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Geometric model of dark energy according to projected hyperconical universes

Robert Monjo

Department of Algebra, Geometry and Topology, Complutense University of Madrid

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RESULTS & DISCUSSION

I. Introduction

A. Standard ΛCDM model

$$ds^2 = dt^2 - a(t)\left(\frac{dr'^2}{1-Kr'^2} + r'^2\, d\Sigma^2\right) \qquad R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R = 8\pi G T_{\alpha\beta} + \Lambda g_{\alpha\beta}$$

$$T_{\alpha\beta} = (p+\rho) u_\alpha u_\beta + p g_{\alpha\beta}$$

Friedmann Equations

$$\boxed{\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3} - \frac{K}{a^2}}$$

$$\boxed{2\left(\frac{\ddot{a}}{a}\right) + \left(\frac{\dot{a}}{a}\right)^2 + \frac{K}{a^2} - \Lambda = -8\pi G p}$$

$$\frac{H^2}{H_0{}^2} = \frac{\rho}{\rho_{crit}} + \frac{\rho_\Lambda}{\rho_{crit}} - \frac{K}{H_0{}^2 a^2} \qquad \rho_{crit} := \frac{3H_0{}^2}{8\pi G}$$

$$\boxed{\frac{H^2}{H_0{}^2} = \Omega_r \left(\frac{a_o}{a}\right)^4 + \Omega_m \left(\frac{a_o}{a}\right)^3 + \Omega_k \left(\frac{a_o}{a}\right)^2 + \Omega_\Lambda}$$

$$\Omega_i := \frac{\rho_i(t_o)}{\rho_{crit}} \qquad \Omega_k := -\frac{K}{H_0{}^2 a_0{}^2}$$

$\Omega_r \approx 8.4\cdot 10^{-5}$, $\Omega_m = 0.3089 \pm 0.0062$, $\Omega_\Lambda = 0.6911 \pm 0.0062$, $\Omega_k \approx 0$

I. Introduction

B. Arnowitt-Deser-Misner (ADM) formalism

Einstein-Hilbert action $$\mathcal{S} = \int d^4x\sqrt{-g}\,(R \boxed{-\Lambda + \mathcal{L}_M})$$

ADM takes these as zero!

Field theories action $$\mathcal{S}[\phi] = \int_M \mathcal{L}[\phi(x), \partial_\mu\phi(x), x]\, \mathrm{d}^n x$$

ADM considers space-time of the universe is **foliated** into a family of **space-like surfaces**.

$$g_{\mu\nu} = \begin{pmatrix} g_{00} & g_{0j} \\ g_{i0} & g_{ij} \end{pmatrix}$$

$$\mathcal{L} = -g_{ij}\partial_t\pi^{ij} + lh + 2l_i\pi^{ij}{}_{;j} - 2\partial_i s^i$$

Conjugate momenta $$\pi^{ij} = \sqrt{-g}\left(\Gamma^0_{pq} - g_{pq}\Gamma^0_{rs}g^{rs}\right)g^{ip}g^{jq}$$ $$l := (g^{00})^{-1/2}$$ lapse

Hamiltonian constraint $$h = -\sqrt{g_s}R^k_k - \frac{1}{\sqrt{g_s}}\left(\frac{1}{2}\pi^2 - \pi^{ij}\pi_{ij}\right)$$ $$l_i := g_{0i}$$ shift

Auxiliar momenta $$s^i = l_{0j}\pi^{ij} - \frac{1}{2}l^{0i}\pi + \nabla^i\sqrt{-g}$$ $$g_s := \det g_{ij}$$ spatial

$$l^i := g^{0i}/g^{00},\ \pi_{ij} := g_{ik}g_{jl}\pi^{kl},\ \pi = g_{ij}\pi^{ij}$$ $$g := \det g_{\mu\nu}$$ total det

