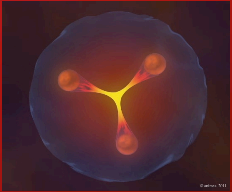
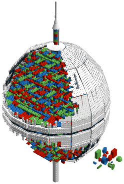
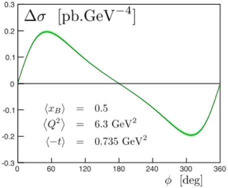
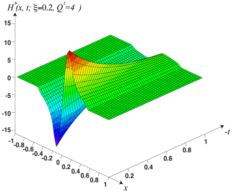
DE LA RECHERCHE À L'INDUSTRIE



Covariant extension of the GPD overlap representation







Getting to Grips with QCD | Hervé MOUTARDE

Sep. 19$^{\text{th}}$, 2018

universite PARIS-SACLAY

1. What are the theoretical constraints on GPDs?
2. How to generically build GPD models satisfying all theoretical constraints?
3. How can we compare models to experimental data?

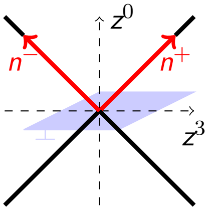
Theoretical constraints on GPDs

Spin-0 Generalized Parton Distribution.

Definition and simple properties.

$$H^q_\pi(x, \xi, t) = \frac{1}{2} \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \left\langle \pi, P + \frac{\Delta}{2} \middle| \bar{q}\left(-\frac{z}{2}\right) \gamma^+ q\left(\frac{z}{2}\right) \middle| \pi, P - \frac{\Delta}{2} \right\rangle_{\substack{z^+=0 \\ z_\perp=0}}$$

with $t = \Delta^2$ and $\xi = -\Delta^+/(2P^+)$.



References

Müller *et al.*, Fortschr. Phys. **42**, 101 (1994)
Ji, Phys. Rev. Lett. **78**, 610 (1997)
Radyushkin, Phys. Lett. **B380**, 417 (1996)

- PDF **forward limit**

$$H^q(x, 0, 0) = q(x)$$

Spin-0 Generalized Parton Distribution.

Definition and simple properties.

$$H^q_\pi(x, \xi, t) \quad = \quad \frac{1}{2} \int \frac{\mathrm{d}z^-}{2\pi} \, e^{ixP^+z^-} \left\langle \pi, P + \frac{\Delta}{2} \middle| \bar{q}\left(-\frac{z}{2}\right) \gamma^+ q\left(\frac{z}{2}\right) \middle| \pi, P - \frac{\Delta}{2} \right\rangle_{\substack{z^+=0 \\ z_\perp=0}}$$

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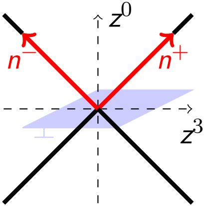

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Ji, Phys. Rev. Lett. **78**, 610 (1997)
Radyushkin, Phys. Lett. **B380**, 417 (1996)

- PDF **forward limit**
- Form factor **sum rule**

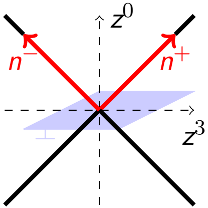
$$\int_{-1}^{+1} dx \, H^q(x, \xi, t) = F^q_1(t)$$

Spin-0 Generalized Parton Distribution.

Definition and simple properties.

$$H^q_\pi(x, \xi, t) \quad = \quad \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ixP^+z^-} \left\langle \pi, P+\frac{\Delta}{2} \middle| \bar{q}\left(-\frac{z}{2}\right)\gamma^+ q\left(\frac{z}{2}\right) \middle| \pi, P-\frac{\Delta}{2} \right\rangle_{\substack{z^+=0\\ z_\perp=0}}$$

with $t = \Delta^2$ and $\xi = -\Delta^+/(2P^+)$.



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Radyushkin, Phys. Lett. **B380**, 417 (1996)

- PDF **forward limit**
- Form factor **sum rule**
- H^q is an **even function** of ξ from time-reversal invariance.

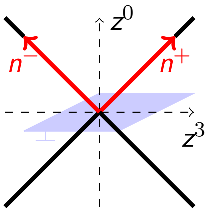
Spin-0 Generalized Parton Distribution.

Definition and simple properties.

$$H^q_\pi(x, \xi, t) \quad =$$

$$\frac{1}{2}\int \frac{\mathrm{d}z^-}{2\pi}\, e^{ixP^+z^-} \left\langle \pi, P+\frac{\Delta}{2} \middle| \bar{q}\left(-\frac{z}{2}\right)\gamma^+ q\left(\frac{z}{2}\right) \middle| \pi, P-\frac{\Delta}{2} \right\rangle_{\substack{z^+=0\\ z_\perp=0}}$$

with $t = \Delta^2$ and $\xi = -\Delta^+/(2P^+)$.



References

Müller *et al.*, Fortschr. Phys. **42**, 101 (1994)
Ji, Phys. Rev. Lett. **78**, 610 (1997)
Radyushkin, Phys. Lett. **B380**, 417 (1996)

- PDF **forward limit**
- Form factor **sum rule**
- H^q is an **even function** of ξ from time-reversal invariance.
- H^q is **real** from hermiticity and time-reversal invariance.

Polynomiality

$$\int_{-1}^{+1} dx\, x^n H^q(x, \xi, t) = \text{polynomial in } \xi$$

Polynomiality

Lorentz covariance

Spin-0 Generalized Parton Distribution.

Not so simple properties.

- **Polynomiality**

Lorentz covariance

- **Positivity**

$$H^q(x, \xi, t) \leq \sqrt{q\left(\frac{x+\xi}{1+\xi}\right) q\left(\frac{x-\xi}{1-\xi}\right)}$$

Spin-0 Generalized Parton Distribution.

Not so simple properties.

- **Polynomiality**

  Lorentz covariance

- **Positivity**

  Positivity of Hilbert space norm

- **Polynomiality**

Lorentz covariance

- **Positivity**

Positivity of Hilbert space norm

- H^q has support $x \in [-1, +1]$.

Spin-0 Generalized Parton Distribution.

Not so simple properties.

- **Polynomiality**

  Lorentz covariance

- **Positivity**

  Positivity of Hilbert space norm

- H^q has support $x \in [-1, +1]$.

  Relativistic quantum mechanics

Spin-0 Generalized Parton Distribution.

Not so simple properties.

- **Polynomiality**

Lorentz covariance

- **Positivity**

Positivity of Hilbert space norm

- H^q has support $x \in [-1, +1]$.

