

# RAR

(Radial Acceleration Relation):  
is it valid?

Chiara Di Paolo



Cosmology 2018

Dubrovnik, October 22<sup>nd</sup> 2018

C. Di Paolo & P. Salucci

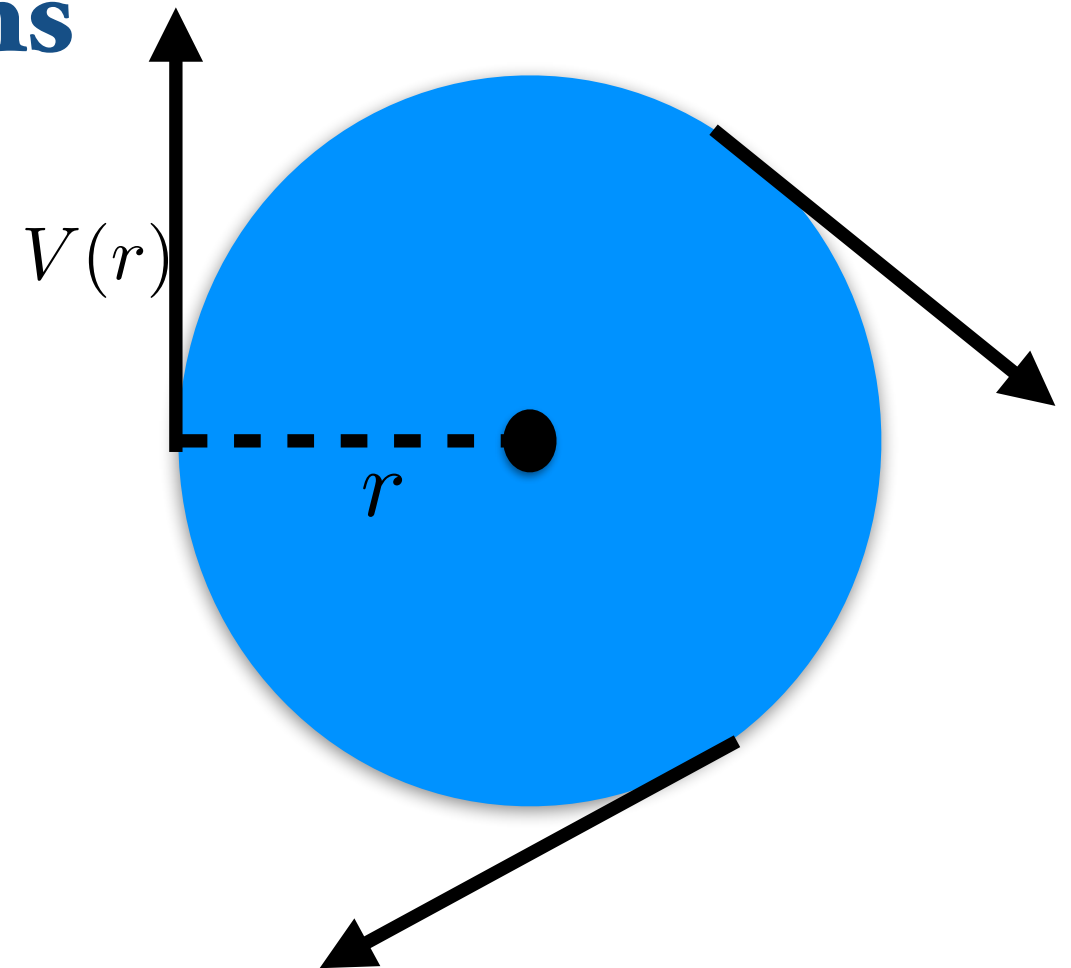
[arXiv:1810.08472](https://arxiv.org/abs/1810.08472)

# Gravitational acceleration

## Rotating systems



stellar disc  
bulge  
HI gaseous disc  
DM halo

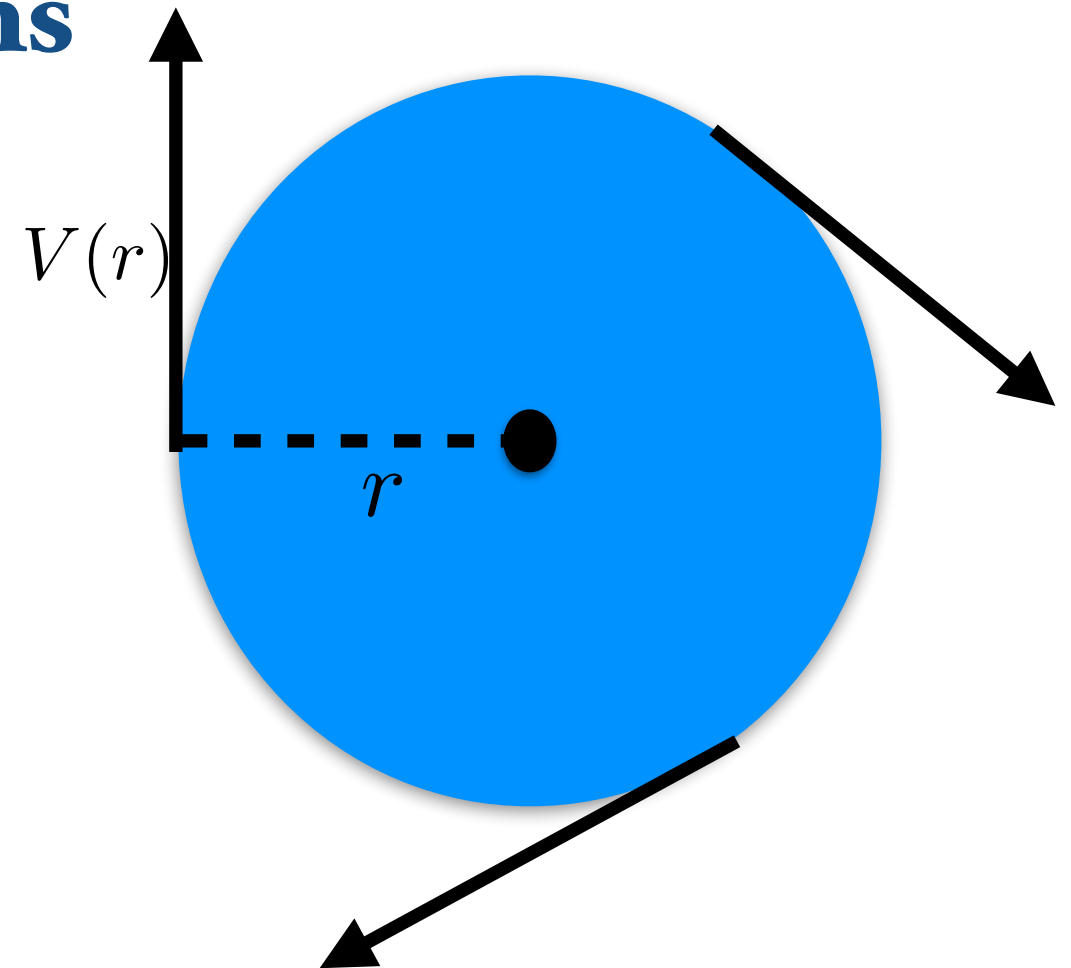


# Gravitational acceleration

## Rotating systems



stellar disc  
bulge  
HI gaseous disc  
DM halo



Gravitational (radial) acceleration :

$$g(r) = \frac{V^2(r)}{r} = \left| -\frac{d\Phi}{dr} \right| = G \frac{M(r)}{r^2} \quad \left\{ M(r) = \underbrace{M_d(r)} + M_{bu}(r) + M_{HI}(r) + \underbrace{M_h(r)} \right.$$

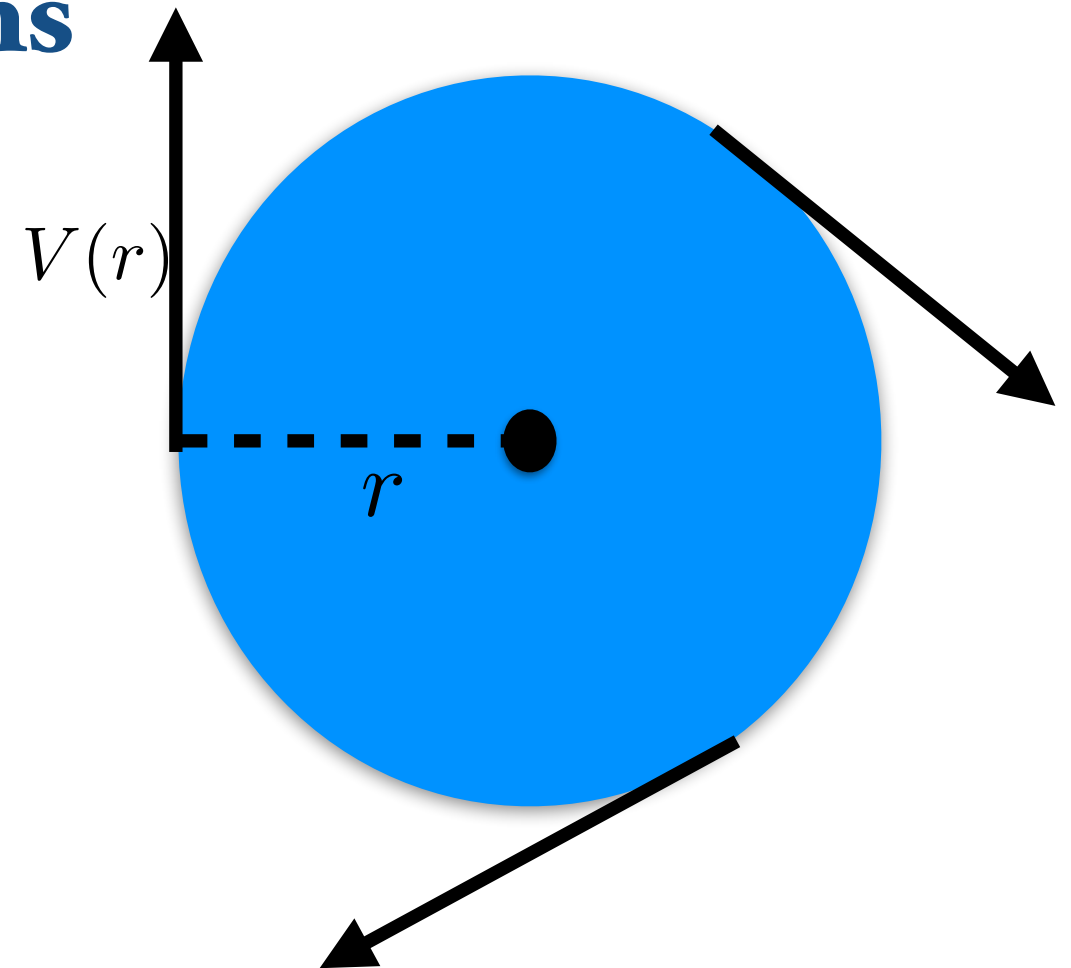
DM halo

# Gravitational acceleration

## Rotating systems



stellar disc  
 bulge  
 HI gaseous disc  
 DM halo



Gravitational (radial) acceleration :

$$g(r) = \frac{V^2(r)}{r} = \left| -\frac{d\Phi}{dr} \right| = G \frac{M(r)}{r^2} \quad \left\{ M(r) = \underbrace{M_d(r) + M_{bu}(r) + M_{HI}(r)}_{\text{Baryonic component}} + \underbrace{M_h(r)}_{\text{DM halo}} \right.$$

Baryonic component :

$$g_b(r) = \frac{V_b^2(r)}{r} = \left| -\frac{d\Phi_b}{dr} \right| = G \frac{M_b(r)}{r^2} \quad \left\{ \begin{array}{l} M_b(r) = \underbrace{M_d(r) + M_{bu}(r) + M_{HI}(r)}_{\text{Baryonic component}} \\ V_b^2(r) = V_d^2(r) + V_{bu}^2(r) + V_{HI}^2(r) \end{array} \right.$$

DM halo



# RAR: Gravitational acceleration relation

McGaugh relation between  
gravitational acceleration  $g(r)$   
and  
baryonic acceleration  $g_b(r)$

153 rotationally supported galaxies  
from SPARC sample,  
2963 circular velocity measurements

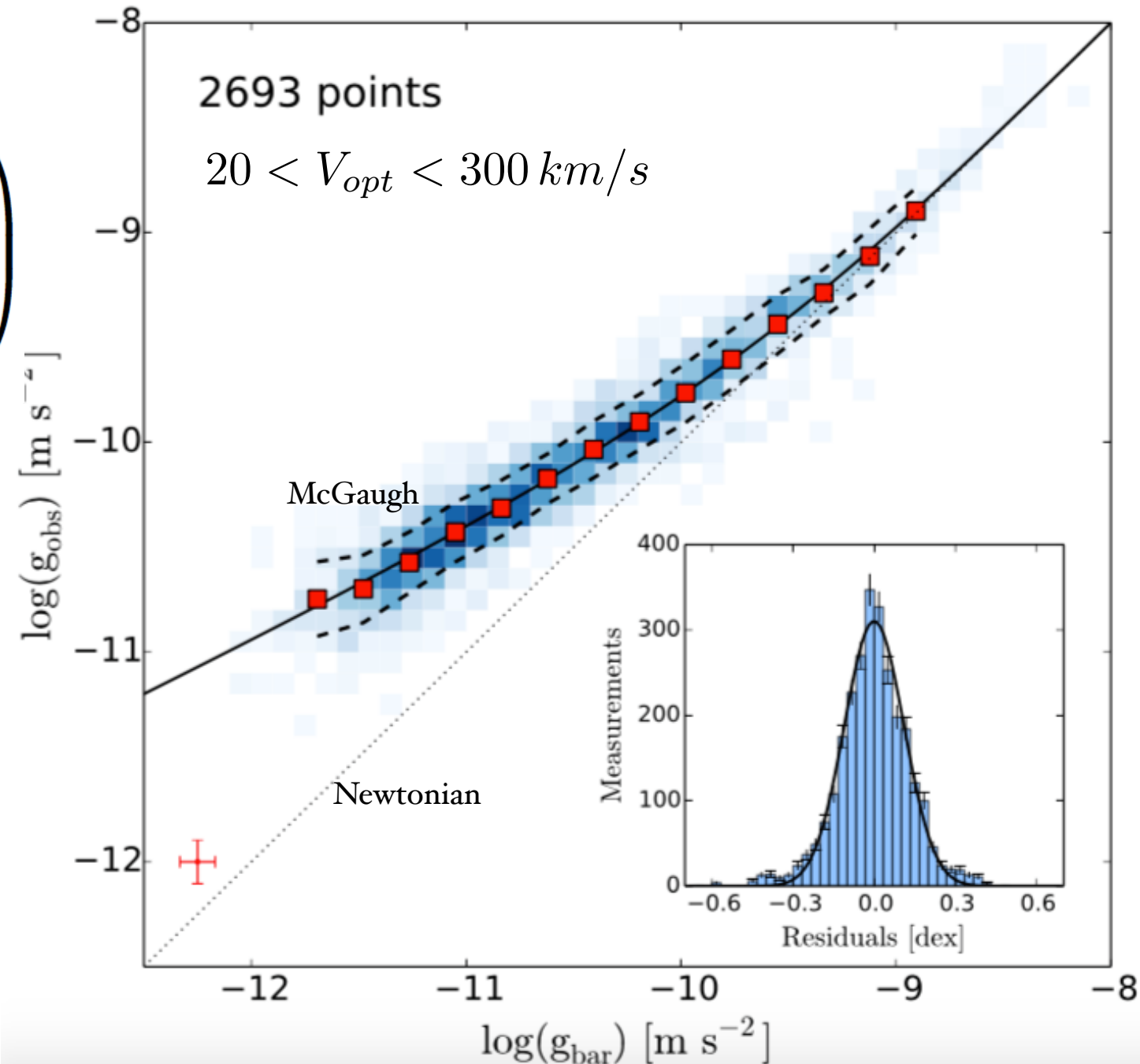
# RAR: Gravitational acceleration relation

McGaugh relation between gravitational acceleration  $g(r)$  and baryonic acceleration  $g_b(r)$

$$\text{Log } g(r) = \text{Log} \left( \frac{g_b(r)}{1 - \exp\left(-\sqrt{\frac{g_b(r)}{g_{\dagger}}}\right)} \right)$$

$$g_{\dagger} = 1.2 \times 10^{-10} \text{ m s}^{-2}$$

153 rotationally supported galaxies from SPARC sample, 2963 circular velocity measurements



McGaugh et al. (2016)

# RAR: Gravitational acceleration relation

McGaugh relation between  
gravitational acceleration  $g(r)$   
and  
baryonic acceleration  $g_b(r)$

$$\text{Log } g(r) = \text{Log} \left( \frac{g_b(r)}{1 - \exp\left(-\sqrt{\frac{g_b(r)}{g_{\dagger}}}\right)} \right)$$

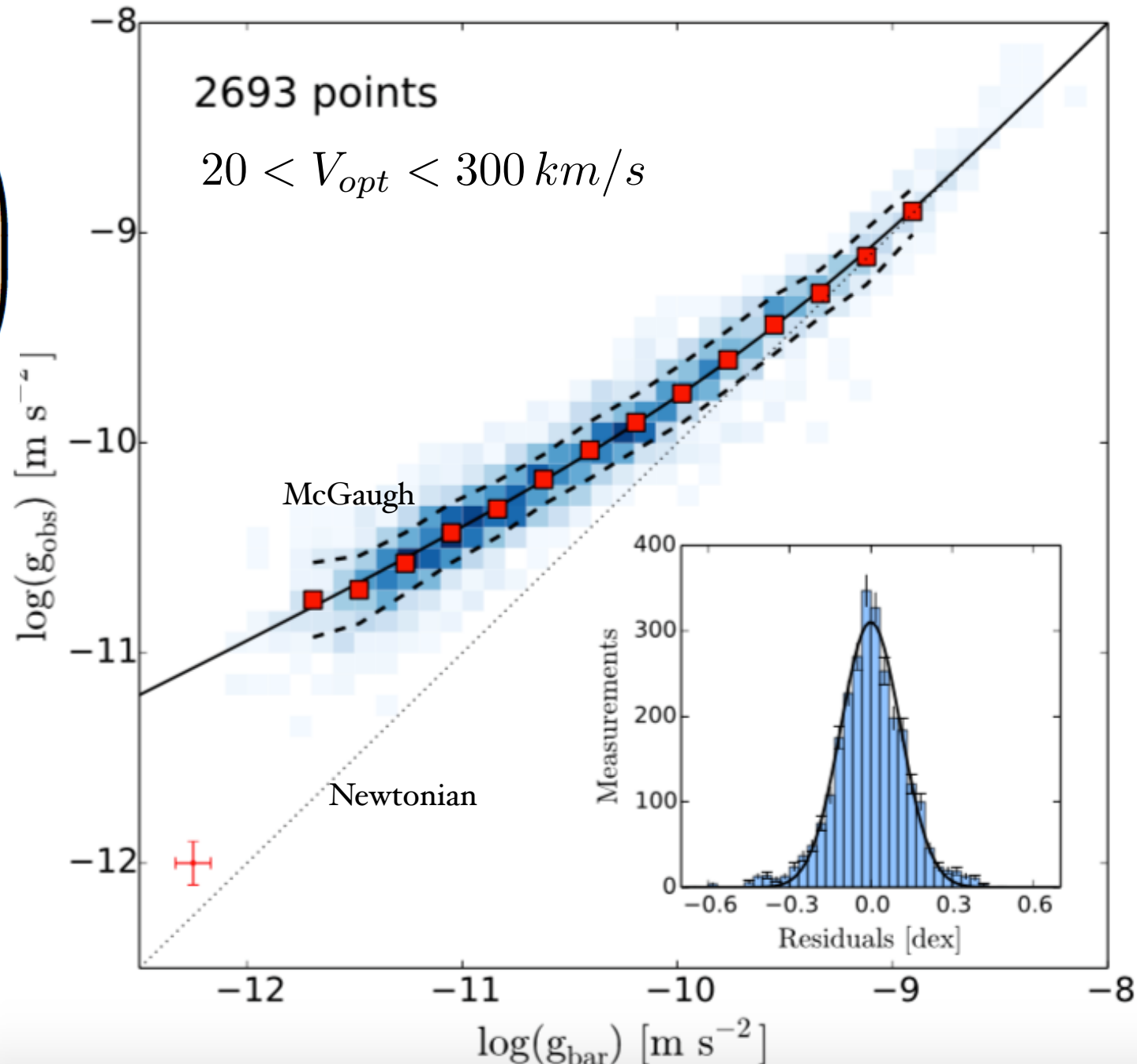
$$g_{\dagger} = 1.2 \times 10^{-10} \text{ m s}^{-2}$$

empirical **universal relation**

$$g = f(g_b)$$

at any radius  
and  
in any object

153 rotationally supported galaxies  
from SPARC sample,  
2963 circular velocity measurements



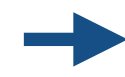
McGaugh et al. (2016)

# RAR: Gravitational acceleration relation

Salucci P.



1000 **normal Spirals**,  
25000 circular velocity measurements



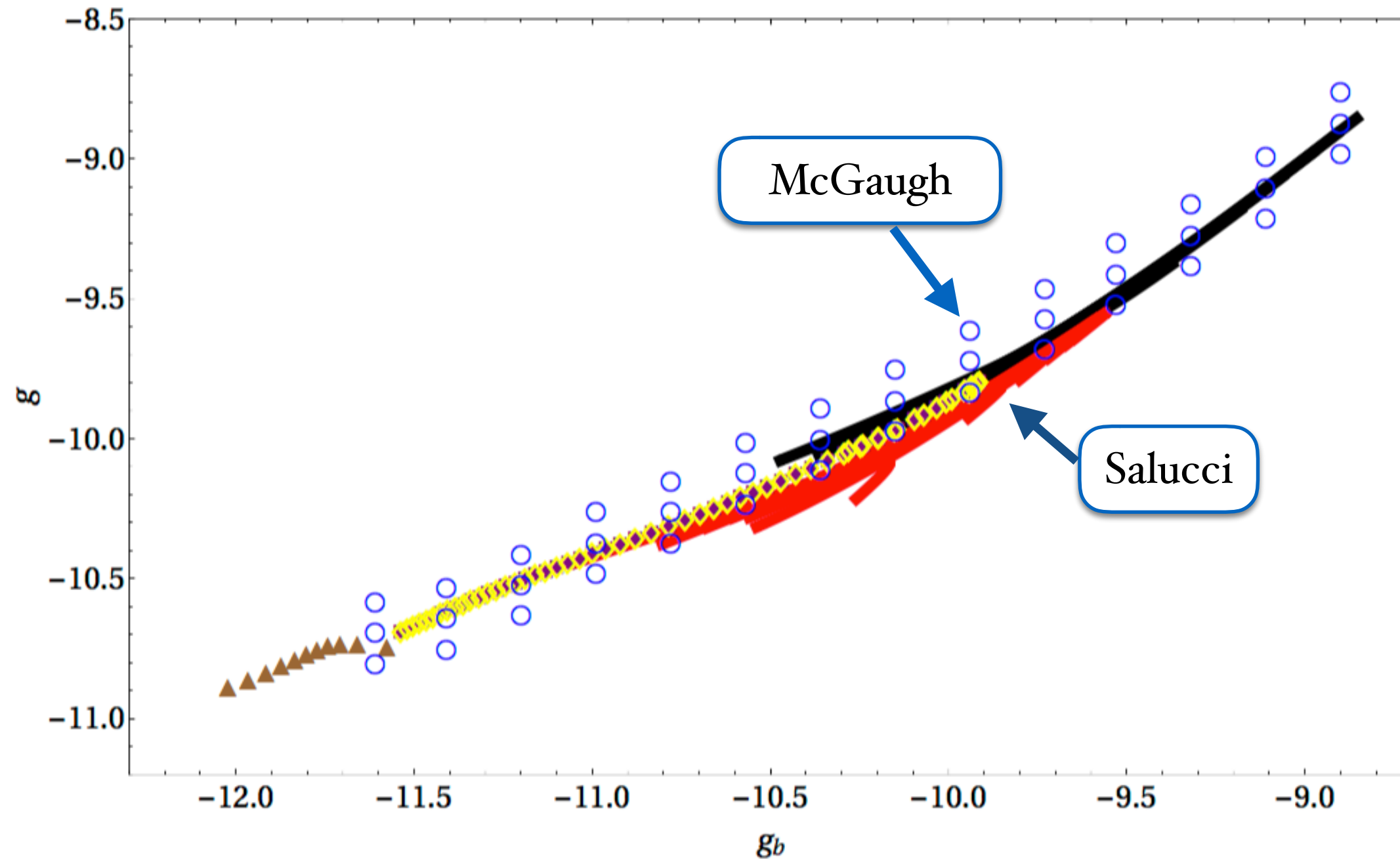
$$70 < V_{opt} < 300 \text{ km/s}$$

# RAR: Gravitational acceleration relation

Salucci P.

