Testing weak-field gravity with wide binary stars

Indranil Banik, MNRAS, 480, 2660

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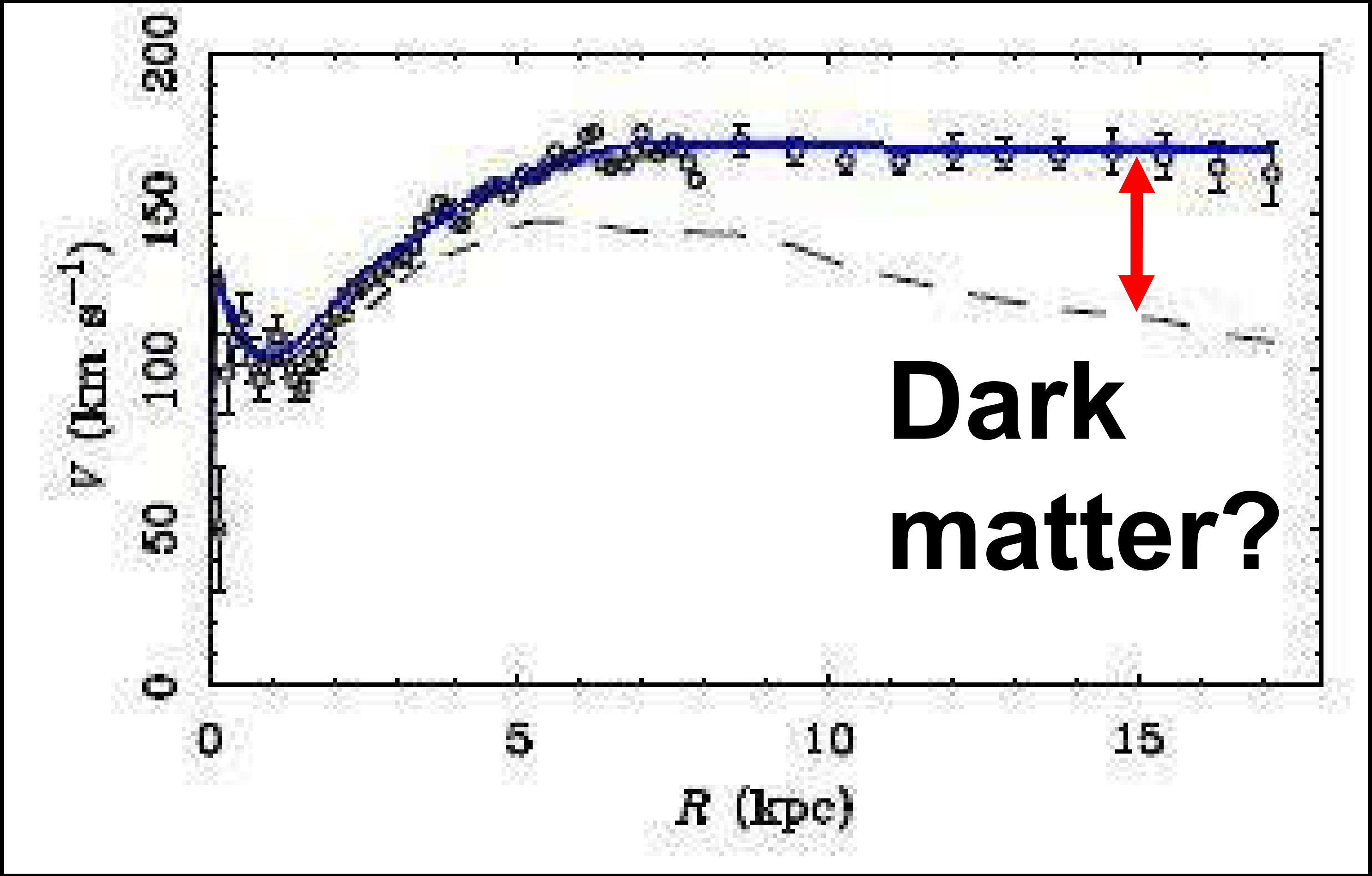
Galaxies

Visible mass × Newtonian Gravity = Acceleration

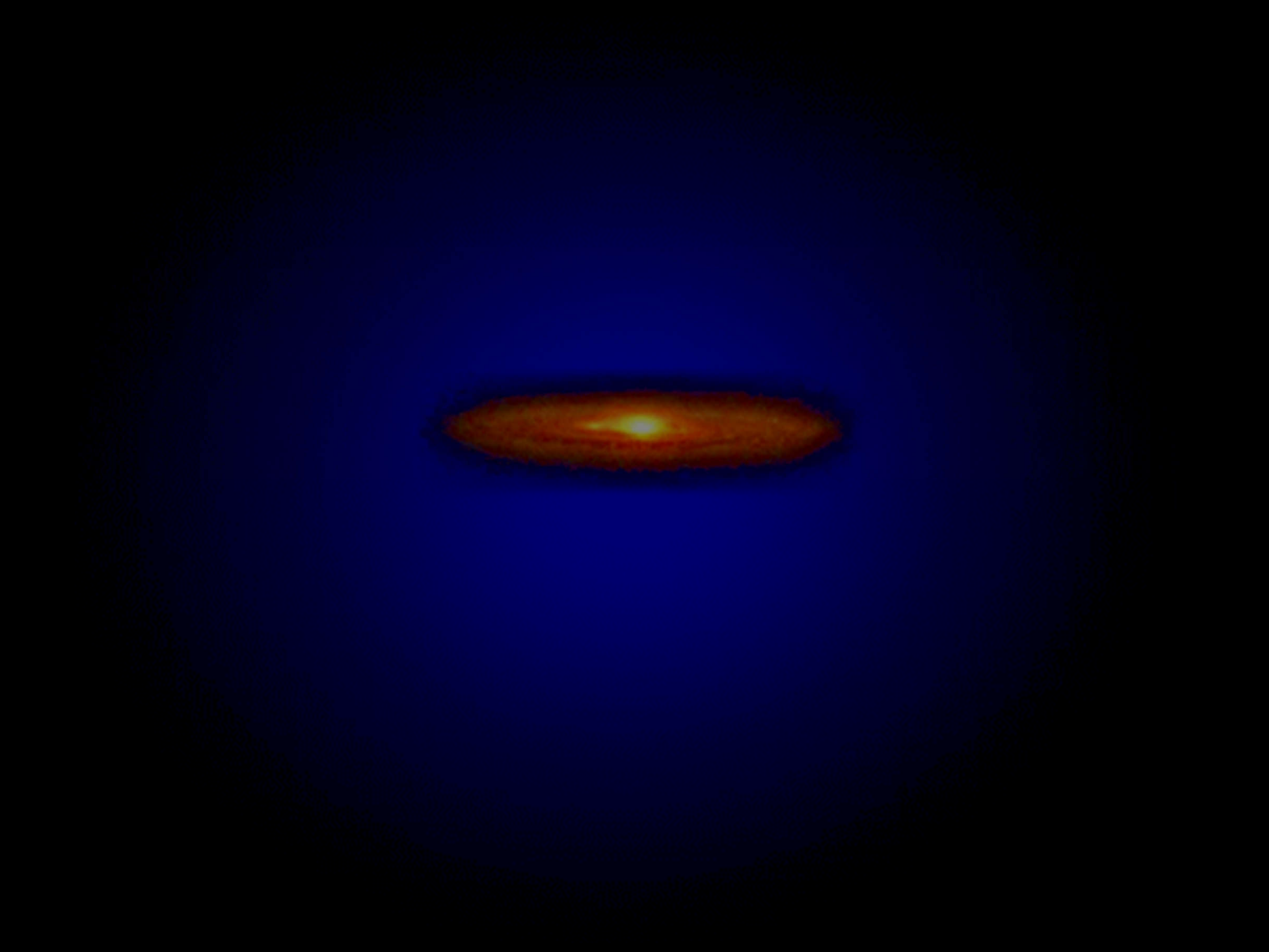
- The observed acceleration is discrepant with this prediction

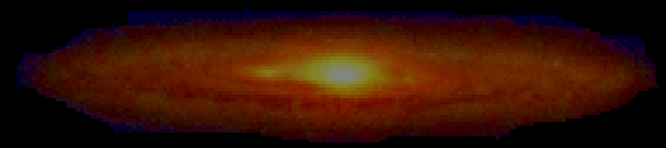


Living Reviews in Relativity, **15**, 10 (2012), by

Famaey & McGaugh

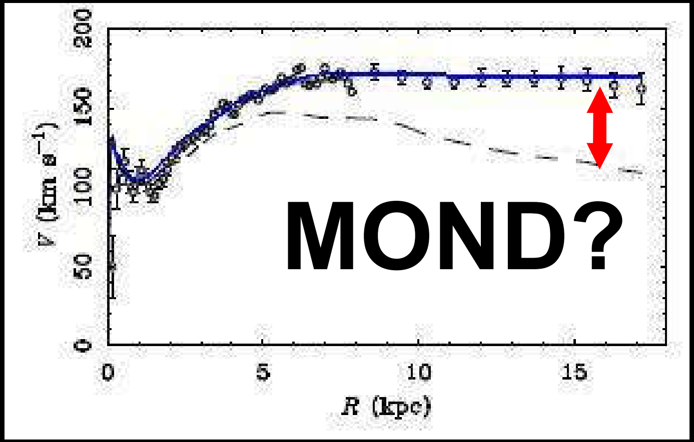
No direct evidence





Introduction

- Modified Newtonian Dynamics (MOND, Milgrom 1983) posits that gravity transitions from inverse square law to inverse distance law for accelerations $< a_0 = 1.2 \times 10^{-10}$ m/s^2
- Designed to explain why galaxy rotation curves flatten instead of decreasing outwards beyond visible extent of galaxies, like in Solar System