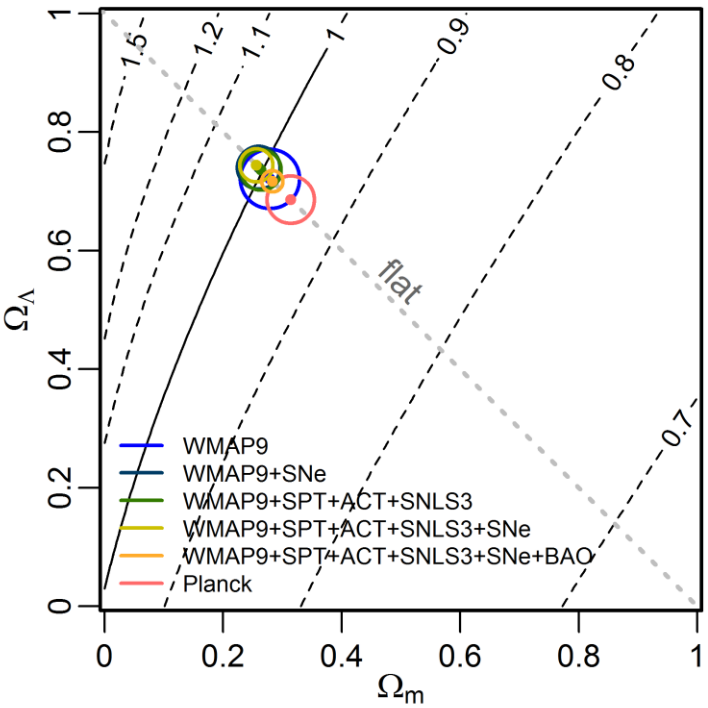
I. Introduction

C. Problems of the current theory

- **Horizon problem**: Why is the temperature isotropic in regions initially disconnected.
- **Flatness problem**: Why is energy density approximately equal to the critical density.
- **Dark energy**: What is the origin of dark energy
- **(Age of universe**: Why is the age of the universe similar to the inverse of the Hubble parameter? Is the linear expansion compatible?)
- **Approximations**:
  - General Relatity valid for cosmologic scales. -> Valid only for local scale?
  - Homogeneous and isotropic fluid, that is, Friedmann-Robertson-Walker (FRW). -> Are inhomogeneous metrics valid?

I. Introduction

C. Motivation

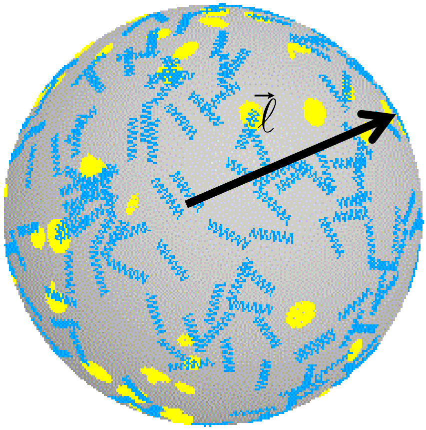


The most simple manifold with linear expansion:

$$S_t^3 = \{\vec{\ell} \in \mathrm{R}^4 : |\vec{\ell}| = R(t)\}$$

$$|\vec{\ell}'| = R(t_o) \quad \Big\Downarrow \quad a(t) := \frac{R(t)}{R(t_o)} = \frac{t}{t_o}$$

$$\boxed{\vec{\ell}(t) = a(t)\vec{\ell}' \in S_t^3}$$



Similar idea

1/*H* = *t* Dirac-Milne model (2012) [k < 0]

$$d\vec{\ell}_{FRW} = a(t)d\vec{\ell}' \quad \Longrightarrow \quad ds^2_{FRW} = dt^2 - a(t)\ d\vec{\ell}'$$

Proper-time preservation: homogeneous case

$$\boxed{d\vec{\ell} = a(t)d\vec{\ell}' + \vec{\ell}'\,da(t)} \quad \Longrightarrow \quad ds^2 = dt^2 - d\vec{\ell}^2 \neq ds^2_{FRW}$$

Local-time preservation: inhomogeneous case

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II. General considerations

A. Definition of hyperconical universes

$$\mathrm{H}^4 \subset M := \mathrm{R}_+ \times \mathrm{R}^4 \subset \mathrm{R}^{1,4}$$

$$\mathrm{H}^4 := \left\{ X \in M \ :\ \|X - O\|_\eta = \beta_0 t,\ \ O \in M,\ \ t \in \mathrm{R}_+ \right\}$$

Linear expansion $\beta_0 = cte$

Coordinates:

$$X = (x^0, \ldots, x^4) := (t_X, \vec{r}, u) \in \mathrm{T}_O M \cong \mathrm{R} \times \mathrm{R}^4$$

$$O = (0\ , \ldots,\ 0) \in \mathrm{T}_O M$$

Choosing *t* := *t*$_X$, the constraint is:

$$\boxed{\|X - O\|_\eta^2 = t^2 - \vec{r}^{\,2} - u^2 = \beta_0^2 t^2} \Rightarrow \boxed{v^2 t^2 - \vec{r}^{\,2} - u^2 = 0}$$

$$v^2 := 1 - \beta_0^2$$

$$S_t^3 = \{(\vec{r}, u) \in \mathrm{R}^4 :\ \vec{r}^{\,2} + u^2 = v^2 t^2\} \subset \mathrm{H}^4$$

- If $\beta_0 < 1$, S_t^3 is a 3-spheroid
- If $\beta_0 > 1$, S_t^3 is a 3-hyperboloid

Wick rotation of *u* (complexification) or change of metric as: diag(-1, 1, 1, 1, -1)

Comoving observer (assuming $\beta_0 < 1$)

$$X = (t, 0, vt) \subset \mathrm{H}^4$$

II. General considerations

B. Hypothesis of local equivalences and projection

Equivalence in time

The local time of an observer in H^4 is the same that in $R^{1,3}$

Let be x_0, $x \in R^{1,3}$ two static points, $x_0 = (t_0, \mathbf{0})$, $x = (t, \mathbf{0})$, with $t > t_0 > 0$