1000 **normal Spirals**,  
25000 circular velocity measurements

$70 < V_{opt} < 300 \text{ km/s}$



Salucci  
&  
McGaugh  
data



agreement

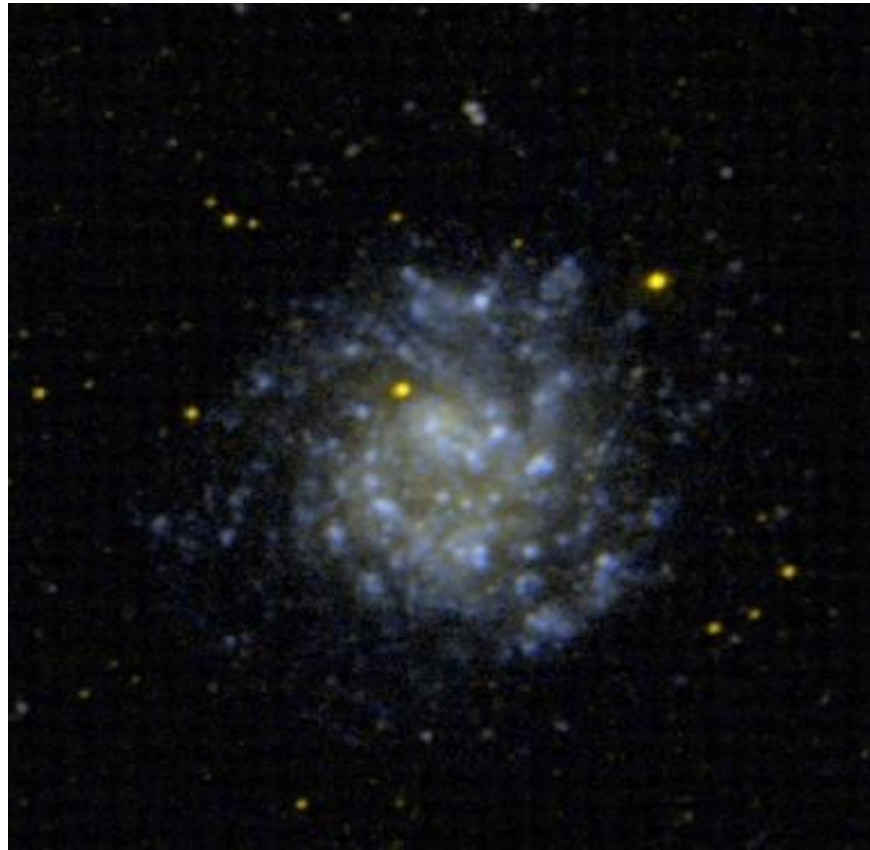
Salucci (2018)

# RAR: Gravitational acceleration relation

36 **dwarf disk galaxies** (Karukes & Salucci)

303 circular velocity measurements

→  $19 < V_{opt} < 61 \text{ km/s}$



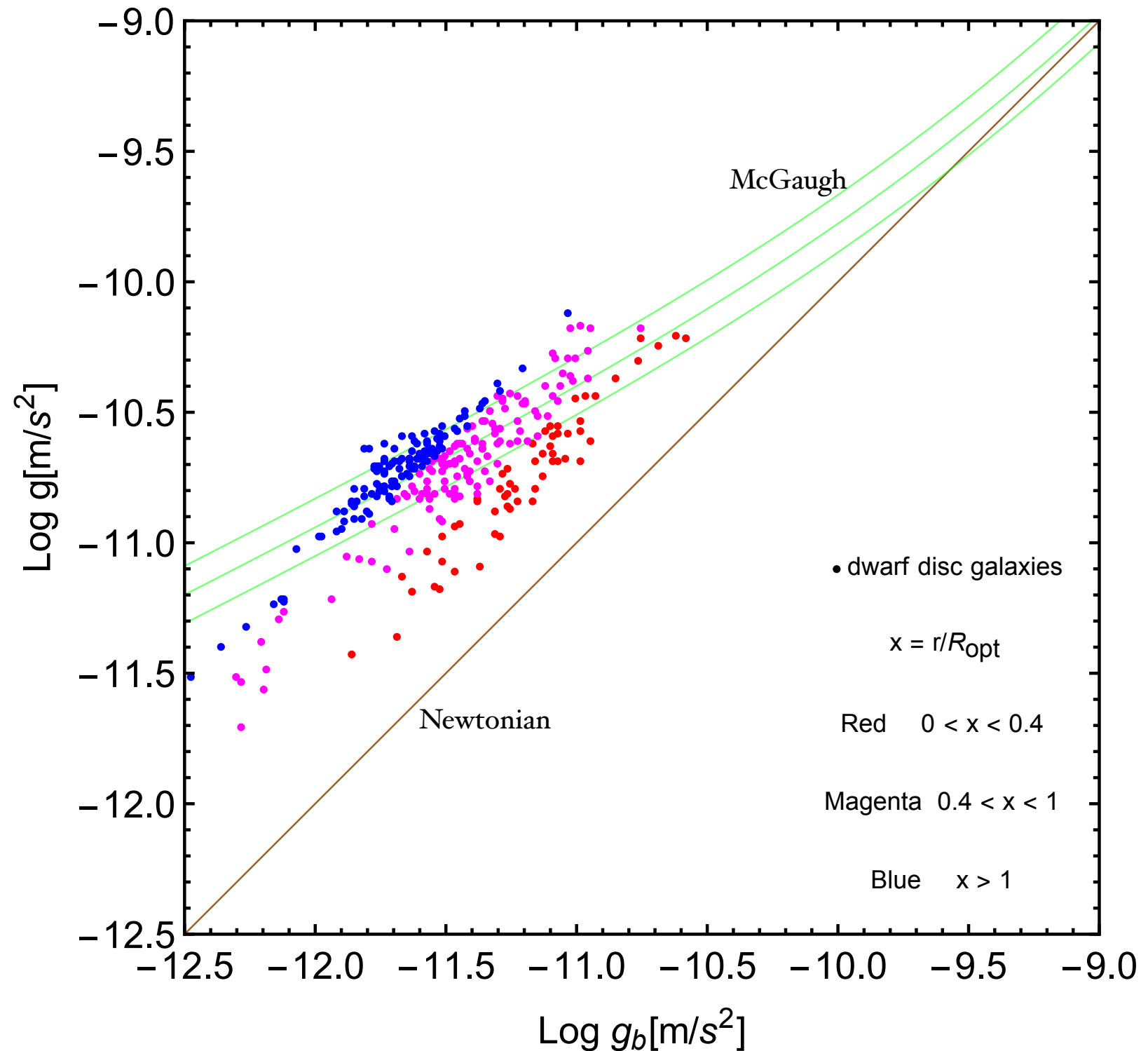
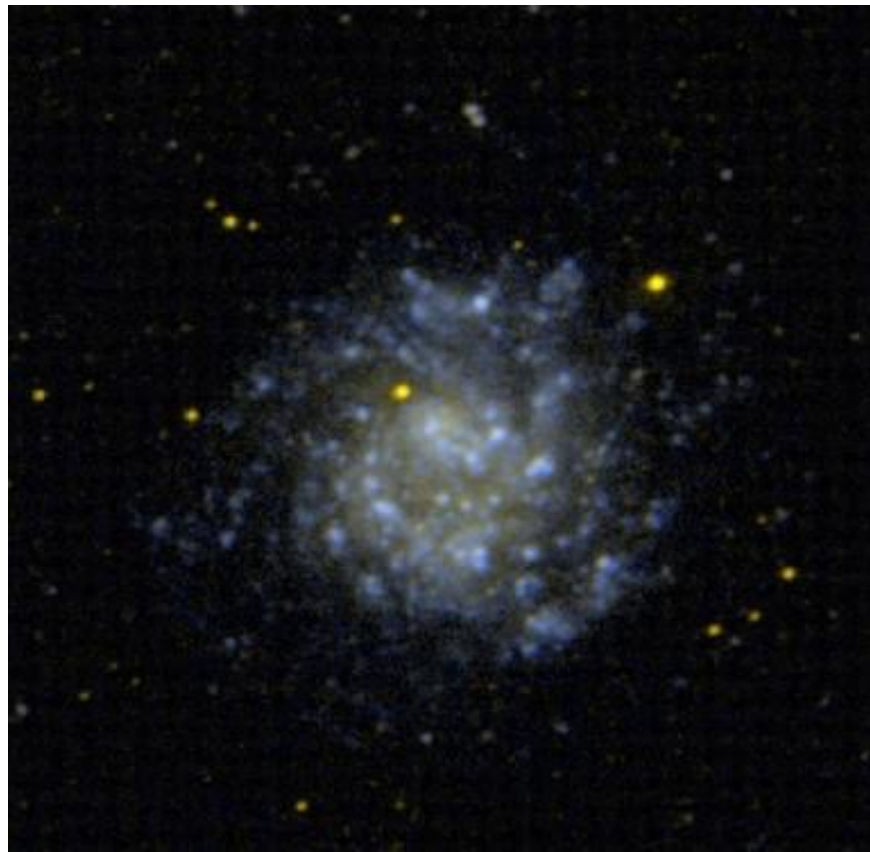


# RAR: Gravitational acceleration relation

36 **dwarf disk galaxies** (Karukes & Salucci)

303 circular velocity measurements

→  $19 < V_{opt} < 61 \text{ km/s}$

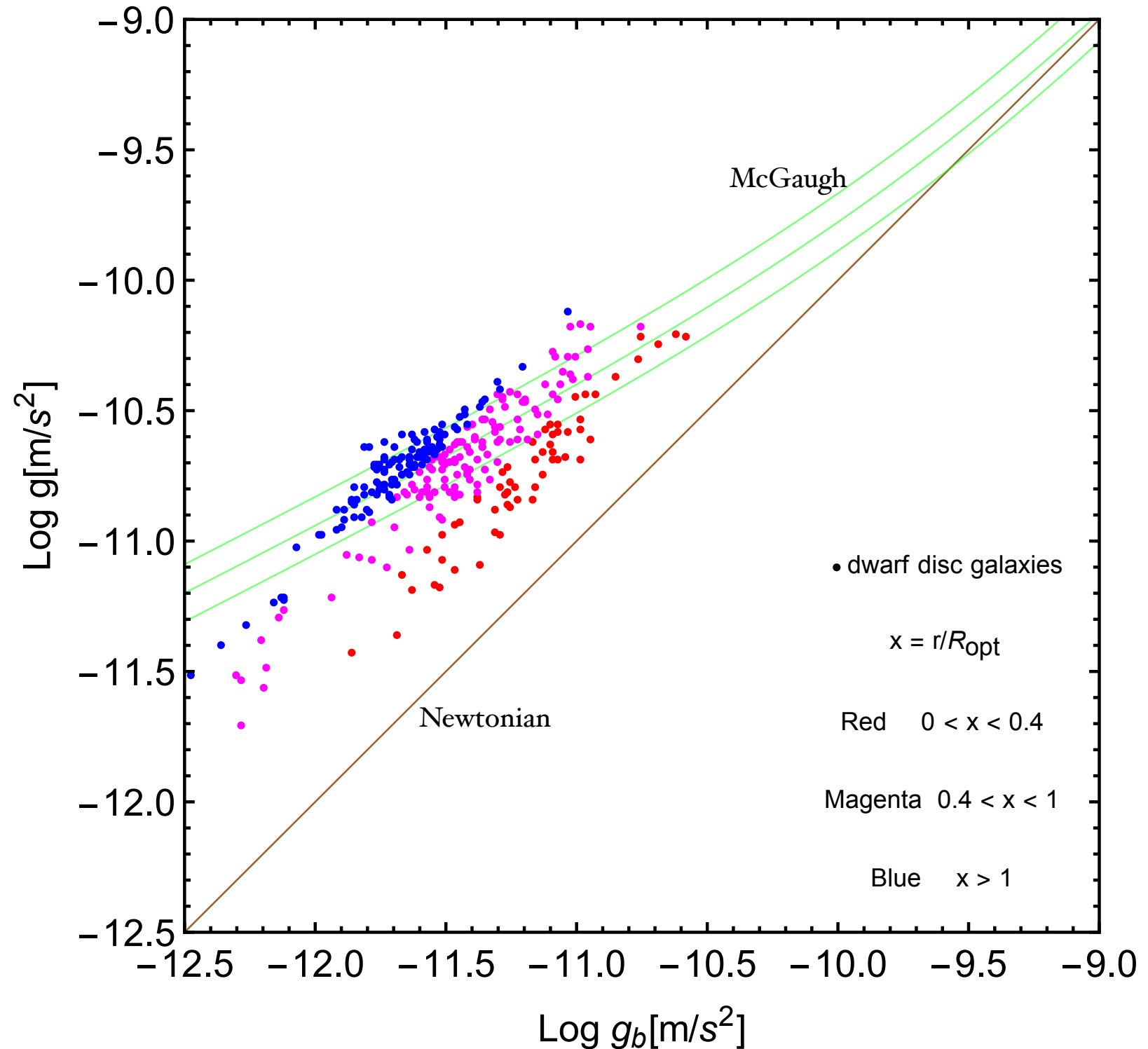
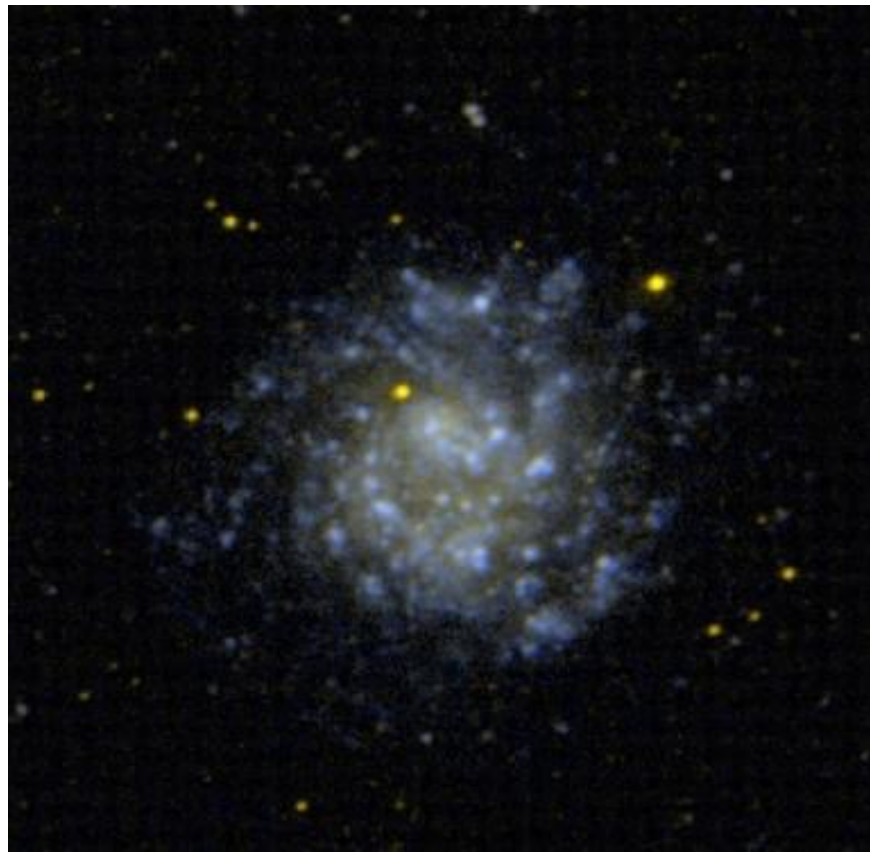


# RAR: Gravitational acceleration relation

36 **dwarf disk galaxies** (Karukes & Salucci)

303 circular velocity measurements

→  $19 < V_{opt} < 61 \text{ km/s}$



a) McGaugh relation  
breaks down

b) radial dependence



# RAR: Gravitational acceleration relation

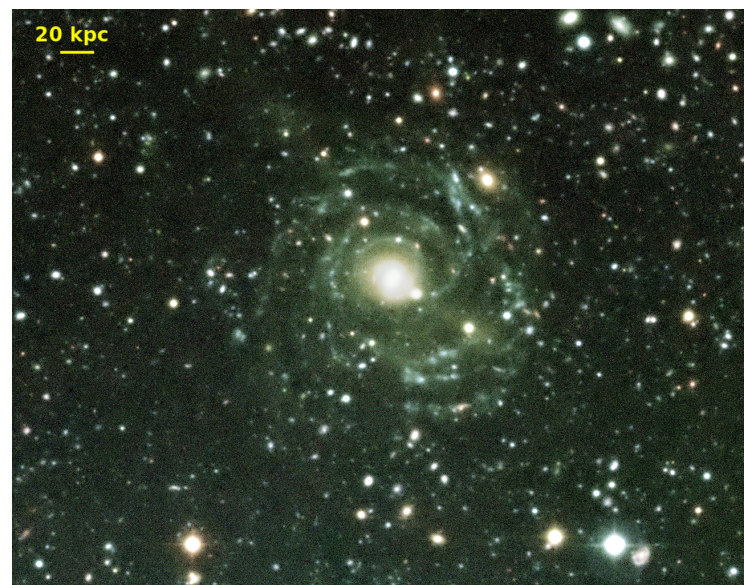
72 **Low Surface Brightness** galaxies

(Di Paolo & Salucci)

1601 circular velocity measurements

$$24 < V_{opt} < 300 \text{ km/s}$$

emit much less  
light per area  
than normal galaxies



# RAR: Gravitational acceleration relation

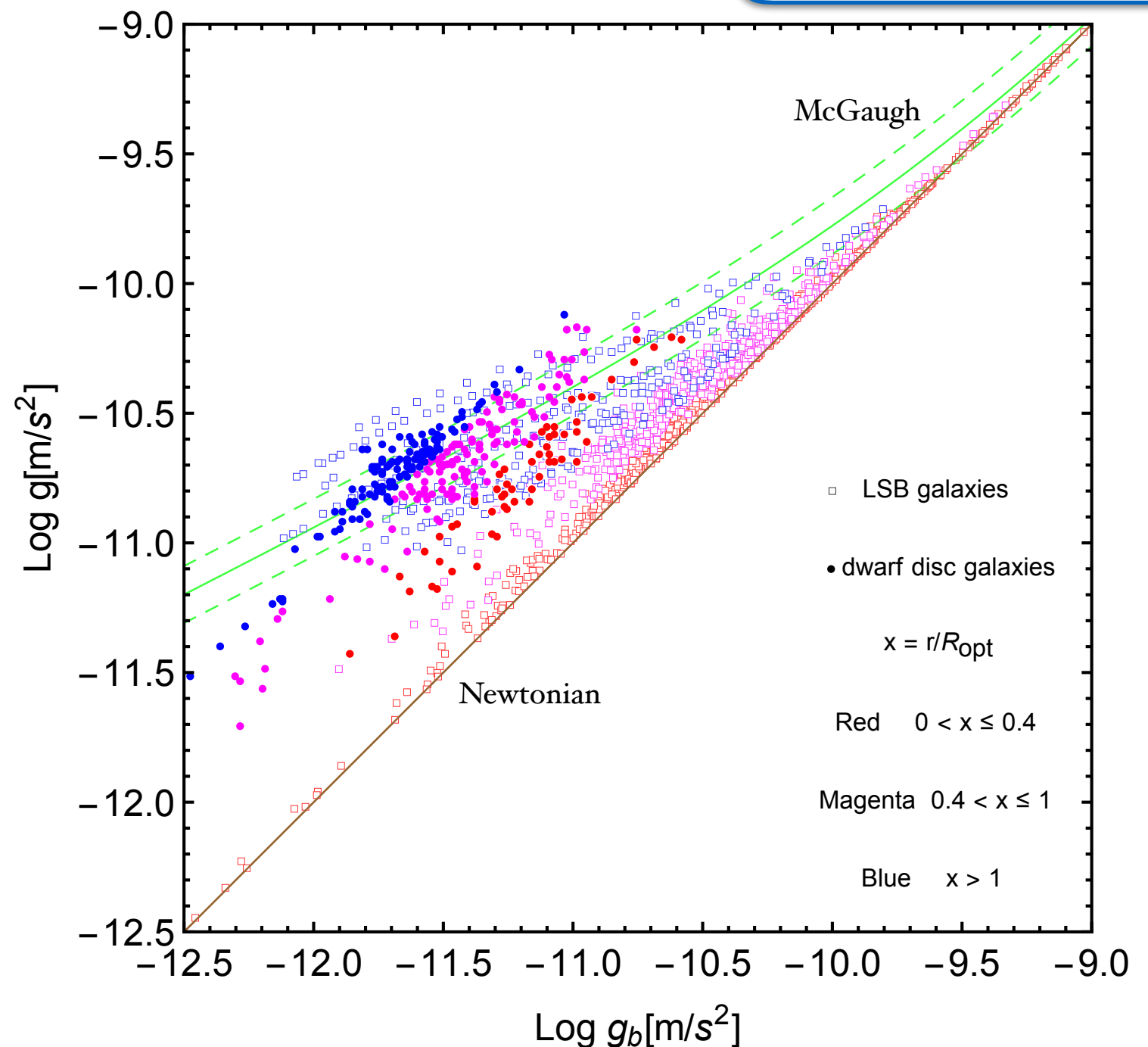
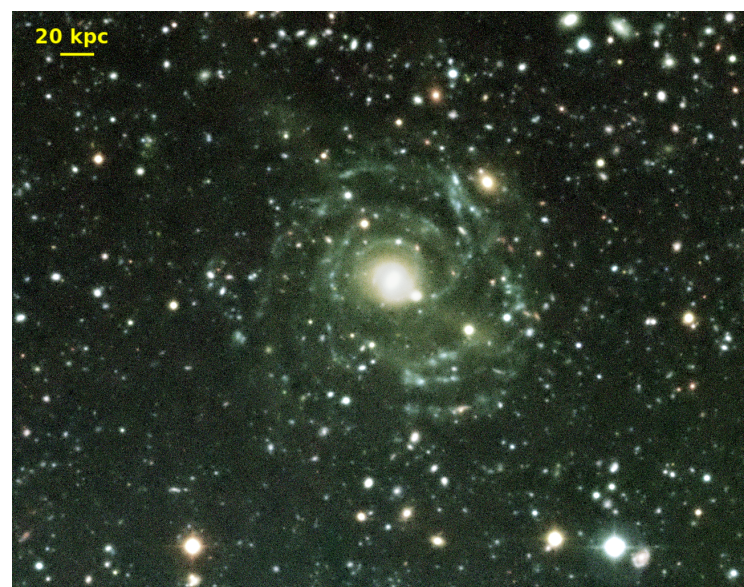
72 **Low Surface Brightness** galaxies

(Di Paolo & Salucci)

1601 circular velocity measurements

$$24 < V_{opt} < 300 \text{ km/s}$$

emit much less  
light per area  
than normal galaxies



# RAR: Gravitational acceleration relation

a) McGaugh relation  
breaks down

b) radial dependence



*Why?*



# RAR: Gravitational acceleration relation

a) McGaugh relation breaks down

b) radial dependence

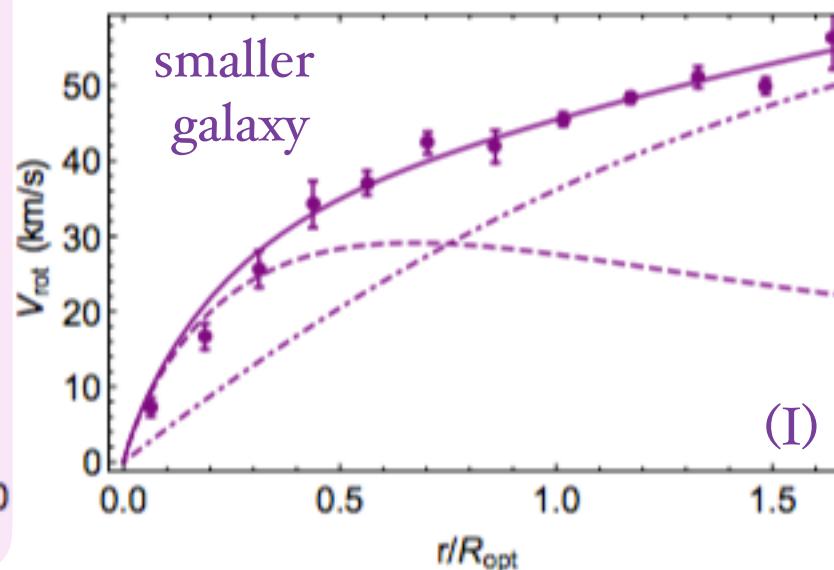
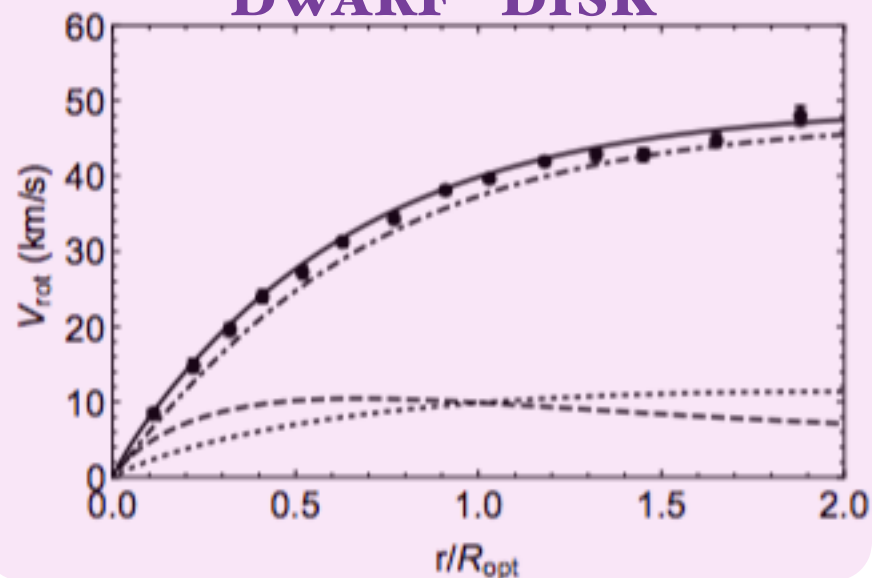


## Why?

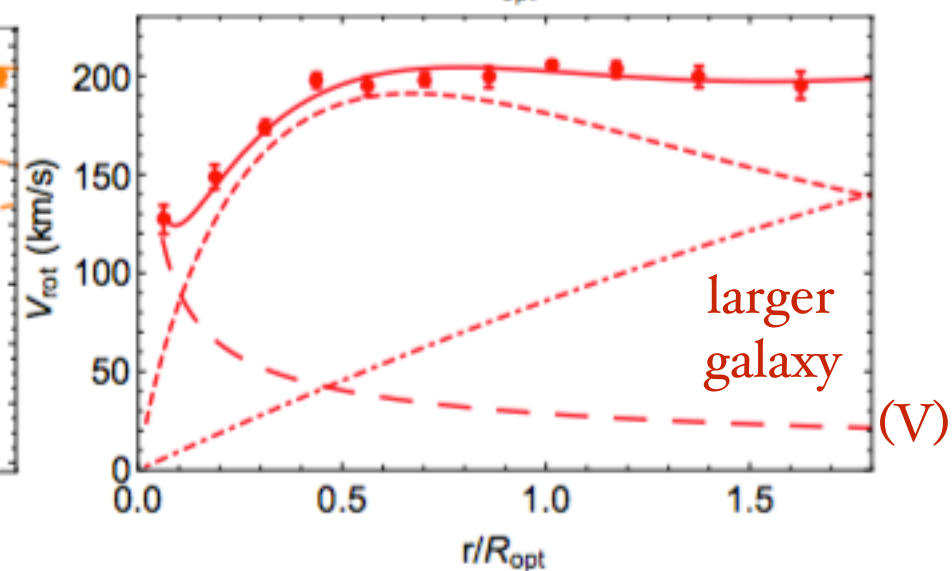
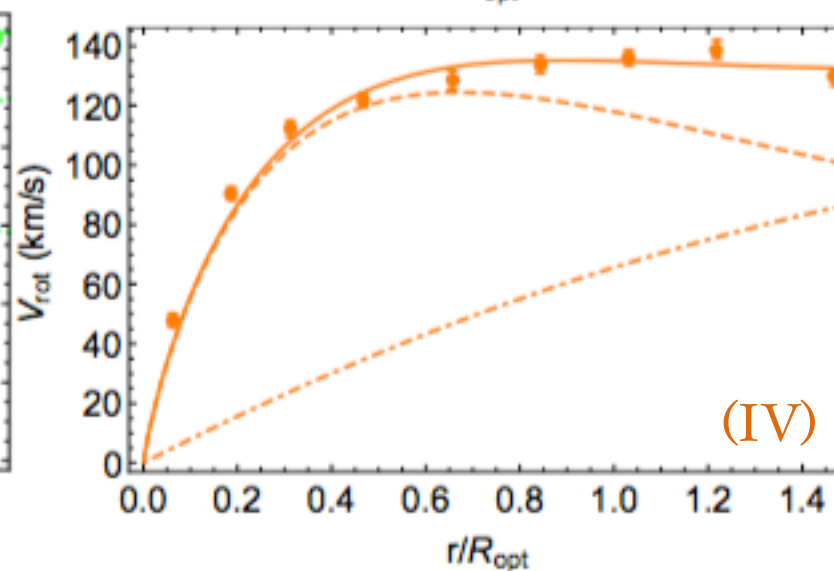
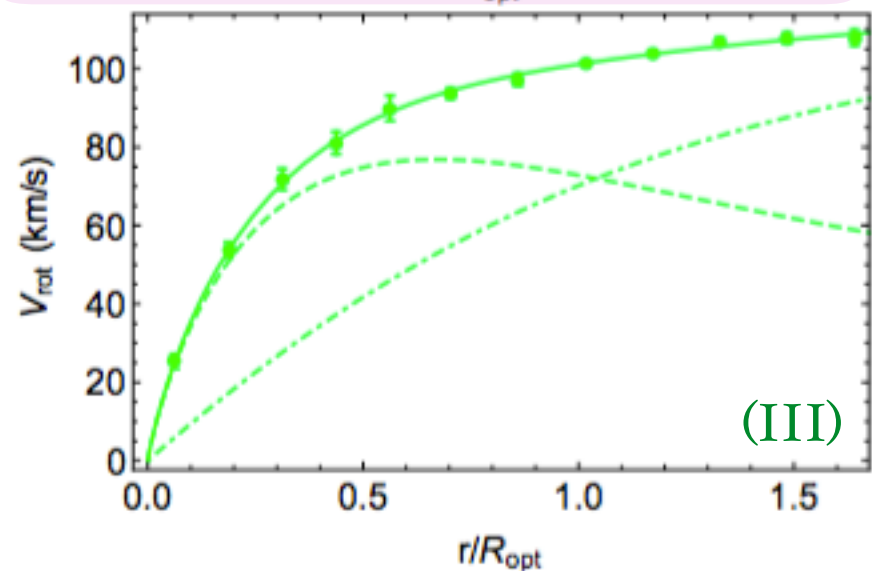
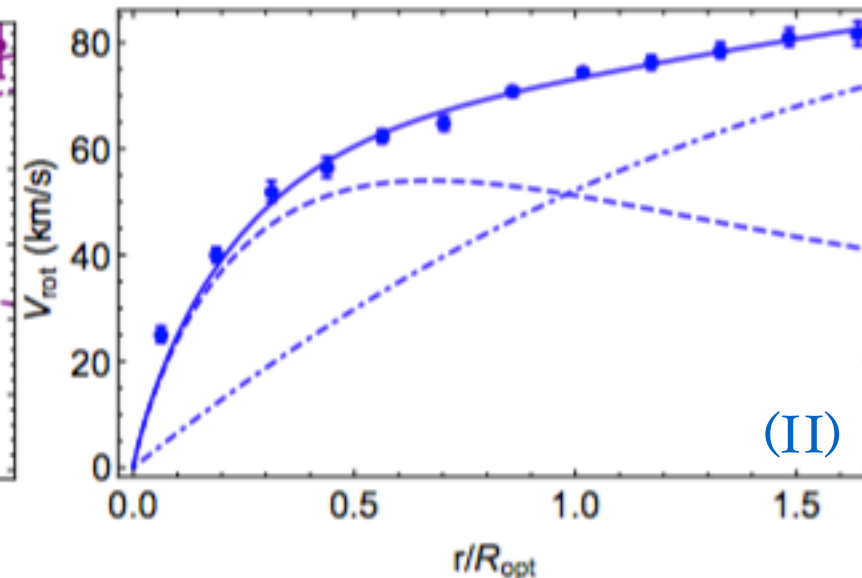
A step back