Relativistic quantum mechanics

- **Soft pion theorem** (pion target)

$$H^q(x, \xi = 1, t = 0) = \frac{1}{2}\phi_\pi^q\left(\frac{1+x}{2}\right)$$

- **Polynomiality**

  Lorentz covariance

- **Positivity**

  Positivity of Hilbert space norm

- H^q has support $x \in [-1, +1]$.

  Relativistic quantum mechanics

- **Soft pion theorem** (pion target)

  Dynamical chiral symmetry breaking

Spin-0 Generalized Parton Distribution.

Not so simple properties.

- **Polynomiality**

Lorentz covariance

- **Positivity**

Positivity of Hilbert space norm

- H^q has support $x \in [-1, +1]$.

Relativistic quantum mechanics

- **Soft pion theorem** (pion target)

Dynamical chiral symmetry breaking

How can we implement *a priori* these theoretical constraints?

- In the following, focus on **polynomiality** and **positivity**.
- Do not discuss the reduction to form factors or PDFs.

Dispersion relations and the lines $x = \pm\xi$.

Relation between $\mathrm{Re}\mathcal{H}(\xi)$ and $H(x = \pm\xi, \xi)$ at leading order.

- Write dispersion relation **at fixed** t **and** Q^2:

$$\mathrm{Re}\mathcal{H}(\xi) = \int_1^\infty \frac{\mathrm{d}\omega}{\pi}\,\mathrm{Im}\,C(\omega)\left\{\int_{-1}^{+1}\mathrm{d}x\left[\frac{1}{\omega\xi - x} - \frac{1}{\omega\xi + x}\right]H\left(x, \frac{x}{\omega}\right) + \mathcal{I}(\omega)\right\}.$$

Diehl and Ivanov, Eur. Phys. J. **C52**, 919 (2007)

- At **leading order** in α_s (no kinematic corrections):

$$\mathrm{Im}\,C(\omega) \propto \pi\Big[\delta(\omega - 1) - \delta(\omega + 1)\Big].$$

- Dispersion relation simplifies to:

$$\mathrm{Re}\mathcal{H}(\xi) \propto \int_{-1}^{+1}\mathrm{d}x\left[\frac{1}{\omega\xi - x} - \frac{1}{\omega\xi + x}\right]H(x, x) + \mathcal{I},$$

$$\mathrm{Im}\mathcal{H}(\xi) \propto H(\xi, \xi) - H(-\xi, \xi).$$

GPD models generically satisfying all theoretical constraints

Polynomiality.

Mixed constraint from Lorentz invariance and discrete symmetries.

- Express Mellin moments of GPDs as **matrix elements**:

$$\int_{-1}^{+1} \mathrm{d}x\, x^m H^q(x, \xi, t)$$

$$= \frac{1}{2(P^+)^{m+1}} \left\langle P + \frac{\Delta}{2} \middle| \bar{q}(0)\gamma^+ (i\overleftrightarrow{D}^+)^m q(0) \middle| P - \frac{\Delta}{2} \right\rangle$$

- Identify the **Lorentz structure** of the matrix element:

linear combination of $(P^+)^{m+1-k}(\Delta^+)^k$ for $0 \leq k \leq m+1$

- Remember definition of **skewness** $\Delta^+ = -2\xi P^+$.
- Select **even powers** to implement time reversal.
- Obtain **polynomiality condition**:

$$\int_{-1}^{1} \mathrm{d}x\, x^m H^q(x, \xi, t) = \sum_{\substack{i=0 \\ \text{even}}}^{m} (2\xi)^i C^q_{mi}(t) + (2\xi)^{m+1} C^q_{mm+1}(t) \ .$$

- Assume the existence of $D^q(z, t)$ such that:

$$\int_{-1}^{+1} \mathrm{d}z \, z^m D(z, t) = C^q_{mm+1}(t) \, .$$

- $H^q(x, \xi, t) - D(x/\xi, t)$ satisfies polynomiality at order m:

$$\int_{-1}^{1} \mathrm{d}x \, x^m \Big(H^q(x, \xi, t) - D(x/\xi, t) \Big) = \sum_{\substack{i=0 \\ \text{even}}}^{m} (2\xi)^i C^q_{mi}(t) \, .$$

- In the Radon transform framework, this is the **Ludwig-Helgason** consistency condition.
- Thus, there exists a function F_D such that:

$$H(x, \xi, t) = D(x/\xi, t) + \int_{\Omega_{\mathrm{DD}}} \mathrm{d}\beta \mathrm{d}\alpha \, F_D(\beta, \alpha, t) \, \delta(x - \beta - \alpha\xi) \, .$$

- The support $\Omega_{\mathrm{DD}} = \{|\alpha| + |\beta| \leq 1\}$ is related to the GPD physical domain $|x|, |\xi| \leq 1$.

- Most general representation of GPD:

$$H^q(x,\xi,t) = \int_{\Omega_{\mathrm{DD}}} \mathrm{d}\beta \mathrm{d}\alpha\, \delta(x-\beta-\alpha\xi)\big(F^q(\beta,\alpha,t)+\xi G^q(\beta,\alpha,t)\big)$$

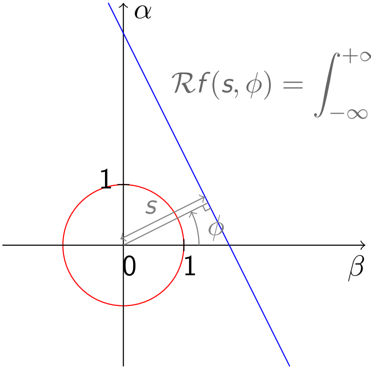
- Support property: $x \in [-1,+1]$.
- Discrete symmetries: F^q is α-even and G^q is α-odd.
- **Gauge**: any representation (F^q, G^q) can be recast in one representation with a single DD f^q:

$$H^q(x,\xi,t) = x \int_{\Omega_{\mathrm{DD}}} \mathrm{d}\beta \mathrm{d}\alpha\, f^q_{\mathrm{BMKS}}(\beta,\alpha,t)\delta(x-\beta-\alpha\xi)$$

Belitsky *et al.*, Phys. Rev. **D64**, 116002 (2001)

$$H^q(x,\xi,t) = (1-x) \int_{\Omega_{\mathrm{DD}}} \mathrm{d}\beta \mathrm{d}\alpha\, f^q_{\mathrm{P}}(\beta,\alpha,t)\delta(x-\beta-\alpha\xi)$$

Pobylitsa, Phys. Rev. **D67**, 034009 (2003)



For $s > 0$ and $\phi \in [0, 2\pi]$:

$$\mathcal{R}f(s, \phi) = \int_{-\infty}^{+\infty} \mathrm{d}\beta \mathrm{d}\alpha \, f(\beta, \alpha) \delta(s - \beta \cos\phi - \alpha \sin\phi)$$

and:

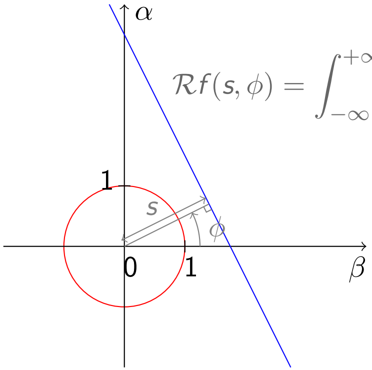
$$\mathcal{R}f(-s, \phi) = \mathcal{R}f(s, \phi \pm \pi)$$

Relation to GPDs:

$$x = \frac{s}{\cos\phi} \text{ and } \xi = \tan\phi$$

Relation between GPD and DD in Belistky *et al.* gauge

$$\frac{\sqrt{1 + \xi^2}}{x} H(x, \xi) = \mathcal{R}f_{\mathrm{BMKS}}(s, \phi) \ .$$



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Relation to GPDs:

$$x = \frac{s}{\cos\phi} \text{ and } \xi = \tan\phi$$

Relation between GPD and DD in Pobylitsa gauge

$$\frac{\sqrt{1+\xi^2}}{1-x} H(x,\xi) = \mathcal{R}f_{\mathrm{P}}(s,\phi) \ .$$

The range of the Radon transform.

The polynomiality property a.k.a. the Ludwig-Helgason condition.

- The Mellin moments of a Radon transform are **homogeneous polynomials** in $\omega = (\sin\phi, \cos\phi)$.
- The converse is also true:

Theorem (Hertle, 1983)

Let $g(s, \omega)$ an even compactly-supported distribution. Then g is itself the Radon transform of a compactly-supported distribution if and only if the ***Ludwig-Helgason consistency condition*** *hold:*

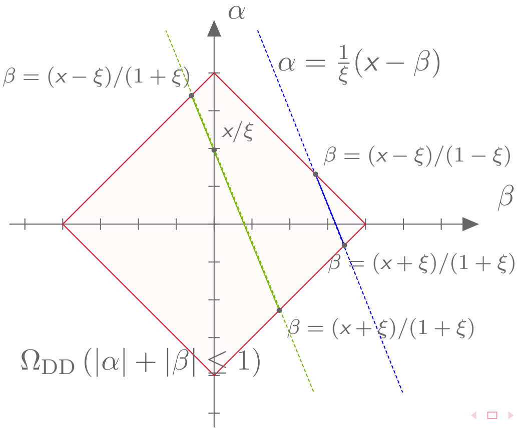
(i) *g is C^∞ in ω,*

(ii) *$\int \mathrm{d}s\, s^m g(s, \omega)$ is a homogeneous polynomial of degree m for all integer $m \geq 0$.*

- Double Distributions and the Radon transform are the **natural solution** of the polynomiality condition.

DGLAP and ERBL regions

$$(x, \xi) \in \text{ DGLAP} \quad \Leftrightarrow \quad |s| \geq |\sin\phi| \; ,$$
$$(x, \xi) \in \text{ ERBL} \quad \Leftrightarrow \quad |s| \leq |\sin\phi| \; .$$



Each point (β, α) with $\beta \neq 0$ contributes to **both** DGLAP and ERBL regions.

Theorem (simple case)

Let f be a compactly-supported summable function defined on $\mathbb{R}^2$ and $\mathcal{R}f$ its Radon transform.
Let $(s_0, \omega_0) \in \mathbb{R} \times S^1$ and U_0 an open neighborhood of ω_0 s.t.:

$$\textit{for all } s > s_0 \textit{ and } \omega \in U_0 \quad \mathcal{R}f(s, \omega) = 0 \; .$$

Then $f(\aleph) = 0$ on the half-plane $\langle \aleph | \omega_0 \rangle > s_0$ of $\mathbb{R}^2$.

Theorem (Boman and Todd Quinto, 1987)

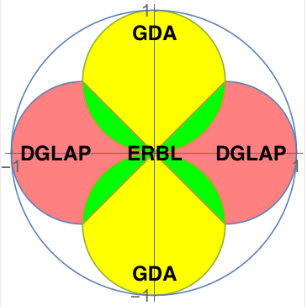
Assume $(s_0, \omega_0) \in \mathbb{R} \times S^{n-1}$ and $f \in \mathcal{E}'(\mathbb{R})$. Let $\mu(\aleph, \omega)$ be a strictly positive real analytic function on $\mathbb{R}^n \times S^{n-1}$ that is even in ω. Let U_0 be an open neighborhood of ω_0. Finally assume $R_\mu(s, \omega) = 0$ for $s > s_0$ and $\omega \in U_0$. Then $f = 0$ on the half space $\langle \aleph | \omega_0 \rangle > s_0$.

Theorem (simple case)

$$\textit{for all } s > s_0 \textit{ and } \omega \in U_0 \quad \mathcal{R}f(s,\omega) = 0 \ .$$

Then $f(\aleph) = 0$ *on the half-plane* $\langle \aleph \,|\omega_0 \rangle > s_0$ *of* $\mathbb{R}^2$.

DGLAP and ERBL regions in polar coordinates



Uniqueness of the extension.

Statement.

Theorem (simple case)

$$\textit{for all } s > s_0 \textit{ and } \omega \in U_0 \quad \mathcal{R}f(s, \omega) = 0 \; .$$

Then $f(\aleph) = 0$ *on the half-plane* $\langle \aleph \,|\omega_0 \rangle > s_0$ *of* $\mathbb{R}^2$.

Consider a GPD H being zero on the DGLAP region.

- Take $\xi_0 = \tan\phi_0 \in [0, 1]$, $x_0 \in]\xi_0, +\infty[$ and s_0 *s.t.* $x_0 \cos\phi_0 > s_0 > \sin\phi_0$.
- $\exists \epsilon > 0$ *s.t.* $s_0 > \sin\phi$ for $|\phi - \phi_0| < \epsilon$.
- Hyp: the underlying DD f has a zero Radon transform for all $\phi \in]\phi_0 - \epsilon, \phi_0 + \epsilon[$ and $s > s_0$ (DGLAP region).
- Then $f(\beta, \alpha) = 0$ for all (β, α) *s.t.* $\beta \cos\phi_0 + \alpha \sin\phi_0 = s > s_0$.
- At last select $s = x_0 \cos\phi_0$ to get $\beta + \alpha\xi_0 = x_0$.
- Cannot constrain the line $\beta = 0$. $\square$

Proof.

