



# SCALAR FIELD (A.K.A. ULTRA LIGHT AXION) DM IN dSph's AND SOME ASPECTS OF STRUCTURE FORMATION

Alma González  
CONACYT/University of Guanajuato  
México

[alma.gonzalez@fisica.ugto.mx](mailto:alma.gonzalez@fisica.ugto.mx)  
<http://fisica.ugto.mx/~gfm/>

# Scalar Field Dark Matter

a.k.a. Axion like Dark Matter, BECDM, FuzzyDM, wave DM, ULA-DM, etc...

- ❖ Dark Matter is described by a scalar field. Can be coupled to the SM: ej. Axion for QCD.
- ❖ Or it can be only coupled gravitationally. We'll work with a real scalar field.

$$\mathcal{L}_\phi = -\frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2$$

$$T_\nu^\mu = g^{\mu\alpha}\phi_\alpha\phi_{,\nu} - \delta_\nu^\mu(V(\phi) + \frac{1}{2}g^{\kappa\lambda}\phi_\kappa\phi_{,\lambda})$$

Hu et. al 2000, Matos & Ureña 2002, P. Sikivie and Yang, 2009. Marsh & Silk 2013, Shive et. al 2014, and many others.

Most recent review on the subject: H. Lam, J. Ostriker, S. Tremaine, Edward Witten arXiv: 1610.08297

It is a field representation, not a particle one.

# Scalar Field Dark Matter

Mass  $m_a \gtrsim 10^{-23} \text{eV}$  to be consistent with CMB and LSS bounds.

The mass of the scalar field sets a cut-off in the mass power spectrum in the small scales (large  $k$ 's).

Self-gravitating objects could form of galactic size. But in general a Halo will be composed of a inner "soliton" and an external cloud.

SFDM halos always have a core density profile, due to the presence of the soliton. The more massive the Axion, the less extended the "soliton"

Clear Predictions/differences with respect to CDM

# Motivation (my version of it)

Missing Satellite "problem" is actually two problems

- **Observational problem:** Determine the precise number of satellite galaxies. Luminosity below detection threshold, non-complete samples, etc.

"Is there a missing satellite problem with CDM? The answer is likely to be not in the era of DES and LSST" Hargis et al. 2014

There is no missing satellite problem.

Kim et al. 2017

- **Theoretical problem:** What makes a halo not to host/produce stars so that they are undetectable. Or else, what inhibits the creation of small halos? Does the answer highly depend on the DM nature?

# Cusp Vs Core Problem. Also two problems

**Theoretical:** No so easy to include baryons on simulations to determine how DM properties+baryons shape the final DM density profile.

Peñarrubia et al. 2012 , Read et al. 2016 Pontzen & Governato, Nature, 2014  
Sawala et al. 2016; Zhu et al. 2016

**Observational problem:** Degeneracies between different effects makes not trivial to recover the "true" density profile.

Walker 2011, Juan C. B. Pineda 2016

dSph's kinematics &  
constraints to Axion DM  
mass

# Dwarf Spheroidal Galaxies & Axion Dark Matter

$$L_* \approx 10^6 L_\odot$$

$$\langle \sigma_* \rangle \approx 10 \text{ km/s}$$

$$\Upsilon_* = 100 - 1000$$

to reproduce  
kinematics with only  
stellar component

We only observe one component of the velocity dispersion along the line of sight.

$$\beta = 0$$

Isotropic

$$\beta$$

Non-isotropic. Not necessarily  
constant anisotropy

# dSph's kinematics: Constraints to axion DM

## Stars and the Jeans eqn.

e.g. Binney and Tremaine  
Assuming spherical symmetry

Relate DM mass, to stellar dist.  $\nu$  and velocity anisotropy  $\beta$ :

Important

$$\frac{1}{\nu} \frac{d}{dr} (\nu \langle v_r^2 \rangle) + 2 \frac{\beta \langle v_r^2 \rangle}{r} = - \frac{GM}{r^2}.$$

Integrate density

Assume constant  $\beta$  and Plummer profile for stars:

$$\nu(r) = \frac{3L}{4\pi r_{\text{half}}^3} \frac{1}{[1 + (r/r_{\text{half}})^2]^{5/2}}.$$

measured for dSphs as single population

→ Projected l.o.s. velocity dispersion with  $\beta$  as free param.

$$\sigma_{\text{los}}^2 = \frac{2G}{I(R)} \int_R^\infty dr' \nu(r') M(r') (r')^{2\beta-2} F(\beta, R, r').$$

