

COSMOLOGICAL CONSTRAINTS ON NON-STANDARD NEUTRINO INTERACTIONS

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Based on work with F. Forastieri, P. Natoli

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THE COSMIC NEUTRINO BACKGROUND

The presence of a background of relic neutrinos (**CνB**) is a basic prediction of the standard cosmological model

- Neutrinos are kept in thermal equilibrium with the cosmological plasma by weak interactions until $T \sim 1 \text{ MeV}$ ($z \sim 10^{10}$);
- Below $T \sim 1 \text{ MeV}$, neutrino free stream keeping an equilibrium spectrum:

$$f_\nu(p) = \frac{1}{e^{p/T} + 1}$$

- Today $T_\nu = 1.9 \text{ K}$ and $n_\nu = 113 \text{ part/cm}^3$ per species
- Free parameters: the three masses (but cosmological evolution mostly depends on their sum)

THE COSMIC NEUTRINO BACKGROUND

Weak cross section: $\sigma \simeq G_F^2 T^2$

Weak interaction rate $\Gamma = n \langle \sigma v \rangle \sim G_F^2 T^5$

Expansion rate $H \simeq \frac{T^2}{m_p}$

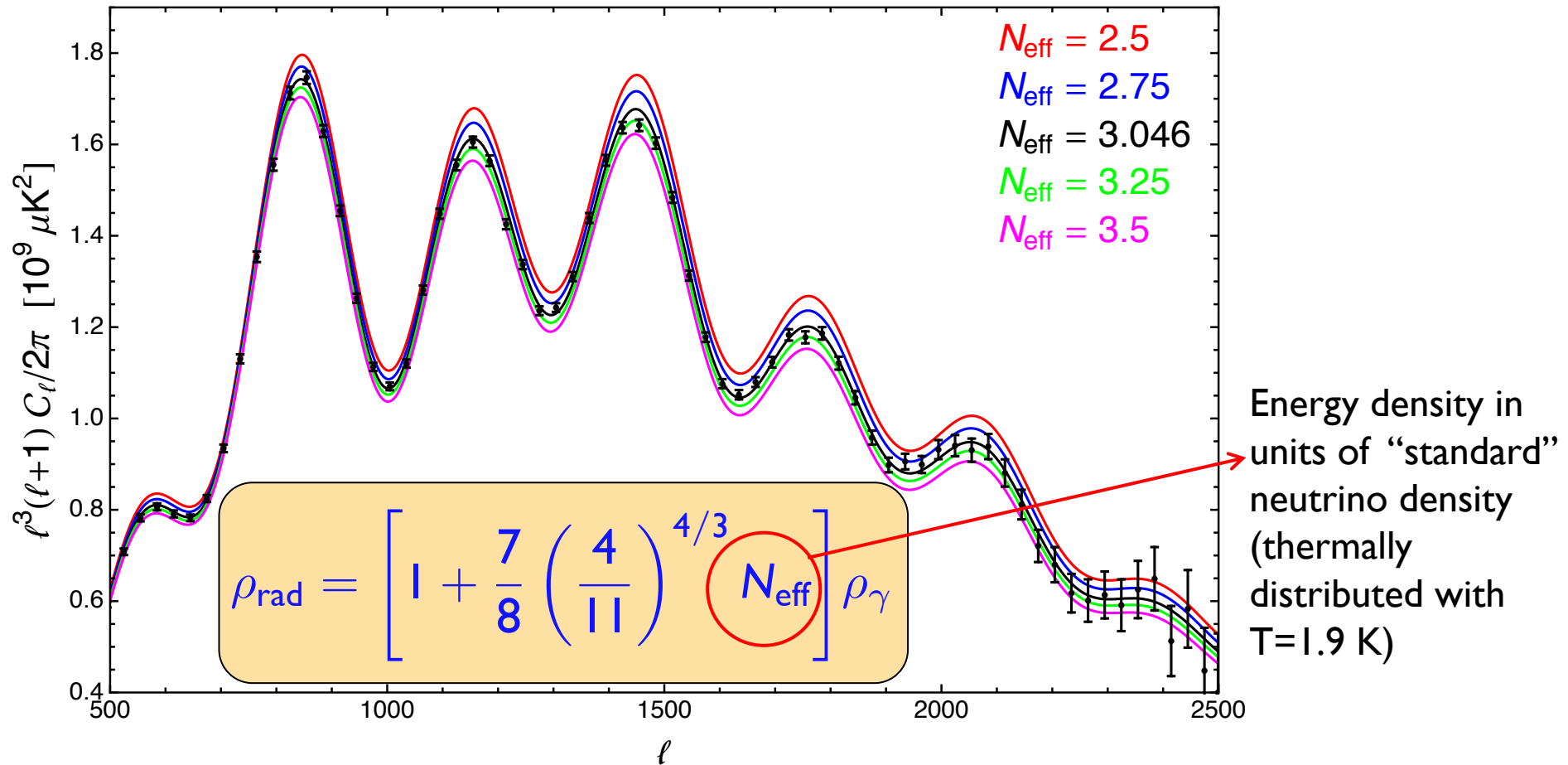
Interactions become ineffective at

$$1 \simeq \frac{\Gamma}{H} \sim G_F^2 T^3 m_p \sim \left(\frac{T}{\text{MeV}} \right)^3$$

Given this, we can use conservation laws to compute the temperature, density, etc... of neutrinos at a given time.

Effective number of relativistic species

Planck 2018 + BAO: $N_{\text{eff}} = 2.99 \pm 0.17$



Due to non-instantaneous decoupling, the standard expectation is $N_{\text{eff}} = 3.046$ (updated calculation gives $N_{\text{eff}} = 3.045$; see de Salas & Pastor 2016)

(note I am showing $\sim \ell^4 C_\ell$, not $\ell^2 C_\ell$)

THE COSMIC NEUTRINO BACKGROUND

The Λ CDM(+ ν) model assumes:

- only weak and gravitational interactions for ν 's;
- no sterile neutrinos or other light relics;
- perfect lepton symmetry (zero chemical potential);
- no entropy generation after neutrino decoupling beyond e^+e^- annihilation;
- neutrinos are stable;
- in general, there are no interactions that could lead to neutrino scattering/annihilation/decay

See M. Gerbino's talk on Friday

THE COSMIC NEUTRINO BACKGROUND

Possible extensions to the standard picture:

- Non-standard interactions, e.g. scalar interactions (scattering will affect neutrino free streaming; decay changes N_{eff})
- Non-thermal distributions, e.g. low-reheating scenarios) ($N_{\text{eff}} < 3.046$, suppression of the spectrum)
- Sterile neutrinos ($N_{\text{eff}} > 3.046$, another free streaming species)
- Large lepton asymmetries ($N_{\text{eff}} > 3.046$, larger average velocity)

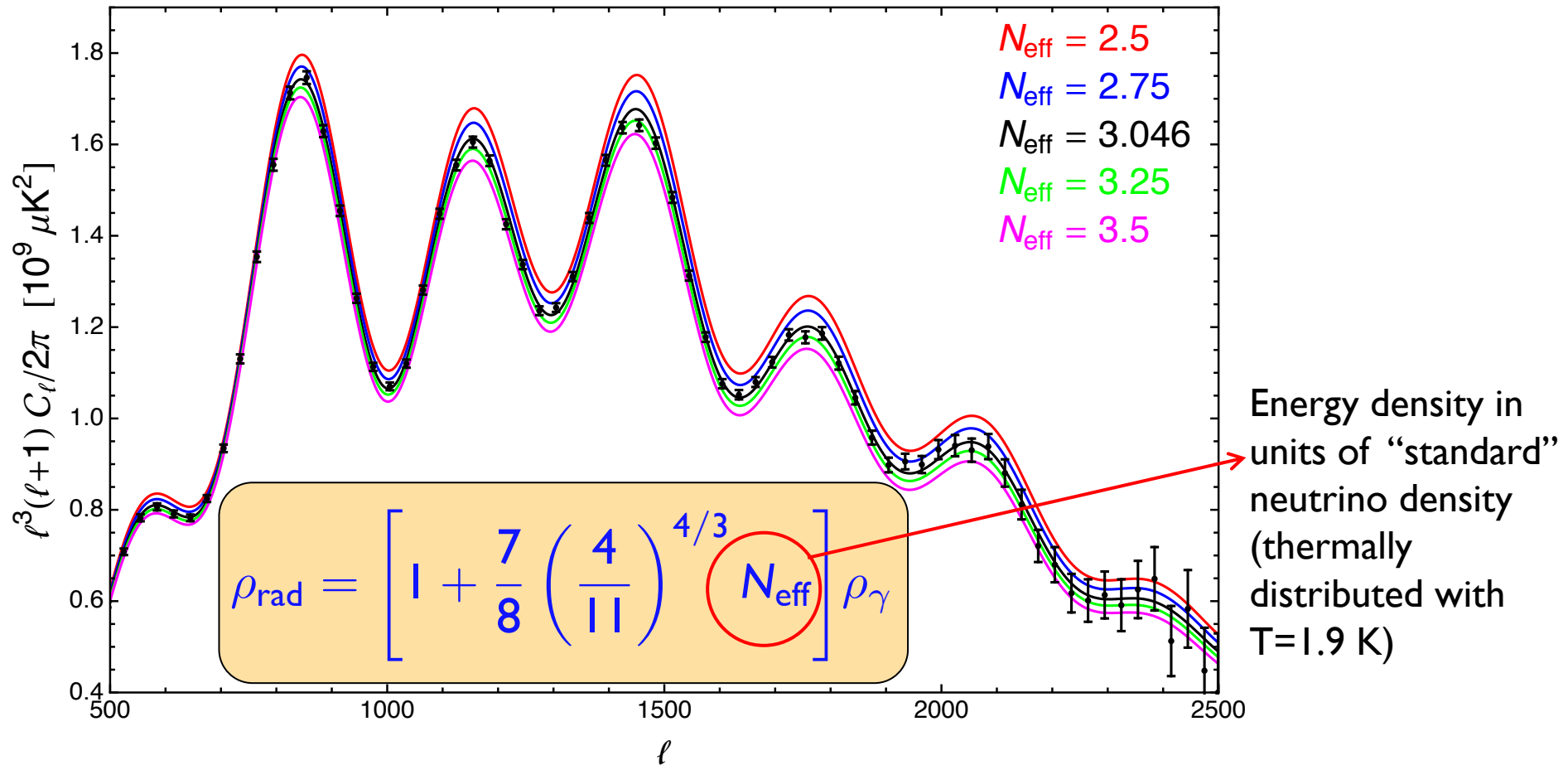
THE COSMIC NEUTRINO BACKGROUND

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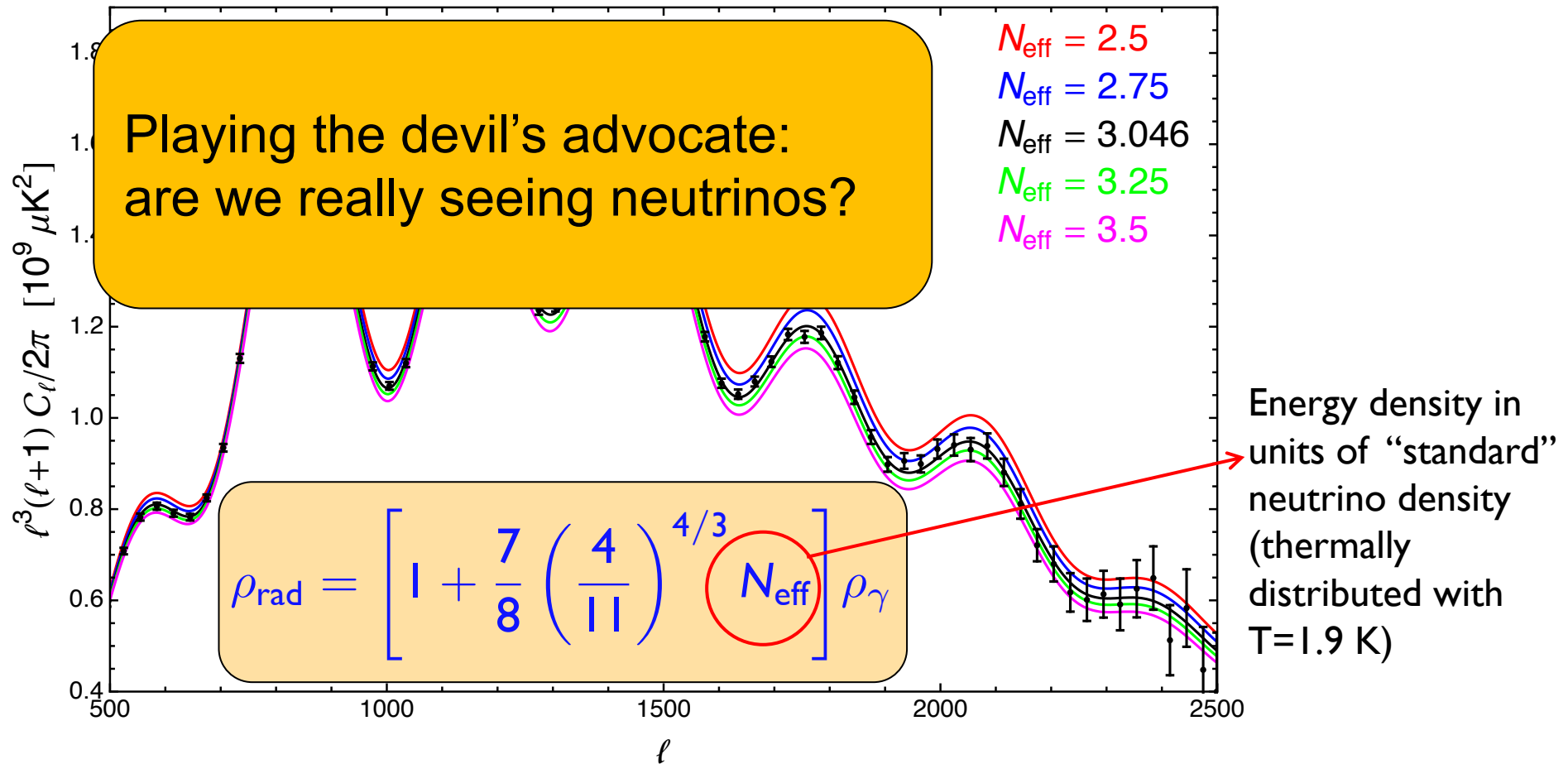


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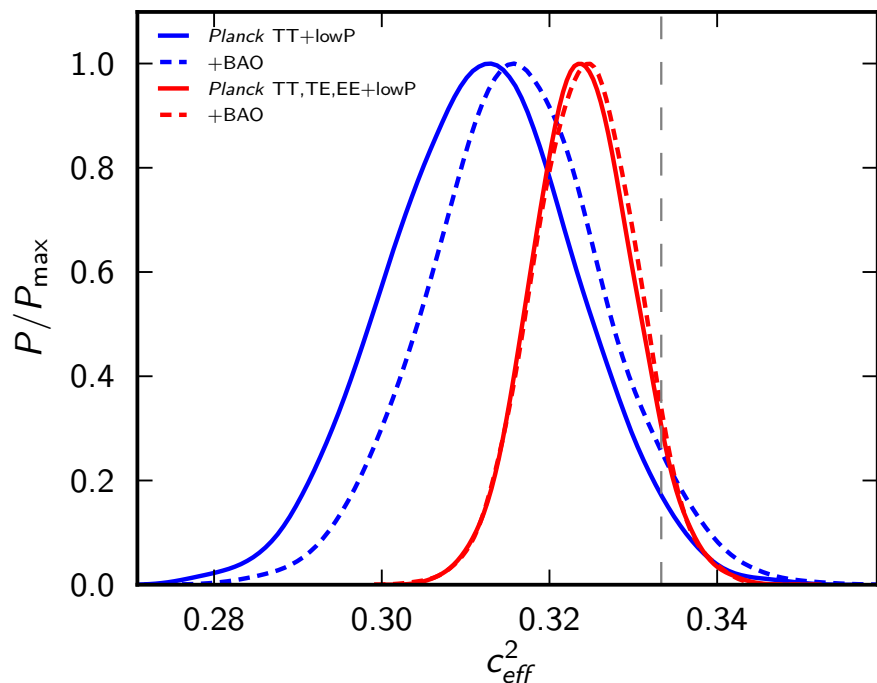


