 - under the umbrella of the Helmholtz alliance “Physics at the Terascale”
- <https://gitlab.desy.de/claus.kleinwort/millepede-ii>

Contact

For information exchange the **Millepede** mailing list anacentre-millepede2@desy.de should be used.



References

1. A New Method for the High-Precision Alignment of Track Detectors, Volker Blobel and Claus Kleinwort, Proceedings of the Conference on Advanced Statistical Techniques in Particle Physics, Durham, 18 - 22 March 2002, Report DESY 02-077 (June 2002) and [hep-ex/0208021](https://arxiv.org/abs/hep-ex/0208021)
2. Alignment Algorithms, V. Blobel, [Proceedings](#) of the LHC Detector Alignment Workshop, September 4 - 6 2006, CERN
3. Software alignment for Tracking Detectors, V. Blobel, NIM A, 566 (2006), pp. 5-13, [doi:10.1016/j.nima.2006.05.157](https://doi.org/10.1016/j.nima.2006.05.157)
4. A new fast track-fit algorithm based on broken lines, V. Blobel, NIM A, 566 (2006), pp. 14-17, [doi:10.1016/j.nima.2006.05.156](https://doi.org/10.1016/j.nima.2006.05.156)
5. Millepede 2009, V. Blobel, [Contribution](#) to the 3rd LHC Detector Alignment Workshop, June 15 - 16 2009, CERN
6. General Broken Lines as advanced track fitting method, C. Kleinwort, NIM A, 673 (2012), pp. 107-110, [doi:10.1016/j.nima.2012.01.024](https://doi.org/10.1016/j.nima.2012.01.024)

Track inputs used for CMS alignment

➤ “Minimum bias” events

- inclusive trigger
- dominantly QCD, $p > 8$ GeV, $p_T > 1$ GeV

➤ Isolated muons

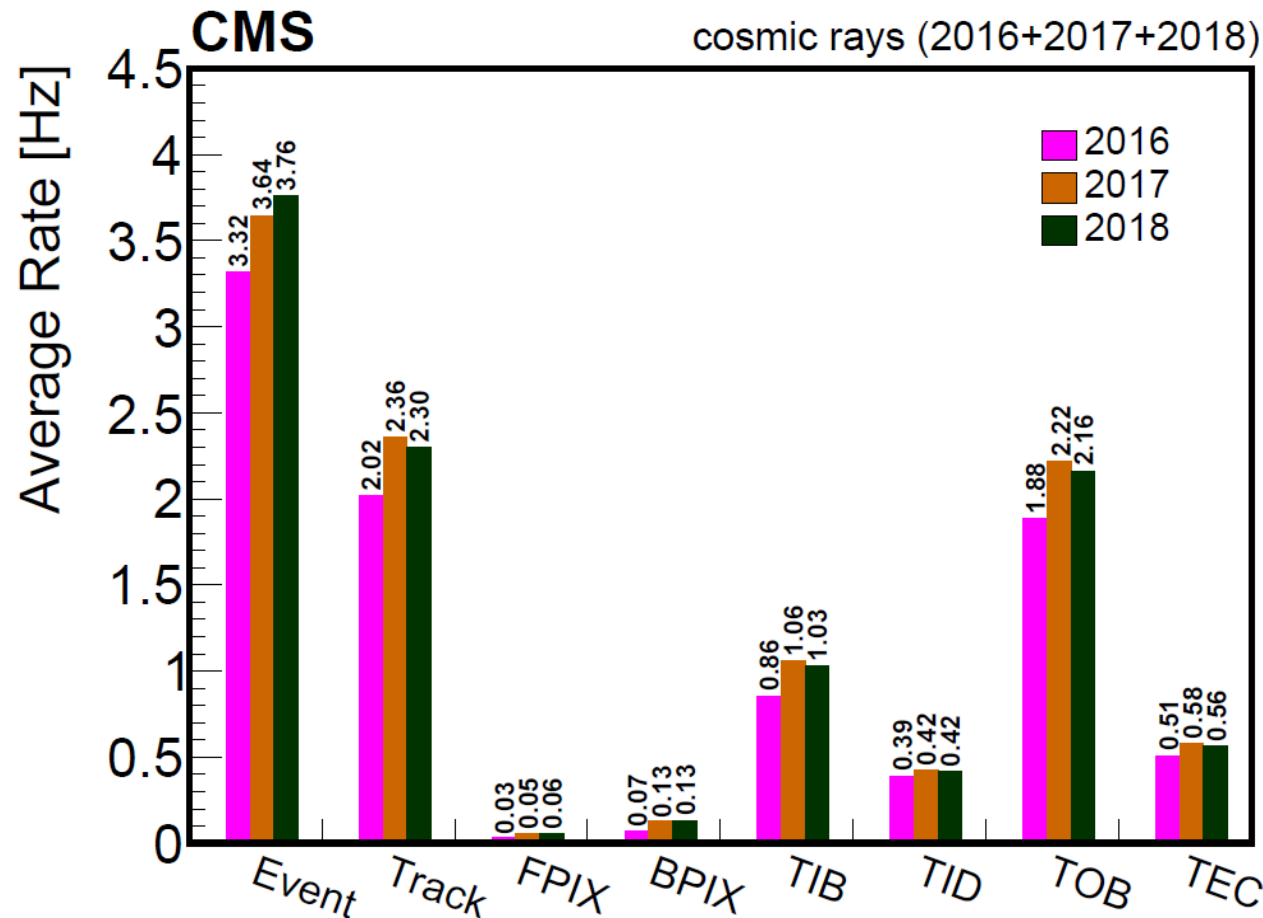
- mostly W decays

➤ Di-muon resonances

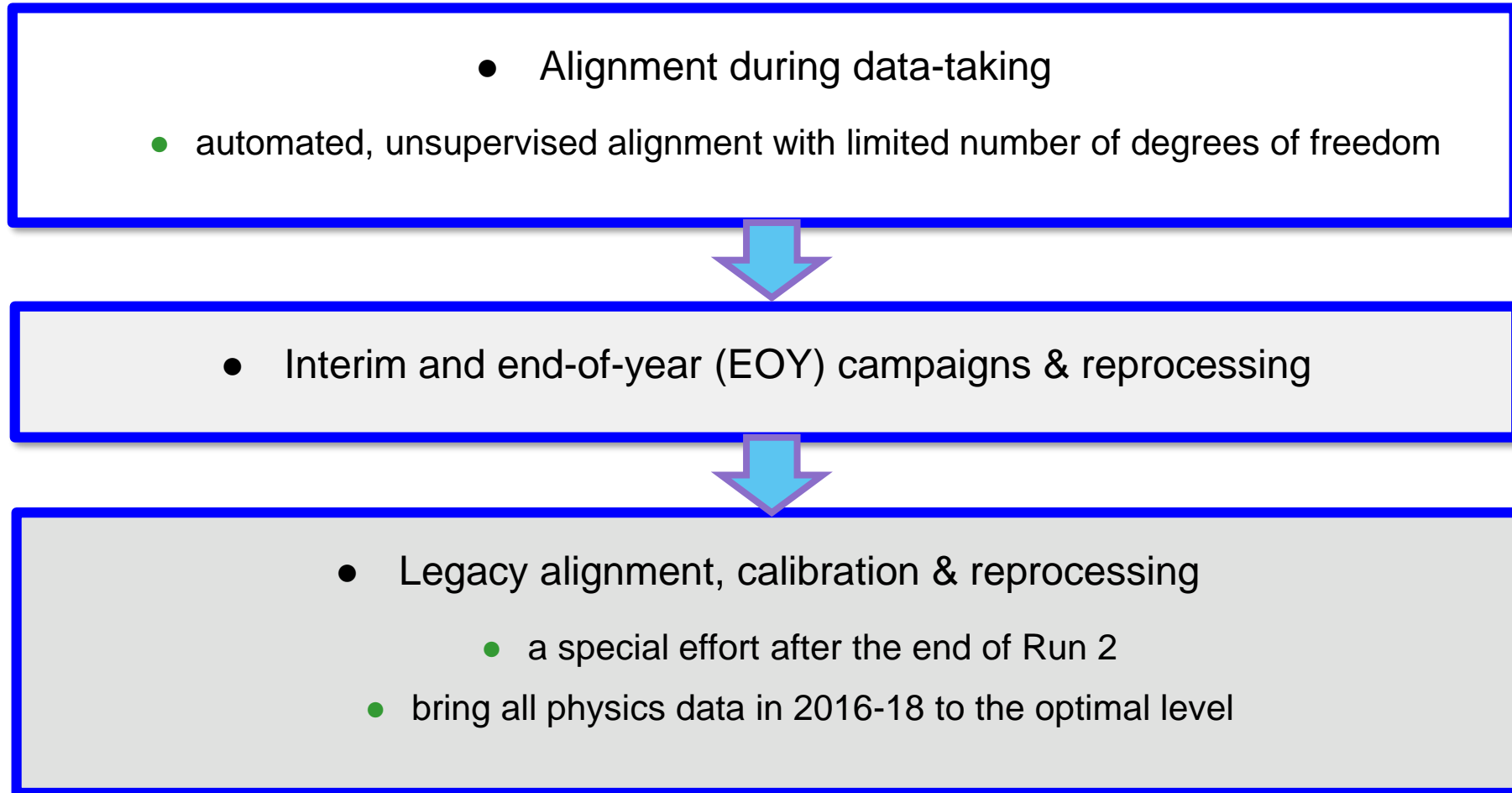
- $Z \rightarrow \mu\mu$, $\Upsilon(1S) \rightarrow \mu\mu$