Let be x_0', $x' \in H^4$ their extension, $x_0 = (t_0, \mathbf{0}, \nu t_0)$, $x = (t, \mathbf{0}, \nu t)$

$$\exists g:\ |x'-x_0'|_g = |x-x_0|_\eta = t-t_0 > 0$$

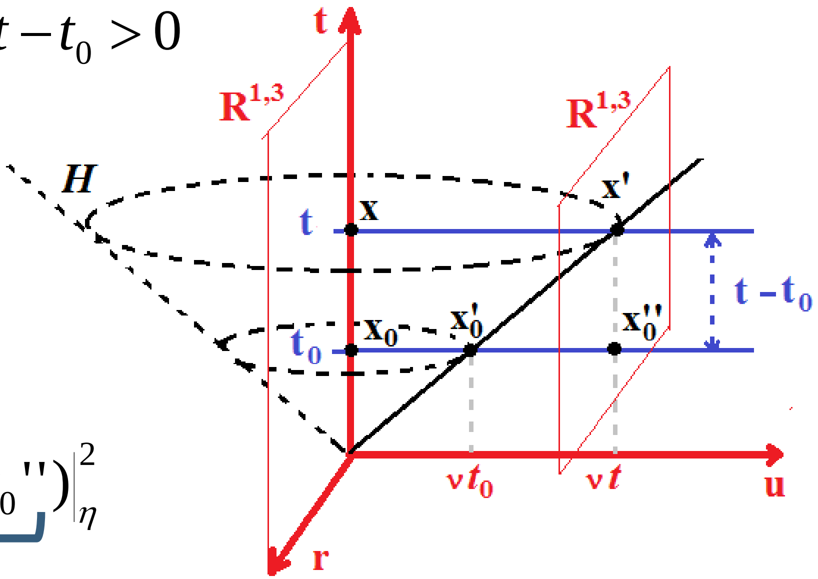
$$\exists x_0'' \in M\ :\ |x'-x_0''|_\eta = |x-x_0|_\eta$$

$x_0'' = (t_0, \mathbf{0}, t/t_0 \cdot \nu t_0)$

$$\exists g:\ |d(x'-x_0')|_g^2 = |d(\underbrace{x'-x_0''}_{})|_\eta^2$$

$$X := x'(\vec{r}) \in \mathrm{H}^4$$

$$X_D := x'(\vec{r}) - x_0'' = \left(t-t_0,\ \ \frac{t}{t_0}\vec{r}',\ \ -\nu t\left(1-\sqrt{1-\frac{r'^2}{\nu^2 t_0^2}}\right)\right)$$



$\boxed{k := \nu^{-2}}$ *Curvature*

II. General considerations

B. Hypothesis of local equivalences and projection

Spatial projection

The local space of an observer in H^4 is the same that in $R^{1,3}$

$$\begin{tikzcd}
H_\eta^{4} \arrow[r, "\mathfrak{I}_t"] \arrow[d, "f"'] & M \subset R_\eta^{1,4} \arrow[dl, "f^\alpha"] \\
R_g^{1,3} &
\end{tikzcd}$$

$$\boxed{f := f^\alpha \circ \mathfrak{I}_t}$$

$$\Lambda CDM \cong f$$

Local-time preservation (Expansion Operator).

$$\mathfrak{I}_\lambda(x') = \mathfrak{I}_\lambda\big((t,s)\big) := \left(t, \frac{\lambda}{t} s\right) = \underset{\in M}{x''}$$

$$s := (\vec{r}, u)$$

∃ Projection (locally conformal but global distortion)

$$f^\alpha\big((t,\vec{r},u)\big) = (\hat{t}, \hat{\vec{r}}) \qquad f^\alpha\big((t,\vec{r},u)\big) \underset{r \ll t}{\approx} (t,\vec{r})$$

Local extension

$$x = (t,\vec{r}) \in R_g^{1,3} \xrightarrow{f^{-1}(g=\eta)} H^4 \ni x' = (t,\vec{r},u)$$

$$x = \left(t, \frac{t}{t_0}\vec{r}'\right) \mapsto \left(t, \frac{t}{t_0}\vec{r}', u\right) \qquad \vec{r}' := \frac{t_0}{t}\vec{r}$$

II. General considerations

B. Hypothesis of local equivalences and projection

Spatial projection

The local space of an observer in H^4 is the same that in $R^{1,3}$

That is, the spatial distance is given by a **locally conformal projection**

Let be *r'* the comoving distance in H^4, the spatial distance measured by an observer is:

$$\hat{r}' := f_{\hat{r}}^{\alpha}(r')$$

where

$$f_{\hat{r}}^{\alpha}(r') \in \{ {}_a f_{\hat{r}}^{\alpha}(r'),\ {}_b f_{\hat{r}}^{\alpha}(r'),\ \ldots \}$$

$${}_a f_{\hat{r}}^{\alpha}(r') = t_0 \gamma' \Delta^{\alpha}(\gamma'/\gamma_{\max}')$$

Distorted stereographic projection

$$\gamma' = \gamma'(r') := \sin^{-1}(r'/t_0)$$

$$\Delta^{\alpha}(x) := \frac{1}{(1-x)^{\alpha}}$$

$${}_b f_{\hat{r}}^{\alpha}(r') = 2 t_0 \tan^{-1} \frac{\Delta^{\alpha}(\gamma'/\gamma_{\max}')}{2}$$

Inverse of distorted stereographic projection

II. General considerations

C. Considerations of compatibility

Features of the model

- Metric *g*
- Ricci curvature *R*
- Evolution: Arnowitt-Deser-Misner (ADM) equations

Theoretical compatibility between the proposed model and ΛCDM

- Equivalence between proper distance of both models.
- Comparison of the Hubble parameter of both models.
- Calculus with Mapple.