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(note I am showing $\sim l^4 C_l$, not $l^2 C_l$)

Probing $C_{\nu B}$ perturbations

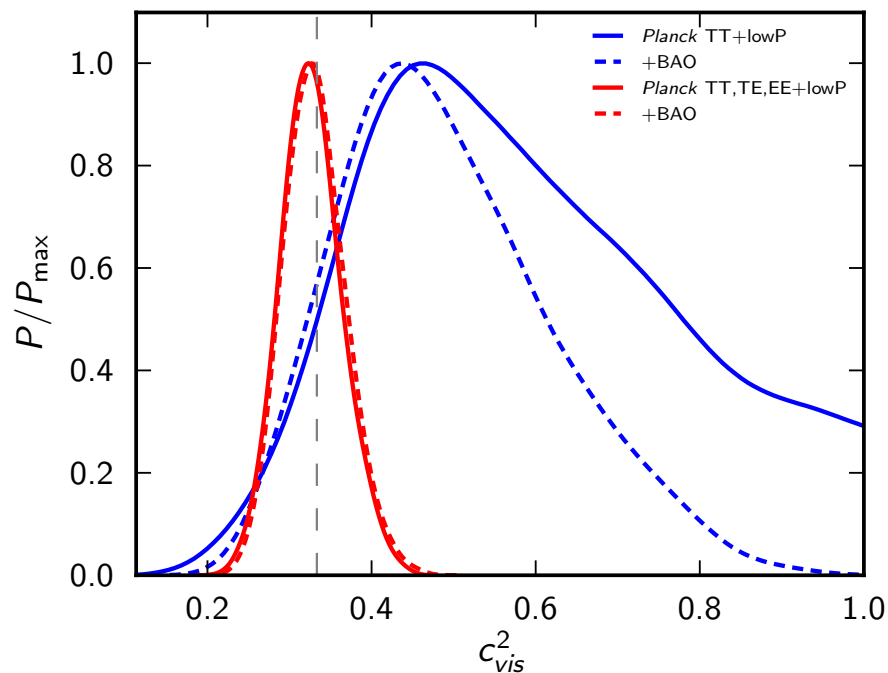
Parameterized by the effective ν sound speed and viscosity
Consistent with free-streaming neutrinos ($c_{\text{vis}}^2 = c_{\text{eff}}^2 = 1/3$)



PlanckTT+lowP+BAO

$$c_{\text{eff}}^2 = 0.316 \pm 0.010$$

$$c_{\text{vis}}^2 = 0.44^{+0.15}_{-0.10}$$

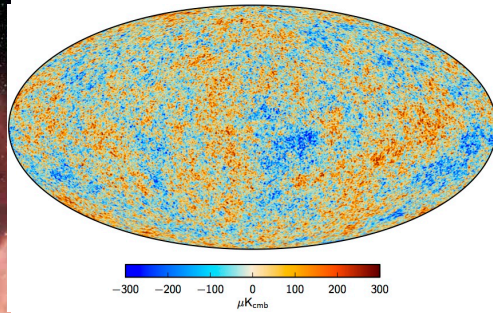


PlanckTT,TE,EE+lowP+BAO

$$c_{\text{eff}}^2 = 0.3242 \pm 0.0059$$

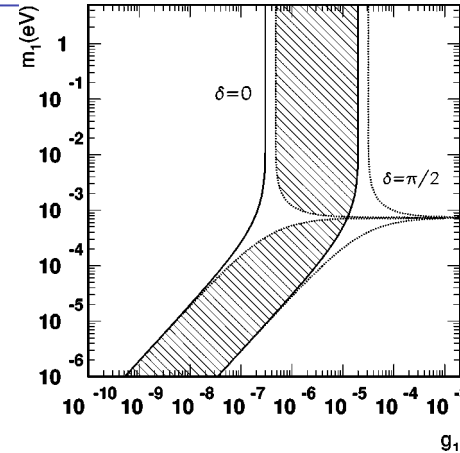
$$c_{\text{vis}}^2 = 0.331 \pm 0.037$$

CONSTRAINTS ON SECRET INTERACTIONS



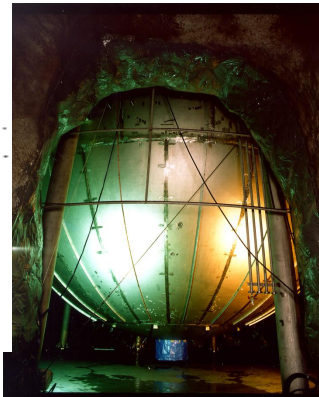
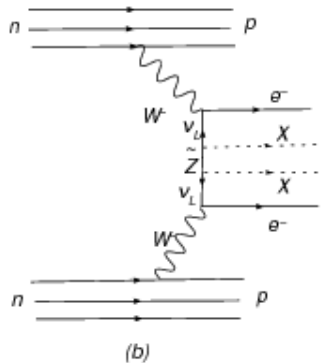
cosmology

$g_{ij} < (\text{few}) \times 10^{-7}$
(mass basis)



Supernovae: $g_{i'j'} < 3 \times 10^{-7}$ or $g_{i'j'} > 2 \times 10^{-5}$
(medium basis)

$$\mathcal{L} \supset h_{ij} \bar{\nu}_i \nu_j \phi + g_{ij} \bar{\nu}_i \gamma_5 \nu_j \phi + h.c. ,$$

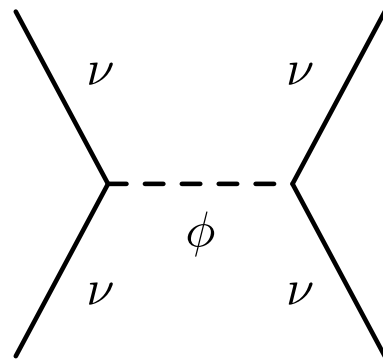
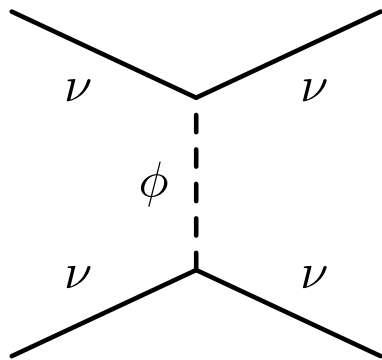


$0\nu 2\beta$ decay

$g_{ee} < (0.8 \div 1.6) \times 10^{-5}$
(flavor basis)

SECRET INTERACTIONS AND COSMOLOGICAL PERTURBATIONS


Collisional processes can suppress stress and affect the perturbation evolution of cosmological neutrinos



In the UR limit:

$$\sigma \sim \frac{g^4}{s} \sim \frac{g^4}{T^2}$$

e.g., in simple majoron models: $\sigma \simeq \frac{1}{32\pi} \frac{g^4}{s}$

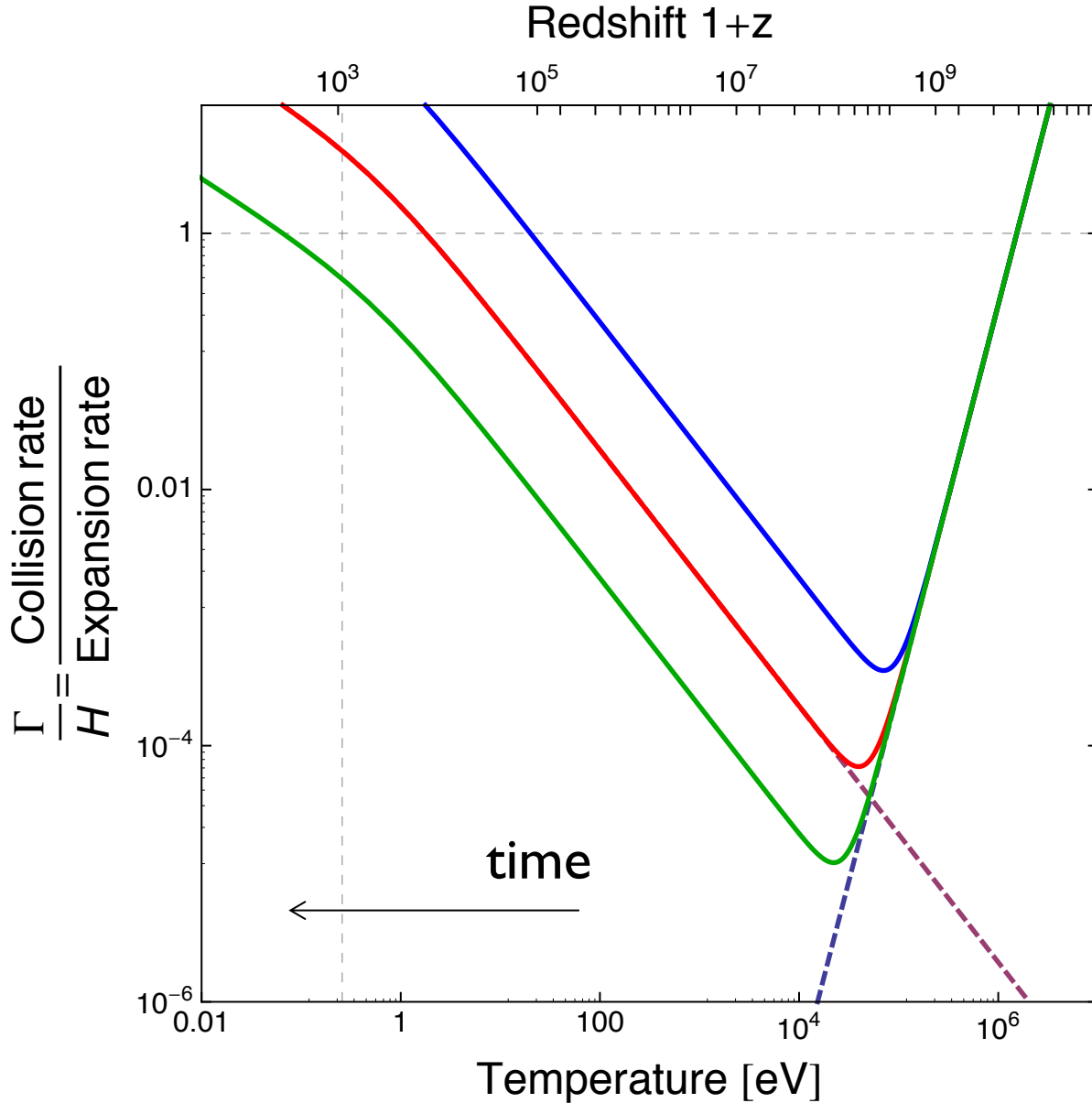
 $\Gamma_{\nu\nu} = \langle \sigma_{\text{bin}} \mathbf{v} \rangle n_{\text{eq}} \propto g^4 T,$

H grows as T^2 (RD) and $T^{3/2}$ (MD) so the ratio Γ/H **increases** with time. Neutrinos **recouple** at low temperatures!

In the following I write generically

$$\Gamma_{\nu\nu} = (\dots) \times \frac{g^4}{T_\nu^2} \times \frac{3\zeta(3)}{2\pi^2} T_\nu^3 = g_{\text{eff}}^4 \times \frac{3\zeta(3)}{2\pi^2} T_\nu$$

SECRET INTERACTIONS AND COSMOLOGICAL PERTURBATIONS



$$g_{\text{eff}} = 1.5 \times 10^{-7}$$

$$g_{\text{eff}} = 2.7 \times 10^{-7}$$

$$g_{\text{eff}} = 5 \times 10^{-7}$$

SECRET INTERACTIONS AND COSMOLOGICAL PERTURBATIONS

Neutrino perturbations in the presence of collisions

$$\frac{\partial \Psi}{\partial \tau} + ik\mu \frac{q}{\epsilon} \Psi + \frac{d \ln f_0}{d \ln q} \left[\dot{\eta} - \frac{\dot{h} + 6\dot{\eta}}{2} \mu^2 \right] = \frac{1}{f_0} \hat{C}[f],$$

Relaxation time approx.: $\hat{C}[f] \simeq -\frac{1}{\tau_c} \delta f$

(massless limit) $\dot{\delta} = -\frac{4}{3}\theta - \frac{2}{3}\dot{h},$

$$\dot{\theta} = k^2 \left(\frac{1}{4}\delta - \Pi \right),$$

No coll. term for monopole and dipole due to conservation of particle number and momentum in $2 \leftrightarrow 2$ processes

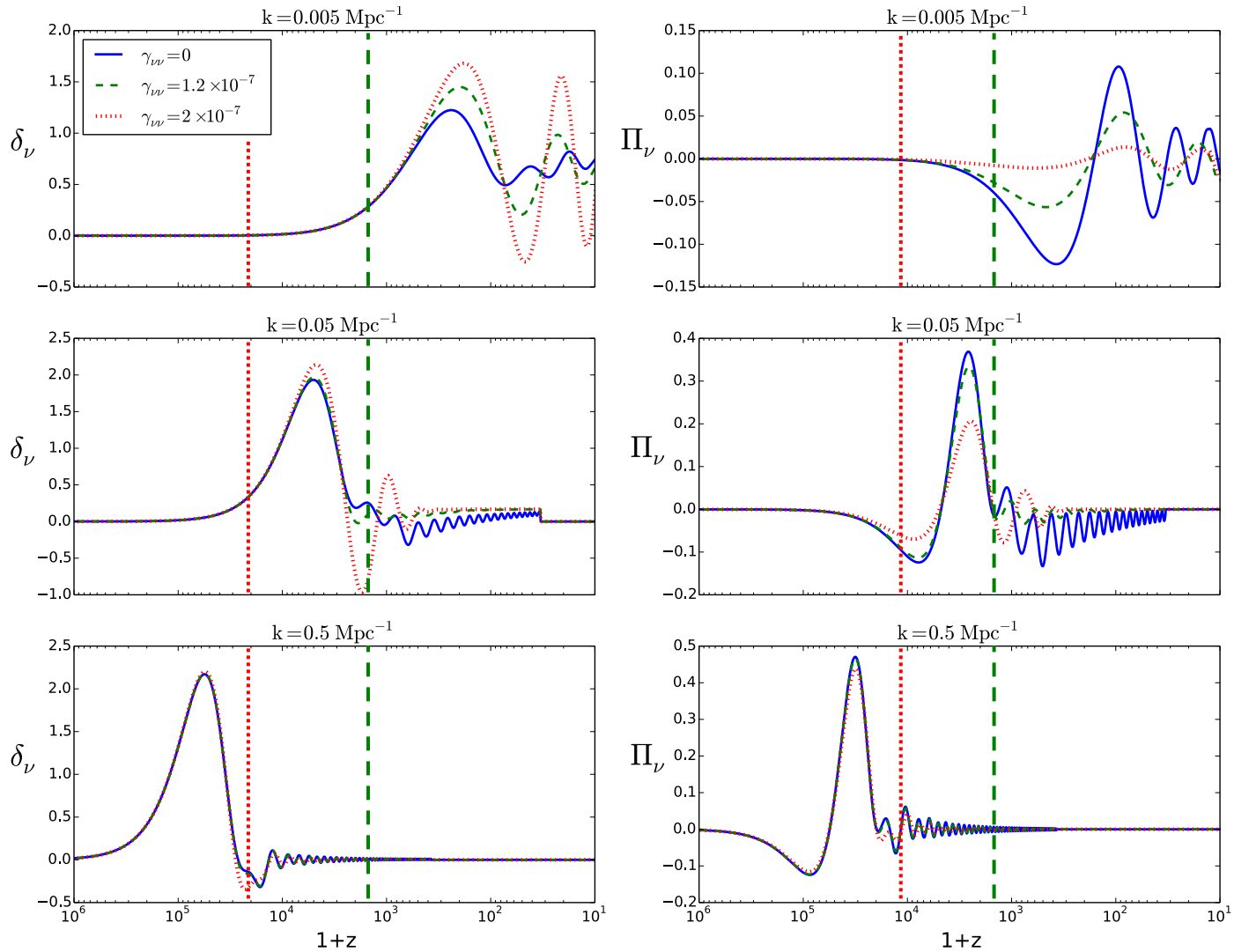
$$\dot{\Pi} = \frac{4}{15}\theta - \frac{3}{10}kF_3 + \frac{2}{15}\dot{h} + \frac{4}{5}\dot{\eta} - a\Gamma\Pi,$$

$$\dot{F}_\ell = \frac{k}{2\ell + 1} \left[\ell F_{\ell-1} - (\ell + 1)F_{\ell+1} \right] - a\Gamma F_\ell \quad (\ell \geq 3).$$