## Rotation Curves of Galaxies

### DWARF DISK



### LSB



# RAR: Gravitational acceleration relation

## Galaxy components :

- Freeman stellar disc
- Burkert (cored) DM halo
- HI gaseous disk (Dwarf Disks)
- Bulge (larger LSB galaxies)

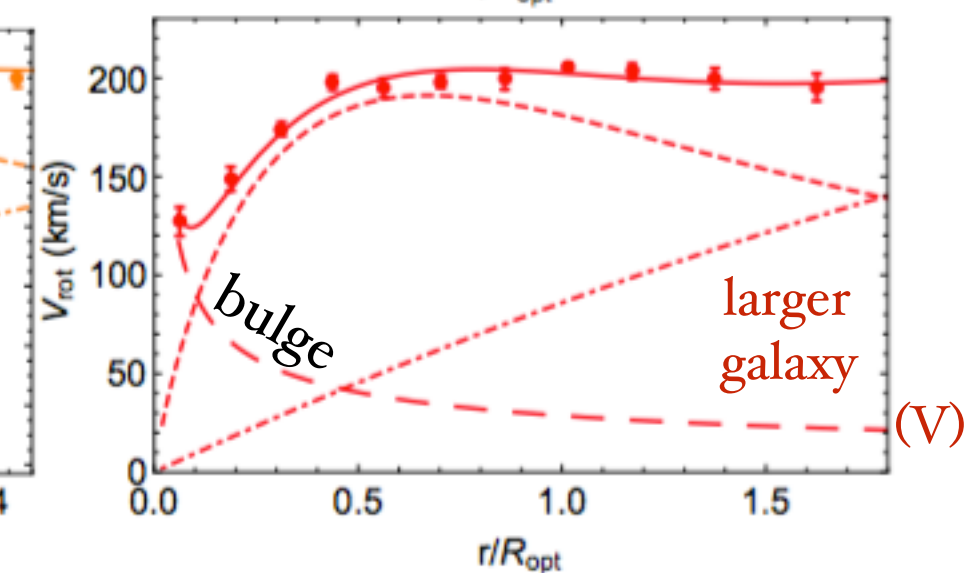
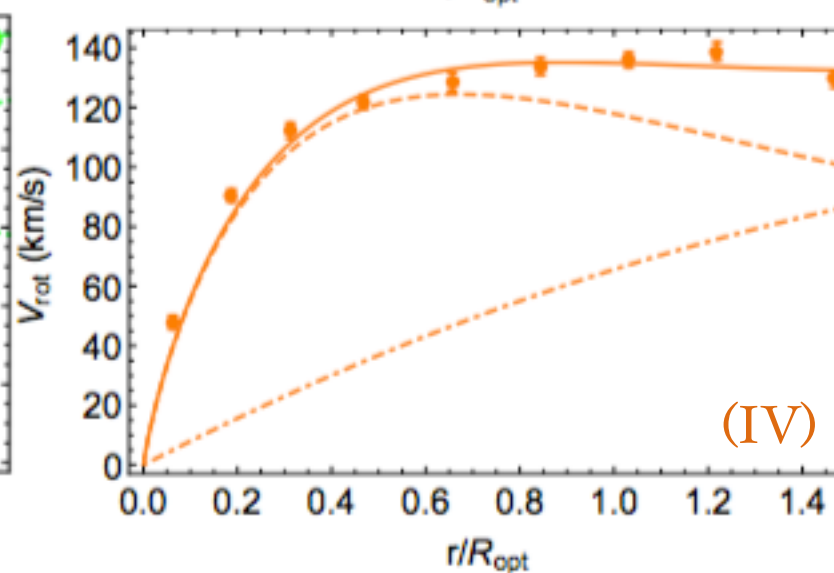
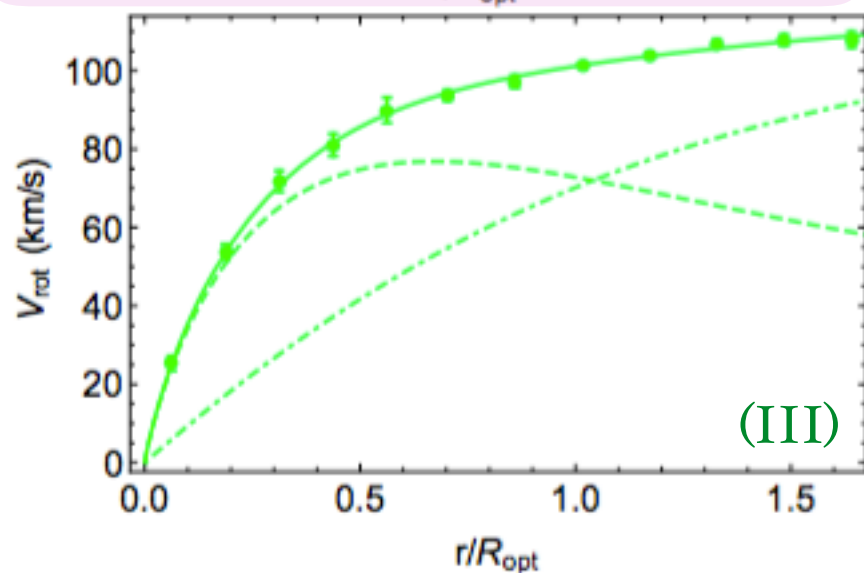
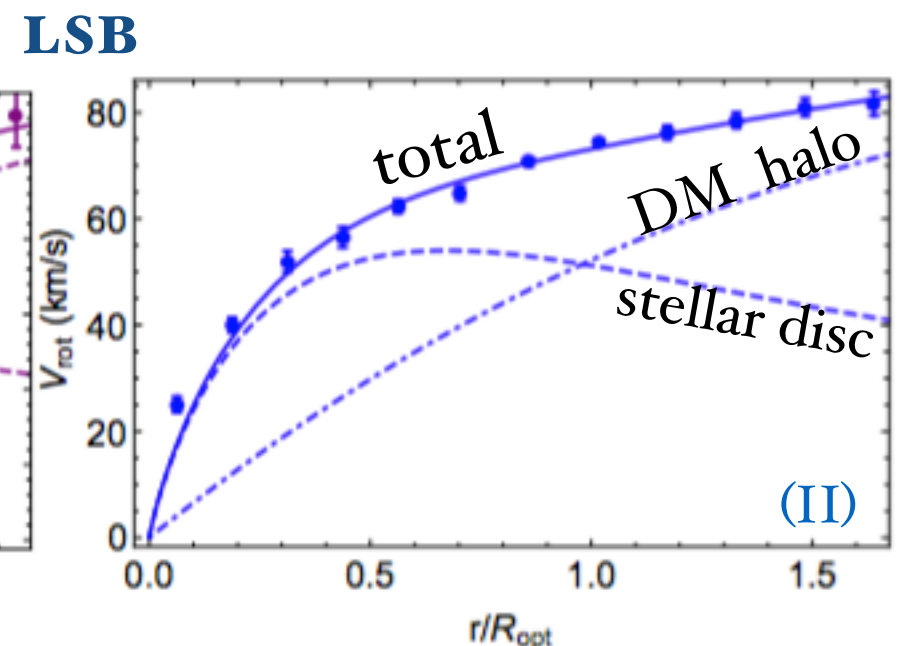
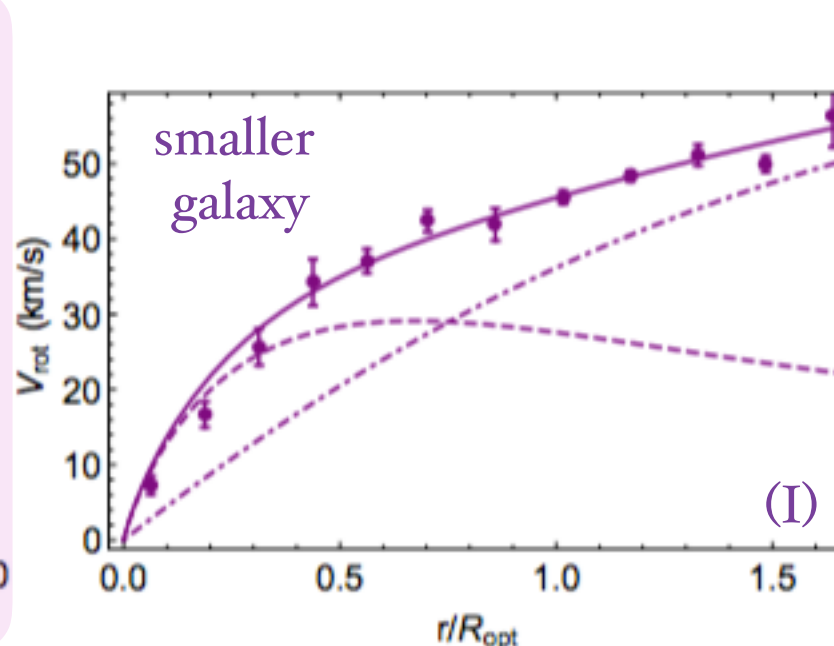
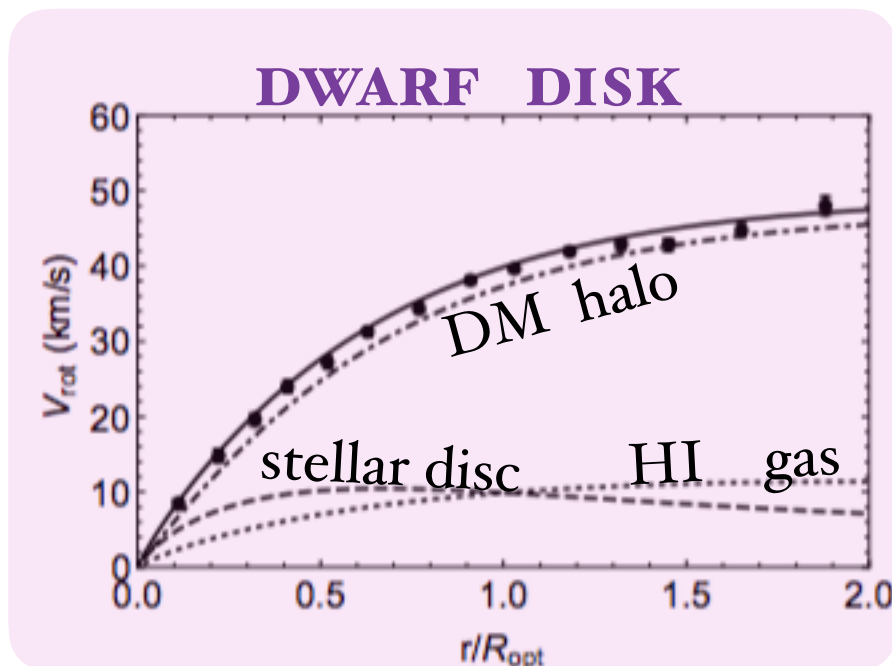
$$V_i^2(r) = G \frac{M_i(r)}{r}$$

Baryonic contribution :

$$V_b^2(r) = V_d^2(r) + V_{bu}^2(r) + V_{HI}^2(r)$$

Total contribution :

$$V^2(r) = V_b^2(r) + V_h^2(r)$$



# RAR: Gravitational acceleration relation

## Galaxy components :

- Freeman stellar disc
- Burkert (cored) DM halo
- HI gaseous disk (Dwarf Disks)
- Bulge (larger LSB galaxies)

$$V_i^2(r) = G \frac{M_i(r)}{r}$$

Baryonic contribution :

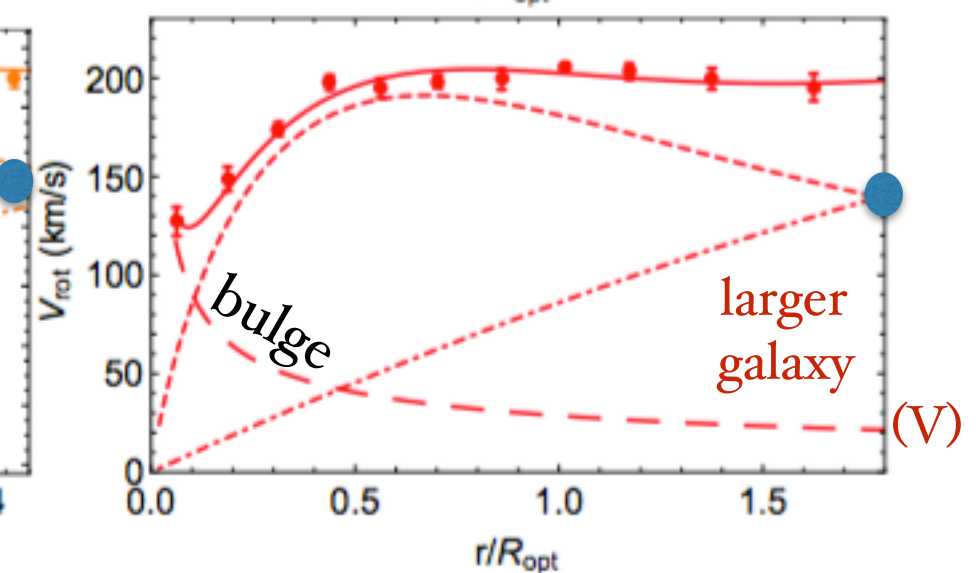
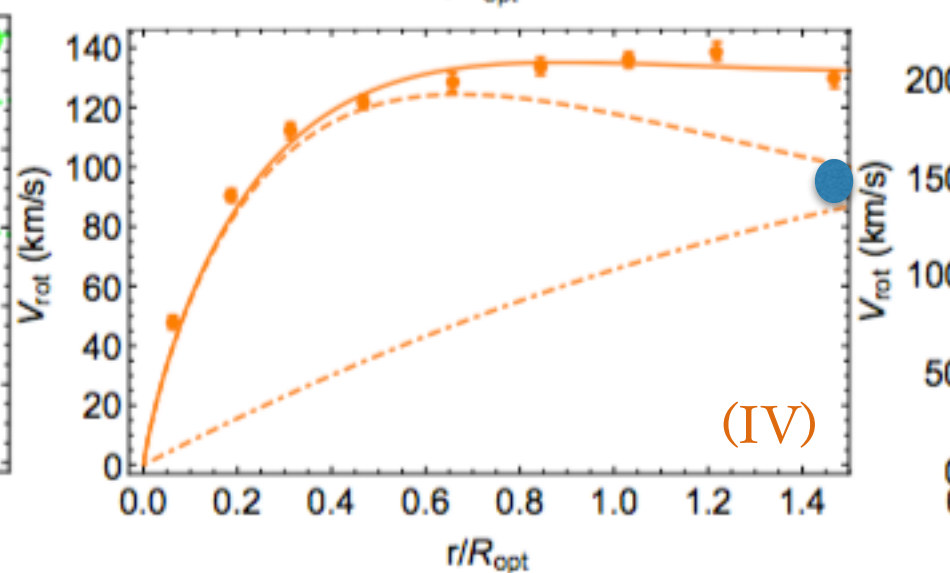
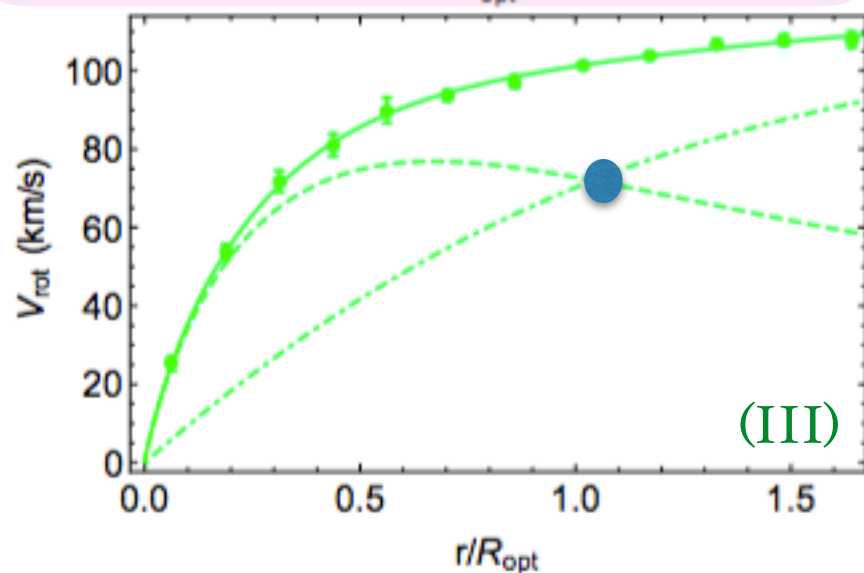
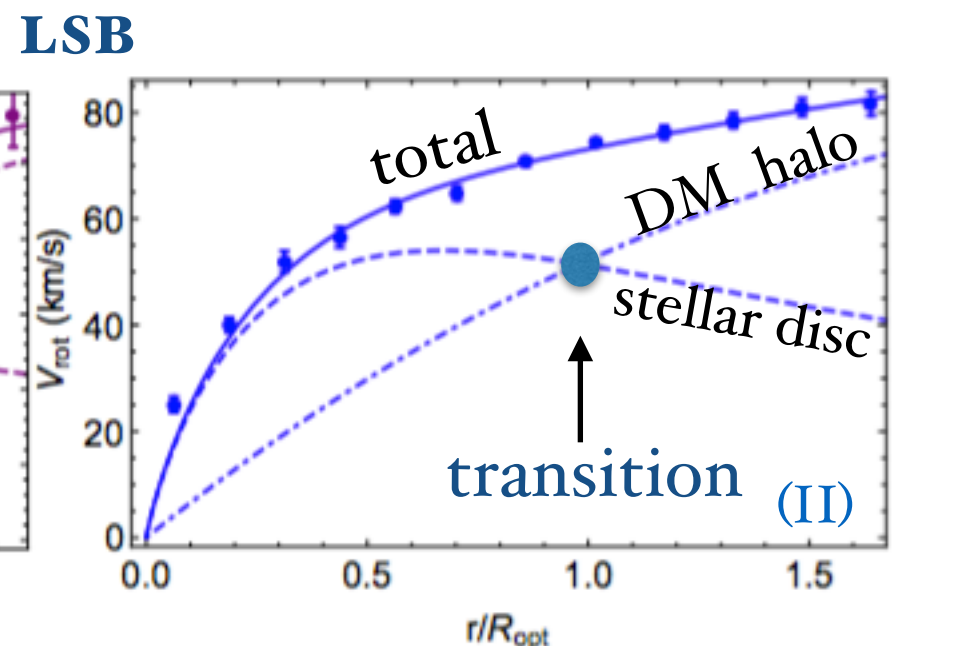
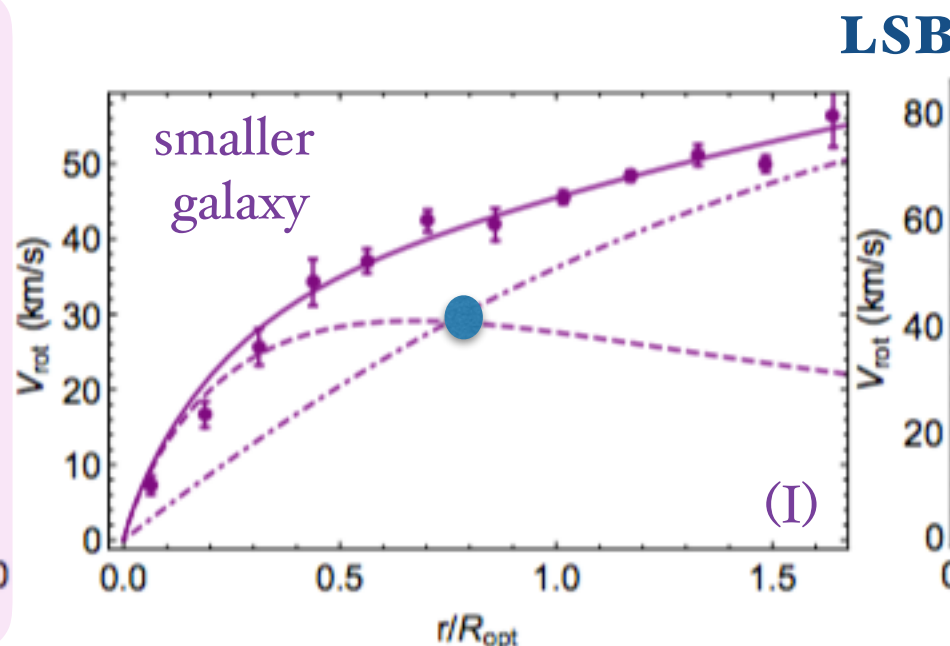
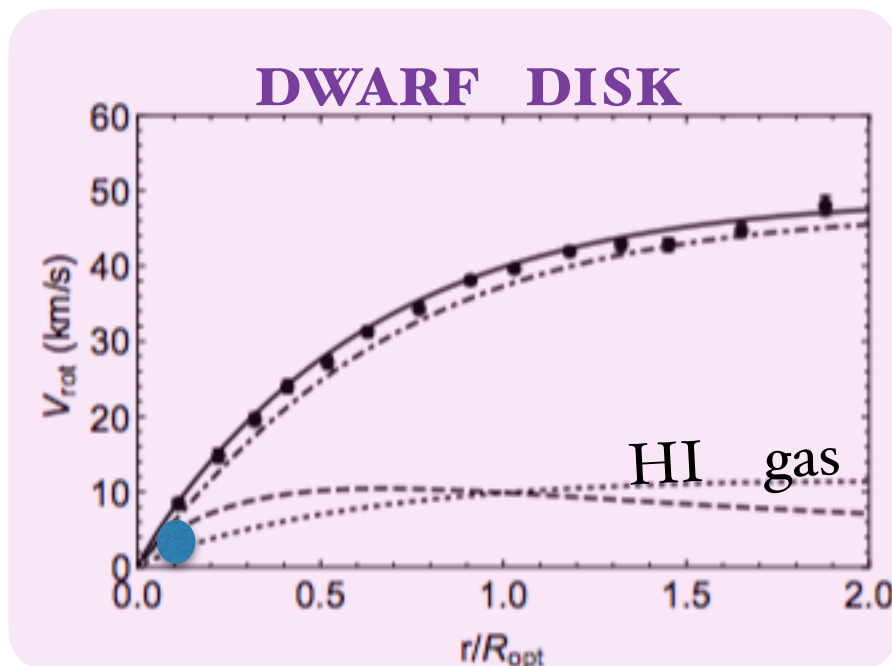
$$V_b^2(r) = V_d^2(r) + V_{bu}^2(r) + V_{HI}^2(r)$$