Observational compatibility

- 580 pairs of (z, μ_{obs}) from Type Ia supernovae (Sne Ia)
  - Supernova Cosmology Project (SCP) Union2.1 database
- Minimisation of χ^2 , using R statistical language, for:

$$\mu_{theo} = 5\log_{10}(r_L H_0) + \tilde{M}(r_L) \quad \Rightarrow \quad \text{Obtaining value of } \boxed{\alpha}$$

- Location of the first CMB acoustic peak

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III. Features of the model

- **A. Metric**
- **B. Redshift-distance relation**
- **C. Ricci curvature**
- **D. Arnowitt-Deser-Misner (ADM) equations**

IV. Co

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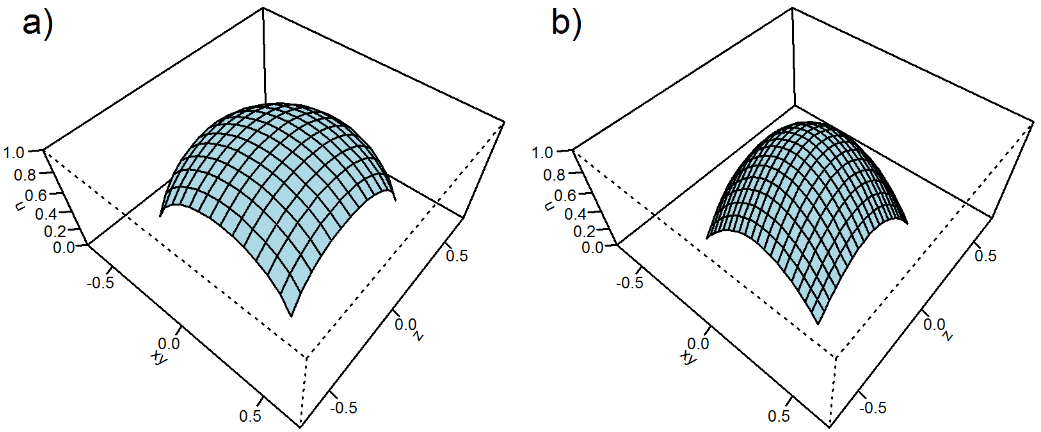
III. Theoreti



A. Metric tensor

Differential line element

$$X_D := x'(\vec{r}) - x_0'' = \left(t - t_0,\quad \frac{t}{t_0}\vec{r}',\quad -vt\left(1-\sqrt{1-\frac{r'^2}{v^2 t_0^2}}\right)\right)$$

$$dt - vt\left(1-\frac{r'^2}{v^2 t_{O'}^2}\right)^{-\frac{1}{2}} \frac{r'}{v^2 t_{O'}^2}dr'$$

$$dX^2|_{D,\eta} = dt^2\left(1+2v^2\sqrt{1-\frac{r'^2}{v^2 t_{O'}^2}}-2v^2\right) - \left(\frac{t}{t_{O'}}\right)^2\left(\frac{dr'^2}{1-\frac{r'^2}{t_{O'}^2}} + r'^2\,d\Sigma^2\right) - 2\frac{t}{t_{O'}}\frac{r'}{t_{O'}}\frac{dr'\,dt}{\sqrt{1-\frac{r'^2}{v^2 t_{O'}^2}}}$$

Metric tensor

Change of coordinates →

$$g_{00} = 1 + 2k^{-1}(b-1)$$

$$b(r') := \sqrt{1-\frac{kr'^2}{t_{O'}^2}}$$

$$g_{r'r'} = -\frac{a^2}{b^2} \qquad g_{\theta\theta} = -a^2 r'^2$$

$$g_{0r'} = -a\frac{\dot{a}r'}{b} \qquad g_{\varphi\varphi} = -a^2 r'^2 \sin^2\theta$$

$$t' := t\sqrt{g_{00}}$$

$$a(t', r') = \frac{t'}{t_0\sqrt{g_{00}}}$$

$$g_{00}' = 1$$

$$g_{r'r'}' = g_{r'r'} - \frac{g_{0r'}^2}{g_{00}} = -a(t', r')^2\,\frac{1-k^{-1}(1-b)^2}{b^2 g_{00}}$$

$$g_{0r'}' = 0$$

III. Theoretical features

B. Redshift-distance relation

Redshift and comoving distance

$$z := \frac{\lambda}{\lambda_0} - 1 = \frac{a_0}{a} - 1$$

$$dt' = -\frac{a}{a_0}\frac{\sqrt{1-k^{-1}(1-b)^2}}{bg_{00}}\,dr'$$

$$\frac{dz}{dr'} = \frac{dz}{dt'}\frac{dt'}{dr'} = \frac{\sqrt{1-k^{-1}(1-b)^2}}{bg_{00}}\,H_{hyp}(z)$$