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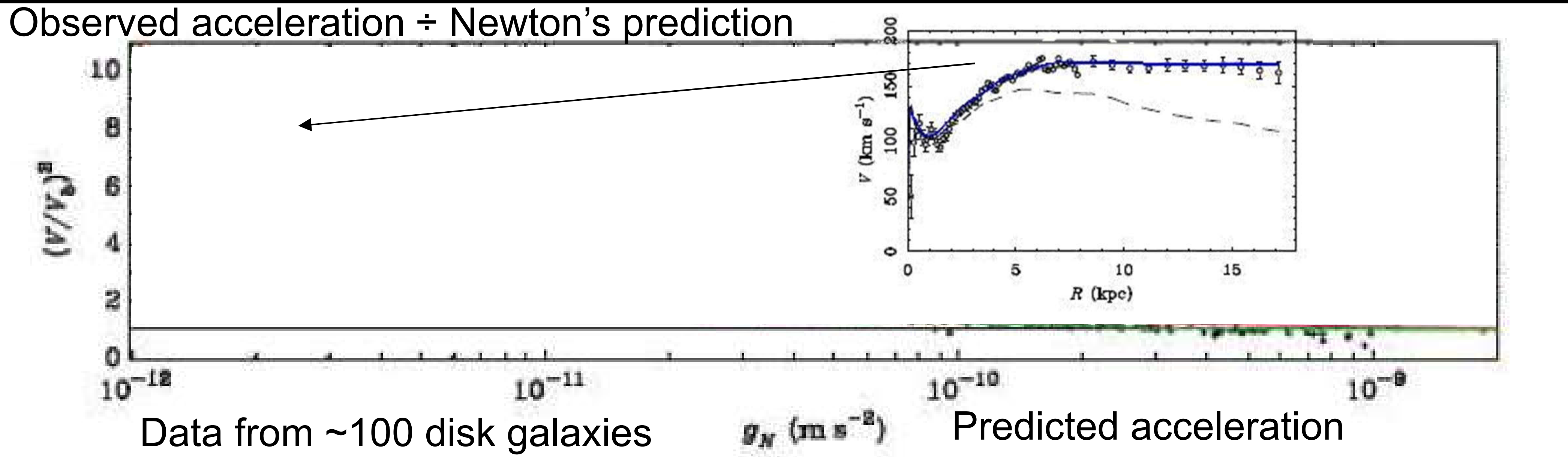
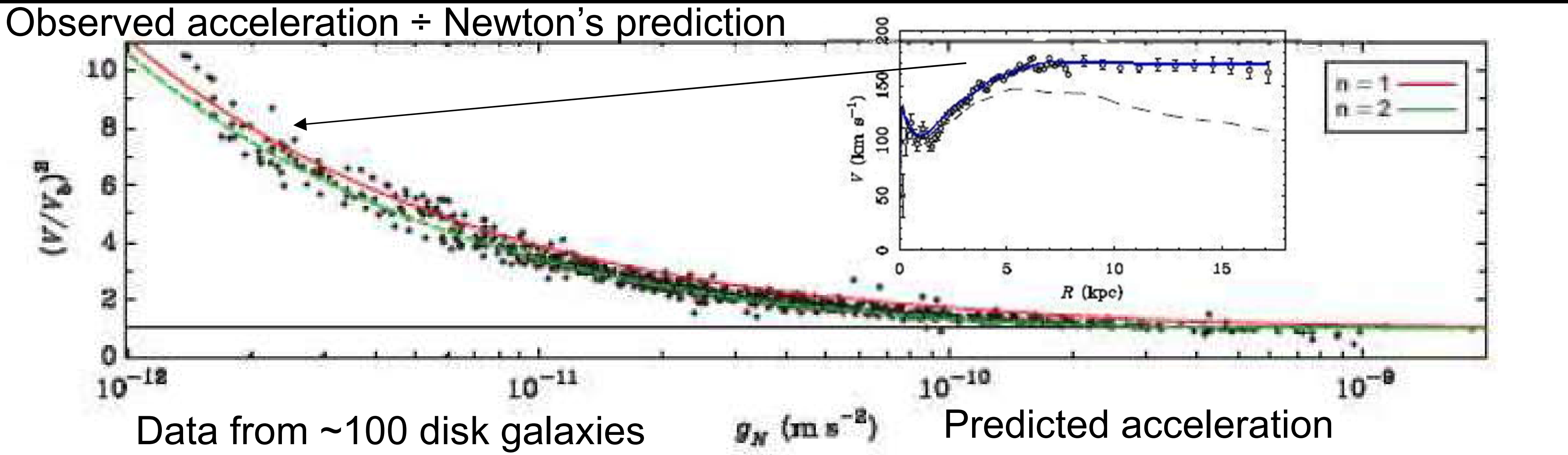
The Acceleration Discrepancy



- Dark matter must exist if the visible mass is insufficient to predict the dynamics
- But if it is sufficient, then extra degree of freedom in invisible mass unnecessary

The Acceleration Discrepancy



- Dark matter must exist if the visible mass is insufficient to predict the dynamics
- But if it is sufficient, then extra degree of freedom in invisible mass unnecessary

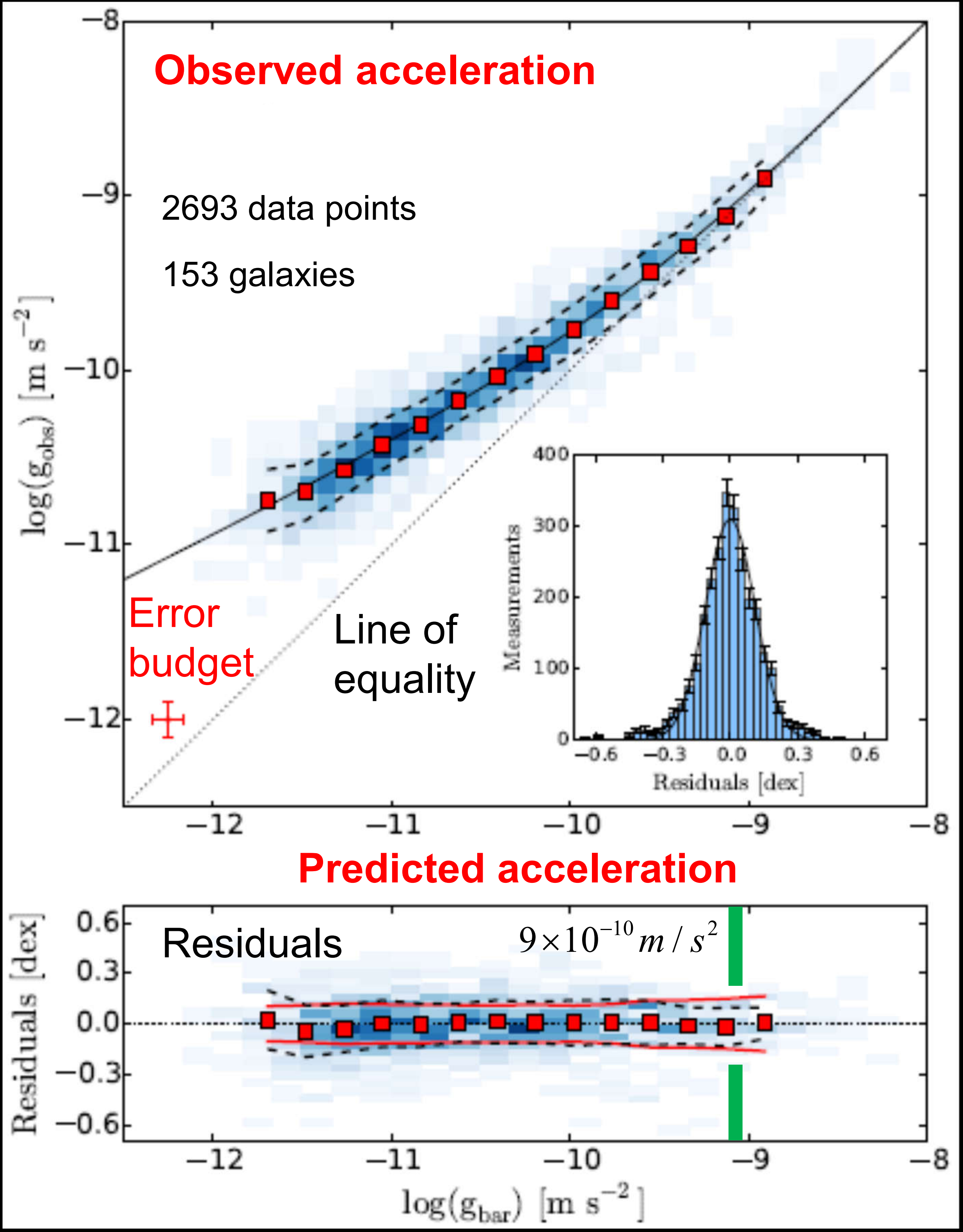
➢ DM disfavored by data (Occam's Razor)

- Gravity may need modifying…

Latest data

Physical Review Letters, 117, 201101 (McGaugh+, 2016)

- Gas fraction has no effect on this relation
- Neither does the surface brightness