➤ Cosmic ray muons

- with / without magnetic field
- dedicated, interfill & during collisions



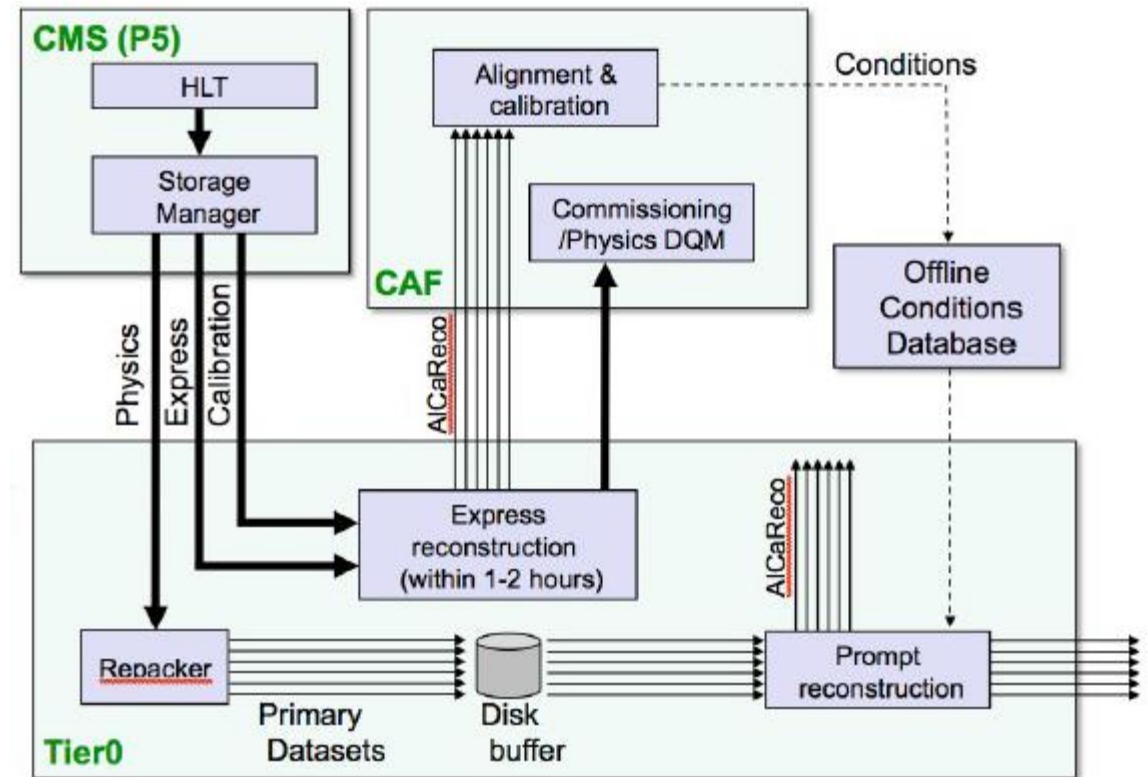
Generation cycles of CMS alignment



Automated alignment

Prompt calibration loop (PCL)

- Restricted to parameters of very high level structures
- Focuses on offsets and angles of pixel tracker:
 - two half-barrels
 - two half cylinders in each endcap
 - 36 parameters in total
- Part of **prompt calibration**, which operates on stream from express reconstruction at the CAF
- **Fast updates** of alignment constants can be provided within 48 hours
 - in time for prompt reconstruction

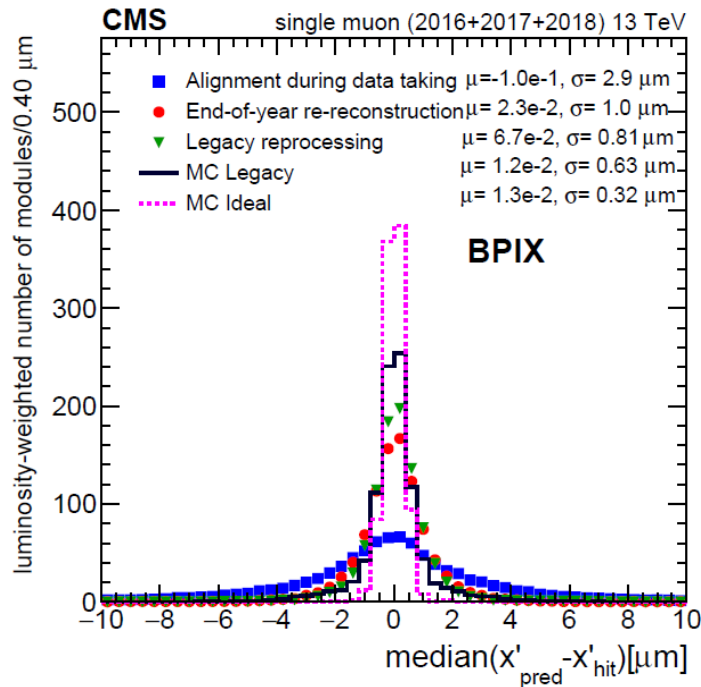


General quality of the alignment

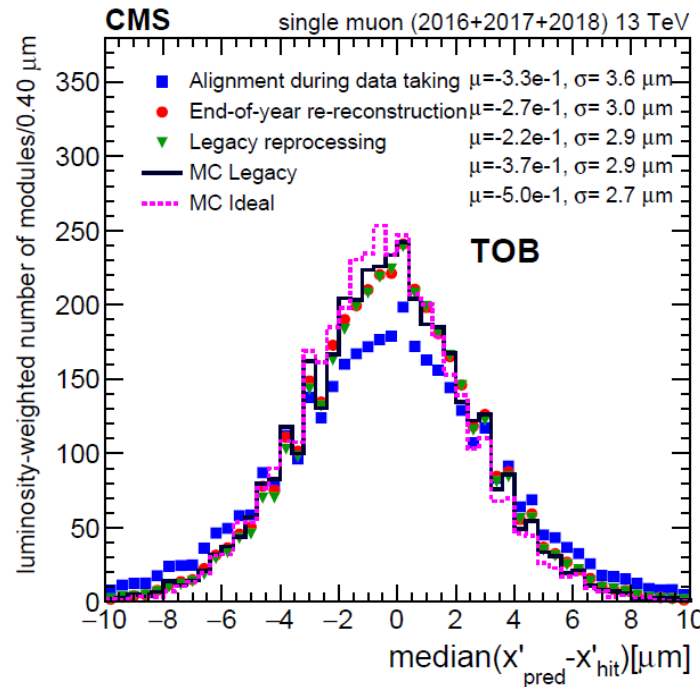
DMR = distributions of the medians of the residual distributions

- Misalignment shows by de-centered distributions of hit residuals → visible in **median**
 - put medians of all residual distributions into one plot → representative of alignment precision
 - expect **narrow peak** for perfect alignment

