



Cosmology 2018 in Dubrovnik

# The effects of scalar meson interactions on Symmetry Energy in RMF theory



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# SYMMETRY ENERGY DEF.

Important for Nuclear Physics and Astrophysics

- Neutron skin thickness in finite nuclei
- Stable nuclei
- Heavy-Ion collisions
- Giant Resonance
- ...

- Supernova explosions
- Neutron emission and cooling of the protoneutron stars
- Mass-Radius relations in NS
- Composition of the crust of NS
- ...

Describes the increase of Energy with increasing asymmetry of matter.

$$E_{sym}(n) = \frac{1}{8} \left. \frac{\partial^2 \epsilon(n, x)}{\partial x^2} \right|_{x=\frac{1}{2}}$$

# EOS of NS matter

Basic inputs:

- $\epsilon(n, x)$  – energy density
- $x$  – proton fraction

Its validity within 1 MeV has been verified by using microscopic many-body theories and phenomenological models + various effective interactions.

Empirical parabolic law:

$$\epsilon(n, x) = \epsilon\left(n, x = \frac{1}{2}\right) + E_{sym}(n)(1 - 2x)^2 + O((1 - 2x)^4)$$

↓  
SNM

Symmetric Nuclear Matter  
(relatively well-determined)

↓  
ANM

Asymmetric Nuclear Matter  
(uncertained)


↓  
(poorly-known)

# IMPORTANCE OF SYMMETRY ENERGY

- Tells us about the structure and composition of a NS outer crust.
- The faster  $E_{sym}$  increases with  $n$ , the more exotic the composition. \*
- The high-density behavior of the  $E_{sym}$  is currently the most uncertain part of the EOS of dense neutron-rich matter.

$E_{sym}$  around saturation density:

$$E_{sym}(n) = (J + Lx + \frac{1}{2} Kx^2 + O(x^3)), \quad J = E_{sym}(n_0)$$



Governs many nuclear physics  
and astrophysics observables

\* Physical Review C 78, 025807 (2008).



# EMPIRICAL VALUES

Significant progress have been made recently in constraining the  $E_{sym}(n)$  around and below  $n_0$ .

- Saturation density:

$$n_0 = 0.16 \text{ fm}^{-3}$$

- Binding Energy for SNM:

$$B = \epsilon\left(n_0, \frac{1}{2}\right) / n_0 - m = -16 \text{ MeV}$$

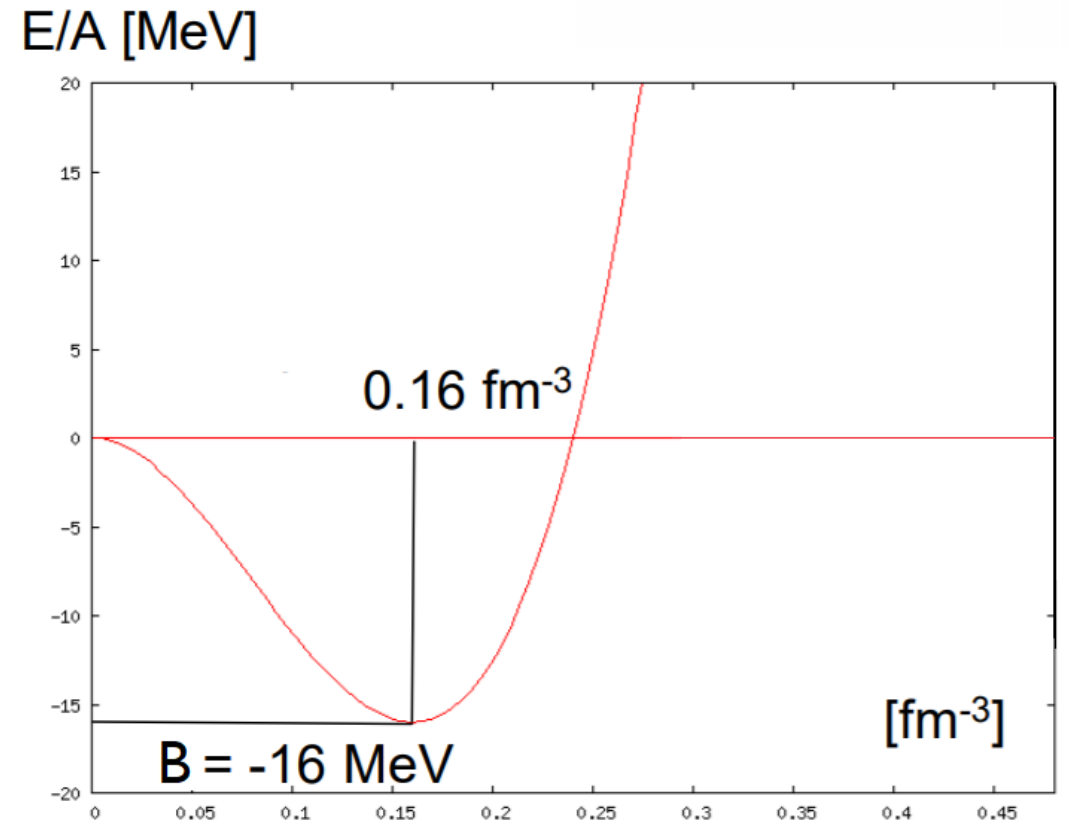
- Incompressibility:

$$K = 230 \text{ MeV}$$

- Slope:

$$L = 40 - 70 \text{ MeV} *$$

$$E_{sym}(n) = (J + Lx + \frac{1}{2} Kx^2 + O(x^3))$$



\* B. A. Li and X. Han, Phys. Lett. B 727, 276 (2013).

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- Incompressibility:

$$K = 230 \text{ MeV}$$

$$\longleftrightarrow K = 9 n^2 \left. \frac{\partial^2 E_{sym}}{\partial n^2} \right|_{n_0}$$

**K:** quantity in the study of nuclei, supernova collapse, neutron stars, and heavy-ion collisions - it is directly related to the curvature of nuclear matter EOS.