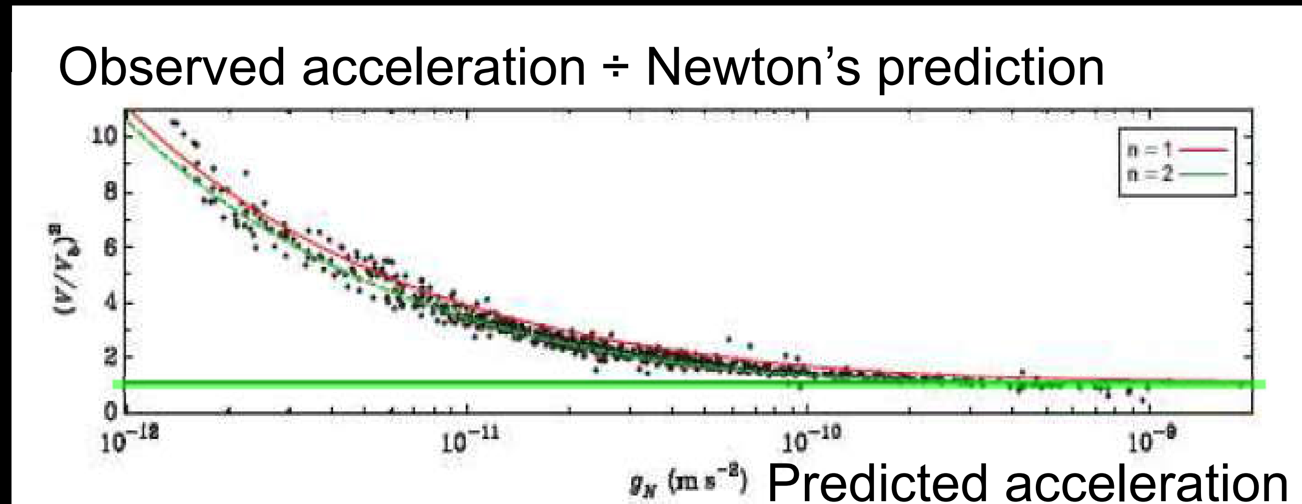
Modified Newtonian Dynamics

- MOND (Milgrom, 1983) says Newtonian dynamics off by some factor, depending on <u>acceleration</u> (or energy density)
- Gives **more** gravity for **same** mass
- E**mpirical** theory (as is Newtonian gravity)
- Flexibility required as we don’t know exactly what happens at low accelerations

Basics Of MOND

$$\mu\left(\frac{|\mathbf{g}|}{a_0}\right)\vec{g} = \vec{g}_N$$

(Spherical symmetry)



$$\mu(x) = 1 \ \text{ when } \ x >> 1$$

$$\mu(x) = x \ \text{ when } \ x << 1, \ \text{ so } \ g = \sqrt{g_N a_0}$$

$$\text{Far from an isolated point mass,} \quad g = \frac{\sqrt{GMa_0}}{r}$$

But is gravity non-Newtonian or is this dark matter?

Wide binary stars

- For Sun, $\frac{GM}{r^2} < a_0$ if $r > 7000$ AU (MOND radius)
- Gravity enhanced ~1.5x at Solar Circle of MW
- Gaia can't accurately resolve line of sight separation as distance accuracy ~80 kAU

➢ Focus on $\tilde{v} \equiv v \div v_{c,Newton}(r_p) = v \div \sqrt{GM/r_p}$

- v = relative velocity of stars, M = total mass
- As sky-projected radius r_p < true radius r, must have $\tilde{v}$ below its true 3D value
- $\tilde{v} < \sqrt{2}$ in Newtonian gravity, limit ~1.7 in MOND
- Stars with high $\tilde{v}$ would strongly support MOND

Doing the test with GAIA data

- Gaia should detect thousands of wide binaries (WBs) within ~150 pc
- Very low chance of random 3D alignment in position and velocity
- At ~0.5 km/s, wide binaries would separate ~10^5 AU in only a few Myr
- But Gaia data can't accurately resolve line of sight separation of components as distance accuracy ~80 kAU – enough to make it likely that system is a binary

MOND governing equations

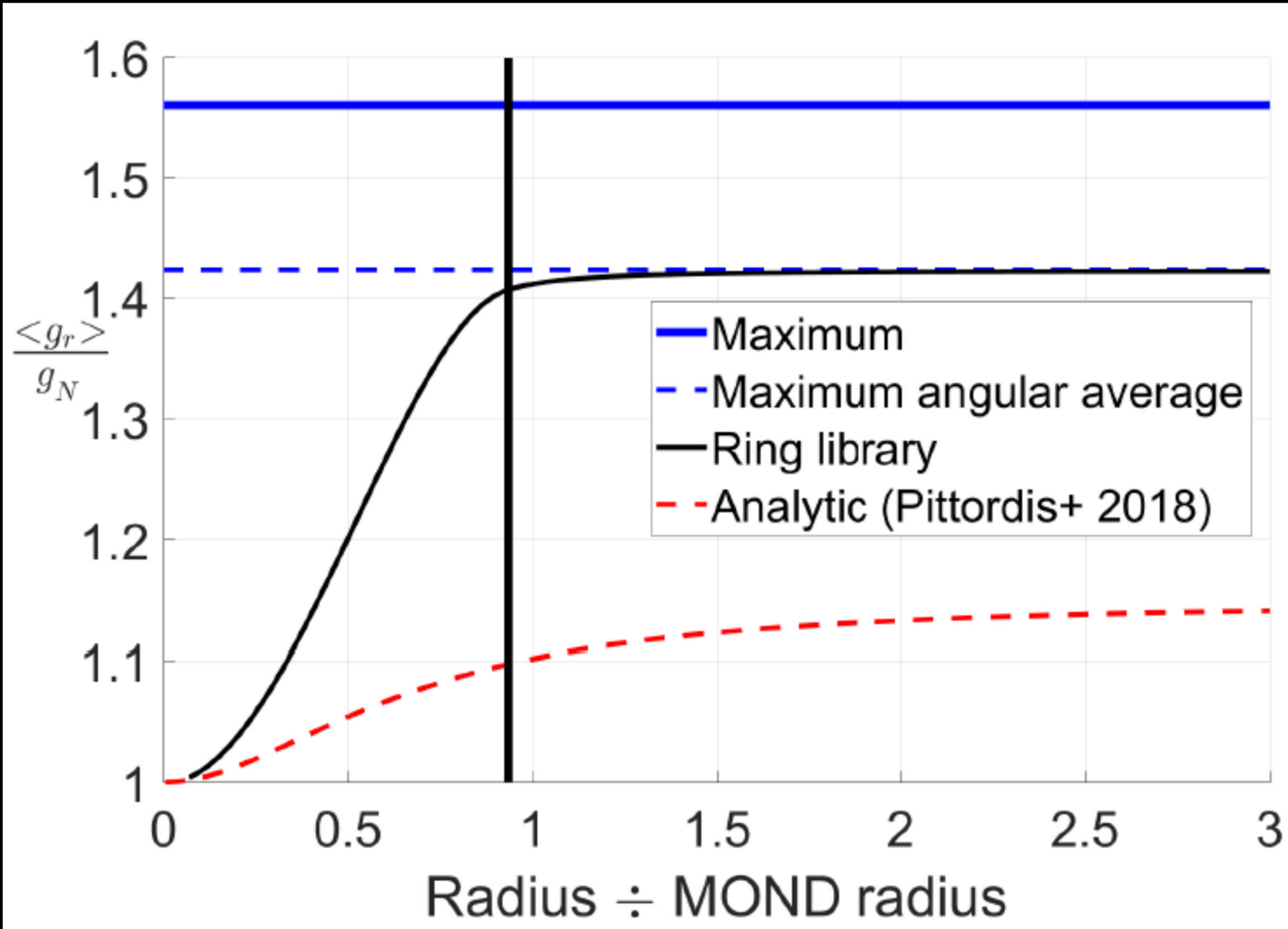
$$\overbrace{\nabla \cdot \boldsymbol{g}}^{\propto \rho_{PDM} + \rho_b} = \nabla \cdot \left[\nu \overbrace{\left(\frac{g_N}{a_0} \right)}^{y} \boldsymbol{g}_N \right]$$

$$\nu(y) = \frac{1}{2} + \sqrt{\frac{1}{4} + \frac{1}{y}}$$

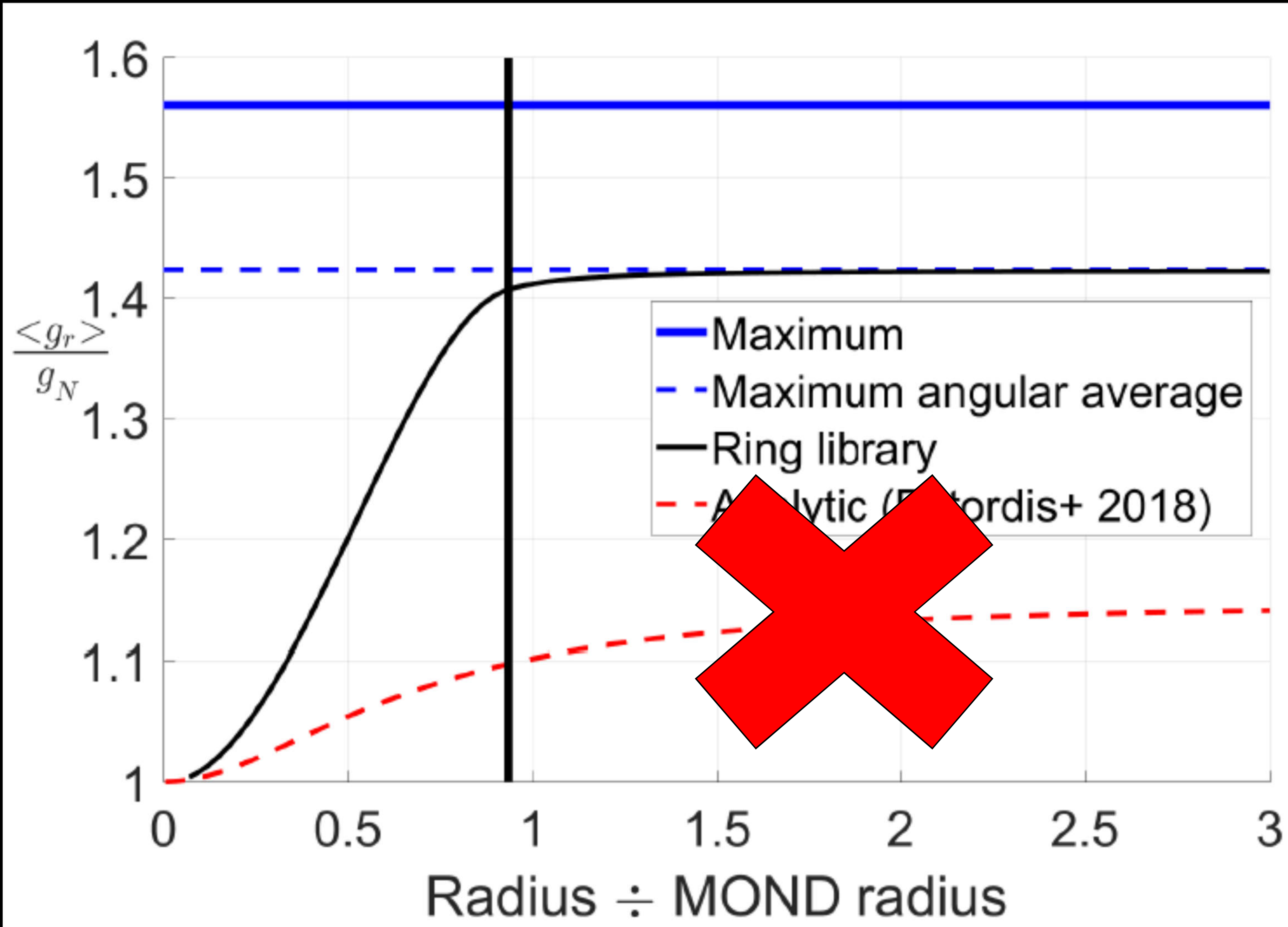
- External field (EF) from rest of Galaxy must be considered as it is $\sim a_0$
- Solve numerically: $\boldsymbol{g}(\boldsymbol{r}) = \int \nabla \cdot \boldsymbol{g}(\boldsymbol{r}') \frac{(\boldsymbol{r} - \boldsymbol{r}')}{4\pi |\boldsymbol{r} - \boldsymbol{r}'|^3} d^3\boldsymbol{r}'$
- Treat binary as single mass + test particle to make g axisymmetric (Banik & Zhao 2018, MNRAS, 480, 2660)