Total contribution :

$$V^2(r) = V_b^2(r) + V_h^2(r)$$

**Baryonic fraction**

$$f_b(r) = \frac{V_b^2(r)}{V^2(r)}$$

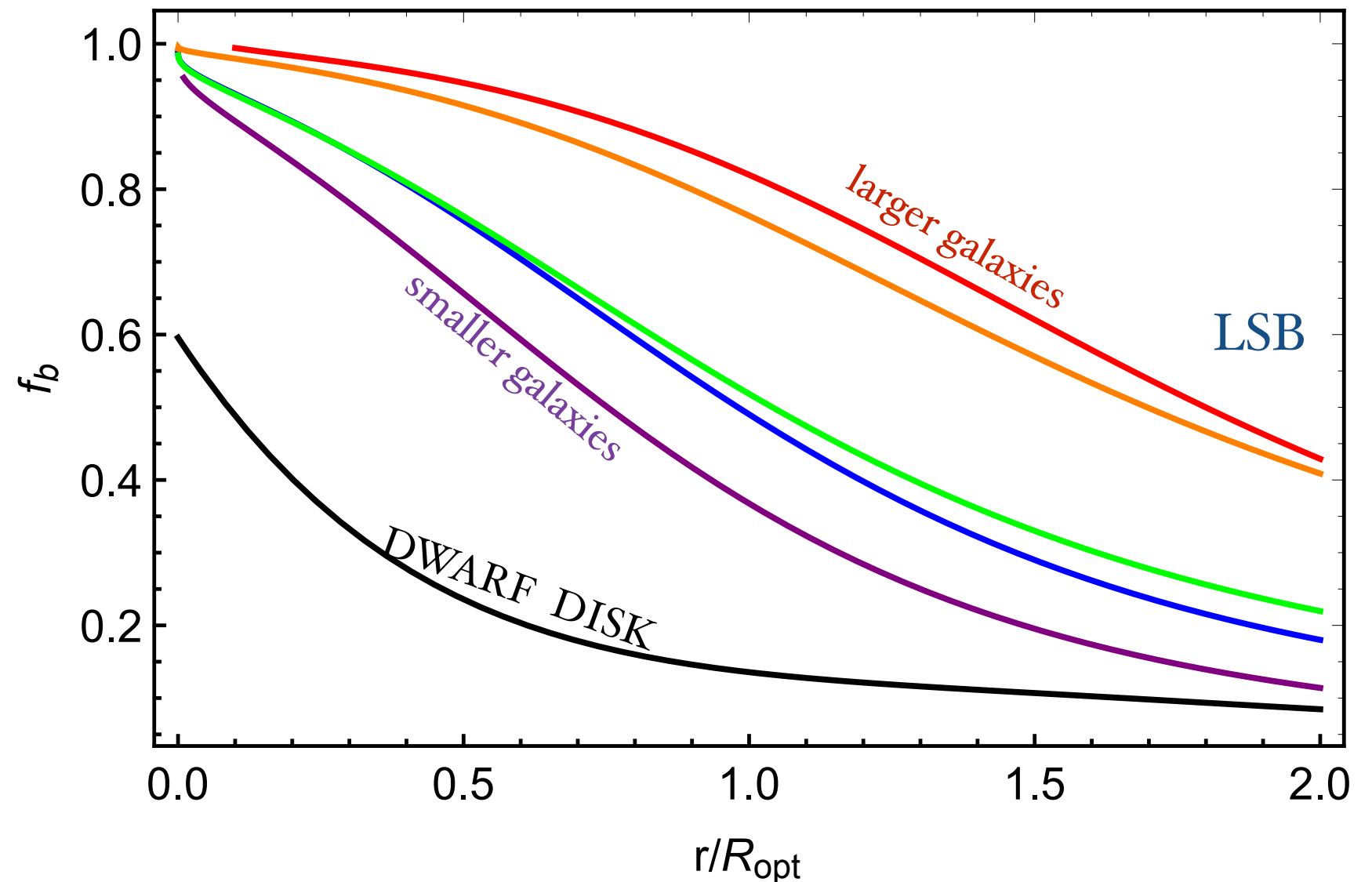


# RAR: Gravitational acceleration relation

## Baryonic fraction

$$f_b(r) = \frac{V_b^2(r)}{V^2(r)}$$

NOTE:  
radial dependence



$R_{opt}$   $\rightarrow$  optical radius  
size of the stellar disk  
( 83 % of total luminosity )

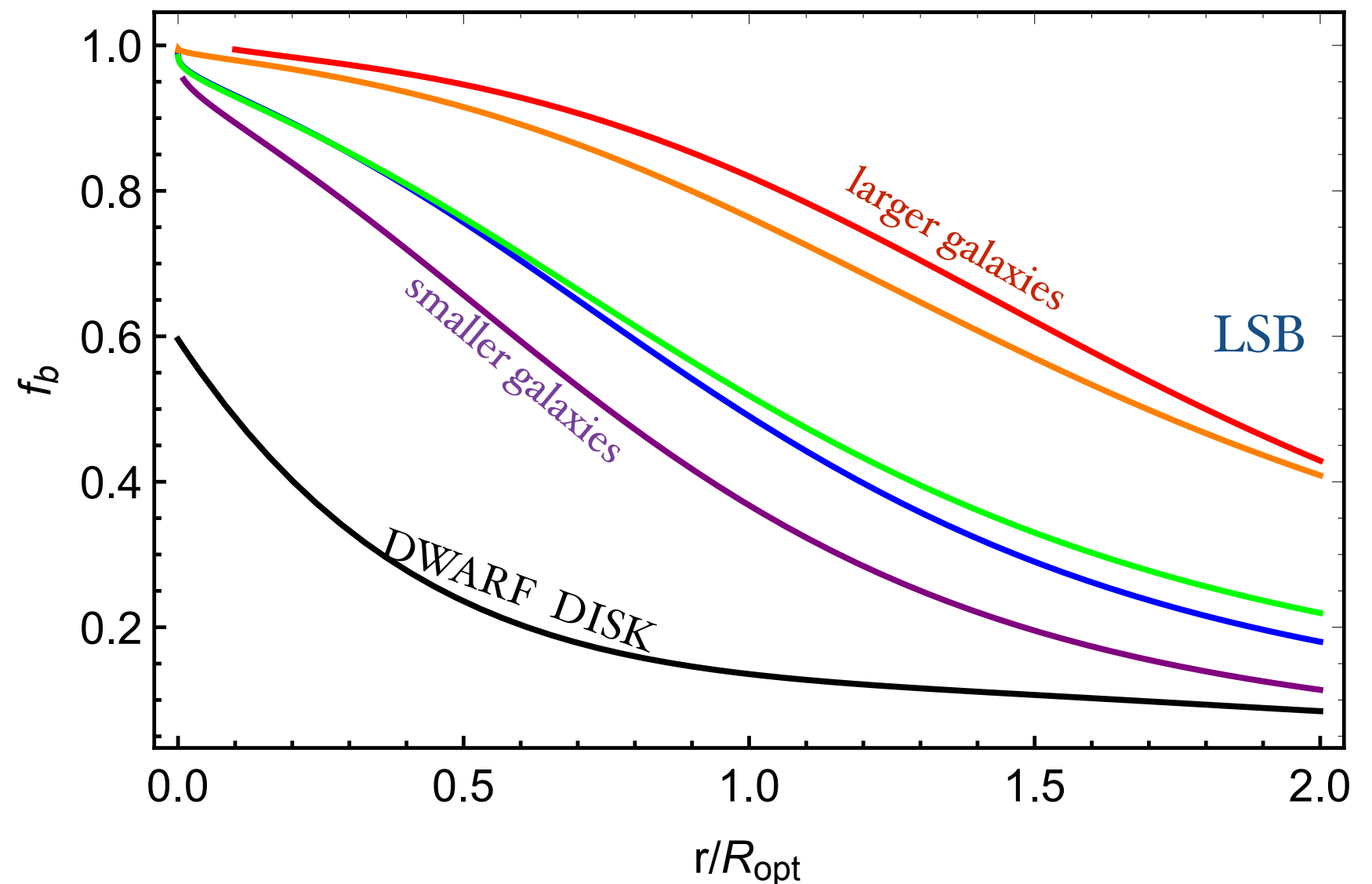


# RAR: Gravitational acceleration relation

## Baryonic fraction

$$f_b(r) = \frac{V_b^2(r)}{V^2(r)}$$

NOTE:  
radial dependence



For all single data measurements  $(r, V(r))$  we evaluate:

$g(r) = V^2(r)/r$	→	only from observations	(errors $\simeq 10\%$ )
$g_b(r) = f_b(r)g(r)$	→	from observations + curves modelling	(errors $\simeq 20 - 30\%$ )

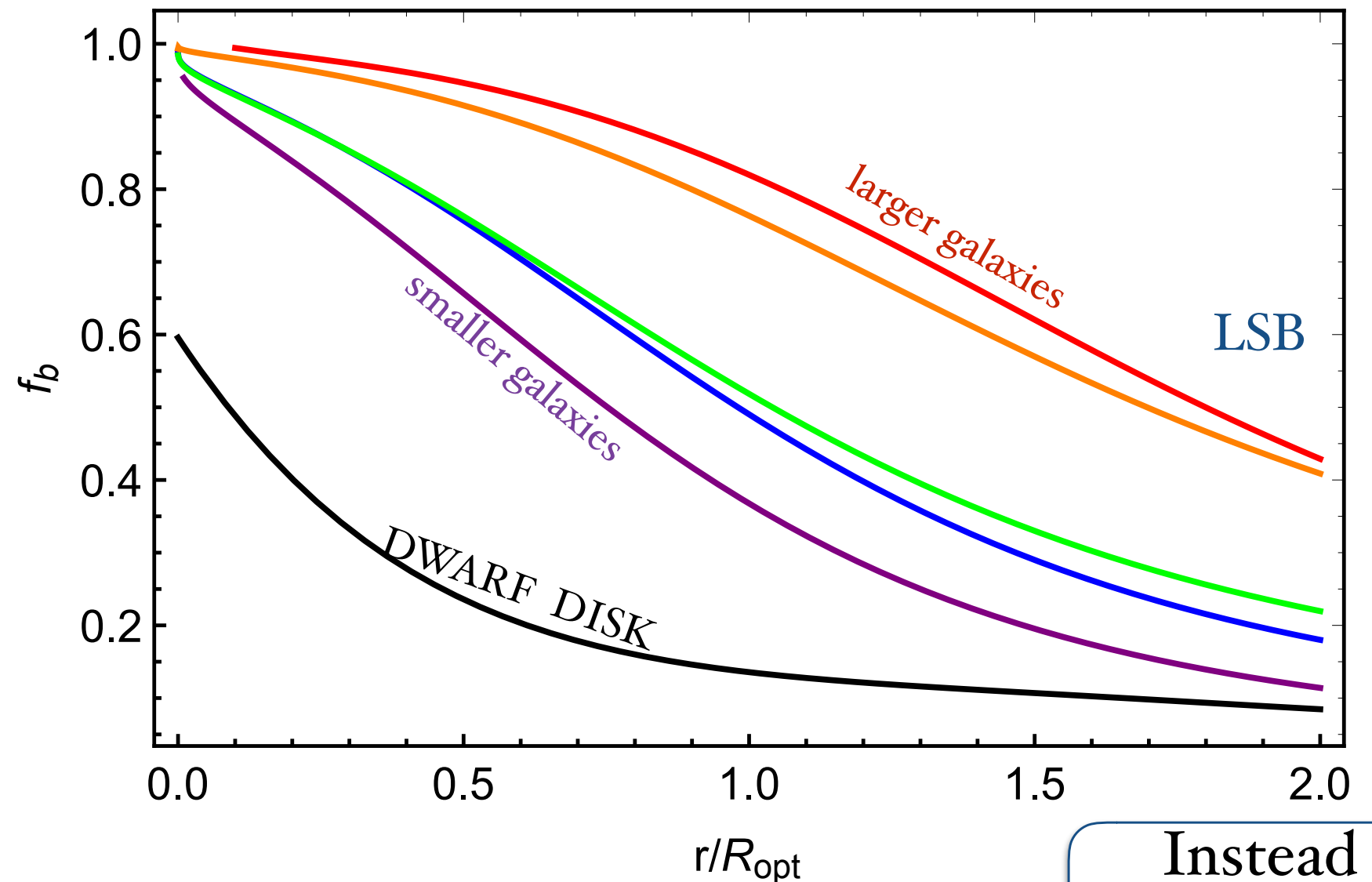


# RAR: Gravitational acceleration relation

## Baryonic fraction

$$f_b(r) = \frac{V_b^2(r)}{V^2(r)}$$

NOTE:  
radial dependence



For all single data measurements  $(r, V(r))$  we evaluate:

$$g(r) = V^2(r)/r \quad \rightarrow \quad \text{only from observations} \quad (\text{errors} \simeq 10\%)$$

$$g_b(r) = f_b(r)g(r) \quad \rightarrow \quad \text{from observations + curves modelling} \quad (\text{errors} \simeq 20 - 30\%)$$

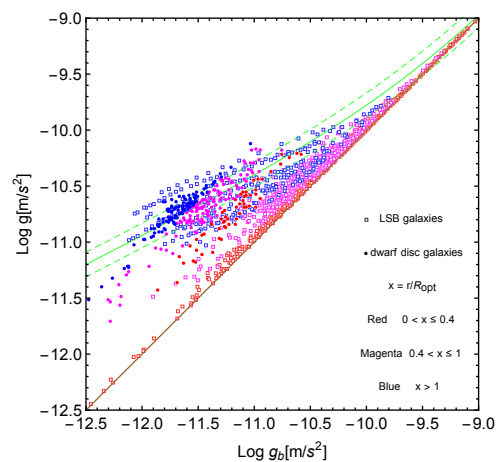
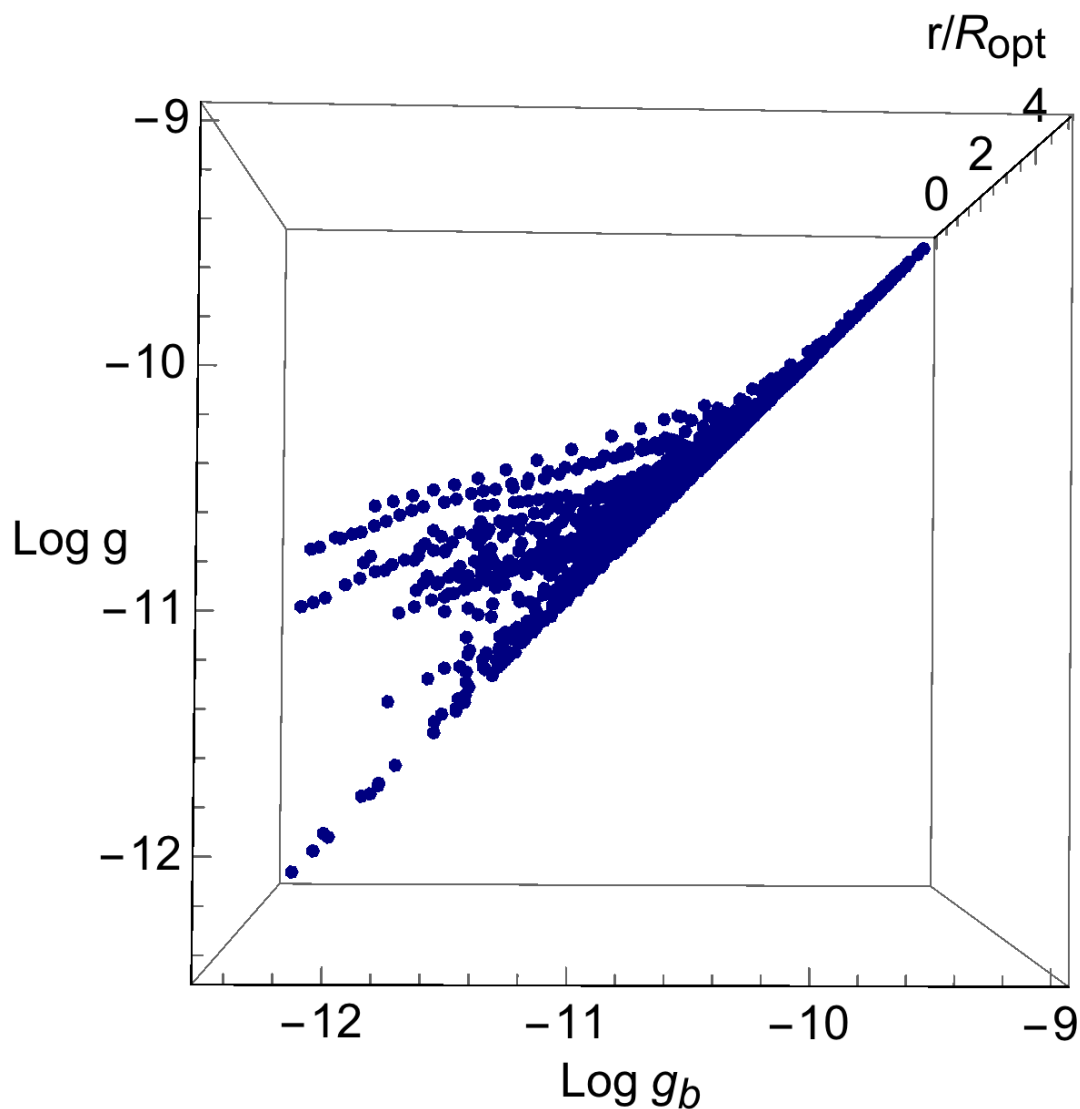
Instead  
McGaugh  
agnostic to  
the presence  
of DM