Projection

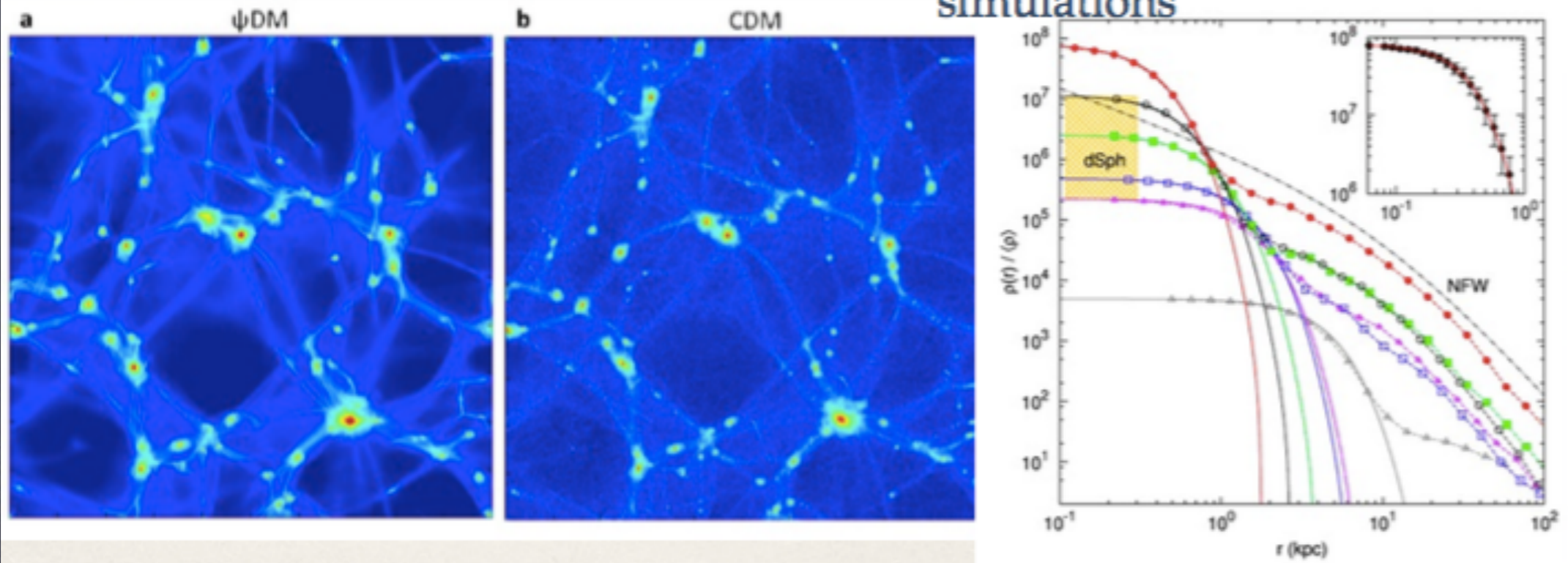
DATA:  
Walker et  
al (2009)

From Plummer



# Axion DM halo model

Recently verified by means of numerical simulations



Hsi-Yu Schive<sup>1</sup>, Tzihong Chiueh & Tom Broadhurst, Nature, 06/2014

$$\rho(r) = \rho_{\text{sol}} \begin{cases} \frac{1}{(1 + (r/r_{\text{sol}})^2)^8} & \text{for } r < r_{\epsilon} \\ \frac{\delta_{\text{NFW}}}{r/r_s (1 + r/r_s)^2} & \text{for } r \geq r_{\epsilon} \end{cases} .$$

where

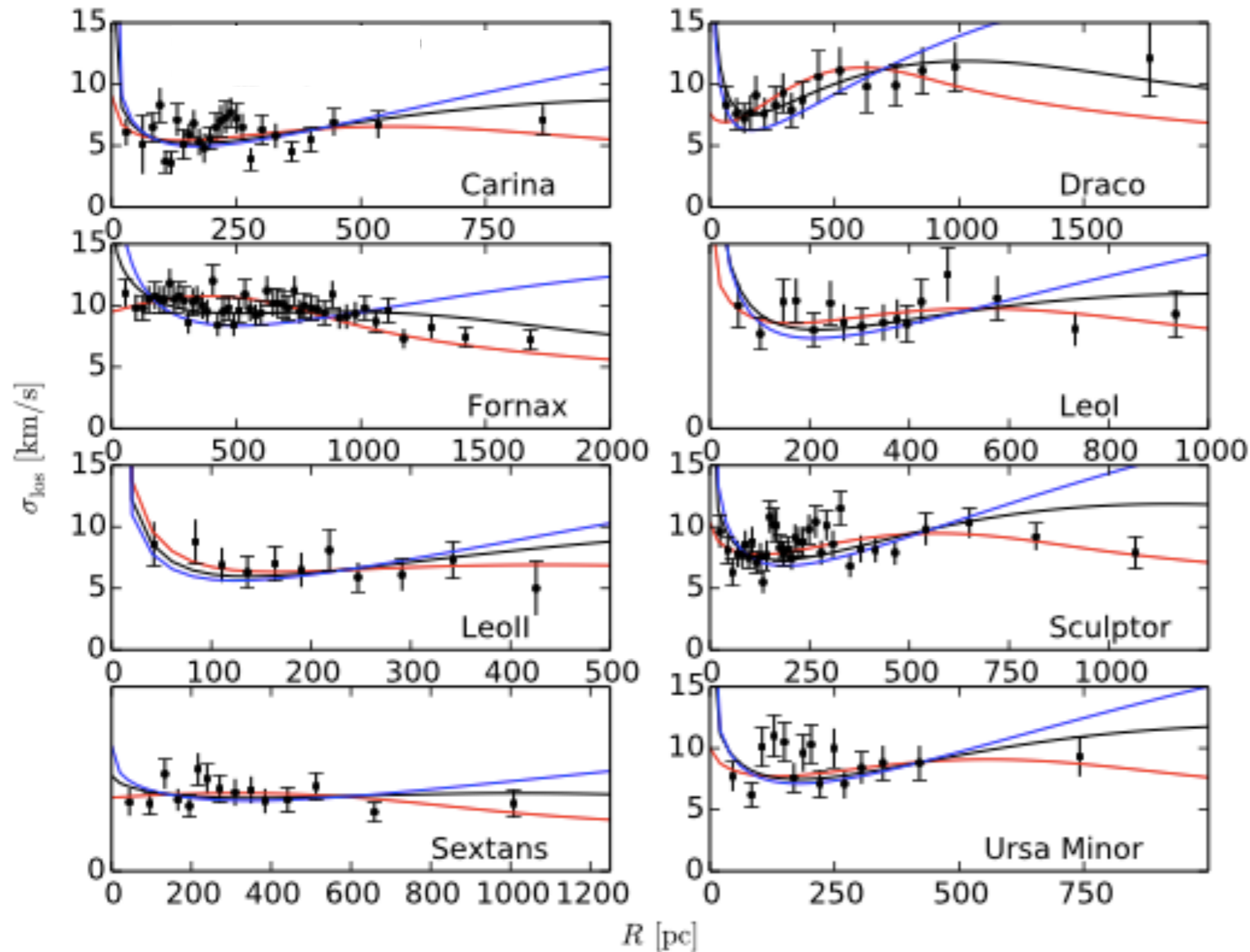
$$r_{\epsilon} = r_{\text{sol}} (\epsilon^{-1/8} - 1)^{1/2} ,$$