Forces in MOND

- For widely separated WBs, EF dominates
- ➢ Get analytic result from Arxiv: 1509.08457
- Force not entirely radial and depends on angle of system relative to EF
- ➢ Can check force library with azimuthally averaged radial force at fixed distance
- ➢ Get inverse square law at long range, but with renormalized G and some angular dependence



1.6
1.5
1.4
1.3
1.2
1.1
1
$\frac{<g_r>}{g_N}$
0
0.5
1
1.5
2
2.5
3
Radius ÷ MOND radius
Maximum
Maximum angular average
Ring library
Analytic (Pittordis+ 2018)



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A lytic (tordis+ 2018)
0
0.5
1
1.5
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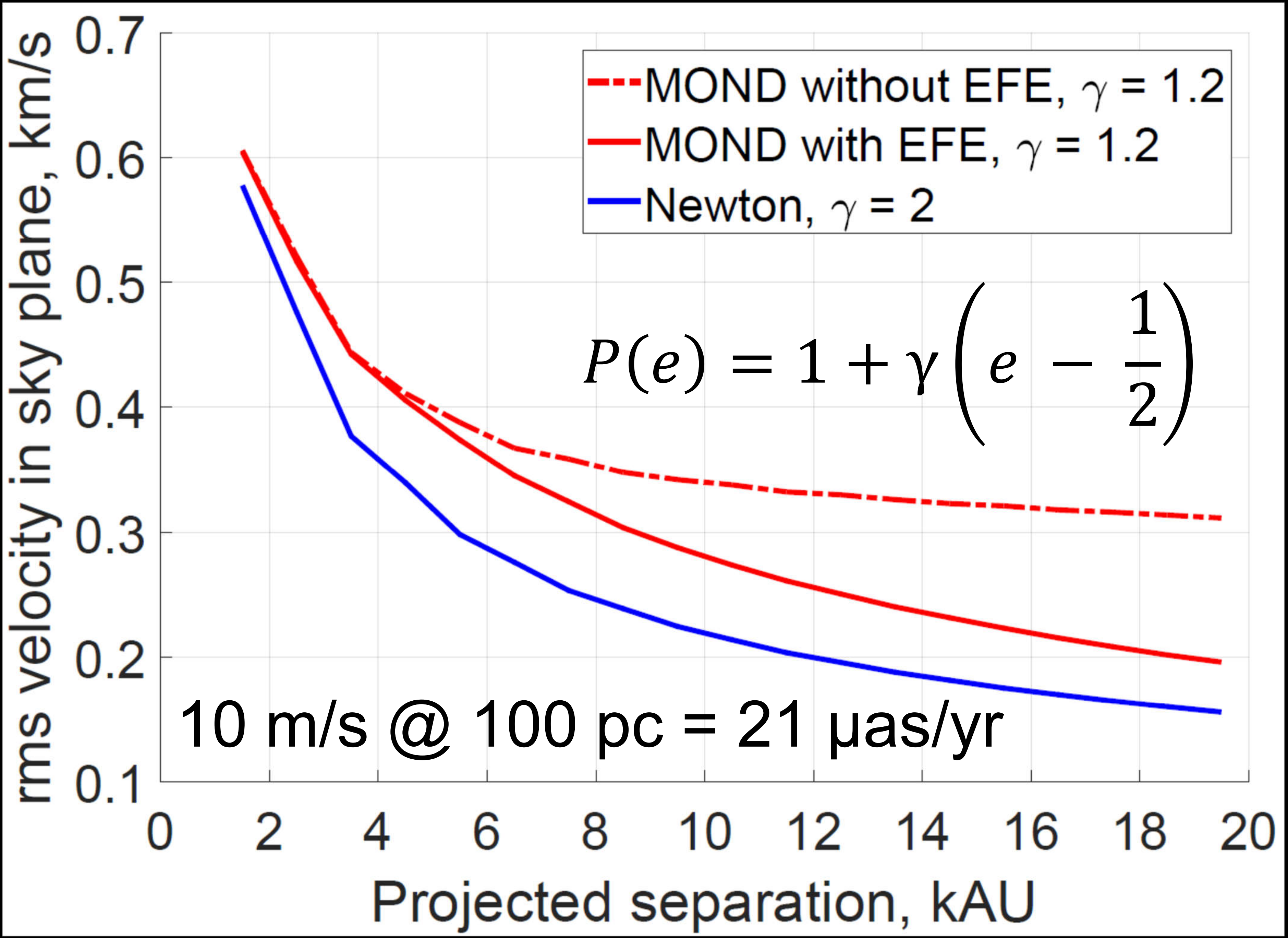
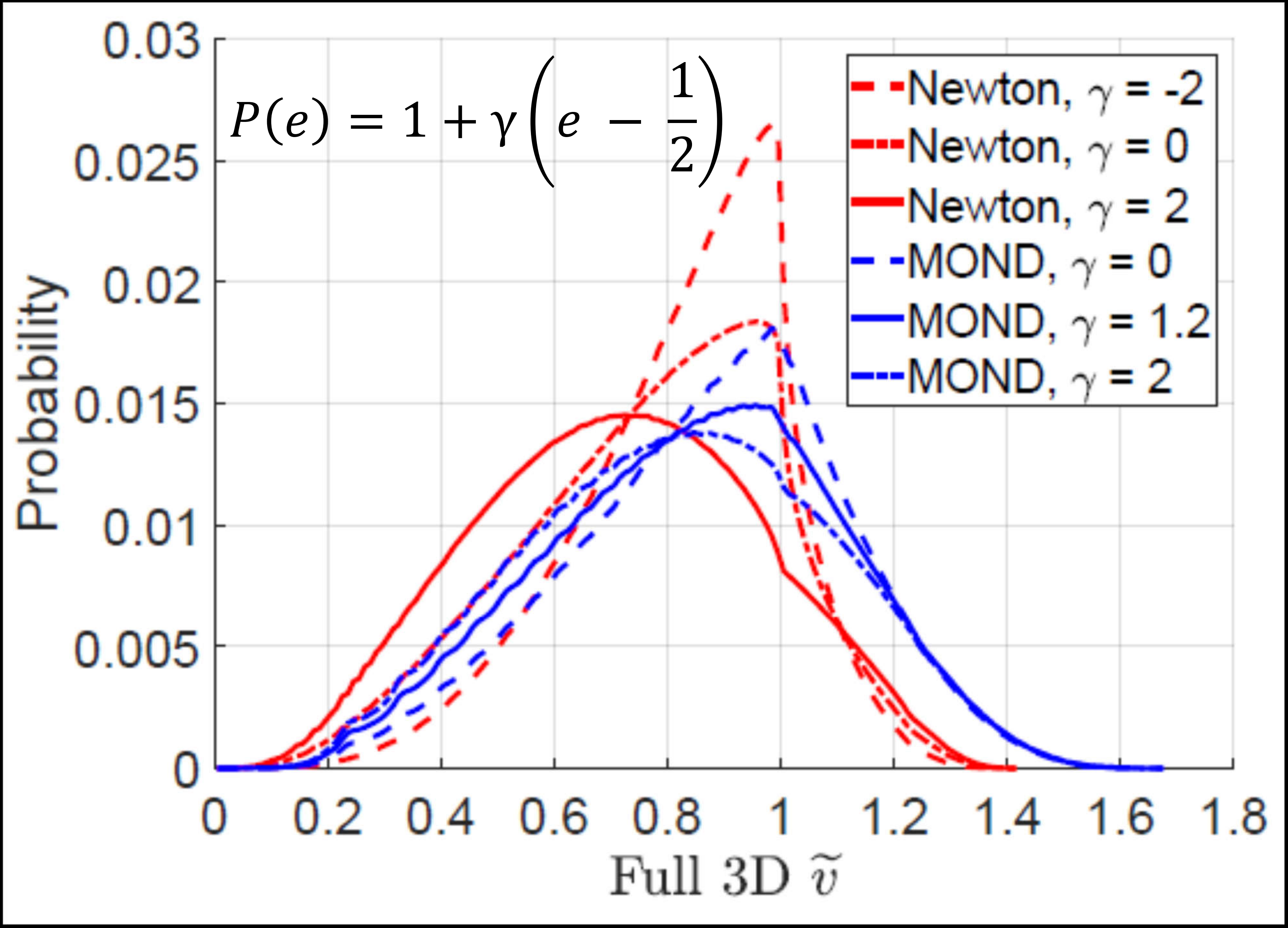
Radius ÷ MOND radius