barrel pixel detector



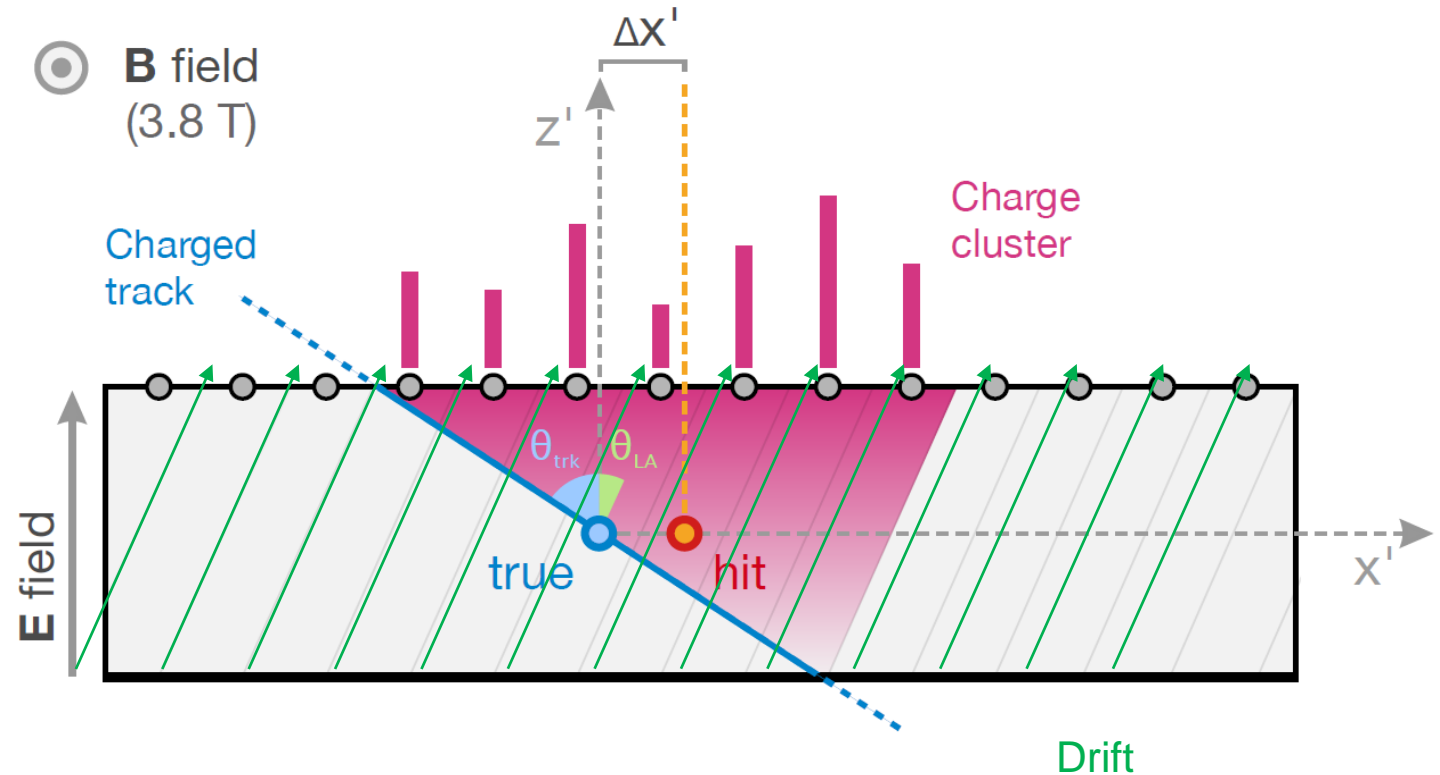
outer barrel strip detector



→ After legacy alignment, **close to ideal**. Also very decent description in MC

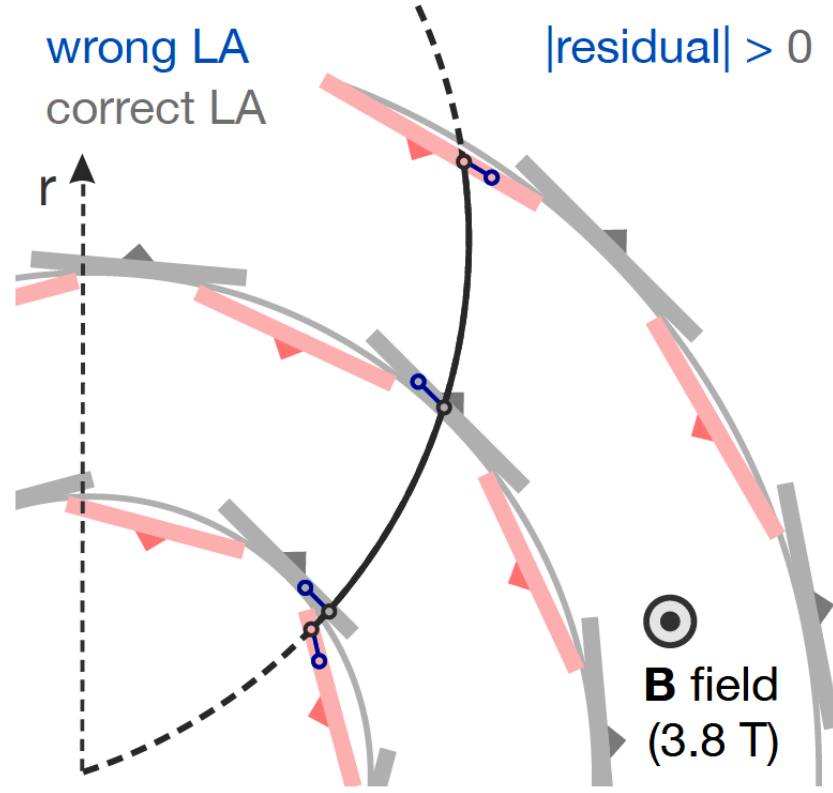
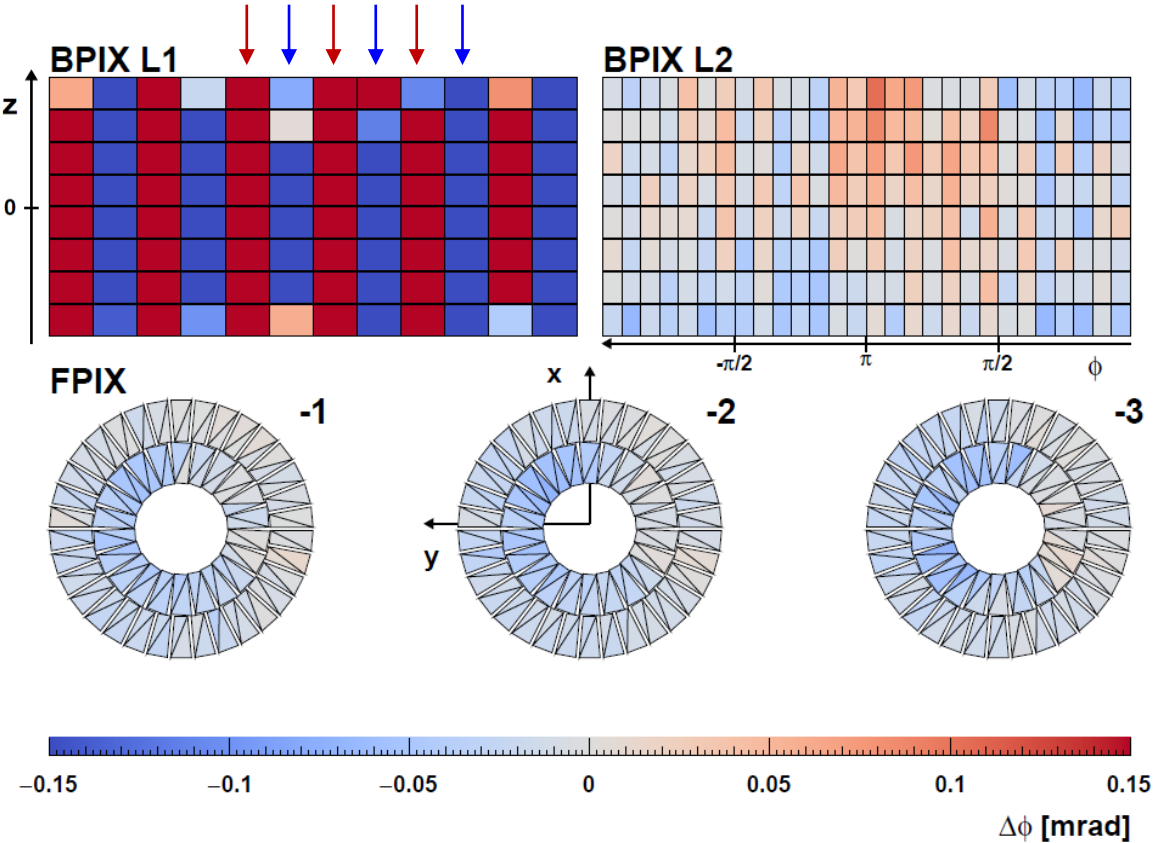
Lorentz angle effects

- Inside the silicon volume, the drift of the charge carriers is **deflected** by the Lorentz angle
 - shifts the apparent cluster position
- While this is addressed in first order by a dedicated Lorentz angle calibration, variations of the Lorentz angle as a function of location and time may result in effects that “look” like a misalignment of the sensor
- Radiation damage may have impact after accumulation of 1 fb^{-1} , while pixel local reconstruction calibration can only be performed after 10 fb^{-1}
- **De-facto corrected** by the alignment procedure



Lorentz angle effects (cont'd)

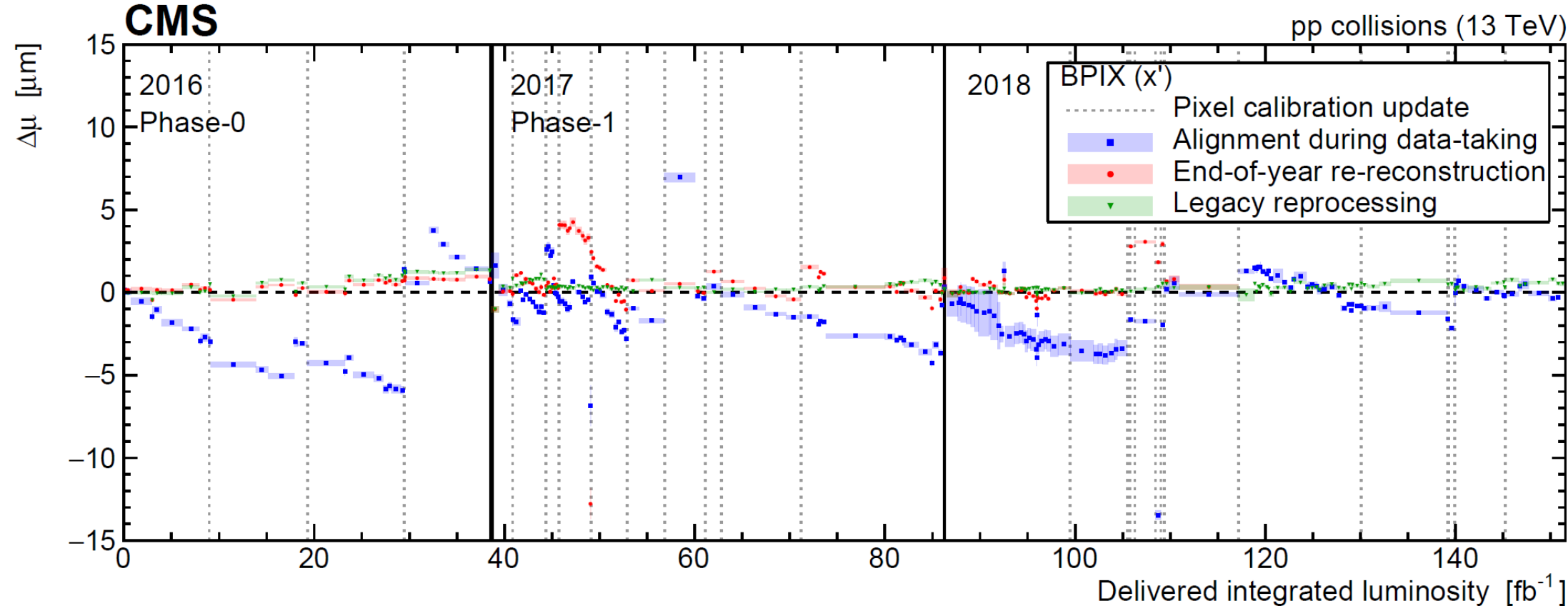
- Large alignment corrections in innermost barrel pixel layer, alternating between adjacent ladders
 - explained by alternating orientations of pixel modules



Lorentz angle effects (cont'd)