Higher order momenta are driven to zero by the collisions

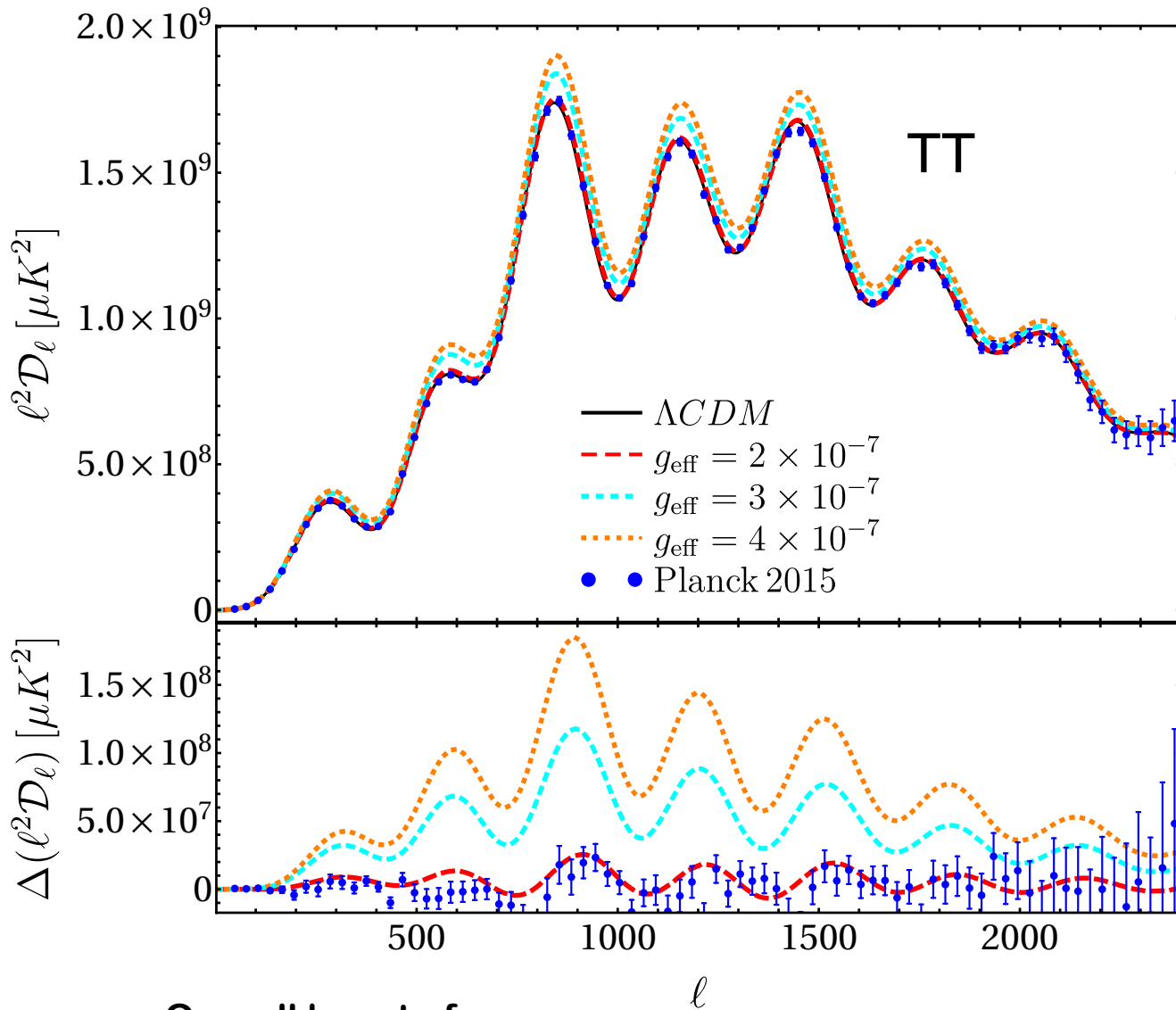
—————> fluctuations are confined to the monopole and dipole

SECRET INTERACTIONS AND COSMOLOGICAL PERTURBATIONS



Higher order momenta are driven to zero by the collisions

→ fluctuations are confined to the monopole and dipole

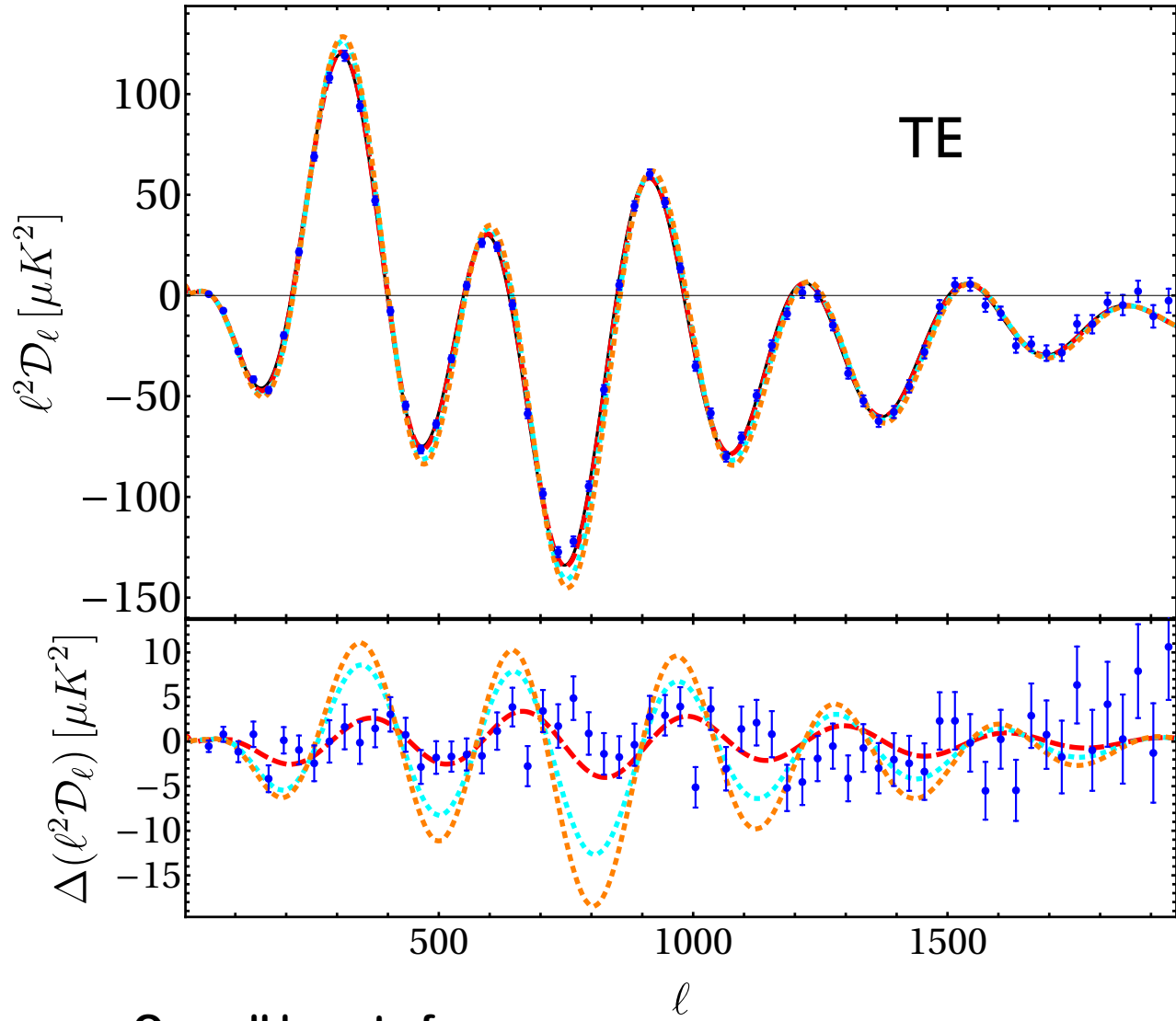


Neutrino momenta affect the gravitational potentials and thus propagate to the photons

Data points are from Planck 2015

Overall boost of the spectrum amplitude + phase shift

(Forastieri, ML, Natoli, 2015; see also Archidiadono, Hannestad 2013; Cyr-Racine, Sigurdson 2013)

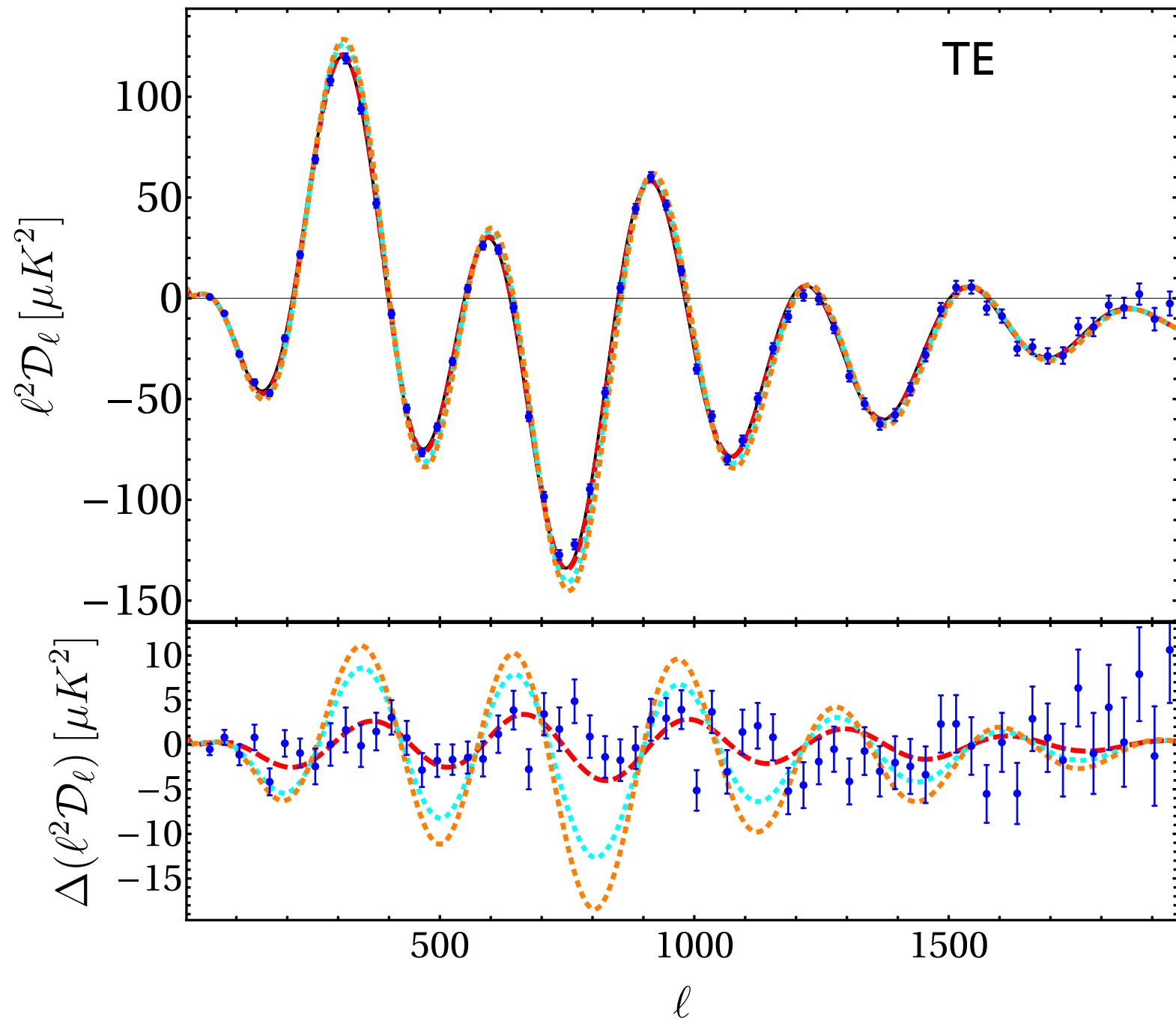


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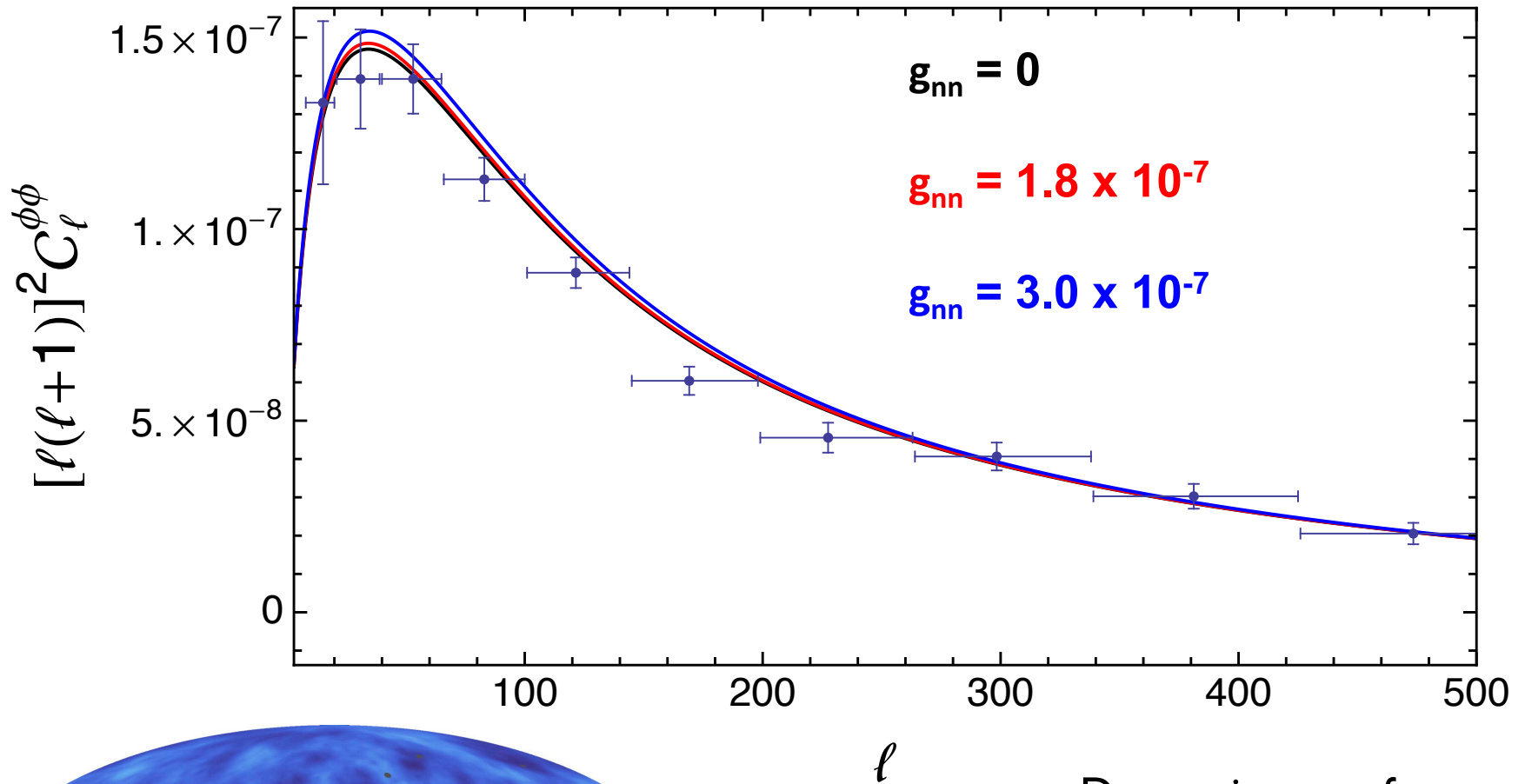
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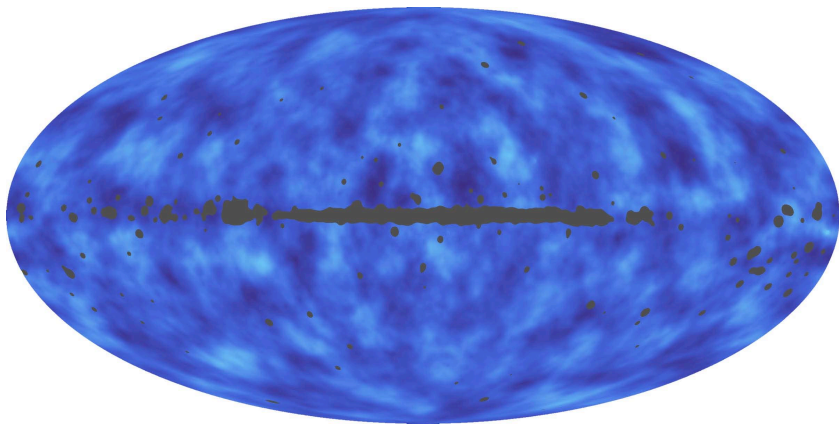
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LENSING AND INTERACTING NEUTRINOS