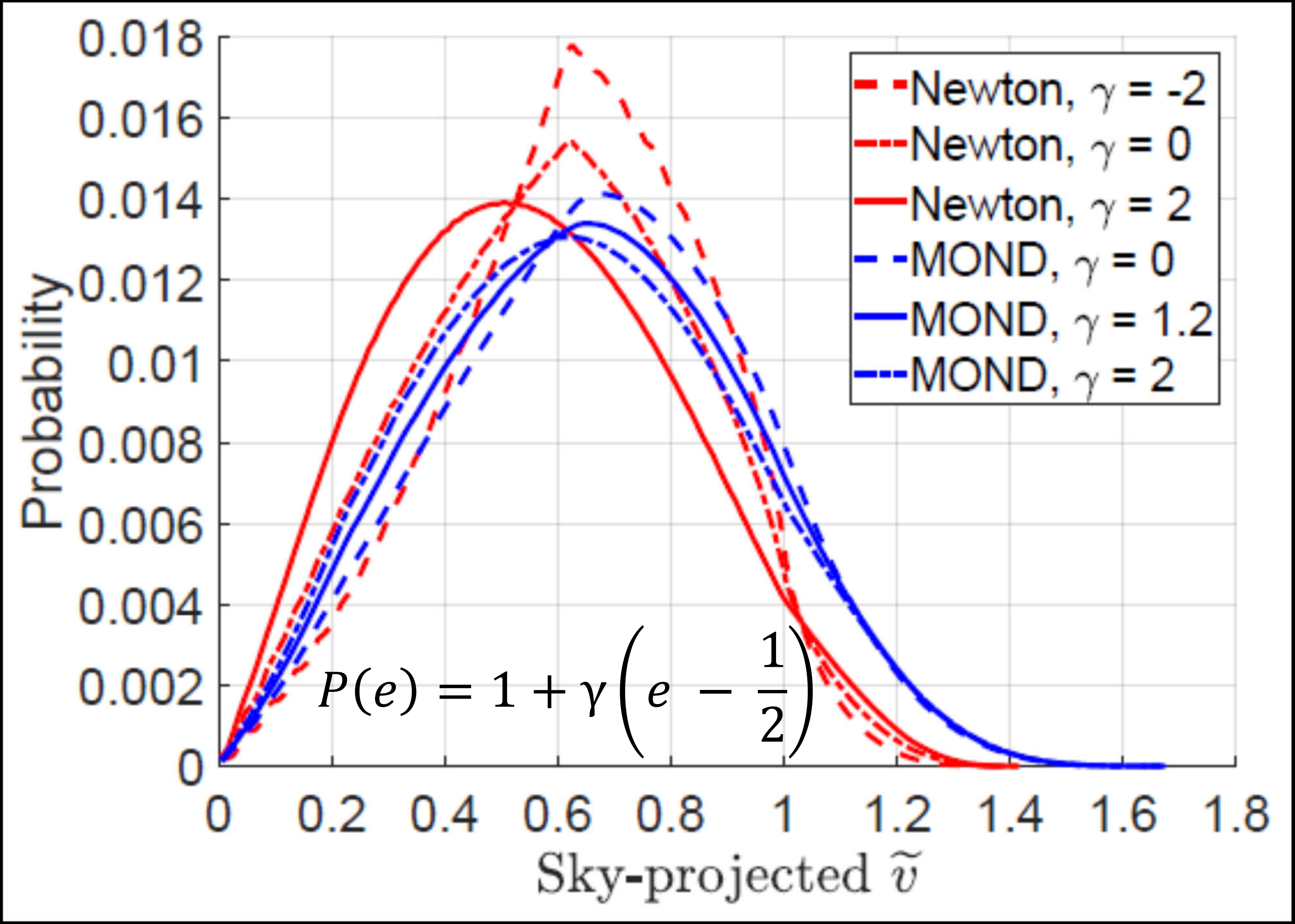
Typical velocity differences



The $\widetilde{v}$ distribution



…without radial velocities

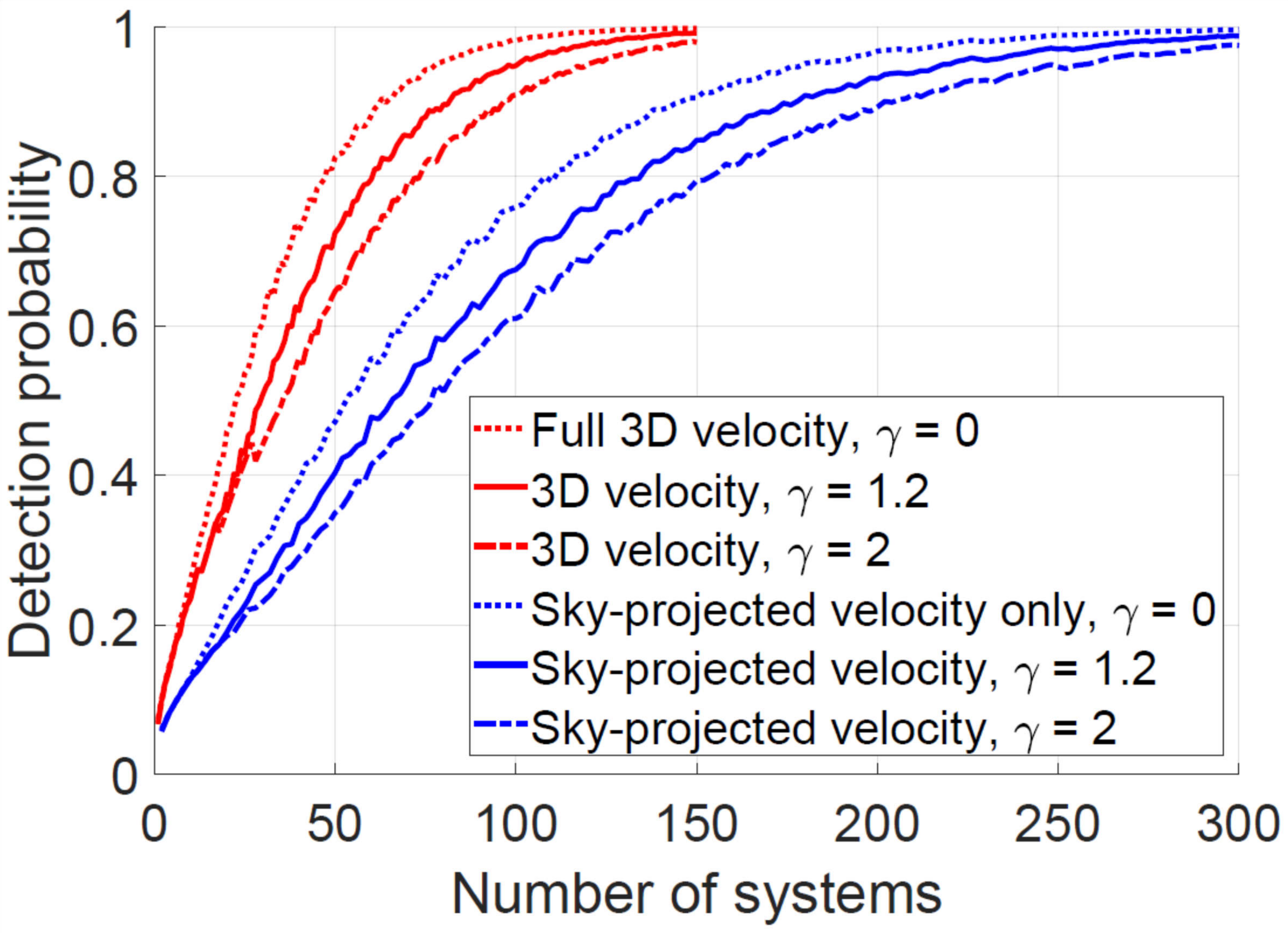


The detection probability

- Integrate for 20 revolutions to get probability distribution over $(r_p, \tilde{v})$
- Try different viewing angles, semi-major axes, eccentricities, masses and orbital planes. Marginalise over their priors.
- For some region e.g. $(r_p > 2\,kAU, \tilde{v} > 1.3)$, get Newtonian binary fraction e.g. 0.1
- Get 99% CL upper limit e.g. ≤15/100
- Get chance of ≥16/100 systems in MOND
- This is the **detection probability** of MOND

Detection probabilities

- Consider previous method applied to all possible ranges of $(r_p, \tilde{v})$.
- Get range which maximises probability. This gives $P_{detection}$ for a particular model (set of model parameters)
- Try different eccentricity distributions γ for Newtonian model to minimise chance of detecting MOND effects. Mimics scientific process if MOND correct – γ will be varied.
- Repeat for different numbers of WB systems