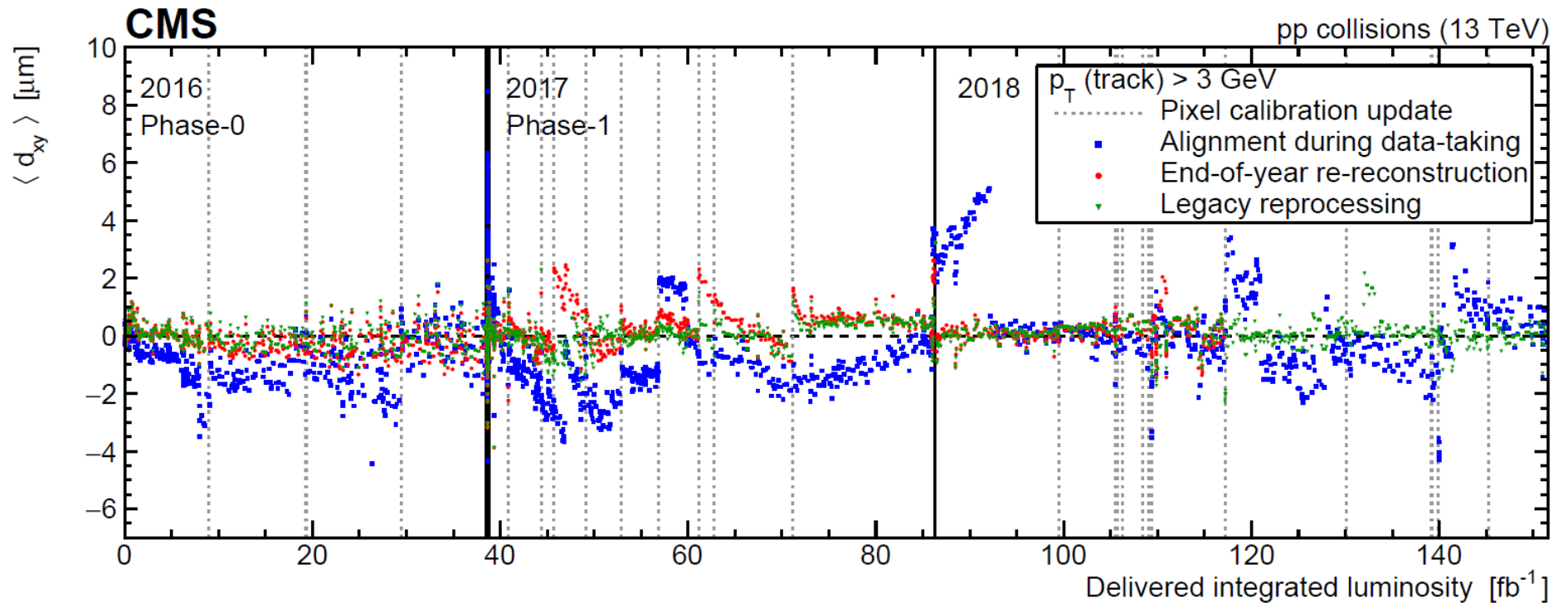
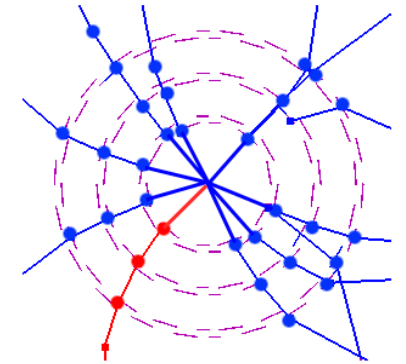
Effect of radiation damage

- Can we see this effect building up? Compare mean values of DMR for modules with electric field pointing inwards and outwards: $\Delta\mu = \mu_{inwards} - \mu_{outwards}$



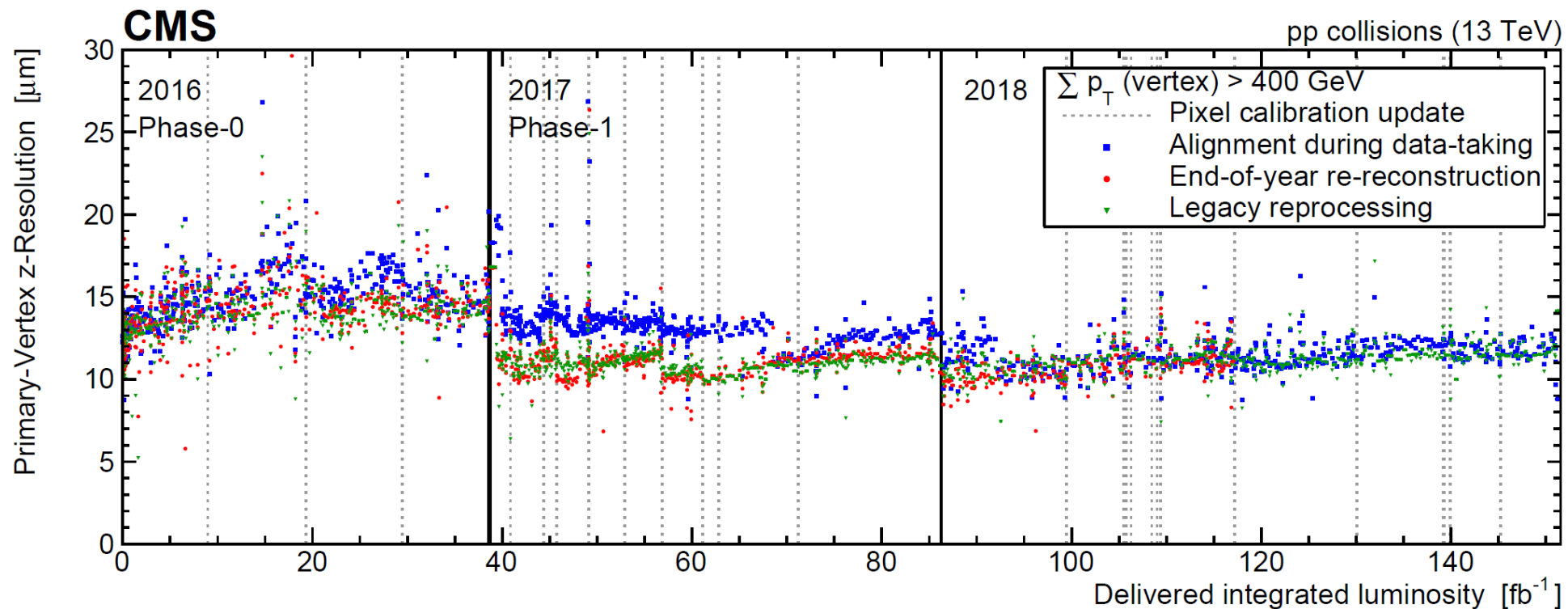
Impact parameter monitoring

- Measured by refitting a primary vertex with one track excluded, and evaluating the latter's impact parameter
 - Initially, in early 2017 suboptimal tracking performance due to commissioning of new pixel tracker
- Generally very good performance after **legacy alignment**



Primary vertex reconstruction performance

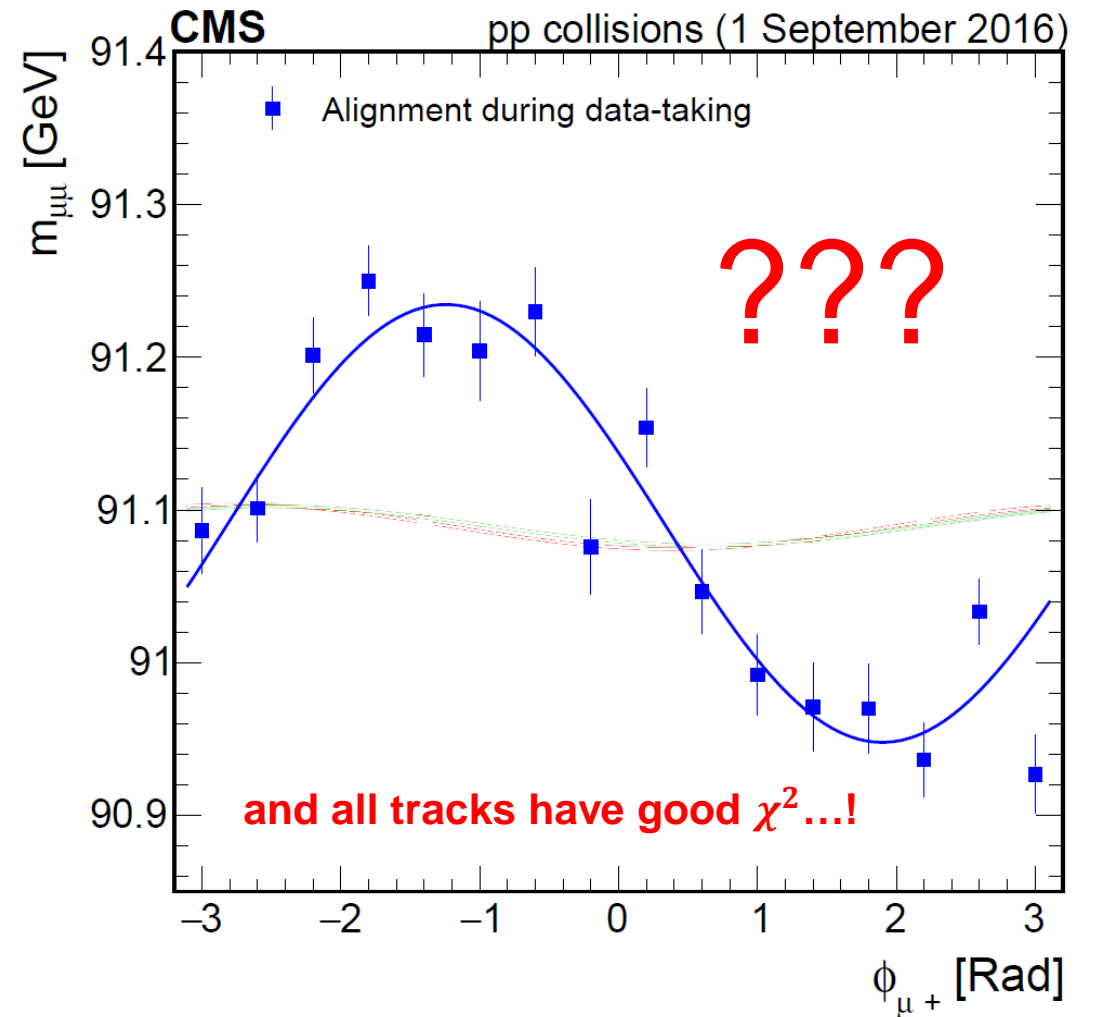
- Measured by splitting a primary vertex into two sub-vertices and studying the residuals
- ➔ After proper alignment, visible improvement due to the new pixel tracker
- Outliers in prompt alignment: short IOV → suboptimal local pixel reconstruction configuration



Systematics of misalignment: weak modes

What is going on here?