- Slope:

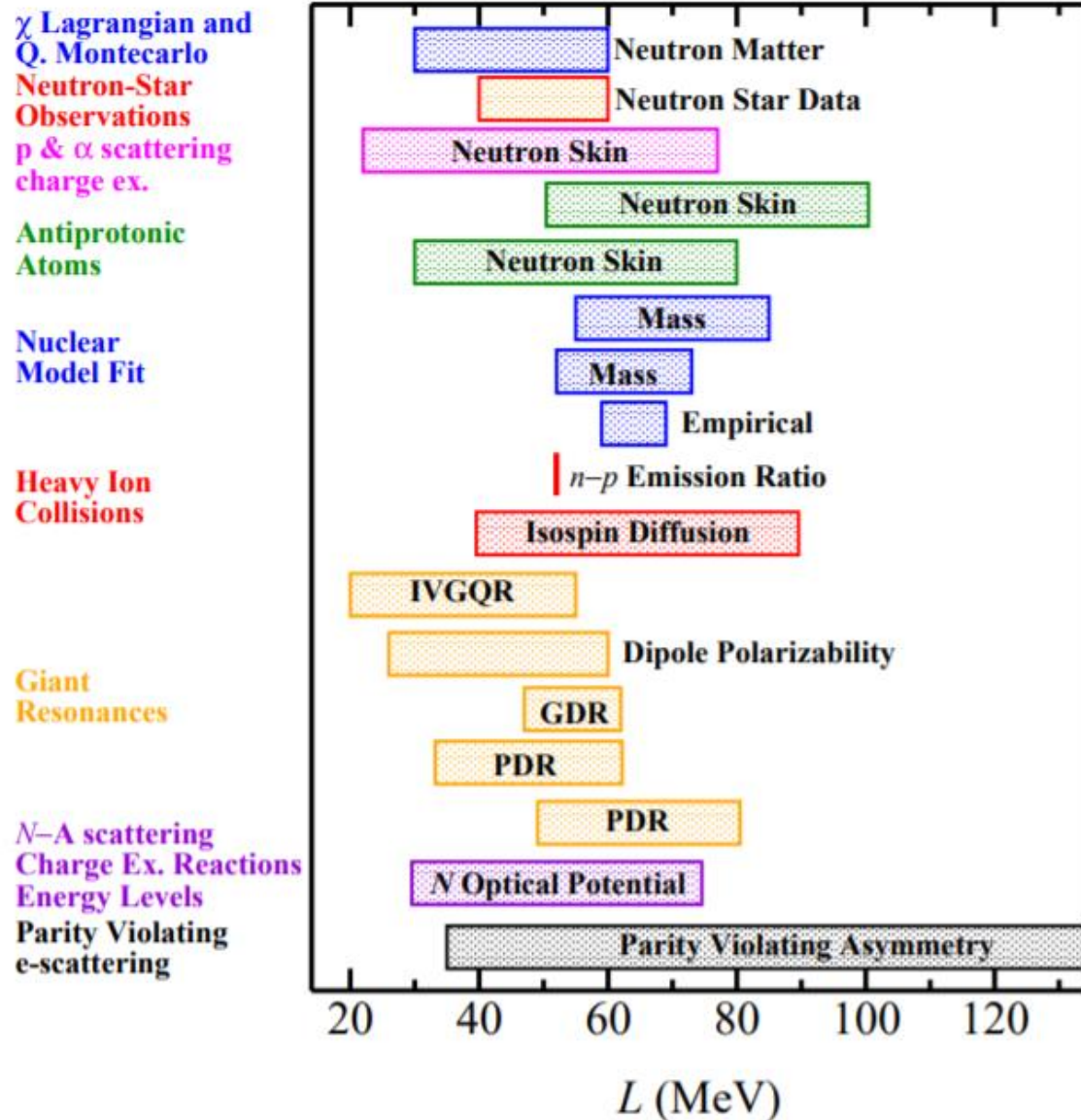
$$L = 40 - 70 \text{ MeV} *$$

$$\longleftrightarrow L = 3n \left. \frac{\partial E_{sym}}{\partial n} \right|_{n_0} \longleftrightarrow$$

**L is important for :** the size of the neutron skin in heavy nuclei, location of the neutron drip line, core-crust transition density and gravitational binding energy of neutron stars.

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# Slope from various studies



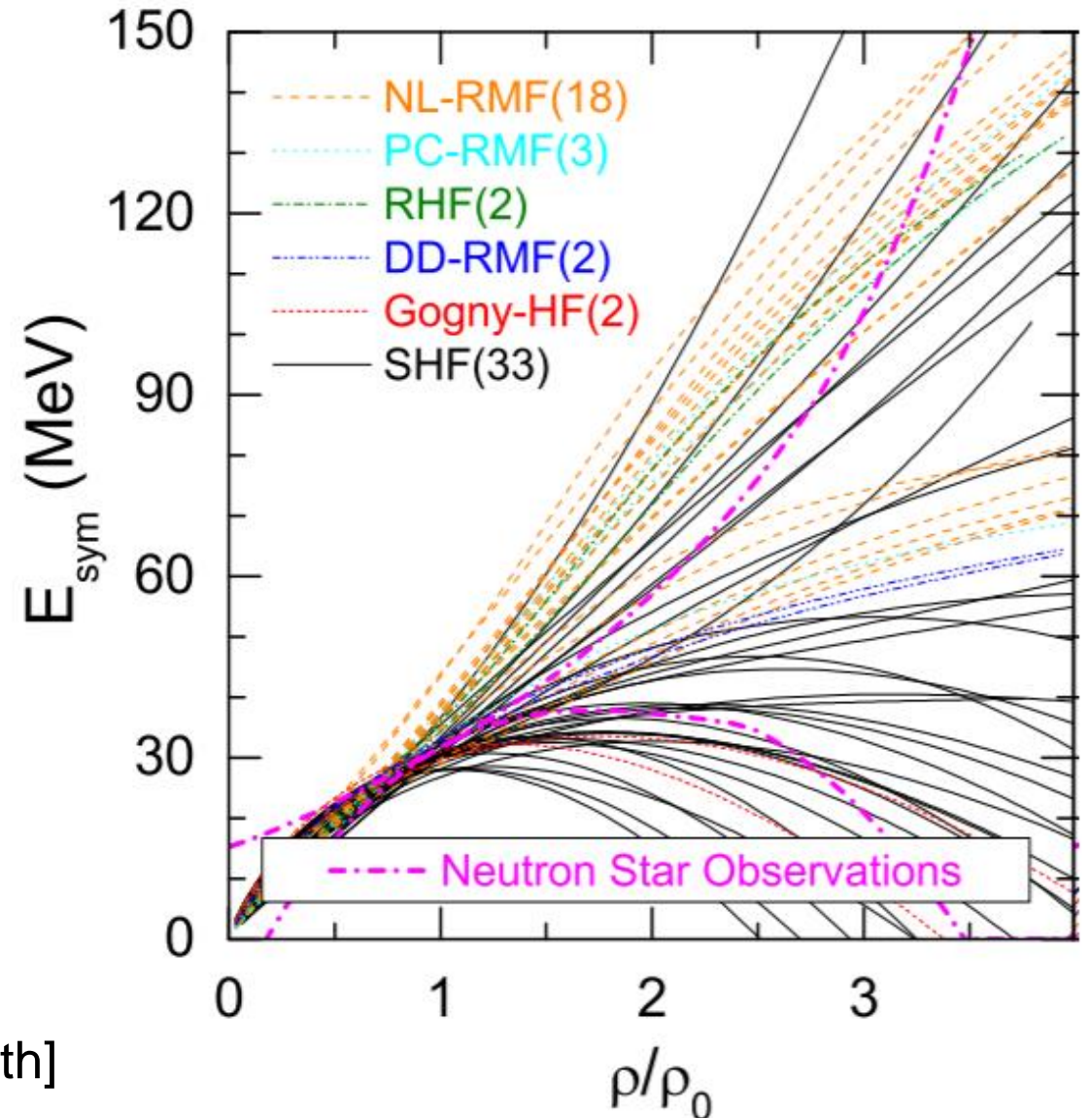
Hebeler *et al.* PRL105 (2010) 161102  
 and Gandolfi *et al.* PRC85 (2012) 032801 (R)  
 Steiner *et al.* Astrophys. J. 722 (2010) 33  
 Lie-Wen Chen *et al.* PRC 82 (2010) 024321  
 Centelles *et al.* PRL 102 (2009) 122502  
 Warda *et al.* PRC 80 (2009) 024316  
 Möller *et al.* PRL 108 (2012) 052501  
 Danielewicz NPA 727 (2003) 233  
 Agrawal *et al.* PRL109 (2012) 262501  
 Famiano *et al.* PRL 97 (2006) 052701  
 Tsang *et al.* PRL 103 (2009) 122701  
 Roca-Maza *et al.* PRC 87 (2013) 034301  
 Roca-Maza *et al.* PRC (2013), in press  
 Trippa *et al.* PRC 77 (2008) 061304(R)  
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 Carbone *et al.* PRC 81 (2010) 041301(R)  
 Xu *et al.* PRC 82 (2010) 054607  
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# RELATIVISTIC MEAN FIELD THEORY (RMF)