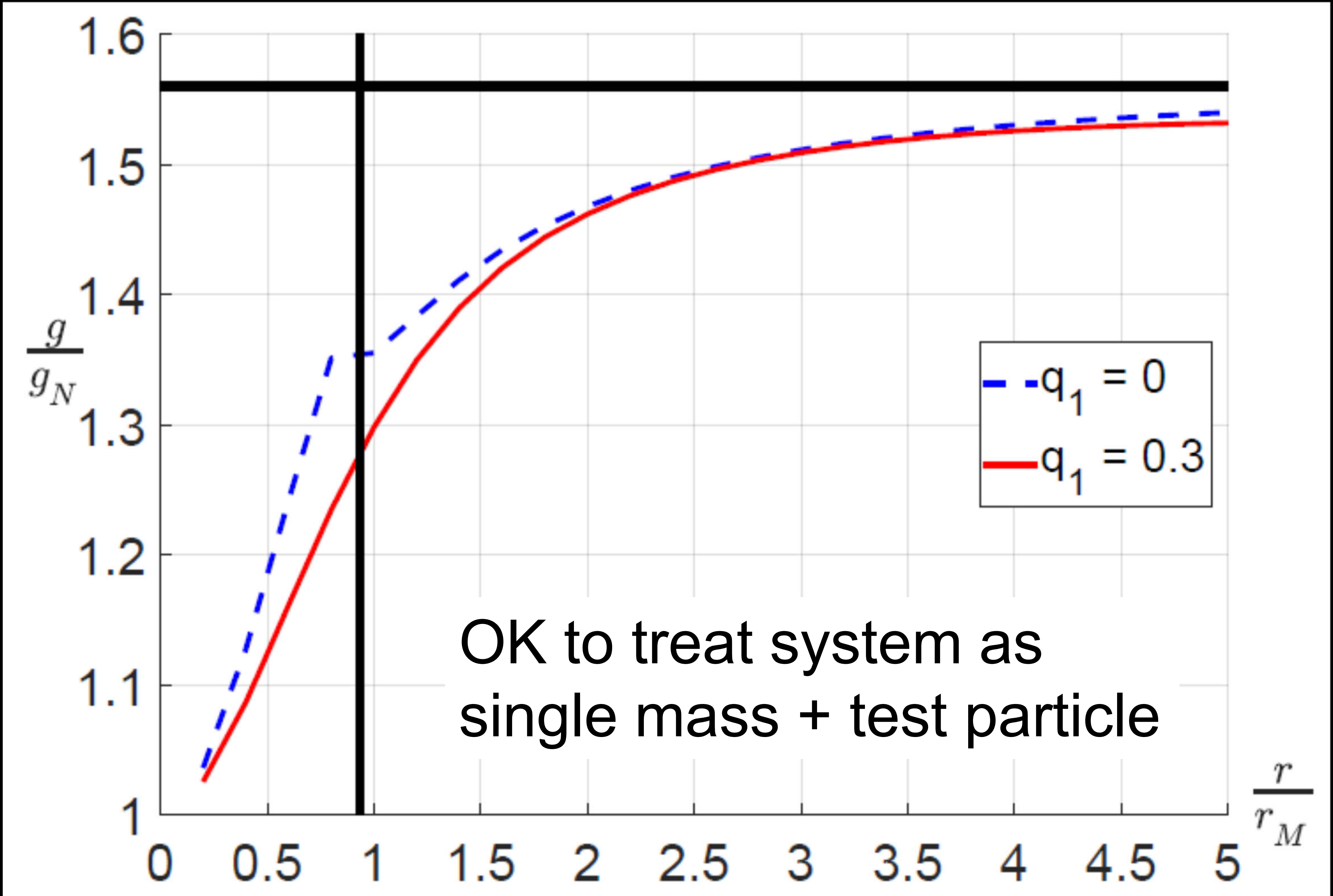


Detection probability
0
0.2
0.4
0.6
0.8
1
0
50
100
150
200
250
300
Number of systems
Full 3D velocity, γ = 0
3D velocity, γ = 1.2
3D velocity, γ = 2
Sky-projected velocity only, γ = 0
Sky-projected velocity, γ = 1.2
Sky-projected velocity, γ = 2

Theoretical uncertainties

- MOND interpolating function constrained within ~10% by extragalactic rotation curves (Lelli+ 2018, ApJ, 836, 152 & P. Li+ ’18)
- Different MOND formulations yield similar results (Arxiv: 1509.08457, Banik & Zhao ’18)
- Treating binary as single mass is OK in Newtonian and EF-dominated regimes. Small error at intermediate separations.
- Consider force between two stars aligned with EF with different q_1, fraction of total system mass M in the less massive star

The wide binary mass ratio



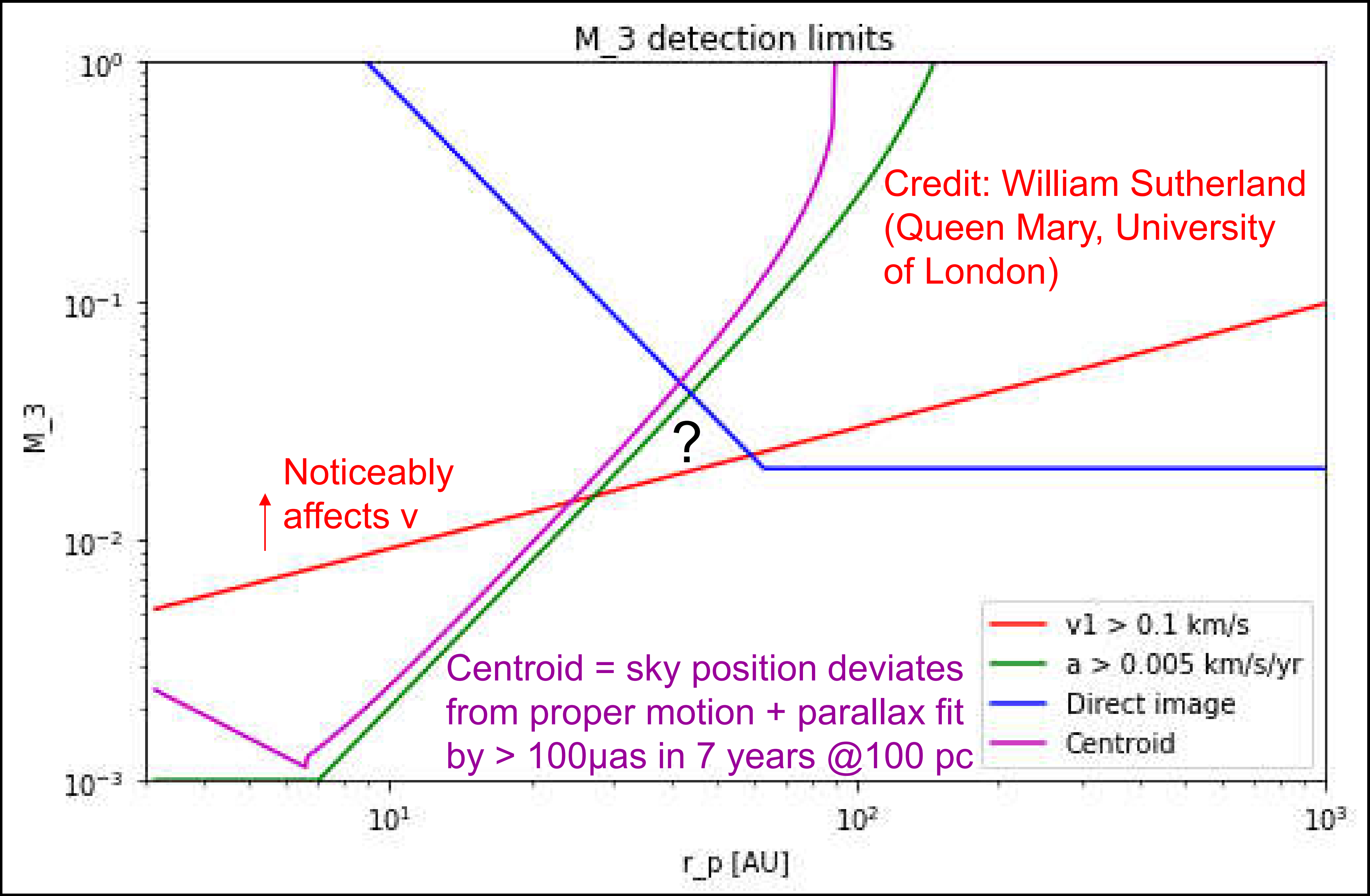
Recently ionised systems

- Some WBs were ionised recently – may hamper the wide binary test
- WB velocity only ~0.3 km/s while interstellar velocity dispersion ~15 km/s
- ➢ Ionisation possible only if field star passed within ~200 AU of star in a WB
- System would disperse within orbital timescale of ~1 Myr
- ➢ Very few WBs ionised so recently that they are still nearby in 3D
- ➢ Similarly, molecular clouds not big problem

