To B or not to B:

Primordial magnetic fields from Weyl anomaly

Takeshi Kobayashi

based on arXiv: 1808.08237
with A. Benevides and A. Dabholkar

Cosmology 2018 in Dubrovnik

Are photons gravitationally produced in the early universe?

cf. inflaton, gravitons

CLASICALLY, NO

$$\mathcal{L} = -\frac{1}{4}\sqrt{-g}\,F_{\mu\nu}F^{\mu\nu}$$

invariant under Weyl transformation: $g_{\mu\nu} \to \Omega^2 g_{\mu\nu}$

Photons do not feel gravity in an FRW universe.

BUT QUANTUM MECHANICALLY ...

Weyl symmetry is anomalous.

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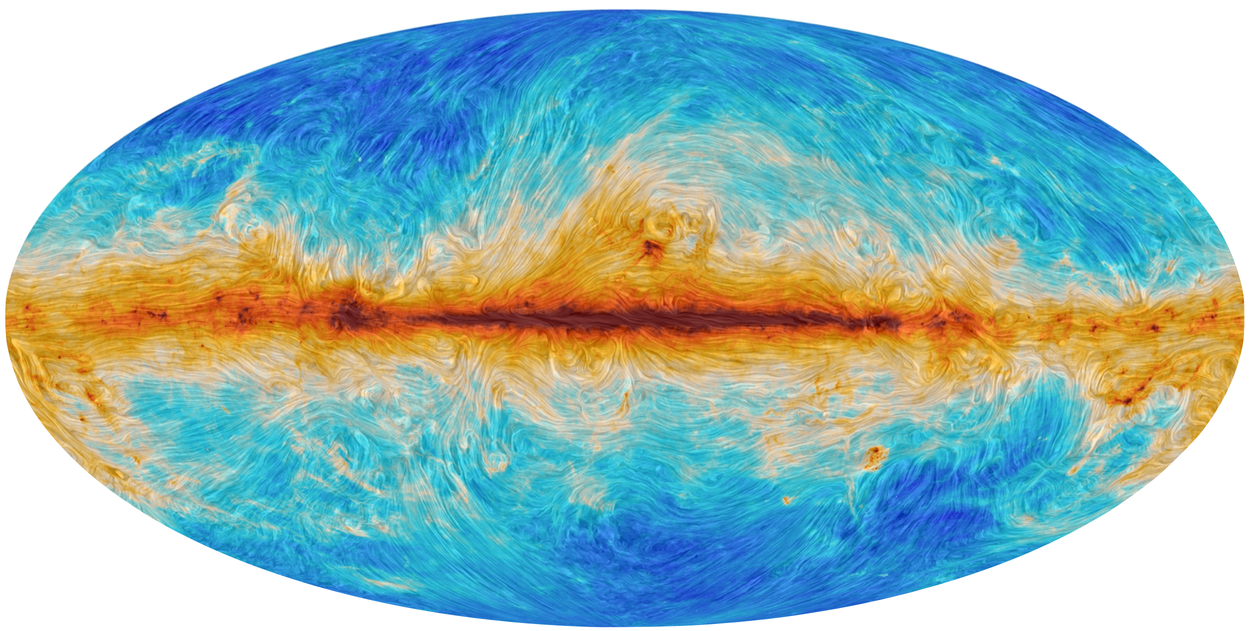
Photons DO feel gravity in an FRW universe.