- Good description of the nuclear matter in higher densities.
- Contains both: nucleonic and mesonic degrees of freedom.

Fig. 60 selected representatives from 6 classes of phenomenological models and/or energy density functional theories including:

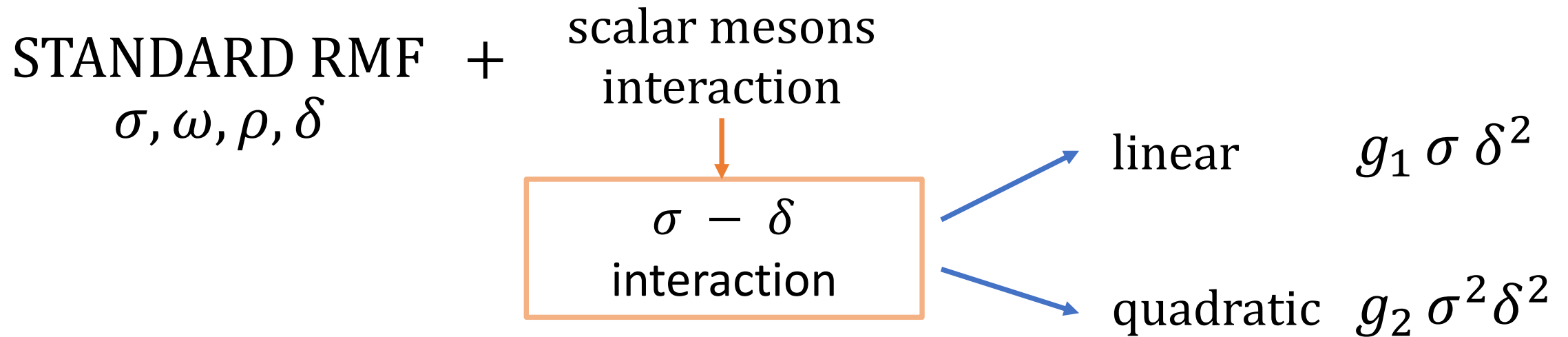
- Relativistic Mean Field (RMF) using 3 different kinds of coupling schemes,
- Relativistic Hartree-Fock (RHF),
- Gogny Hartree-Fock (HF) Skyrme Hartree-Fock





# RELATIVISTIC MEAN FIELD THEORY (RMF)

- Good description of the nuclear matter in higher densities.
- Contains both: nucleonic and mesonic degrees of freedom.



$$\begin{aligned}
 \mathcal{L} = & \frac{1}{2} (\partial_\mu \sigma \partial^\mu \sigma - m_\sigma^2 \sigma^2) - \frac{1}{4} (\partial_\mu \omega_\nu - \partial_\nu \omega_\mu) (\partial^\mu \omega^\nu - \partial^\nu \omega^\mu) + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu - \frac{1}{4} (\partial_\mu \vec{\rho}_\nu - \partial_\nu \vec{\rho}_\mu) (\partial^\mu \vec{\rho}^\nu - \partial^\nu \vec{\rho}^\mu) + \frac{1}{2} m_\rho^2 \vec{\rho}_\mu \vec{\rho}^\mu \\
 & + \frac{1}{2} \partial_\mu \vec{\delta} \partial^\mu \vec{\delta} - \frac{1}{2} m_\delta^2 \vec{\delta}^2 + \bar{\psi} (i \partial_\mu \gamma^\mu - m) \psi + g_\sigma \sigma \bar{\psi} \psi - g_\omega \omega_\mu \bar{\psi} \gamma^\mu \psi - \frac{1}{2} g_\rho \vec{\rho}_\mu \bar{\psi} \gamma^\mu \vec{\tau} \psi + g_\delta \vec{\delta} \bar{\psi} \vec{\tau} \psi - U(\sigma) + \mathcal{L}_{\sigma\delta},
 \end{aligned}$$

# RELATIVISTIC MEAN FIELD THEORY (RMF)

Step 1. Lagrangian  $\mathcal{L} = \frac{1}{2} (\partial_\mu \sigma \partial^\mu \sigma - m_\sigma^2 \sigma^2) - \frac{1}{4} (\partial_\mu \omega_\nu - \partial_\nu \omega_\mu) (\partial^\mu \omega^\nu - \partial^\nu \omega^\mu) + \dots$

Step 2. Energy density (in effective masses language)

$$\epsilon = \frac{2}{(2\pi)^3} \left( \int_0^{k_p} d^3k \sqrt{k^2 + m_p^{*2}} + \int_0^{k_n} d^3k \sqrt{k^2 + m_n^{*2}} \right) + \frac{1}{2} C_\sigma^2 (m - \bar{m}^*)^2 + \frac{1}{2} C_\omega^2 n^2 + \frac{1}{8} C_\rho^2 (2x - 1)^2 n^2 + \frac{1}{8} C_\delta^2 (\Delta m^*)^2 + 2g_\alpha (\Delta m^*)^2 (m - \bar{m}^*)^\alpha + U(m - \bar{m}^*)$$

Coupling constants of each meson ( $C_i^2 = \frac{g_i^2}{m_i^2}$ ,  $i = \sigma, \omega, \rho, \delta$ )

Effective masses  $m_p^* = m - g_\sigma \bar{\sigma} - g_\delta \bar{\delta}_3$ ,  
 $m_n^* = m - g_\sigma \bar{\sigma} + g_\delta \bar{\delta}_3$ .