- Track-based alignment of trackers with a large number of individual modules (~17,000 in case of CMS) has potential for large systematic effects
- For example, in reconstructed $Z \rightarrow \mu\mu$ decays, position of the mass peak should not (!) depend on azimuth angle of a muon
 - “weak modes”
- **Control of weak modes is one of the greatest challenges in alignment**



What are weak modes?

- As mentioned, track-based alignment can be described as solving a huge linear equation system:

$$\begin{matrix} \bullet & \begin{pmatrix} \square & \cdots & \square \\ \vdots & \ddots & \vdots \\ \square & \cdots & \square \end{pmatrix} & \underbrace{\begin{pmatrix} \Delta p_1 \\ \vdots \\ \Delta p_n \end{pmatrix}}_{\text{alignment parameters}} & = & \begin{pmatrix} \square \\ \vdots \\ \square \end{pmatrix} \\ & C' & & & & b' \end{matrix}$$

In CMS, C' is typically a matrix with 50,000 - 200,000 rows and columns

- The matrix C' reflects also the (inverse) **covariance matrix** of the alignment parameters
- In practice, we may find that some of the **eigenvalues** of this matrix are close to zero → **infinite uncertainty**
- The eigenvalues are associated to **eigenvectors** ("modes"), i.e. linear combinations of alignment parameters, that are only weakly constrained by our computation
 - "weak modes"
 - total χ^2 remains (almost) unchanged when this parameter combination is varied

But why does this happen...?

- A weak mode corresponds to a certain **geometry transformation** (= coherent set of alignment corrections)
- In track-based alignment, we detect misalignment by **incompatibility** of the reconstructed hit positions with the **track model**
- The geometry transformation of a weak mode is such that it transforms **all valid tracks into other valid tracks**
 - track sample is **invariant** under this transformation
 - no change of total χ^2
- The helix trajectory in cylindrical coordinates (track from origin, assuming $d_0 = z_0 = \phi_0 = 0$):
$$r = -2 QR \sin \phi \approx -2 QR \phi$$
$$z = -2 QR \phi \cot \theta$$
- Within validity of $\sin \phi \approx \phi$ approximation, **any linear transformation** in (r, ϕ, z) space results in a weak mode

Helix track parameters:

QR : signed curvature radius

$\cot \theta$: dip angle

d_0 : transverse impact parameter

z_0 : longitudinal " "

Classification and diagnosis of weak modes

For collision tracks

	Δz	Δr	$\Delta \phi$
	<i>z expansion</i>	<i>bowing</i>	<i>twist</i>
vs. z	$\Delta z = \epsilon z$ overlap	$\Delta r = \epsilon r (z_0^2 - z^2)$ overlap	$\Delta \phi = \epsilon z$ $Z \rightarrow \mu\mu$
	<i>telescope</i>	<i>radial</i>	<i>layer rotation</i>
vs. r	$\Delta z = \epsilon r$ cosmics	$\Delta r = \epsilon r$ overlap	$\Delta \phi = \epsilon r$ cosmics
	<i>skew</i>	<i>elliptical</i>	<i>sagitta</i>
vs. ϕ	$\Delta z = \epsilon \cos(\phi + \phi_0)$ cosmics	$\Delta r = \epsilon r \cos(2\phi + 2\phi_0)$ cosmics	$\Delta \phi = \epsilon \cos(\phi + \phi_0)$ cosmics

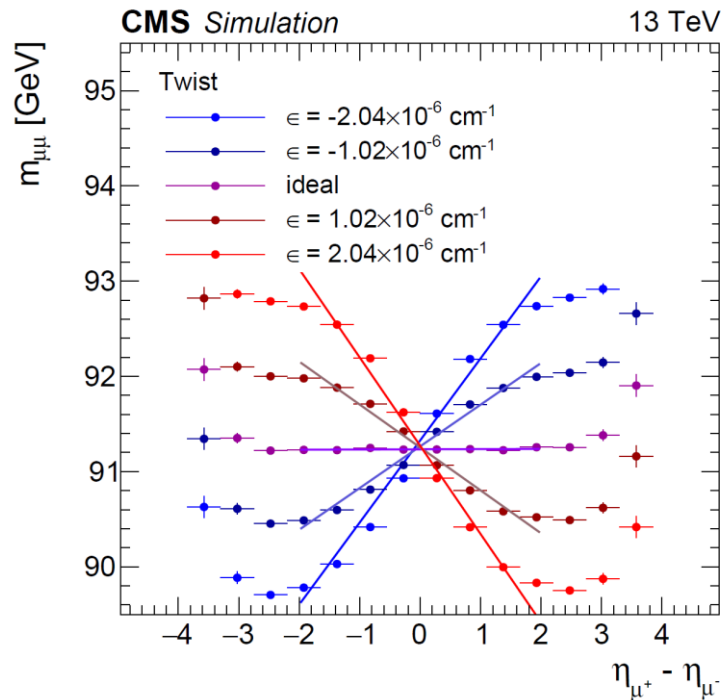
- Overlap validation: check relative hit positions in sensor overlaps (not shown)
- Cosmics validation: split cosmic muon track by hemispheres, compare parameters of sub-tracks
- $Z \rightarrow \mu\mu$ validation: check for dependence of Z mass peak on muon parameters

Classification and diagnosis of weak modes (cont'd)

Test with simulation: a few examples

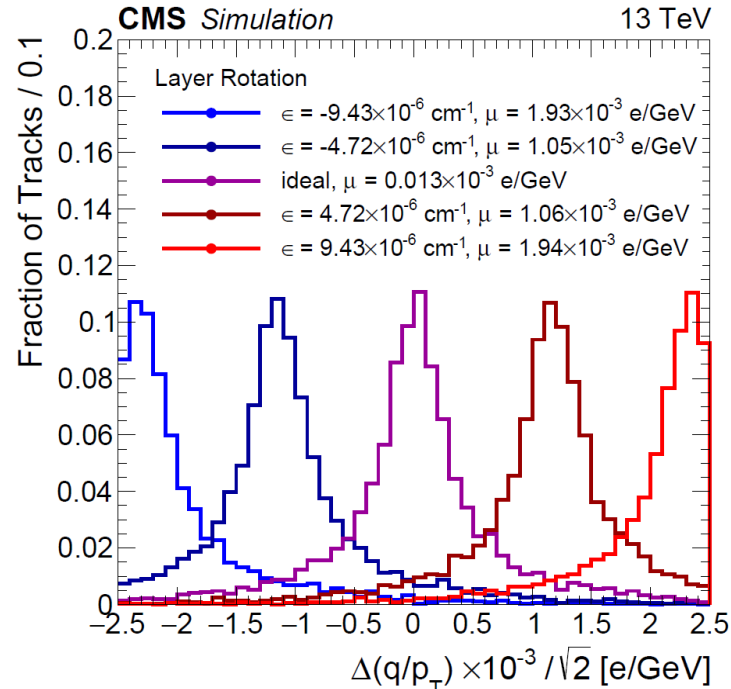
$Z \rightarrow \mu\mu$ validation

Twist: $m_{\mu\mu}$ vs $\Delta\eta$

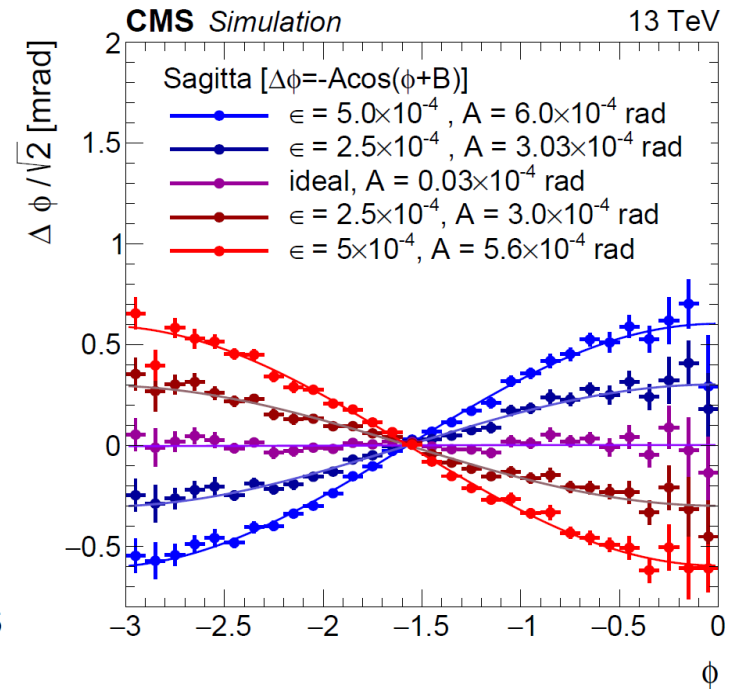


Cosmic ray track validation („cosmic splitting“)