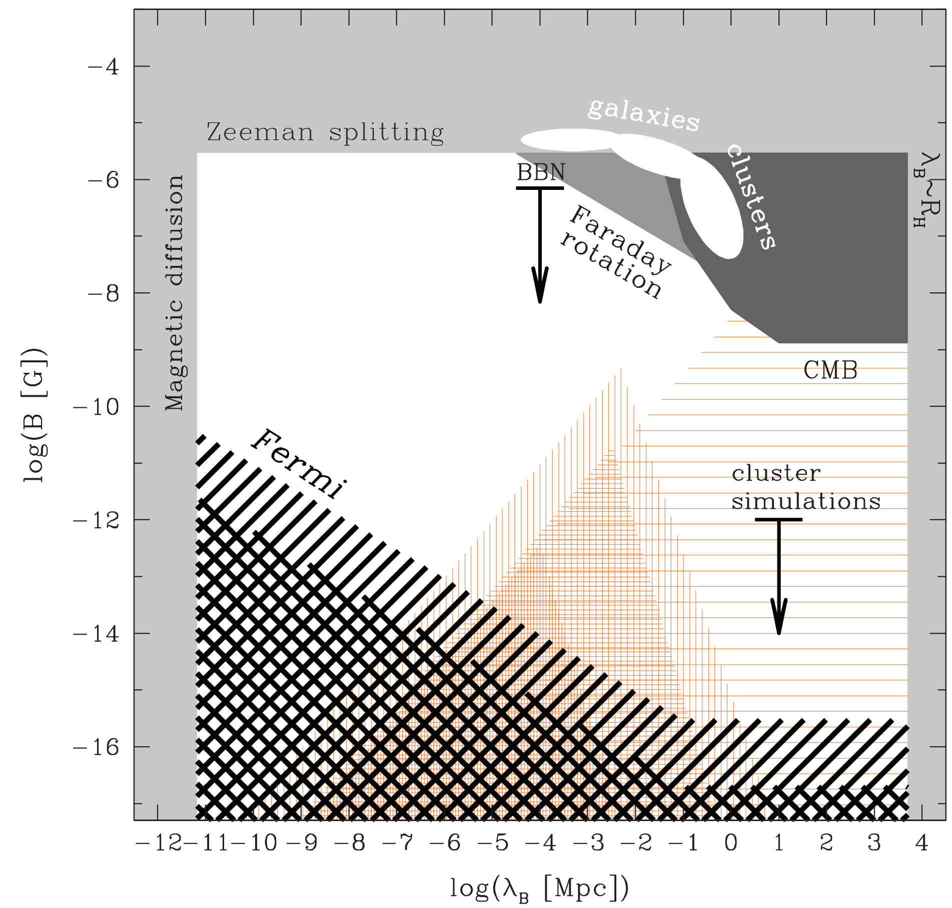
IMPLICATIONS FOR OUR UNIVERSE

cosmological photon production

→ primordial magnetic fields



ESA and the Planck Collaboration



A. Neronov and I. Vovk, Science 328 (2010) 73

galactic B

seed field of $B \sim 10^{-20}\mathrm{G}$

(hints of) extragalactic B

$B \gtrsim 10^{-15}\mathrm{G}$ at $\gtrsim \mathrm{Mpc}$

PRIMORDIAL B FROM WEYL ANOMALY

- Intrinsic to the SM, so the produced B (if any) serves as an irreducible contribution to the B of our universe

- Many studies on this topic since Dolgov '93, but with little consensus on the amplitude of B

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- I will show that there is actually NO B from Weyl anomaly