$$H_{hyp}(z) = \frac{1}{t'} = \frac{1+z}{t_0}$$

$$\xi_k\left(\frac{r'}{t_0}\right) := \int_0^z \frac{\sqrt{1-k^{-1}(1-b)^2}}{bg_{00}}\frac{dr'}{t_0} = \int_0^z \frac{dz'}{1+z'}$$

$$r_{hyp}' = t_0\;\xi_k^{-1}\left(\ln(1+z)\right)$$

$$\hat{r}_{hyp}' = f_r^{\alpha}\left(r_{hyp}'\right)$$

III. Theoretical features

C. Ricci curvature

$$R_{\alpha\beta} := R^{\mu}_{\alpha\mu\beta} = -\partial_{\beta}\Gamma^{\mu}_{\mu\alpha} + \partial_{\mu}\Gamma^{\mu}_{\alpha\beta} - \Gamma^{\mu}_{\rho\beta}\Gamma^{\rho}_{\alpha\mu} + \Gamma^{\mu}_{\rho\mu}\Gamma^{\rho}_{\alpha\beta} \qquad \Gamma^{\mu}_{\alpha\beta} = \frac{1}{2} g^{\mu\nu}\left(\partial_{\alpha} g_{\nu\beta} + \partial_{\beta} g_{\alpha\nu} - \partial_{\nu} g_{\alpha\beta}\right)$$

locally

$$R_{r'r'} = -\left(\frac{\dot a}{a}\right)^2 \frac{2k^2}{2b-b^2+k-1} g_{r'r'}$$

$$R_{\theta\theta} = -\left(\frac{\dot a}{a}\right)^2 \frac{(4b-b^2+2k-3)k^2}{2b-b^2+k-1} g_{\theta\theta}$$

$$R_{\varphi\varphi} = -\left(\frac{\dot a}{a}\right)^2 \frac{(4b-b^2+2k-3)k^2}{2b-b^2+k-1} g_{\varphi\varphi}$$

$$\boxed{R_{ij} \approx -2k\left(\frac{\dot a}{a}\right)^2 g_{ij}} \quad \Rightarrow \quad \boxed{R \approx -6k\left(\frac{\dot a}{a}\right)^2}$$

k ≡ 1

$$R^{FRW}_{ij} = -\left[\cancel{\frac{\ddot a}{a}} + 2\left(\frac{\dot a}{a}\right)^2 + \cancel{\frac{2K}{a^2}}\right] g^{FRW}_{ij} \quad \Rightarrow \quad R^{FRW} \approx -6\left(\frac{\dot a}{a}\right)^2$$

K ≡ 0

III. Theoretical features

D. Arnowitt-Deser-Misner (ADM) equations

$$\mathcal{S} = \int d^4x \mathcal{L} = \int d^4x \sqrt{-g}\,(R - \boxed{R_u} + \mathcal{L}_M) \qquad R_u \approx -\frac{6k_o}{t^2} = -\frac{k_o}{2}\left(g^{ij}\partial_t g_{ij}\right)^2$$

$$\mathcal{L} = -g_{ij}\partial_t \pi^{ij} + lh + 2l_i \pi^{ij}{}_{;j} - 2\partial_i s^i + \sqrt{-g}R_u$$

$$\pi^{ij} \;=\; \sqrt{-g}\left[\left(\Gamma^0_{pq} - g_{pq}\Gamma^0_{rs}g^{rs}\right) g^{ip}g^{jq} - \frac{\partial R_u}{\partial(\partial_t g_{ij})}\right]$$

$$\pi^{ii} \approx \sqrt{-g}\left[\left(-g_{ii}\frac{k_o r'^2}{2t_{\hat{o}}^2 t} + g_{ii}\delta^m_m \frac{k_o r'^2}{2t_{\hat{o}}^2 t}\right)(g^{ii})^2 + k_o (g^{ii})^2 \partial_t g_{ii}\right] \approx$$

$$\approx \sqrt{-g}\frac{k_o g^{ii}}{t}\left(\frac{k_o r'^2}{t_{\hat{o}}^2} + 2\right) \approx 2\sqrt{-g}\frac{k_o g^{ii}}{t} = \sqrt{-g}k_o (g^{ii})^2 \partial_t g_{ii}$$

$$\partial_t g_{ij} = 2\frac{1}{\sqrt{-g_s g^{00}}}\left(\pi_{ij} - \frac{1}{2}\pi g_{ij}\right) + \nabla_i g_{0j} + \nabla_j g_{0i}$$

$$\partial_t g_{ii} = \frac{1}{\sqrt{-g_s g^{00}}}\pi^{ii} {g_{ii}}^2$$

→ $\boxed{k_o = k = 1}$

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IV. Compatibility

A. Equivalent proper distance: Obtaining cosmological parameters

$$\hat{r}_{\Lambda CDM}' = \hat{r}_{hyp}'$$

$$t_0 \equiv 1$$

$$\hat{r}_{\Lambda CDM}' = r_{\Lambda CDM}' = \int_0^z \frac{1}{H_{\Lambda CDM}(z')}\,dz'$$