Layer rotation: distribution of $\Delta(\frac{q}{p_T})$



Sagitta: $\Delta\phi$ vs ϕ

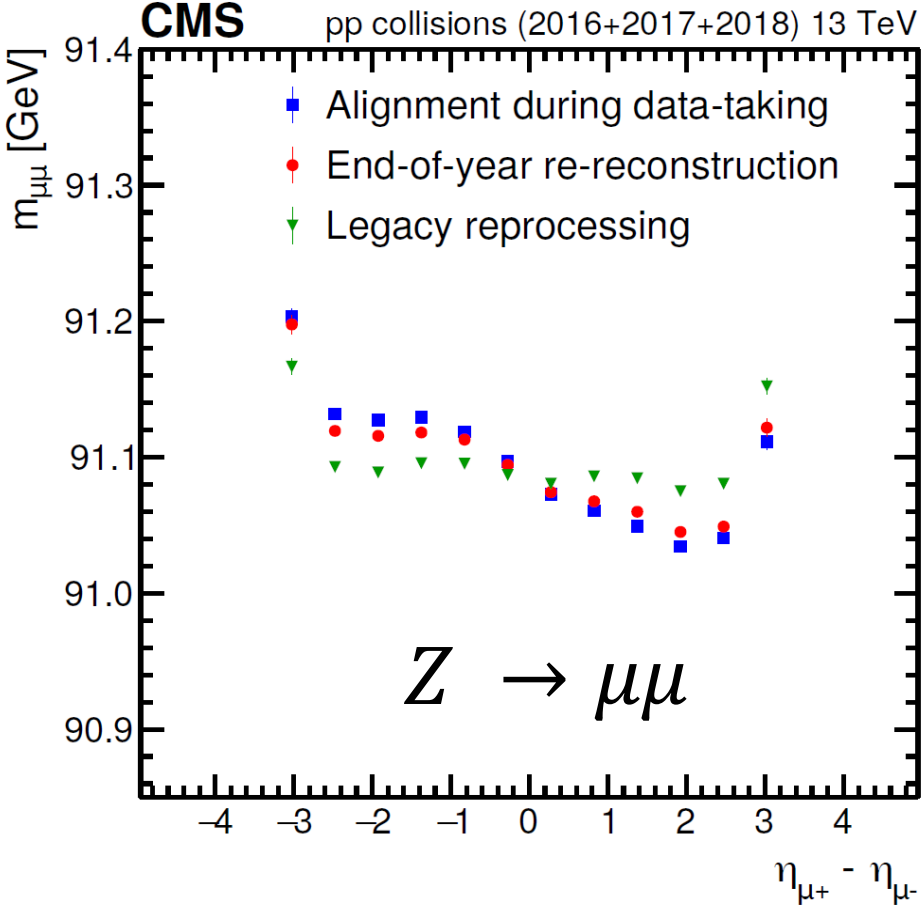
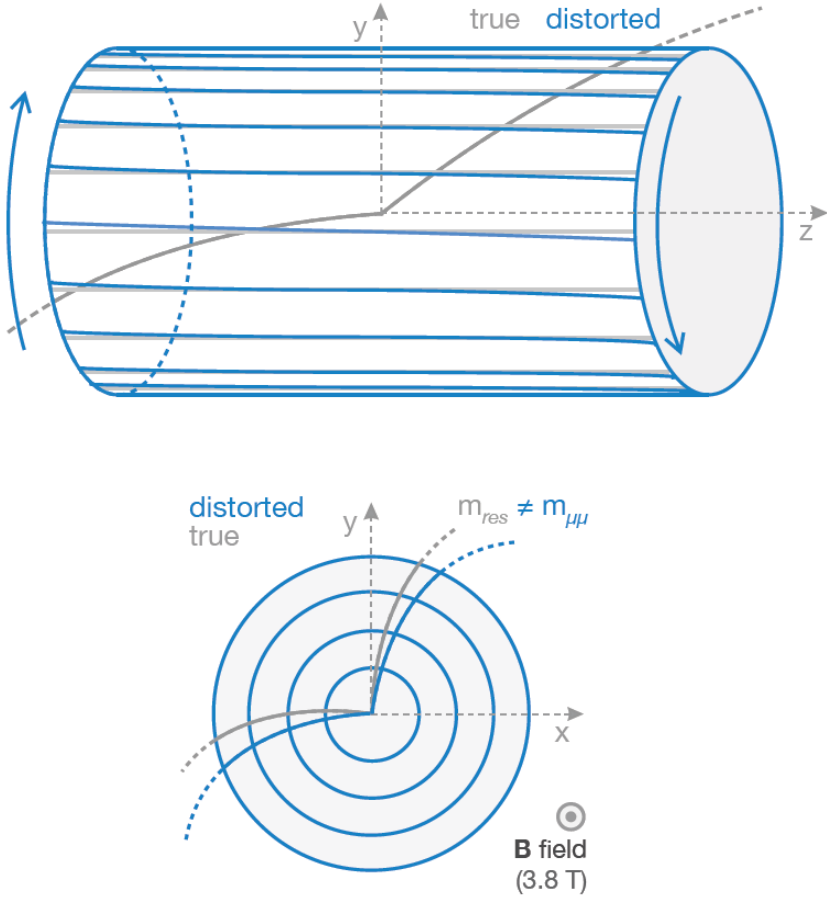


➔ Demonstrates the power of $Z \rightarrow \mu\mu$ and cosmic ray events to identify & control weak modes

How to control weak modes: a strategy

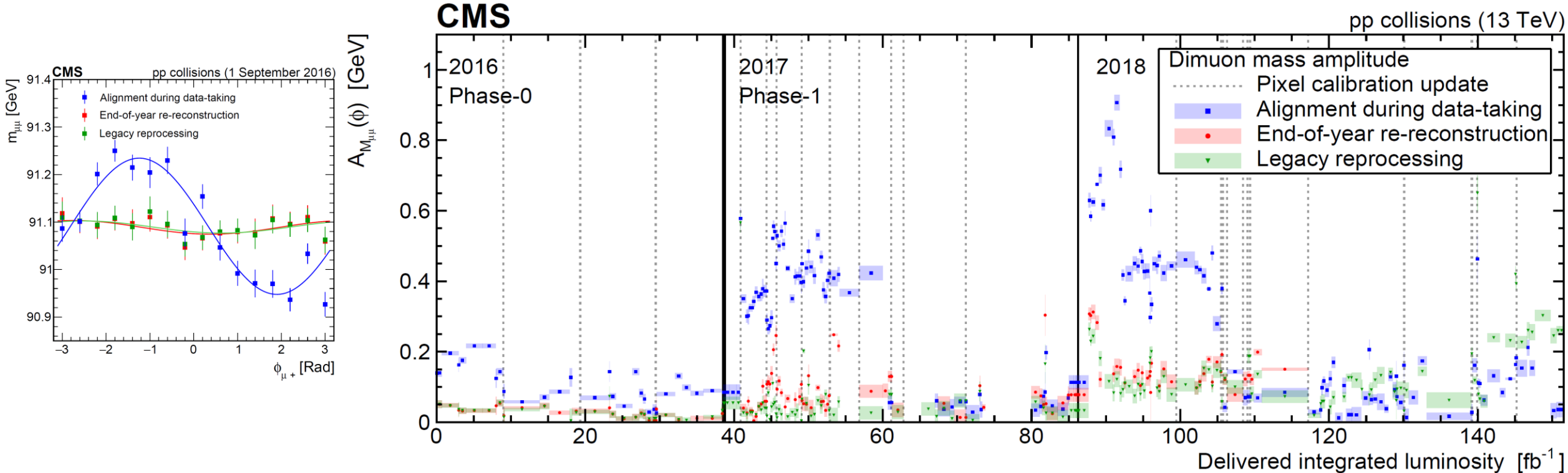
- Include tracks in the alignment which do not pass through the detector center
 - ➔ cosmic muons, recorded both with magnetic field on and off
- Include track combinations having mass and vertex constraints
 - $Z \rightarrow \mu\mu$
 - $\Upsilon(1S) \rightarrow \mu\mu$
- If all else fails: apply counter-transformation in form of a constraint

Practical example: correction of a twist weak mode



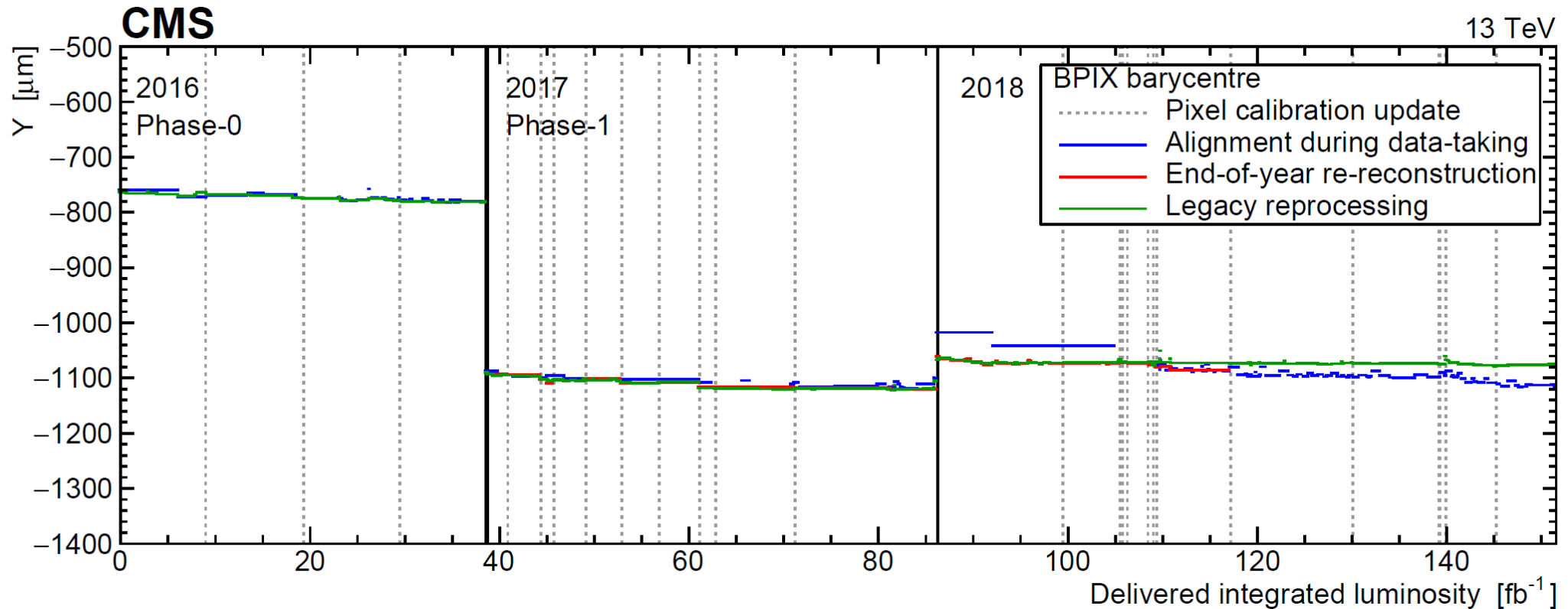
- Sizable twist in alignment during data-taking → resolved in legacy alignment