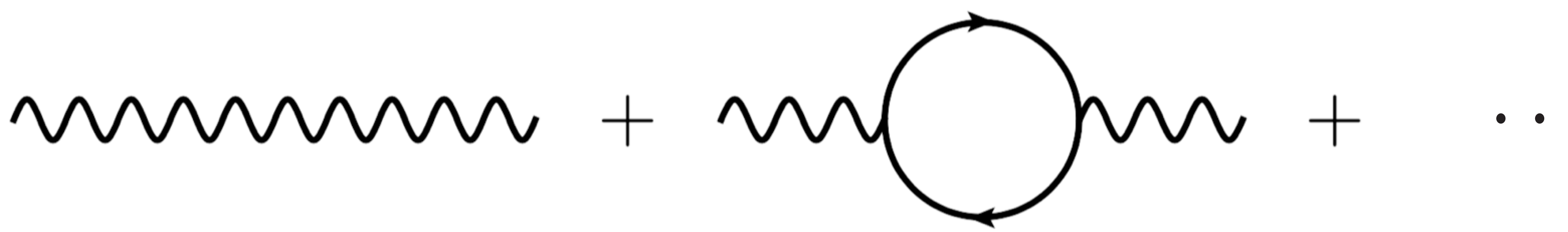
PLAN OF THE TALK

1. quantum effective action in curved spacetime

2. quantum/classical nature of photons

Quantum Effective Action

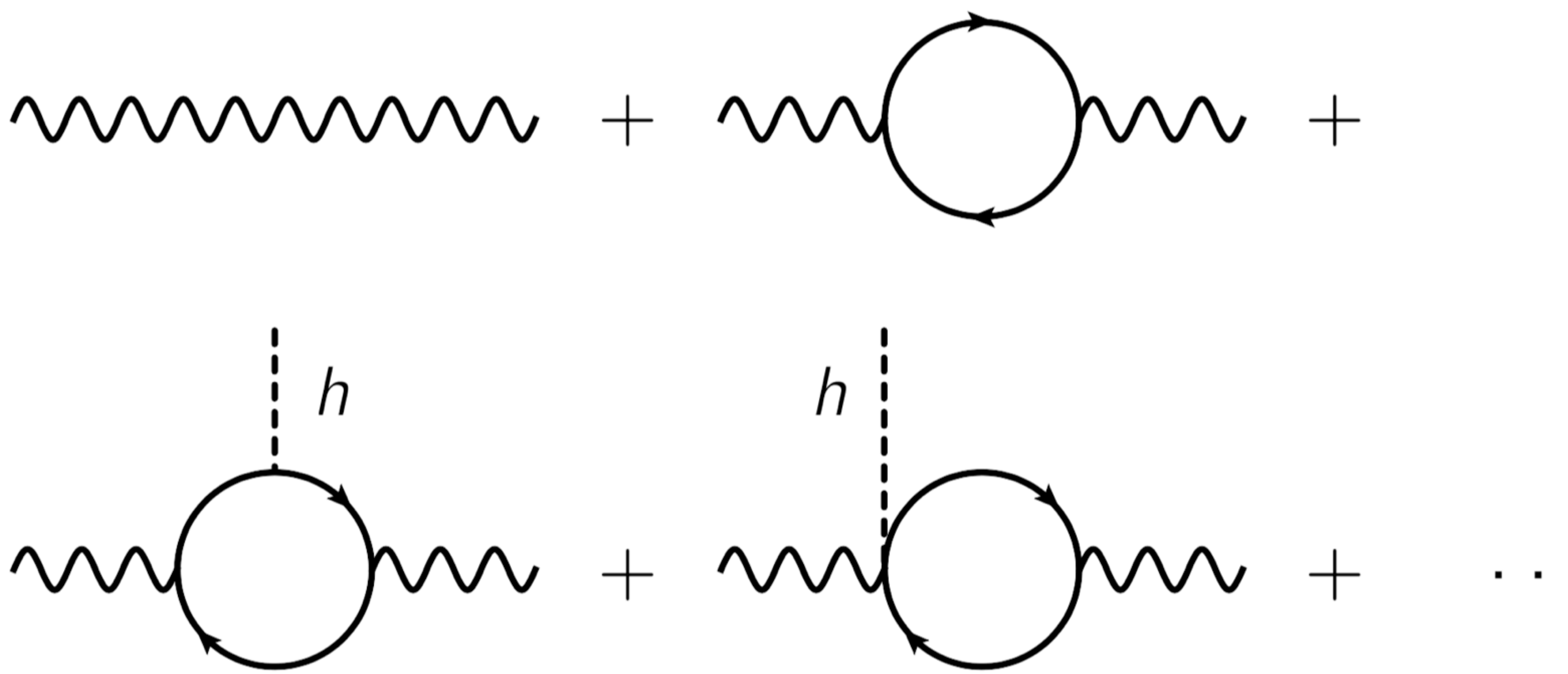
QED EFFECTIVE ACTION IN FLAT SPACE



$$S = -\frac{1}{4e^2}\int d^4x\, F_{\mu\nu}\left[1 - \tilde{\beta}\log\left(\frac{-\partial^2}{M^2}\right)\right]F^{\mu\nu}$$

$$\tilde{\beta} = \frac{d\log e}{d\log M} > 0$$

QED EFFECTIVE ACTION IN CURVED SPACE



Barvinsky, Vilkovisky '83 ~

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

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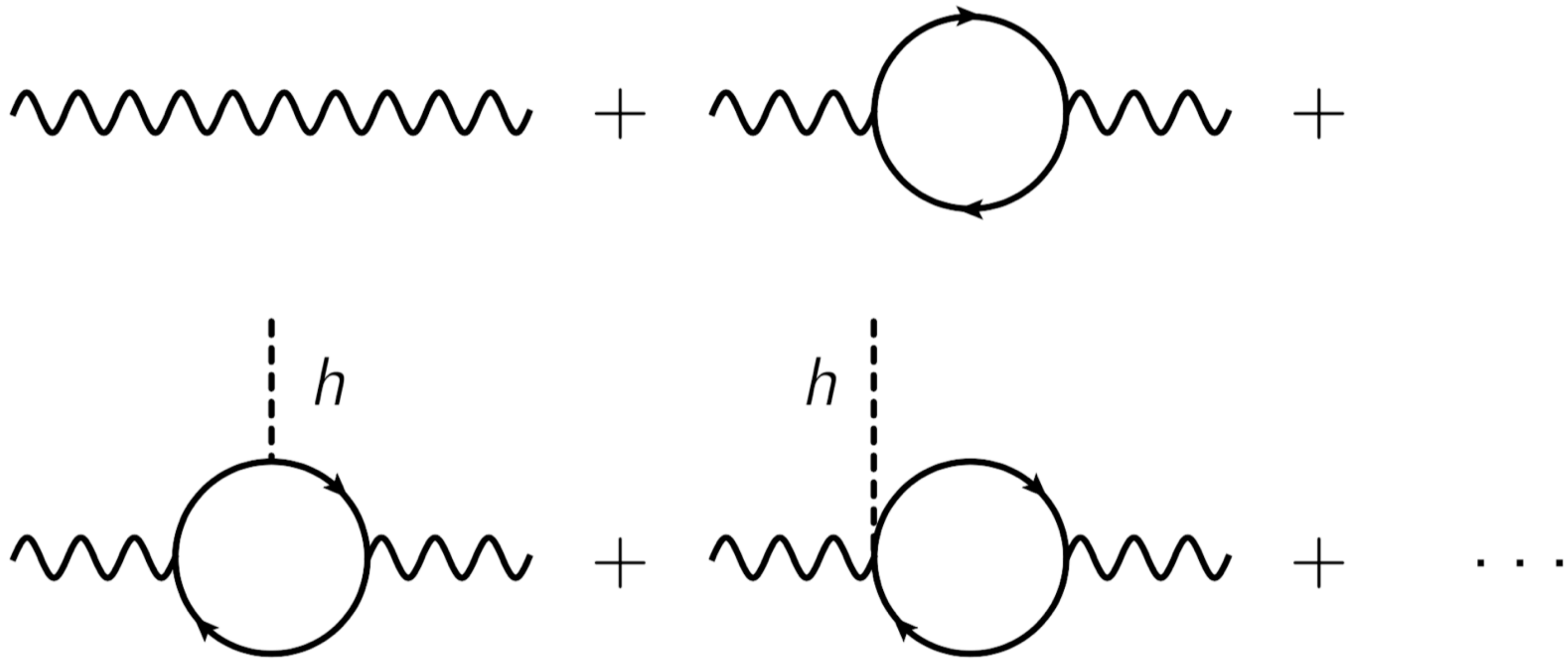


