On the streaming model for redshift-space distortions

Based on Kuruvilla & Porciani, MNRAS, 2018

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Introduction

- Redshift surveys provide us with 3D maps of the universe.
- Observed redshift is however affected by both the cosmological expansion and peculiar motions along the line-of-sight.

$$1 + z_{\rm obs} \approx (1 + z_{\rm cos})(1 + \frac{v_{\rm los}}{c})$$

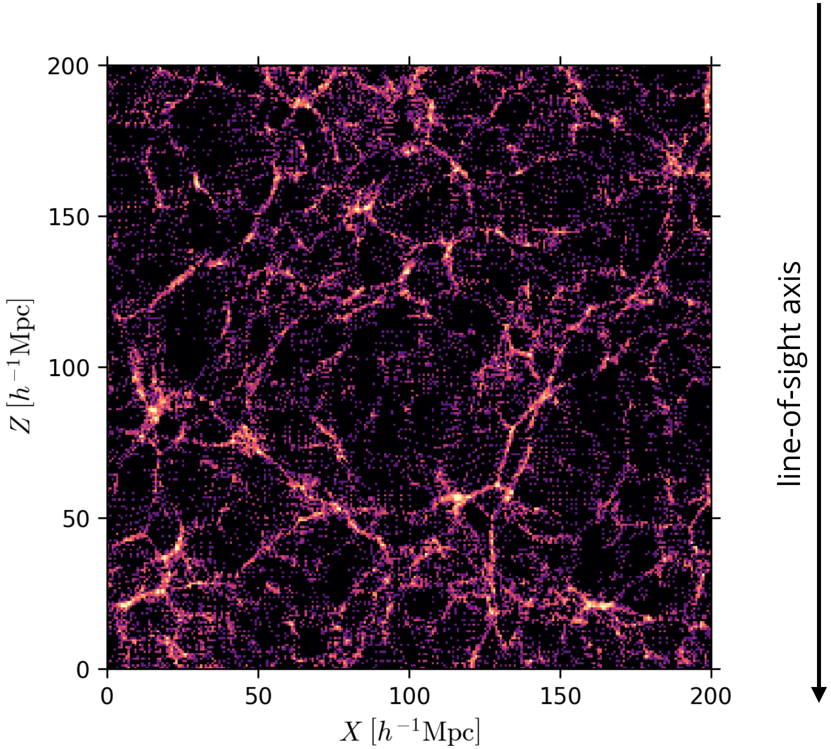
- Thus the 3D maps are distorted which is known as the 'redshift-space distortions' (RSD).

Real space

(Unobserved)

Redshift space

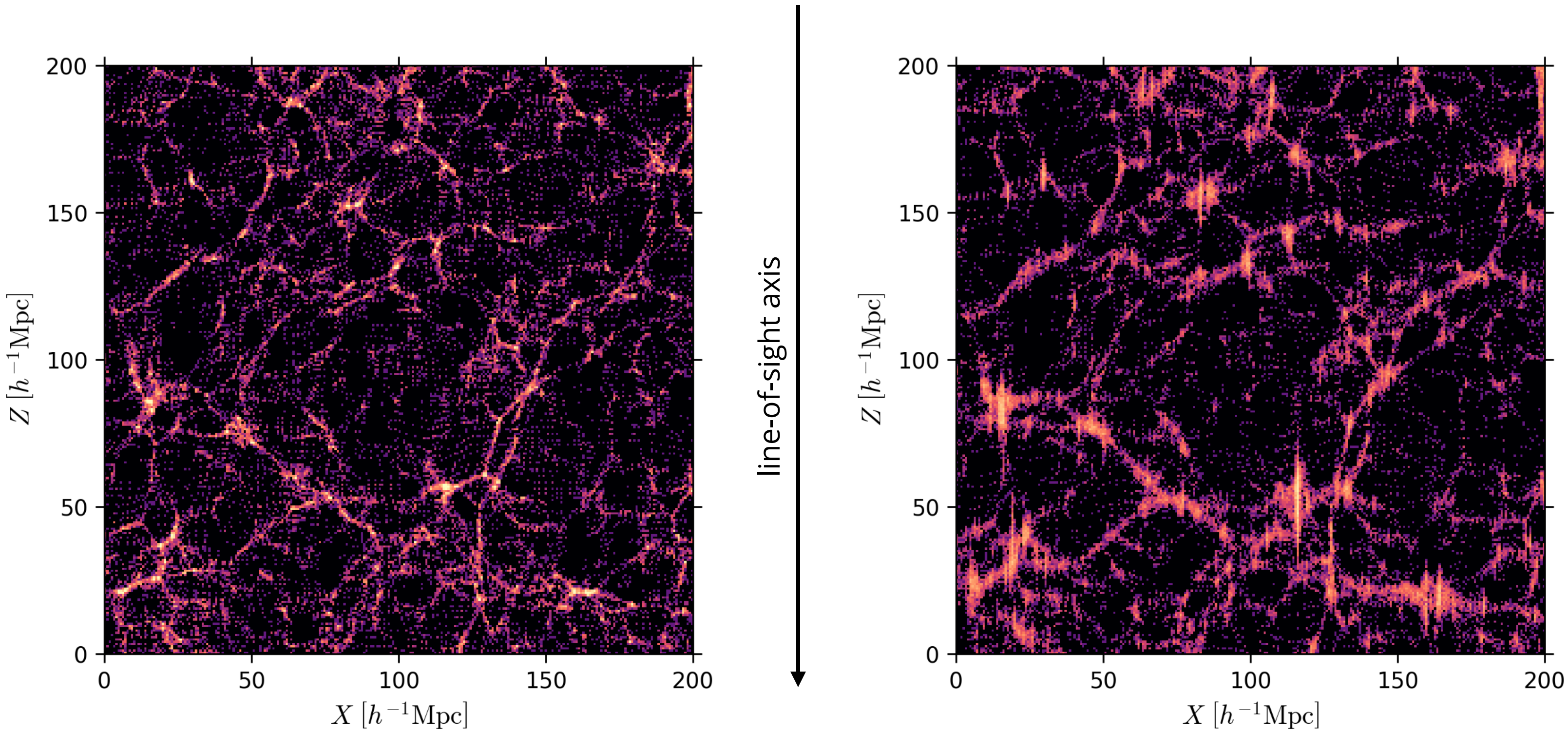
(Observed)



$$\vec{s} = \vec{x} + (\vec{v} \cdot \hat{z})\hat{z}$$

Real space
(Unobserved)

Redshift space
(Observed)



$$\vec{s} = \vec{x} + (\vec{v} \cdot \hat{z})\hat{z}$$

Real space
(Unobserved)

Redshift space
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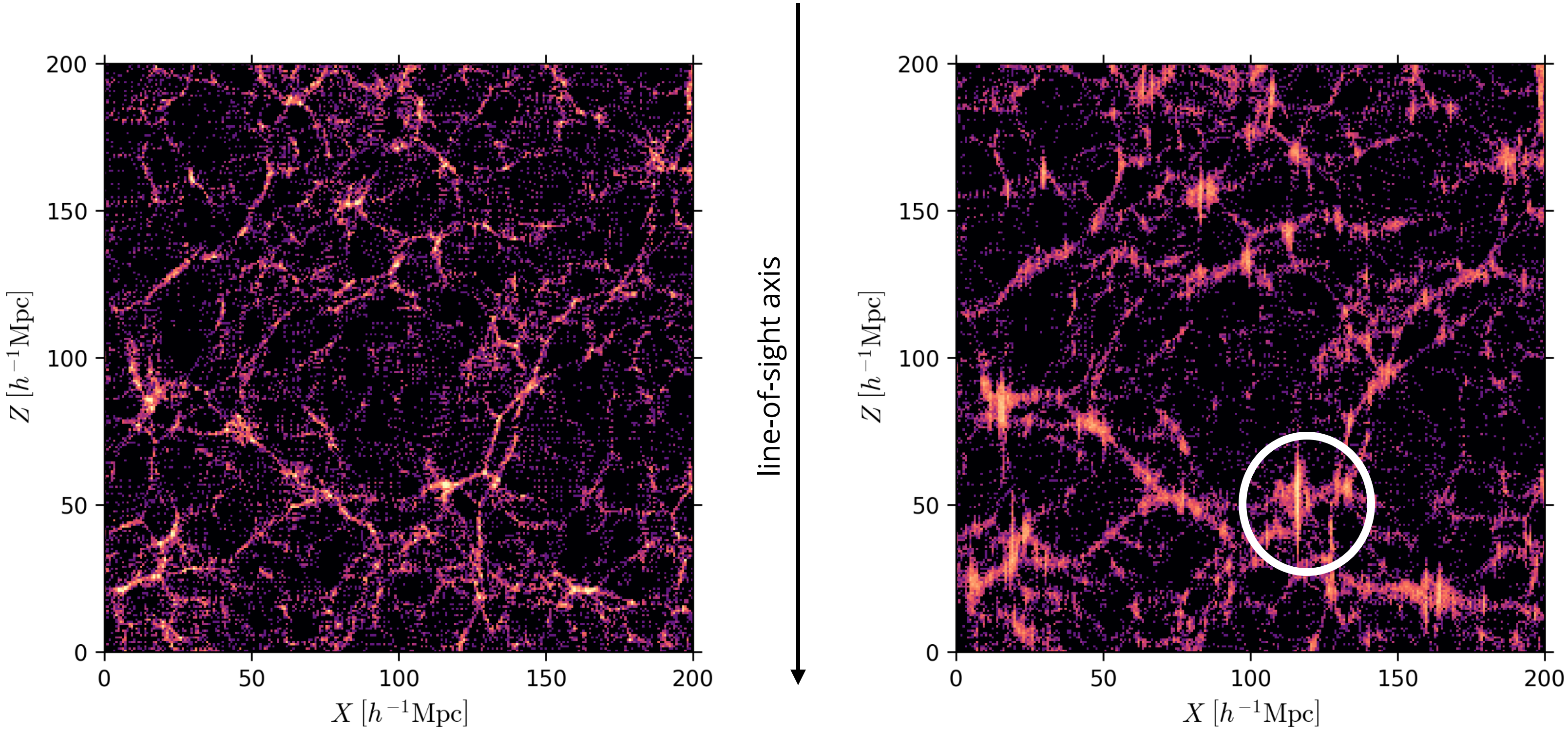
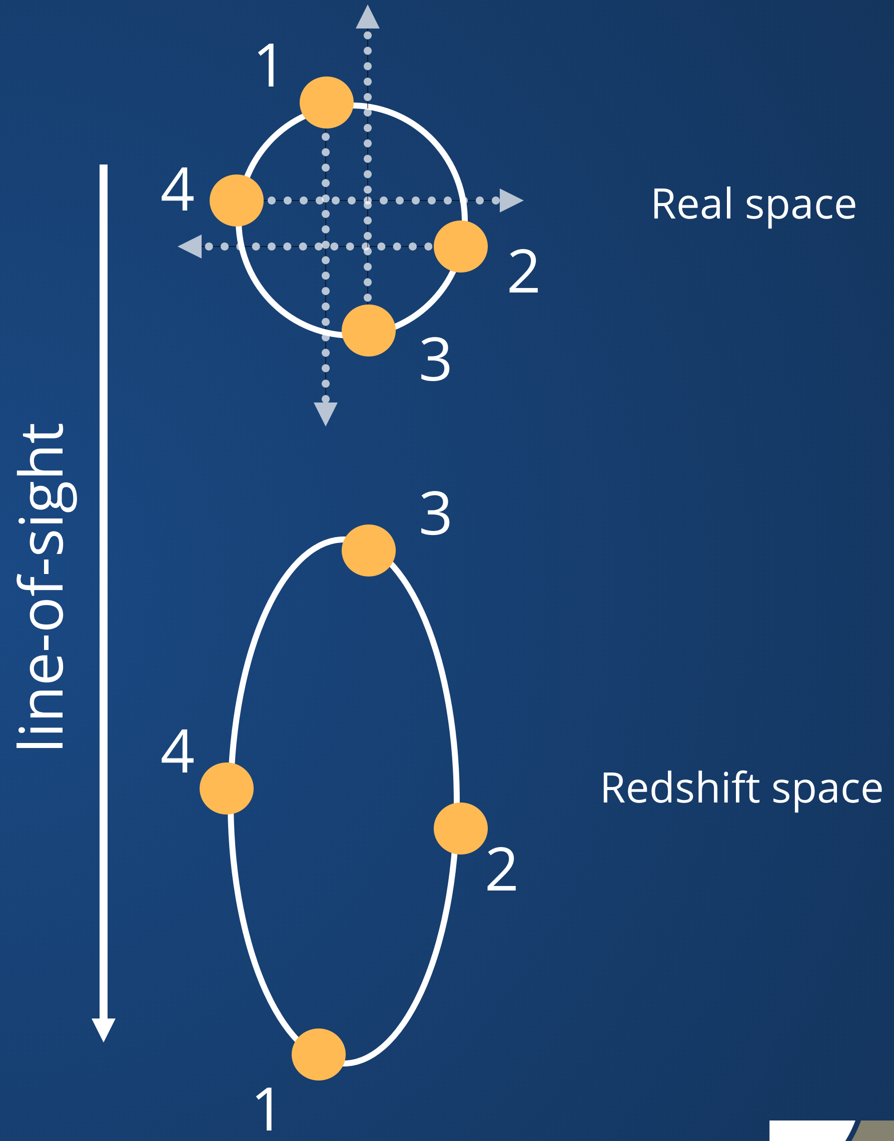


$$\vec{s} = \vec{x} + (\vec{v} \cdot \hat{z})\hat{z}$$

Finger-of-God effect

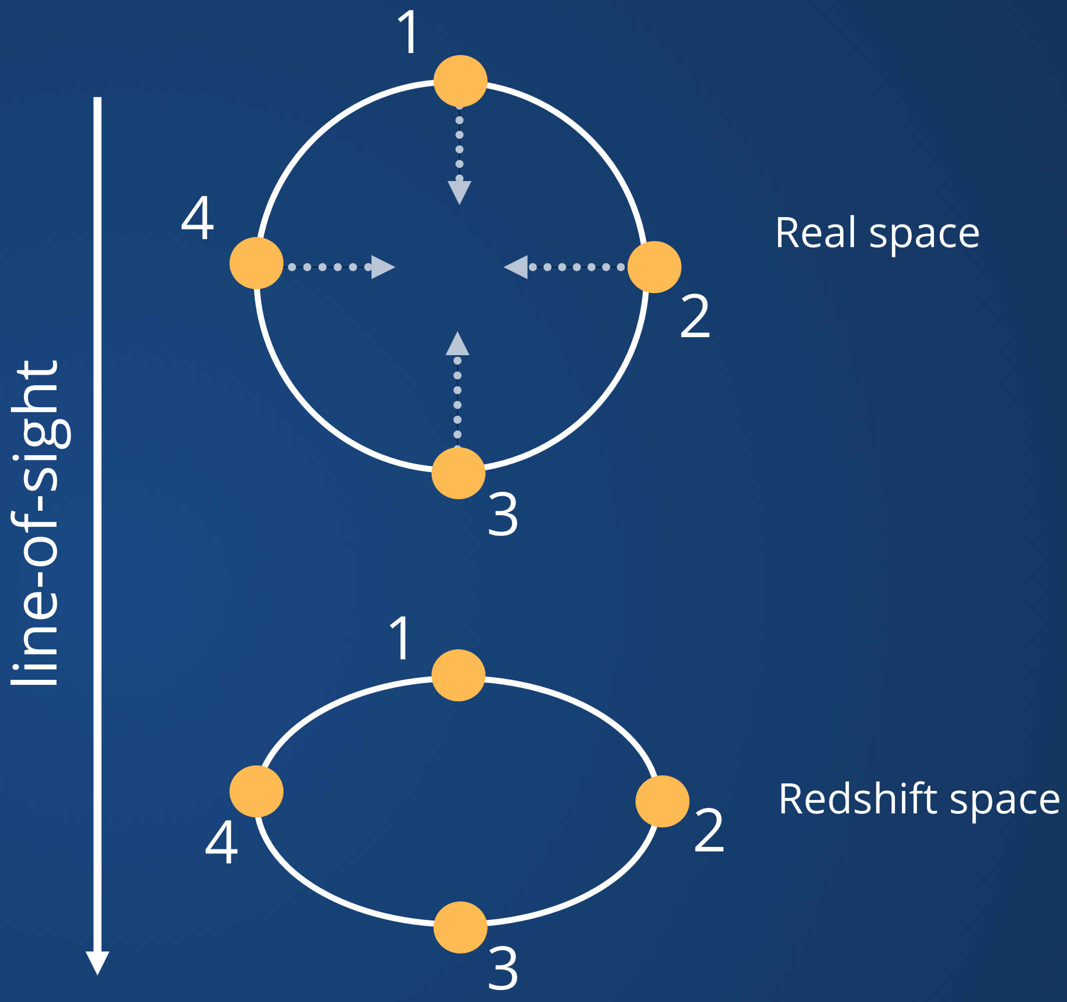
- Non-linear effect
- Collapsed structures appear highly elongated along the los