Proton fraction ( $x = \frac{n_p}{n}$ )

# SYMMETRY ENERGY

Step 1. Lagrangian  $\mathcal{L} = \frac{1}{2} (\partial_\mu \sigma \partial^\mu \sigma - m_\sigma^2 \sigma^2) - \frac{1}{4} (\partial_\mu \omega_\nu - \partial_\nu \omega_\mu) (\partial^\mu \omega^\nu - \partial^\nu \omega^\mu) + \dots$

Step 2. Energy density (in effective masses language)

$$\epsilon = \frac{2}{(2\pi)^3} \left( \int_0^{k_p} d^3k \sqrt{k^2 + m_p^{*2}} + \int_0^{k_n} d^3k \sqrt{k^2 + m_n^{*2}} \right) + \frac{1}{2} \frac{1}{C_\sigma^2} (m - \bar{m}^*)^2 + \frac{1}{2} C_\omega^2 n^2 + \frac{1}{8} C_\rho^2 (2x - 1)^2 n^2 + \frac{1}{8} \frac{1}{C_\delta^2} (\Delta m^*)^2 + 2g_\alpha (\Delta m^*)^2 (m - \bar{m}^*)^\alpha + U(m - \bar{m}^*)$$

Step 3. Symmetry energy

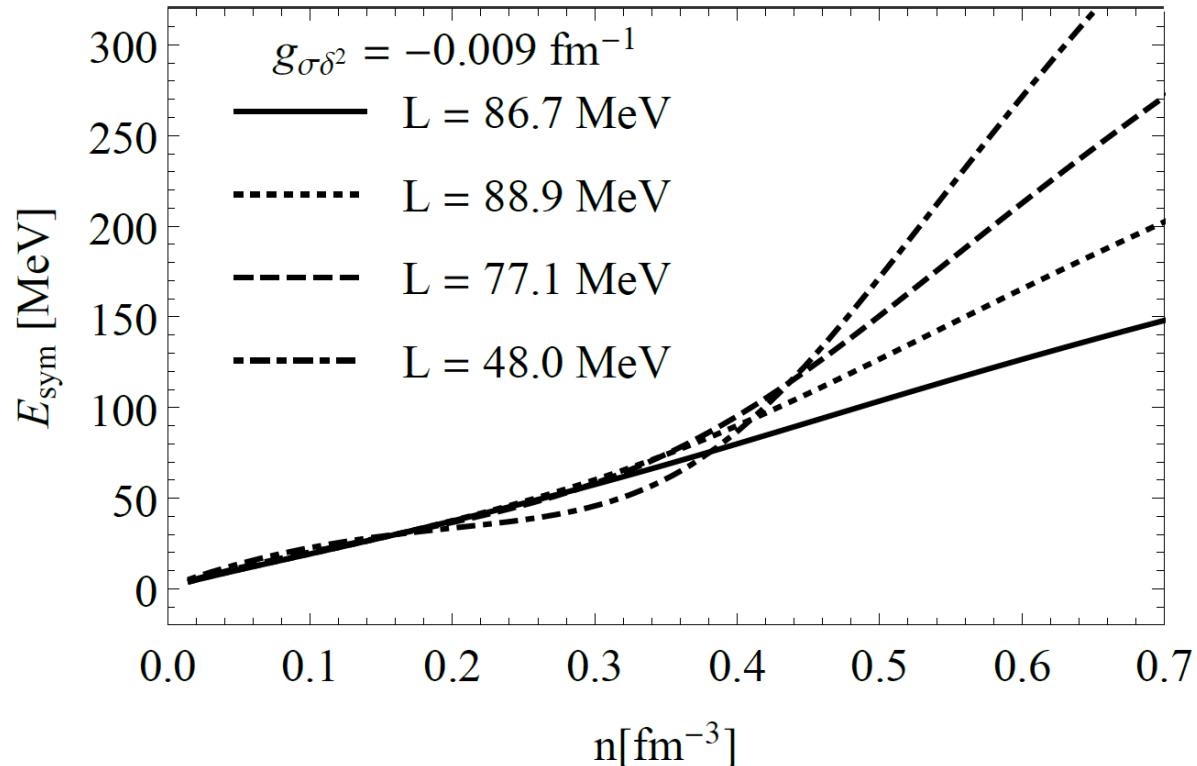
$$E_{sym}(n) = \frac{1}{8} C_\rho^2 n + \frac{k_0^2}{6\sqrt{k_0^2 + m_0^{*2}}} - C_\delta^2 \frac{m_0^{*2} n}{2(k_0^2 + m_0^{*2}) (1 + C_\delta^2 A - 8g_\alpha C_\delta^2 (m - m_0^*)^\alpha)}$$