- Identify the matrix element defining a GPD as an **inner product** of two different states.
- Apply Cauchy-Schwartz inequality, and identify PDFs at specific kinematic points, *e.g.*:

$$|H^q(x,\xi,t)| \leq \sqrt{\frac{1}{1-\xi^2} q\left(\frac{x+\xi}{1+\xi}\right) q\left(\frac{x-\xi}{1-\xi}\right)}$$

- This procedures yields **infinitely many inequalities** stable under LO evolution.

Pobylitsa, Phys. Rev. **D66**, 094002 (2002)

- The **overlap representation** guarantees *a priori* the fulfillment of positivity constraints.

- Decompose an hadronic state $|H; P, \lambda\rangle$ in a Fock basis:

$$|H; P, \lambda\rangle = \sum_{N,\beta} \int [\mathrm{d}x \mathrm{d}\mathbf{k}_\perp]_N \psi_N^{(\beta,\lambda)}(x_1, \mathbf{k}_{\perp 1}, \ldots, x_N, \mathbf{k}_{\perp N}) \, |\beta, k_1, \ldots, k_N\rangle$$

- Derive an expression for the pion GPD in the DGLAP region $\xi \leq x \leq 1$:

$$H^q(x, \xi, t) \propto \sum_{\beta,j} \int [\mathrm{d}\bar{x} \mathrm{d}\bar{\mathbf{k}}_\perp]_N \delta_{j,q} \delta(x - \bar{x}_j) \left(\psi_N^{(\beta,\lambda)}\right)^* (\hat{x}', \hat{\mathbf{k}}'_\perp) \psi_N^{(\beta,\lambda)}(\tilde{x}, \tilde{\mathbf{k}}_\perp)$$

with $\tilde{x}, \tilde{\mathbf{k}}_\perp$ (resp. $\hat{x}', \hat{\mathbf{k}}'_\perp$) generically denoting incoming (resp. outgoing) parton kinematics.

Diehl *et al.*, Nucl. Phys. **B596**, 33 (2001)

- Similar expression in the ERBL region $-\xi \leq x \leq \xi$, but with overlap of N- and $(N+2)$-body LFWFs.

Overlap representation.
Advantages and drawbacks.

- Physical picture.
- Positivity relations are fulfilled **by construction**.
- Implementation of **symmetries of N-body problems**.

What is not obvious anymore

What is *not* obvious to see from the wave function representation is however the **continuity of GPDs at** $x = \pm\xi$ and the **polynomiality** condition. In these cases both the DGLAP and the ERBL regions must cooperate to lead to the required properties, and this implies **nontrivial relations between the wave functions** for the different Fock states relevant in the two regions. An *ad hoc* Ansatz for the wave functions would **almost certainly lead** to GPDs that **violate the above requirements**.

Diehl, Phys. Rept. **388**, 41 (2003)

Implementing Lorentz covariance.

Extend an overlap in the DGLAP region to the whole GPD domain.

For **any model of LFWF**, one has to address the following three questions:

1. Does the extension exist?
2. If it exists, is it unique?
3. How can we compute this extension?

Modeling strategy

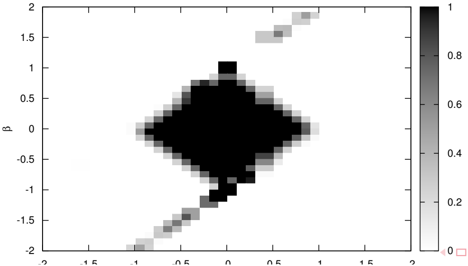
1. Ensure positivity by modeling the DGLAP region as an overlap of LFWFs.
2. Ensure polynomiality by inverting the Radon transform to identify an underlying DD.

Chouika *et al.*, Eur. Phys. J. **C77**, 906 (2017)

Ill-posedness in the sense of Hadamard.

A first glimpse at the inverse Radon transform.

- Numerical evaluation ***almost unavoidable*** (polar vs cartesian coordinates).
- Ill-posedness by **lack of continuity**.
- The **unlimited** Radon inverse problem is **mildly** ill-posed while the **limited** one is **severely** ill-posed.
- Even if it existed, an analytic expression of the invert Radon transform would be of **limited practical use**.



Mezrag, PhD dissertation (2015)

Fully discrete case

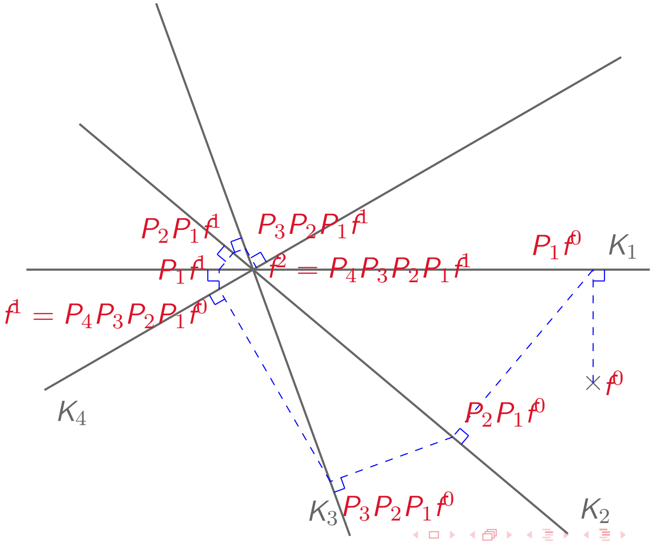
Assume f piecewise-constant with values f_m for $1 \leq m \leq M$.
For a collection of lines $(L_n)_{1 \leq n \leq N}$ crossing Ω_{DD}, the Radon transform writes:

$$g_n = \mathcal{R}f = \int_{L_n} f = \sum_{m=1}^{M} f_m \times \mathrm{Measure}(L_n \cap C_m) \quad \text{for } 1 \leq n \leq N$$

A discretized problem

Consider $N+1$ Hilbert spaces H, H_1, ..., H_N, and a family of continuous surjective operators $R_n : H \to H_n$ for $1 \leq n \leq N$.
Being given $g_1 \in H_1$, ..., $g_n \in H_n$, we search f solving the following system of equations:

$$R_n f = g_n \quad \text{for } 1 \leq n \leq N$$



$P_2P_1f^1$
$P_3P_2P_1f^1$
P_1f^0
K_1
P_1f^1
$f^2 = P_4P_3P_2P_1f^1$
$f^1 = P_4P_3P_2P_1f^0$
f^0
$P_2P_1f^0$
K_4
K_3
$P_3P_2P_1f^0$
K_2

Inverse problem: geometrical interpretation.

Towards a numerical evaluation (3/3).

And if the input data are inconsistent?