Kaiser effect

- Linear effect

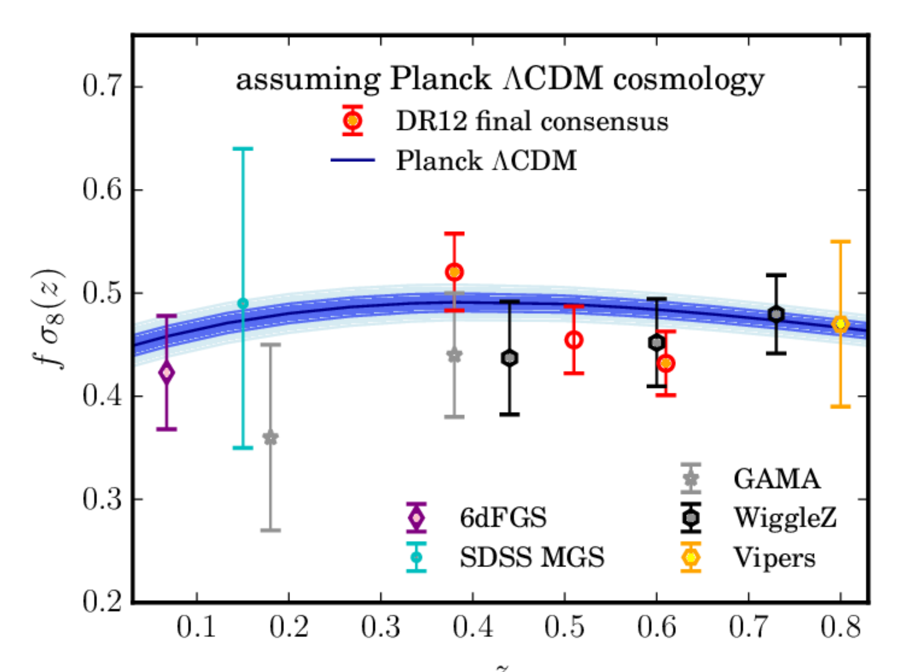
- Linear structures appear squeezed along the los



Motivation

- Tool to test theories of gravity.
- Growing interest in extending RSD studies to smaller scales as a test of modified gravity and interacting dark energy models.

(Jennings et al. 12; Marulli et al. 12; Hellwing et al. 14; Taruya et al. 14; Zu et al. 14; Xu 15; Barreira et al. 16; Sabiu et al. 16; Arnalte-Mur et al. 17)

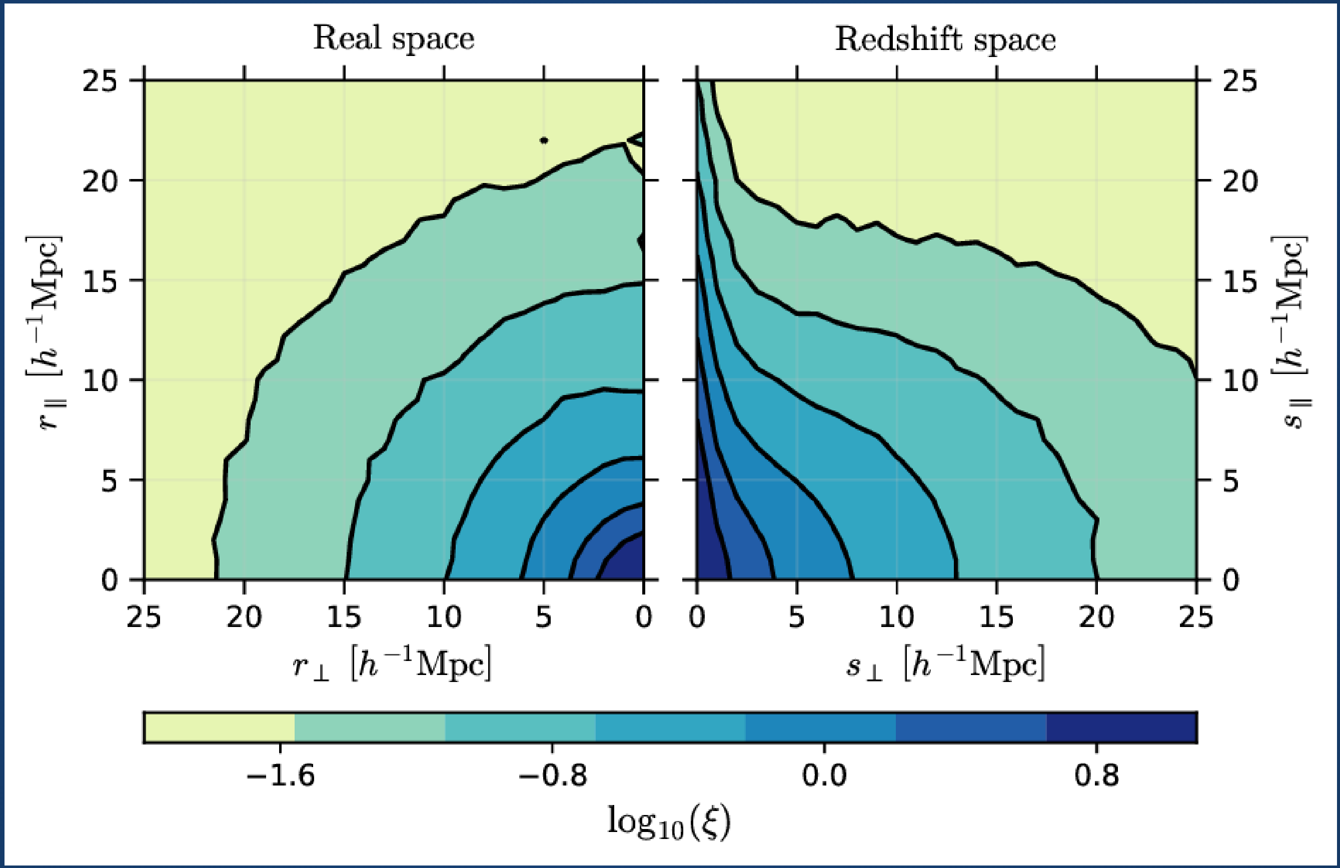


Alam et. al 2017

Check out Motonari Tonegawa's poster also!

How can we model RSD?

Streaming Model



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Streaming Model

$$1 + \xi_{\rm s}(s_\perp, s_\parallel) = \int [1 + \xi_{\rm r}(r)]\, \mathcal{P}(w_\parallel \mid \vec{r}\,)\, {\rm d}r_\parallel$$

Peebles 80, Fisher 95, Scoccimarro 04

- Assumes plane-parallel approximation.
- For wide-angle effects, see:

  Szalay et al. 98, Szapudi 04, Papai & Szapudi 08, Bertacca et al. 12, ...

Streaming Model

$$1 + \xi_{\rm s}(s_\perp, s_\parallel) = \int [1 + \xi_{\rm r}(r)]\, \mathcal{P}(w_\parallel \mid \vec{r}\,)\, {\rm d}r_\parallel$$

Peebles 80, Fisher 95, Scoccimarro 04

$\xi_{\rm s}(s_\perp, s_\parallel)$: Anistropic redshift-space correlation function

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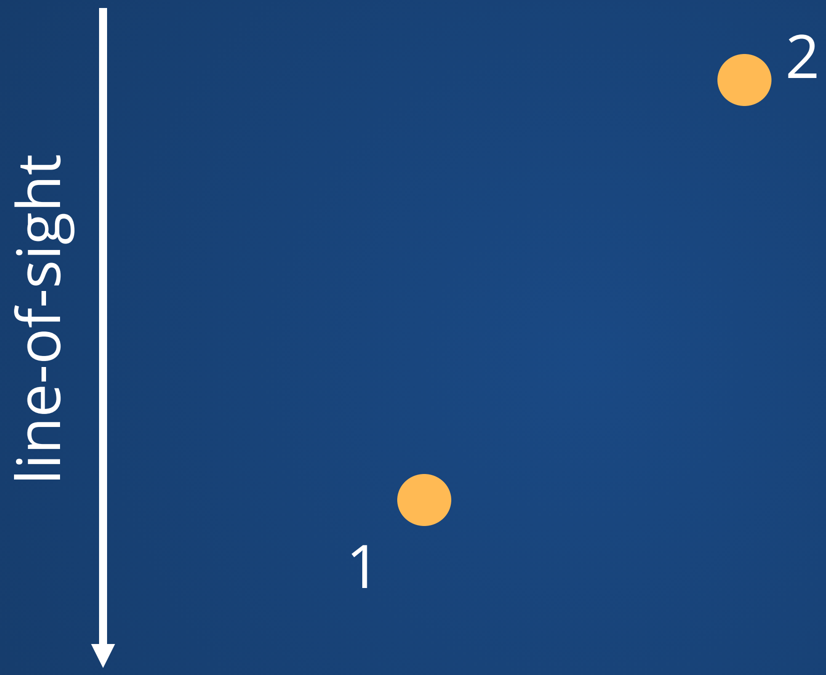
Streaming Model

$$1 + \xi_{\mathrm{s}}(s_\perp, s_\parallel) = \int [1 + \xi_{\mathrm{r}}(r)] \, \mathcal{P}(w_\parallel \mid \vec{r}\,) \, \mathrm{d}r_\parallel$$

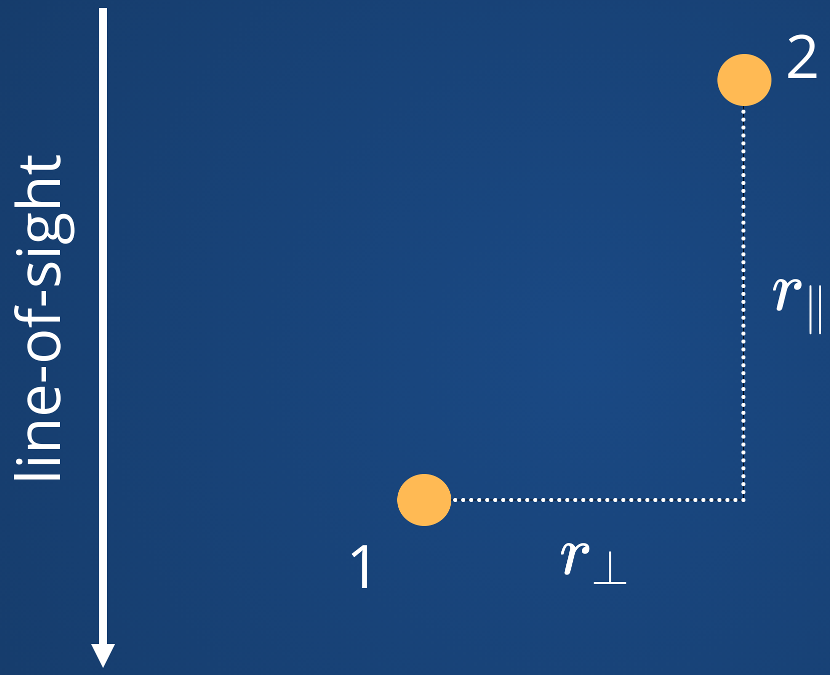
Peebles 80, Fisher 95, Scoccimarro 04

$\xi_{\mathrm{r}}(r)$: Isotropic real-space correlation function

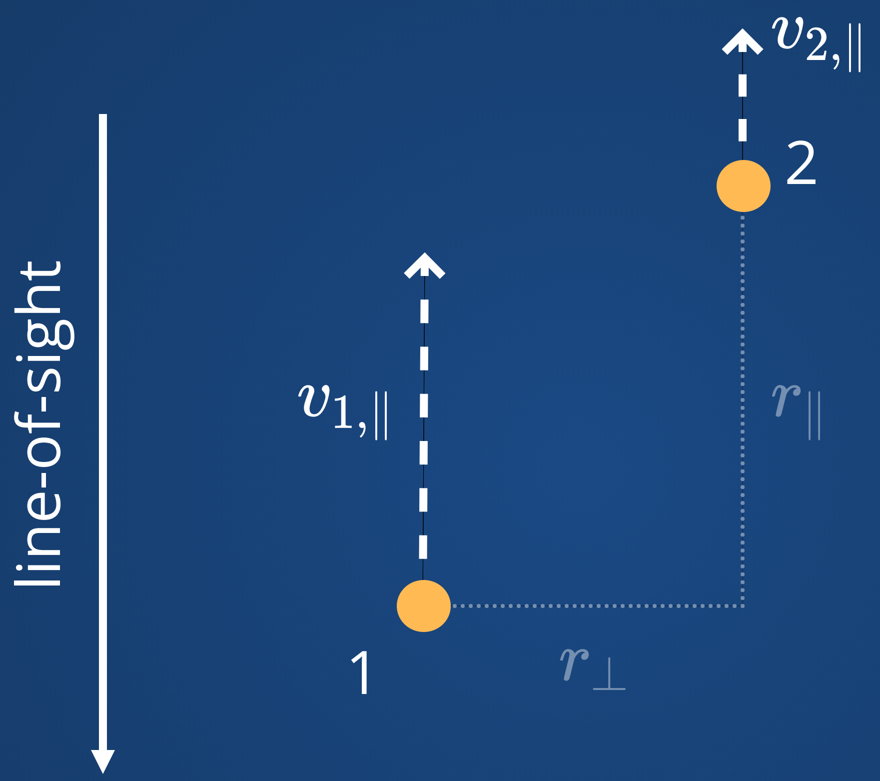
Pairwise velocity



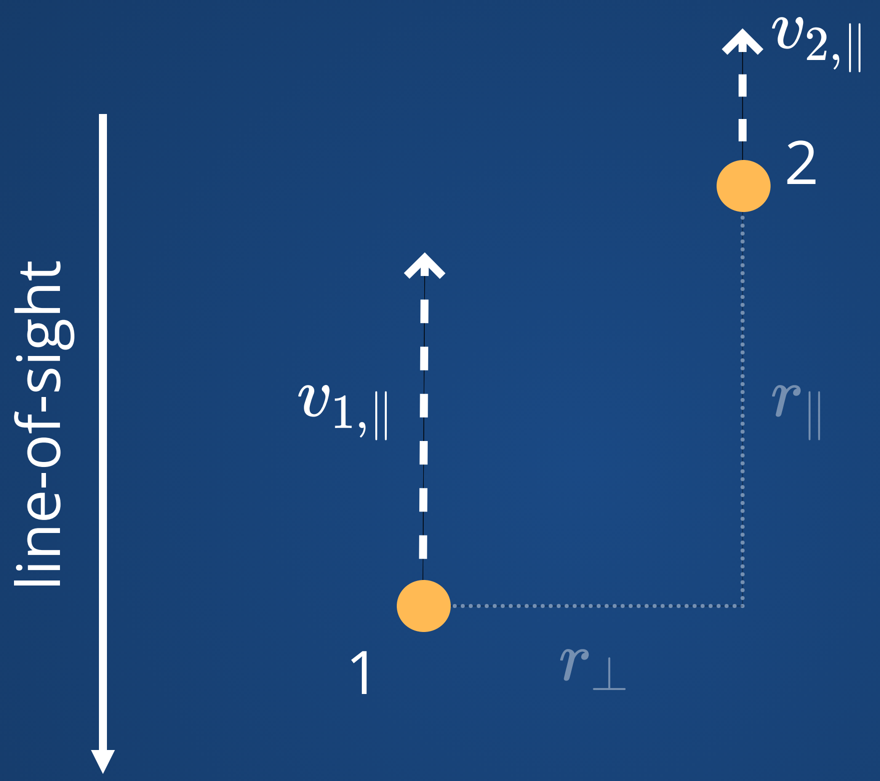
Pairwise velocity



Pairwise velocity



Pairwise velocity



$$w_{\parallel}(r_{\perp}, r_{\parallel}) = v_{2,\parallel} - v_{1,\parallel}$$

Streaming Model

$$1 + \xi_{\mathrm{s}}(s_\perp, s_\parallel) = \int [1 + \xi_{\mathrm{r}}(r)]\, \mathcal{P}(w_\parallel \mid \vec{r}\,)\, \mathrm{d}r_\parallel$$