Barvinsky, Vilkovisky '83 ~

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

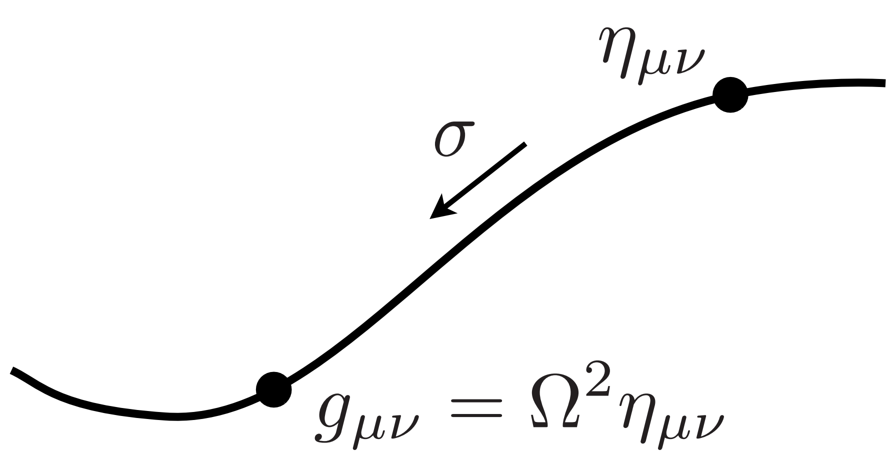
Perturbative expansion valid if $R^2 \ll \nabla^2 R,$

which is NOT the case in cosmology.

BEYOND WEAK GRAVITY

Curvature expansion can be resummed to all orders for classically Weyl-invariant theories in Weyl-flat spacetimes.

Bautista, Benevides, Dabholkar '17



$$S[g, A] = S_{\text{flat}}[\eta, A] + \int_0^1 d\sigma \int d^4x \sqrt{-\Omega^{2\sigma}\eta}\ (\log \Omega)\ \mathcal{B}\left[\Omega^{2\sigma}\eta, A\right]$$

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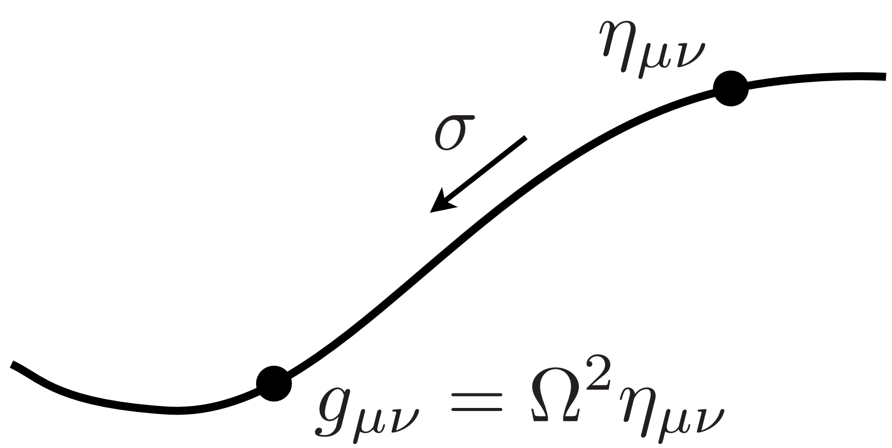

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$$= -\frac{1}{4e^2}\int d^4x\, F_{\mu\nu}\left[1 - \tilde{\beta}\log\left(\frac{-\partial^2}{\Omega^2 M^2}\right)\right]F^{\mu\nu}$$

IN AN FRW UNIVERSE

$$ds^2 = a(\tau)^2 \left(-d\tau^2 + d\boldsymbol{x}^2\right)$$

$$S = -\frac{1}{4}\int d^4x\, d^4y\, \mathcal{I}^2(x,y)\, F_{\mu\nu}(x)F^{\mu\nu}(y)$$

$$\mathcal{I}^2(x,y) = \frac{1}{e^2}\int \frac{d^4k}{(2\pi)^4} e^{ik\cdot(x-y)} \left[1 - \tilde{\beta}\log\left(\frac{k^2}{a(\tau)^2 M^2}\right)\right]$$

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logarithmic running of coupling with physical momentum k/a

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logarithmic running of coupling with physical momentum k/a