# $g$ , $g_b$ , $x$ test

*LSB*

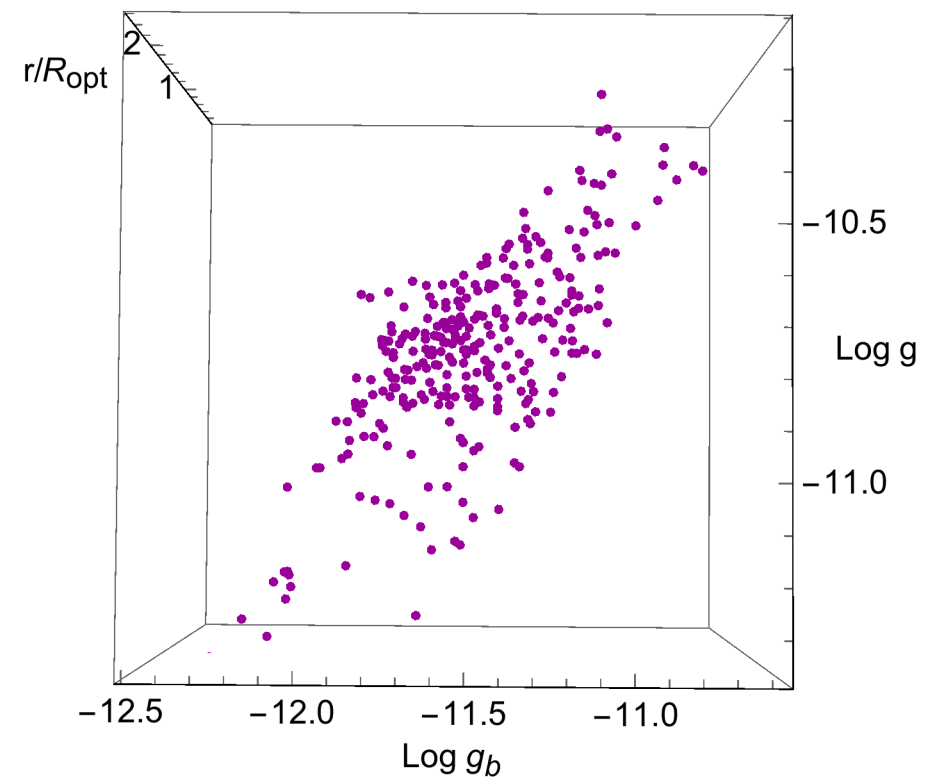
$$x = r / R_{opt}$$

$g$  ,  $g_b$  ,  $x$  test



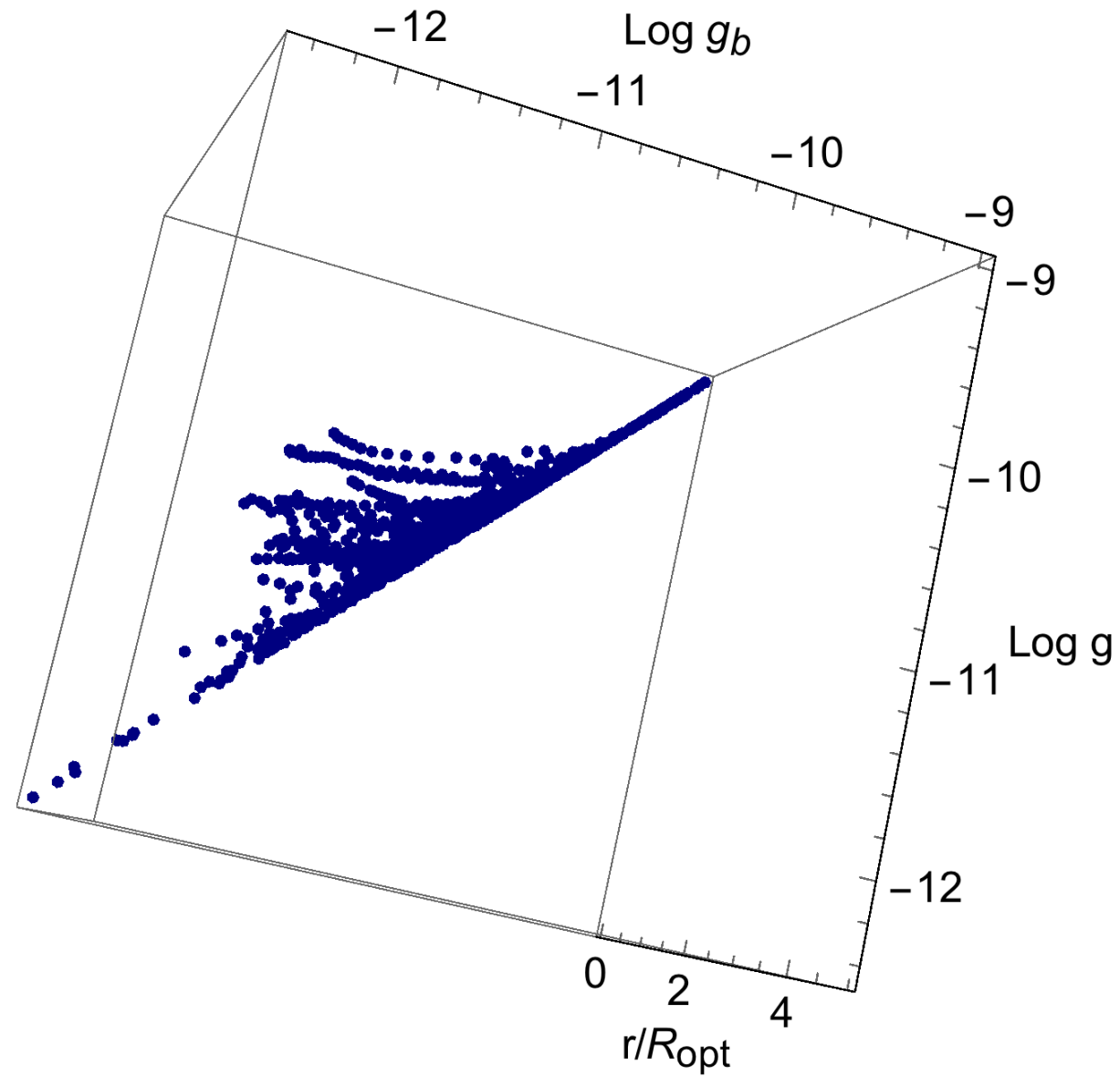
radial dependence

*Dwarf disks*



# $g$ , $g_b$ , $x$ test

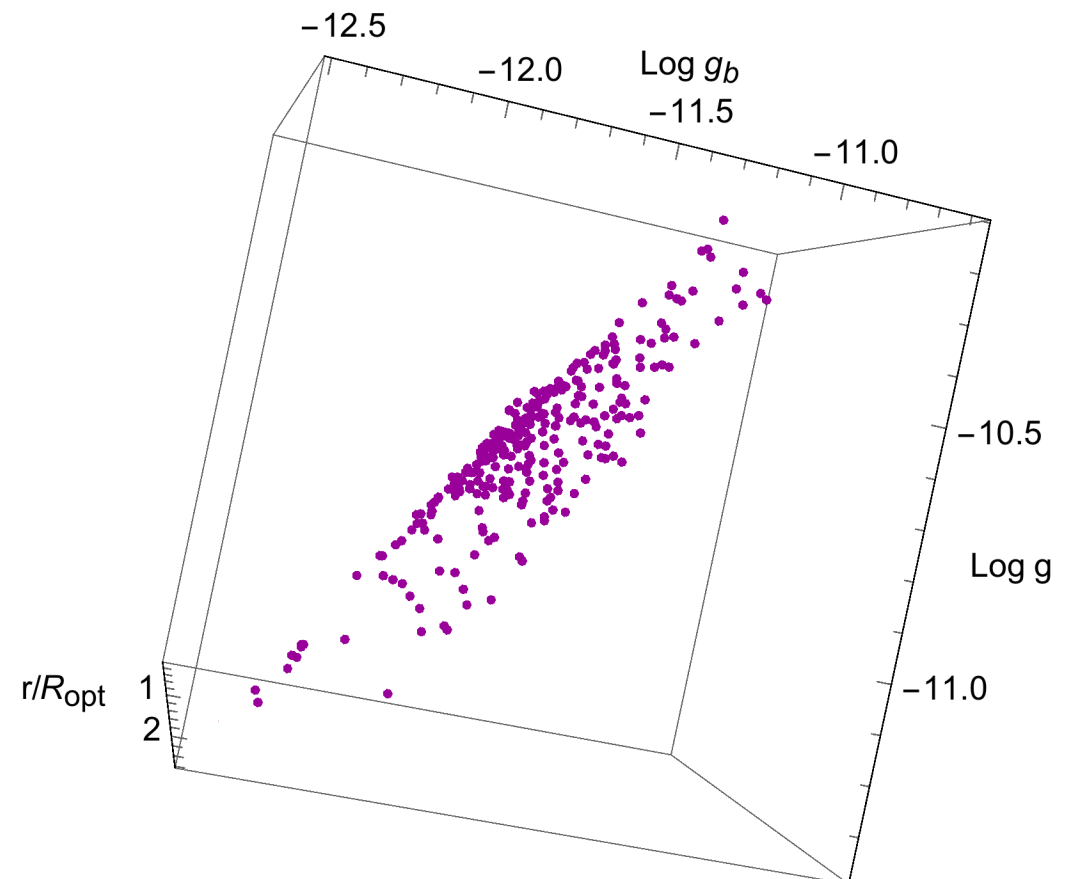
*LSB*



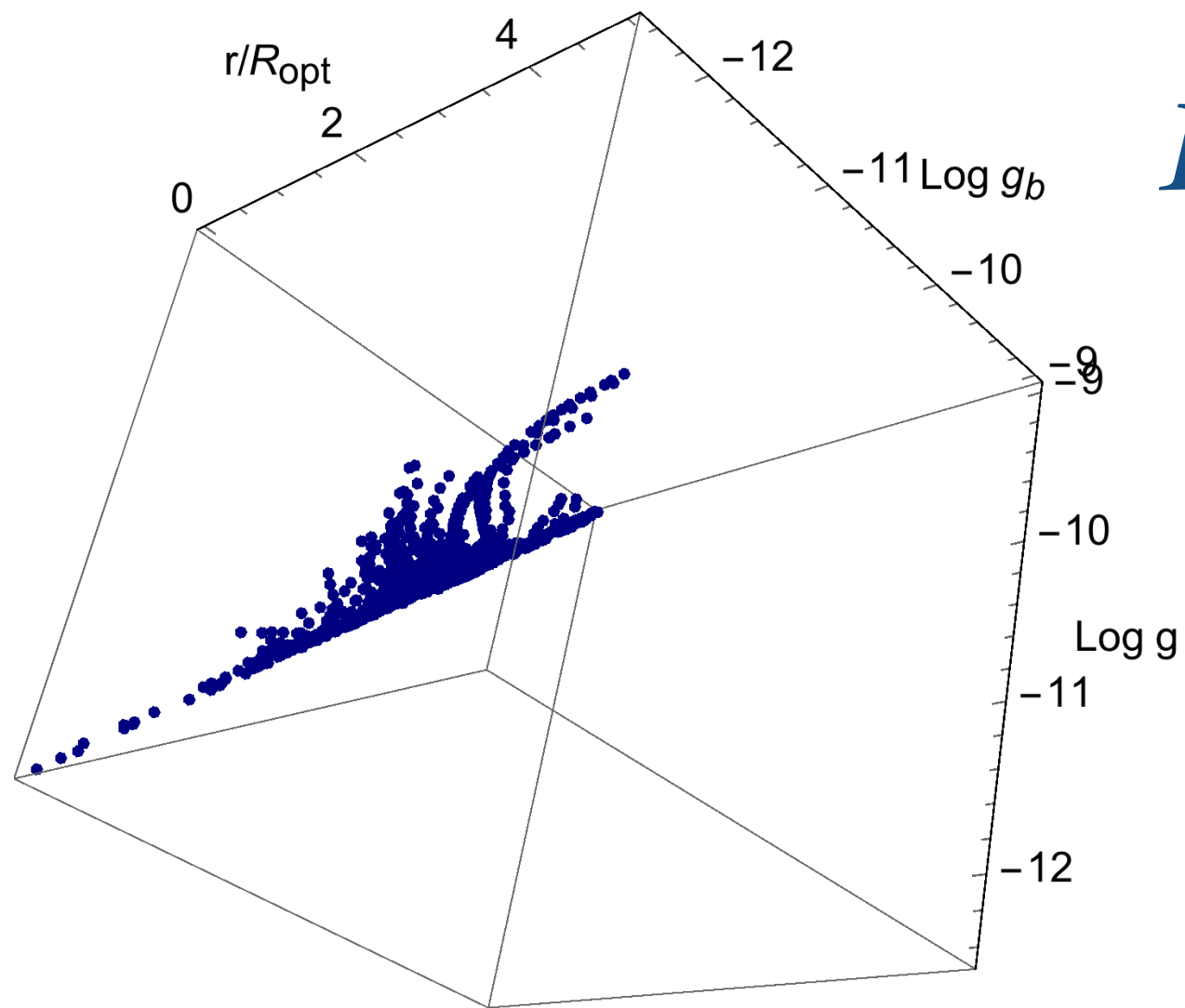
$$x = r/R_{\text{opt}}$$

$g$  ,  $g_b$  ,  $x$  test

*Dwarf disks*



# $g, g_b, x$ test

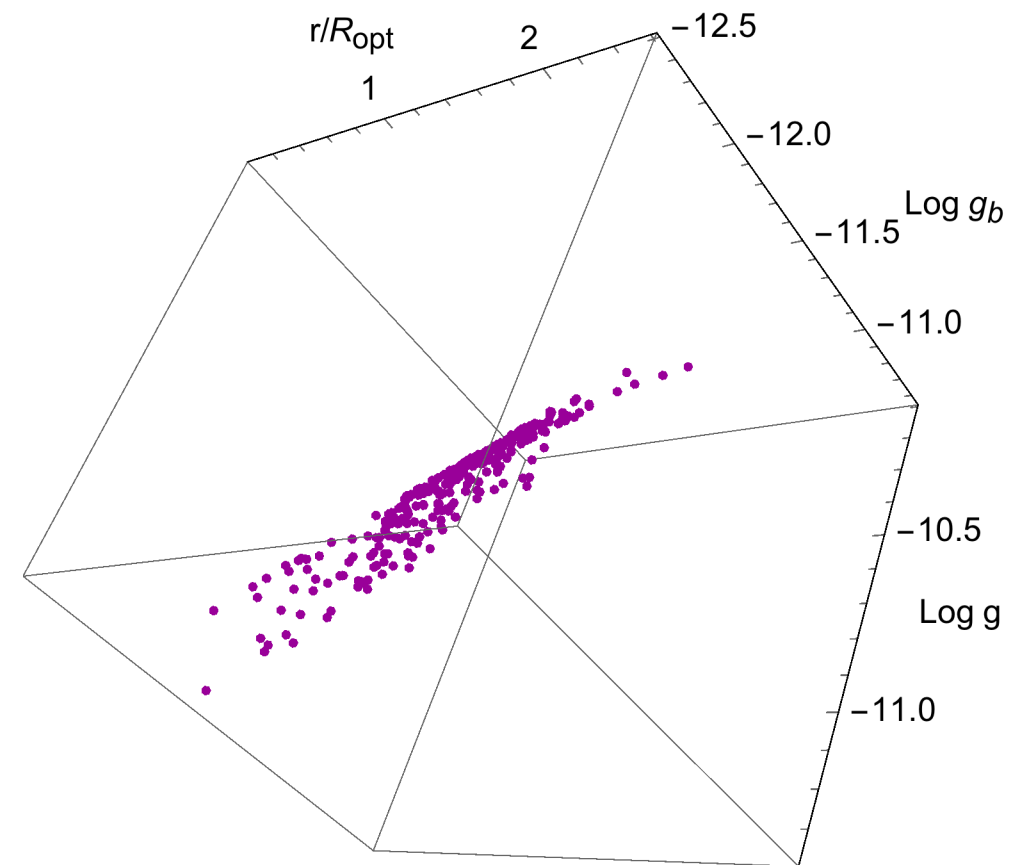


*LSB*

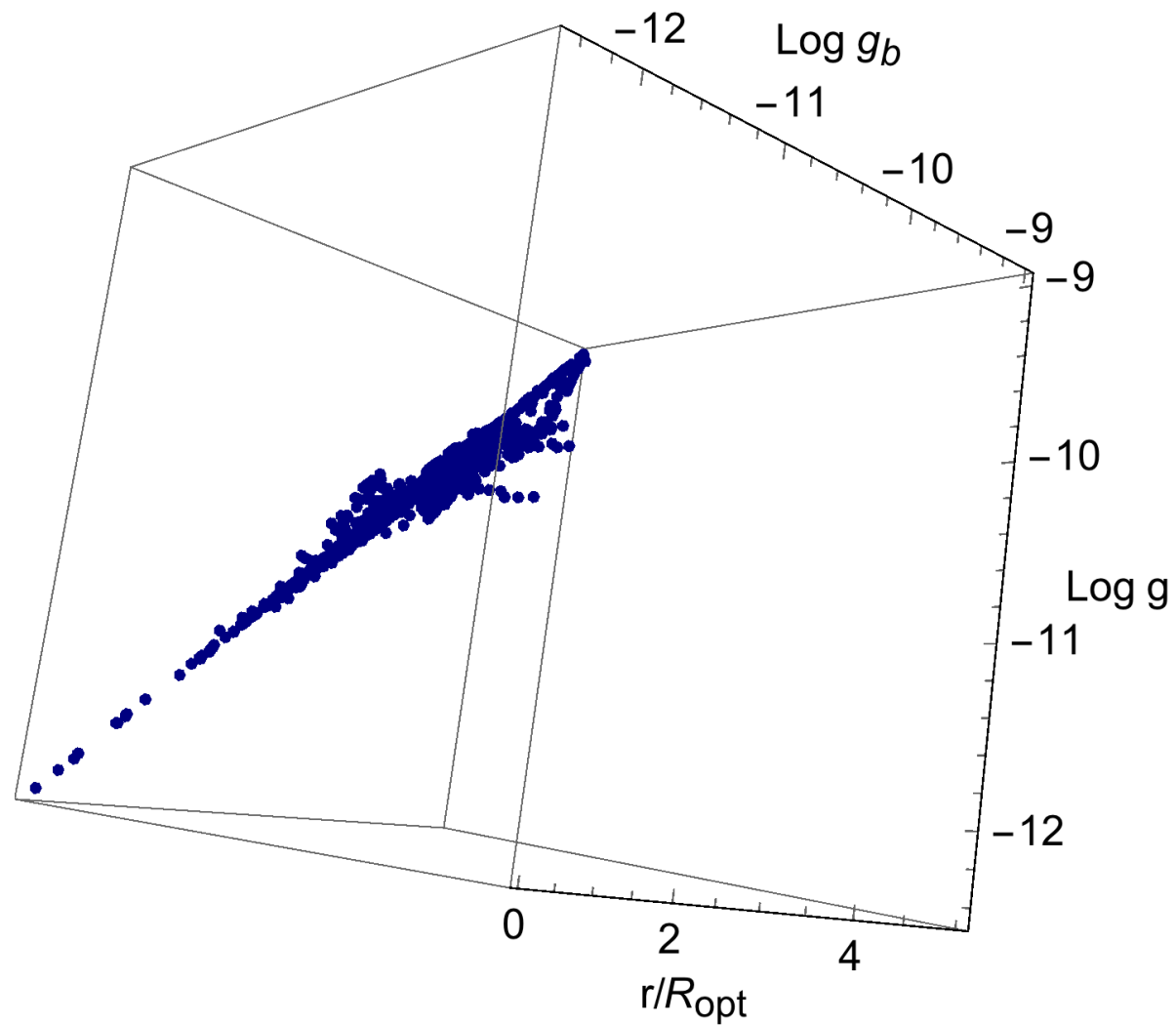
$$x = r/R_{opt}$$

$g, g_b, x$  test

*Dwarf disks*



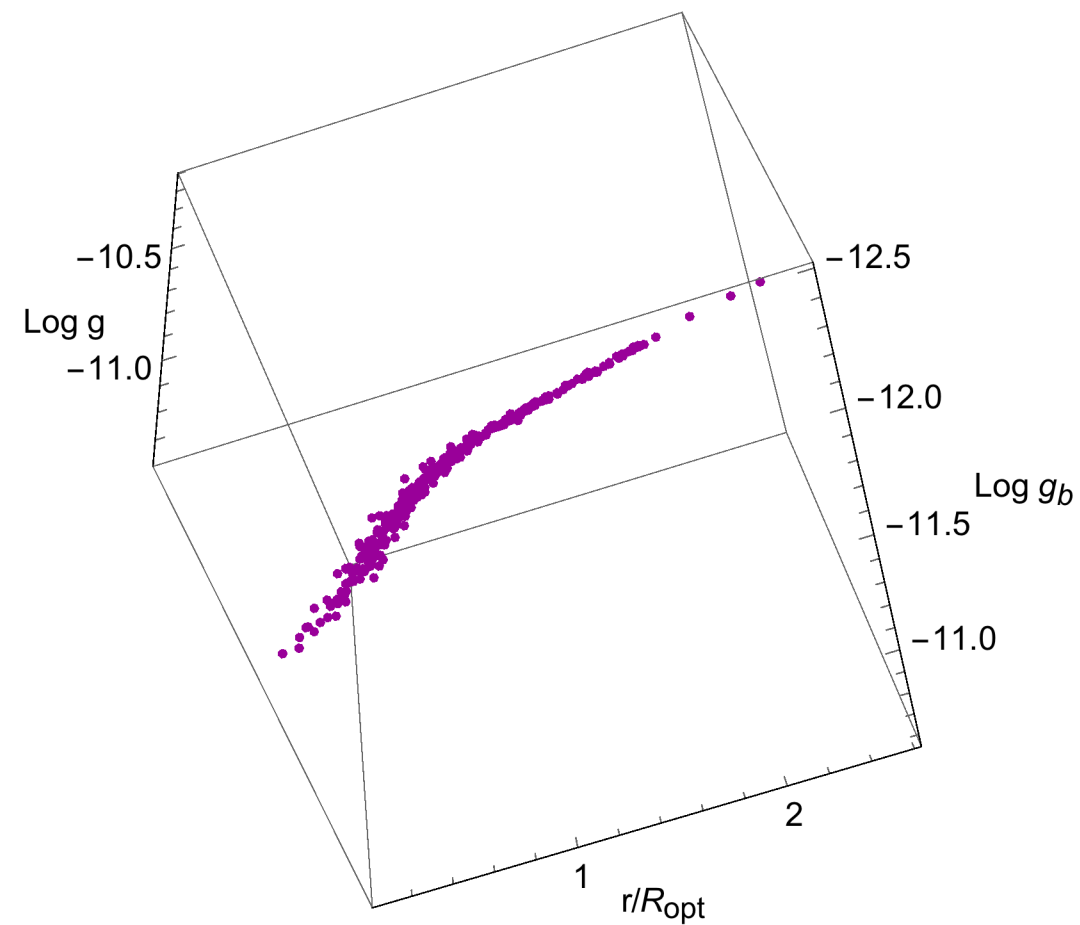
# $g, g_b, x$ test



*LSB*

$$x = r/R_{\text{opt}}$$

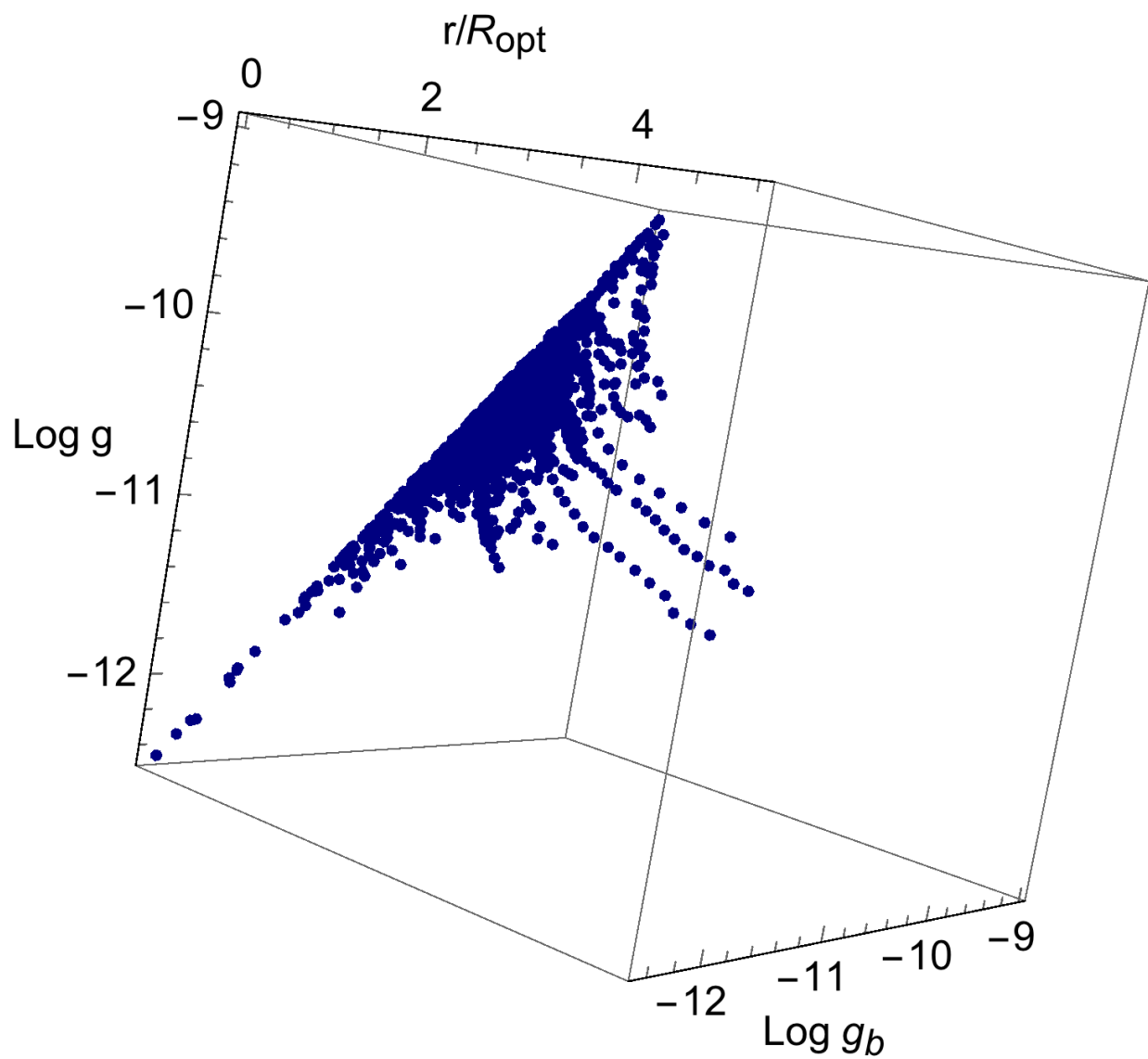
$g, g_b, x$  test



*Dwarf disks*

# $g$ , $g_b$ , $x$ test

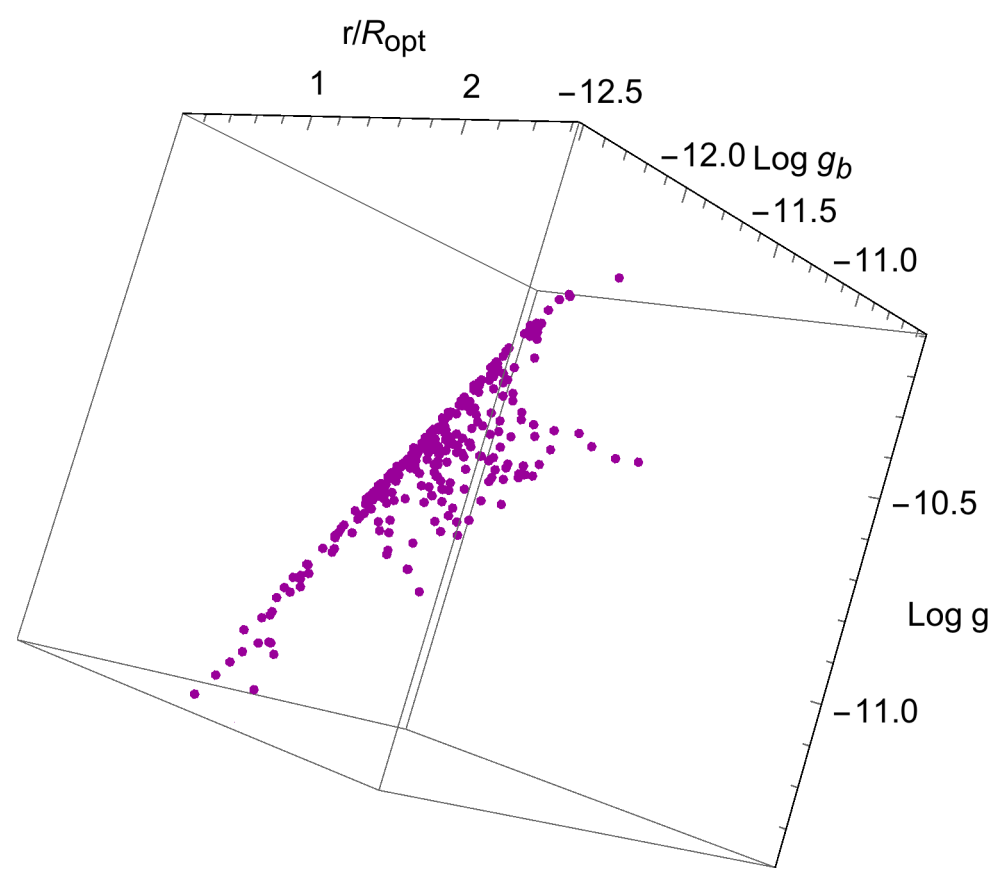
*LSB*



$$x = r/R_{\text{opt}}$$

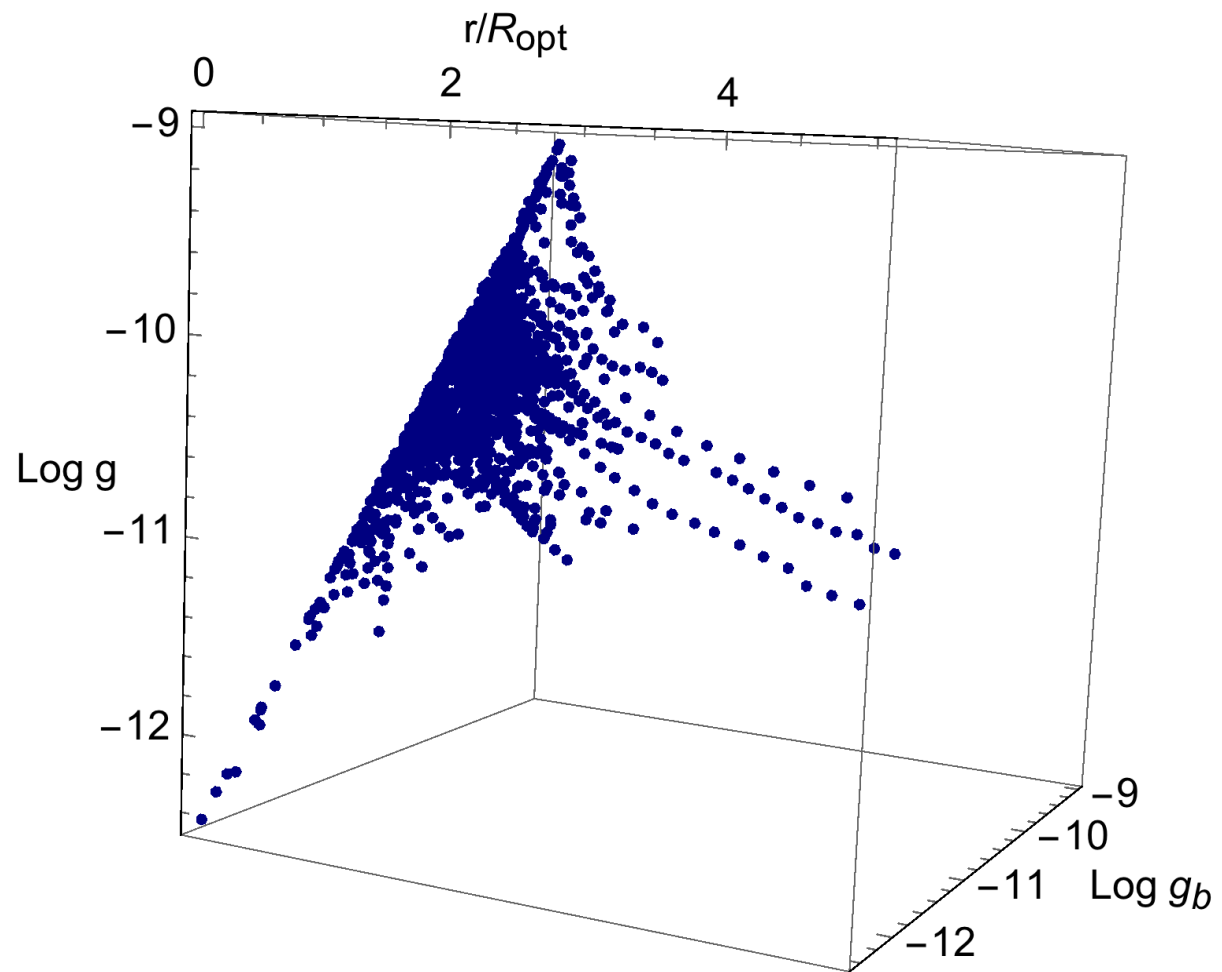
$g$  ,  $g_b$  ,  $x$  test

*Dwarf disks*



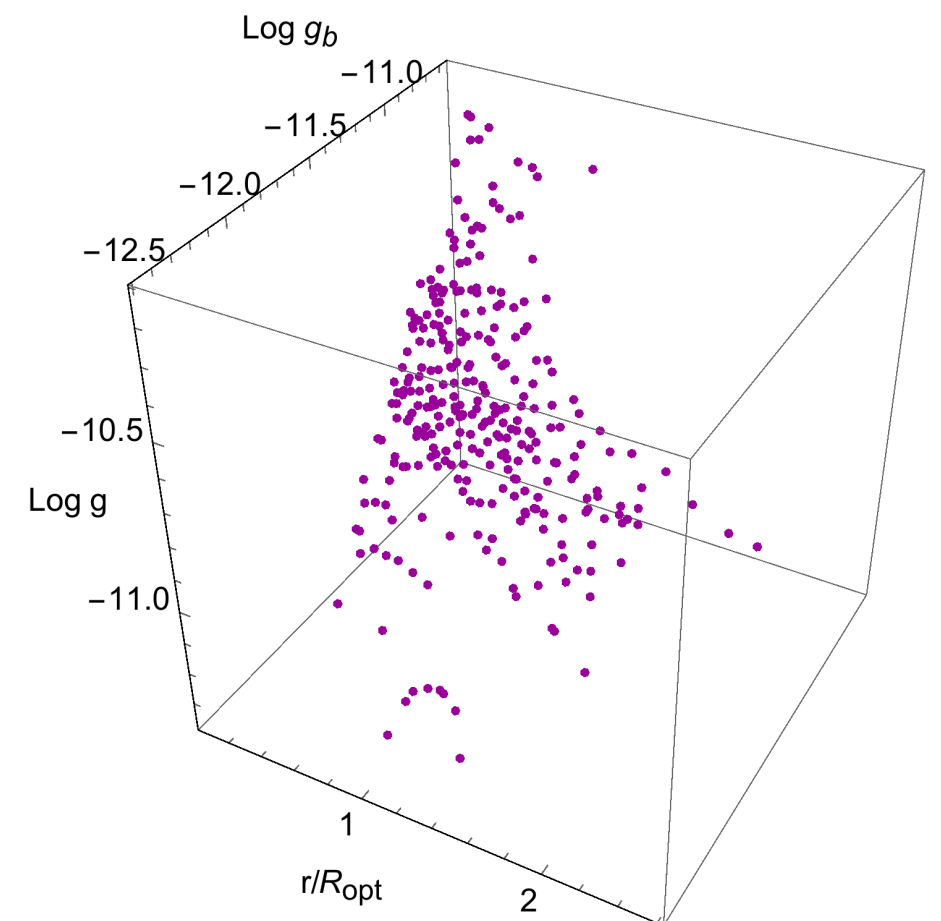
# $g$ , $g_b$ , $x$ test

*LSB*



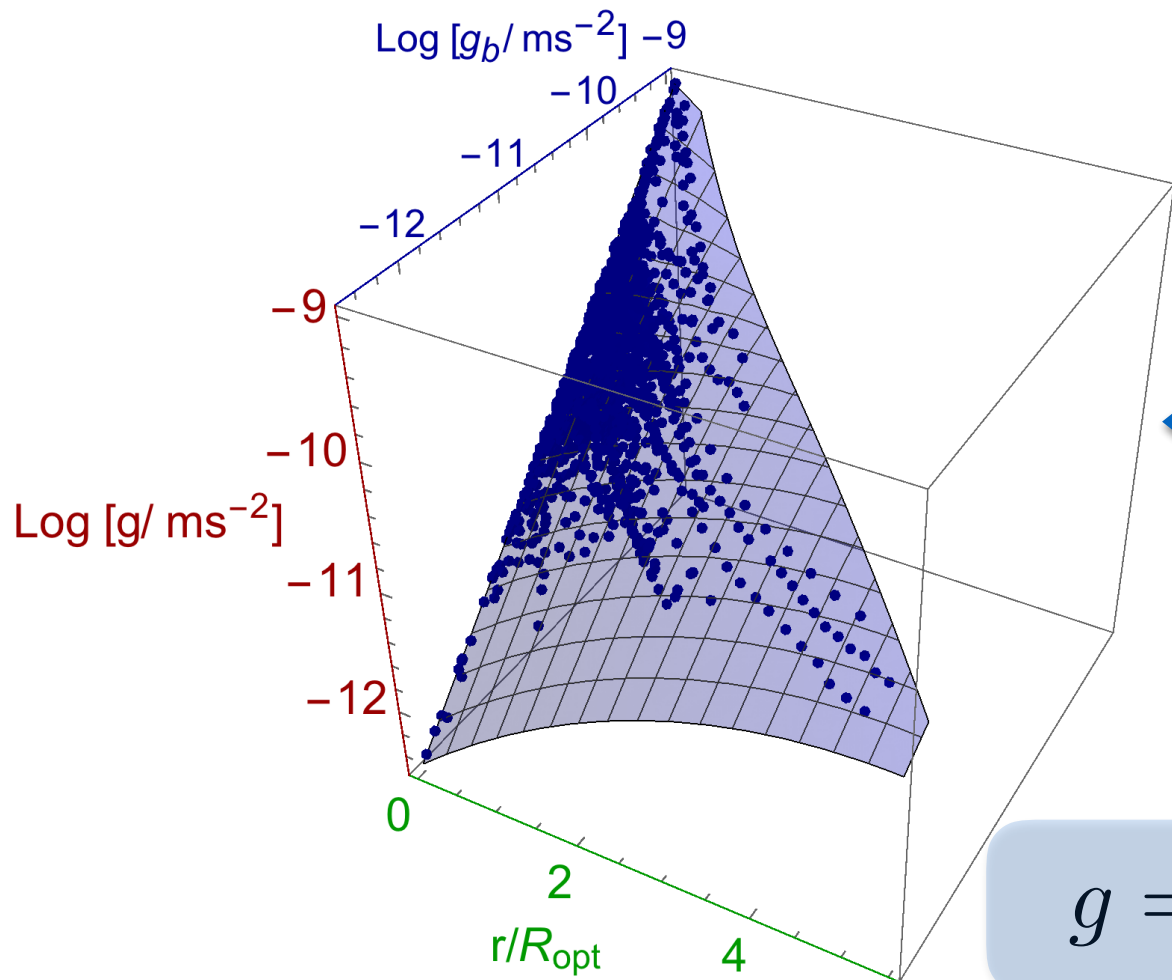
$$x = r/R_{\text{opt}}$$

*$g$  ,  $g_b$  ,  $x$  test*



*Dwarf disks*

# $g$ , $g_b$ , $x$ test



*LSB*

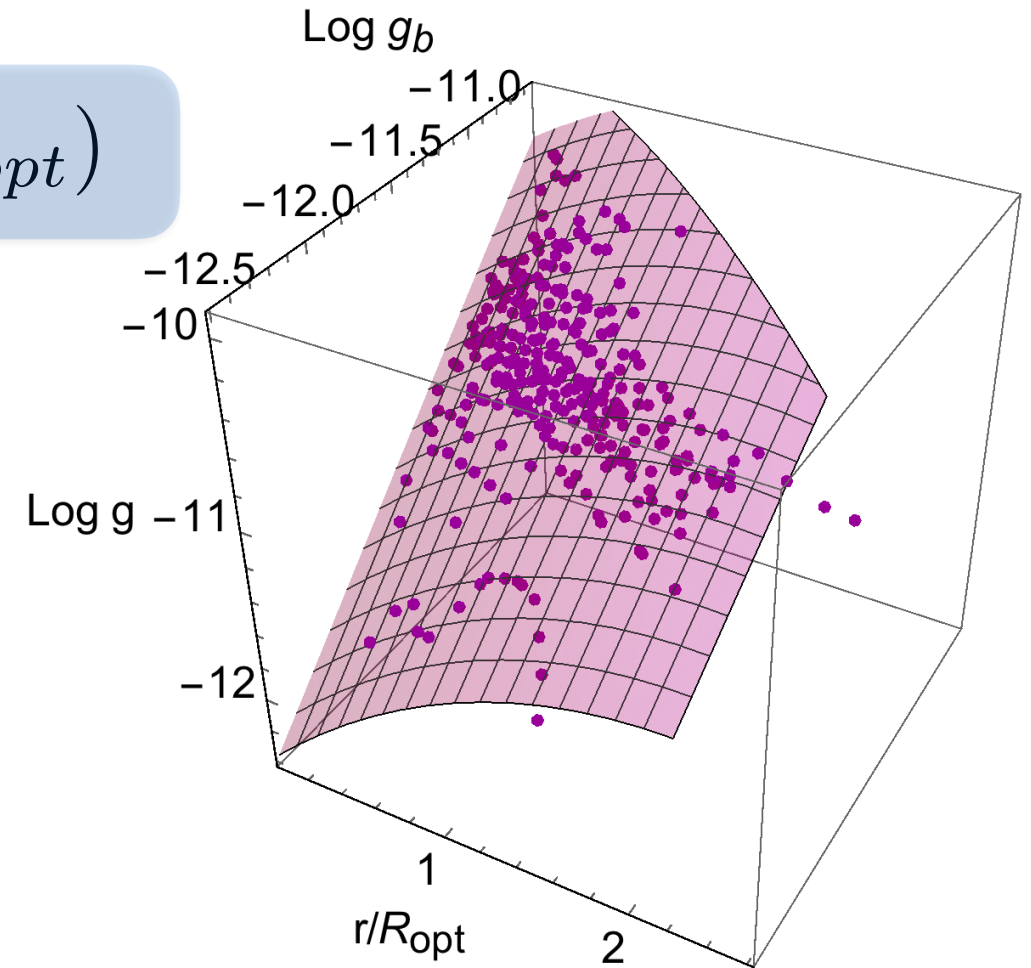
$$x = r / R_{opt}$$

$$\begin{aligned} \text{Log } g_{LSB}(x, \text{Log } g_b) = & (1 + a x) \text{Log } g_b + \\ & + b x \text{Log} [1 - \exp(-\sqrt{g_b(r)/g_{\dagger}})] \\ & + c x + d x^2 \quad , \end{aligned}$$

$$g = f(g_b, r/R_{opt})$$