$$\hat{r}_{hyp}' = f_r^{\alpha}(r_{hyp}') = \\ = f_r^{\alpha} \circ \xi_k^{-1} \circ \ln(1+z)$$

$$H_{hyp} = 1+z$$

$$\int_0^z \frac{1}{H_{\Lambda CDM}(z')}\,dz' = \hat{r}_{hyp}' =: \int_0^z \frac{1}{\hat{H}_{hyp}(z')}\,dz'$$

$$H_{\Lambda CDM} = H_0\sqrt{\Omega_r(1+z)^4 + \Omega_m(1+z)^3 + \Omega_\Lambda}$$

$$\hat{H}_{hyp} := \left(\frac{d}{dz} \circ f_r^{\alpha} \circ \xi_k^{-1} \circ \ln(1+z)\right)^{-1}$$

IV. Compatibility

A. Equivalent proper distance: Obtaining cosmological parameters

$$H_\Lambda^{(2)} = \sqrt{\Omega_r + \Omega_m + \Omega_\Lambda} + \frac{4\Omega_r + 3\Omega_m}{\sqrt{\Omega_r + \Omega_m + \Omega_\Lambda}} \frac{z}{2} + $$

$$+ \frac{8\Omega_r^2 + (24\Omega_\Lambda + 12\Omega_m)\Omega_r + 12\Omega_m\Omega_\Lambda + 3\Omega_m^2}{(\Omega_r + \Omega_m + \Omega_\Lambda)^{3/2}} \frac{z^2}{8}$$

$$\hat{H}^{(2)} = 1 + \frac{\gamma_k - 2\alpha\sqrt{k}}{\gamma_k} z + \frac{5\alpha^2 k - 2\alpha\sqrt{k}\gamma_k - 3\alpha k + m\gamma_k^2}{2\gamma_k^2} z^2$$

$$m = \begin{cases} 2 & \text{if} \quad f^{\alpha} =_a f^{\alpha} \\ \dfrac{5}{2} & \text{if} \quad f^{\alpha} =_b f^{\alpha} \end{cases}$$

$$\boxed{\alpha,\ \Omega_\Lambda,\ \Omega_m,}\ \Omega_r \approx 0,\ \ k \equiv 1$$

IV. Compatibility

A. Equivalent proper distance: Obtaining cosmological parameters

$$\Omega_\Lambda = \frac{3\alpha^2 k + 2\alpha\sqrt{k}\gamma_k - \alpha k}{2\gamma_k^2} + \omega_\Lambda$$

$$\Omega_m = \frac{2\alpha k(1-3\alpha)}{\gamma_k^2} + \omega_m$$

$$\Omega_r = \frac{9\alpha^2 k - 2\alpha\sqrt{k}\gamma_k - 3\alpha k}{2\gamma_k^2} + \omega_r$$

$$\Omega_{\mathrm{r}} \approx 9.0 \pm 0.5 \cdot 10^{-5}$$

$$k \equiv 1 \equiv t_0$$

$$\boxed{\begin{aligned} \alpha &= 0.2830219501(1) \pm c_\alpha i \\ \Omega_\Lambda &= 0.6937181(2) \pm c_\omega i \\ \Omega_m &= 0.306192(6) \mp c_\omega i \end{aligned}}$$

$$(\omega_\Lambda, \omega_m, \omega_r) = \begin{cases} \left(\frac{1}{2},\ 0,\ \frac{1}{2}\right), & f^\alpha =_a f^\alpha \\ \left(\frac{6+k}{12}, \frac{-k}{3}, \frac{2+k}{4}\right), & f^\alpha =_b f^\alpha \end{cases}$$