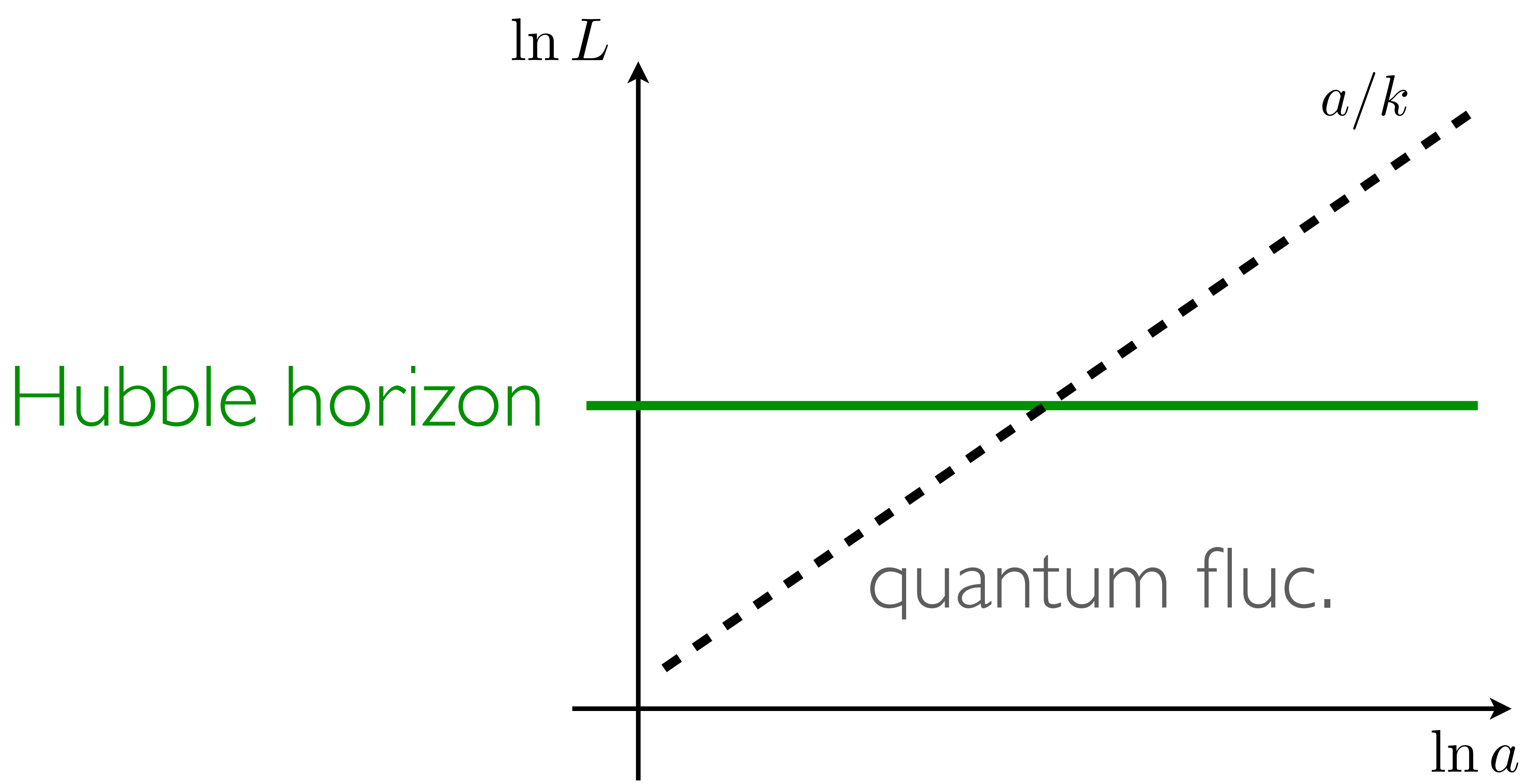
→ logarithmic dependence on a

Gauge Field Evolution

REVIEW: MASSLESS SCALARS IN DS

$$S = -\frac{1}{2}\int d^4x\, a(\tau)^2\, \partial_\mu\phi\, \partial^\mu\phi$$ (indices raised with $\eta^{\mu\nu}$)

$$\phi_k'' + 2\frac{a'}{a}\phi_k' + k^2\phi_k = 0$$



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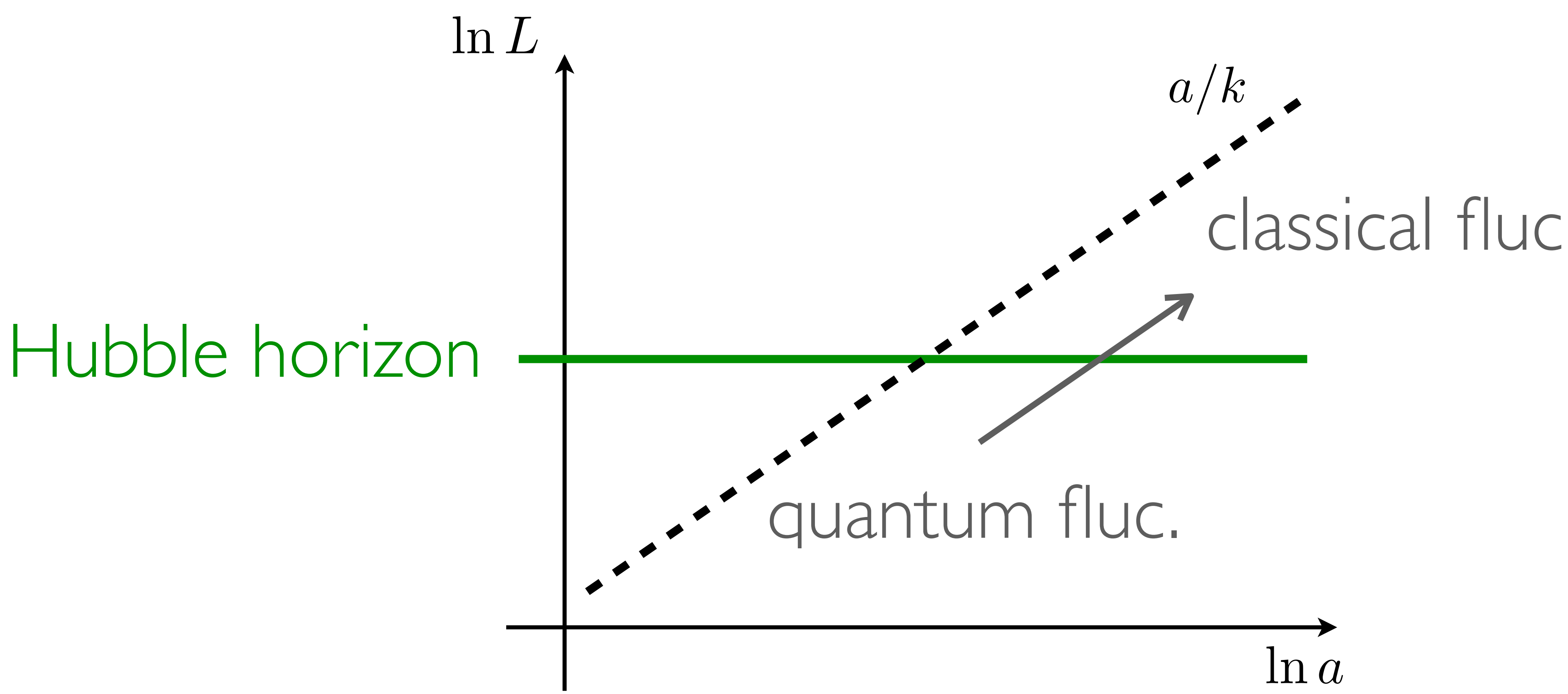