and

$$\delta_{\text{NFW}} = \epsilon \rho_{\text{sol}} \left( \frac{r_{\epsilon}}{r_s} \left( 1 + \frac{r_{\epsilon}}{r_s} \right)^2 \right) .$$

$$r_{\text{sol}} = \left[ \frac{\rho_{\text{sol}}}{2.42 \times 10^9 \text{ M}_{\odot} \text{ kpc}^{-3}} \left( \frac{m_a}{10^{-22} \text{ eV}} \right)^2 \right]^{-0.25}$$

2 free parameters per halo + free anisotropy.  
we treat the axion mass as universal parameter.



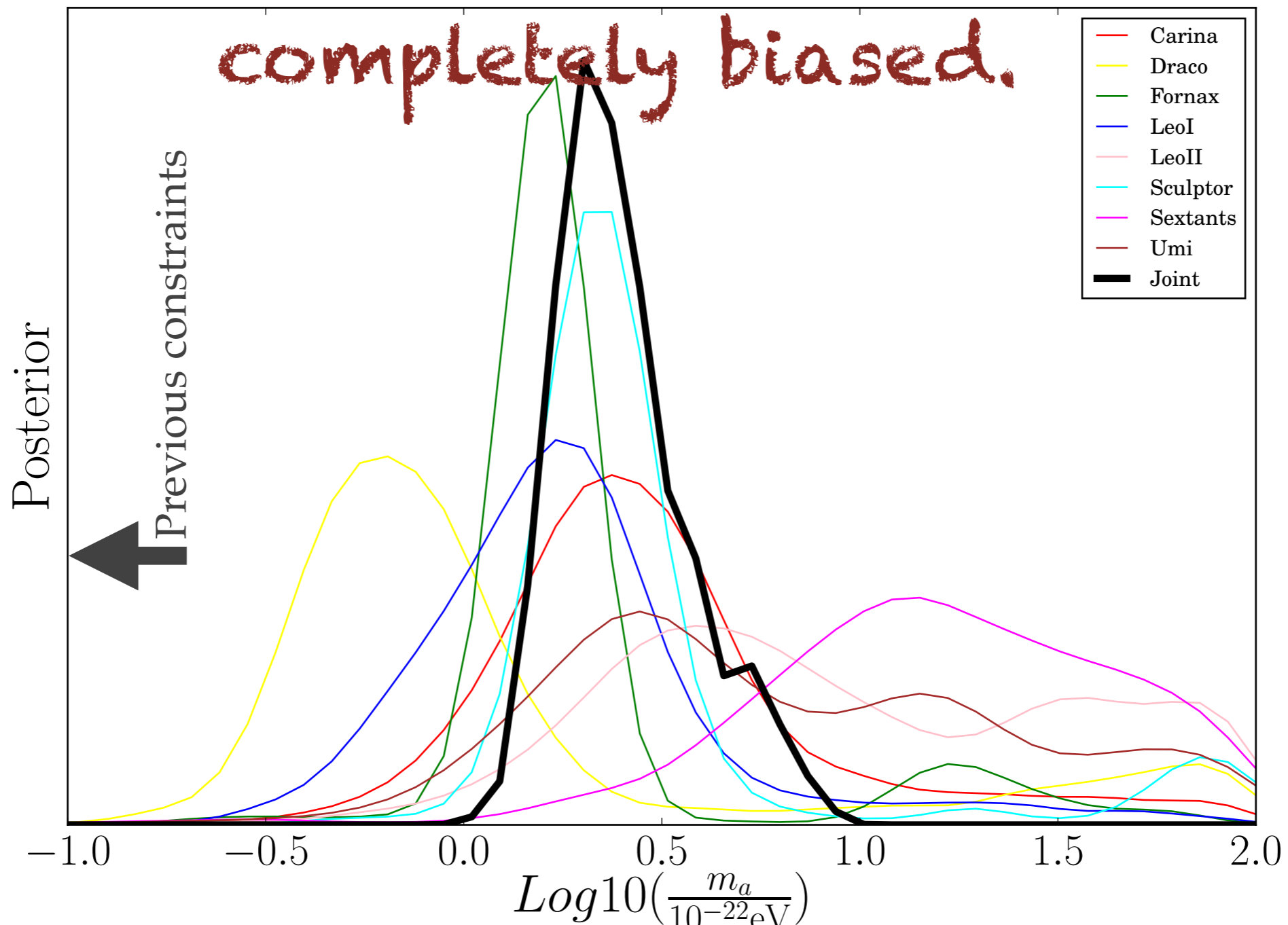
Axion  
Like DM,  
soliton  
only with  
a radius  
of  $\sim 2$  kpc