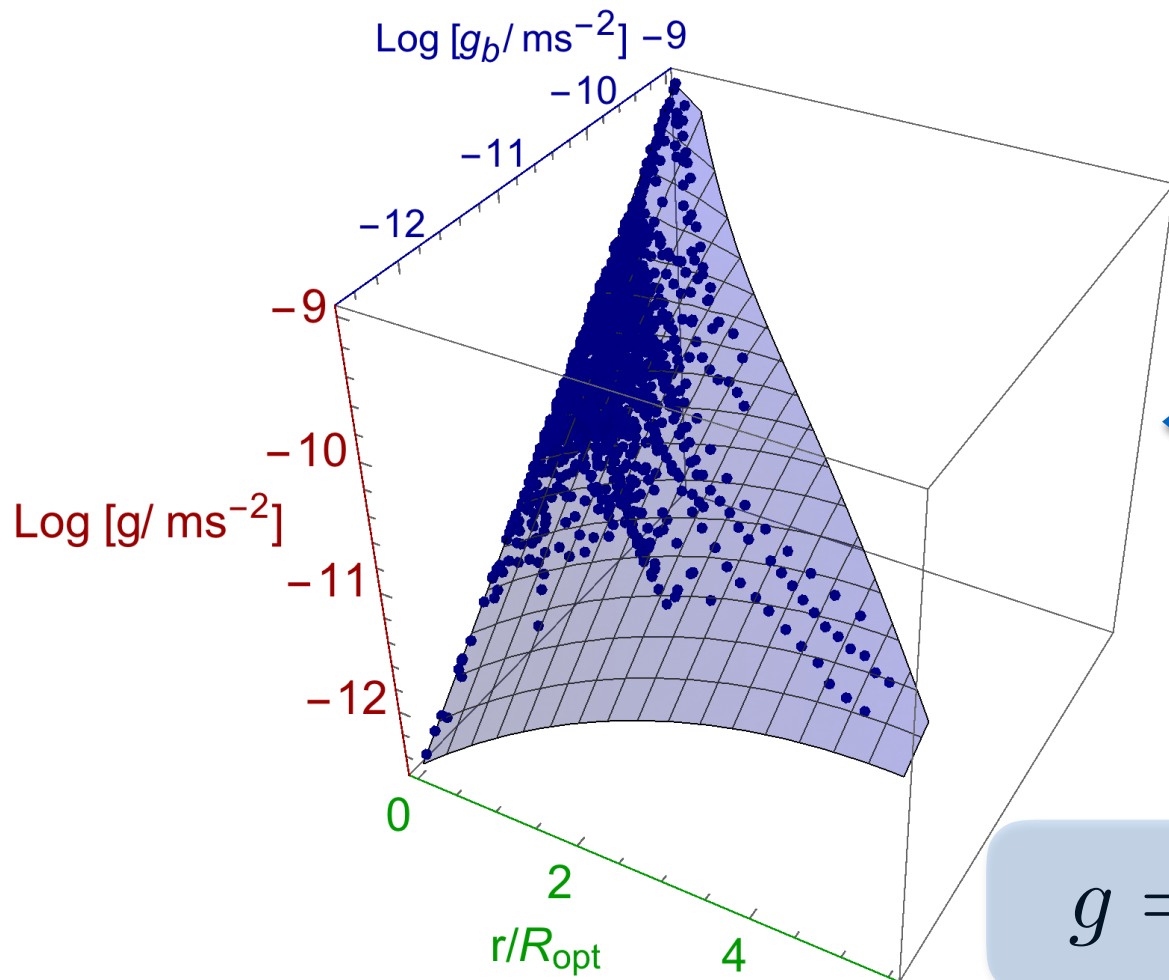
$$\begin{aligned} \text{Log } g_{DD}(x, \text{Log } g_b) = \\ \text{Log } g_{LSB} \left( \frac{x}{l} + h, \frac{\text{Log } g_b}{m} + n \right) + q \end{aligned}$$

*Dwarf disks*





# $g, g_b, x$ test



## LSB

$$x = r / R_{opt}$$

$$\begin{aligned} \text{Log } g_{LSB}(x, \text{Log } g_b) = & (1 + ax) \text{Log } g_b + \\ & + bx \text{Log} [1 - \exp(-\sqrt{g_b(r)/g_{\dagger}})] \\ & + cx + dx^2, \end{aligned}$$

a	b	c	d
-0.95	1.79	-9.01	-0.05

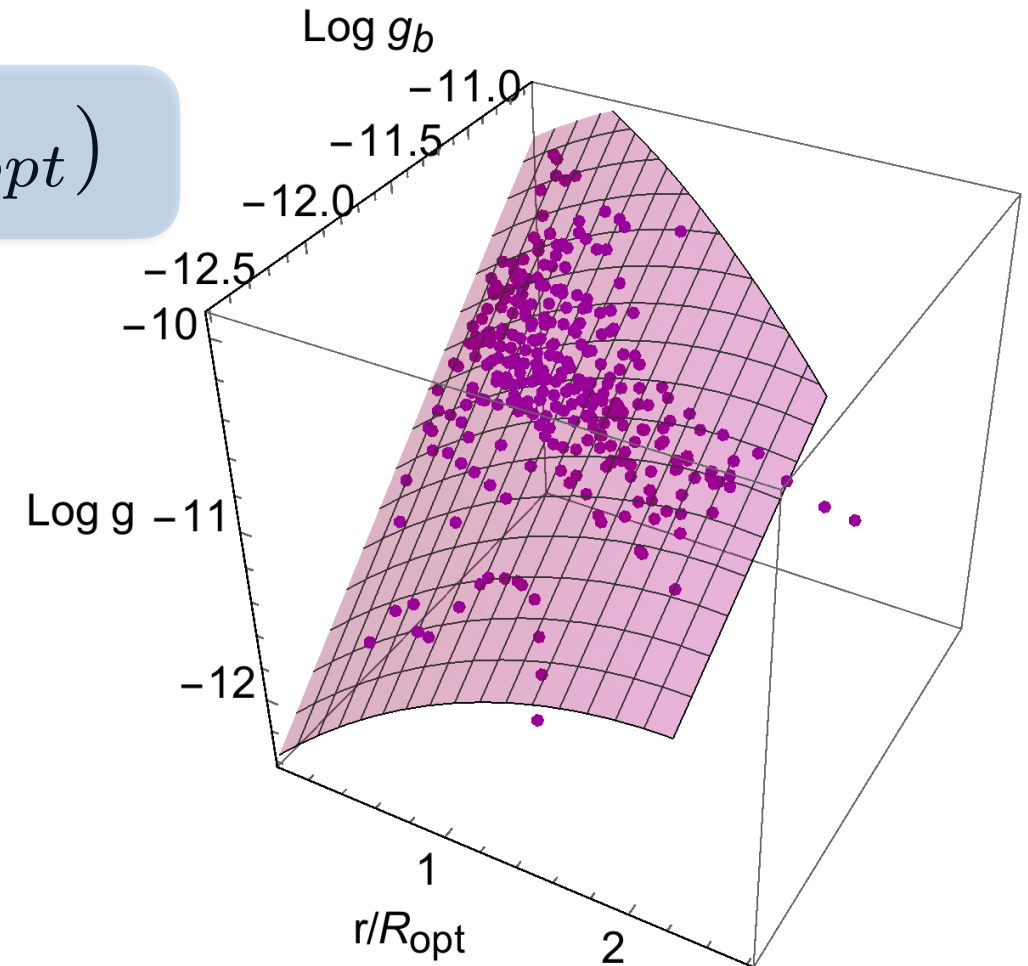
$$\sigma : 0.31 \rightarrow 0.05 \text{ dex}$$

$$g = f(g_b, r/R_{opt})$$

$$\begin{aligned} \text{Log } g_{DD}(x, \text{Log } g_b) = \\ \text{Log } g_{LSB} \left( \frac{x}{l} + h, \frac{\text{Log } g_b}{m} + n \right) + q \end{aligned}$$

l	h	m	n	q
0.49	2.41	0.74	1.72	1.19

$$\sigma : 0.17 \rightarrow 0.03 \text{ dex}$$

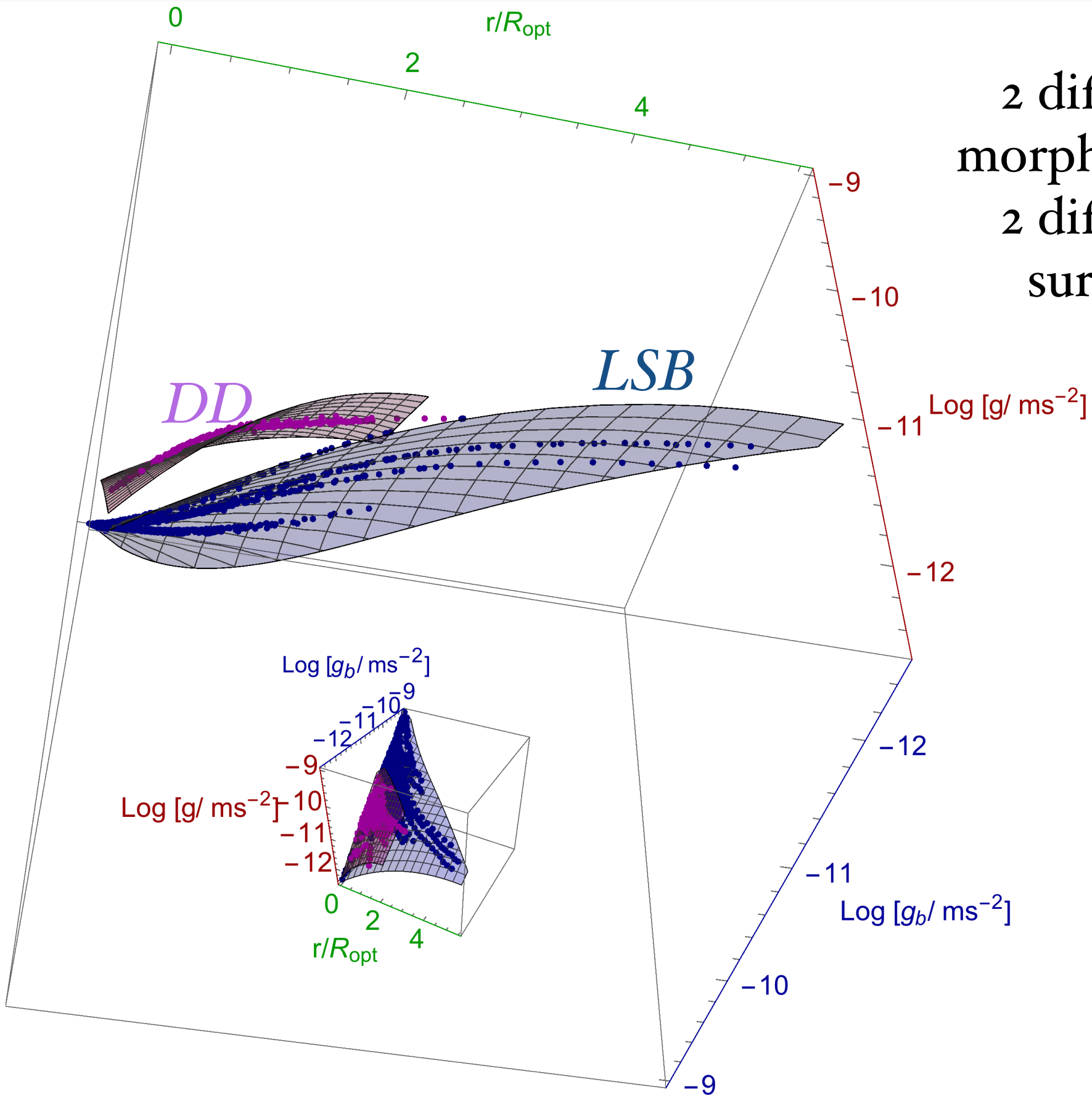


## Dwarf disks

# $g$ , $g_b$ , $x$ test

2 different morphologies,  
2 different surfaces

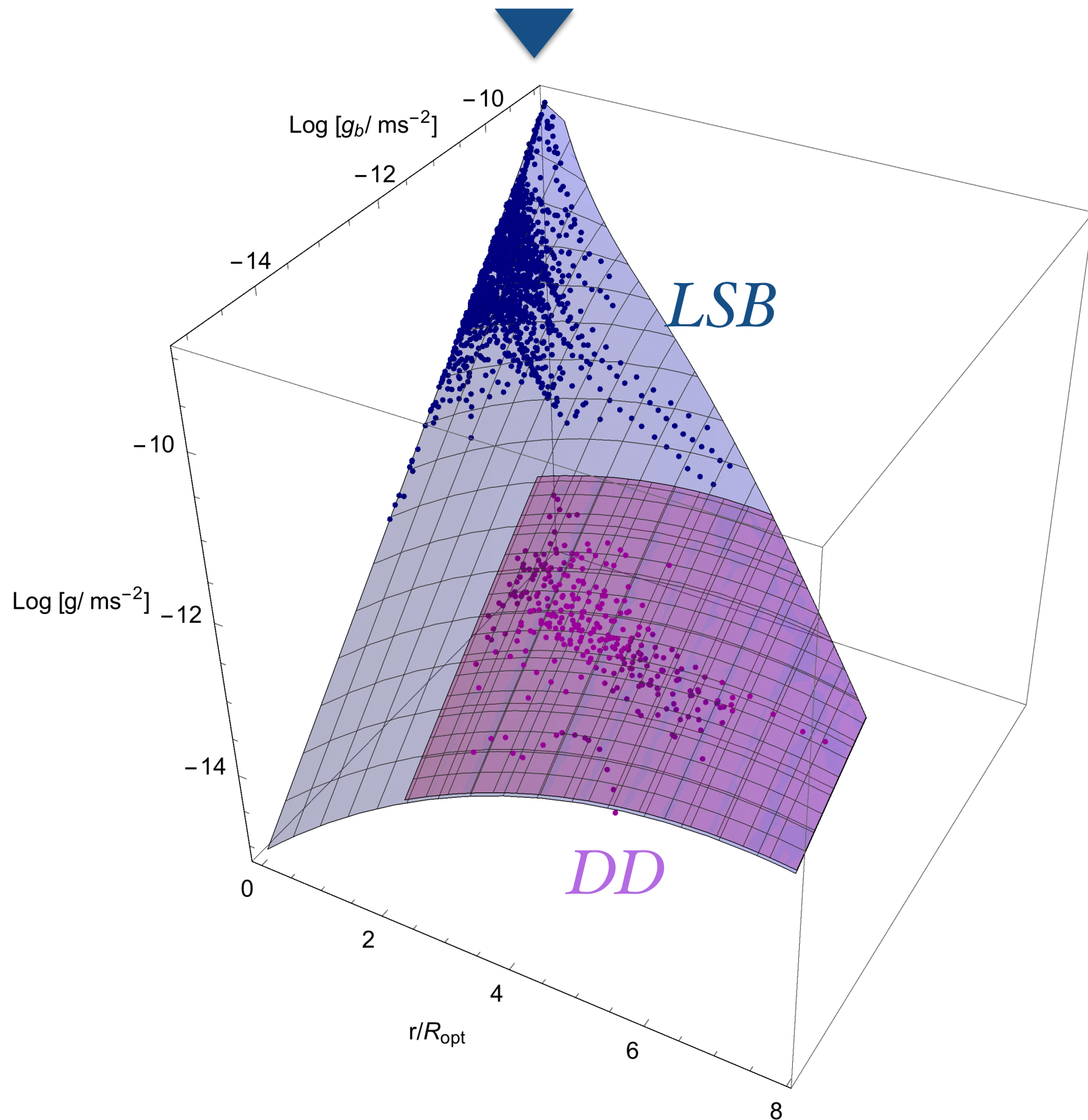
$$x = r/R_{opt}$$



# $g$ , $g_b$ , $x$ test

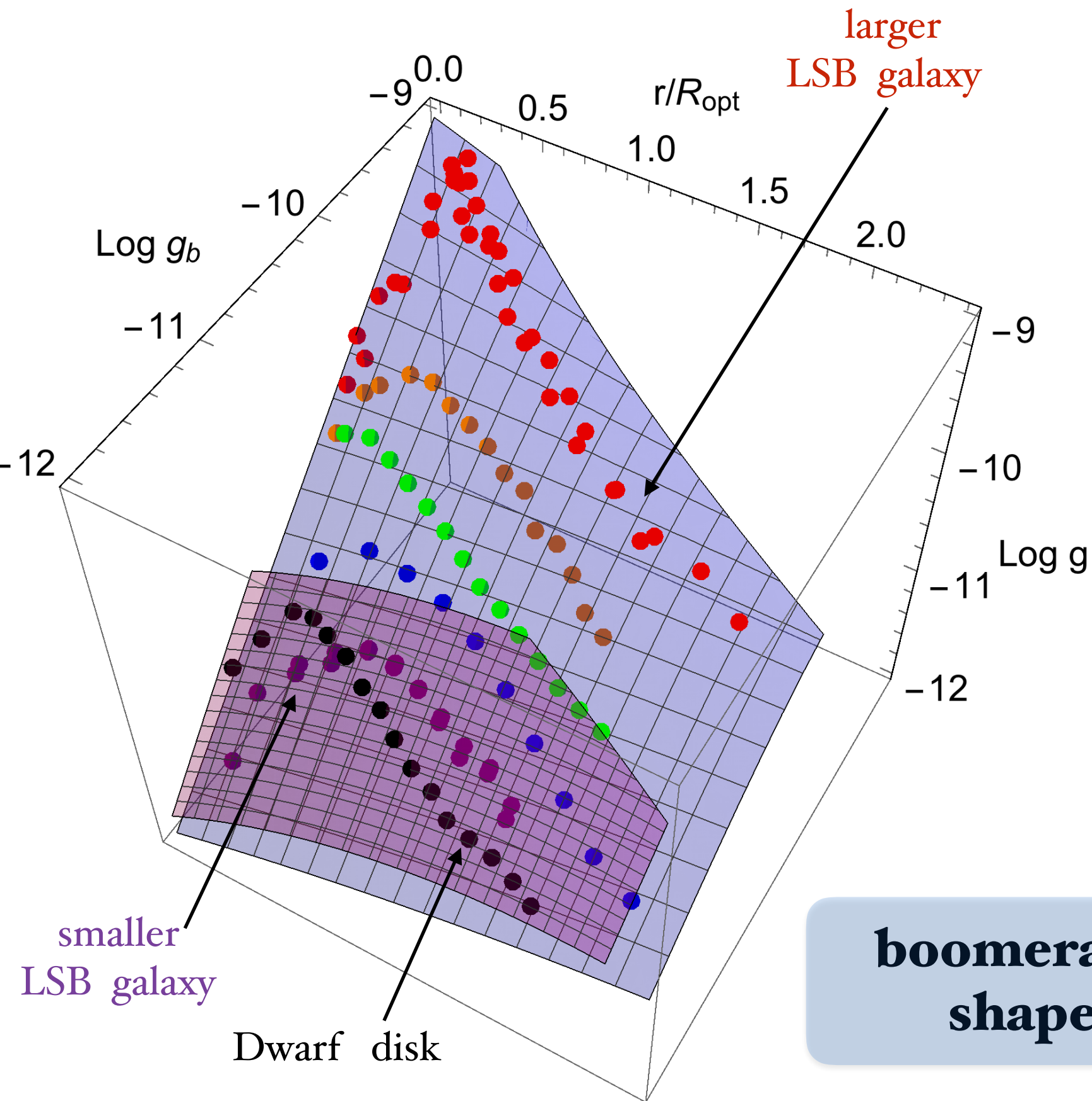
Simple translations and/or dilations

$$x = r / R_{opt}$$



universal  
relation

# $g$ , $g_b$ , $x$ single galaxies test



a) larger galaxies  $\blacktriangleright$  higher  $g$  and  $g_b$

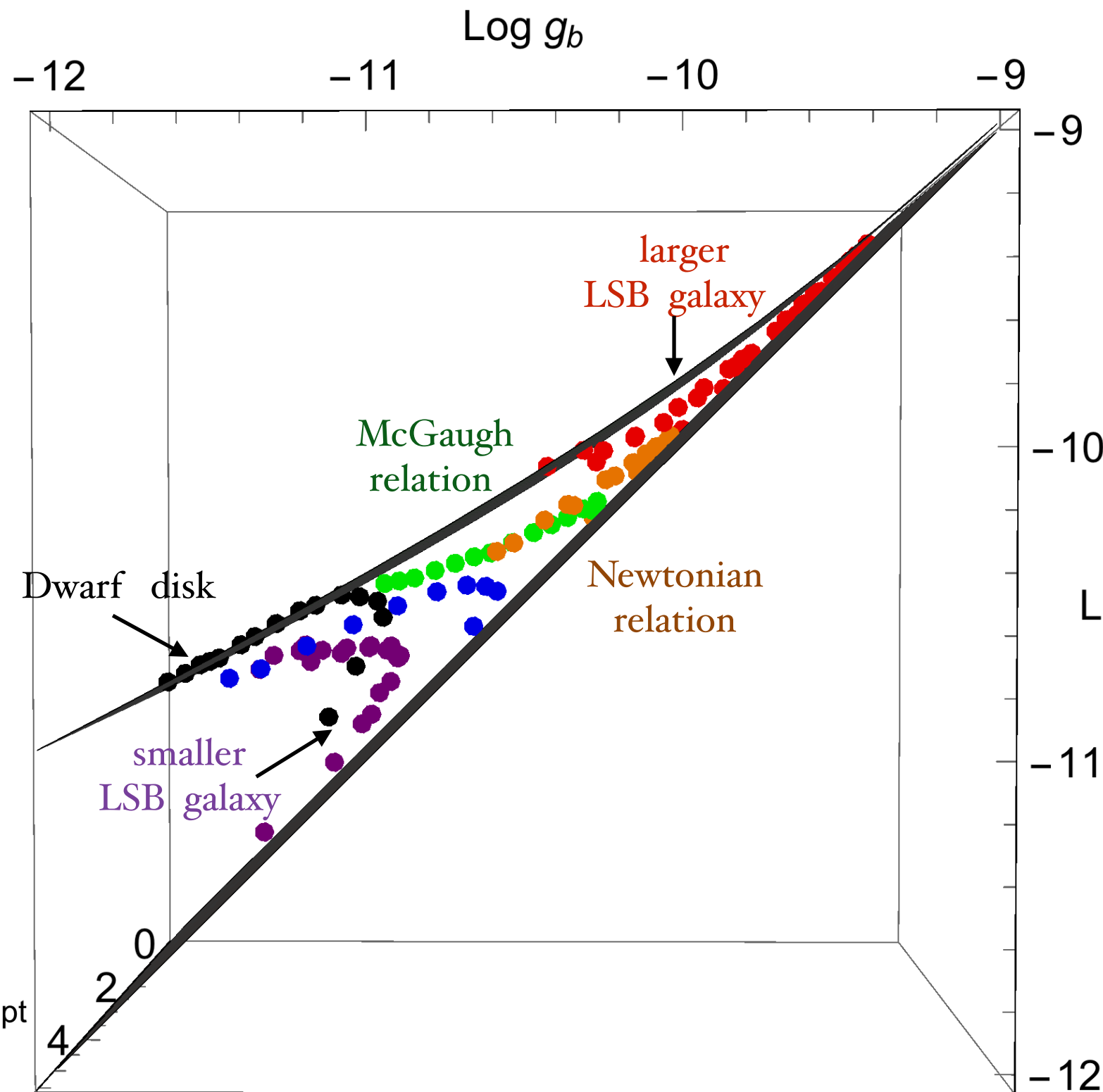
b)  $g \gtrsim g_b$  growth till  $R_D$   
(stellar disk scale length)