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PHOTONS

$$S = -\frac{1}{4}\int d^4x\, I(\tau)^2\, F_{\mu\nu}F^{\mu\nu}$$

$$I(\tau)^2 = \frac{1}{e^2}\left[1 + 2\tilde{\beta}\log\left(\frac{a(\tau)}{a_\star}\right)\right]$$

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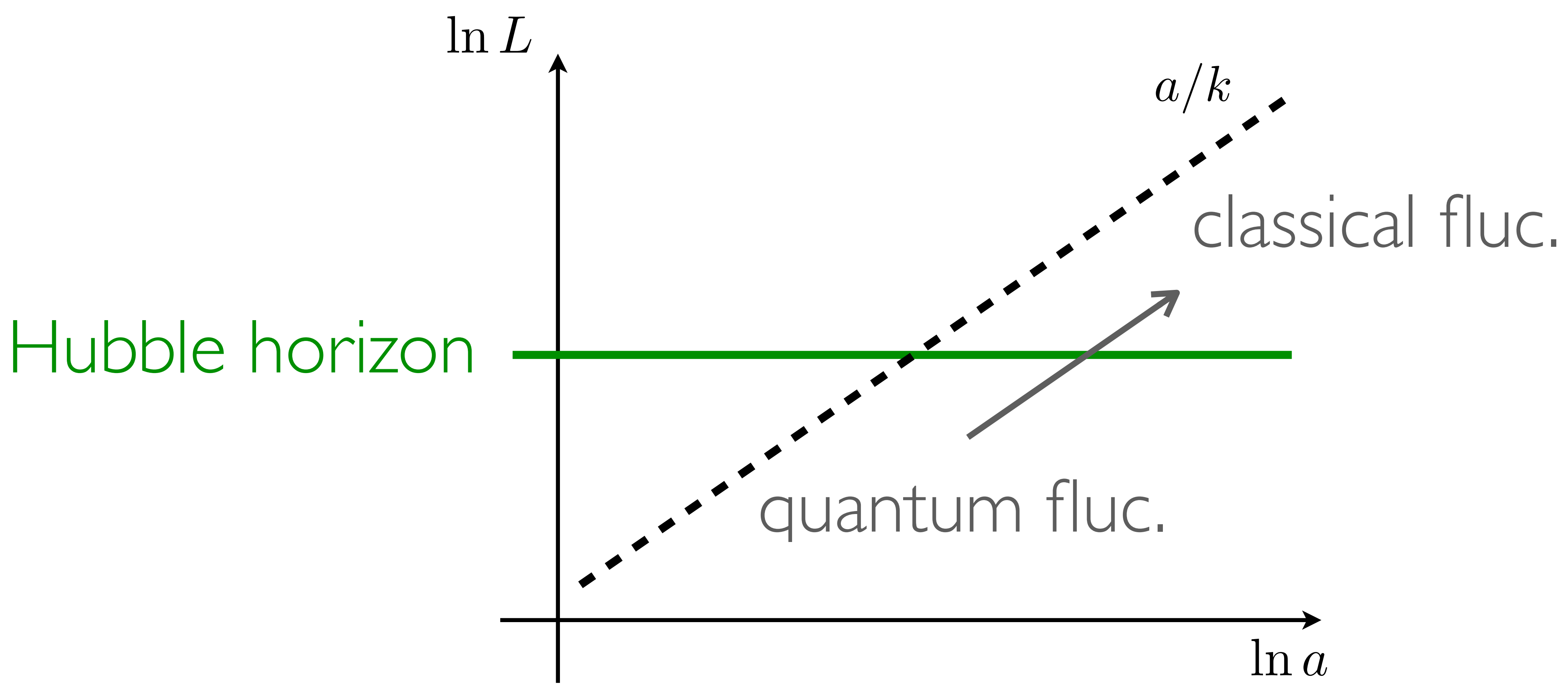