First done (for different profile in the `rcr_e`) in  
A. Diez-Tejedor, AXGM, S. Profumo, 1404.1054v2. Now done with  
the ULA+NFW profile AXGM et al. 2016 and Chen. et. al 2016.

# Soliton+NFW halo model

## Joint/Individual Analysis Comparison

Looks nice but is completely biased.



We were not the only deceived ones

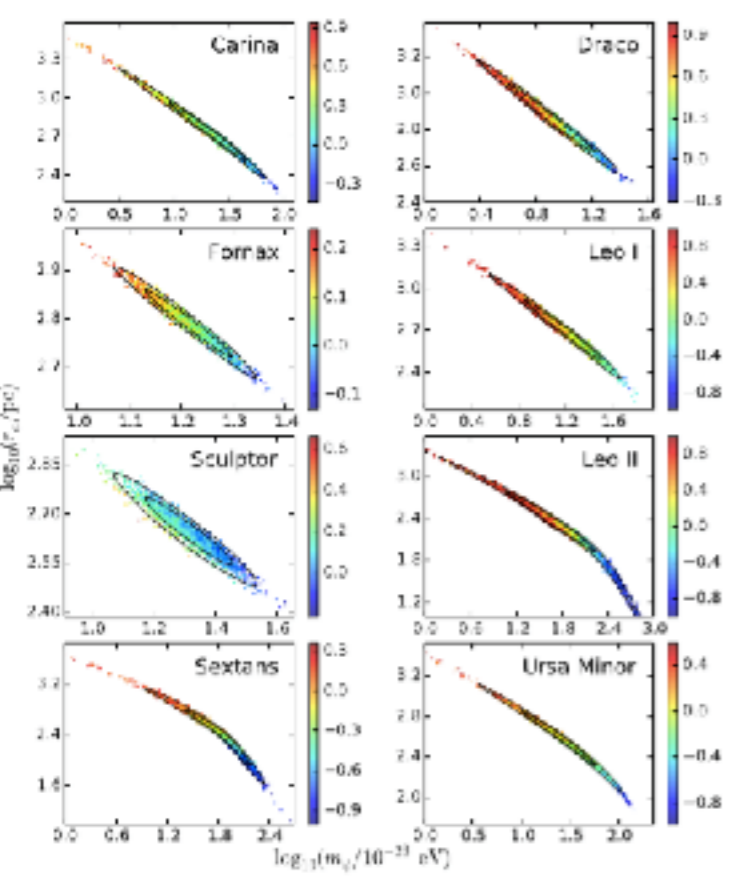


Figure 2. Posterior distributions of  $m_2$  and  $r_2$  colored by  $\beta$  for each dSph in our MCMC analysis. Contours show the  $1\sigma$  and  $2\sigma$  confidence regions. The confidence intervals of the model parameters for each dSph are also listed in Table 1.