Data points are from
Planck 2015

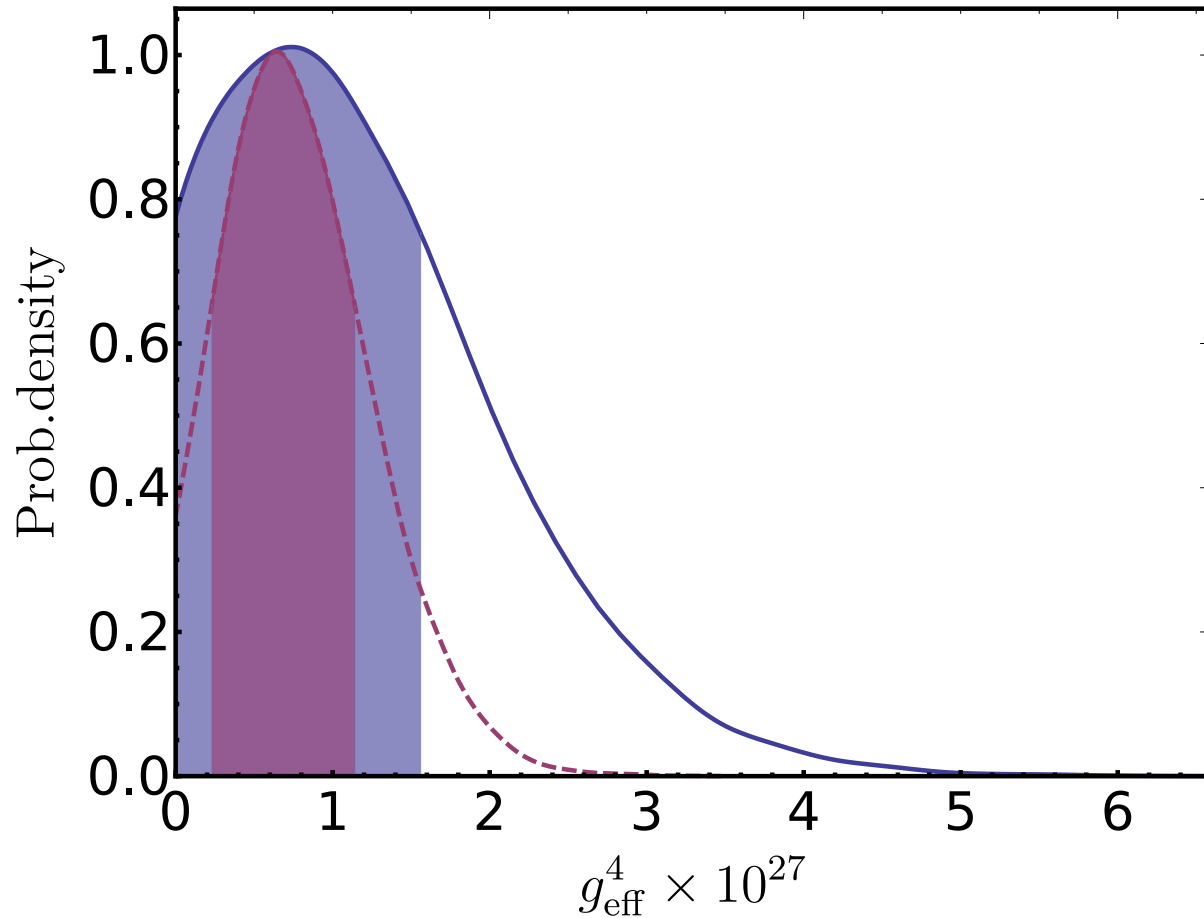


Lensing potential estimated from the
four-point correlation function

CONSTRAINTS FROM PLANCK 2015

PlanckTT(TEEE) +lowP

$$g_{\text{nn}}^4 < 2.90 \text{ (1.70)} \times 10^{-27}$$



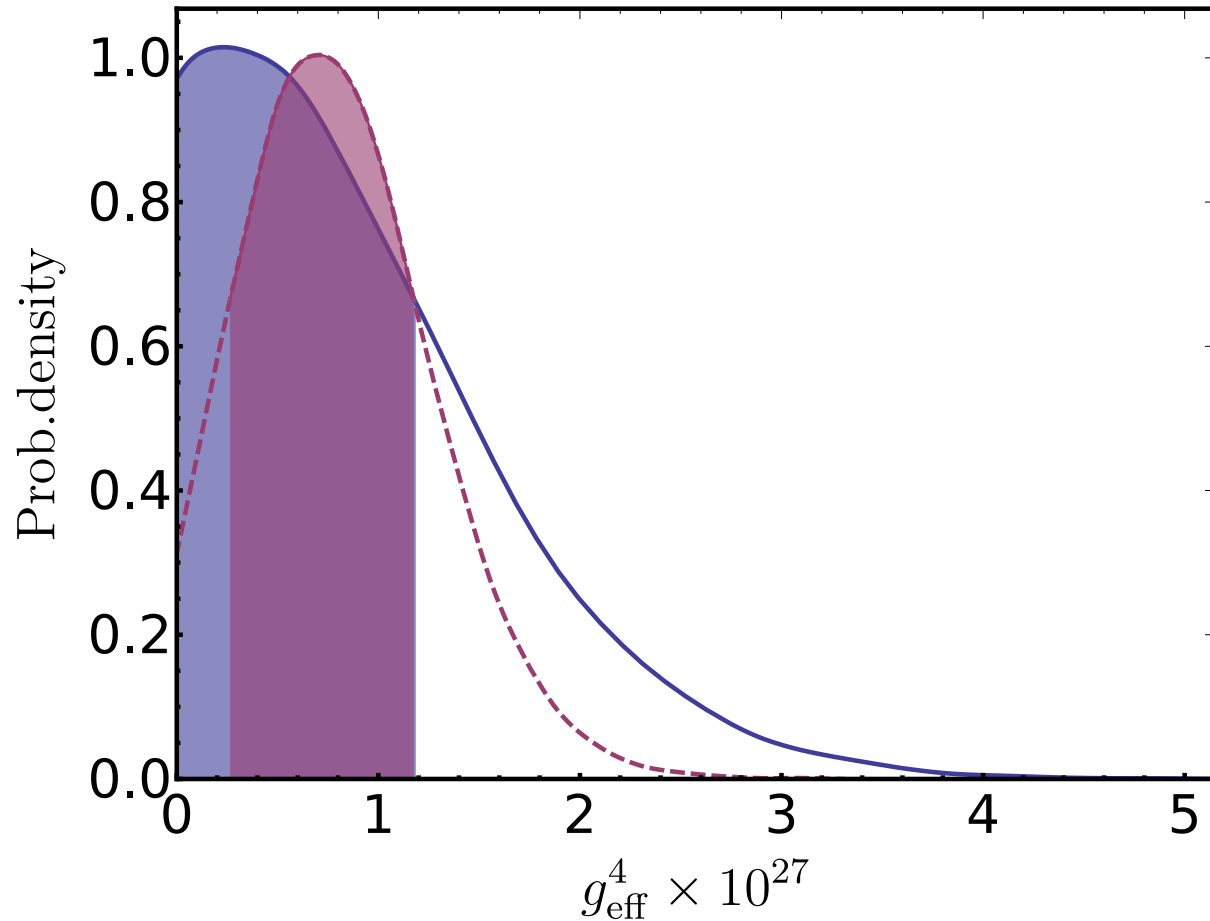
Limits are 95% CL

Forastieri, ML, Natoli

CONSTRAINTS FROM PLANCK 2015

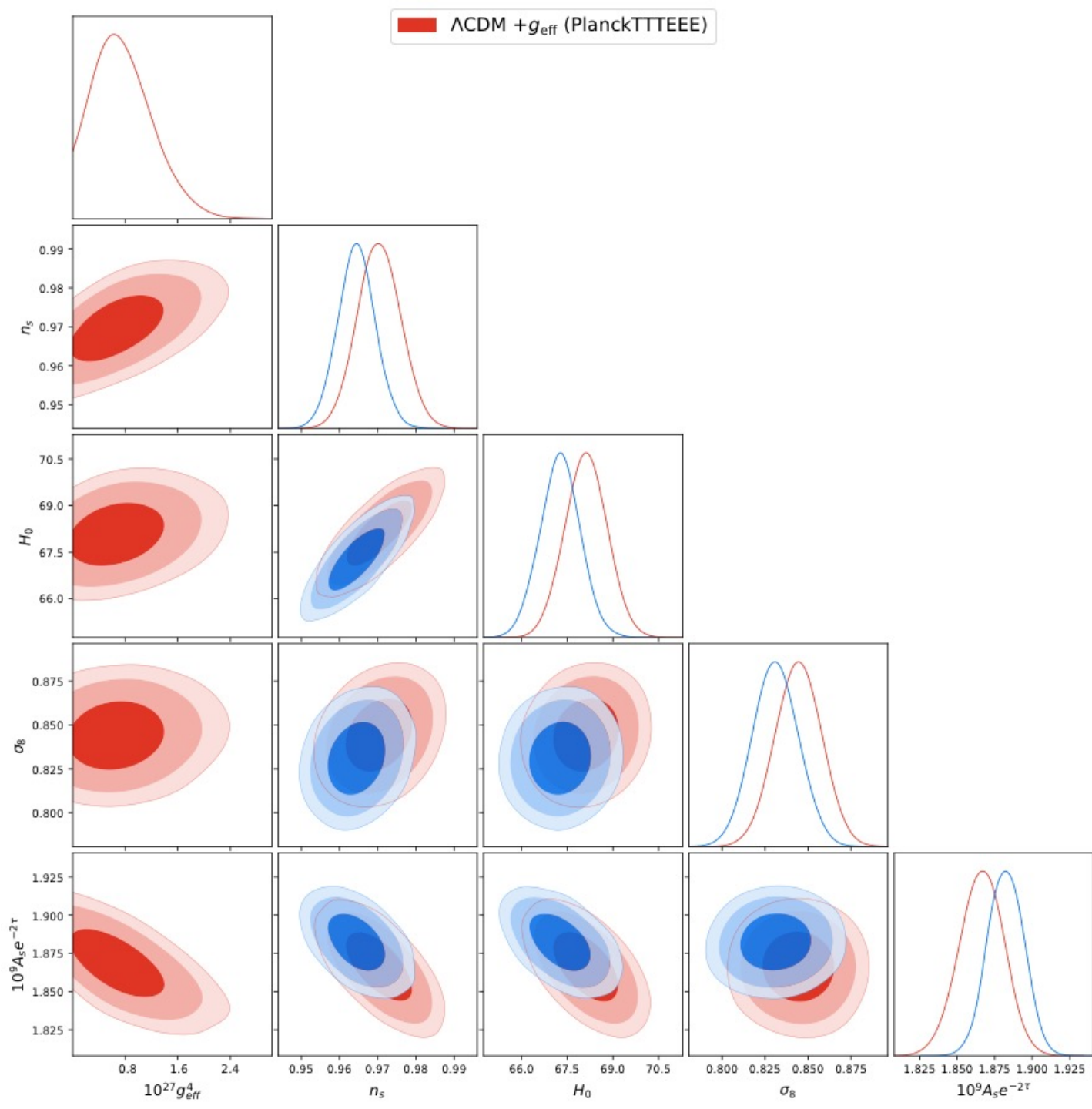
PlanckTT(TEEE) +lowP+lensing

$$g_{\text{nn}}^4 < 2.35 \text{ (1.64)} \times 10^{-27}$$

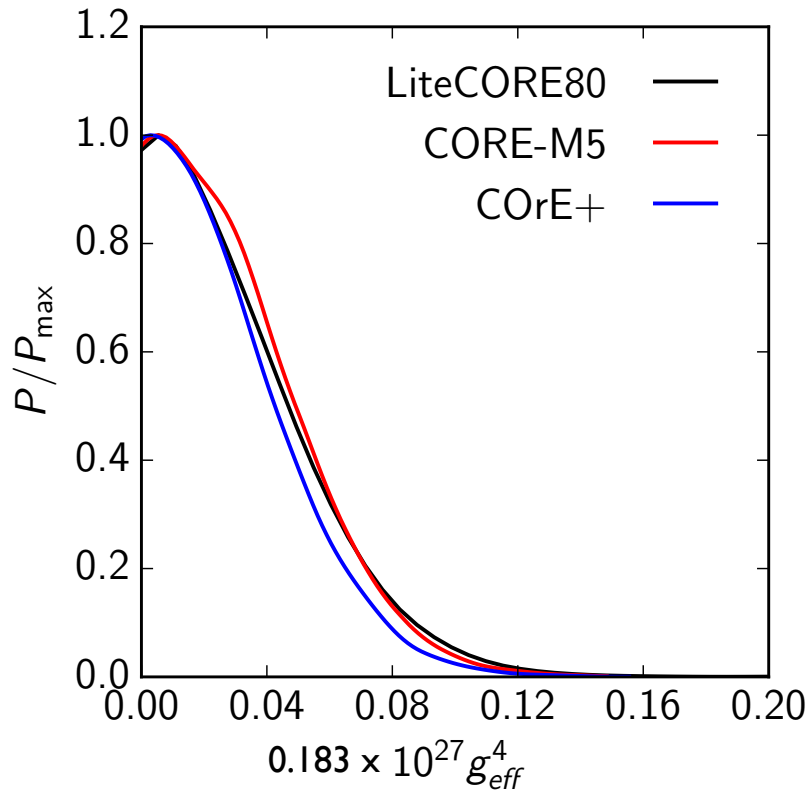


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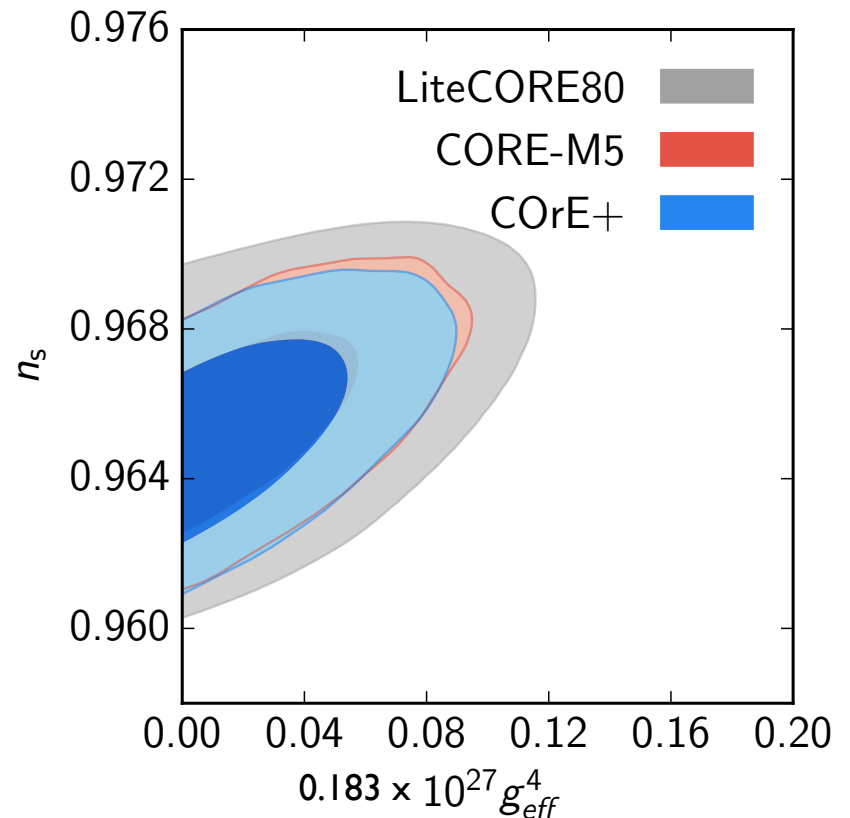