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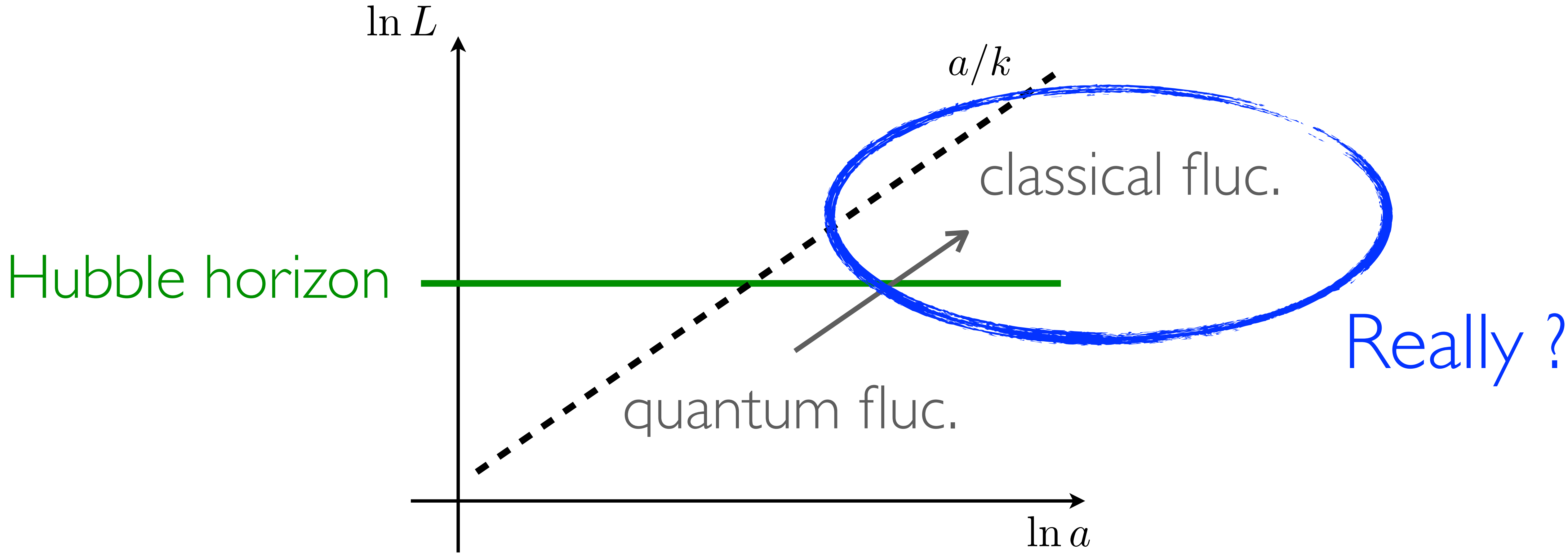


MEASURES OF 'QUANTUMNESS'

Grishchuk, Sidorov '90
Maldacena '15 Green, TK '15

1. Bogoliubov coefficient

$|\beta_{\boldsymbol{k}}|^2$: number of created photons per comoving phase volume

2. quantumness measure

$$\kappa_k \equiv \left| \frac{\langle 0|\phi_{\boldsymbol{k}}\, \phi_{-\boldsymbol{k}}(\tau)|0\rangle\, \langle 0|\pi_{\boldsymbol{k}}\, \pi_{-\boldsymbol{k}}|0\rangle}{[\phi_{\boldsymbol{k}}, \pi_{-\boldsymbol{k}}]^2} \right|^{1/2} \quad \begin{cases} \sim 1 & \text{: quantum} \\ \gg 1 & \text{: classical} \end{cases}$$