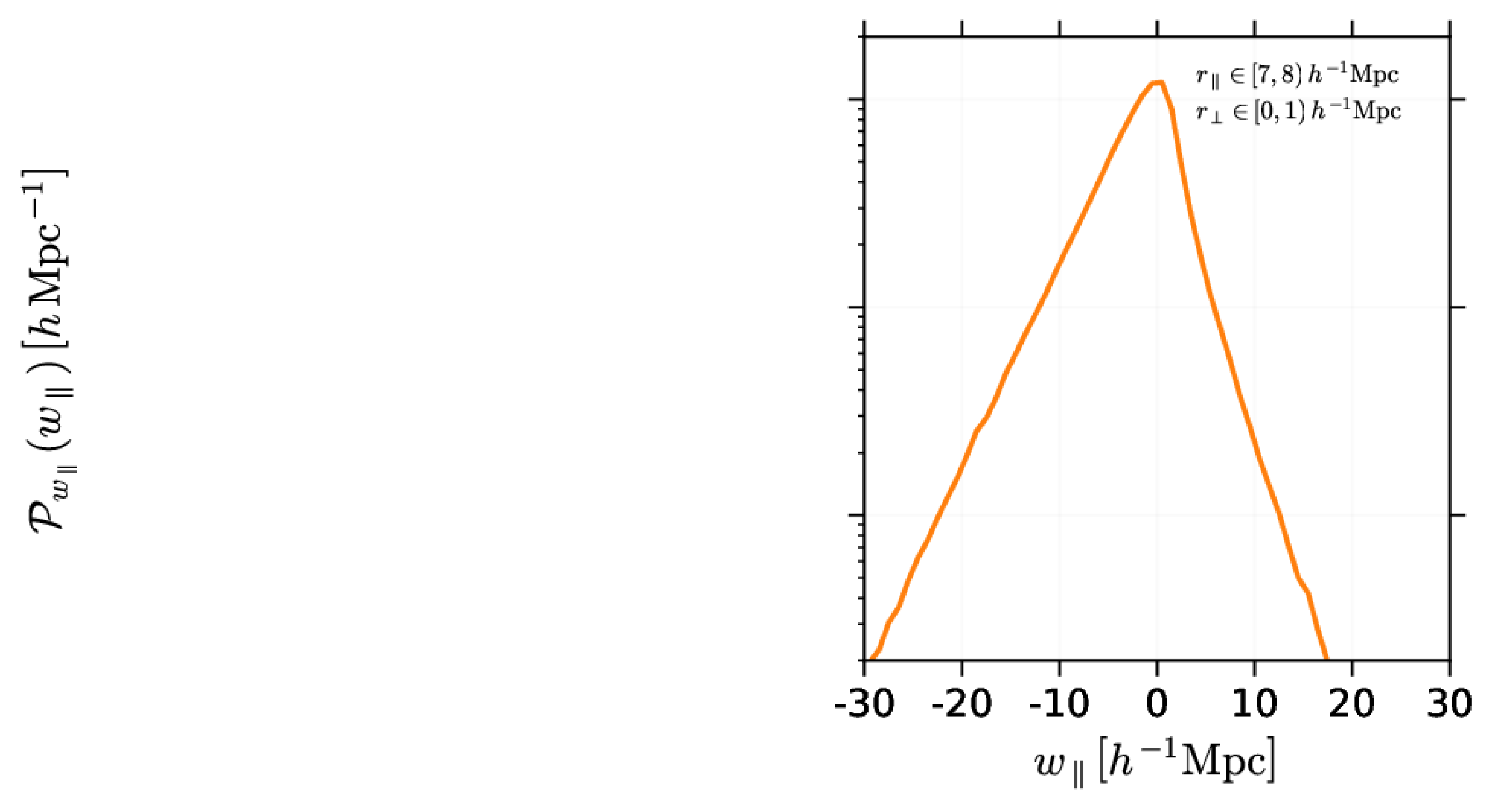
Peebles 80, Fisher 95, Scoccimarro 04

$$w_\parallel = (\vec{v}_2 - \vec{v}_1) \cdot \hat{z}$$

$\mathcal{P}(w_\parallel \mid \vec{r}\,)$: Relative line-of-sight velocity distribution

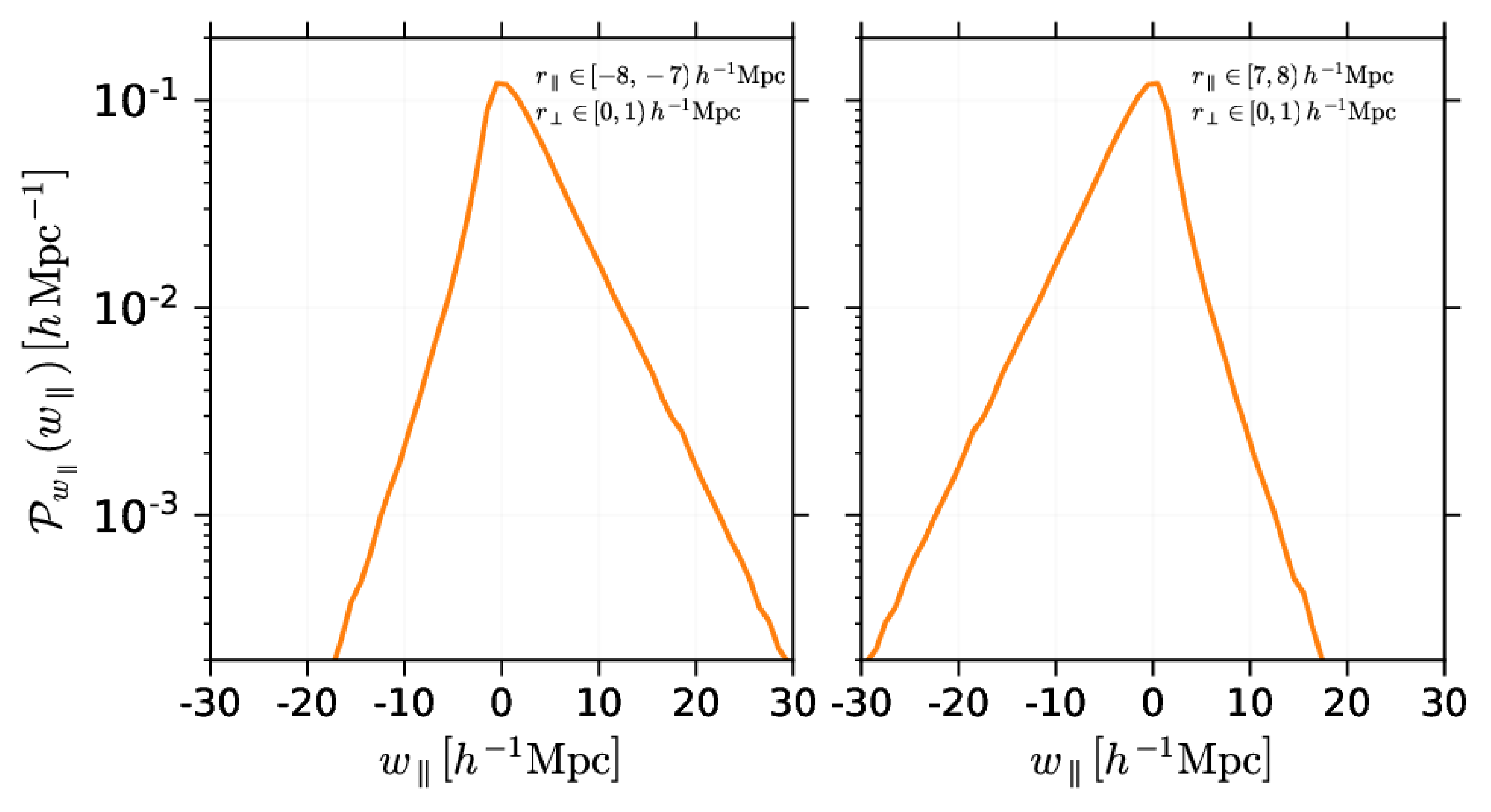
Line-of-sight PDF

$$w_\parallel = (\vec{v}_2 - \vec{v}_1) \cdot \hat{z}, \quad \vec{v} = \vec{u}/aH$$

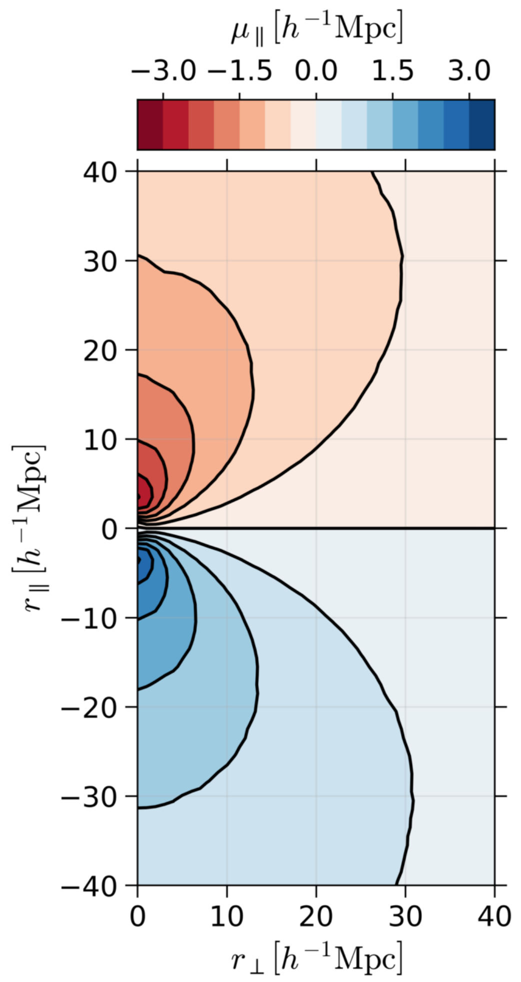


Line-of-sight PDF

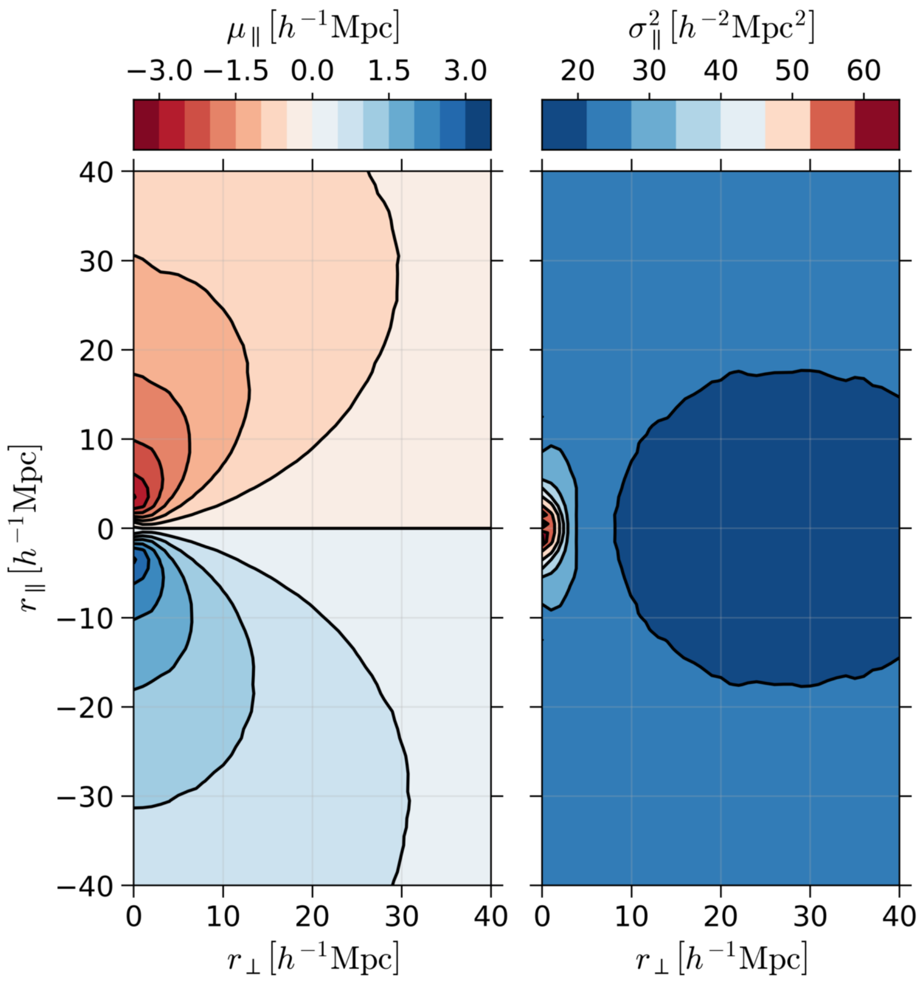
$$w_{\parallel} = (\vec{v}_2 - \vec{v}_1) \cdot \hat{z}, \quad \vec{v} = \vec{u}/aH$$