Step 4. Slope  $L$

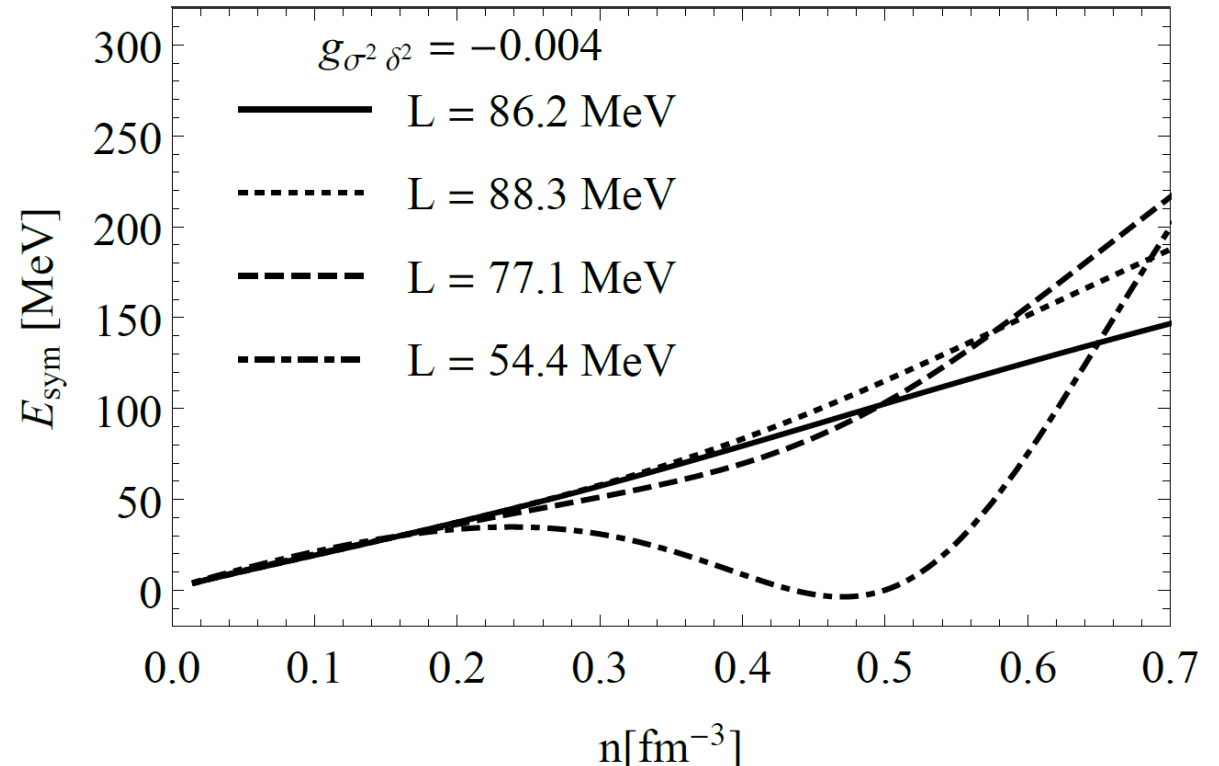
Step 5. Precedure:  $C_\delta^2 = 0.5, 1.5, 2.5, 3.5 \rightarrow C_\rho^2 \rightarrow L$

# RESULTS

linear



quadratic



With  $\sigma - \delta$  interaction the  $E_{\text{sym}}$  can be close to zero.

Better control of the slope (negative  $g_{\alpha}$  gives sufficiently small  $L$ )

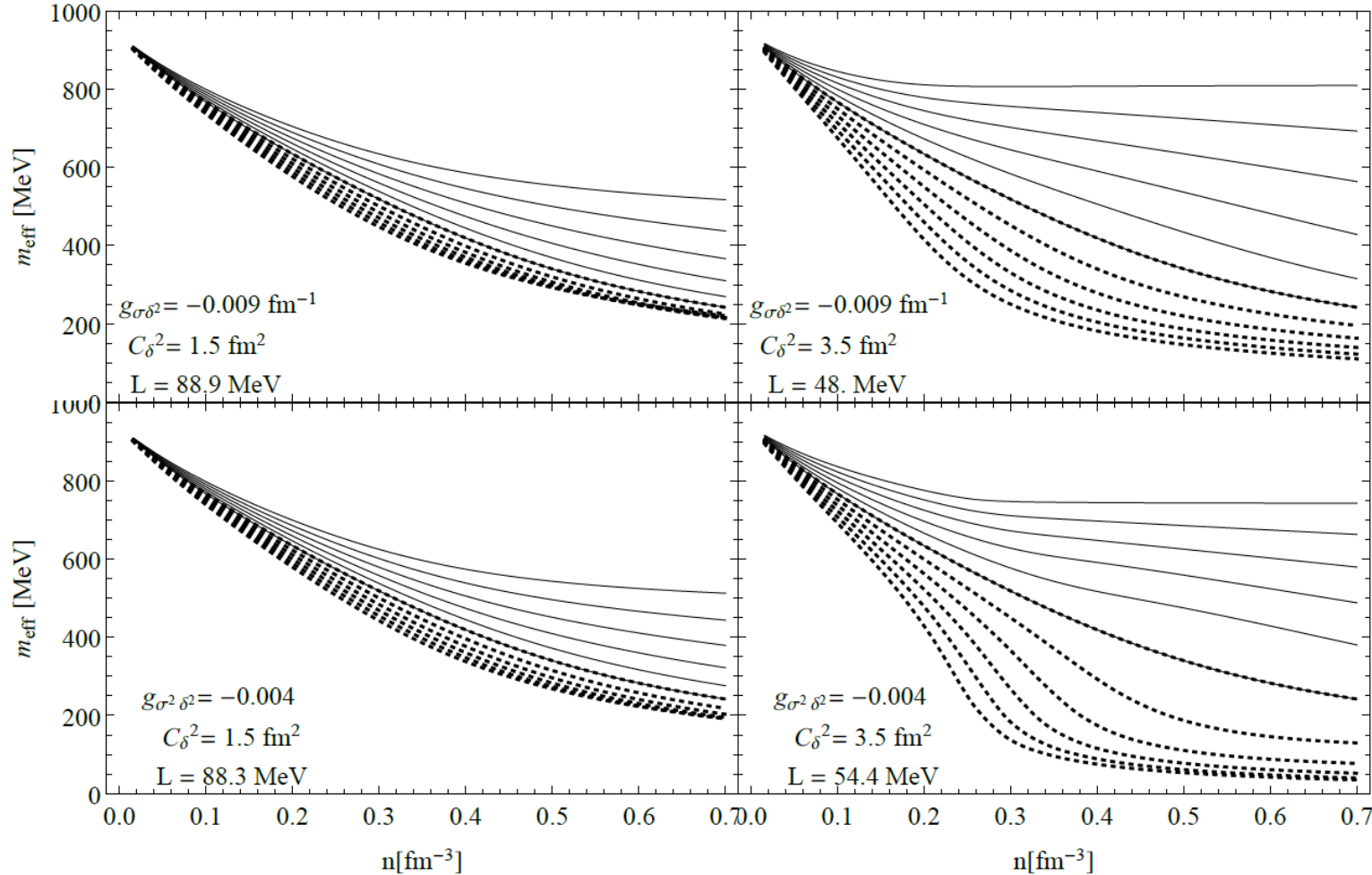


# RESULTS

$C_\delta^2$  - small

$C_\delta^2$  - big

linear



quadratic

The presence of  $\delta$ -meson makes the Effective nucleon mass splitting.



$m_p$  i  $m_n$  are no longer the same!