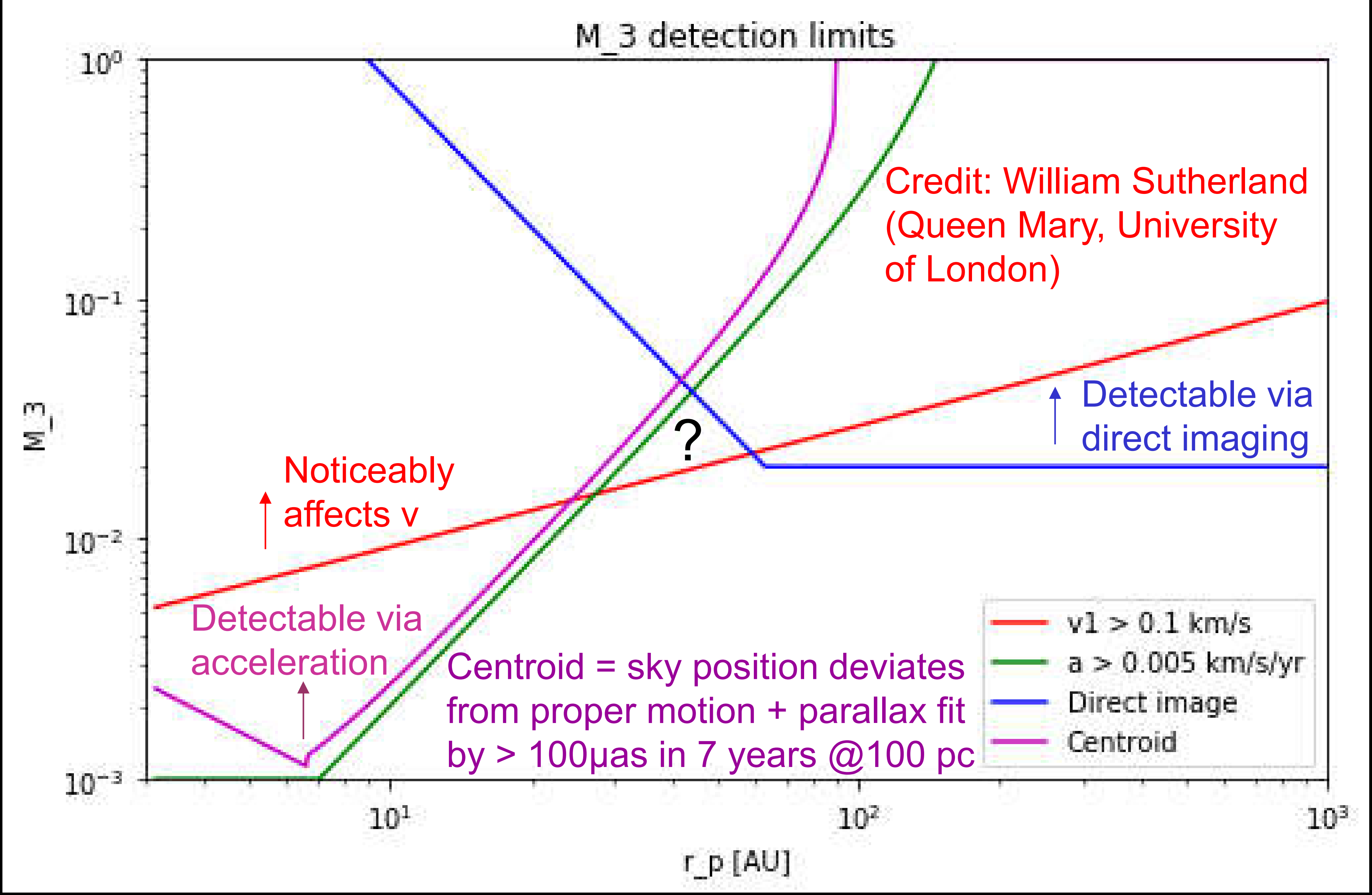
Undetected third star

- System could have a third undetected low-mass star
- Must be quite close to one of the WB stars to significantly affect its velocity
- Would cause a large orbital acceleration
- ➢ May be detectable as radial velocity trend or as acceleration of sky position (astrometry)
- Technique fails further out, but then star must be more massive, so easier to detect directly

Closing the gap

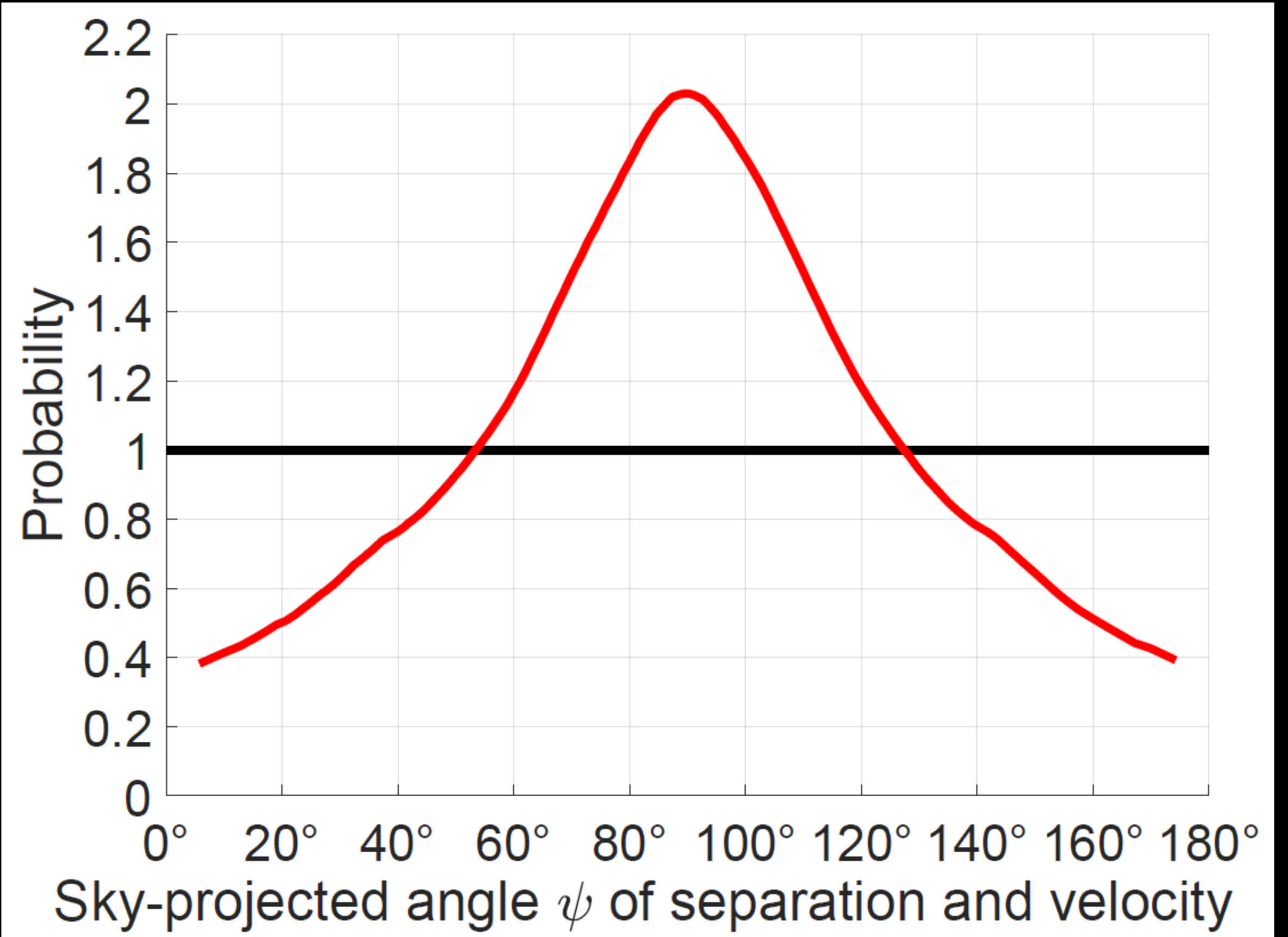


Closing the gap

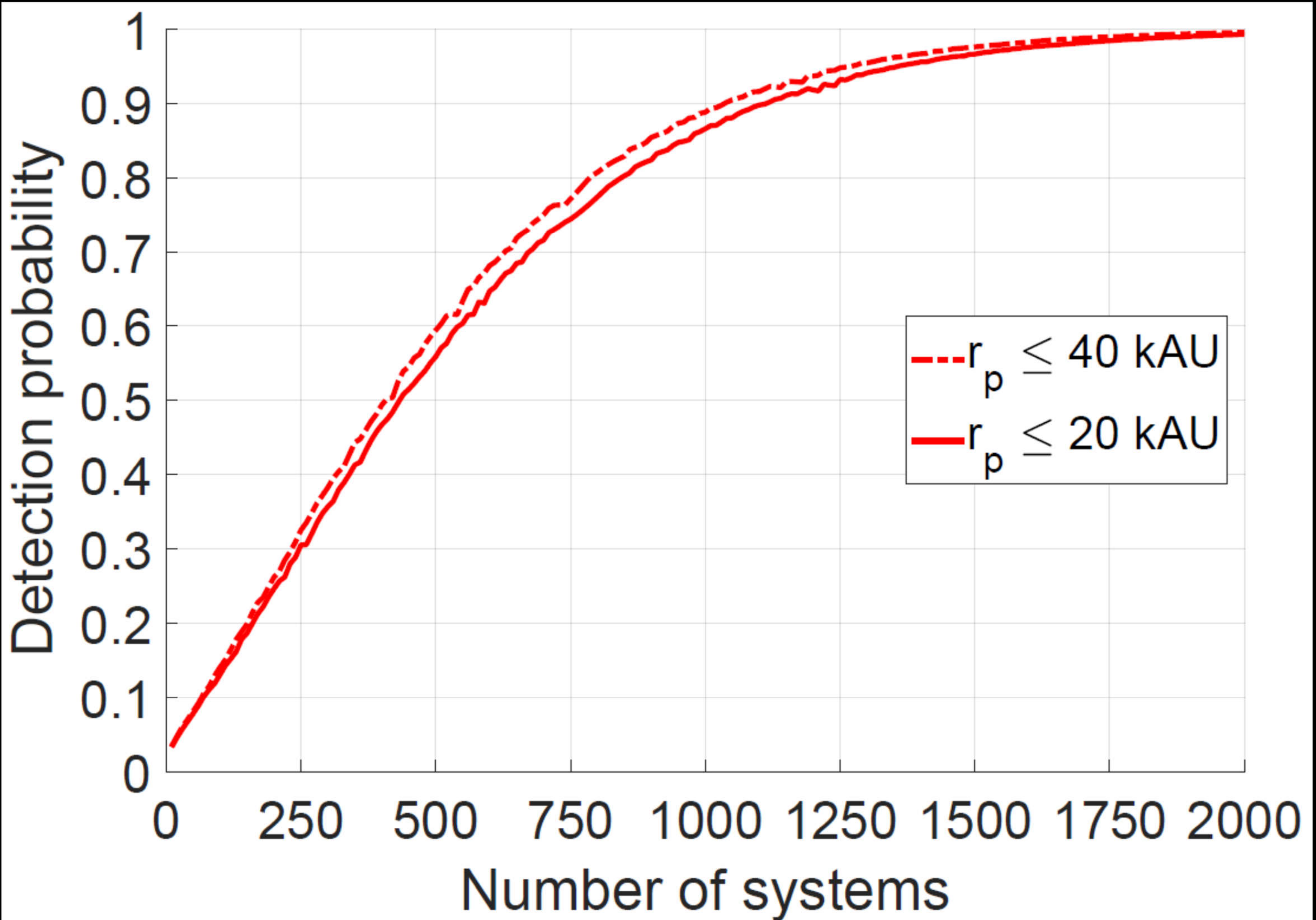


Are relative velocities orbital?