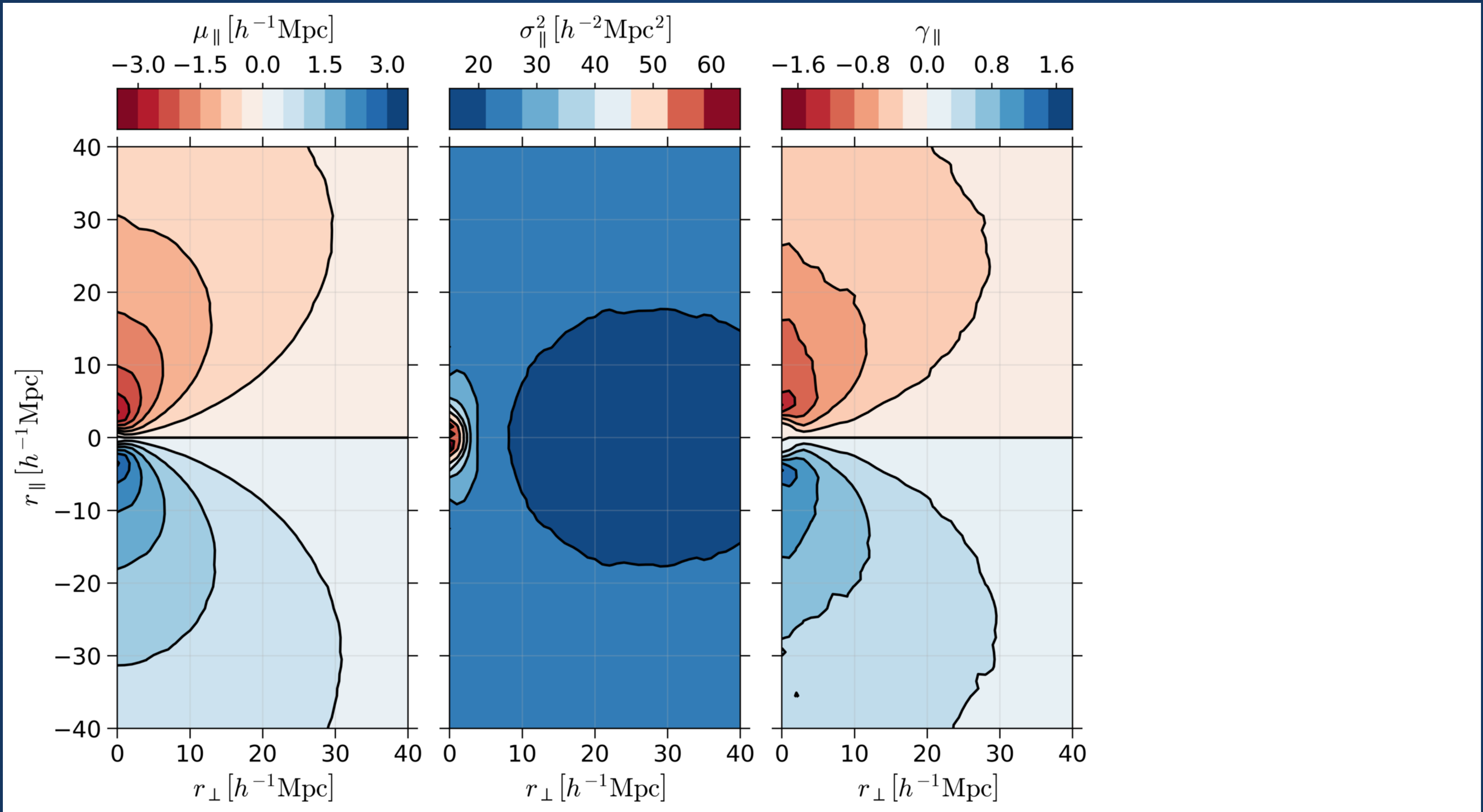
Moments


$\mu_{\parallel}\,[h^{-1}\mathrm{Mpc}]$
−3.0 −1.5 0.0 1.5 3.0
40
30
20
10
0
−10
−20
−30
−40
$r_{\parallel}\,[h^{-1}\mathrm{Mpc}]$
0 10 20 30 40
$r_{\perp}\,[h^{-1}\mathrm{Mpc}]$

Moments

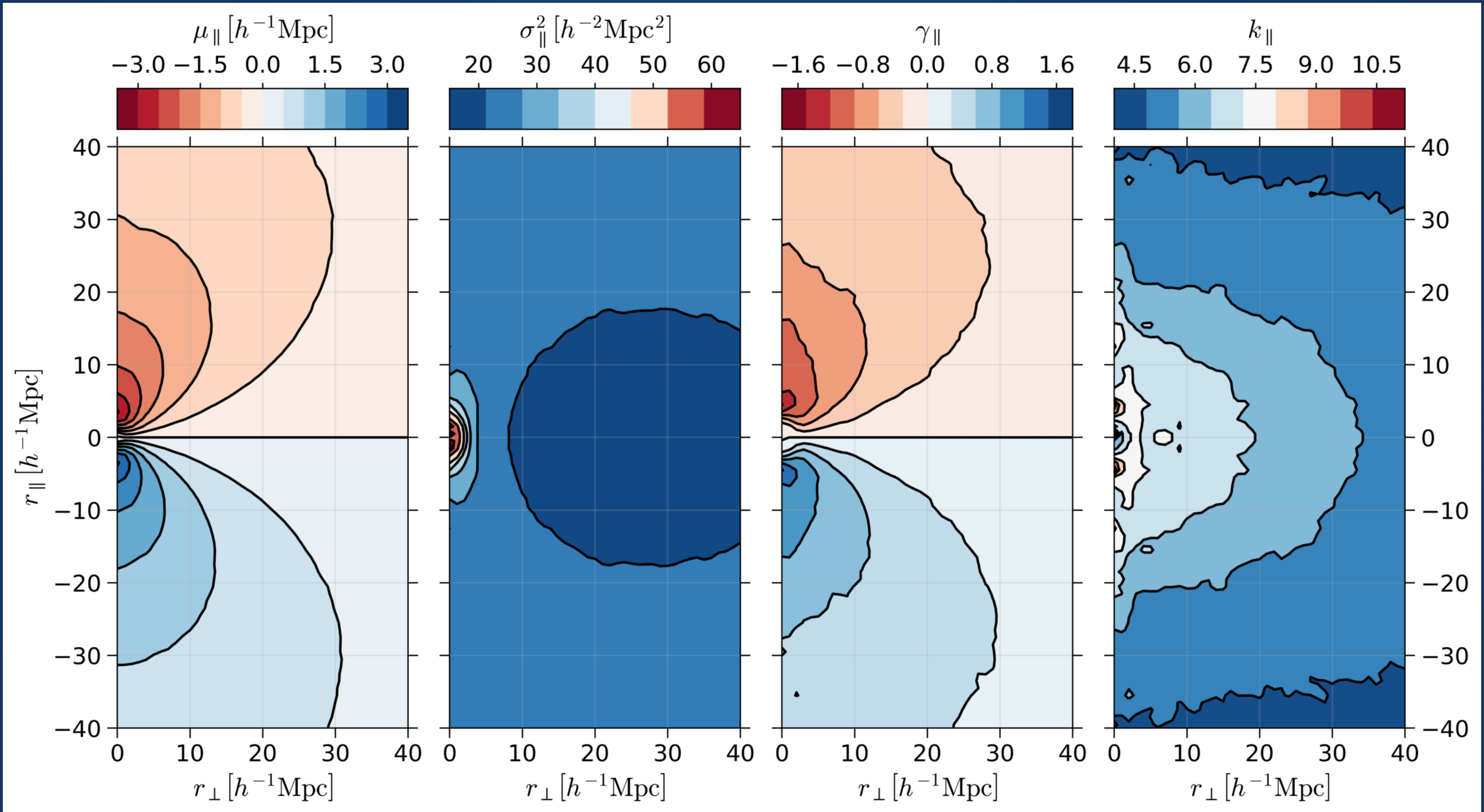

$\mu_\parallel\,[h^{-1}\mathrm{Mpc}]$
−3.0 −1.5 0.0 1.5 3.0
$\sigma^2_\parallel\,[h^{-2}\mathrm{Mpc}^2]$
20 30 40 50 60
$r_\parallel\,[h^{-1}\mathrm{Mpc}]$
40 30 20 10 0 −10 −20 −30 −40
0 10 20 30 40
$r_\perp\,[h^{-1}\mathrm{Mpc}]$
0 10 20 30 40
$r_\perp\,[h^{-1}\mathrm{Mpc}]$

Moments



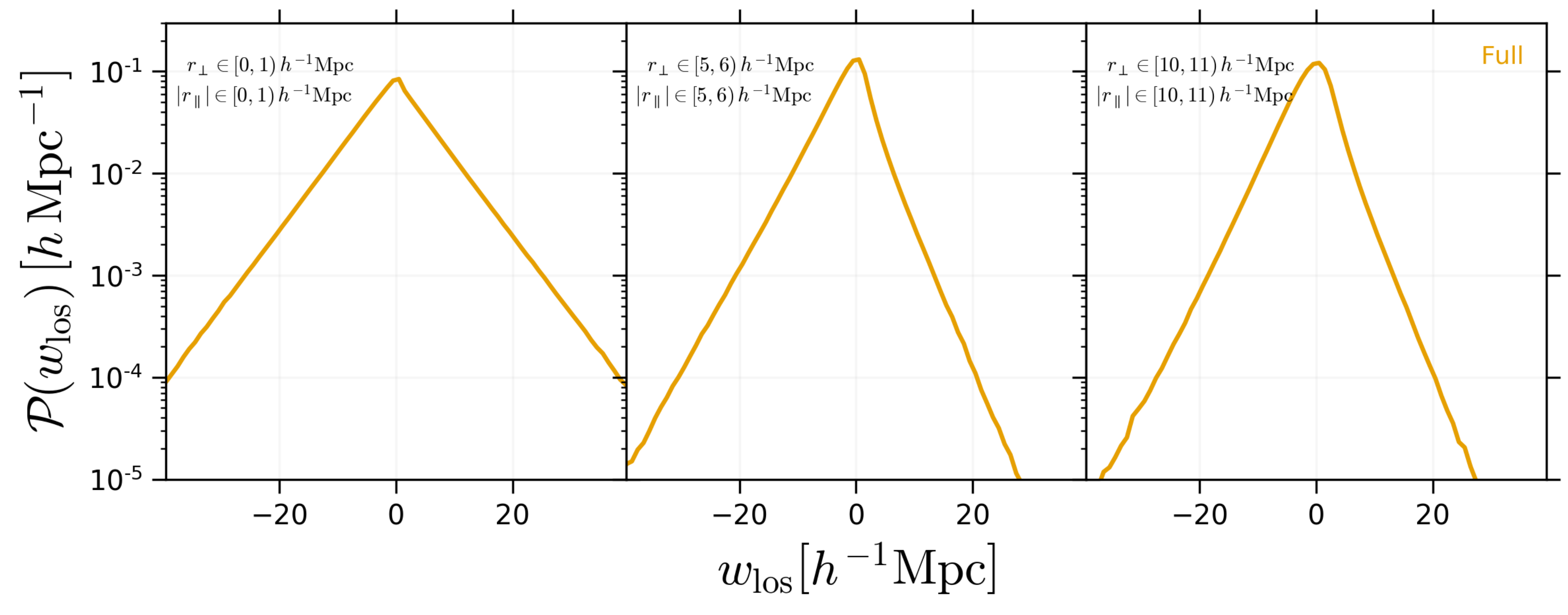
$\mu_\parallel\,[h^{-1}\mathrm{Mpc}]$
−3.0 −1.5 0.0 1.5 3.0
$\sigma^2_\parallel\,[h^{-2}\mathrm{Mpc}^2]$
20 30 40 50 60
$\gamma_\parallel$
−1.6 −0.8 0.0 0.8 1.6
$r_\parallel\,[h^{-1}\mathrm{Mpc}]$
40 30 20 10 0 −10 −20 −30 −40
0 10 20 30 40
$r_\perp\,[h^{-1}\mathrm{Mpc}]$

Moments



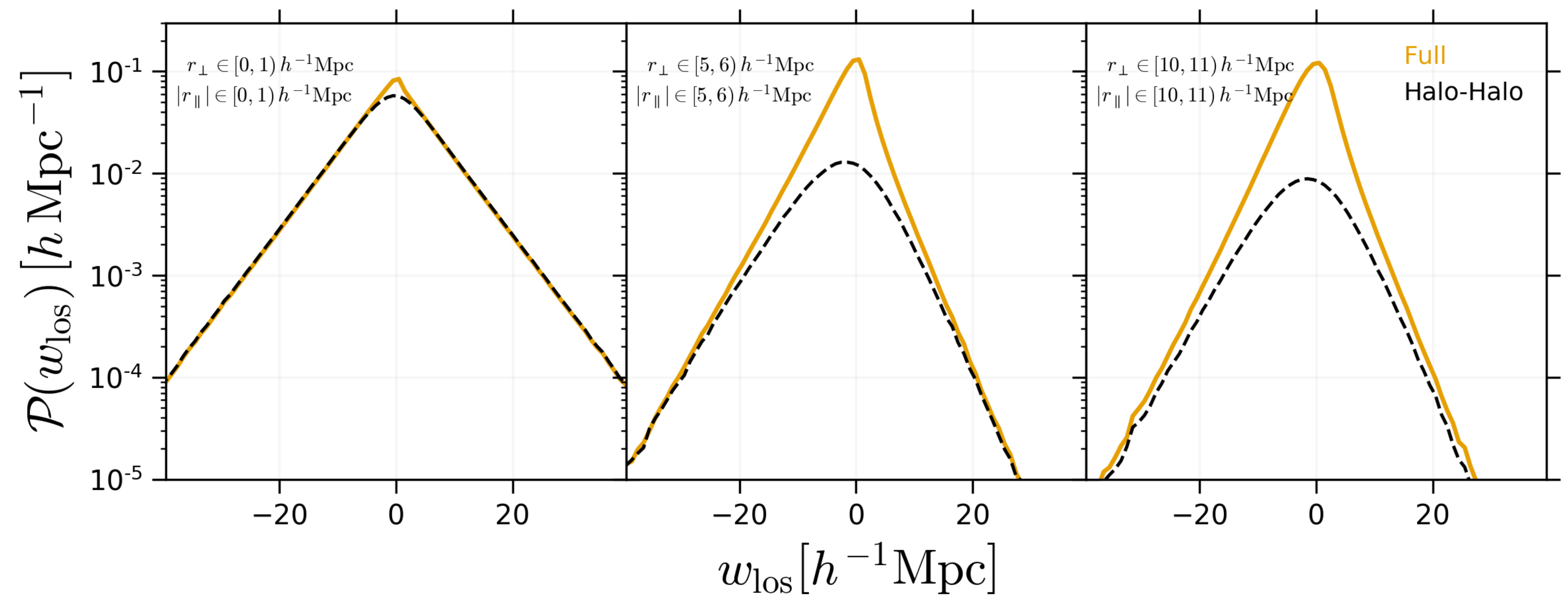
$\mu_{\parallel}\,[h^{-1}\mathrm{Mpc}]$
−3.0 −1.5 0.0 1.5 3.0
$\sigma^2_{\parallel}\,[h^{-2}\mathrm{Mpc}^2]$
20 30 40 50 60
$\gamma_{\parallel}$
−1.6 −0.8 0.0 0.8 1.6
$k_{\parallel}$
4.5 6.0 7.5 9.0 10.5
$r_{\parallel}\,[h^{-1}\mathrm{Mpc}]$
40 30 20 10 0 −10 −20 −30 −40
0 10 20 30 40
$r_{\perp}\,[h^{-1}\mathrm{Mpc}]$