# STARS

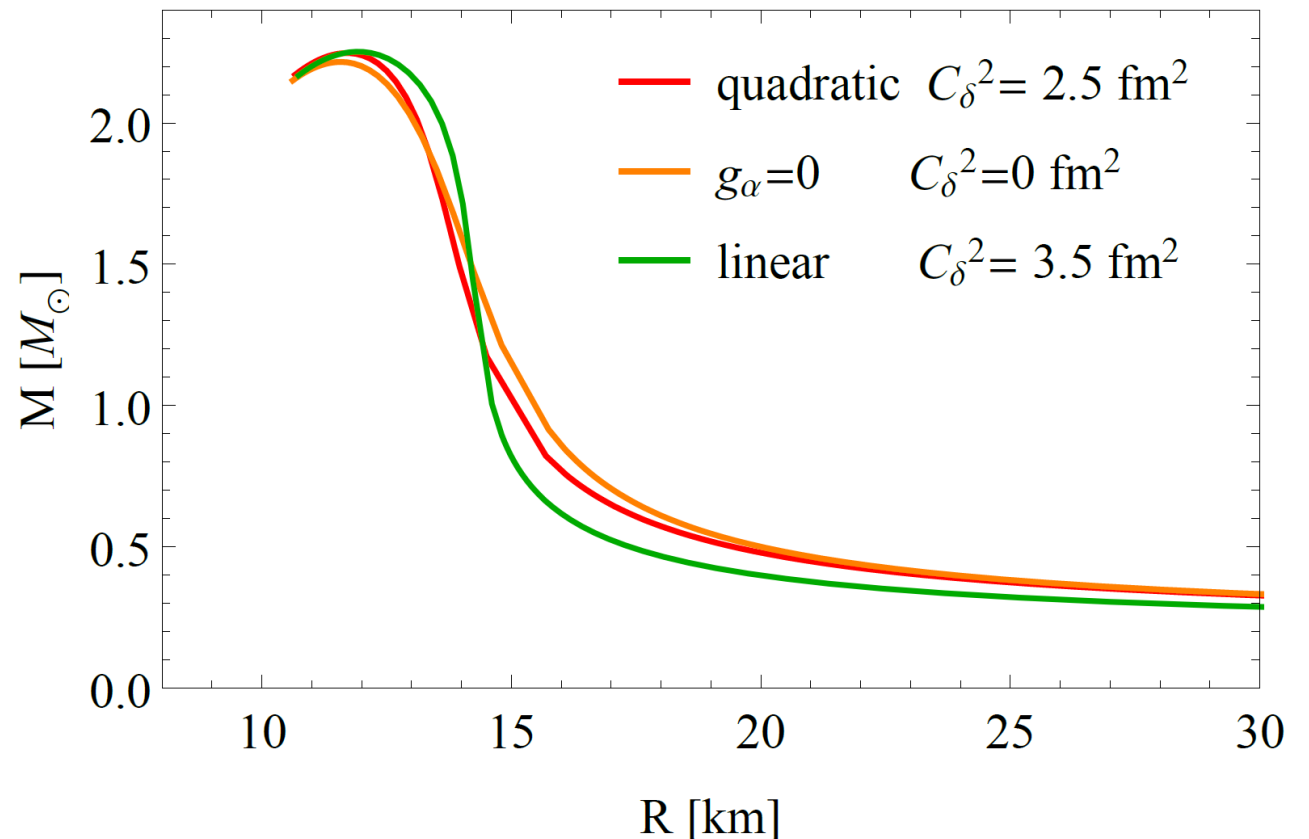
Step 1. EOS ( $\beta$ -equilibrium + charge neutrality)

Step 2. TOV  $P'(x) = -G(P(R) + \rho(R)) \frac{M(R) + 4\pi r^3}{R(c^4 - 2GM(R))}$   
 $M'(R) = 4\pi r^2 \rho(R)$



Structure of the star:

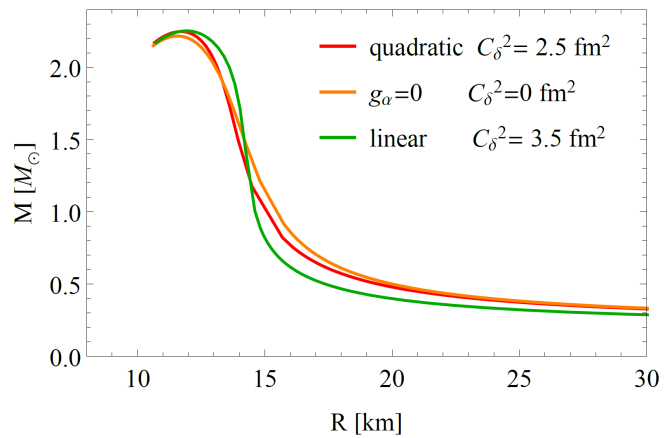
- M vs R
- M( $\rho$ )
- $\rho(R)$
- ...



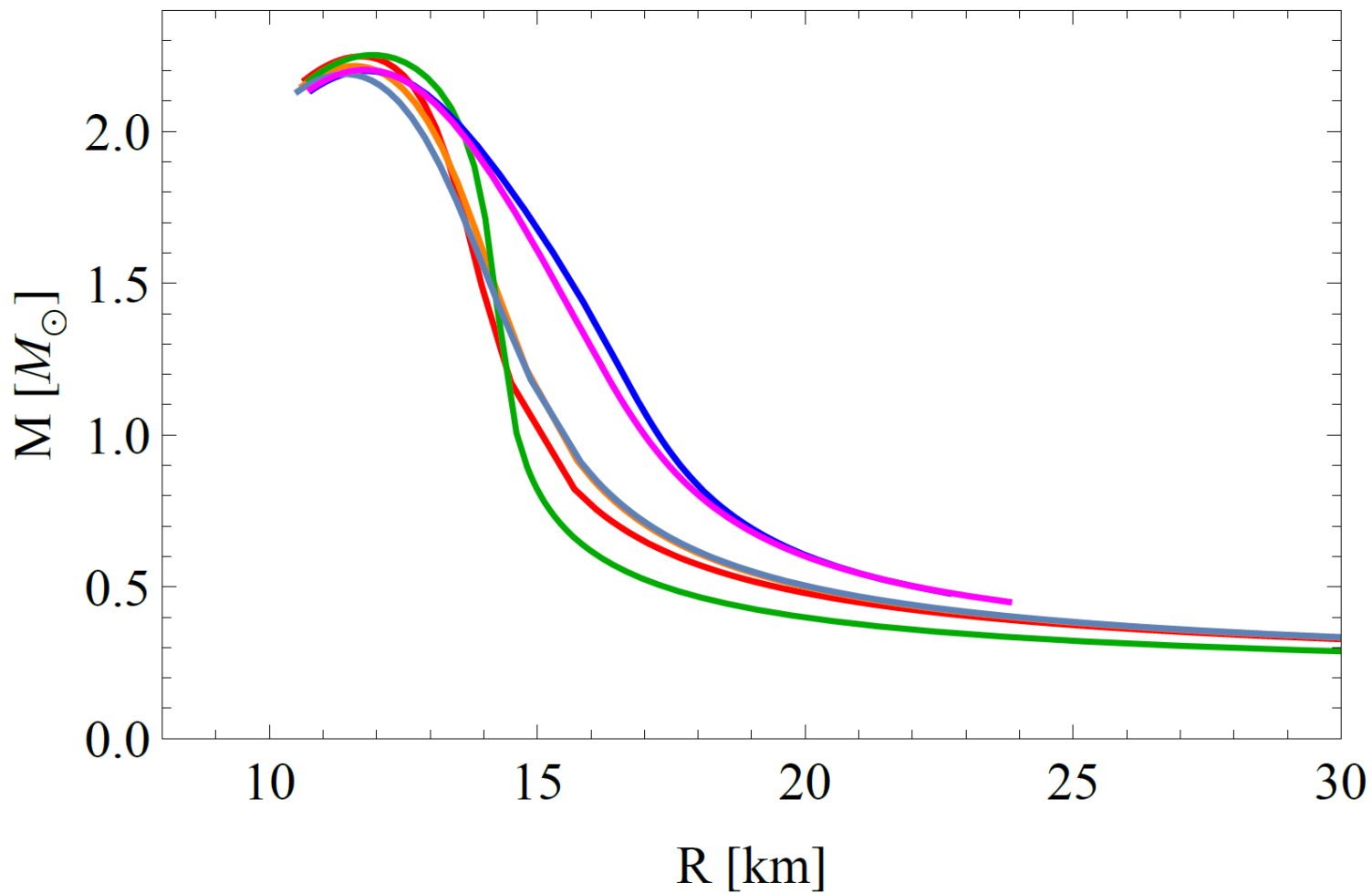
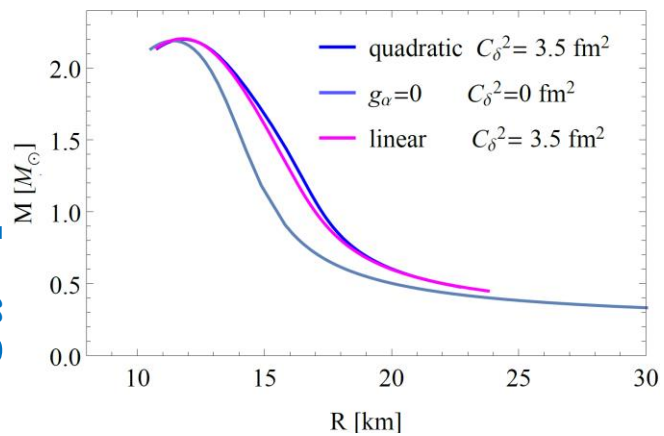
# STARS