Dimuon mass validation: evolution with time



➔ Large initial amplitudes in data-taking alignment are resolved in the end-of-year and legacy alignment

Barycenter of barrel pixel detector

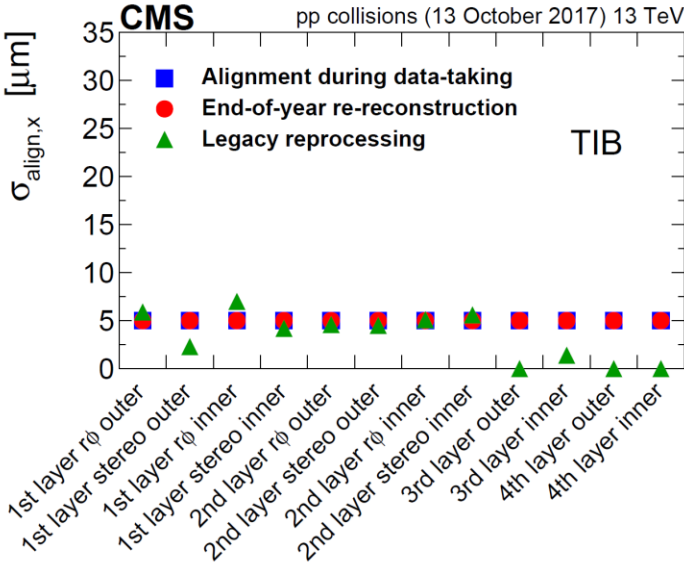
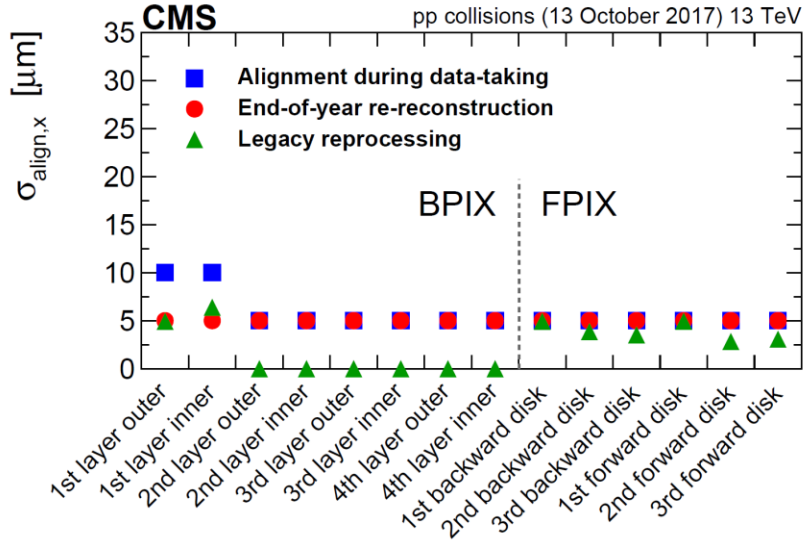


- Very good stability (at level of few microns)
- Changes in winter shutdowns due to (re-)insertions of pixel tracker
- Reprocessing cures an artificial drop due to radiation damage effects

How precise are the alignment parameters?

APU = Alignment parameter uncertainty

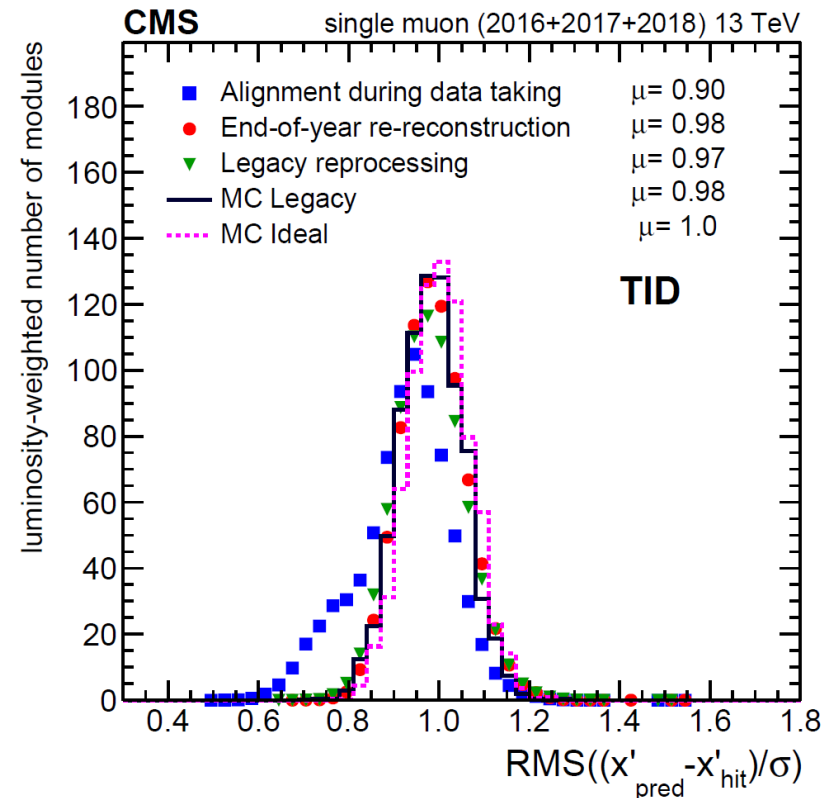
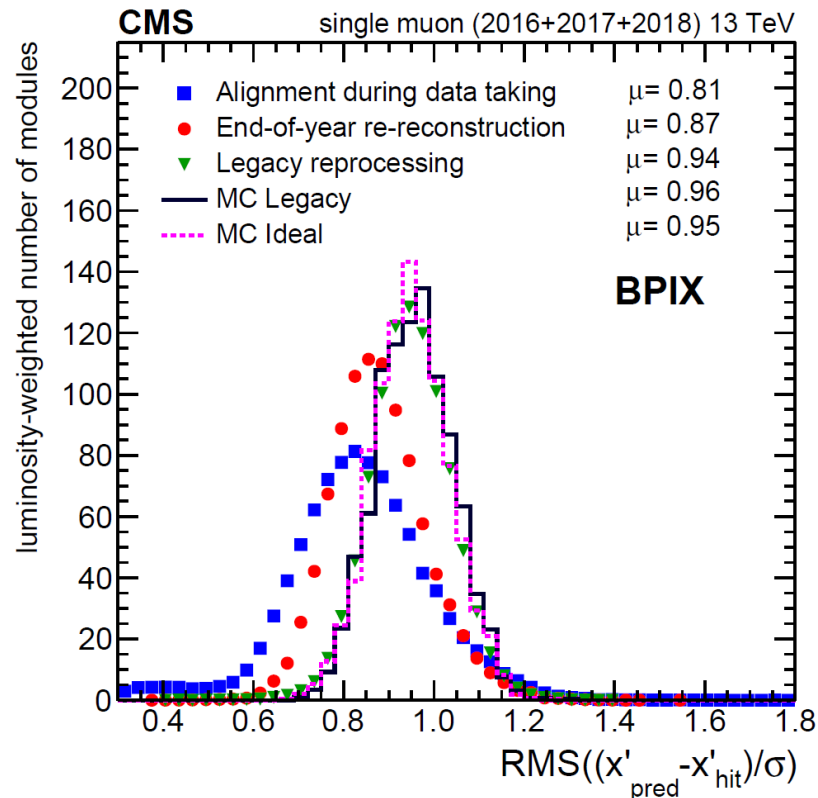
- Direct error estimation by matrix inversion usually not feasible, since matrix too large
- Obtained by studying distributions of normalized residuals: $\frac{x'_{hit} - x'_{track}}{\sigma}$, where $\sigma = \sqrt{\sigma_{hit}^2 + \sigma_{track}^2 + \sigma_{align}^2}$
 - adjust σ_{align} such that distributions become unit normal \rightarrow iterative procedure



\rightarrow Very good control of alignment precision

A direct look at normalized residuals

- Important test: check RMS width of the normalized residuals
 - After the legacy alignment, it is centered close to 1, and agrees well with MC
- shows both correct alignment and correct assignment of alignment parameter uncertainties



Summary

- Alignment is a key driver for physics performance
- Methodology has evolved enormously to a new level, to meet LHC challenges
- Powerful alignment workflows are in place
 - still a huge effort year by year; always new challenges surfacing
- For Run 3, first alignments have already been produced from cosmic runs, and even first collisions
 - start thinking about alignment Phase 2 tracker

⇒ Alignment is not static...

it continues to be challenging... *and interesting!*

Further reading

- V. Blobel and C. Kleinwort, "A New method for the high precision alignment of track detectors", <https://inspirehep.net/conferences/973991>, <https://arxiv.org/abs/hep-ex/0208021>
- CMS Collaboration, "Description and performance of track and primary-vertex reconstruction with the CMS tracker", 2014 *JINST* **9** P10009
- CMS Collaboration, "Alignment of the CMS tracker with LHC and cosmic ray data", 2014 *JINST* **9** P06009
- CMS Collaboration, "Strategies and performance of the CMS silicon tracker alignment during LHC Run 2", [arxiv:2111.08757](https://arxiv.org/abs/2111.08757) (2021), accepted for publication in *NIM A*
- R. Mankel, "Pattern recognition and event reconstruction in particle physics experiments", *Rept.Prog.Phys.* **67** (2004) 553

Backup

The sparse matrix C of a simultaneous fit of alignment parameters (global) and track parameters (local) is a large matrix, that can be reduced to a smaller matrix for the alignment parameters only using Schur complements (**no approximation!**).