Components



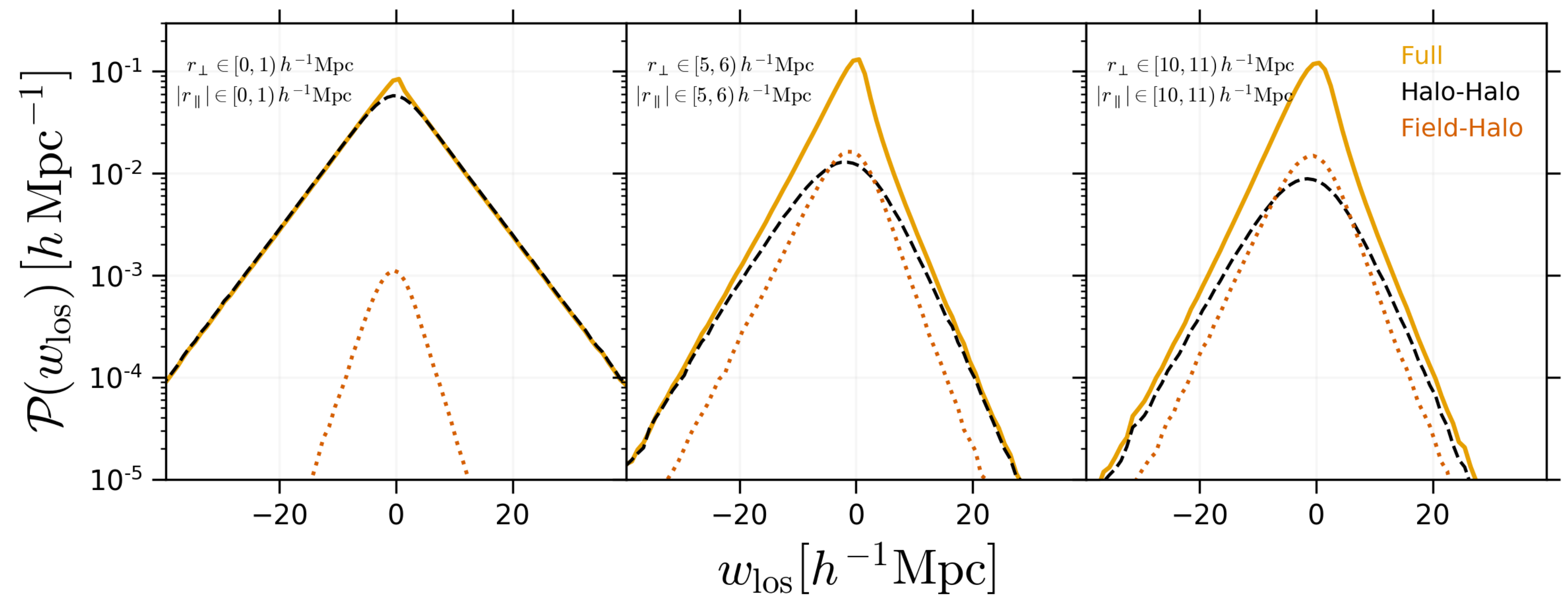
Full
$r_\perp \in [0, 1)\, h^{-1}\mathrm{Mpc}$
$|r_\parallel| \in [0, 1)\, h^{-1}\mathrm{Mpc}$
$r_\perp \in [5, 6)\, h^{-1}\mathrm{Mpc}$
$|r_\parallel| \in [5, 6)\, h^{-1}\mathrm{Mpc}$
$r_\perp \in [10, 11)\, h^{-1}\mathrm{Mpc}$
$|r_\parallel| \in [10, 11)\, h^{-1}\mathrm{Mpc}$
$\mathcal{P}(w_{\mathrm{los}})\, [h\,\mathrm{Mpc}^{-1}]$
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
−20
0
20
$w_{\mathrm{los}}\, [h^{-1}\mathrm{Mpc}]$

Components



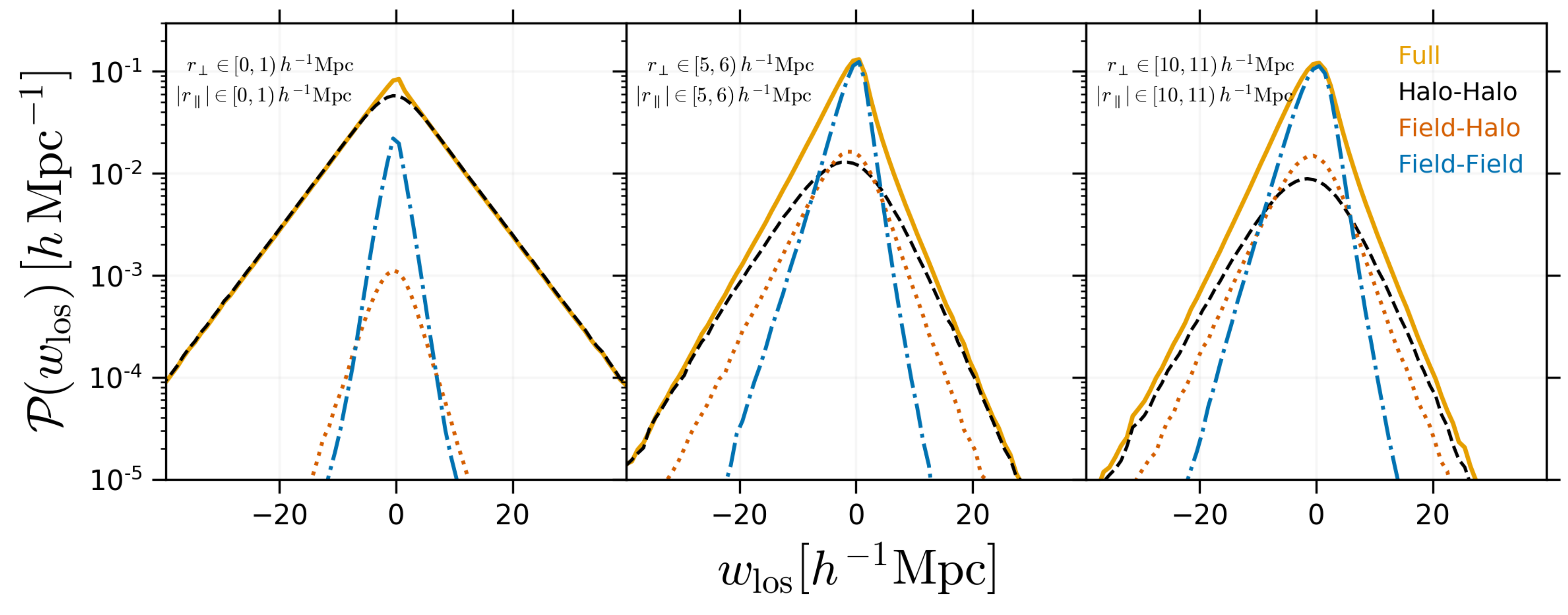
$r_\perp \in [0,1)\, h^{-1}\mathrm{Mpc}$
$|r_\parallel| \in [0,1)\, h^{-1}\mathrm{Mpc}$
$r_\perp \in [5,6)\, h^{-1}\mathrm{Mpc}$
$|r_\parallel| \in [5,6)\, h^{-1}\mathrm{Mpc}$
$r_\perp \in [10,11)\, h^{-1}\mathrm{Mpc}$
$|r_\parallel| \in [10,11)\, h^{-1}\mathrm{Mpc}$
Full
Halo-Halo
$\mathcal{P}(w_{\mathrm{los}})\,[h\,\mathrm{Mpc}^{-1}]$
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
−20
0
20
$w_{\mathrm{los}}\,[h^{-1}\mathrm{Mpc}]$

Components



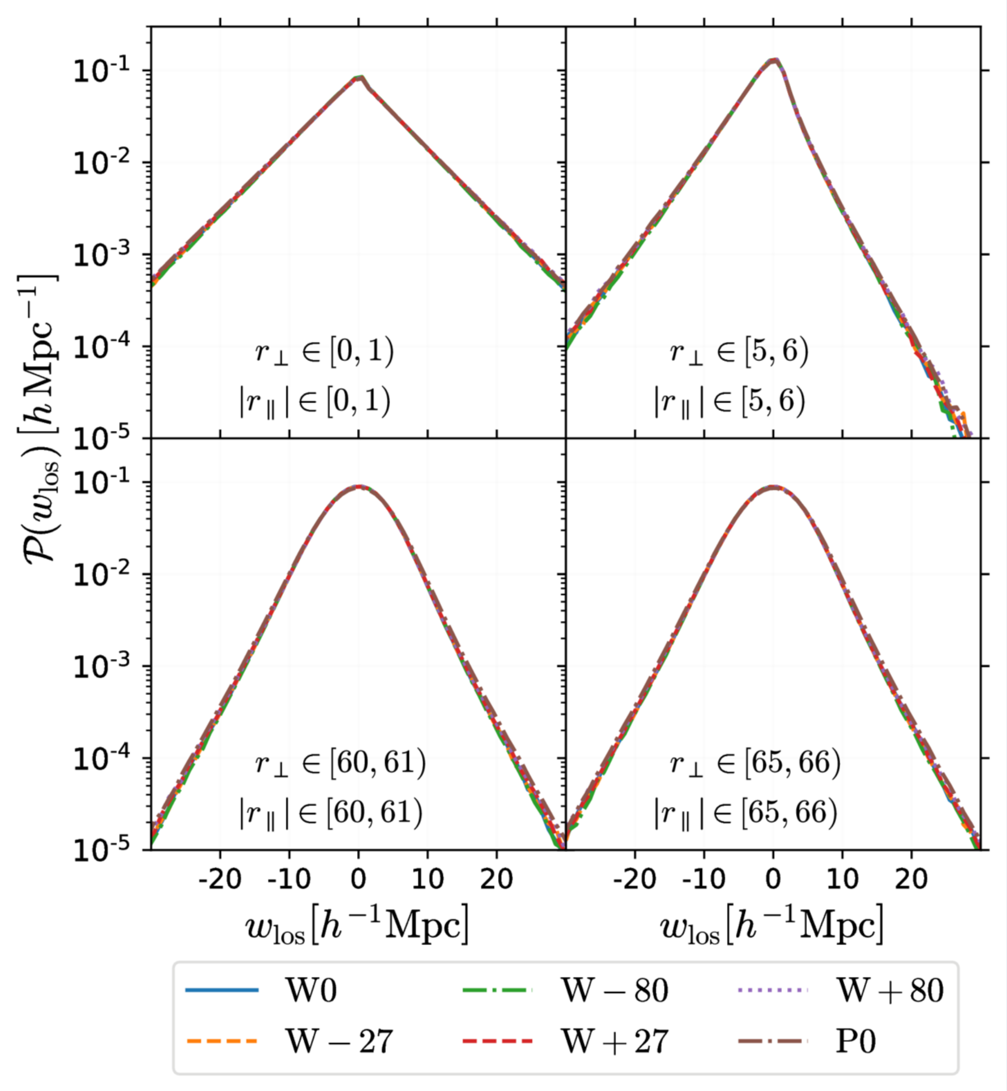
$r_\perp \in [0,1)\, h^{-1}$Mpc
$|r_\parallel| \in [0,1)\, h^{-1}$Mpc
$r_\perp \in [5,6)\, h^{-1}$Mpc
$|r_\parallel| \in [5,6)\, h^{-1}$Mpc
$r_\perp \in [10,11)\, h^{-1}$Mpc
$|r_\parallel| \in [10,11)\, h^{-1}$Mpc
Full
Halo-Halo
Field-Halo
$\mathcal{P}(w_{\rm los})\, [h\, {\rm Mpc}^{-1}]$
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
−20
0
20
$w_{\rm los}\, [h^{-1}{\rm Mpc}]$

Components



$r_\perp \in [0,1)\,h^{-1}\mathrm{Mpc}$
$|r_\parallel| \in [0,1)\,h^{-1}\mathrm{Mpc}$
$r_\perp \in [5,6)\,h^{-1}\mathrm{Mpc}$
$|r_\parallel| \in [5,6)\,h^{-1}\mathrm{Mpc}$
$r_\perp \in [10,11)\,h^{-1}\mathrm{Mpc}$
$|r_\parallel| \in [10,11)\,h^{-1}\mathrm{Mpc}$
Full
Halo-Halo
Field-Halo
Field-Field
$\mathcal{P}(w_{\mathrm{los}})\,[h\,\mathrm{Mpc}^{-1}]$
$w_{\mathrm{los}}\,[h^{-1}\mathrm{Mpc}]$

Cosmology dependence



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What is the form of $\mathcal{P}(w_{\parallel} \mid \vec{r}\,)$?

Streaming Model

$\mathcal{P}(w_{\parallel} \mid \vec{r}\,)$ was assumed to be an exponential. (Davis & Peebles 83)

Current: Gaussian streaming model (GSM) (Reid & White 11)

GSM used in: (Reid et al. 12, Samushia et al. 14; Alam et al. 17, Chuang et al. 17, Satpathy et al. 17)

Streaming Model

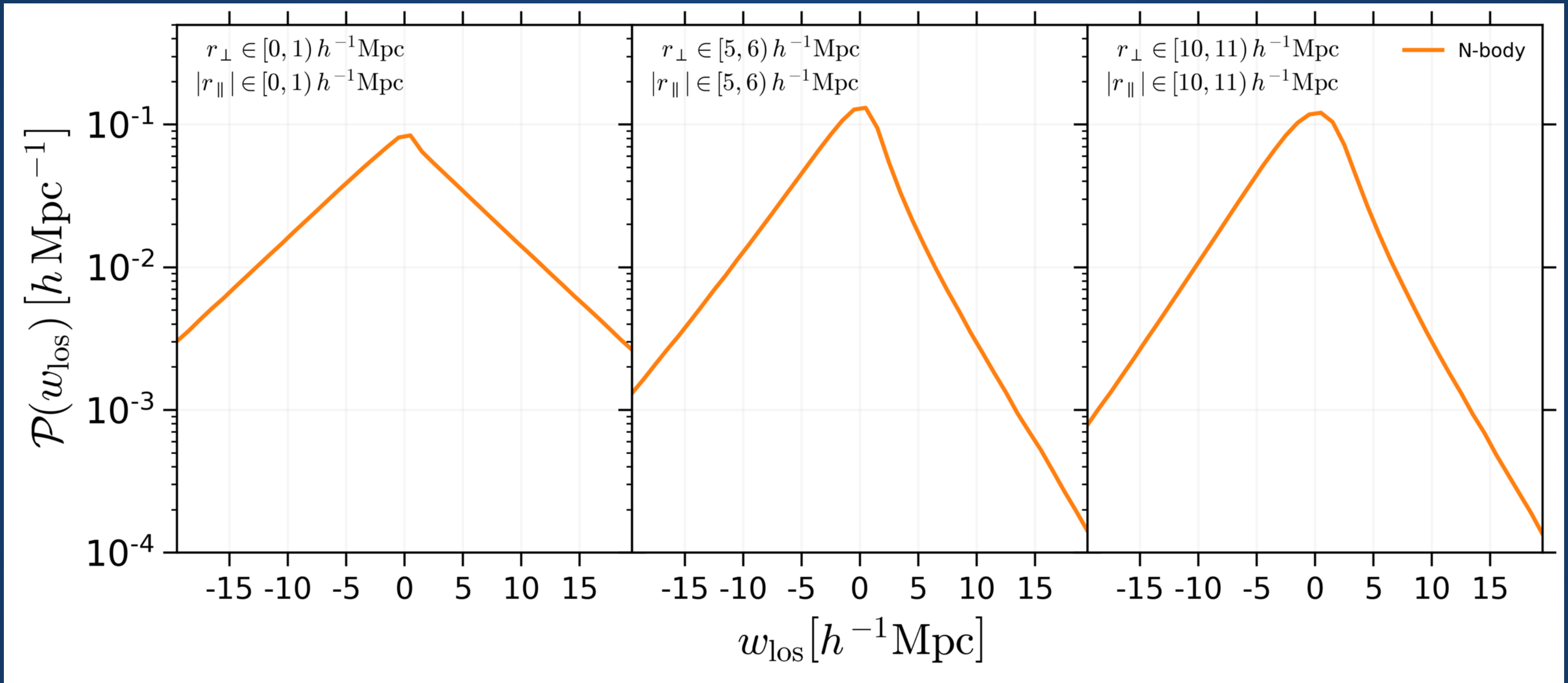
Other developements include:

1. Edgeworth streaming model (Uhlemann et al. 15)
   - Includes skewness

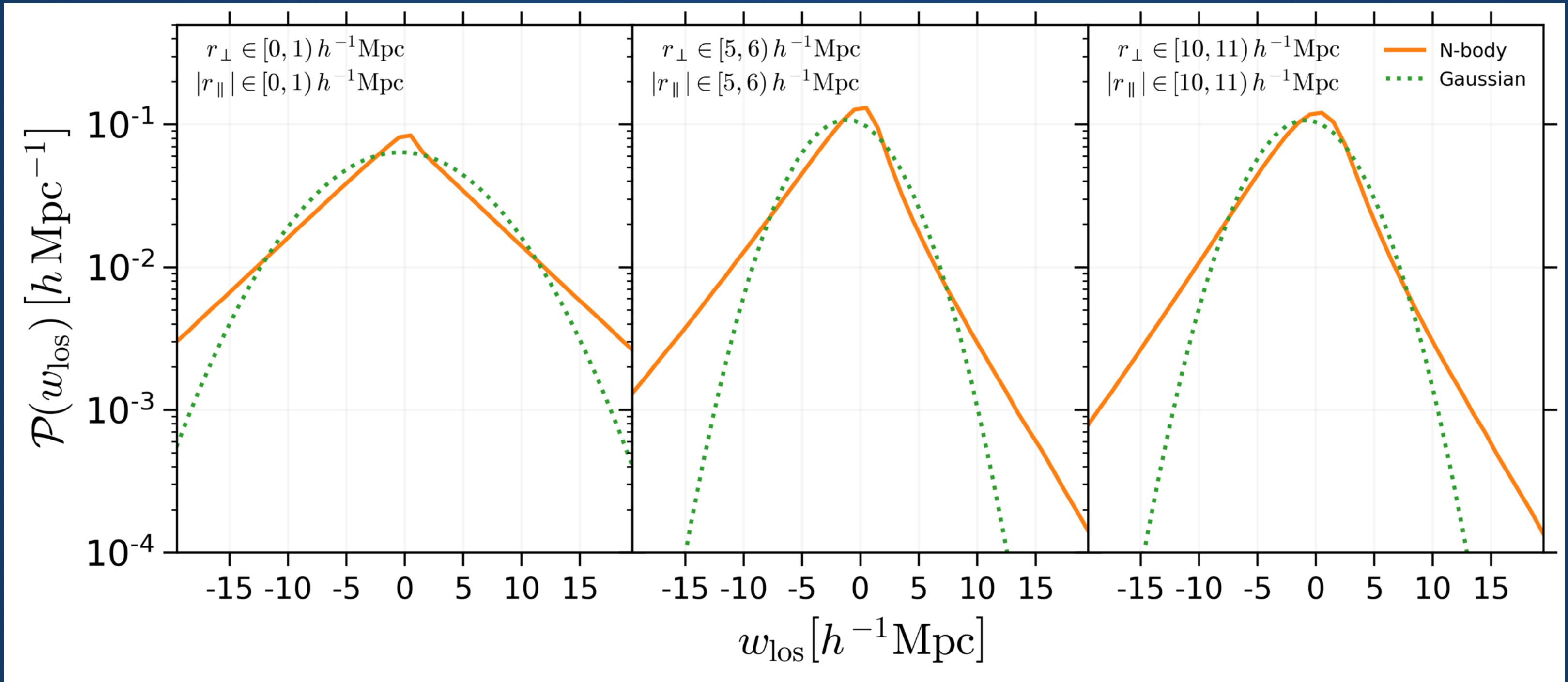
2. Halo model prescription (Sheth & Diaferio 01, Tinker 07)

3. Superposition of Gaussians or quasi-Gaussians. (Bianchi et al. 15, 16)

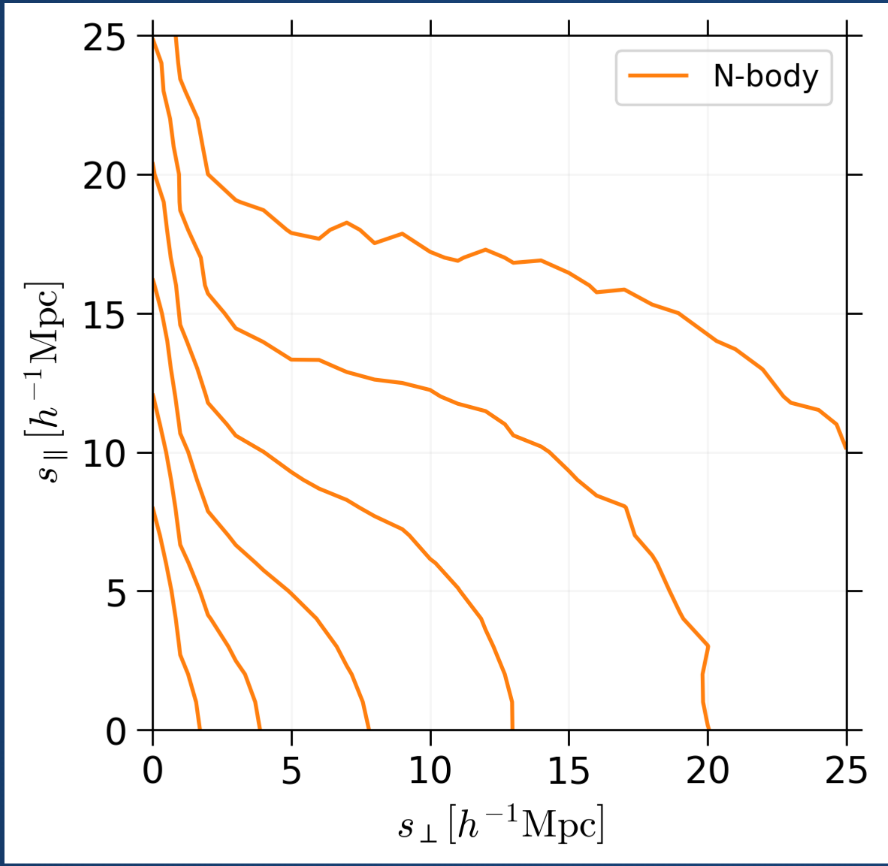
Status Quo - PDF



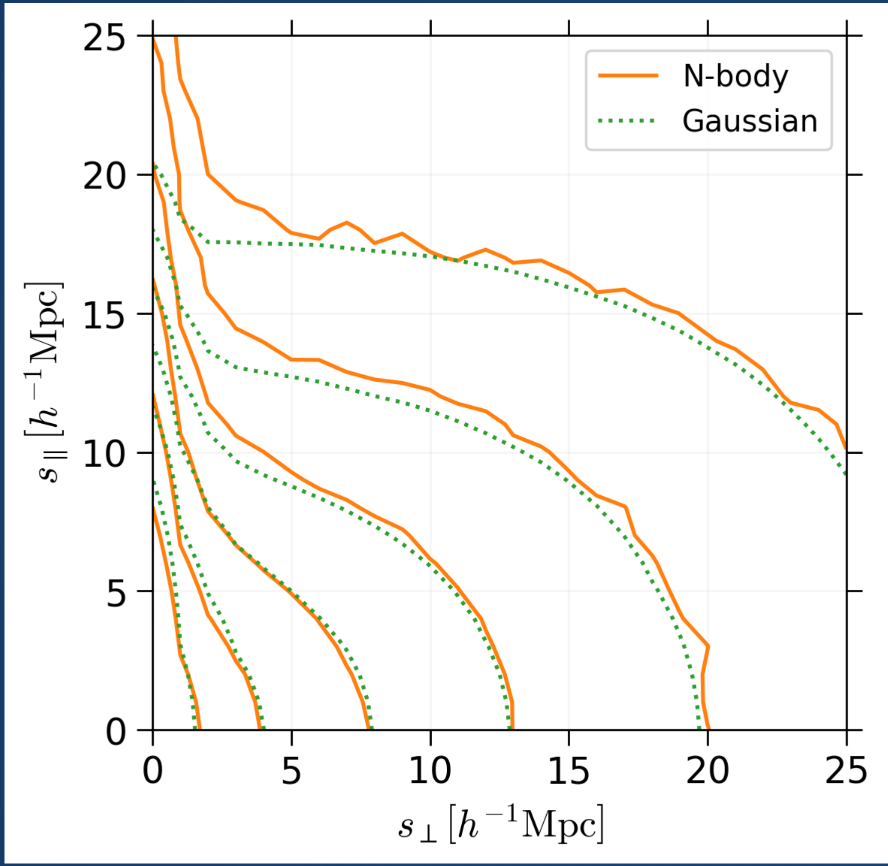
Status Quo - PDF



Status Quo - Correlation

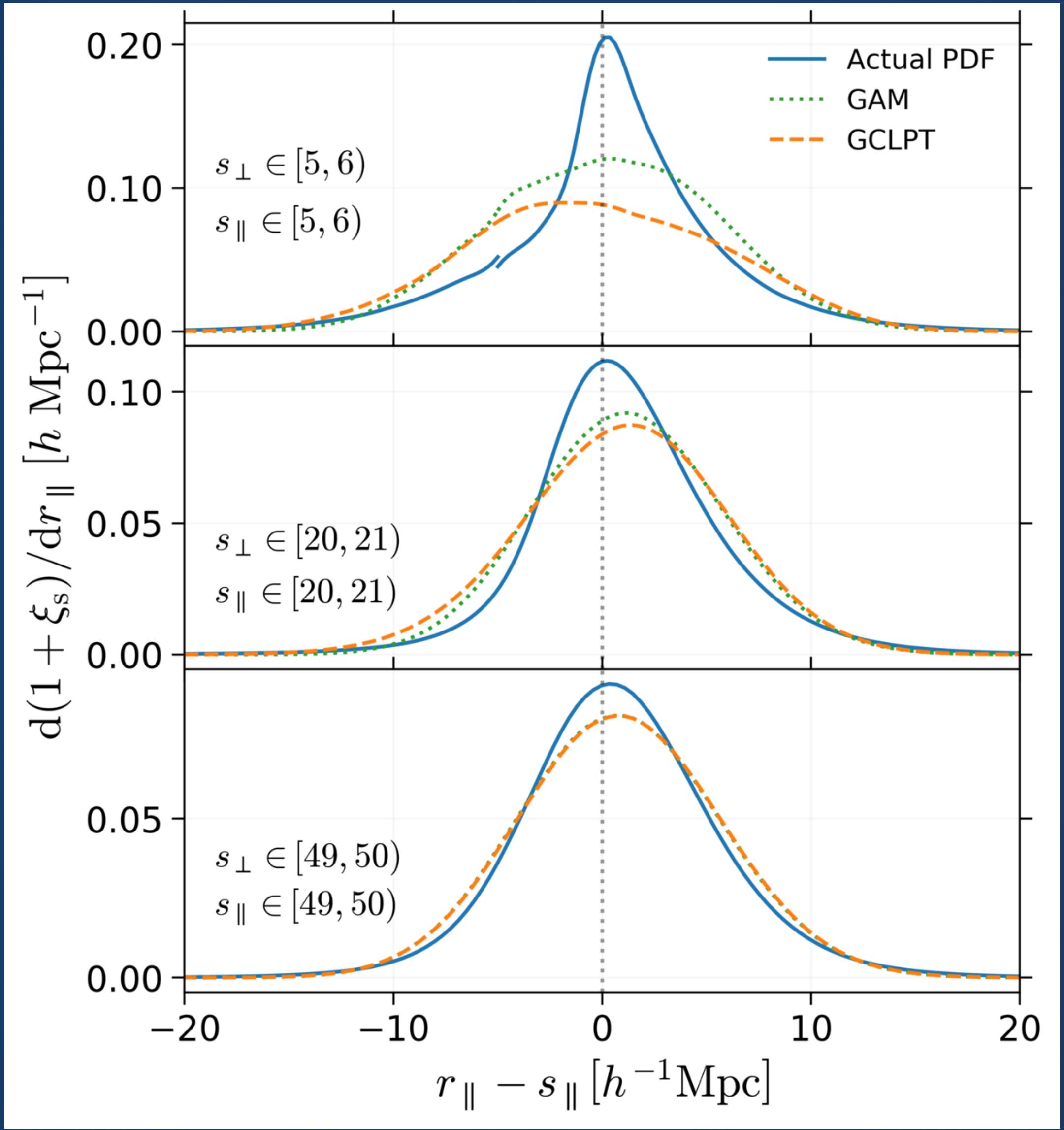


Status Quo - Correlation



MCMC fitting to find the best values.

Two wrongs make it right!



Objectives

- As one probes smaller scales, there is a trade-off between decreasing statistical error and increasing theoretical uncertainty.
- We want to try to bring the theoretical uncertainty down on smaller scales.

New Fitting Model

"Truth is much too complicated to allow anything but approximations."

-- John von Neumann

Generalised Hyperbolic Distribution

We wanted the following characteristics:

1. Unimodal.

2. Quasi-exponential tails.

3. Highly tunable lower order cumulants.

4. Gaussian distribution in some limit.

Generalised Hyperbolic Distribution

GHD is a 5 parameter distribution

$$\mathcal{P}_{w_\parallel}(x;\alpha,\beta,\delta,\lambda,\mu) \propto e^{\beta(x-\mu)}\, K_{\lambda-\frac{1}{2}}\left(\alpha\sqrt{\delta^2+(x-\mu)^2}\right)$$

Generalised Hyperbolic Distribution

$$\mathcal{P}_{w_{\|}}(x; \alpha, \beta, \delta, \lambda, \mu) \propto e^{\beta(x-\mu)} K_{\lambda-\frac{1}{2}}\left(\alpha\sqrt{\delta^2 + (x-\mu)^2}\right)$$

Broadly speaking,

λ defines various subclasses and influences the tails,

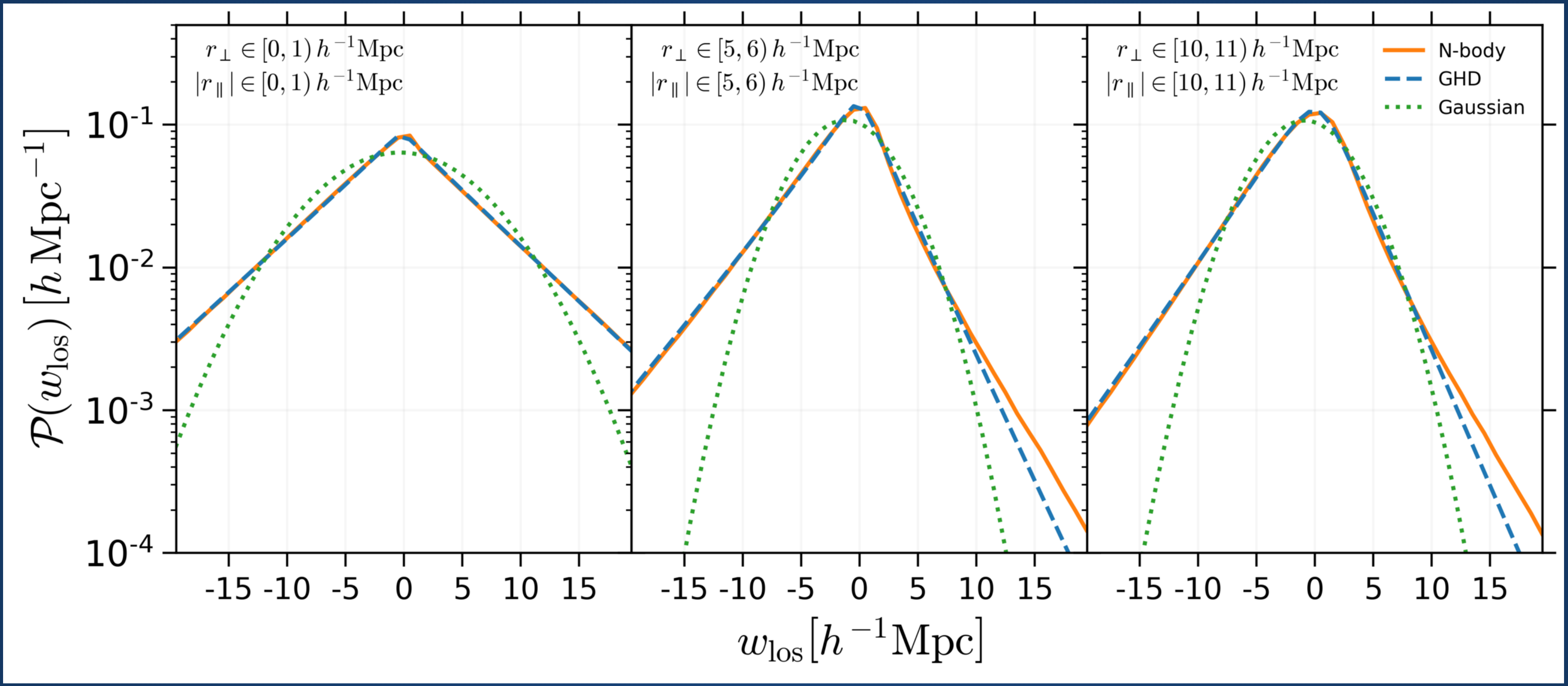
α modifies the shape (i.e. variance and kurtosis),

β the skewness

δ the scale and

μ shifts the mean value.