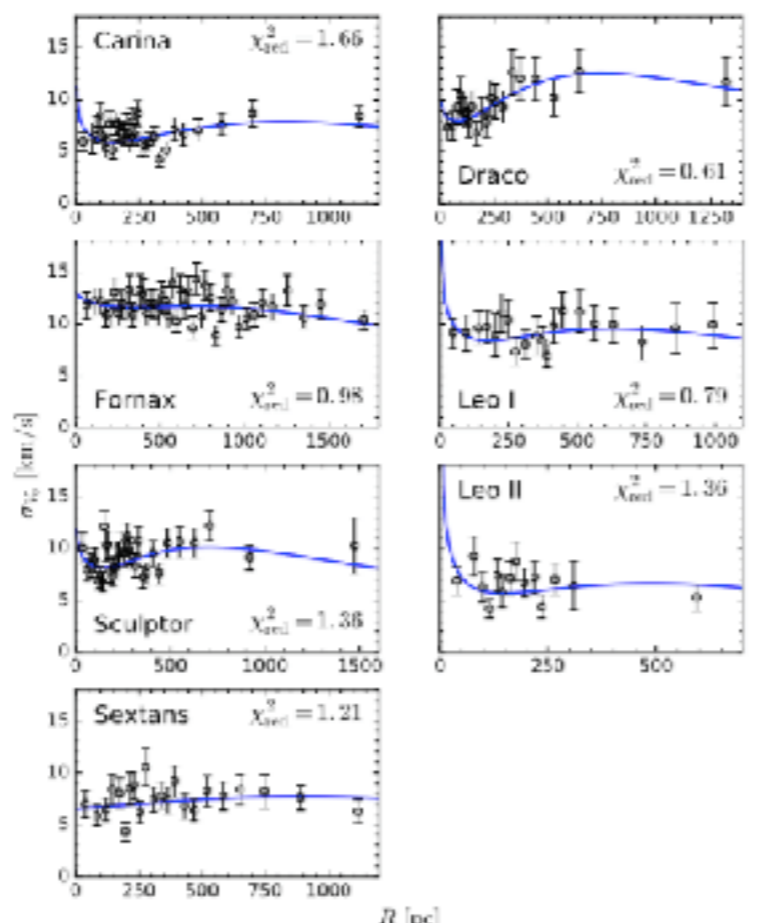
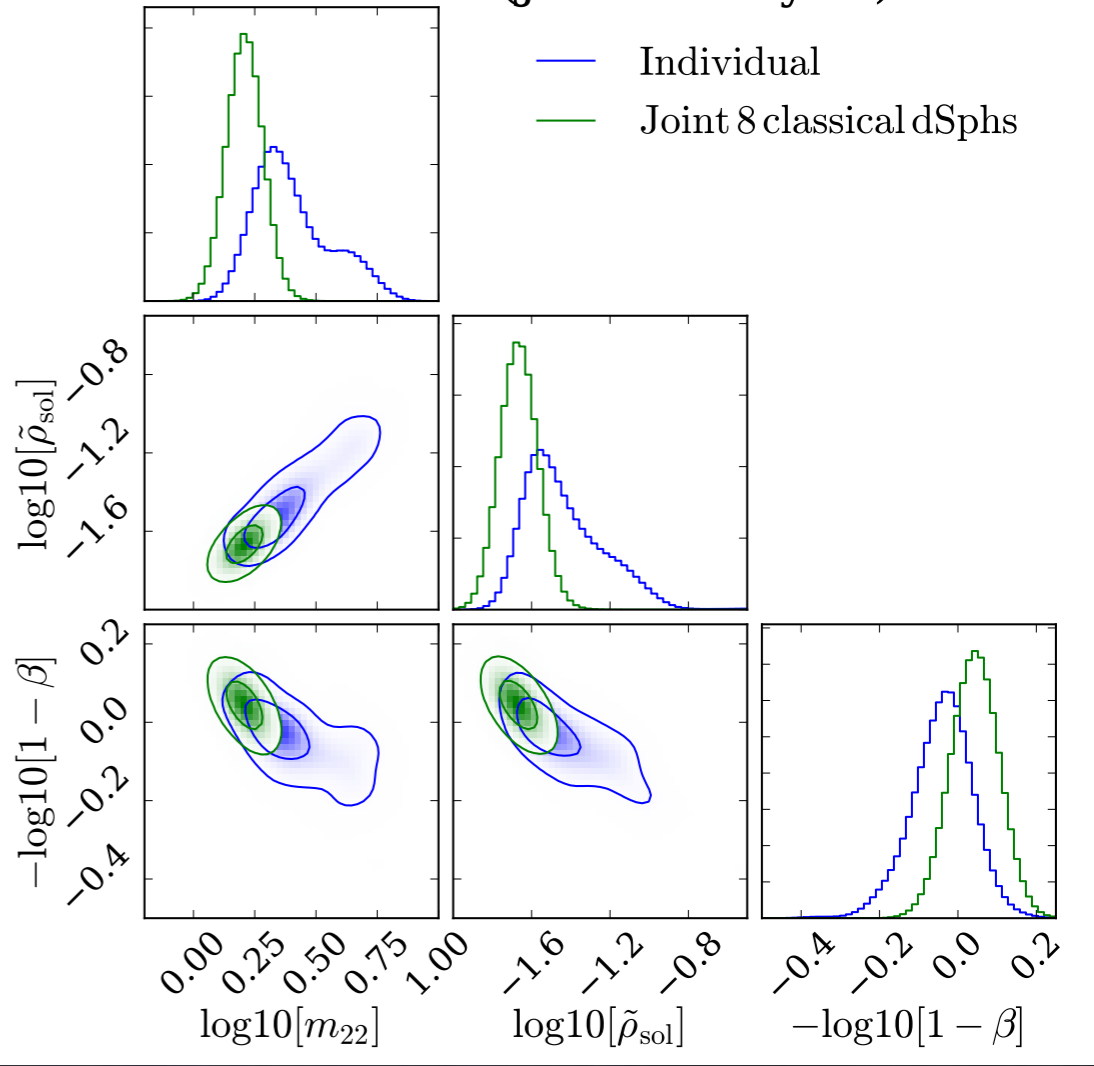


Figure 3. Same as Figure 1 but for the observational data set of Walker et al. (2007). The confidence intervals of the model parameters for each dSph are listed in Table 1.