SCALAR ν INTERACTIONS IN THE CMB SPECTRUM



$$g_{\text{eff}}^4 < 0.4 \times 10^{-27}$$

Future generation CMB can improve limits by nearly one order of magnitude



Core Parameters paper
arXiv: 1612.00021

MAJORON MODELS

A simple realization of scalar neutrino interactions is found in Majoron models, in which neutrino masses arise from the spontaneous violation of lepton number

$$\mathcal{L}_V = y_V \bar{L} \Phi V_R - \frac{1}{2} y_\sigma \sigma \overline{V_R^c} V_R + h.c.$$

When the scalar singlet σ acquires a vev $v_1 \gg v_\Phi$ it generates the large mass term M (a Majorana mass term for the rh neutrinos) in the see saw mass matrix

$$M = \frac{y_\sigma v_1}{\sqrt{2}}$$

Diagonalization of the mass matrix yields small neutrino masses $m_\nu \sim v_\Phi^2/v_1$ and an interaction term between the neutrino mass eigenstates and the majoron $J = \text{Im}(\sigma)$

$$\mathcal{L}_Y = \frac{iJ}{2} g_{ij} \bar{\nu}_i \gamma_5 \nu_j \quad \text{with} \quad g_{ij} \simeq \frac{m_{\nu,i}}{v_1} \delta_{ij} \sim \frac{v_\Phi^2}{v_1^2} \delta_{ij}$$

Our results on the scattering rate imply that the scale of lepton number breaking

$$v_1 > 300 \text{ TeV}$$

SUMMARY

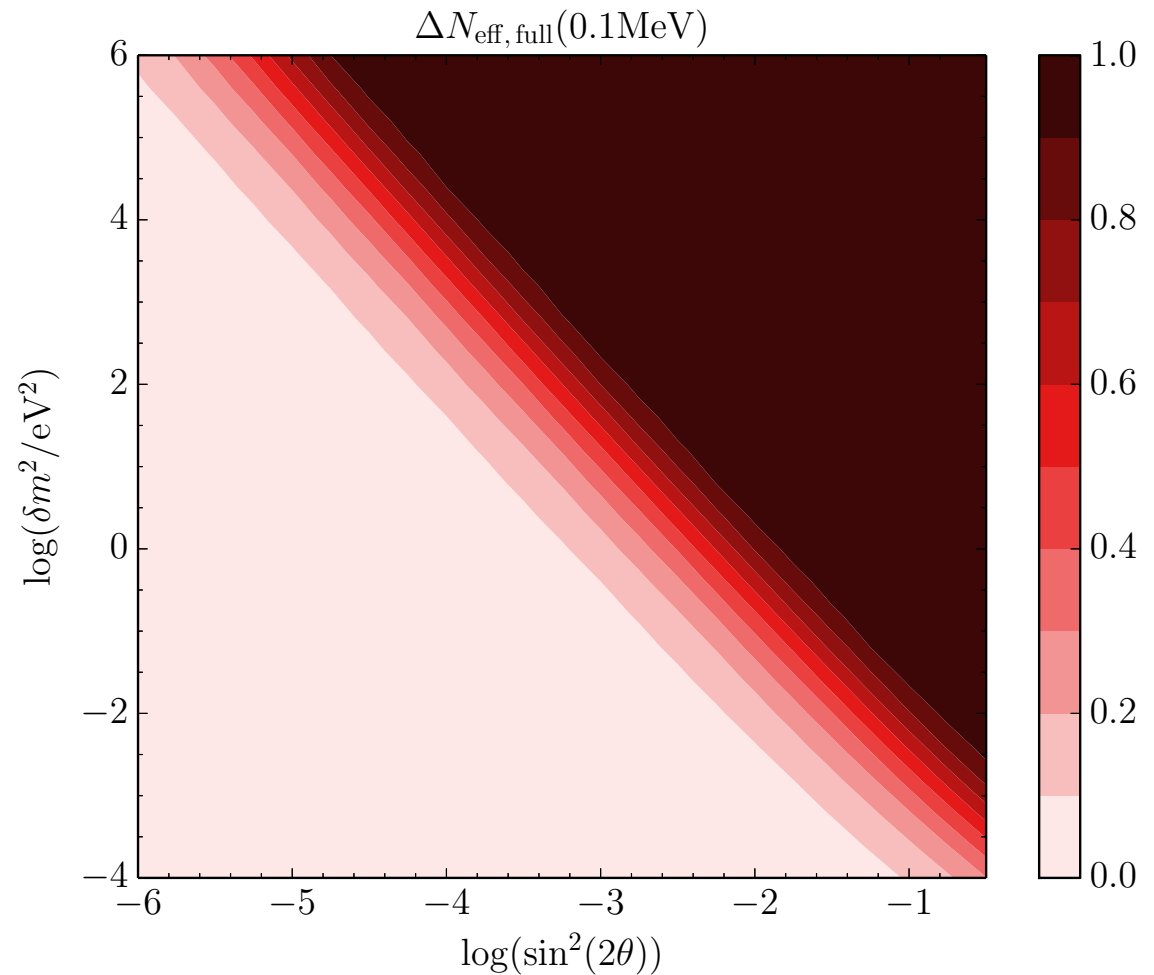
- Cosmological observations are in good agreement with the standard picture of the evolution of the neutrino background;
- the precision of the available data allows to test non-standard scenarios with high accuracy;
- the strength of neutrino scalar interactions is constrained by CMB observations at the 10^{-7} level ($z_{\text{rec}} < 4000$ from PlanckTT+lowP+lensing);
- For a simple majoron model (with diagonal couplings) $g < 7 \times 10^{-7}$ from PlanckTT+lowP+lensing...
- ...corresponding to a scale of lepton number breaking above ~ 300 TeV
- Limits from future experiments might improve by one order of magnitude

BACKUP SLIDES

1 eV for the real mass is allowed by Planck.

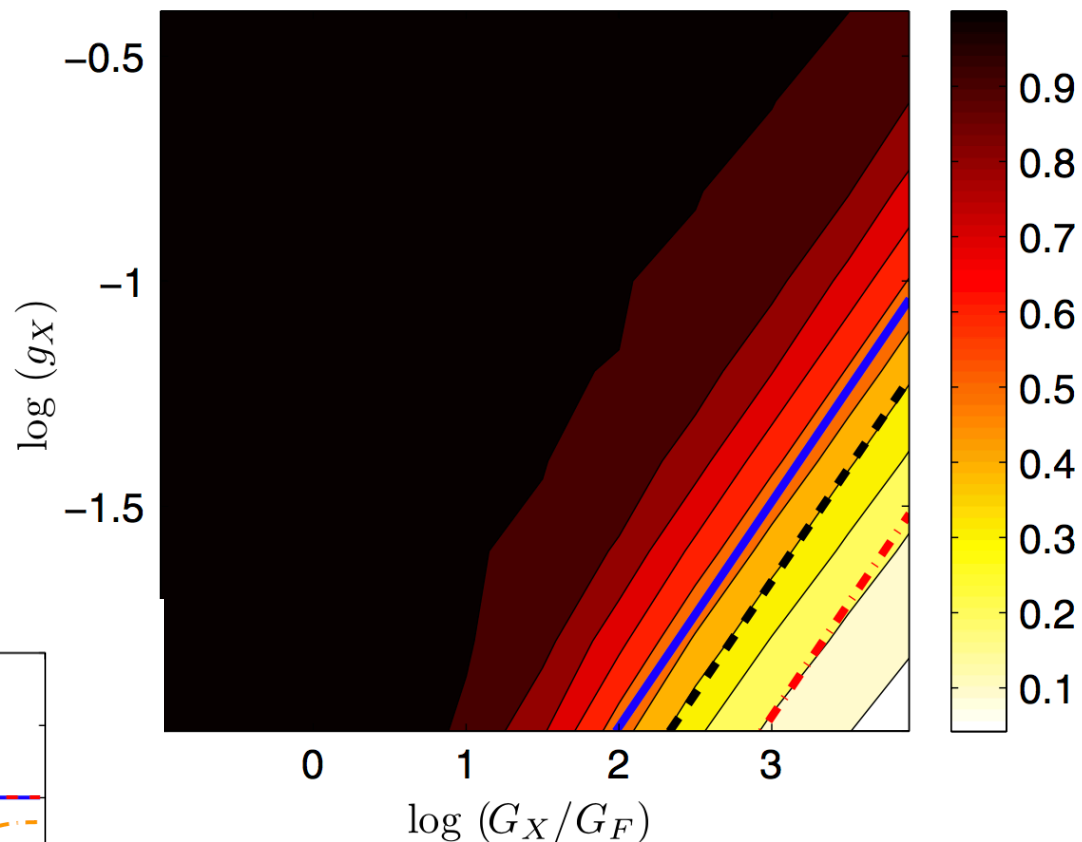
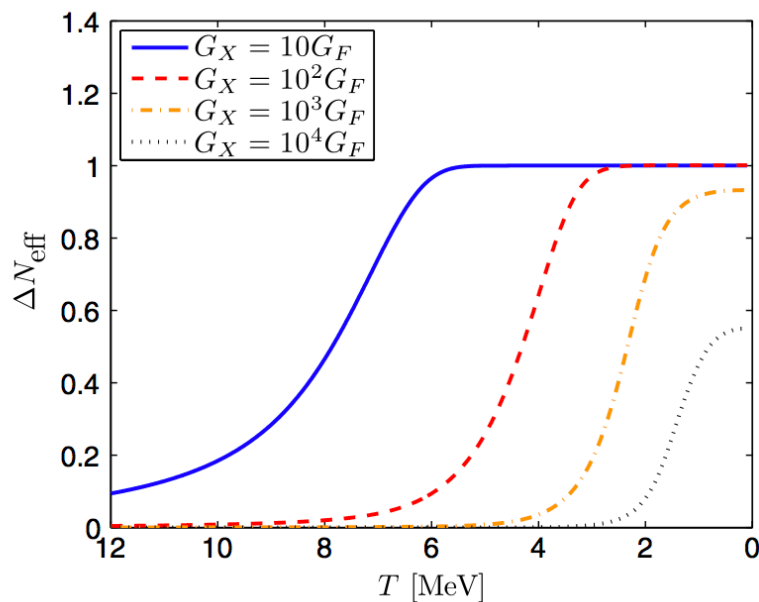
However, for $m_s \sim 1$ eV and $\sin^2 2\theta \sim 0.1$ (the preferred SBL solution) full thermalization ($\Delta N_{\text{eff}} \sim 1$) is expected.

This is at odds with Planck constraints



Hannestad et al.
2015

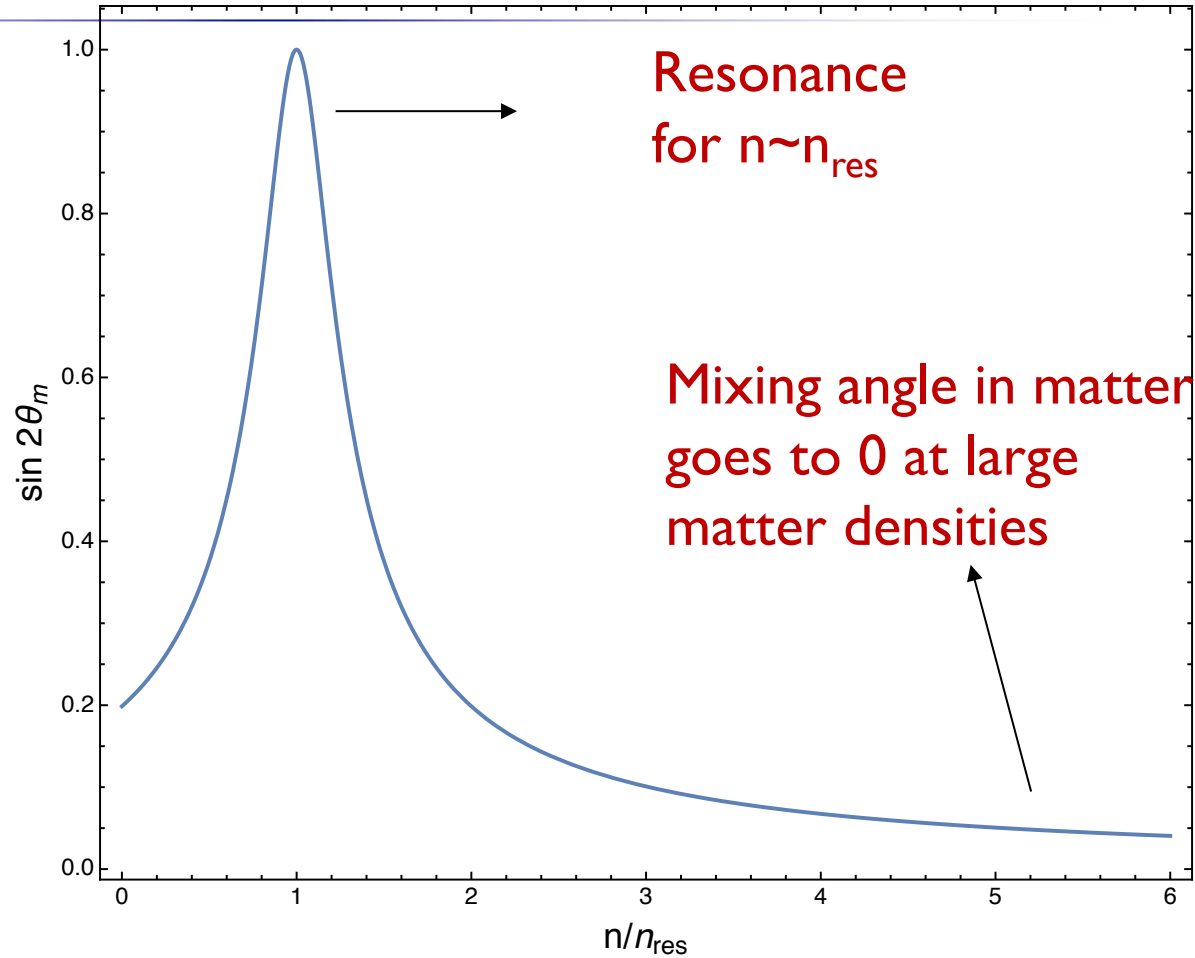
A possible solution:
new (“secret”)
neutrino interactions in
the sterile sector can
prevent production in
the early Universe



Hannestad et al.
2014

MSW effect

Mixing angle in matter is changed due to the effect of neutrino – matter (usually electrons) scattering



In ordinary matter (i.e., weak interactions)
$$n_{\text{res}} = \frac{\Delta m^2 \cos 2\theta}{2E\sqrt{2}G_F}$$

For secret vector interactions, $G_F \rightarrow G_X$

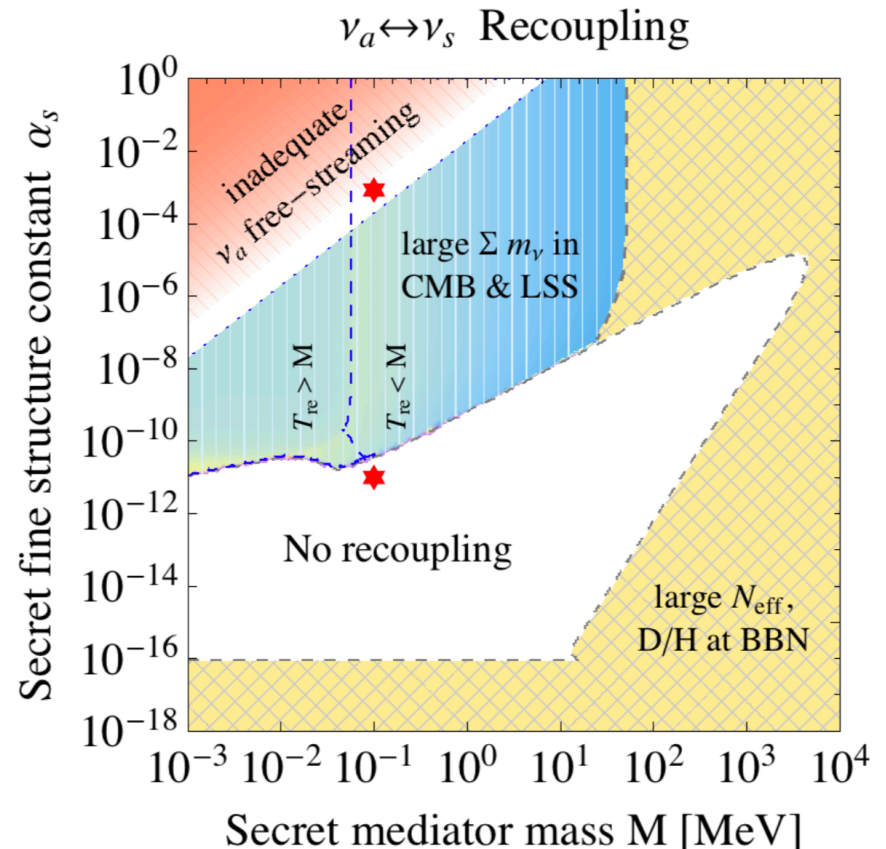
Interactions mediated by a massive gauge boson X
 (Hannestad et al. 2014; Dasgupta & Kopp 2014; Bringmann et al
 2014; Mirizzi et al 2015; Chu, Dasgupta, Kopp 2015)

$$\mathcal{L}_s = g_X \bar{\nu}_s \gamma_\mu \frac{1}{2} (1 - \gamma_5) \nu_s X^\mu$$

Production of sterile neutrinos is
 suppressed BUT “secret”
 collisions can still lead to a
 significant late-time ($T \ll 1$ MeV)
 abundancy

Tensions with CMB/LSS?