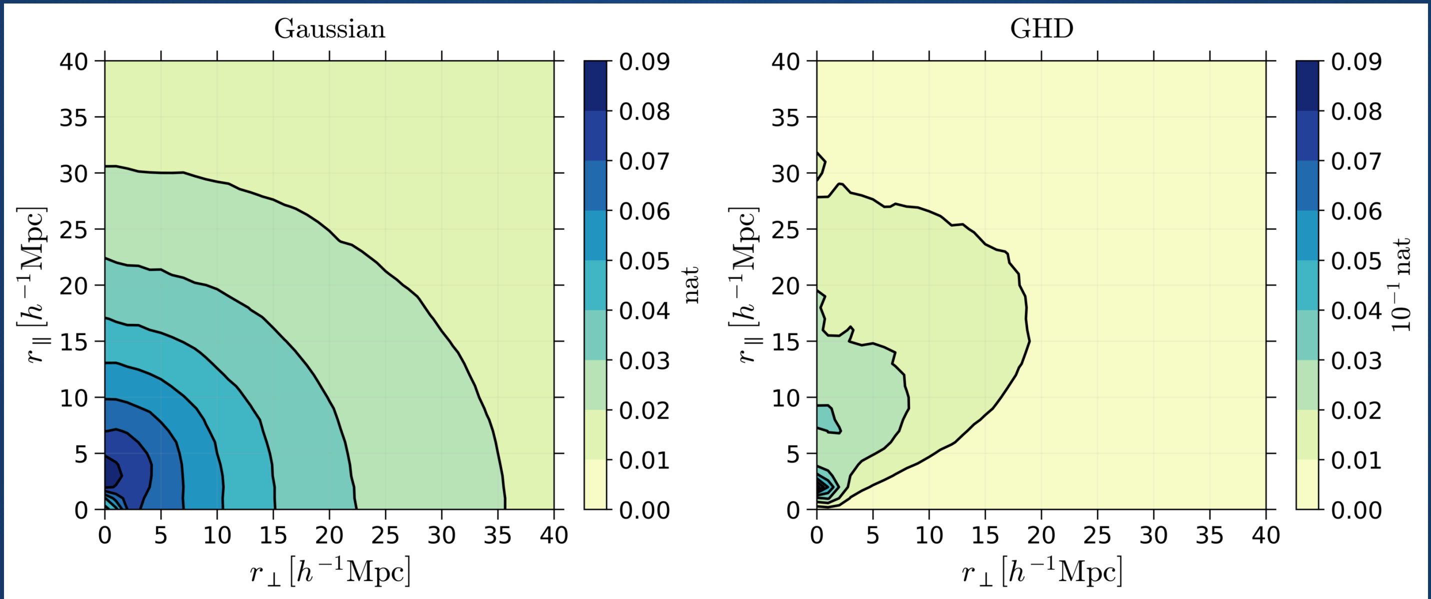
Generalised Hyperbolic Distribution



Kullback-Leibler Divergence

Quantifies the information loss by approximating the true distribution with a functional form

$$D_{\mathrm{KL}}(\mathcal{P}||\mathcal{Q}) = \sum_i \mathcal{P}(i) \log\left(\frac{\mathcal{P}(i)}{\mathcal{Q}(i)}\right)$$



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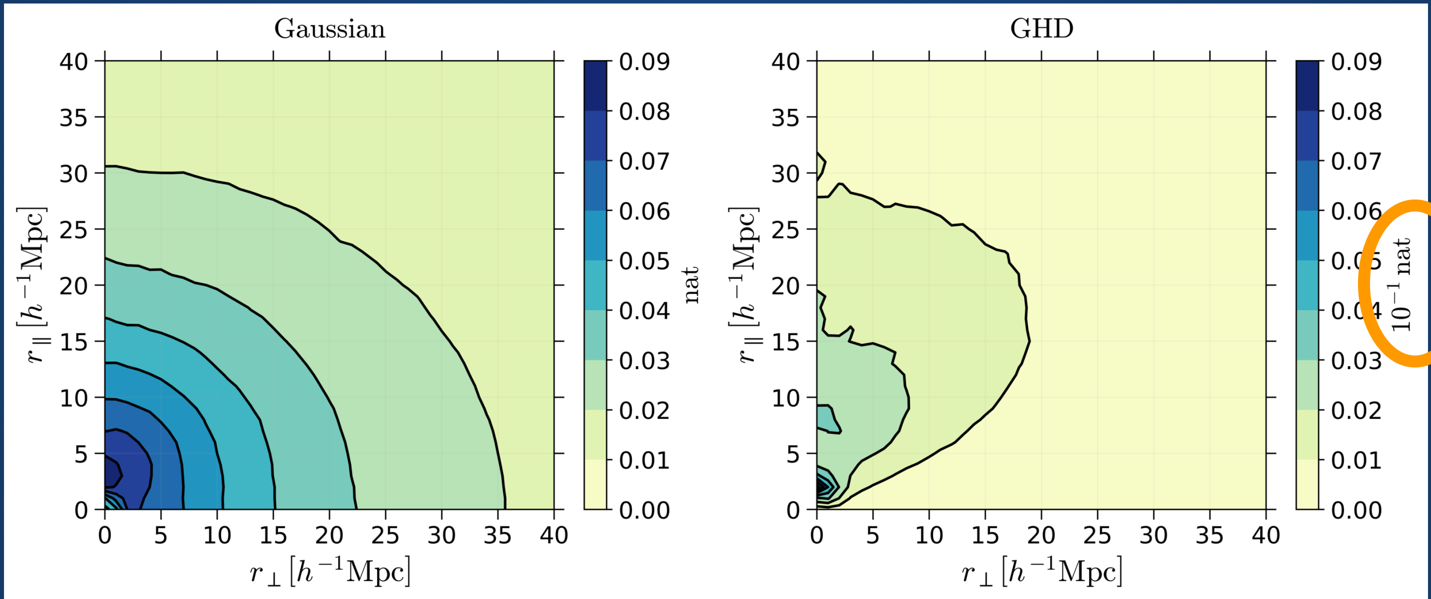
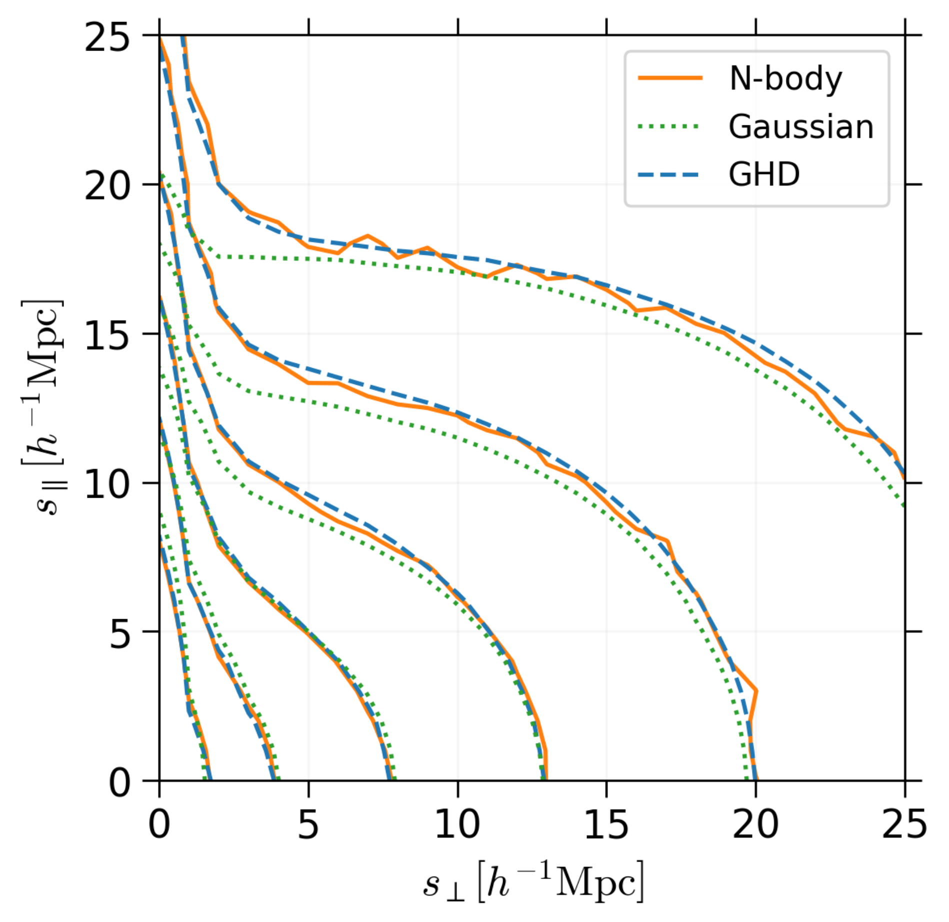


GHD is nearly a lossless approximation.

Generalised Hyperbolic Distribution



"With four parameters I can fit an elephant, and with five I can make him wiggle his trunk."

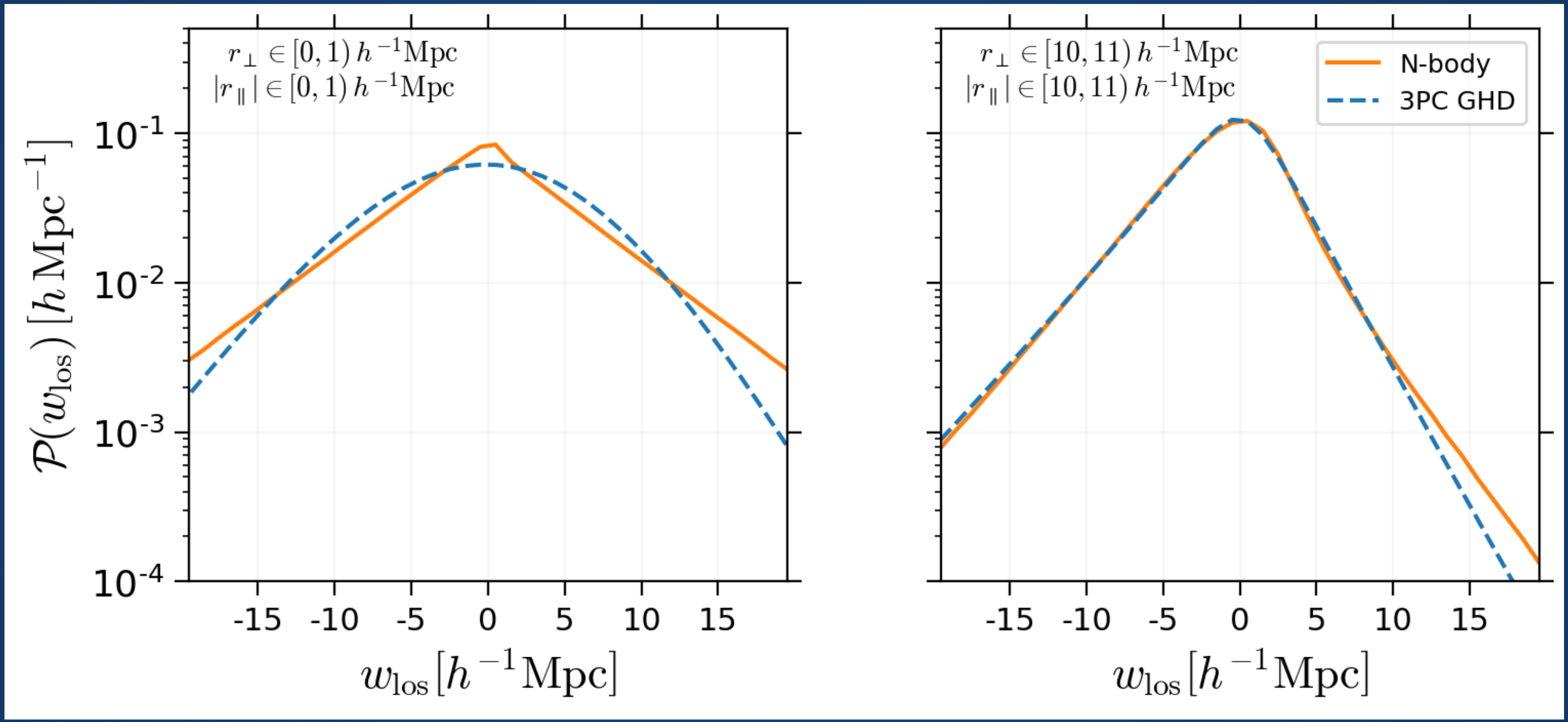
-- John von Neumann

"With four parameters I can fit an elephant, and with five I can make him wiggle his trunk."

-- John von Neumann

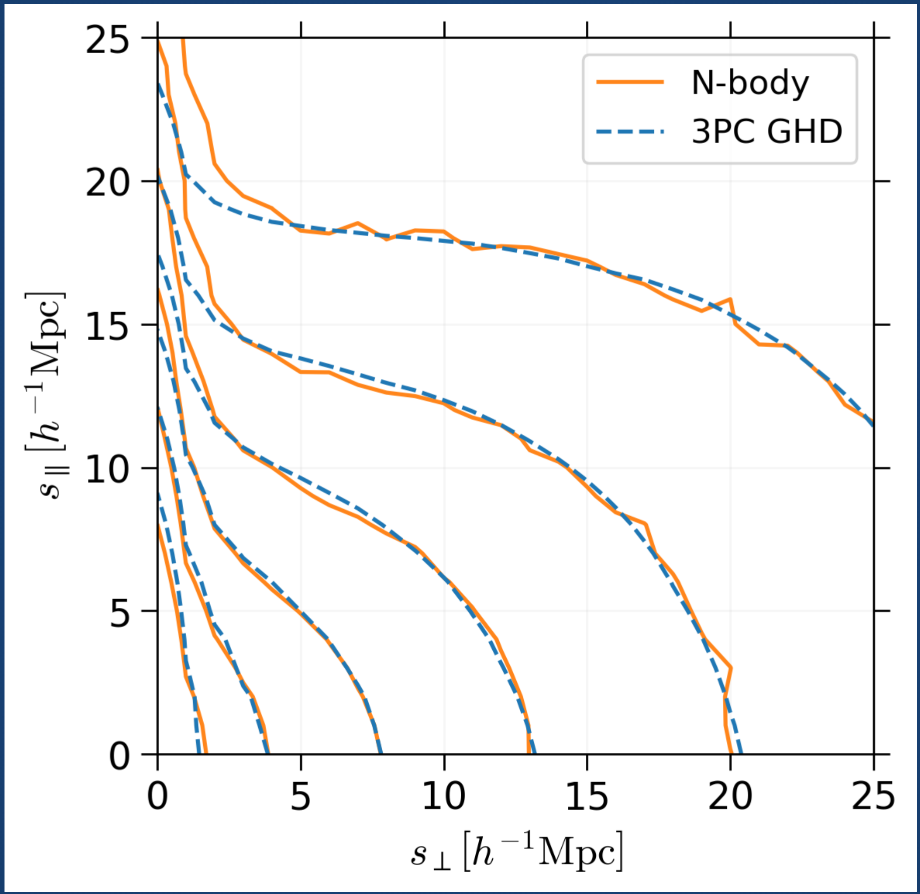
"Drawing an elephant with four complex parameters" by Jurgen Mayer et al. 09

Dimension Reduction - PCA

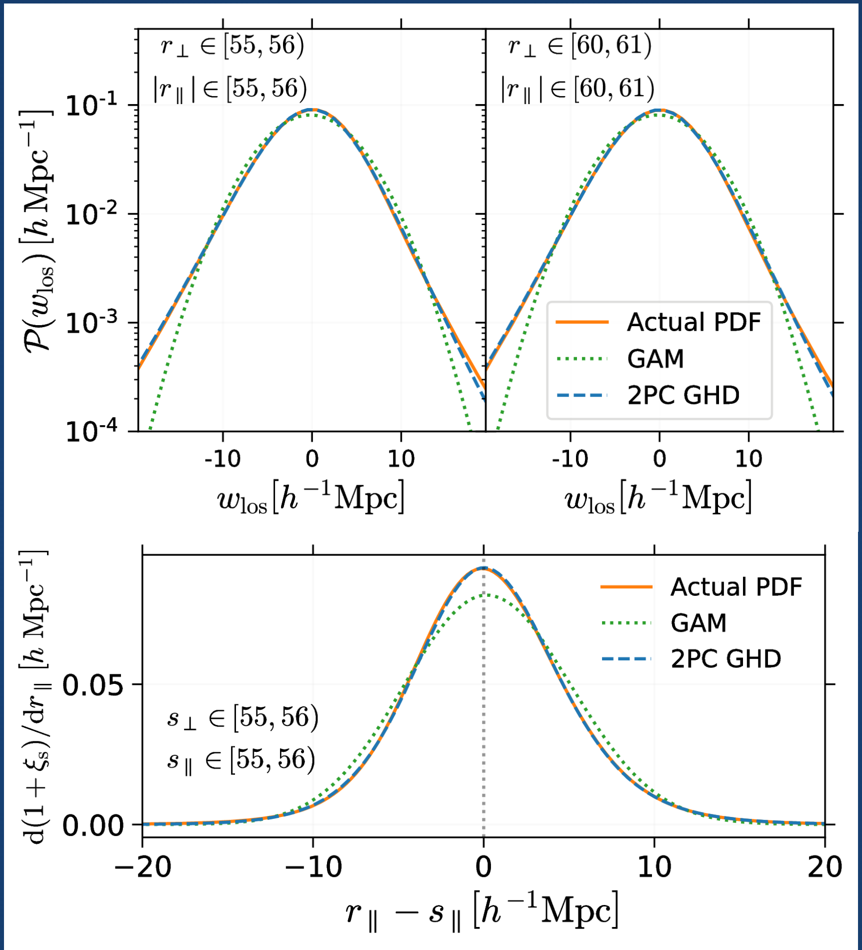


3 principal components

Dimension Reduction - PCA



Dimension Reduction - PCA



2 principal components of GHD at large scales.