Chen, et. al  
1606.09030v1

Fornax (Jeans Analysis)



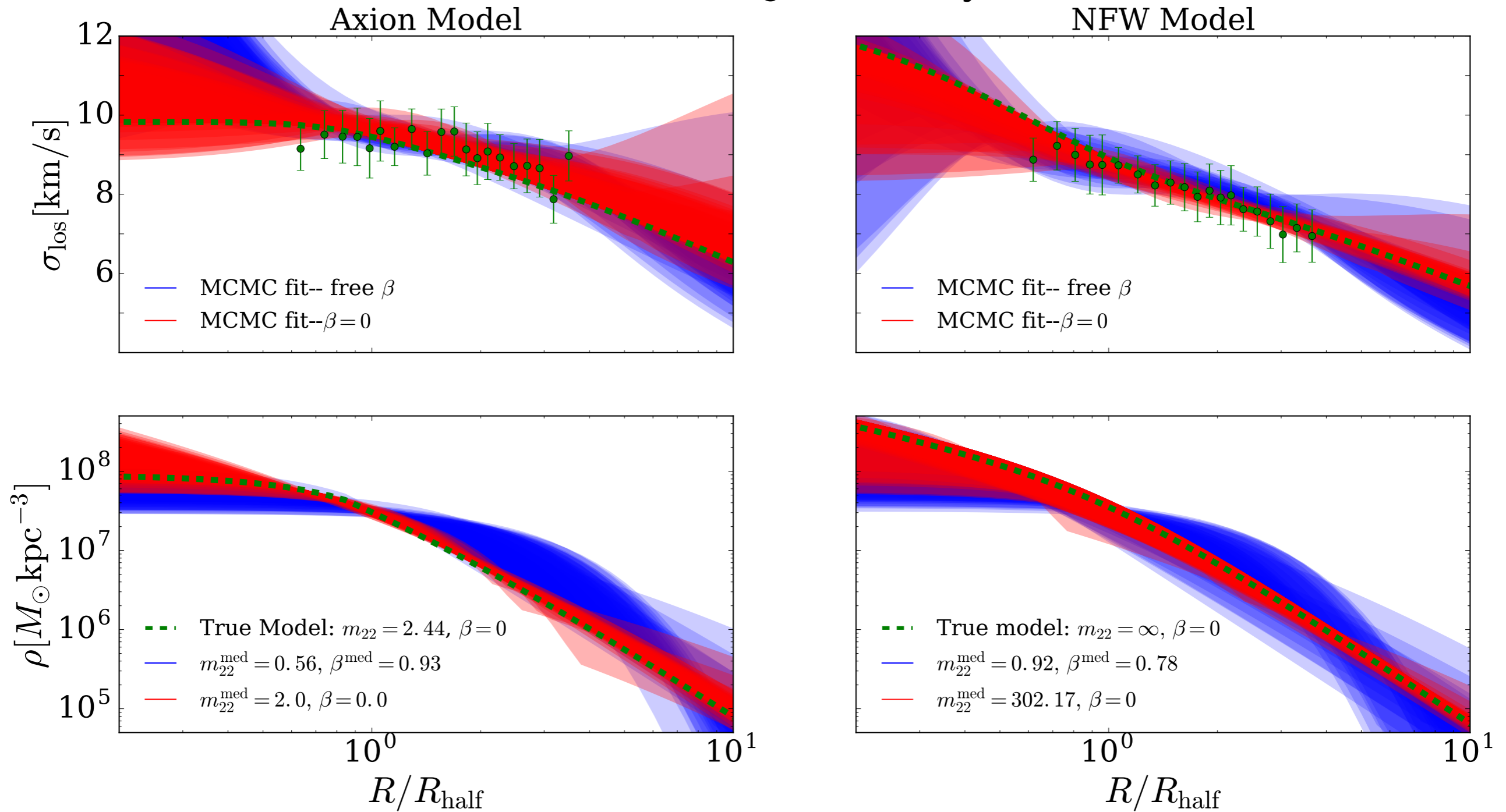
But we knew it before  
we try to publish ;D