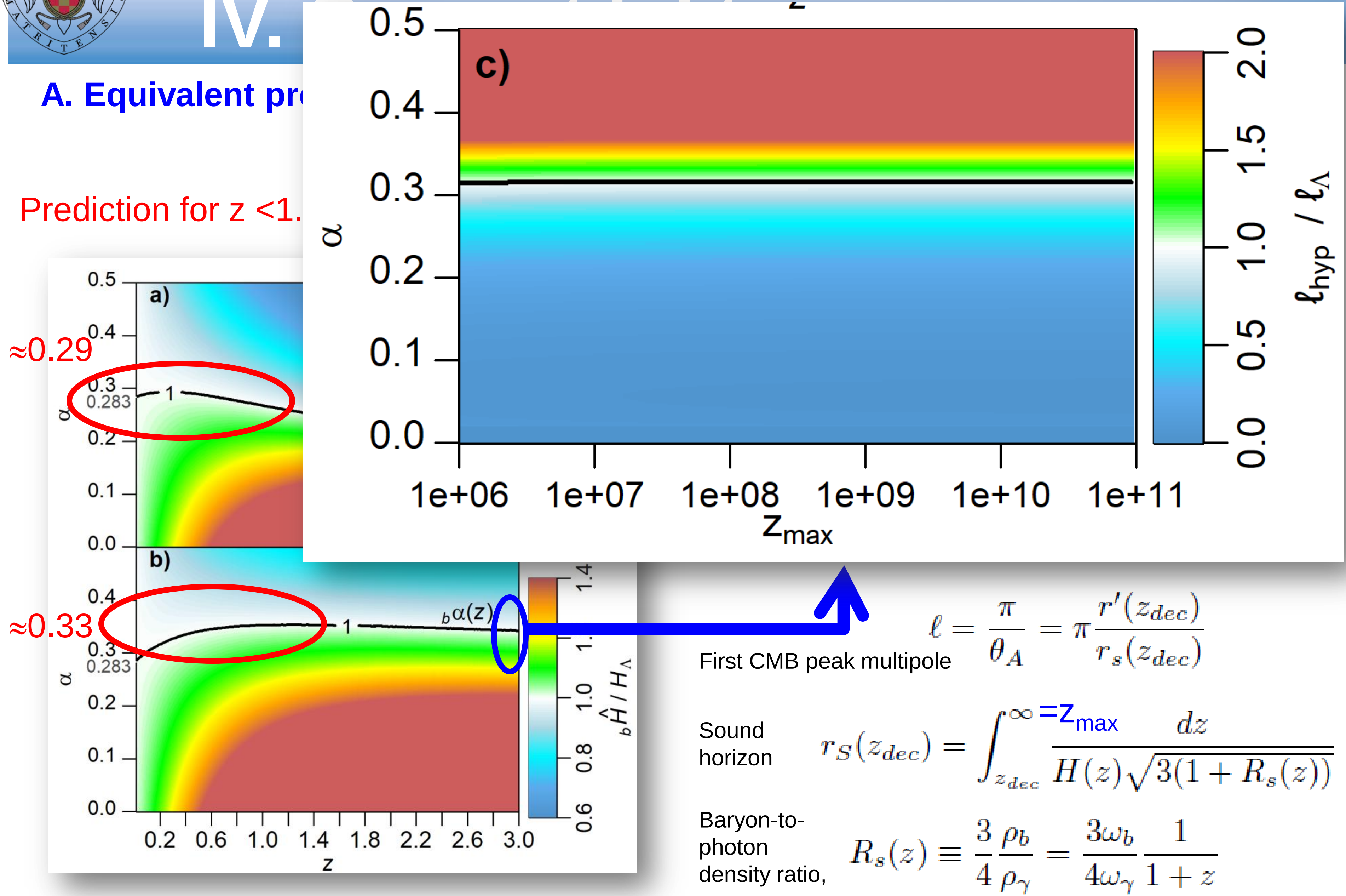
$$(c_\alpha, c_\omega) = \begin{cases} (0.204263(4), 0.260076(4)), & f^\alpha =_a f^\alpha \\ (0.320386(2), 0.407928(3)), & f^\alpha =_b f^\alpha \end{cases}$$

$$\boxed{\exists f^\alpha \rightarrow c_\alpha \quad such \quad as \quad c_\omega = 0}$$

IV.

A. Equivalent pr

Prediction for z <1.



First CMB peak multipole $$\ell = \frac{\pi}{\theta_A} = \pi \frac{r'(z_{dec})}{r_s(z_{dec})}$$

Sound horizon $$r_S(z_{dec}) = \int_{z_{dec}}^{\infty \, = z_{max}} \frac{dz}{H(z)\sqrt{3(1+R_s(z))}}$$

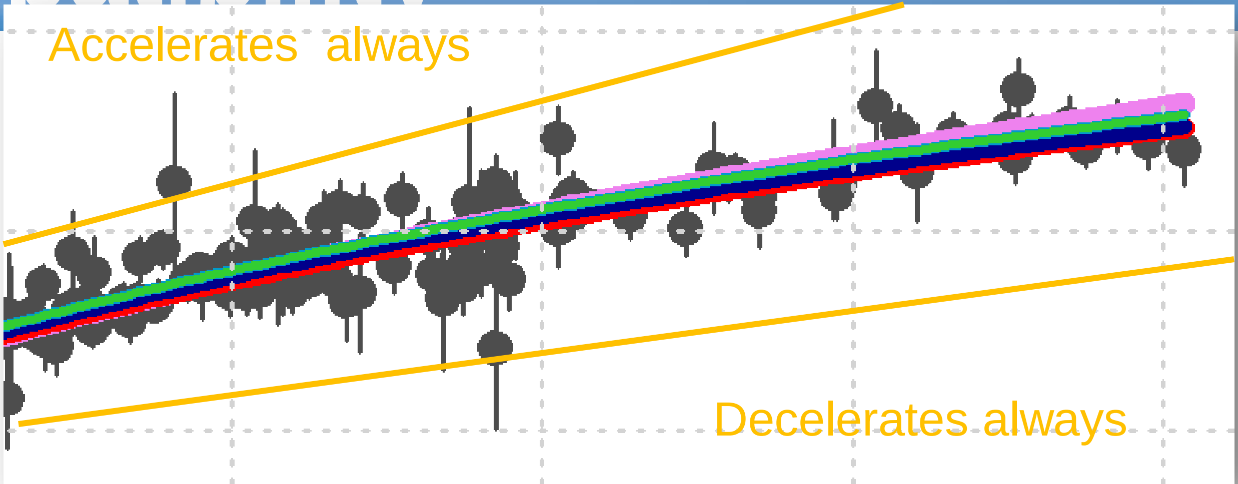
Baryon-to-photon density ratio, $$R_s(z) \equiv \frac{3}{4}\frac{\rho_b}{\rho_\gamma} = \frac{3\omega_b}{4\omega_\gamma}\frac{1}{1+z}$$