The mechanism also reduces
 $N_{\text{eff}} = 2.7$



STERILE PRODUCTION AT $T < 1 \text{ MeV}$

For $g_x > 10^{-2}$ and $M_x < 10 \text{ MeV}$, it is still possible to copiously produce neutrinos at low ($T < 1 \text{ MeV}$) temperatures, through an interplay between vacuum oscillations and collisions (“*scattering-induced decoherence*”)
(Saviano et al 2014; Mirizzi et al 2015;)

Relaxation rate to chemical equilibrium:

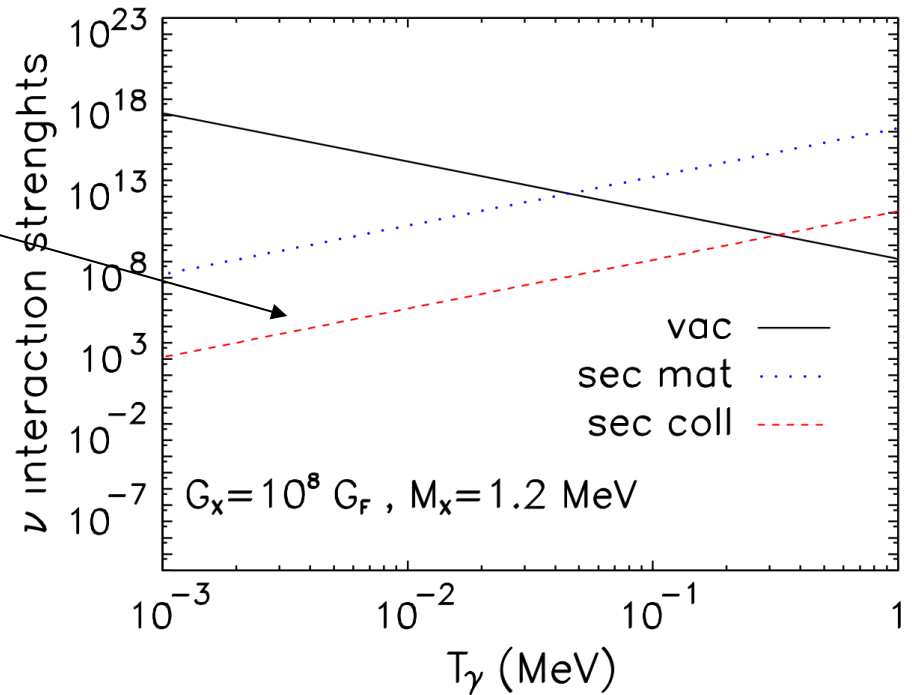
$$\Gamma_t \simeq \langle P(\nu_\alpha \rightarrow \nu_s) \rangle_{\text{coll}} \Gamma_X.$$

Number conservation and flavour equilibration imply

$$n_{s,\text{after}} = n_{a,\text{after}} = \frac{3}{4} n_{a,\text{before}}$$

Then collisions lead to thermalization and

$$T_\nu = \left(\frac{3}{4}\right)^{1/3} T_\nu^{\text{std}} \quad \longrightarrow \quad N_{\text{eff}} = 4 \times \left(\frac{3}{4}\right)^{4/3} \simeq 2.7$$



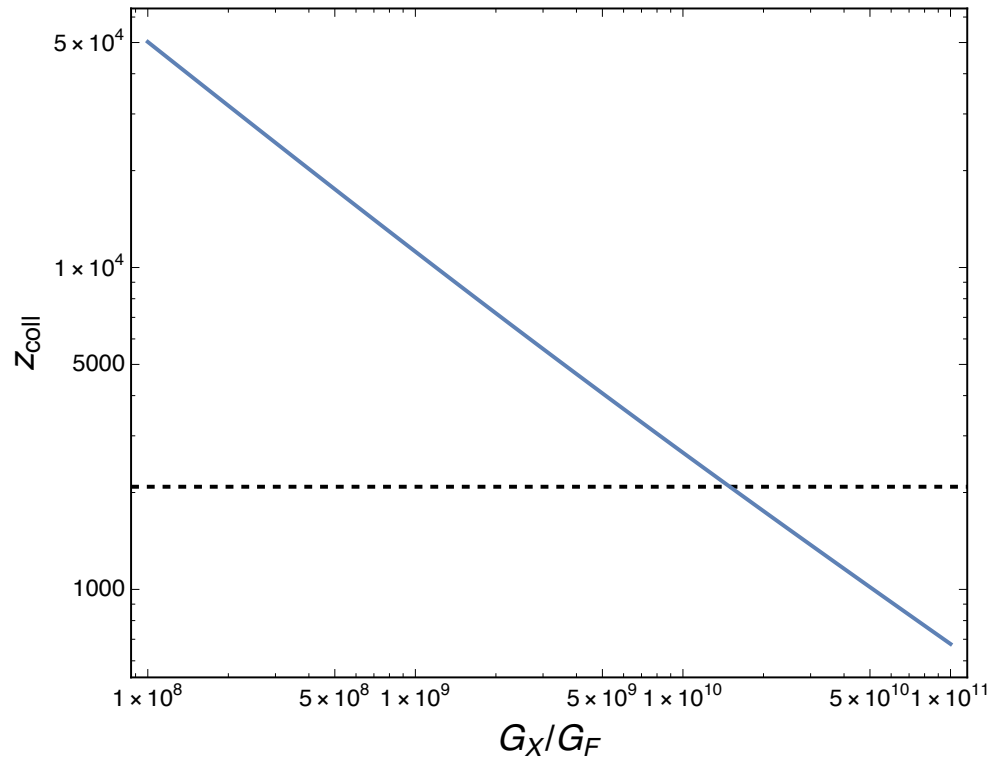
FREE-STREAMING OF INTERACTING STERILES

However, for $m_s \sim 1\text{eV}$ and $T_n \sim (3/4)^{1/3} T_n^{\text{std}}$, the density of free-streaming species is possibly too large

Problem with structure formation?

If G_X is large enough ($> 10^{10} G_F$) free-streaming is suppressed until the sterile state becomes non-relativistic.

Large G_X will leave an imprint in CMB spectrum (see e.g. Cyr-Racine & Sigurdson 2014; Lancaster et al 2016; for active neutrinos)



SECRET INTERACTIONS AND COSMOLOGICAL PERTURBATIONS

In arXiv:1704.00626 we have studied the effect of collisions in the sterile sector on the evolution of cosmological perturbations and on the CMB spectrum.

Starting point is the collisional Boltzmann eqn for neutrinos (monopole and dipole of the collision term are $= 0$)

$$\frac{\partial \Psi_i}{\partial \tau} + i \frac{q(\vec{k} \cdot \hat{n})}{\epsilon} \Psi_i + \frac{d \ln f_0}{d \ln q} \left[\dot{\phi} - i \frac{q(\vec{k} \cdot \hat{n})}{\epsilon} \psi \right] = -\Gamma_{ij} \Psi_j ,$$

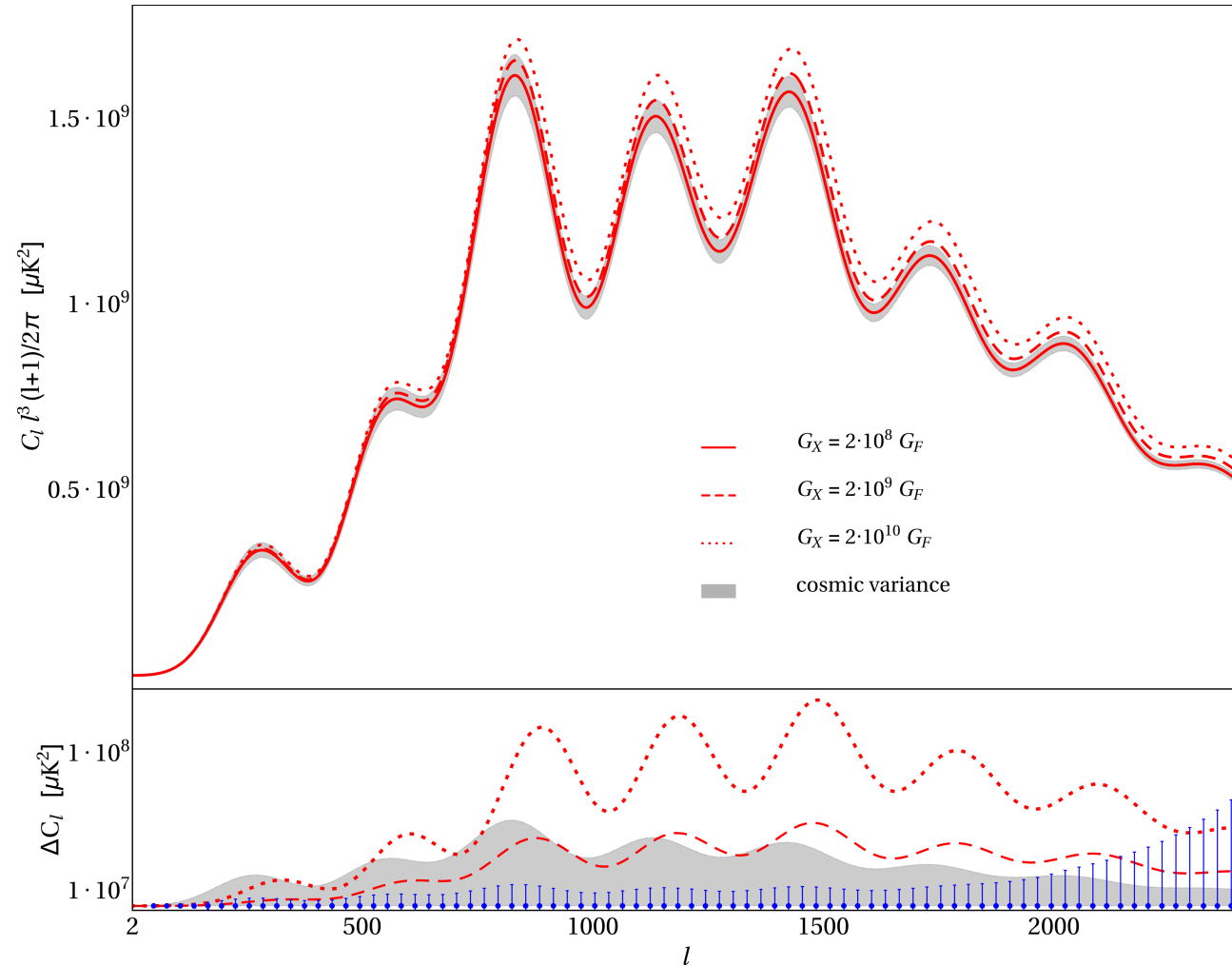
$$\Gamma_{ij} = \begin{bmatrix} \sin^2 \theta_s & 0 & 0 & \sin \theta_s \cos \theta_s \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \sin \theta_s \cos \theta_s & 0 & 0 & \cos^2 \theta_s \end{bmatrix} (3/2)(\zeta(3)/\pi^2) aG_X^2 T_\nu^5 .$$

We have assumed $\nu_s \simeq \sin \theta_s \nu_1 + \cos \theta_s \nu_4$, with $\theta_s = 0.1$

SECRET INTERACTIONS AND COSMOLOGICAL PERTURBATIONS

Collisions push power towards the lowest multipoles (ell=0, 1)

Increase in density and pressure fluctuations below a critical scale



Λ CDM ($N_{\text{eff}}=2.7$)