Gonzalez et. al  
arXiv:1609.05856

# What is wrong? Density Profile Vs Anisotropy

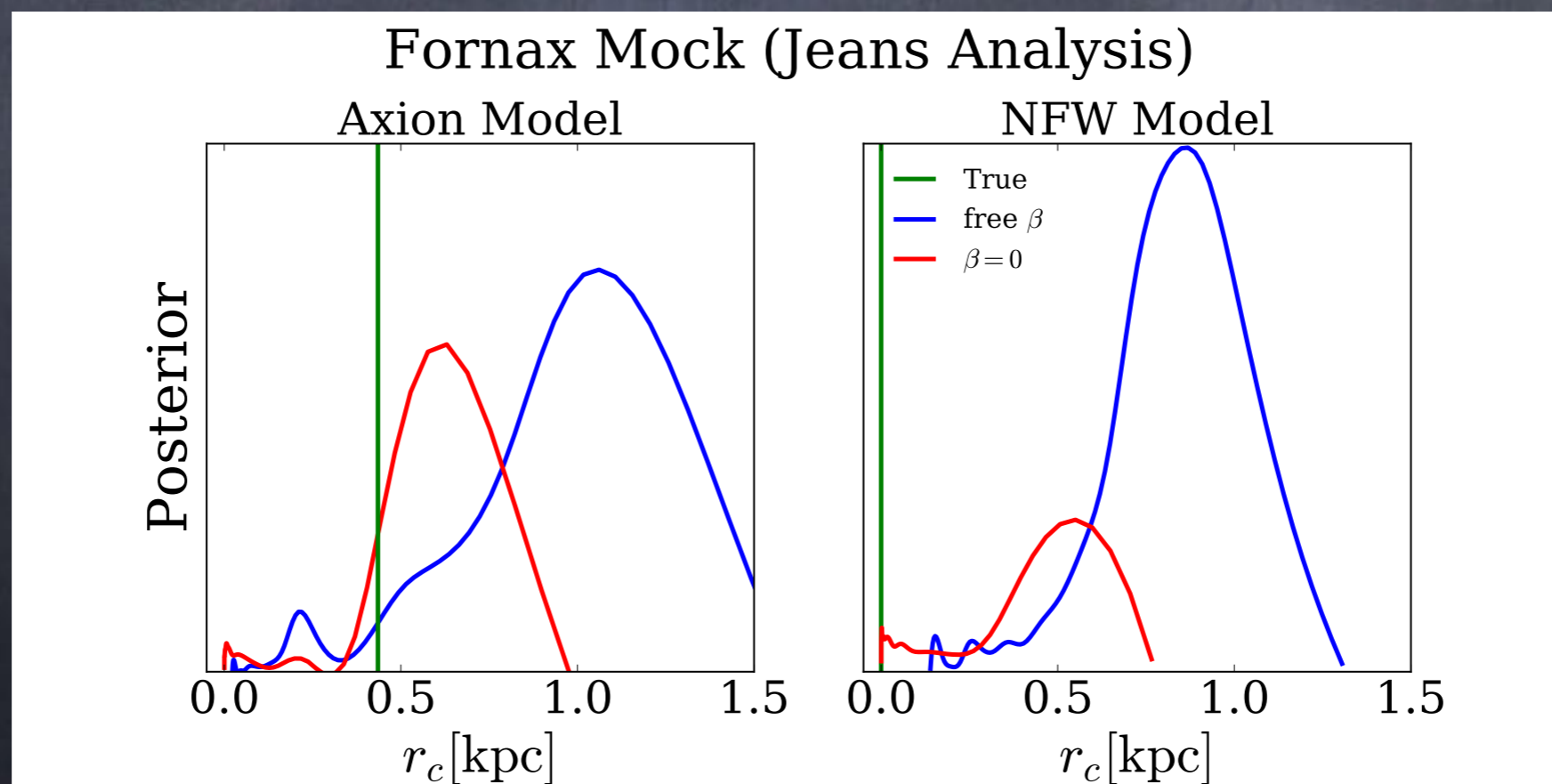
Test: Apply Jeans analysis using the axion density profile to simulated galaxies with different density profiles and isotropy.