$$R_D = R_{\text{opt}}/3.2$$

c)  $g > g_b$  decrease beyond  $R_D$

**boomerang shape**

# $g$ , $g_b$ , $x$ single galaxies test



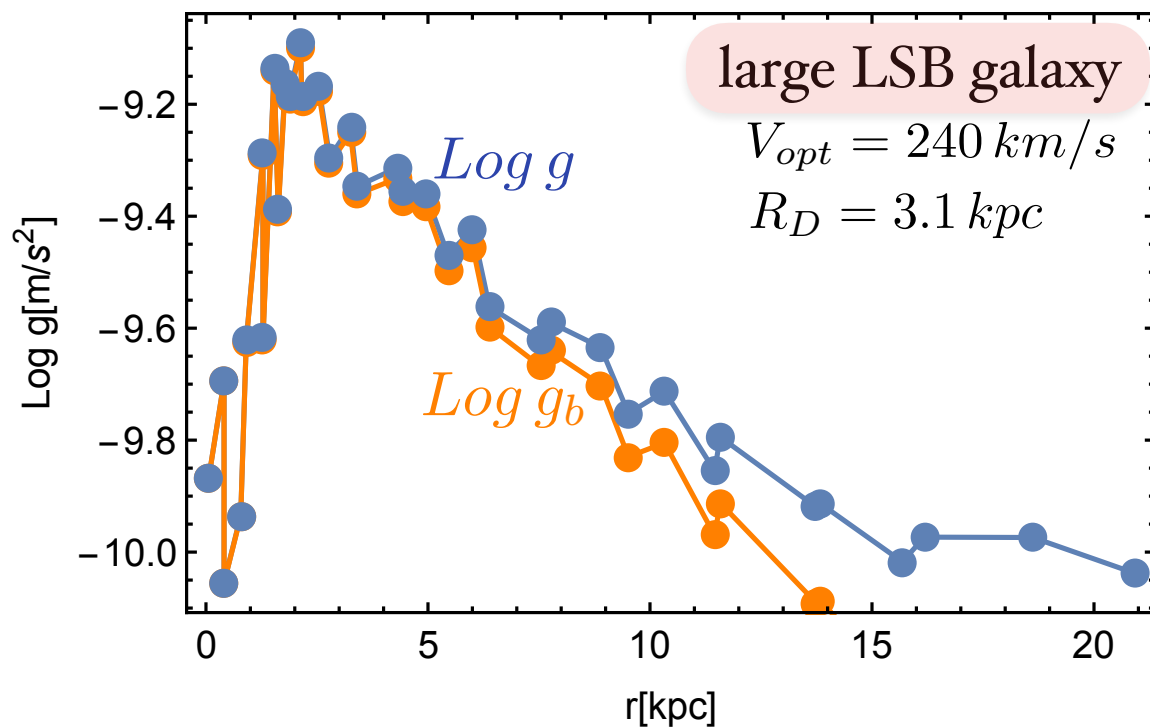
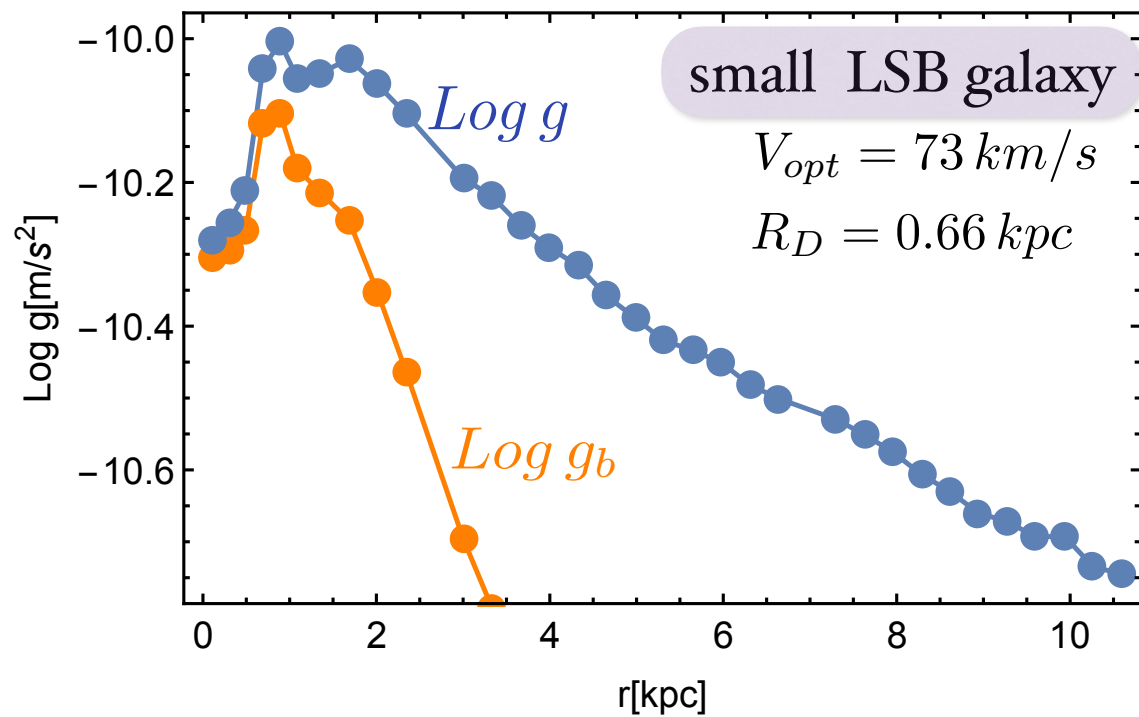
a) larger galaxies  $\blacktriangleright$  higher  $g$  and  $g_b$

b)  $\underline{g \gtrsim g_b}$  growth till  $R_D$   
 Log  $g$  (stellar disk scale length)

c)  $\underline{g > g_b}$  decrease beyond  $R_D$



# $g$ , $g_b$ , $x$ single galaxies test



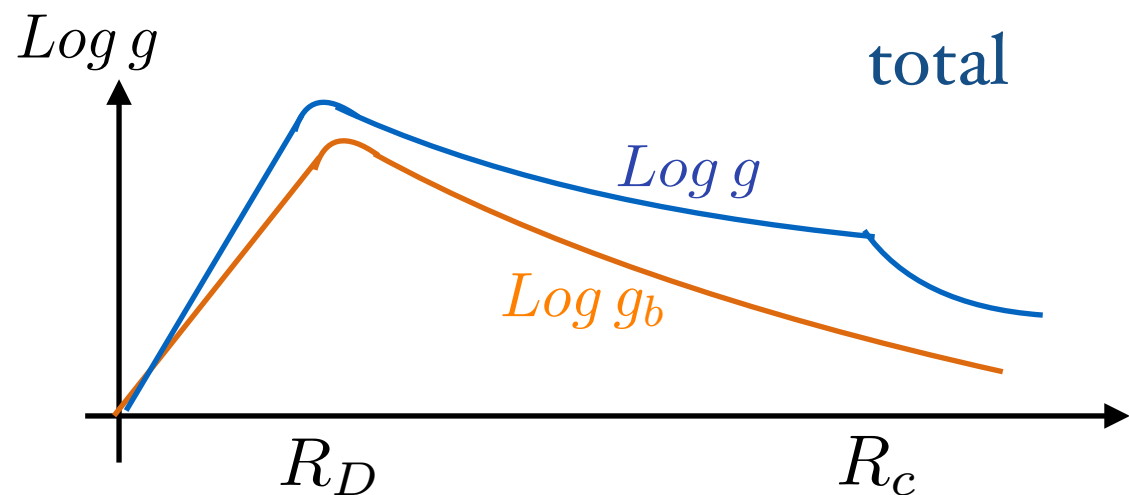
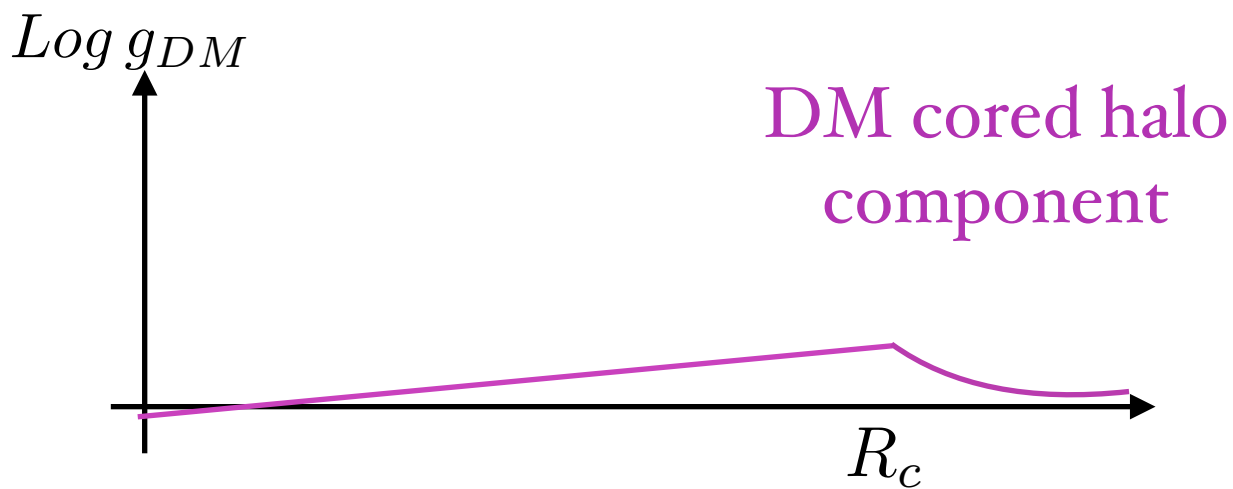
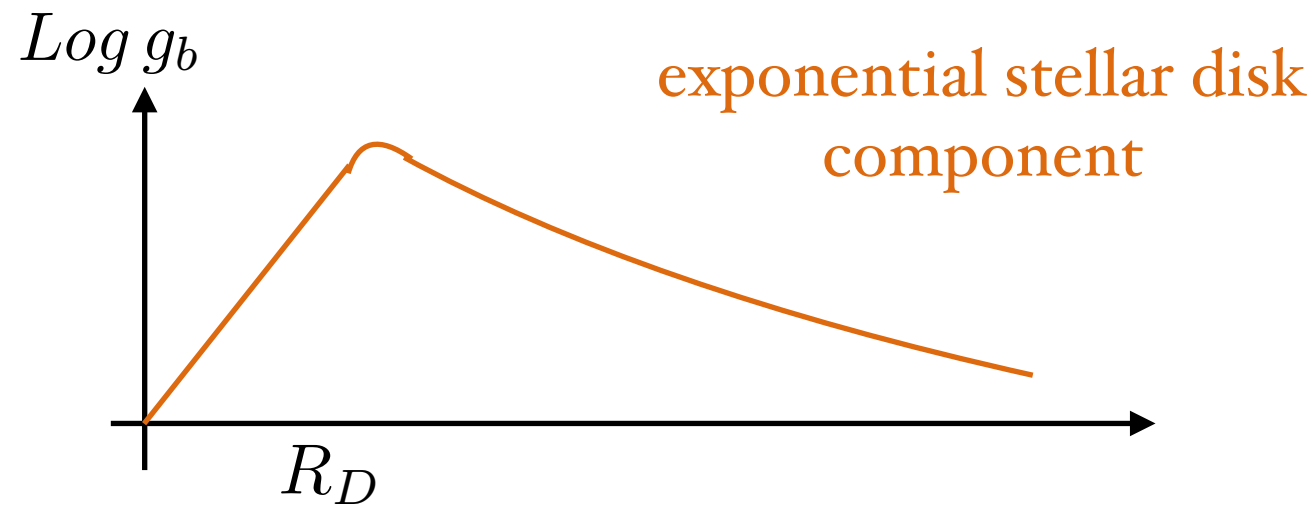
a) larger galaxies  $\blacktriangleright$  higher  $g$  and  $g_b$

b)  $g \gtrsim g_b$  growth till  $R_D$   
(disk scale length)

c)  $g > g_b$  decrease beyond  $R_D$

d)  $g > g_b$  especially evident  
in the smallest galaxies

# $g$ , $g_b$ , $x$ interpreting the evidence



a) larger galaxies  $\blacktriangleright$  higher  $g$  and  $g_b$

b)  $g \gtrsim g_b$  growth till  $R_D$  (disk scale length)

$\downarrow$   
 $\%$  baryonic matter  $>$   $\%$  dark matter inside  $R_D$

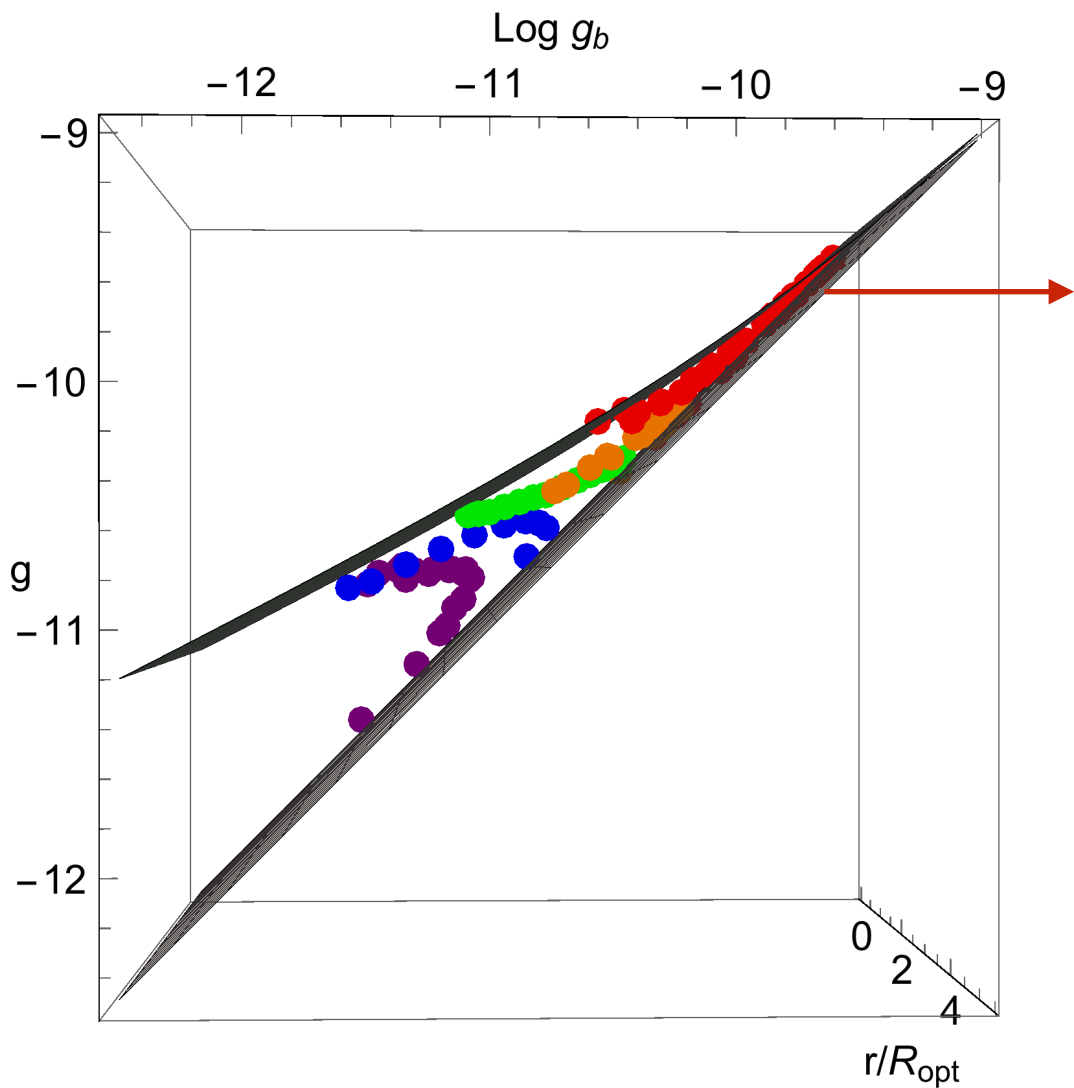
The bulk of matter at high density is inside  $R_D$

c)  $g > g_b$  decrease beyond  $R_D$

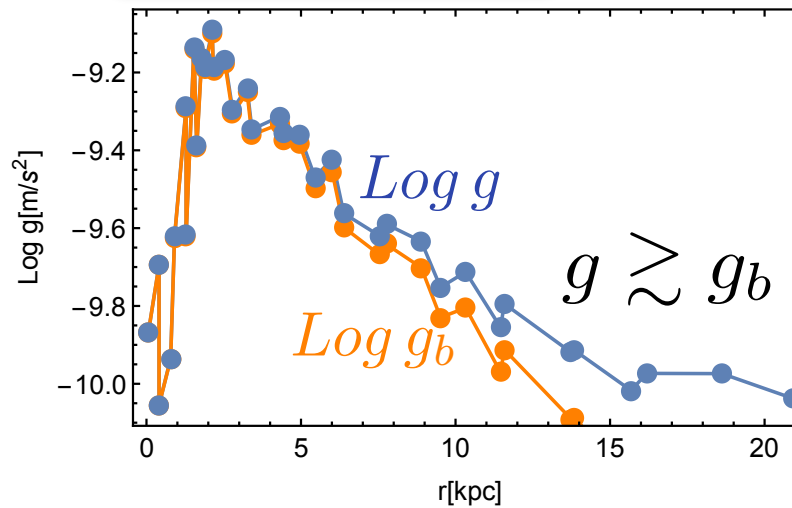
$\downarrow$   
 $\%$  baryonic matter  $<$   $\%$  dark matter in external region

Matter at low density in external region

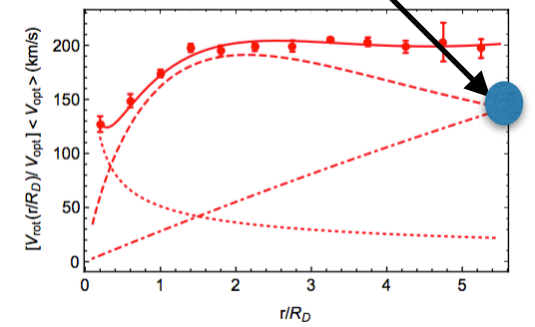
# $g$ , $g_b$ , $x$ interpreting the evidence



large LSB galaxy



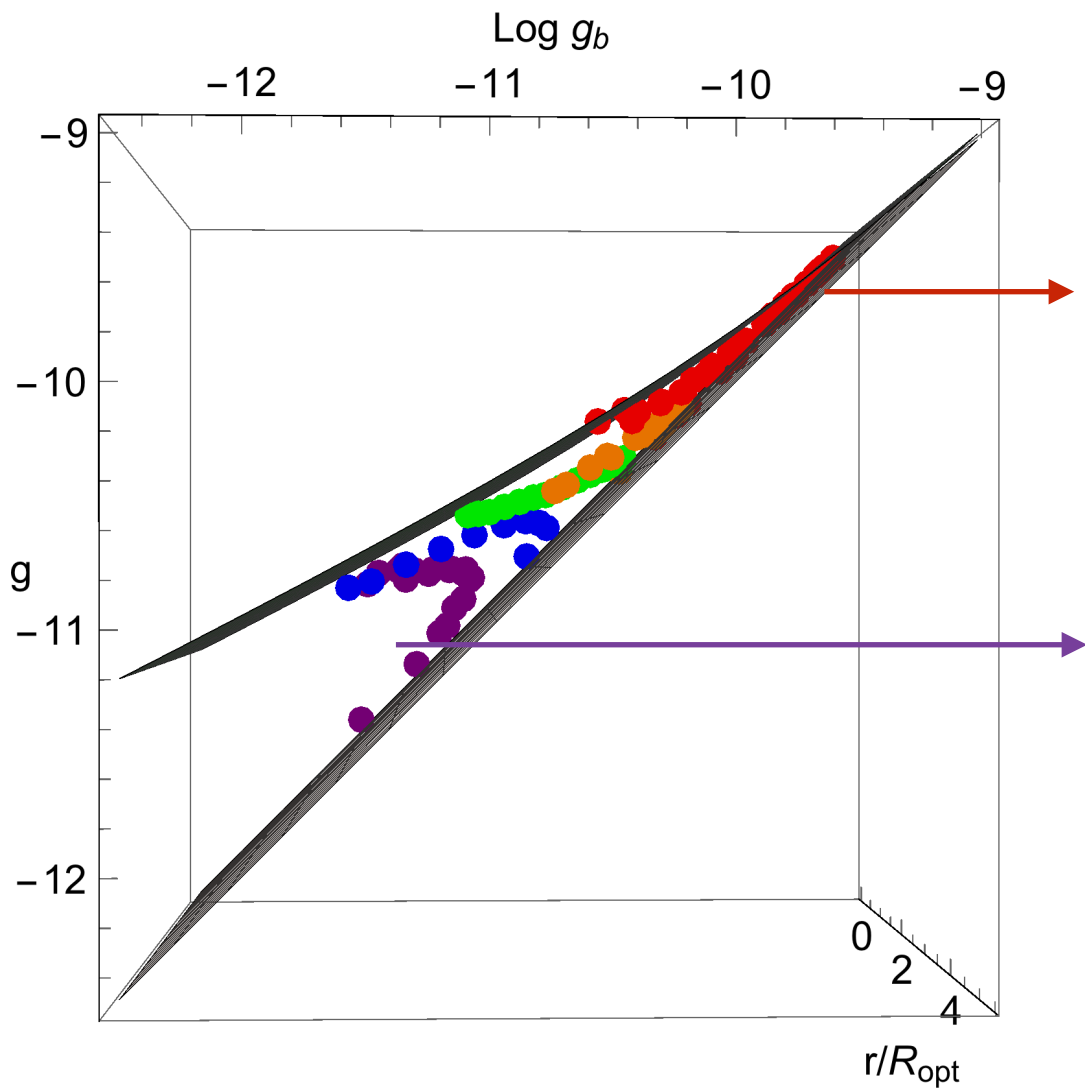
transition



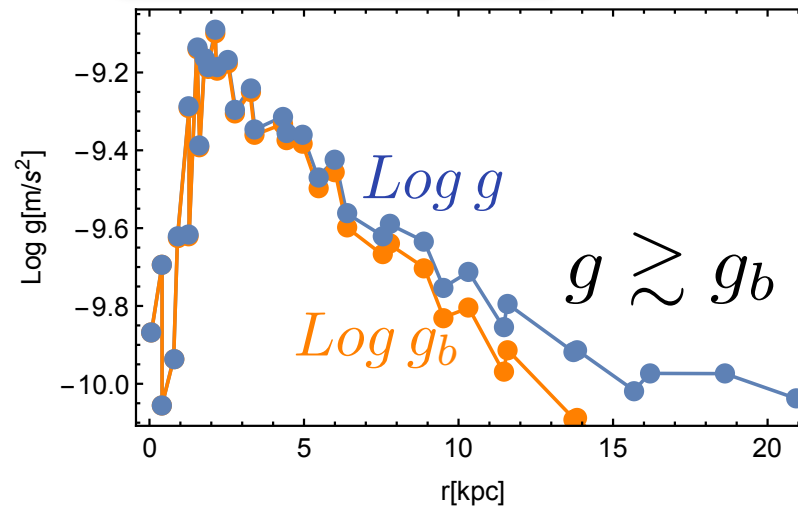
baryonic matter dominant beyond  $\sim R_D$   $g \gtrsim g_b$