The matrix C^{total} , a 8 030 100 × 8 030 100 matrix (several 100 Tera Bytes) ...

$$C^{\text{total}} = \left(\begin{array}{cccc|cccc|cccc|cccc|c}
x & & & & & & & & & & & & & & & \dots \\
& x & & & & x & x & x & & & & x & x & x & & \dots \\
& & x & & & & & & & & & & & & & \dots \\
& & & x & & & x & x & x & & & x & x & x & & \dots \\
& & & & x & & & & & & & & & & & \dots \\
& & & & & x & x & x & & & x & x & x & & & \dots \\
& & & & & & & & & & & & & & & \dots \\
& & & & & & & & & & x & x & x & & & \dots \\
& & & & & & & & & & x & x & x & & & \dots \\
& & & & & & & & & & & & & & & \dots \\
x & & & & & x & x & x & & & & & & & & \dots \\
x & & & & & x & x & x & & & & & & & & \dots \\
x & & & & & x & x & x & & & & & & & & \dots \\
& & x & & & & & & & & & x & x & x & & \dots \\
& & & x & & & & & & & & & x & x & x & \dots \\
& & & & x & & & & & & & & & x & x & x \\
& & & & & & & & & & & & & & & \dots \\
& & & & & & & & & & & & & & & \dots \\
& & & & & & & & & & & & & & & \dots \\
& & & & & & & & & & & & & & & \dots
\end{array} \right) \rightarrow C^{\text{global}} = \left(\begin{array}{cccc|cccc}
x & & & & & & & \\
& x & & & & & & \\
& & x & & & & & \\
& & & x & & & & \\
& & & & x & & & \\
& & & & & x & x & x \\
& & & & & & x & x & x \\
& & & & & & & & x & x & x \\
& & & & & & & & & x & x & x \\
& & & & & & & & & & x & x \\
& & & & & & & & & & & x & x
\end{array} \right)$$

Element $(C^{\text{global}})_{jk} \neq 0$, if parameters j and k in same local fit.

Note: the inverse of a sparse matrix (= covariance matrix) is dense; all parameters are correlated!

...is reduced to a (sparse) 47 655 × 47 655 matrix C^{global} for the global parameters.

NIM A 461 (2001) 162–167

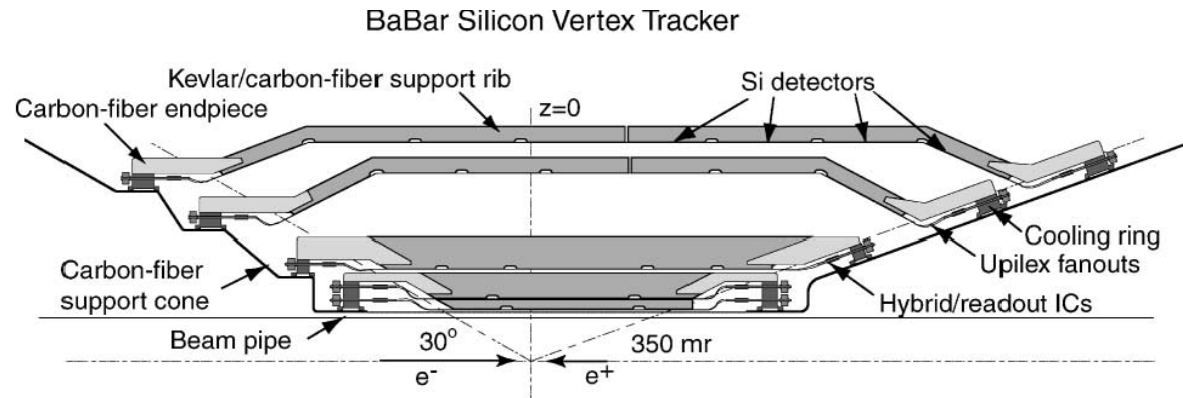


Fig. 1. SVT layout: rz cross-sectional view. The modules of layers 4 and 5 are “bent” towards the beam axis to increase angular coverage and to reduce the crossing angle of low-angle tracks. Note the asymmetry of the detector with respect to $z = 0$.

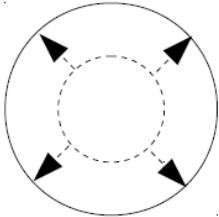
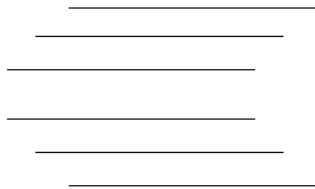
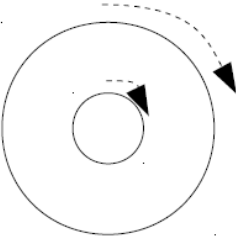
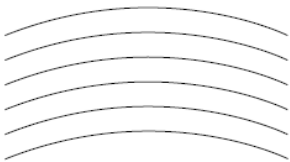
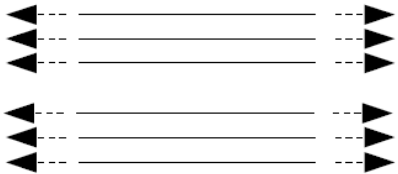
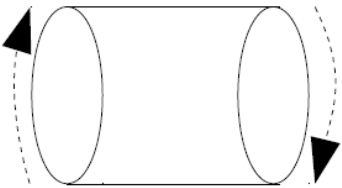
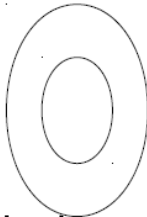

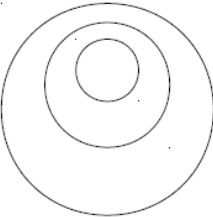
Table 1
Layer structure of the BaBar SVT

Layer	Radius (mm)	Modules/layer	Si Wafers/module	ϕ pitch (μm)	z pitch (μm)
1	32	6	4	50 or 100	100
2	40	6	4	55 or 110	100
3	54	6	6	55 or 110	100
4a	124	8	7	100	210
4b	127	8	7	100	210
5a	140	9	8	100	210
5b	144	9	8	100	210

Classification of weak modes

For collision tracks

-

	Δr	Δz	$r \Delta \phi$
$\propto r$	 <p>radial</p>	 <p>telescope</p>	 <p>curl</p>
$\propto z$	 <p>bow</p>	 <p>z expansion</p>	 <p>twist</p>
$\propto \phi$	 <p>elliptical</p>	 <p>skew</p>	 <p>sagitta</p>

Adapted from: Alessio Bonato,
<https://indico.cern.ch/getFile.py/access?contribId=11&sessionId=2&resId=0&materialId=slides&confId=137973>

Weak modes and track parameter transformations

Twist:

- $\phi \rightarrow \phi + k z$
- $\cot \theta \rightarrow \cot \theta$
- $\frac{1}{QR} \rightarrow \frac{1}{QR} - 2k \cot \theta$

r expansion:

- $r \rightarrow r + k r$
- $\cot \theta \rightarrow \frac{1}{1+k} \cot \theta$
- $QR \rightarrow (1+k) QR$

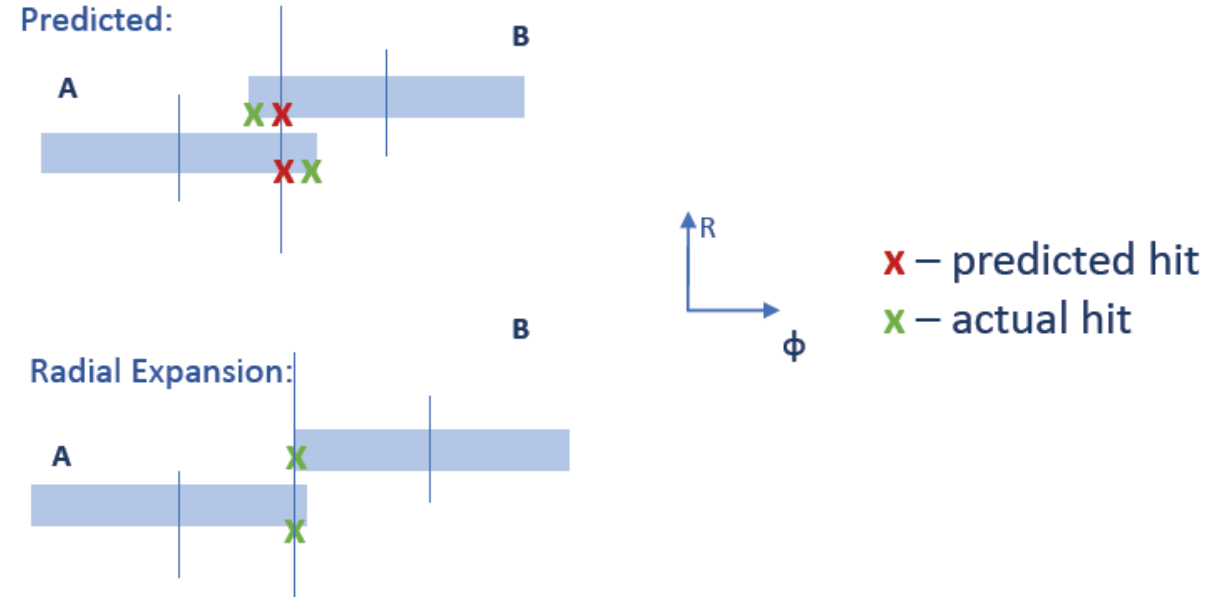
z expansion:

- $z \rightarrow z + k z$
- $\cot \theta \rightarrow (1+k) \cot \theta$
- $QR \rightarrow QR$

- In general, weak modes cause track parameters (momentum, direction) to change
→ **affect physics**

Overlap validation

Overlap validation (radial and z expansion, bowing)



- Millepede-II timing

Table 3: Examples of PEDE wall time (time taken from start of the program to end) for some larger alignment campaigns on a dedicated test machine (Intel Xeon E5-2667 @ 3.2 GHz, 256 GB memory @ 51 GB/s).

Number of global parameters	Number of constraints	Number of records	Matrix size [GB] (sparse)	Wall time [s] (10 threads)
217500	138	4.46×10^7	44	8.4×10^3
213900	1782	2.90×10^7	85	6.8×10^3
576000	942	5.20×10^7	218	4.4×10^4