- Systems with highest $\tilde{v}$ should be moving ~ within sky plane and **near pericentre**
- ➢ WB separation and relative velocity should be orthogonal
- Focus on angle ψ between these directions
- ψ should be isotropic if relative velocity due to undetected star near star in WB
- For WB orbital motion, ψ should be anisotropic and prefer values near 90°



2.2
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
Probability
0°
20°
40°
60°
80°
100°
120°
140°
160°
180°
Sky-projected angle ψ of separation and velocity



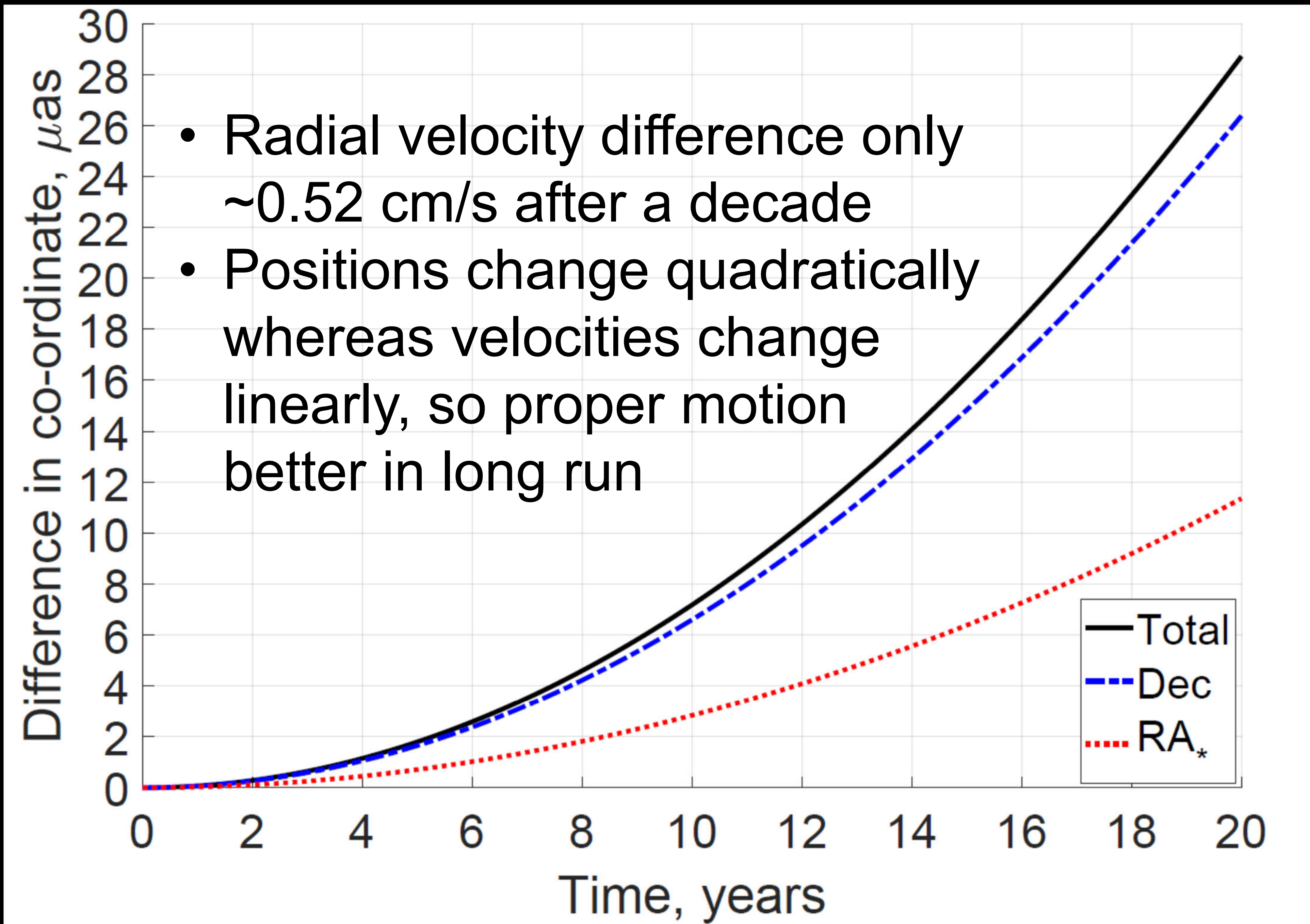
Detection probability
Number of systems
$r_p \leq 40$ kAU
$r_p \leq 20$ kAU
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
250
500
750
1000
1250
1500
1750
2000

Closest wide binary = nearest star

- Proxima Centauri is on a ~13000 AU orbit around α Cen A & B (~17 AU apart) – Kervella 2017 (A & A, 598, Letter 7)
- Can treat system as a central point mass + test particle (Proxima Cen), embedded in external field from Galaxy
- Orbit usually stable over several Gyr, though some orbital orientations would be unstable (observed one is stable)
- Gravity on Proxima Cen ~40% higher in MOND
- Testable with proper motions?

Theia mission (Arxiv: 1707.03148)

- Radial velocity difference only ~0.52 cm/s after a decade
- Positions change quadratically whereas velocities change linearly, so proper motion better in long run

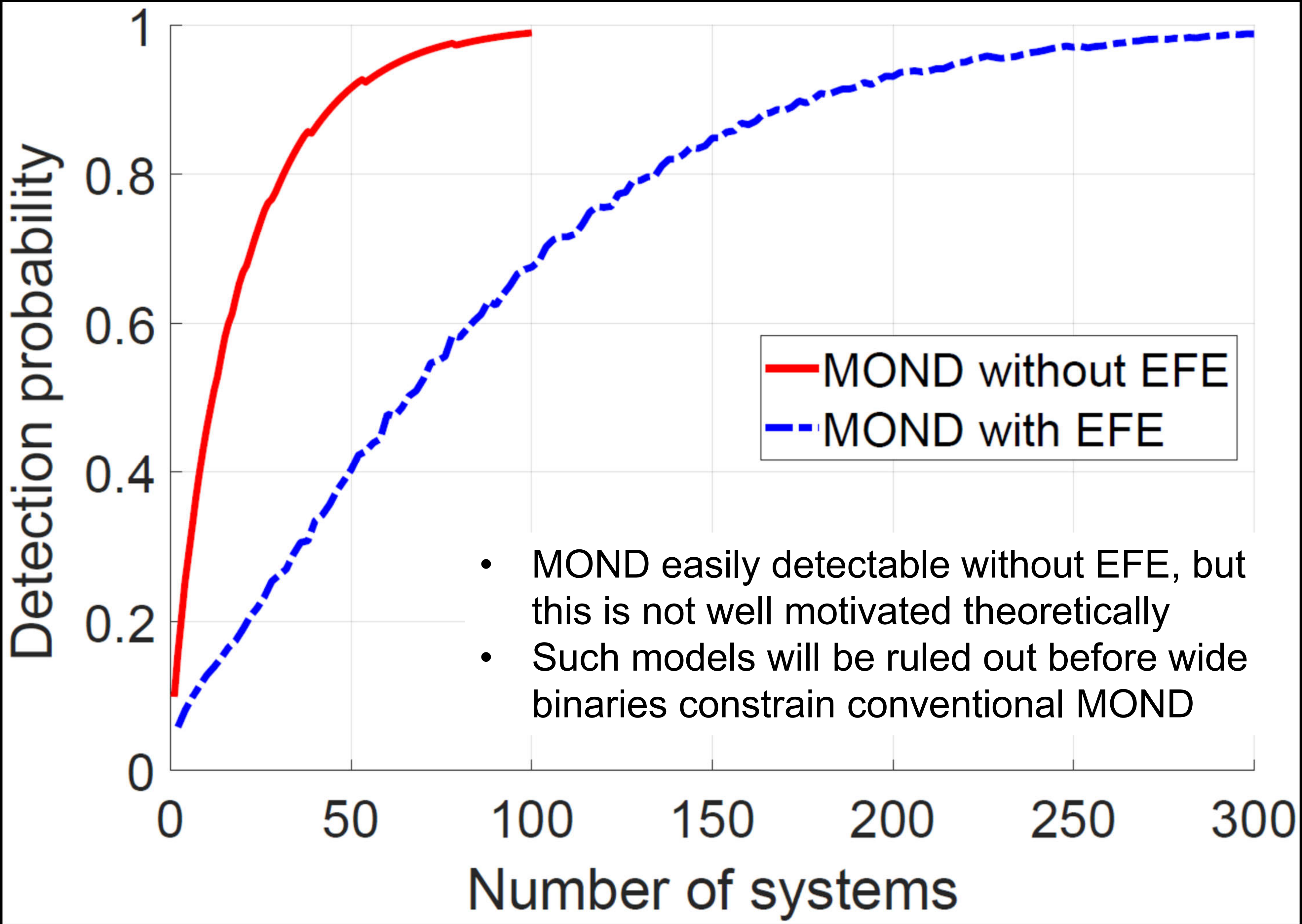


Conclusions

- Gravity may deviate from Newtonian inverse square law at low accelerations (like in MOND)
- Motivation is from galaxies, but can be tested using wide binary stars with separations ~(3-20) kAU
- Orbital velocities ~300 m/s, but ~20% faster in MOND (with EFE). Detectable with ~500 systems.
- Huge excess without EFE that could be detected easily, but not well motivated theoretically
- Main systematic is third faint star in system
- Testable by direct imaging (large separation & mass) + secular RV/PM acceleration (close-in & low-mass)

➢ May need follow-up of high relative velocity systems

Sky-projected velocities



- MOND easily detectable without EFE, but this is not well motivated theoretically
- Such models will be ruled out before wide binaries constrain conventional MOND