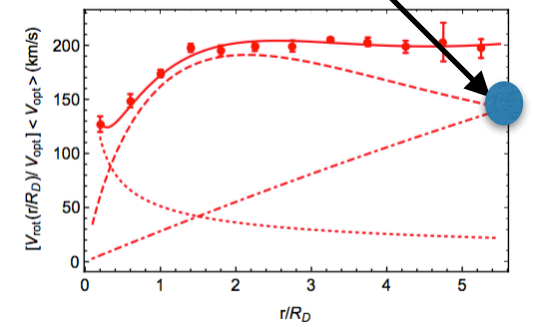
# $g$ , $g_b$ , $x$ interpreting the evidence



large LSB galaxy

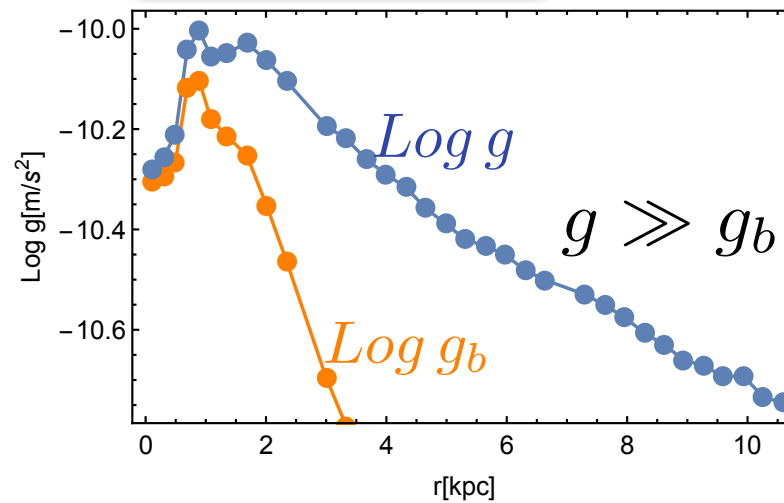


transition

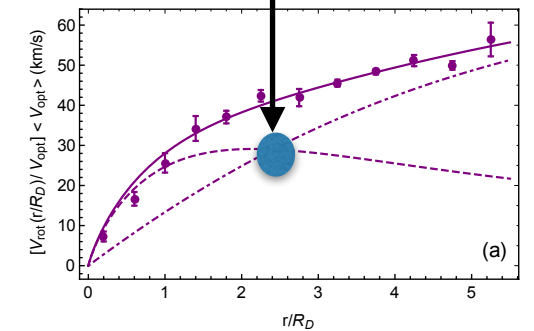


baryonic matter dominant beyond  $\sim R_D$   $g \gtrsim g_b$

small LSB galaxy



transition

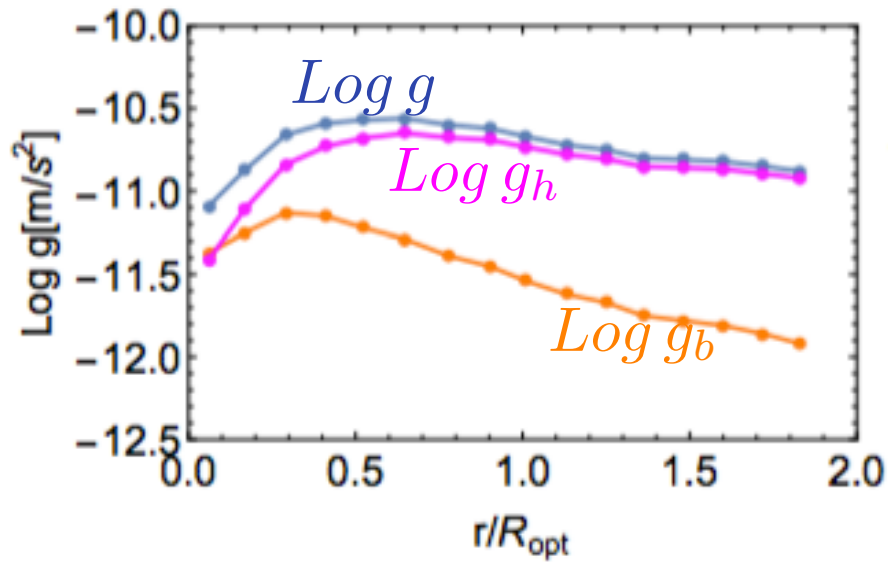


baryonic matter dominant till  $\sim R_D$   $g \gg g_b$

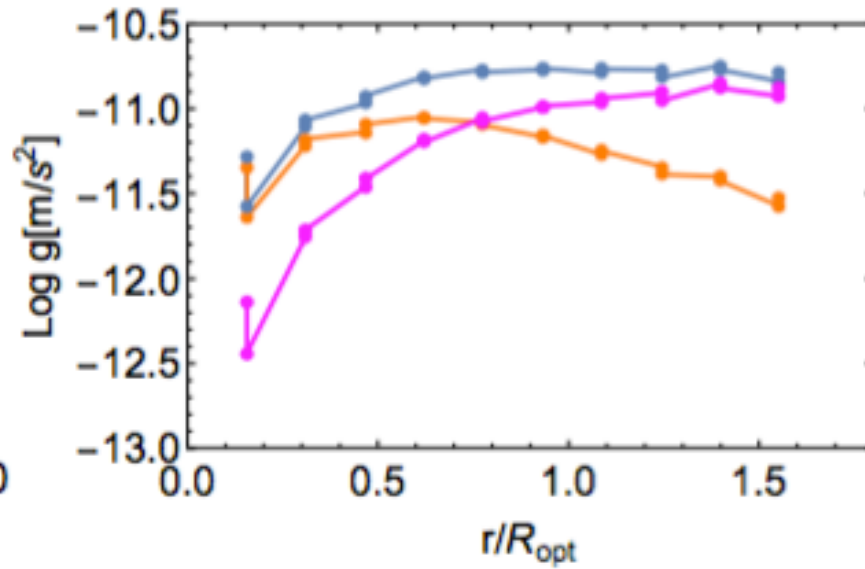
# $g$ , $g_b$ , $x$ interpreting the evidence

For completeness:

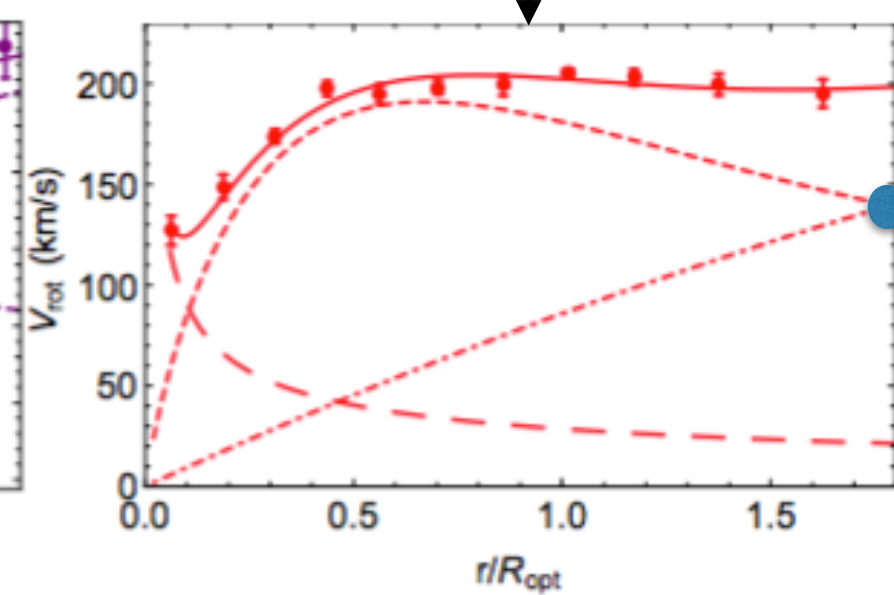
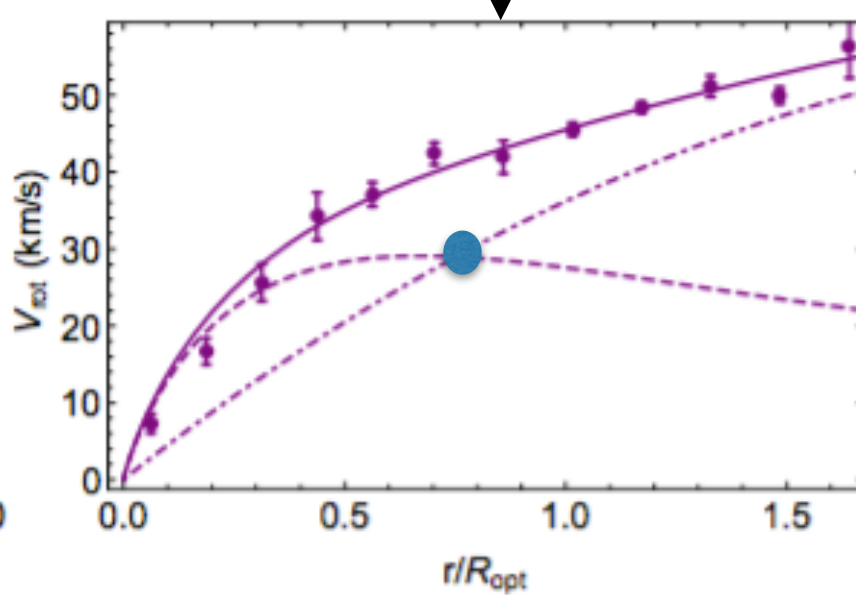
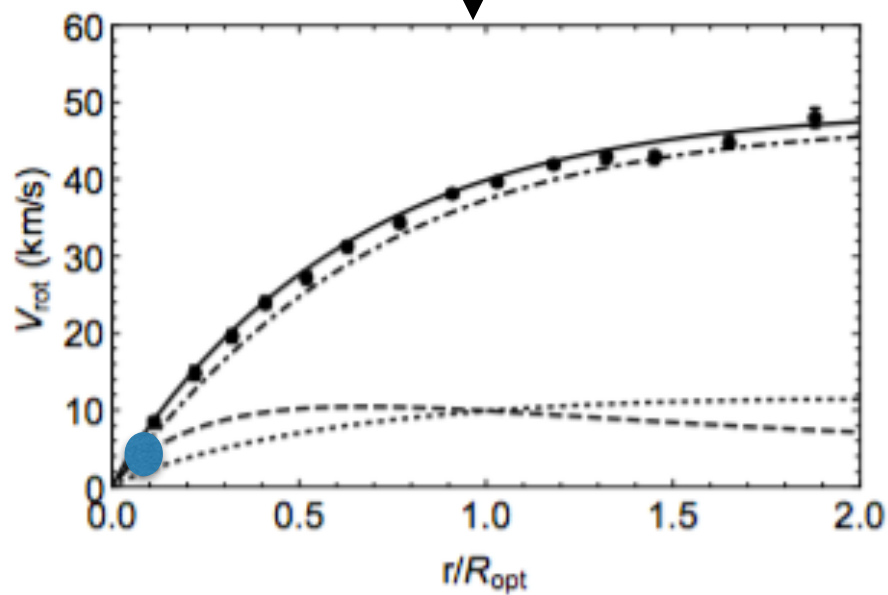
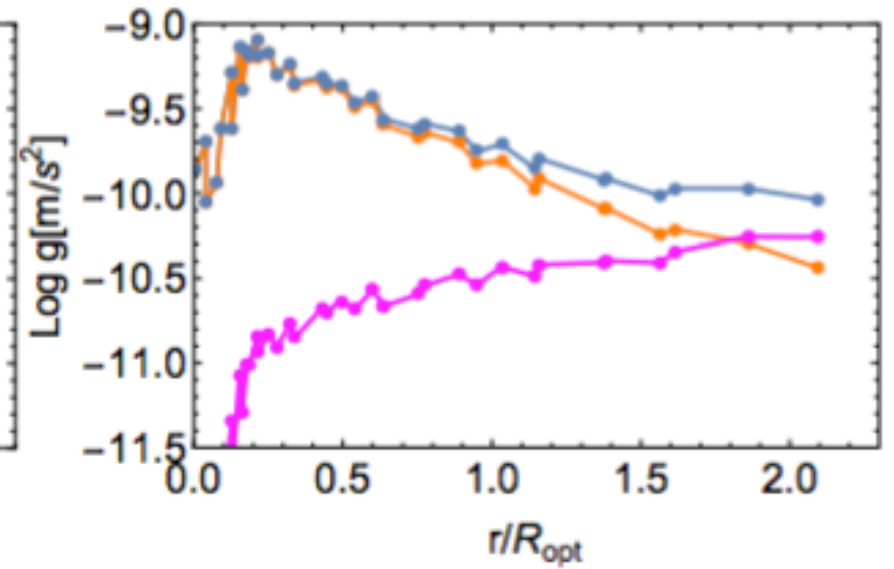
Dwarf Disk



small LSB galaxy



large LSB galaxy



dark matter dominated  
in central region

$\rightarrow g \gg g_b$  everywhere in Dwarf Disk

# Many Consequences

- ~ enough data both inside and outside  $R_D$  to see the boomerang shape



McGaugh , Salucci  
only one side

McGaugh relationship  
not universal



dependence on the radial coordinate

# Many Consequences

- ~ enough data both inside and outside  $R_D$  to see the boomerang shape ▶
  - McGaugh, Salucci only one side
  - McGaugh relationship not universal

dependence on the radial coordinate

## ~ Baryonic fraction

a) narrow boomerang

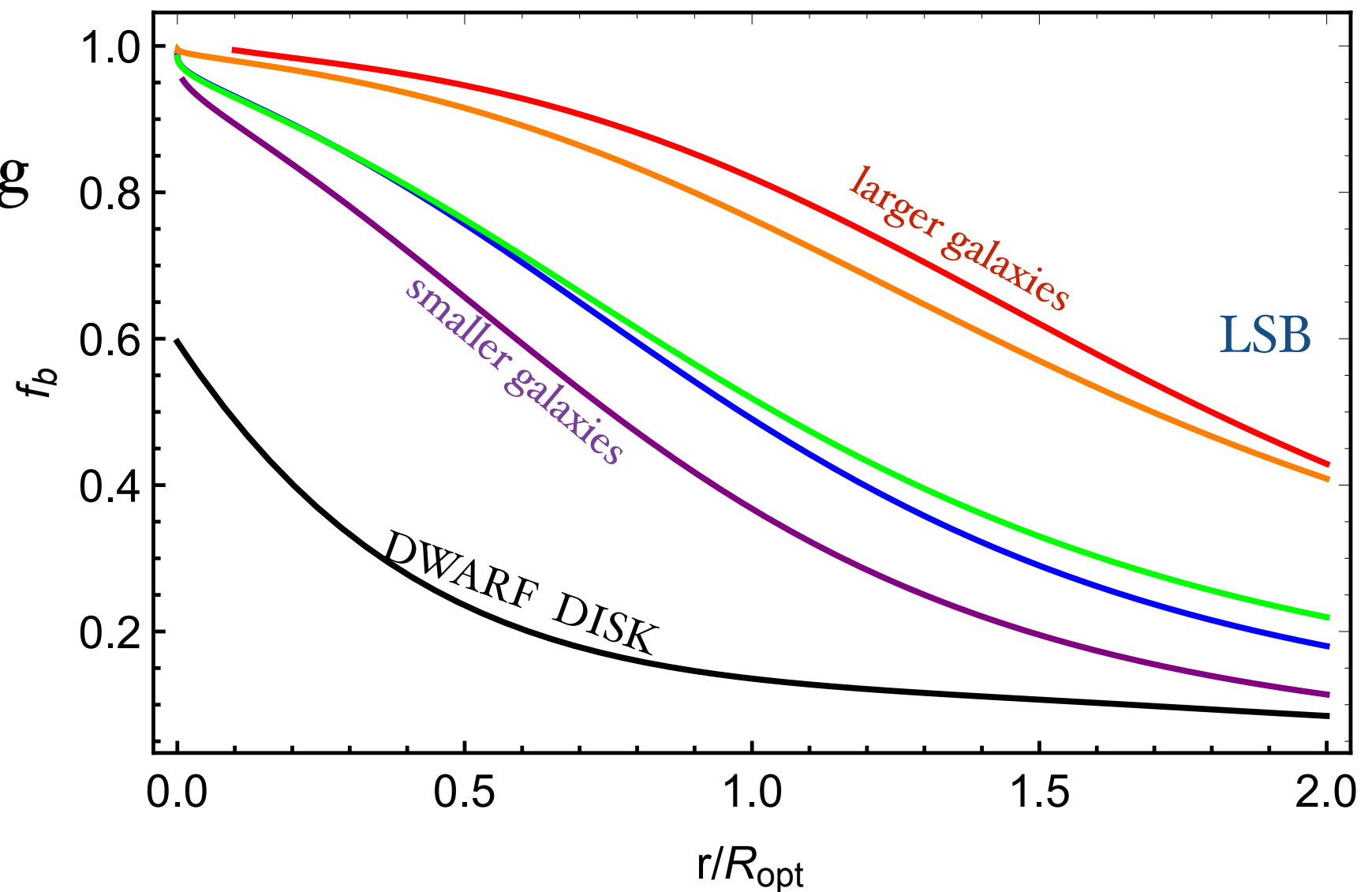
for small  $\frac{\Delta f_b}{\Delta r}$

larger galaxies

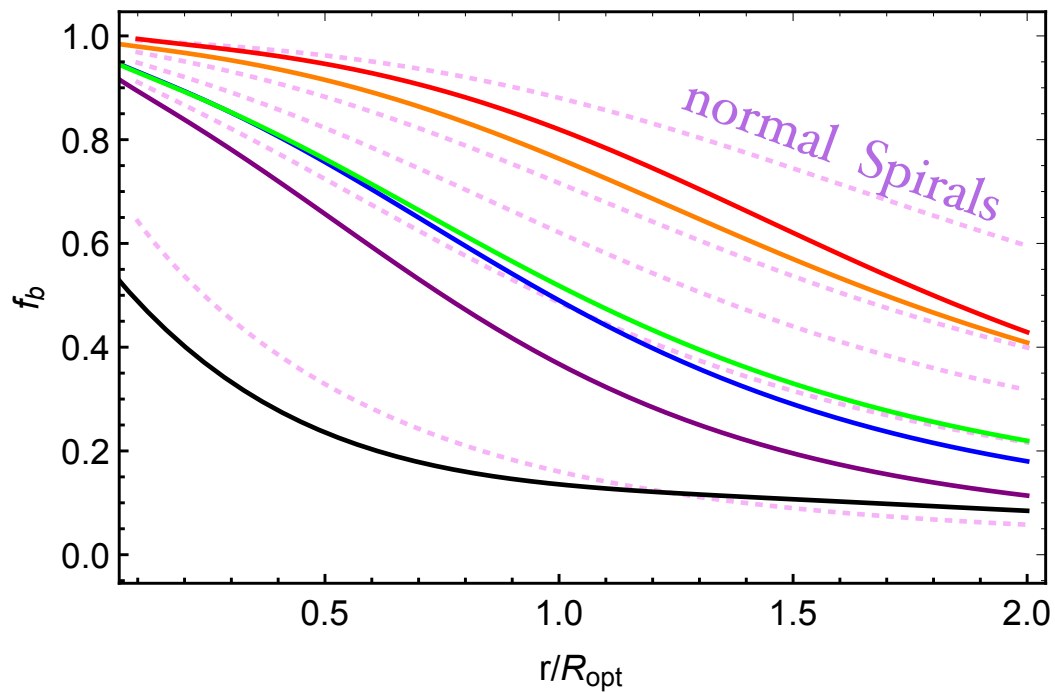
b) large boomerang

for large  $\frac{\Delta f_b}{\Delta r}$

smaller galaxies



# Many Consequences



Even if

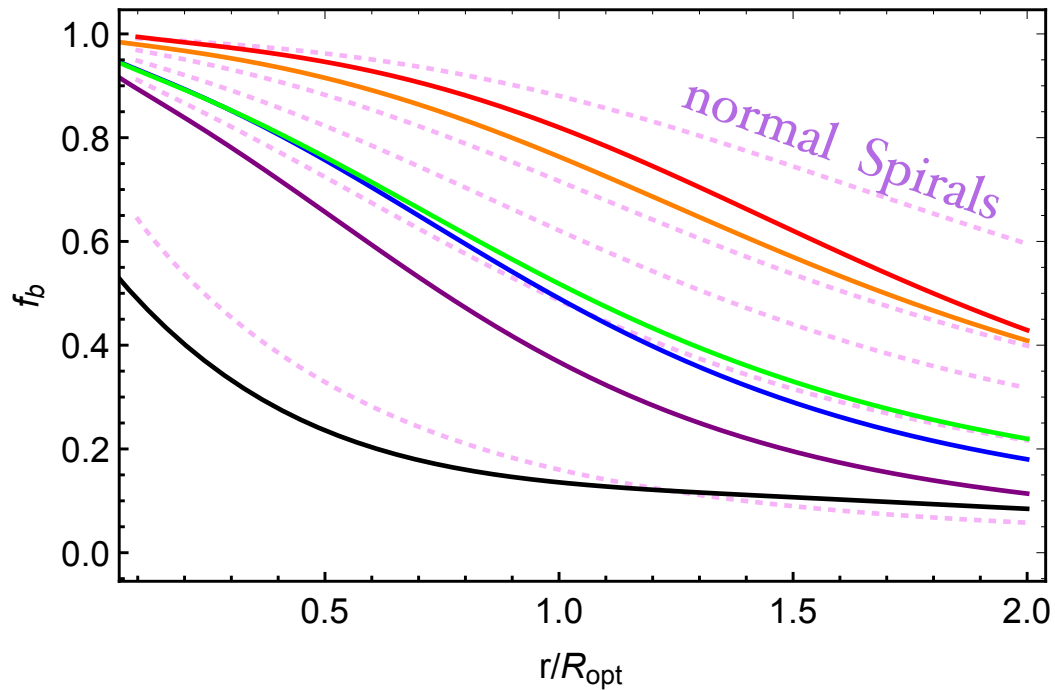
$$f_{b,LSB}(r/R_{opt}) \simeq f_{b,spirals}(r/R_{opt})$$

LSB more extended  
than normal Spirals



$$f_{b,LSB}(r) > f_{b,spiral}(r)$$

# Many Consequences



Even if

$$f_{b,LSB}(r/R_{opt}) \simeq f_{b,spirals}(r/R_{opt})$$

LSB more extended  
than normal Spirals

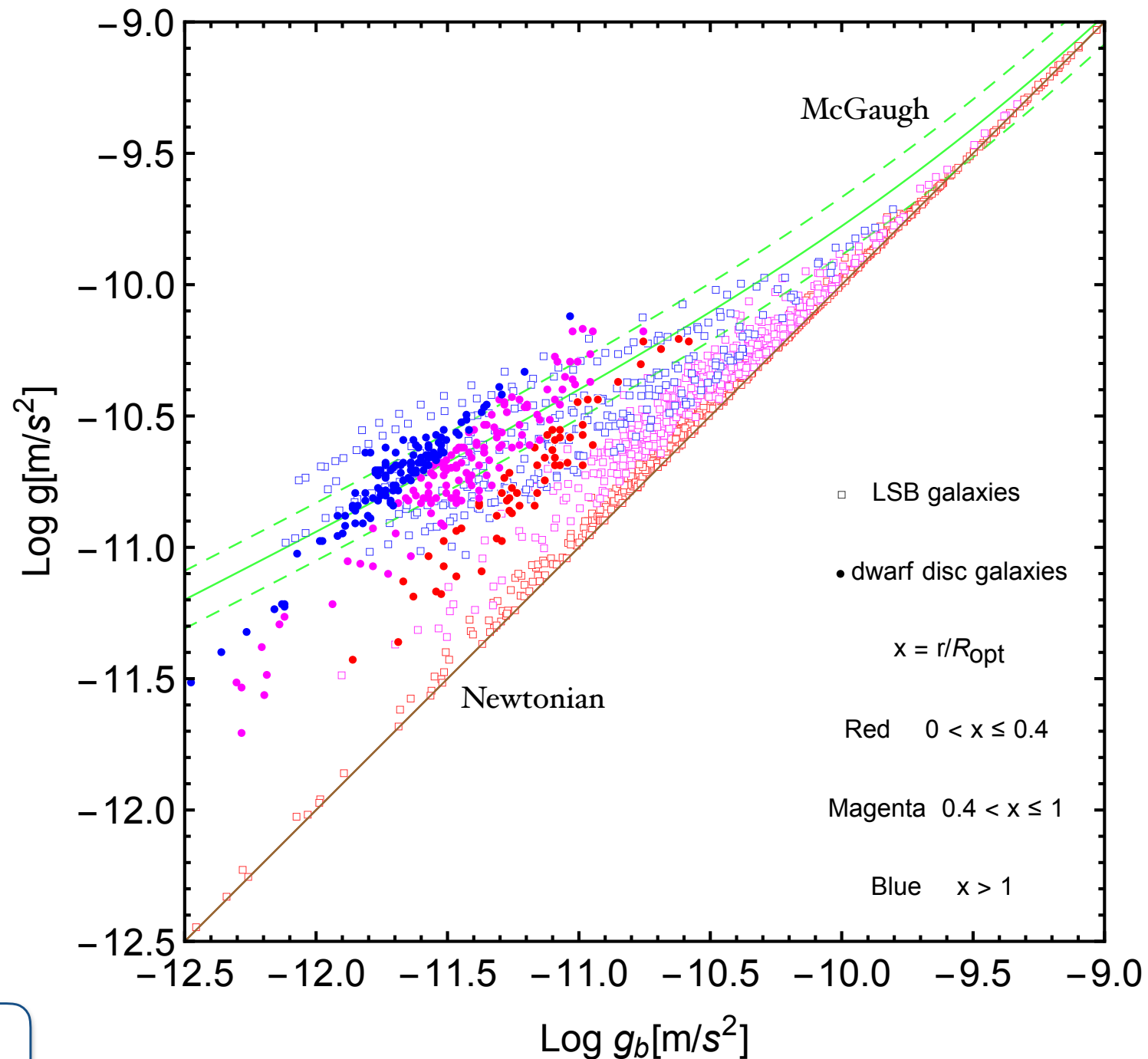


$$f_{b,LSB}(r) > f_{b,spiral}(r)$$



Same  $g_b \rightarrow$  different  $g$

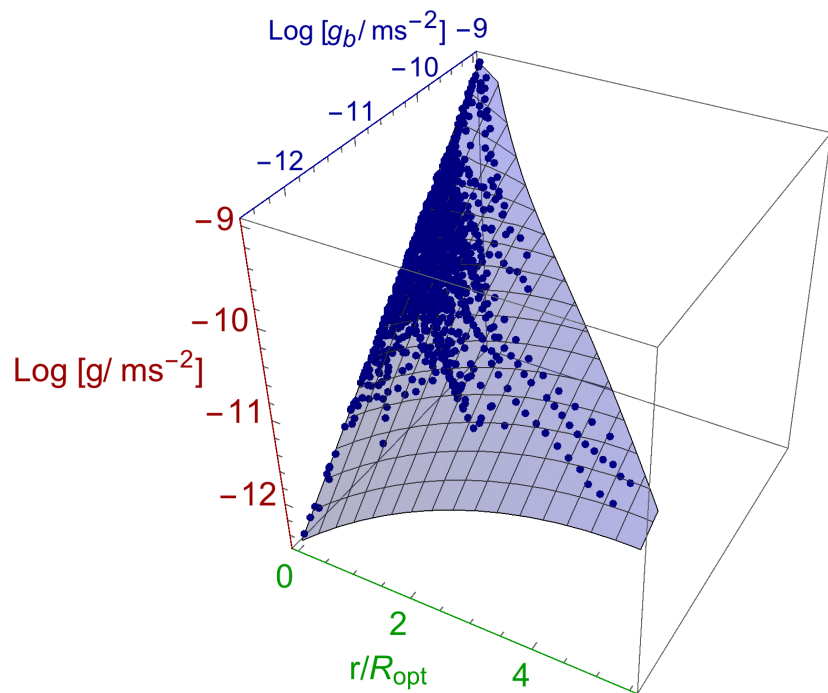
Failure of McGaugh relation in LSB



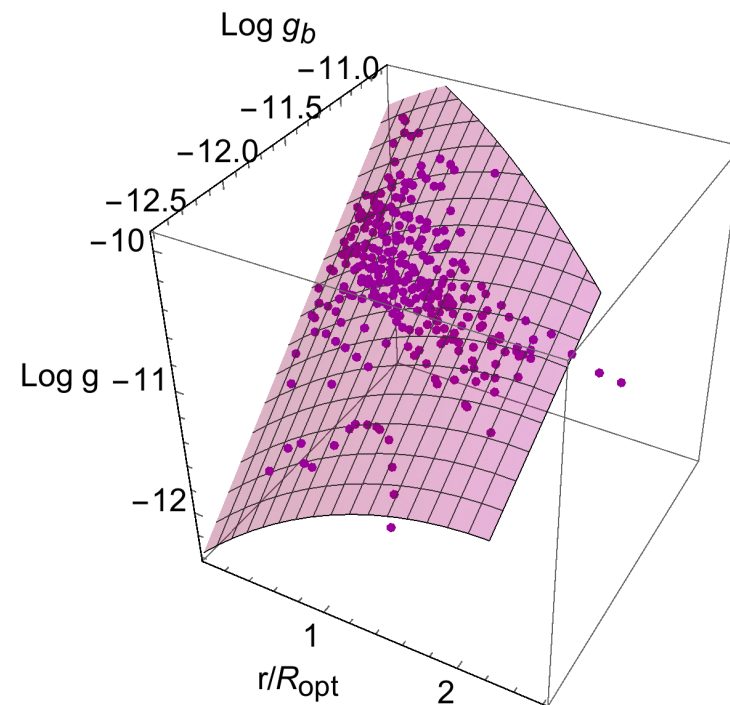
# Conclusions

~ McGaugh relationship (**RAR**):  $g = f(g_b)$  at any radius and in any object

## LSB



## Dwarf Disks



evidence of  
radial dependence

$$g = f(g_b, r/R_{\text{opt}})$$

+

morphology dependence

~ **phenomenological  
understanding**

of our results and  
McGaugh results

→ in the standard  
**baryonic + dark matter** scenario  
and  
mass distribution properties



*Thanks for  
the attention*