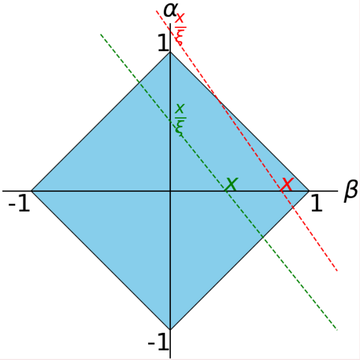
- Instead of solving $g = \mathcal{R}f$, find f such that $\|g - \mathcal{R}f\|_2$ is **minimum**.
- The solution **always exists**.
- The input data are **inconsistent** if $\|g - \mathcal{R}f\|_2 > 0$.

Comparing models to experimental data

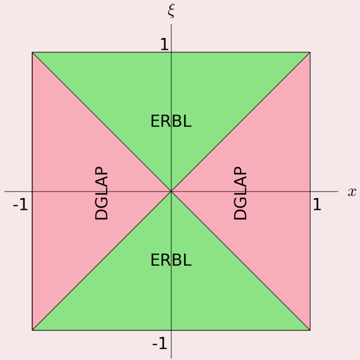
PARtonic
Tomography
Of
Nucleon
Software

How can we get a DD from a GPD in the DGLAP region?

DD



GPD



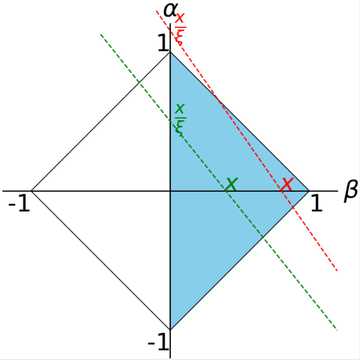
Computation of the extension.

Problem reduction.

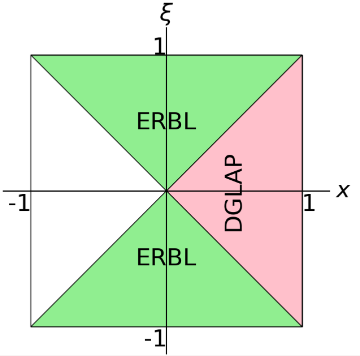
How can we get a DD from a GPD in the DGLAP region?

- Restrict to quark GPDs ($\beta > 0$).

DD



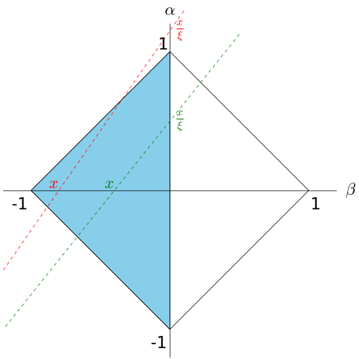
GPD



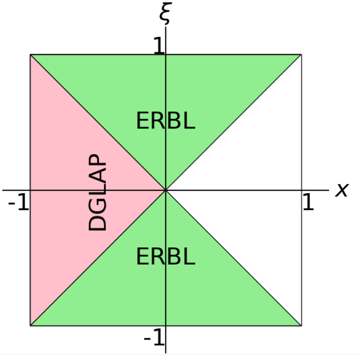
How can we get a DD from a GPD in the DGLAP region?

- Restrict to quark GPDs ($\beta > 0$).
- Only ERBL region "sees" both $\beta > 0$ and $\beta < 0$.

DD



GPD



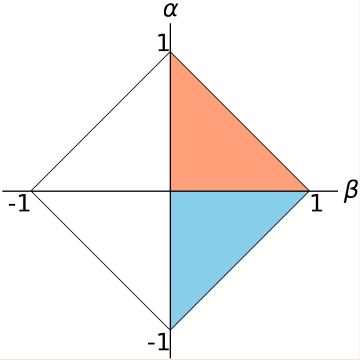
Computation of the extension.

Problem reduction.

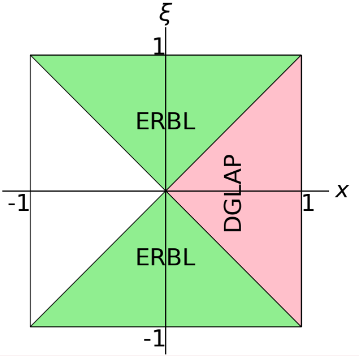
How can we get a DD from a GPD in the DGLAP region?

- Restrict to quark GPDs ($\beta > 0$).
- Only ERBL region "sees" both $\beta > 0$ and $\beta < 0$.
- Use α-parity of the DD.

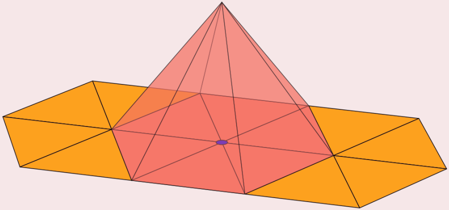
DD



GPD



Example of a P1 basis function



- Discretize the DD on a mesh with $n \simeq 800$ triangular cells.
- Compute the Radon transform of a P1 basis function.
- Sample $m \simeq 4n$ (x, ξ)-lines intersecting the DD support.
- Solve a linear system $AX = B$ with A a sparse $m \times n$ matrix.
- Adopt an iterative regularization method: LSMR.

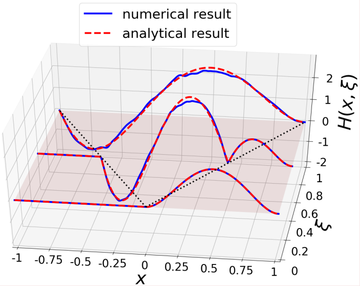
Fong and Saunders, arXiv:1006.0758

Examples - benchmarks (1/4).
Algebraic Bethe-Salpeter model.

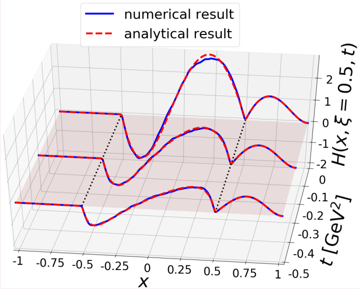
$$\Psi_{l=0}(x, \mathbf{k}_\perp) = 8\sqrt{15}\,\pi \frac{M^3}{\left(\mathbf{k}_\perp^2 + M^2\right)^2}(1-x)\,x\,,$$

$$i\,k_\perp^j\,\Psi_{l=1}(x, \mathbf{k}_\perp) = 8\sqrt{15}\,\pi \frac{k_\perp^j\,M^2}{\left(\mathbf{k}_\perp^2 + M^2\right)^2}(1-x)\,x\,, \qquad j = 1, 2$$

$H(x, \xi, t=0)$



$H(x, \xi = 0.5, t)$



Chouika *et al.*, Eur. Phys. J. **C77**, 906 (2017)

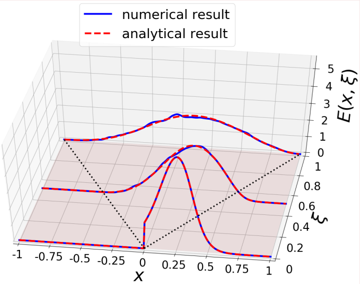
Examples - benchmarks (2/4).