+ m_s + G_X

$G_X < 2.8$ (1.97) $\times 10^{10} G_F$

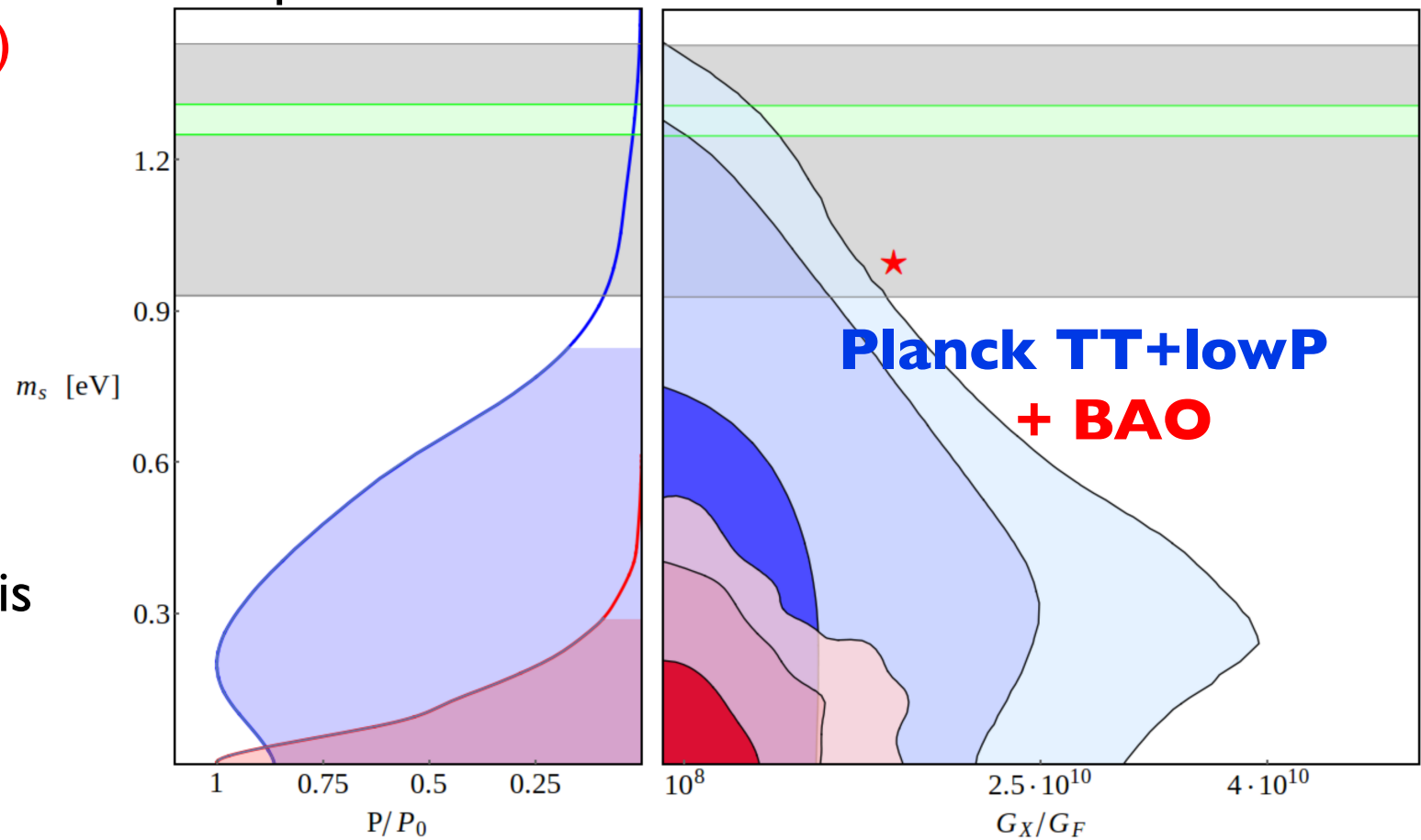
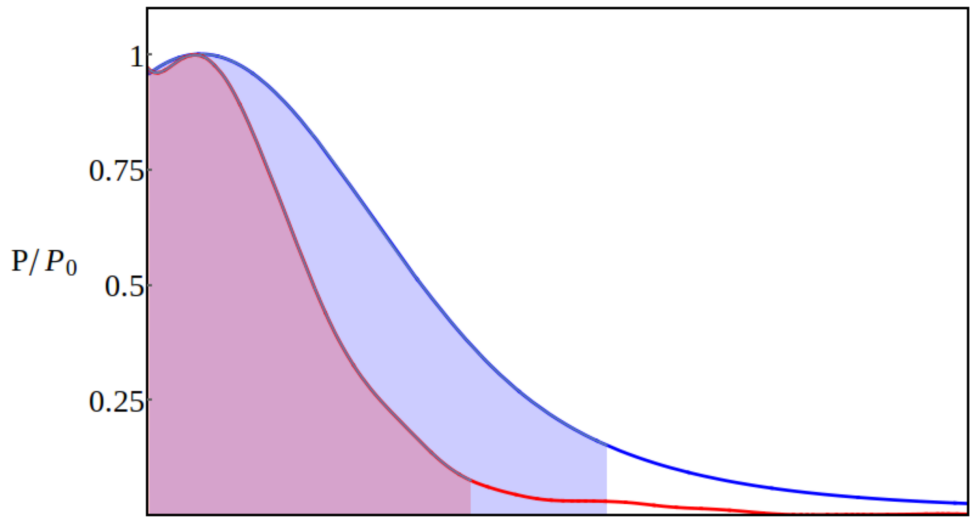
$m_s < 0.82$ (0.29) eV

$H_0 = 62.6 \pm 1.8$ km/s/Mpc

(65.3 \pm 0.7)

($G_X > 10^8$
is always
assumed)

The mass
constraint is
still there!



Parameter	Λ CDM	SACDM_GX0	SACDM	SACDM_Broad	SACDM_Narrow
χ^2_{\min}	11265.1	11272.8	11269.0	11275.2	11277.6

Table 4. Best-fit χ^2 values for the models under consideration, for the PlanckTT+lowP dataset.

The model is mildly disfavoured ($\Delta\chi^2 = 4$) with respect to standard LCDM

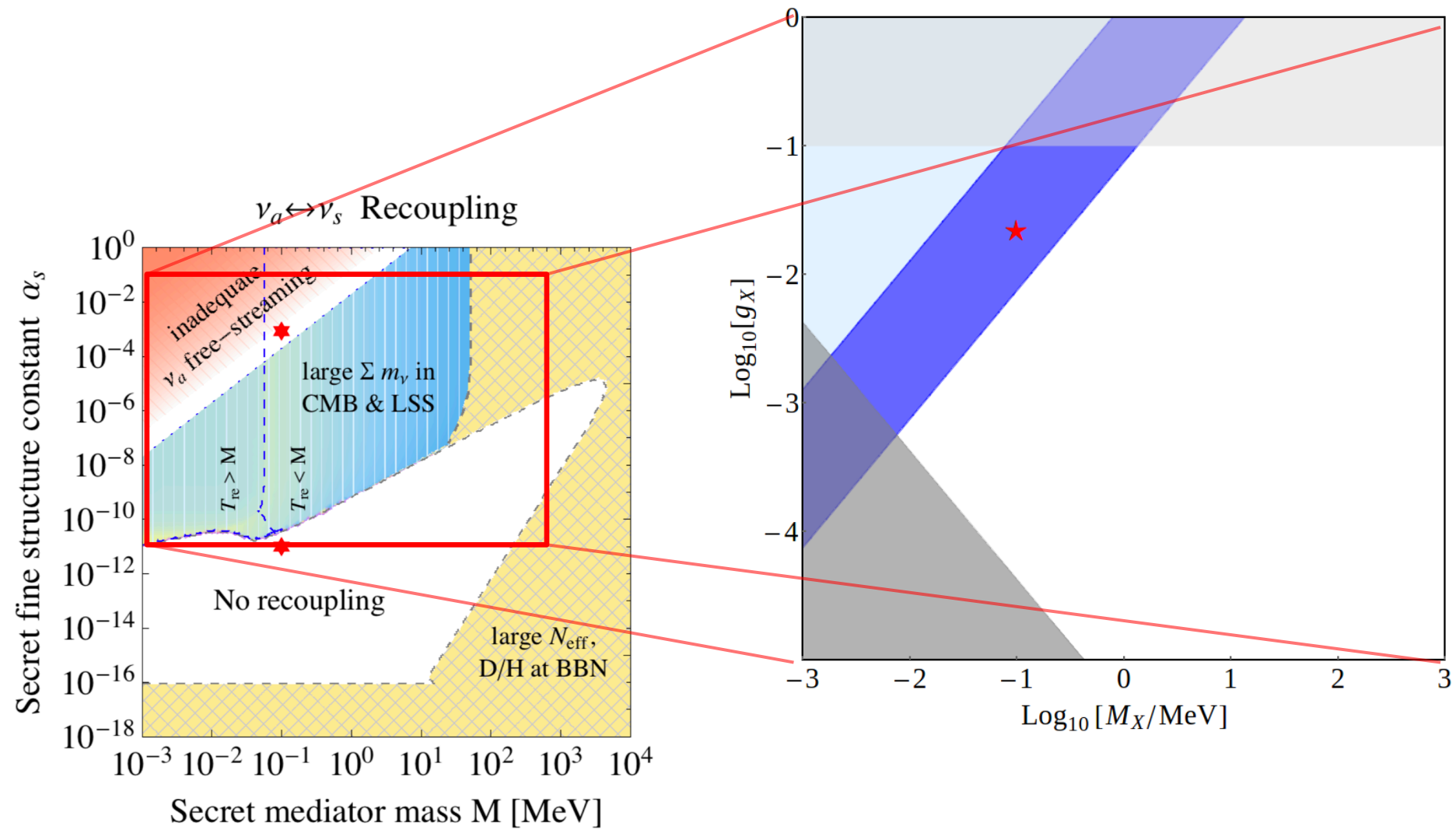
(mainly because of low N_{eff}) – this is independent of SBL anomalies

If we impose a “large” (\sim eVish) sterile neutrino mass, as per SBL anomalies, the model becomes strongly disfavoured:

$$0.93 \text{ eV} < m_s < 1.43 \text{ eV}, \quad \Delta\chi^2 = 10.1$$

$$m_s = 1.27 \pm 0.03 \text{ eV}, \quad \Delta\chi^2 = 12.5$$

(note that this numbers do not take into account H0 tension)



THE COSMIC NEUTRINO BACKGROUND

Perturbations of non-interacting neutrinos evolve according to:

$$\frac{\partial \Psi}{\partial \tau} + ik_{\mu} \frac{q}{\epsilon} \Psi + \frac{d \ln f_0}{d \ln q} \left[\dot{\eta} - \frac{\dot{h} + 6\dot{\eta}}{2} \mu^2 \right] = 0$$

In the massless limit, after integrating over momentum and expanding the angular dependence:

$$\dot{\delta} = -\frac{4}{3}\theta - \frac{2}{3}\dot{h},$$

$$\dot{\theta} = k^2 \left(\frac{1}{4}\delta - \Pi \right),$$

$$\dot{\Pi} = \frac{4}{15}\theta - \frac{3}{10}kF_3 + \frac{2}{15}\dot{h} + \frac{4}{5}\dot{\eta},$$

$$\dot{F}_{\ell} = \frac{k}{2\ell + 1} \left[\ell F_{\ell-1} - (\ell + 1) F_{\ell+1} \right] \quad (\ell \geq 3).$$

THE MAJORON MODEL

As a concrete example, in models in which neutrinos acquire mass through spontaneous breaking of lepton number, they couple to the NG boson of the broken symmetry – the Majoron:

$$\begin{aligned} \mathcal{L}_Y = & Y_u \bar{Q}_L \Phi^* u_L^c + Y_d \bar{Q}_L \Phi d_L^c + Y_e \bar{L}_L \Phi e_L^c + \\ & + Y_\nu \bar{L}_L \Phi^* \nu_L^c + \tilde{Y}_\nu L_L^T \Delta L_L + \frac{Y_R}{2} \nu_L^c \nu_L^c \sigma + H.c., \end{aligned}$$

In the see-saw limit $\langle \Delta \rangle \ll \langle \Phi \rangle \ll \langle \sigma \rangle$ the majoron is the following combination of the Higgs fields:

$$J \propto v_3 v_2^2 \Im(\Delta^0) - 2v_2 v_3^2 \Im(\Phi^0) + v_1 (v_2^2 + 4v_3^2) \Im(\sigma)$$

SECRET NEUTRINO INTERACTIONS

Consider a new (“hidden”) neutrino (pseudo)scalar interaction mediated by a light boson (like e.g. in Majoron models):

$$\mathcal{L} \supset h_{ij} \bar{\nu}_i \nu_j \phi + g_{ij} \bar{\nu}_i \gamma_5 \nu_j \phi + h.c. ,$$

This induces processes like

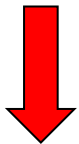
- neutrino-neutrino scattering
- neutrino-neutrino annihilation to phi's
- neutrino decay (needs off-diagonal couplings)
- neutrinoless double beta decay.

THE COSMIC NEUTRINO BACKGROUND

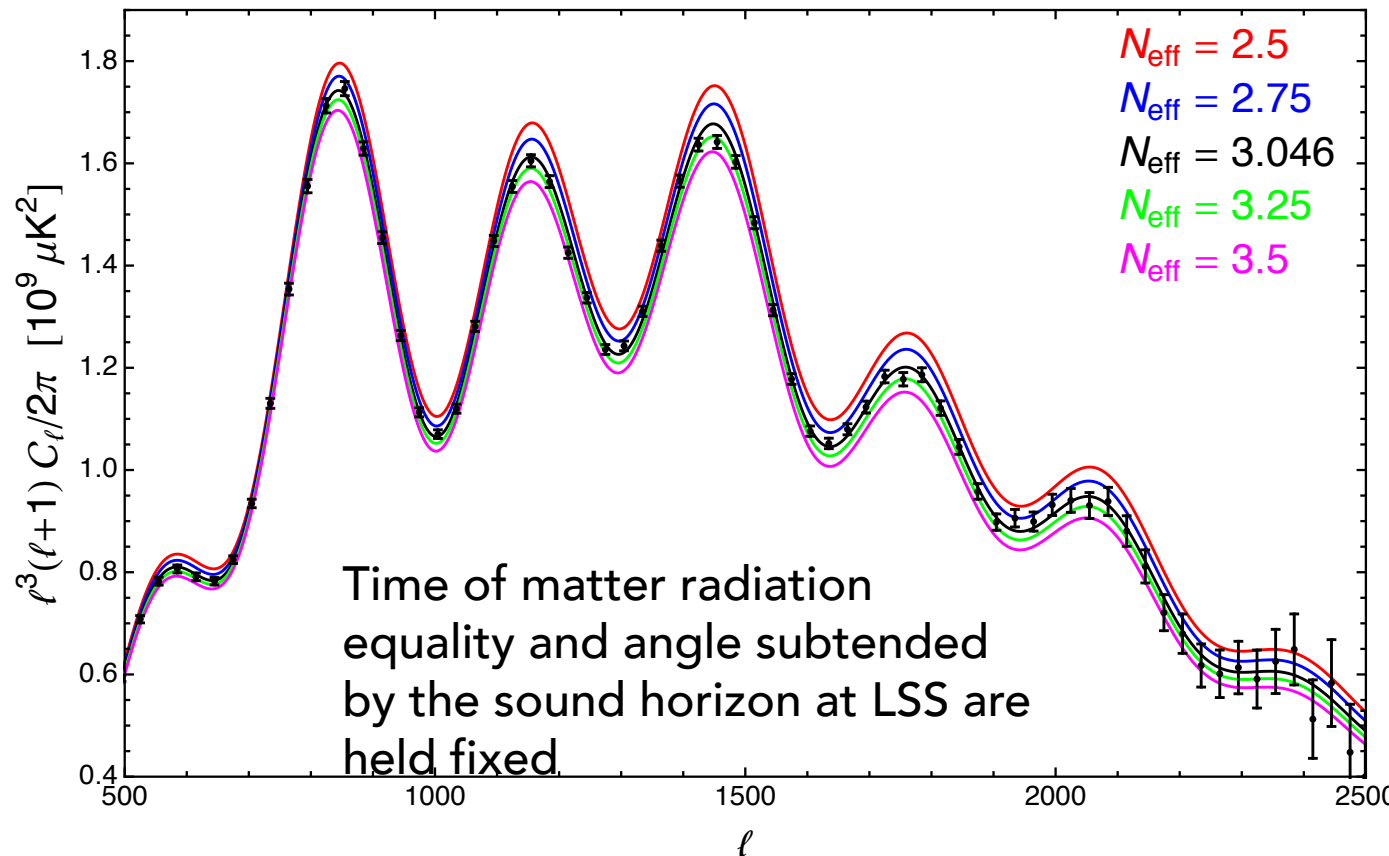
$$\rho_{\text{rad}} = \left[1 + \frac{7}{8} \left(\frac{4}{11} \right)^{4/3} N_{\text{eff}} \right] \rho_{\gamma}$$

Energy density in units of
“standard” neutrino density
(thermally distributed with
 $T=1.9$ K)

Increasing N_{eff} makes
the Universe younger
at recombination and
increases the angular
scale of the photon
diffusion length



increased Silk damping
and reduced power in
the damping tail.



(note I am showing $\sim l^4 C_l$, not $l^2 C_l$)

STERILE NEUTRINO PARAMETERIZATION

A full description of the sterile sector would require to specify (for each sterile species) its mass m_s and the **full form of the distribution function**.

Two notable cases are often considered:

- thermally distributed with arbitrary temperature T_s ;
- à la Dodelson-Widrow: distributed proportionally to active neutrinos with an arbitrary scaling factor χ_s (depends on the mixing angle).