Maximum masses:  $M = 2.19 - 2.25 M_{\odot}$   
Radius:  $R = 11.40 - 11.97 \text{ km}$

$g_{\alpha}$  - negative



$g_{\alpha}$  - positive

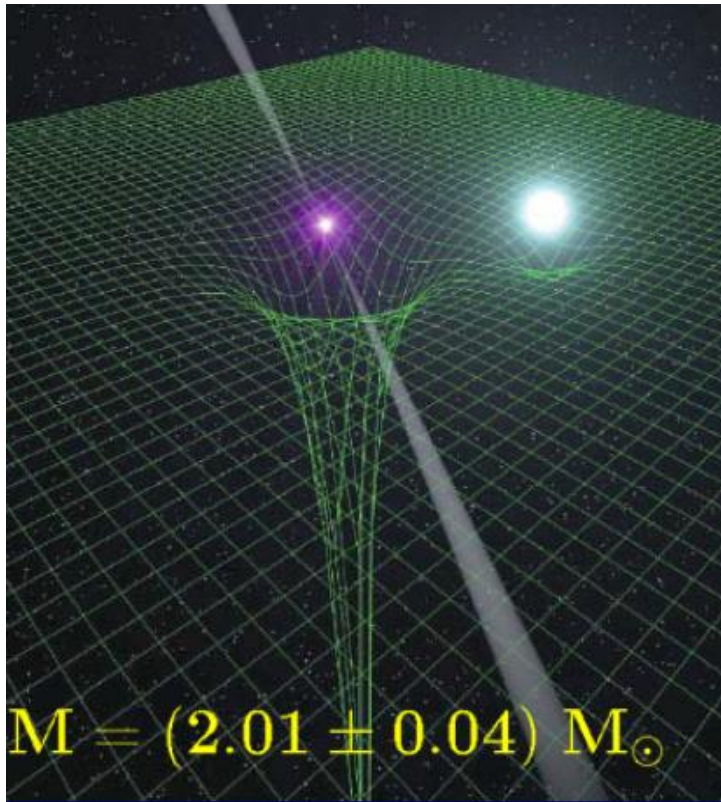


# STARS

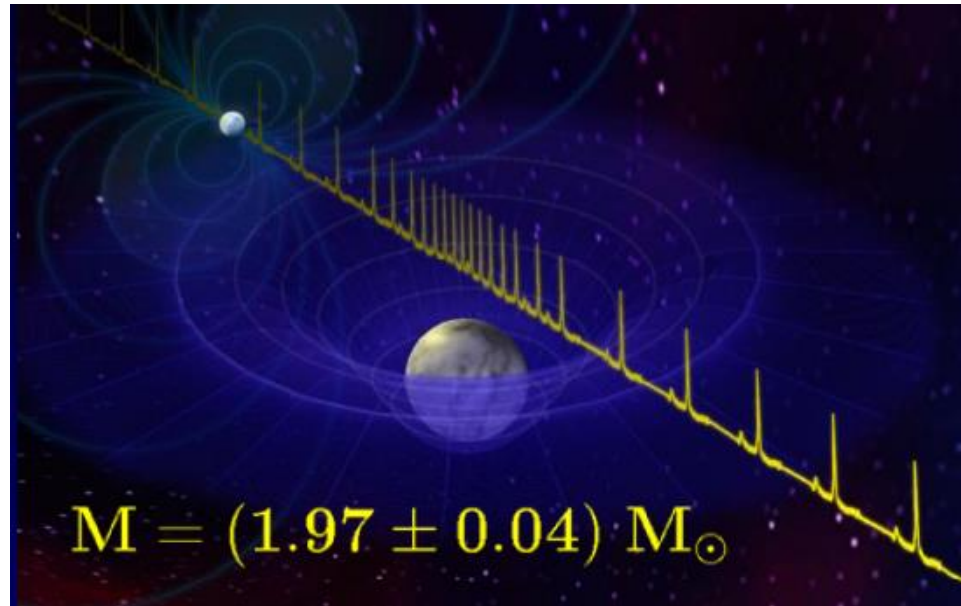
g-linear,  $C_\delta^2 = 3.5 \text{ fm}^2 \rightarrow M = 2.25 M_\odot$  &  $R = 11.93 \text{ km}$   
g-quadratic,  $C_\delta^2 = 2.5 \text{ fm}^2 \rightarrow M = 2.24 M_\odot$  &  $R = 11.86 \text{ km}$