Algebraic spectator model.

$$\varphi(x, \mathbf{k}_\perp) = \frac{gM^{2p}}{\sqrt{1-x}} x^{-p} \left(M^2 - \frac{\mathbf{k}_\perp^2 + m^2}{x} - \frac{\mathbf{k}_\perp^2 + \lambda^2}{1-x} \right)^{-p-1}$$

Hwang and Müller, Phys. Lett. **B660**, 350 (2008)

$H(x, \xi, t = 0)$



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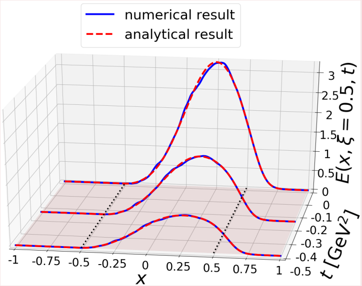
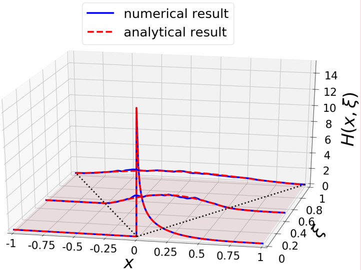


Chouika *et al.*, Eur. Phys. J. **C77**, 906 (2017)

Radyushkin DD Ansatz with phenomenological PDF:

$$q_{\mathrm{Regge}}(x) = \frac{35\,(1-x)^3}{32\,\sqrt{x}}\,.$$

$H(x, \xi, t = 0)$



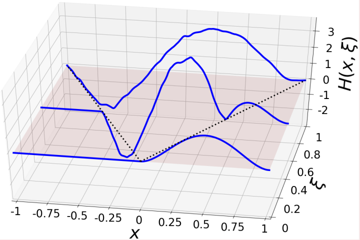
Chouika *et al.*, Eur. Phys. J. **C77**, 906 (2017)

Examples - benchmarks (4/4).

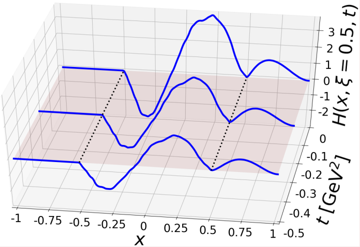
Gaussian wave function model.

$$\Psi\left(x, \mathbf{k}_\perp^2\right) = \frac{4\sqrt{15}\,\pi}{M}\sqrt{x(1-x)}\,e^{-\frac{\mathbf{k}_\perp^2}{4M^2(1-x)x}}\,.$$

$H(x, \xi, t = 0)$



$H(x, \xi = 0.5, t)$



Chouika *et al.*, Eur. Phys. J. **C77**, 906 (2017)

Experimental data and phenomenology	Full processes
Computation of amplitudes	Small distance contributions
First principles and fundamental parameters	Large distance contributions

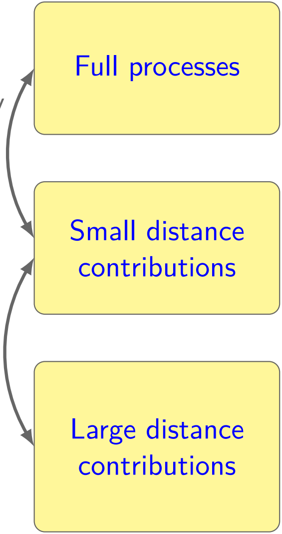
Computing chain design.

Differential studies: physical models and numerical methods.

Experimental data and phenomenology

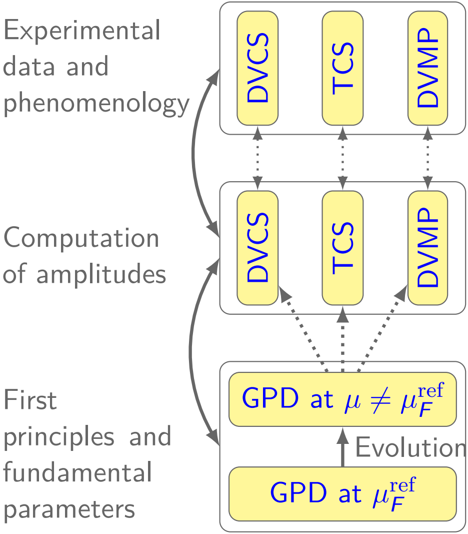
Computation of amplitudes

First principles and fundamental parameters



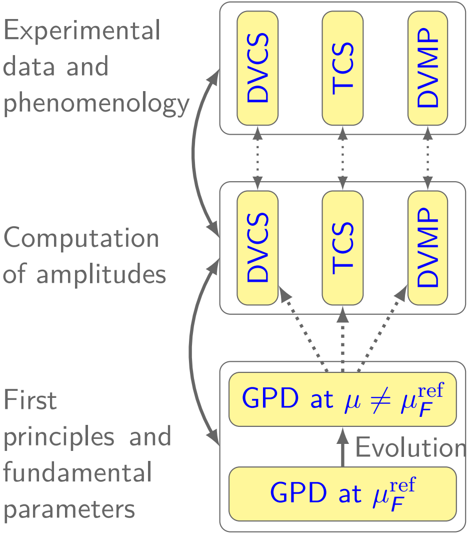
Computing chain design.

Differential studies: physical models and numerical methods.



Computing chain design.

Differential studies: physical models and numerical methods.



- Many observables.
- Kinematic reach.

Computing chain design.

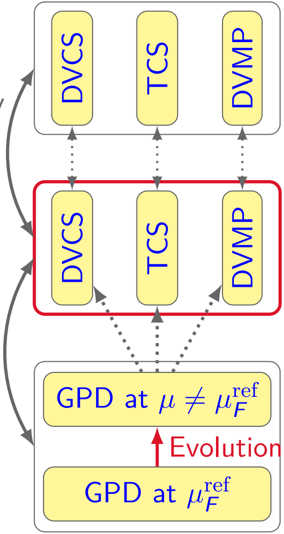
Differential studies: physical models and numerical methods.

Experimental data and phenomenology

Need for modularity

Computation of amplitudes

First principles and fundamental parameters



- Many observables.
- Kinematic reach.