RESULTS

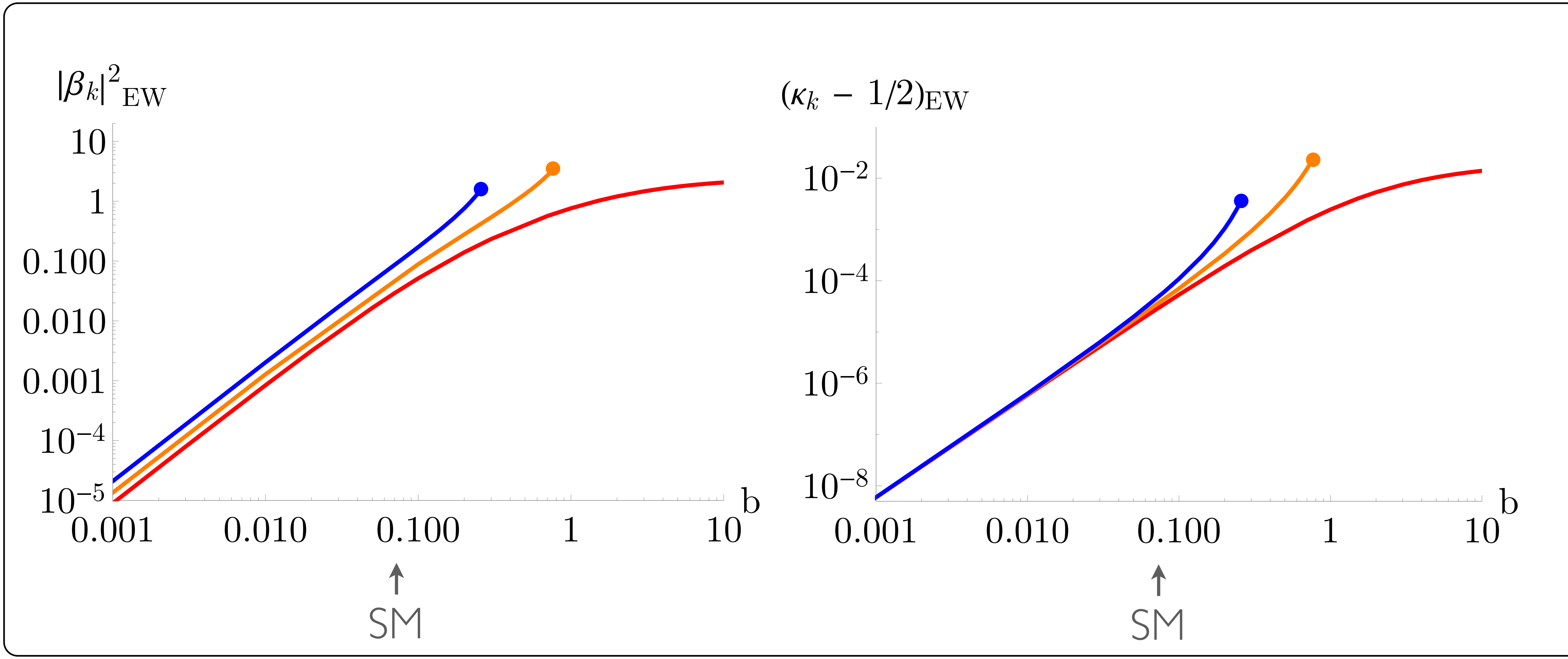
$H_{\rm inf}$ = 10^{14} GeV

$H_{\rm inf}$ = 10^{6} GeV

$H_{\rm inf}$ = 1 GeV

$$\frac{k}{a_0} = (10\,\text{Gpc})^{-1} \qquad b = \frac{\text{Tr}\,(Q^2)}{6\pi^2}$$

endpoints of curves are set by Landau pole



RESULTS

- $H_{\rm inf} = 10^{14}$ GeV (blue)
- $H_{\rm inf} = 10^{6}$ GeV (orange)
- $H_{\rm inf} = 1$ GeV (red)

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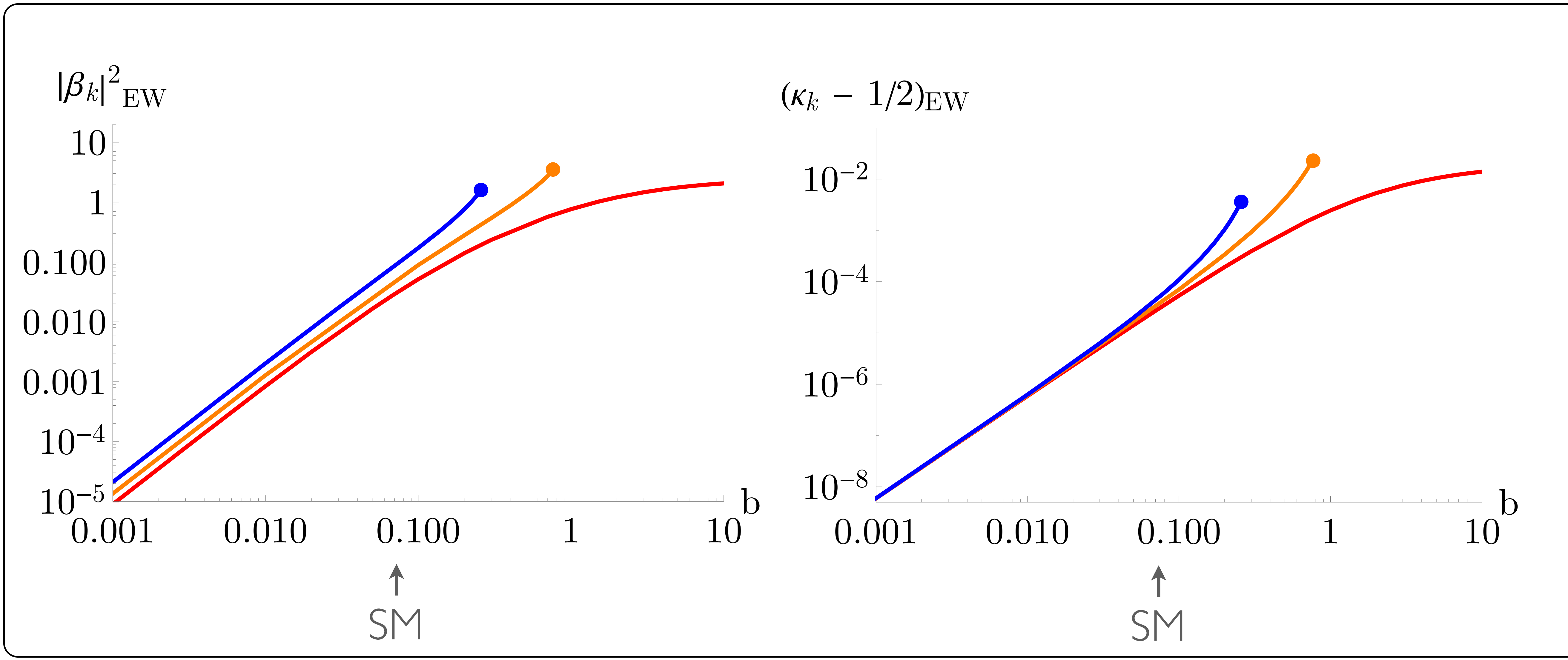

Logarithmic background dependence CANNOT convert vacuum fluc of gauge field into classical magnetic fields.

SUMMARY

No B

- Primordial manetic fields do not arise from QED Weyl anomaly, irrespective of # of massless charged particles in the theory

- Quantum effective action beyond the weak gravity regime should be used for cosmological studies

- It should not be taken for granted that quantum fluc become classical