## Highest well-known masses of NS

NS in a binary system with a WD



Pulsar J0348+0432



Pulsar J1614-2230



# STARS

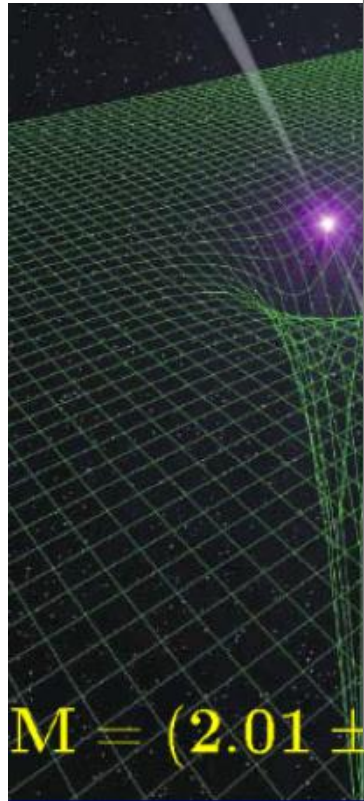
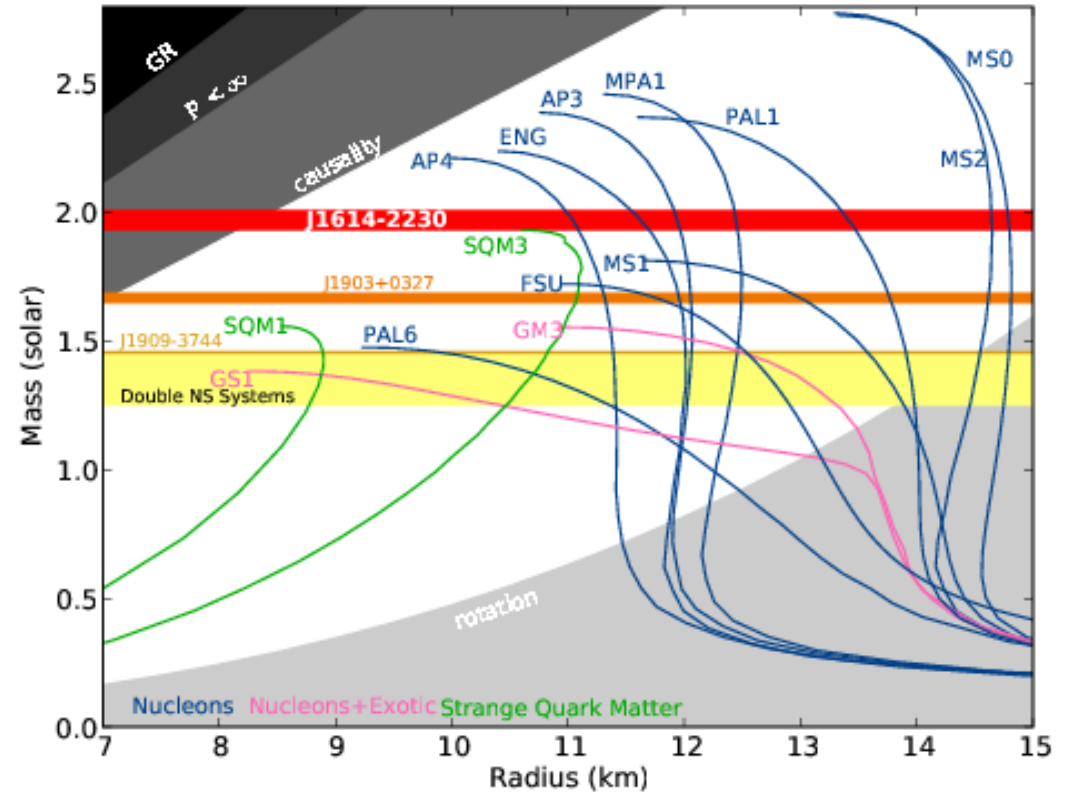
$$\begin{aligned} \text{g-linear, } C_{\delta}^2 &= 3.5 \text{ fm}^2 \rightarrow M = 2.25 M_{\odot} \text{ \& } R = 11.93 \text{ km} \\ \text{g-quadratic, } C_{\delta}^2 &= 2.5 \text{ fm}^2 \rightarrow M = 2.24 M_{\odot} \text{ \& } R = 11.86 \text{ km} \end{aligned}$$

## Highest well-known masses of NS

Neutron star (NS) mass-radius diagram for several typical NS equations of state.

Horizontal bands show the observational constraint:

- from J1614--2230 mass measurement,
- similar measurements for two other millisecond pulsars
- and the range of observed masses for double NS binaries



Pulsar J03448+0432

Pulsar J1614-2230

Thank You

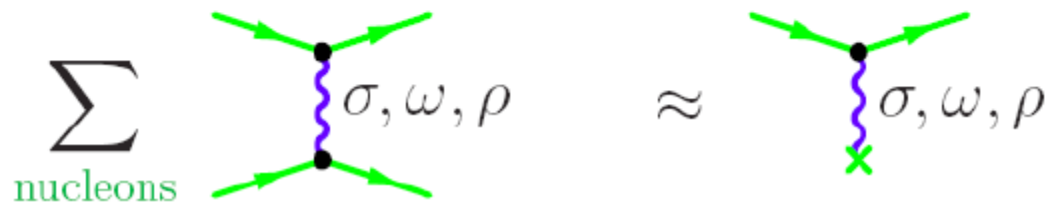
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# BACK-UP

## RMF THEORY

$$\begin{aligned}
 \mathcal{L} = & \sum_N \bar{N} \left[ i (\hat{\partial} + i g_{\omega N} \hat{\omega} + i g_{\rho N} \tau \hat{\rho}) \right] - (m - g_{\sigma N} \sigma) N \\
 & + \underbrace{\frac{1}{2} (\partial_{\mu} \sigma \partial^{\mu} \sigma - m_{\sigma}^2 \sigma^2)}_{\text{scalar}} - U(\sigma) \\
 & - \underbrace{\frac{1}{4} \omega_{\mu\nu} \omega^{\mu\nu} + \frac{1}{2} m_{\omega} \omega_{\mu} \omega^{\mu}}_{\text{vector}} - \underbrace{\frac{1}{4} \rho_{\mu\nu} \rho^{\mu\nu} + \frac{1}{2} \rho_{\mu} \rho^{\mu}}_{\text{iso-vector}}
 \end{aligned}$$

**medium:** mean-field approximation



$$\sigma(r, t) = \sigma$$

$$\omega_{\mu}(r, t) = \delta_{\mu,0} \omega_0$$

$$\rho_{\mu}^a(r, t) = \delta^{a,3} \delta_{\mu,0} \rho_0^{(3)}$$

constant fields