- Perturbative approximations.
- Physical models.
- Fits.
- Numerical methods.
- Accuracy and speed.

Computing chain design.

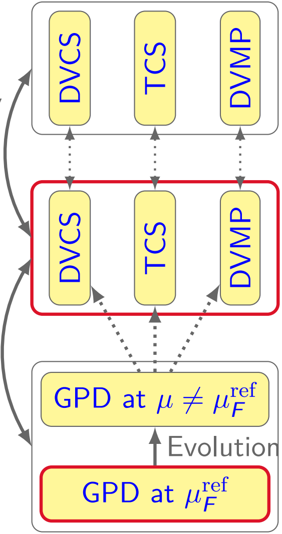
Differential studies: physical models and numerical methods.

Experimental data and phenomenology

Need for modularity

Computation of amplitudes

First principles and fundamental parameters



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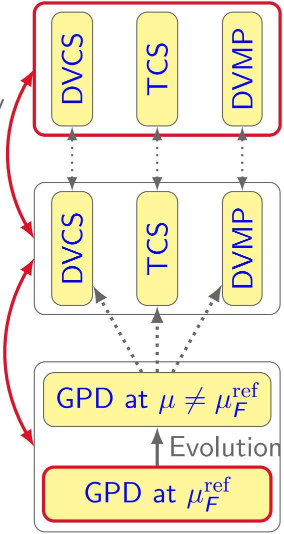

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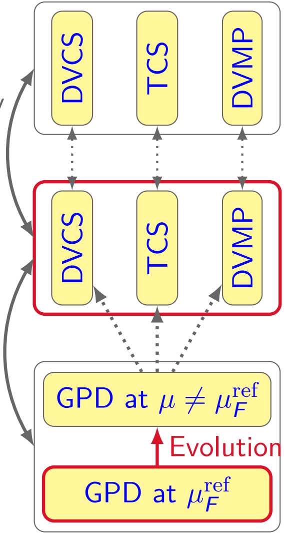

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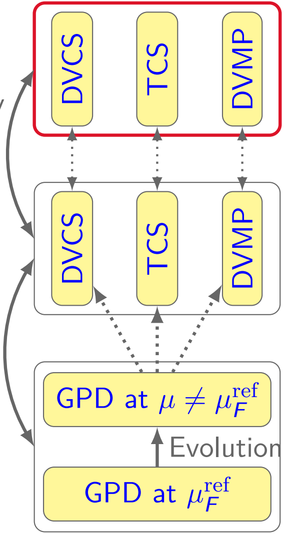

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- Accuracy and speed.

Open source release.

Publicly available on CEA GitLab server.

partons.cea.fr

PARTONS

PARtonic Tomography Of Nucleon Software

Main Page | Download | Tutorials + | Reference documentation + | About +

Main Page

What is PARTONS?

PARTONS is a C++ software framework dedicated to the phenomenology of Generalized Parton Distributions (GPDs). GPDs provide a comprehensive description of the partonic structure of the nucleon and contain a wealth of new information. In particular, GPDs provide a description of the nucleon as an extended object, referred to as 3-dimensional nucleon tomography, and give an access to the orbital angular momentum of quarks.

Table of Contents
- What is PARTONS?
- Get PARTONS
- Configure PARTONS
- How to use PARTONS
- Publications and talks
- License
- Contact and newsletter

PARTONS provides a necessary bridge between models of GPDs and experimental data measured in various exclusive channels, like Deeply Virtual Compton Scattering (DVCS) and Hard Exclusive Meson Production (HEMP). The experimental programme devoted to study GPDs has been carrying out by several experiments, like HERMES at DESY (closed), COMPASS at CERN, Hall-A and CLAS at JLab. GPD subject will be also a key component of the physics case for the expected Electron Ion Collider (EIC).

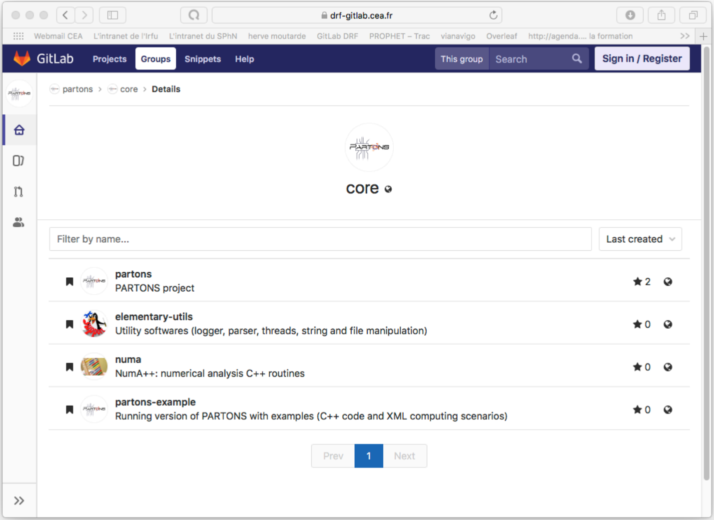
PARTONS is useful to theorists to develop new models, phenomenologists to interpret existing measurements and to experimentalists to design new experiments.

Get PARTONS

Berthou *et al.*, Eur. Phys. J. **C78**, 478 (2018).

Open source release.

Publicly available on CEA GitLab server.



Berthou *et al.*, Eur. Phys. J. **C78**, 478 (2018)

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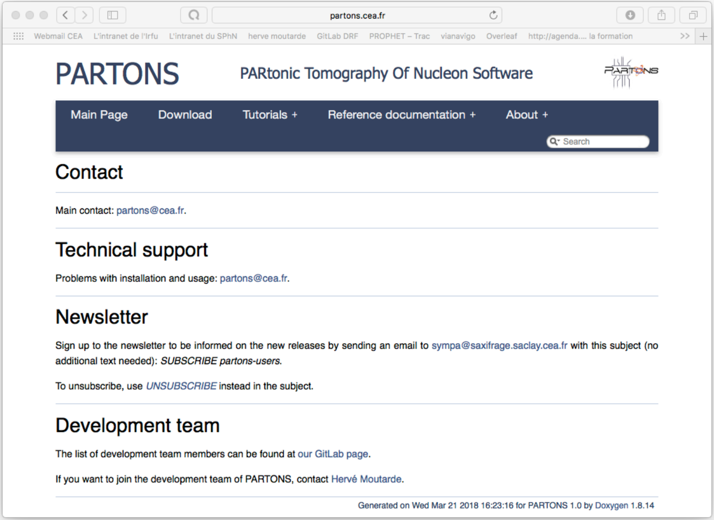

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Technical support

Problems with installation and usage: partons@cea.fr.