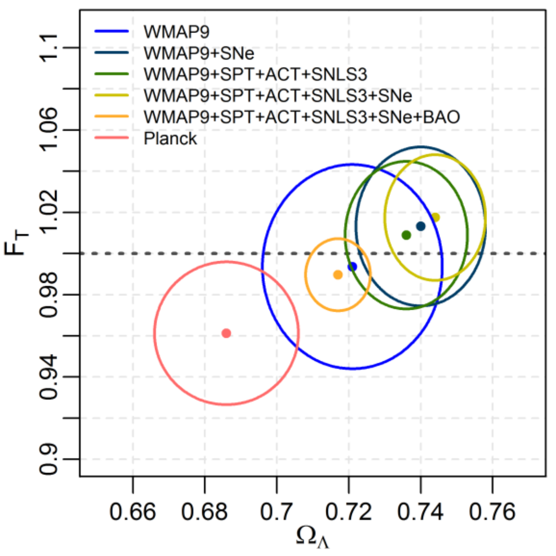
IV. Compatibility

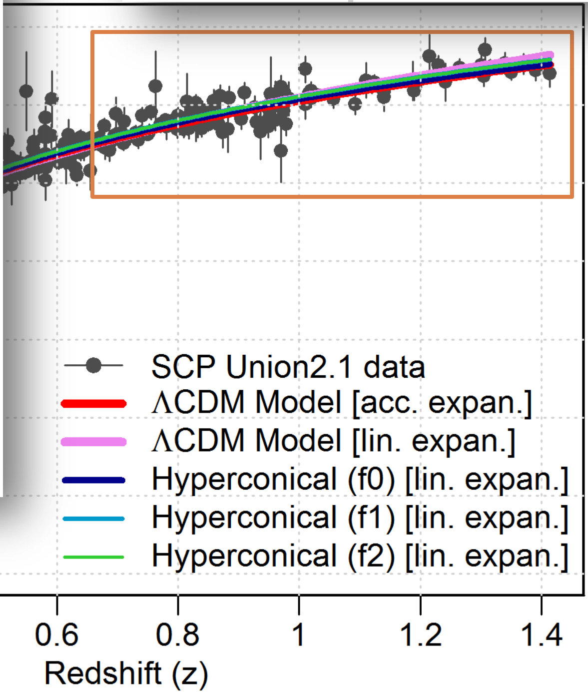
B. Observational compatib

“Equilibrium” region:
Decelerated in the past and accelerates now

≅ Linear always ??







Observation for z < 1.4

k = 1

$_a\alpha = 0.30 \pm 0.01$ ($\chi_0^2 = 562$)

$_b\alpha = 0.36 \pm 0.02$ ($\chi_0^2 = 562$)

$$_0 f_{\hat{r}}^{\alpha}(r') = r'$$

$$_a f_{\hat{r}}^{\alpha}(r') = t_0 \gamma' \Delta^{\alpha}(\gamma' / \gamma_{\max}')$$

$$_b f_{\hat{r}}^{\alpha}(r') = 2t_0 \tan^{-1} \frac{\Delta^{\alpha}(\gamma' / \gamma_{\max}')}{2}$$

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V. Conclusions

- We have considered an **inhomogeneous model** with linear expansion that **does not depends on the matter** content.
- The Hyperconical model is **consistent with $k \equiv 1$**, and thus **freedom** for fitting is given by (locally conformal) **spatial projections f^{α}**.
- However, there exists a unique local **projection $f^{\alpha}(r \ll t)$ compatible with the ΛCDM model** (that is, where model provides real values).
- Thanks to this compatibility, the Hyperconical model **predicts** that **$\Omega_{\Lambda} = 0.6937181(2)$ and $\Omega_m = 0.306192(6)$** for $t_o \equiv 1 \equiv k$ and $\Omega_r = (9.0 \pm 0.5) \cdot 10^{-5}$, **which is compatible** with the Planck Mission results ($\Omega_{\Lambda} = 0.6911 \pm 0.0062$ and $\Omega_m = 0.3089 \pm 0.0062$).
- **Prediction distortion** parameter (α) is **in agreement with the fitted** local and regional values. Moreover, they obtain the same statistical significance that the ΛCDM model.

V. References

Description of the inhomogeneous model used:

R. Monjo, Phys. Rev. D, 96, 103505 (2017).
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Questions

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email: rmoncho@ucm.es