Part III: Application

(Ongoing)

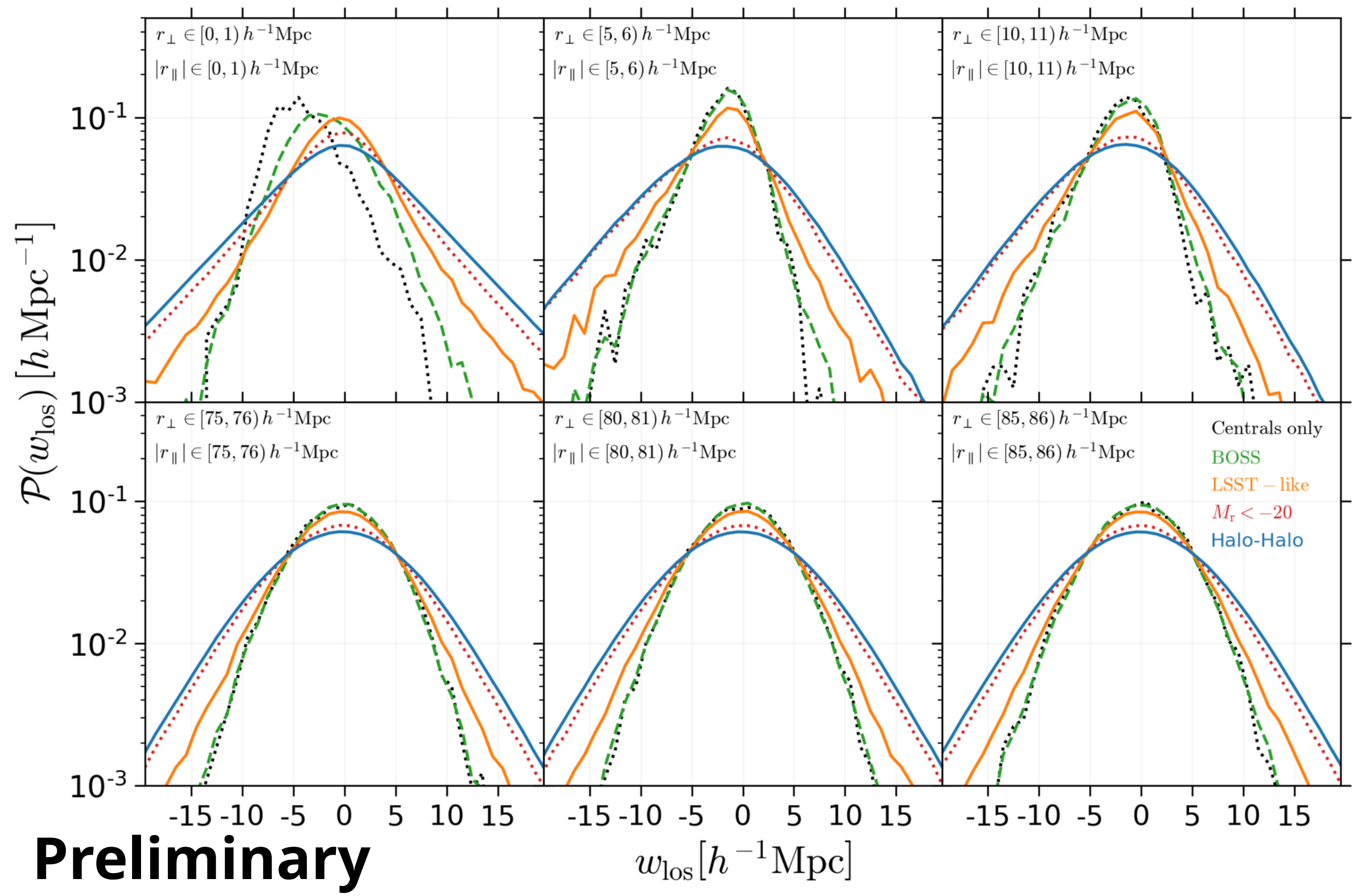
Centrals & Satellites

Our analysis for now focused on DM particles which showcases the extreme case of the pairwise distribution.

However in a galaxy redshift survey, we will be observing galaxies.

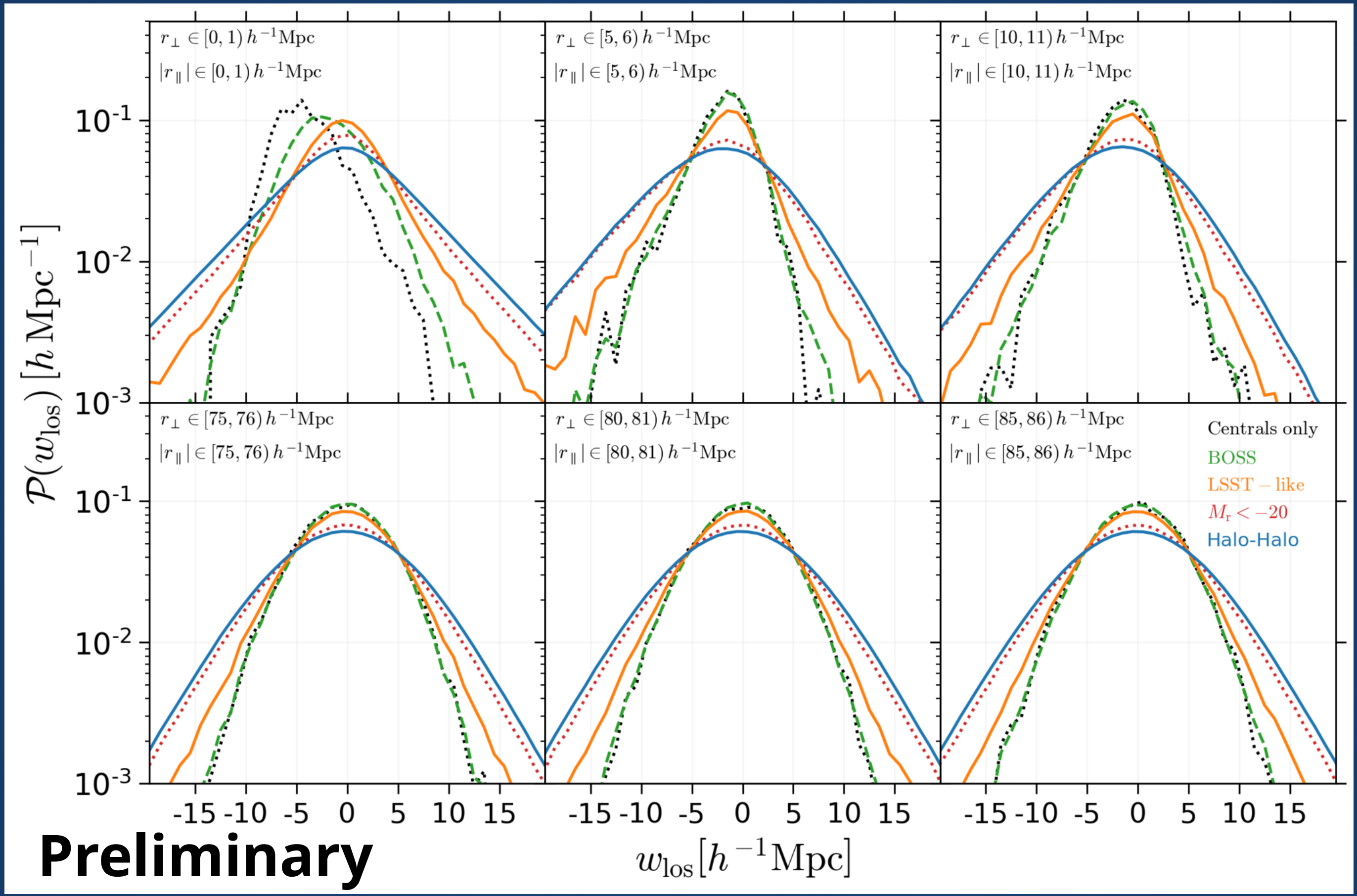
So for a galaxy sample, will Gaussian PDF be enough?

Centrals & Satellites



For future surveys, we will need better functional forms - answer is GHD?

Centrals & Satellites



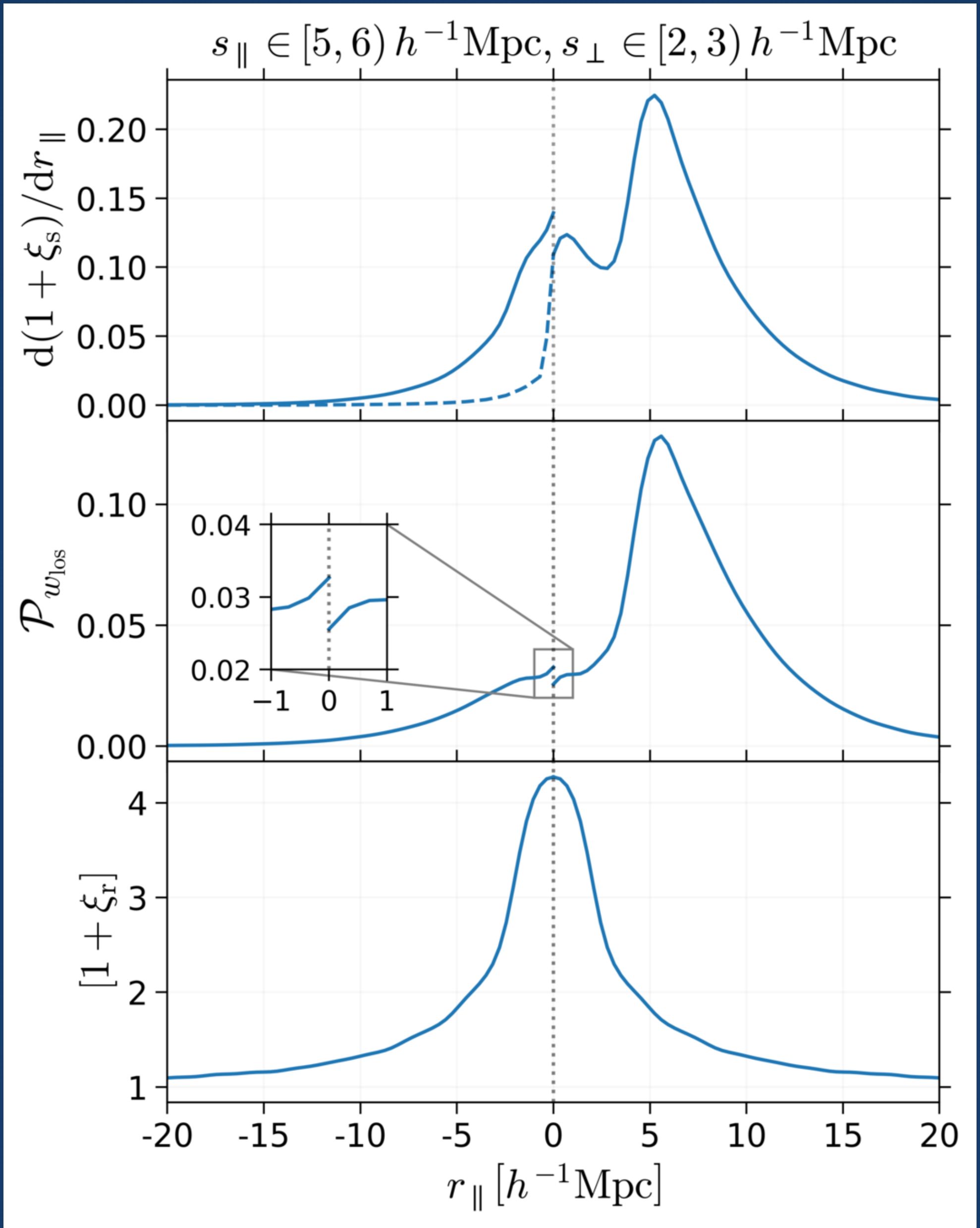
(Optimistically) It should be possible to reduce the GHD to 2 parameters for the galaxy sample, which will outperform the Gaussian approximation.

Conclusions

- Newly introduced GHD is a nearly lossless approximation to the pairwise distribution.
- GHD reproduces the redshift-space correlation function for DM particles accurately.
- Future surveys like Euclid, LSST requires better model than GSM to exploit them to their full potential.

Thank you for your attention.

Backup Slides


$s_\parallel \in [5,6)\,h^{-1}$Mpc, $s_\perp \in [2,3)\,h^{-1}$Mpc
$d(1+\xi_s)/dr_\parallel$
0.20
0.15
0.10
0.05
0.00
$\mathcal{P}_{w_{los}}$
0.10
0.05
0.00
0.04
0.03
0.02
−1
0
1
$[1+\xi_r]$
4
3
2
1
-20
-15
-10
-5
0
5
10
15
20
$r_\parallel\,[h^{-1}$Mpc]

Generalised Hyperbolic Distribution

GHD is a 5 parameter distribution

$$\mathcal{P}_{w_\parallel}(x;\alpha,\beta,\delta,\lambda,\mu) = C\left[\delta^2+(x-\mu)^2\right]^{\frac{\lambda-1/2}{2}} e^{\beta(x-\mu)}\, K_{\lambda-\frac{1}{2}}\left(\alpha\sqrt{\delta^2+(x-\mu)^2}\right)$$

where $C = \dfrac{\left(\alpha^2-\beta^2\right)^{\frac{\lambda}{2}}}{\sqrt{2\pi}\,\alpha^{\lambda-1/2}\,\delta^{\lambda}\,K_\lambda\left[\delta\sqrt{\alpha^2-\beta^2}\right]}$