Newsletter

Sign up to the newsletter to be informed on the new releases by sending an email to sympa@saxifrage.saclay.cea.fr with this subject (no additional text needed): *SUBSCRIBE partons-users*.

To unsubscribe, use *UNSUBSCRIBE* instead in the subject.

Development team

The list of development team members can be found at our GitLab page.

If you want to join the development team of PARTONS, contact Hervé Moutarde.

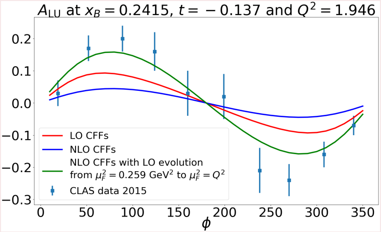
Generated on Wed Mar 21 2018 16:23:16 for PARTONS 1.0 by Doxygen 1.8.14

Berthou *et al.*, Eur. Phys. J. **C78**, 478 (2018)

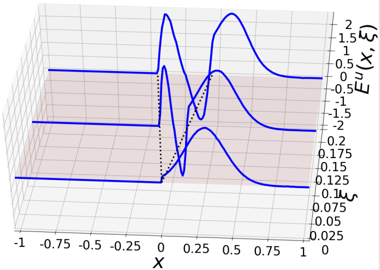
Phenomenology from a qqq wavefunction.

Preliminary results: tests with the χQSM.

CLAS 2015 data



E^u at $t = -0.34$ GeV^2



- Only LO phenomenology achievable without extension to ERBL region.
- Computation of **various DVCS observables** in the valence region under **different pQCD assumptions** with PARTONS.

Chouika, PhD thesis (2018)

Conclusion

Conclusion and prospects.

Putting all the pieces together.

- We can now build generic GPD models satisfying *a priori* **all theoretical constraints**.
- We now have tools to **systematically relate** these models to **experimental data**. Open source release under GPLv3.0. of the PARTONS framework.
- We now have an **operating fitting engine** for global CFF fits.

New studies become possible!

- Global GPD fits.
- Energy-momentum structure of hadrons.
- Quantitative impact of nonperturbative QCD ingredients on 3D hadron structure studies.
- ???

Appendix

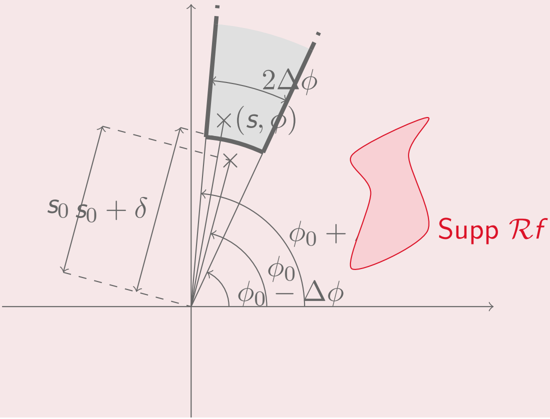
Proof (elements)

Let $\aleph_0 = (\mathfrak{b}, \mathfrak{a}) \in \mathbb{R}^2$, $s \in \mathbb{R}$ and $\delta > 0$ such that $\langle \aleph_0 \, | \omega_0 \rangle = \mathfrak{b} \cos \phi_0 + \mathfrak{a} \sin \phi_0 = s > s_0 + \delta$.
Denote $\mathcal{B}$ a ball containing $\aleph_0$ and the support of f, which is bounded by assumption.
We will show that $f = 0$ in a neighborhood of $\aleph_0$ in $\mathcal{B}$.

Step 1

Identification of a neighborhood T of $\aleph_0$ *s.t.*:

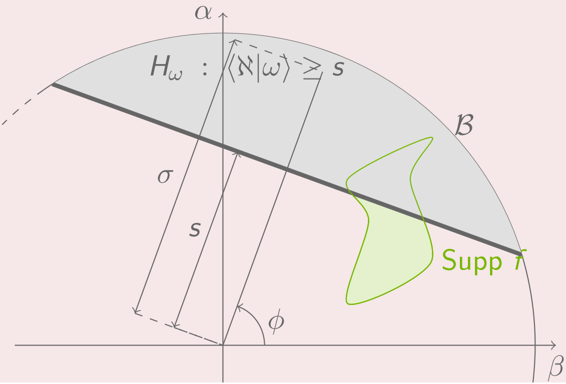
$$\forall s > s_0 + \delta \, , \quad \forall \omega \in T \, , \quad \int_{\mathbb{R}^2} \mathrm{d}\aleph \, \delta(s - \langle \aleph \, | \omega \rangle) f(\aleph) = 0 \, .$$



Step 2

Prove by induction on the multi index $\mathfrak{m} = (m, n)$ that, for all nonnegative integers m, n and $\omega \in T$:

$$\int_{[s_0+\delta,+\infty[} \mathrm{d}s \int_{\mathcal{B}} \mathrm{d}\aleph \, \delta(s - \langle \aleph | \omega \rangle) \aleph^{\mathfrak{m}} \langle \aleph | \omega \rangle^k f(\aleph) = 0 \, .$$

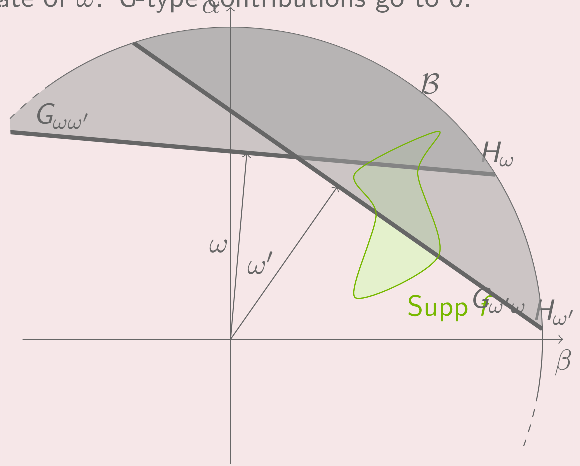


Uniqueness of the extension.

Proof.

Step 2 (cont')

Induction step: infinitesimal change of the first cartesian coordinate of ω. G-type contributions go to 0.



Step 3

Apply previous result with $k = 0$ and $\omega = \omega_0$, let δ goes to 0:

$$\text{for all } n, m \geq 0 \quad \int_{H_{\omega_0}} \mathrm{d}\beta \mathrm{d}\alpha \, \beta^m \alpha^n f(\beta, \alpha) = 0 \; .$$

Conclude by injectivity of the Fourier transform from $L^1(\mathbb{R}^2)$ into the set of continuous functions on $\mathbb{R}^2$.

Back to uniqueness statement.

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