This two models are equivalent from the point of view of cosmological observations as they can be remapped in the same effective model

STERILE NEUTRINO PARAMETERIZATION

In this phenomenological reparameterization

$$m_s^{\text{eff}} \equiv (94.1 \Omega_s h^2) \text{ eV}$$

Effective mass
(sets non-relativistic
energy density)

Effective number of
degrees of freedom
(sets relativistic energy
density)

$$\Delta N_{\text{eff}} = \begin{cases} (T_s/T_\nu)^4 & \text{thermal} \\ \chi_s & \text{DW} \end{cases}$$

To go back to the real
mass:

$$m_s = \begin{cases} m_s^{\text{eff}} (T_s/T_\nu)^{-3} = m_s^{\text{eff}} / \Delta N_{\text{eff}}^{3/4} & \text{thermal} \\ m_s^{\text{eff}} / \chi_s = m_s^{\text{eff}} / \Delta N_{\text{eff}} & \text{DW} \end{cases}$$

PLANCK CONSTRAINTS ON MASSLESS STERILE NEUTRINOS

Planck constraints on N_{eff} alone (can be regarded as a massless limit for the sterile)

$$N_{\text{eff}} = 3.13 \pm 0.32 \quad (\text{PlanckTT+lowP})$$

$$N_{\text{eff}} = 3.15 \pm 0.23 \quad (\text{PlanckTT+lowP+BAO})$$

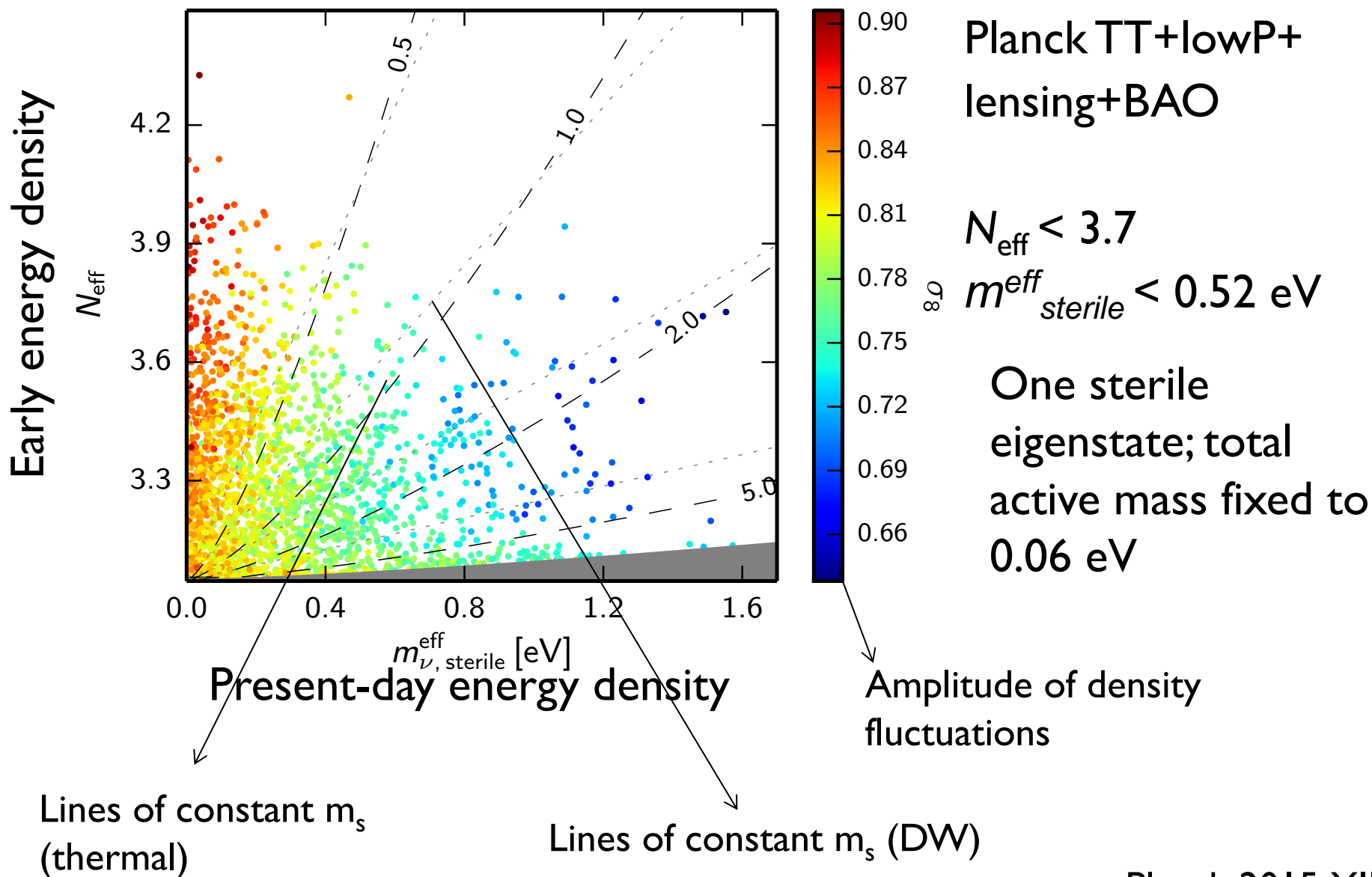
$$N_{\text{eff}} = 2.99 \pm 0.20 \quad (\text{PlanckTT,TE,EE+lowP})$$

$$N_{\text{eff}} = 3.04 \pm 0.18 \quad (\text{PlanckTT,TE,EE+lowP+BAO})$$

(uncertainties are 68% CL)

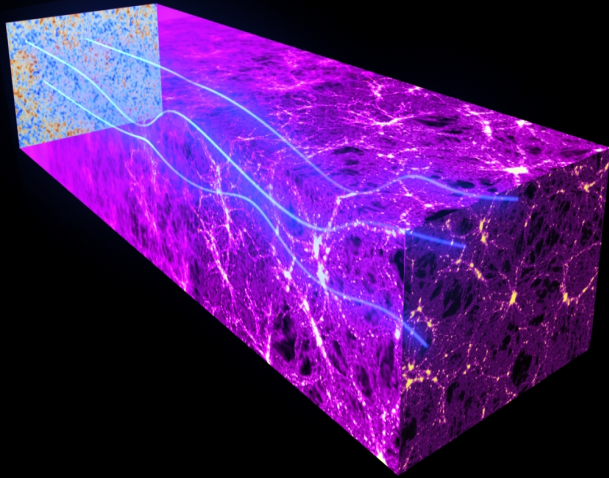
**$N_{\text{eff}} = 4$ (i.e., one extra thermalized neutrino)
*is excluded at between ~ 3 and 5 sigma.***

PLANCK CONSTRAINTS ON MASSIVE STERILE NEUTRINOS



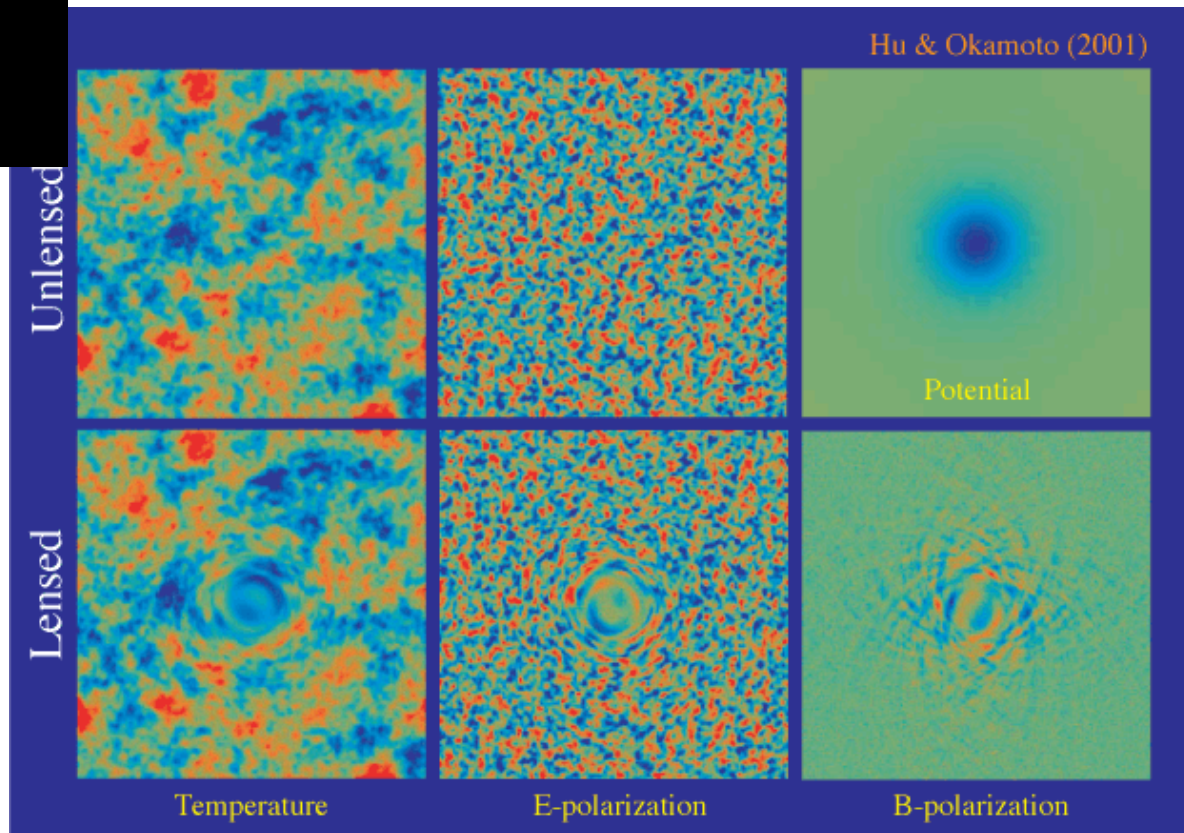
CMB LENSING

The CMB anisotropy pattern is distorted (“blurred”) by the weak lensing effect due to the intervening structures between us and the last scattering surface

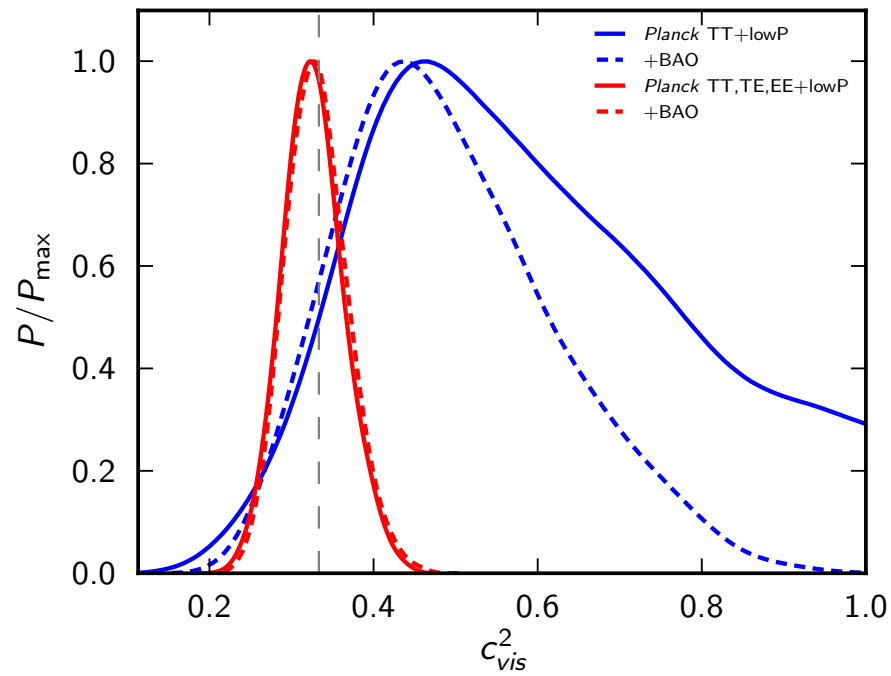
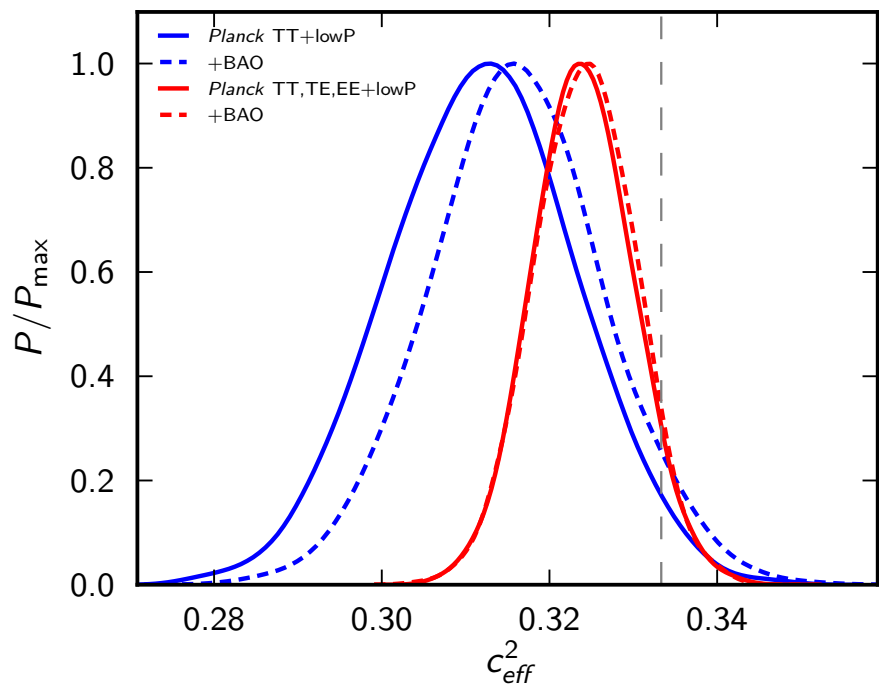


The effect is relevant at small scales (\sim % effect at sub-degree scales) and results in a smearing of the power spectrum at high multipoles. It also induces a non-gaussian signal.

CMB lensing probes the matter distribution of the Universe.



Probing $C_{\nu B}$ perturbations



Planck Collaboration: Large-scale polarization and reionization

Parameter	PlanckTT+lowP	PlanckTT+SIMlow	PlanckTTTEEE+lowP	PlanckTTTEEE+SIMlow
	95 % limits	95 % limits	95 % limits	95 % limits
Ω_K	$-0.052^{+0.049}_{-0.055}$	$-0.053^{+0.044}_{-0.046}$	$-0.040^{+0.038}_{-0.041}$	$-0.039^{+0.032}_{-0.034}$
Σm_ν [eV]	< 0.715	< 0.585	< 0.492	< 0.340
N_{eff}	$3.13^{+0.64}_{-0.63}$	$2.97^{+0.58}_{-0.53}$	$2.99^{+0.41}_{-0.39}$	$2.91^{+0.39}_{-0.37}$
Y_P	$0.252^{+0.041}_{-0.042}$	$0.242^{+0.039}_{-0.040}$	$0.250^{+0.026}_{-0.027}$	$0.244^{+0.026}_{-0.026}$
$dn_s/d \ln k$	$-0.008^{+0.016}_{-0.016}$	$-0.004^{+0.015}_{-0.015}$	$-0.006^{+0.014}_{-0.014}$	$-0.003^{+0.014}_{-0.013}$
$r_{0.002}$	< 0.103	< 0.111	< 0.0987	< 0.111
w	$-1.54^{+0.62}_{-0.50}$	$-1.57^{+0.61}_{-0.49}$	$-1.55^{+0.58}_{-0.48}$	$-1.59^{+0.58}_{-0.46}$
A_L	$1.22^{+0.21}_{-0.20}$	$1.23^{+0.20}_{-0.18}$	$1.15^{+0.16}_{-0.15}$	$1.15^{+0.13}_{-0.12}$

b). Constraints on 1-parameter extensions of the base Λ CDM model obtained using the PlanckTT likelihood in combination with other data sets.

THE COSMIC NEUTRINO BACKGROUND


The neutrino energy density is expressed in terms of the effective number of relativistic species

$$\rho_{\text{rad}} \equiv \rho_{\nu} + \rho_{\gamma} = \left[1 + \frac{7}{8} \left(\frac{4}{11} \right)^{4/3} N_{\text{eff}} \right] \rho_{\gamma}$$

assuming the standard thermal history, $N_{\text{eff}} = 3.046$ for the three active neutrinos (Mangano et al., 2005).

The only unknown parameter is the mass.

$$\rho_{\nu} = \sum_{\nu} m_{\nu} n_{\nu} = \left(\sum_{\nu} m_{\nu} \right) \frac{1}{4\pi^3} \int f(p) d^3 p$$



$$\Omega_{\nu} = \sum_{\nu} \frac{\rho_{\nu}}{\rho_c} = \frac{\sum_{\nu} m_{\nu}}{93.14 h^2 \text{ eV}}$$