## Fornax Mock (Jeans Analysis)

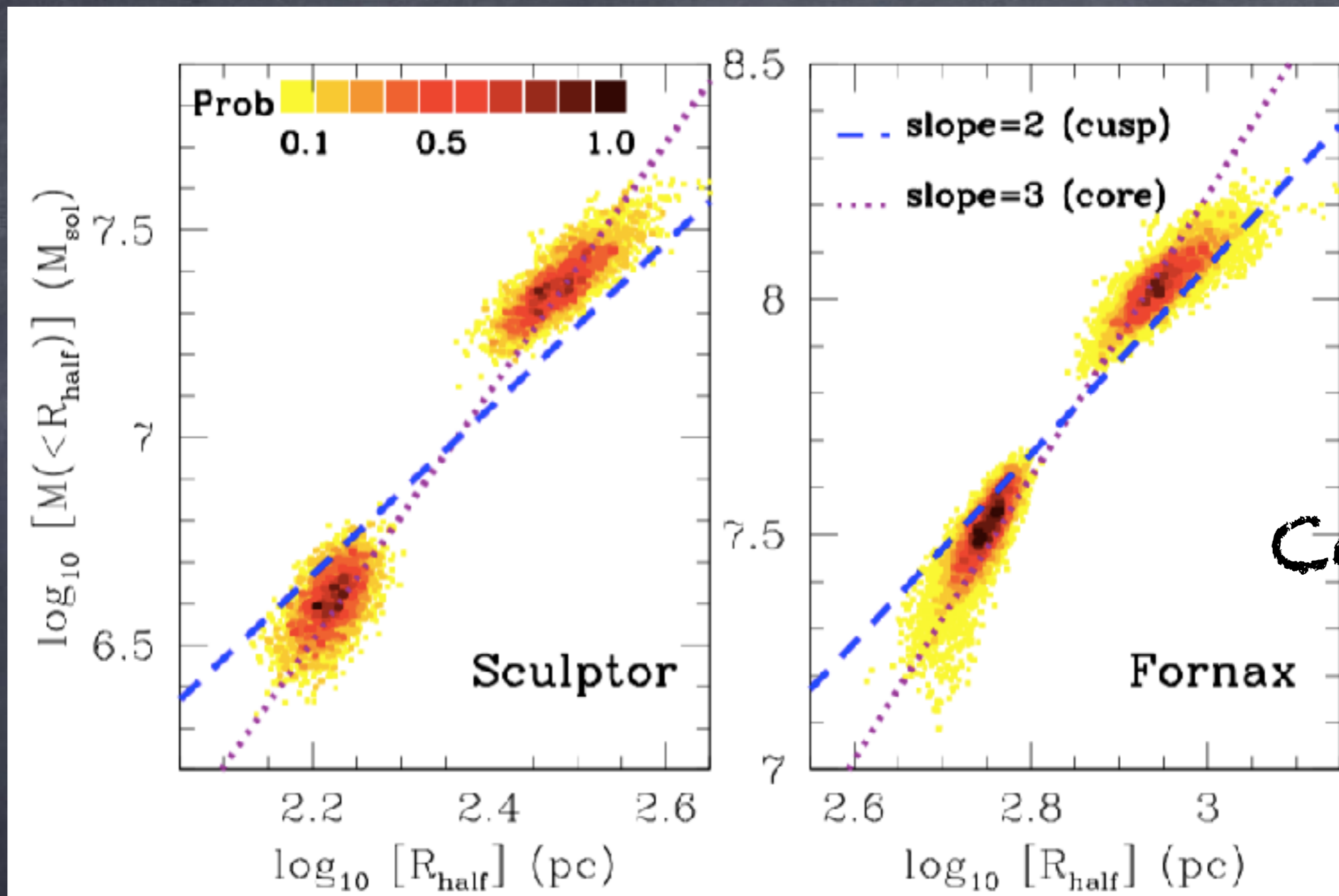


Result:

- In analyzing the galaxy mock with the corresponding density profile, we get a result biased from the correct one.
- More important, we always recover a large core for the axion density model, even when a cusp was in the simulated galaxy. We can no trust inference from real data since we don't know neither the true profile nor the true anisotropy...



Now what?... review another observable

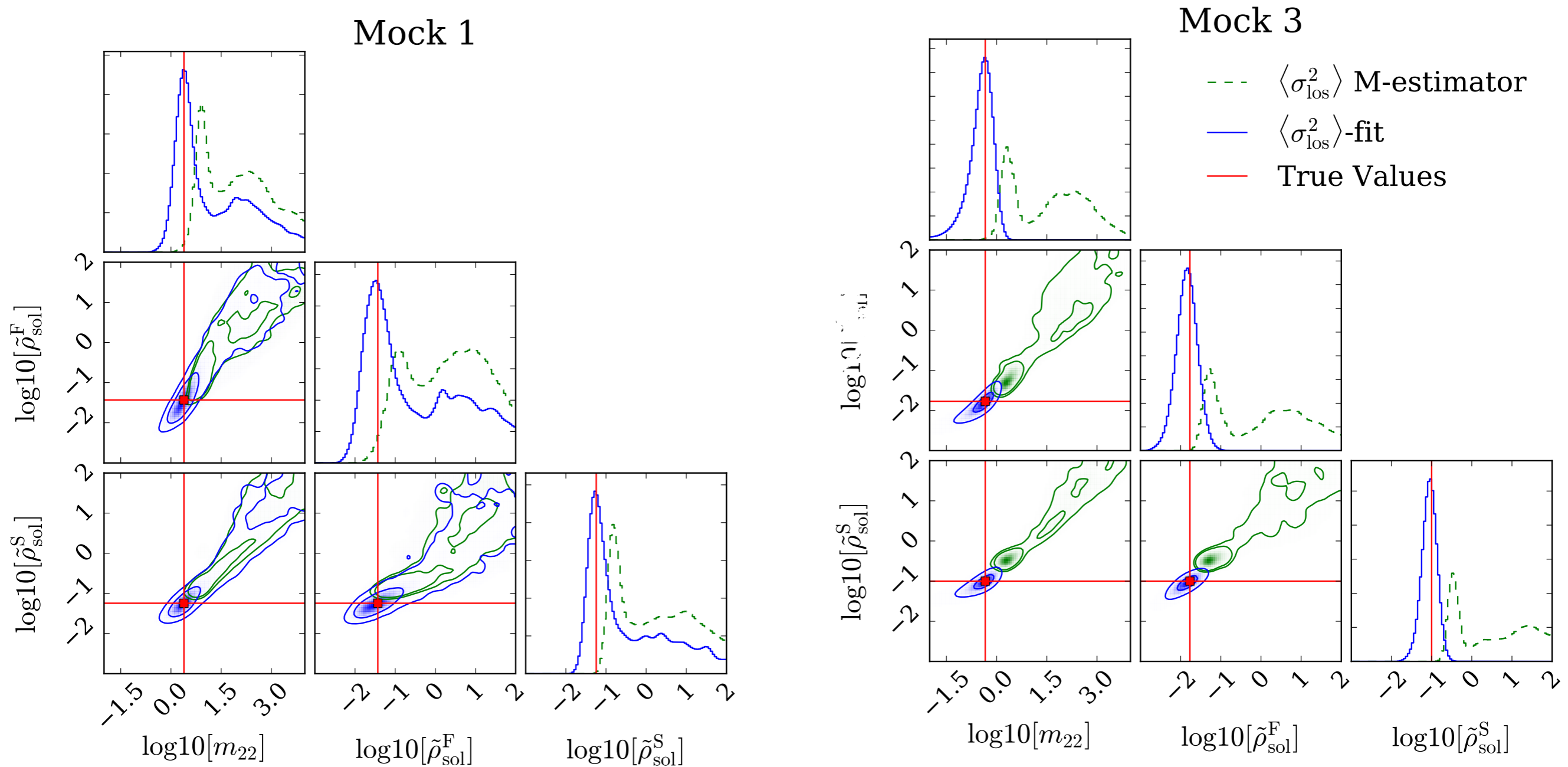


Walker & Peñarrubia 2011

Use a simple Mass estimator

$$\langle \sigma_{\text{los}}^2(r_{\text{half}}) \rangle = \frac{2GM(<r_{\text{half}})}{5r_{\text{half}}}$$

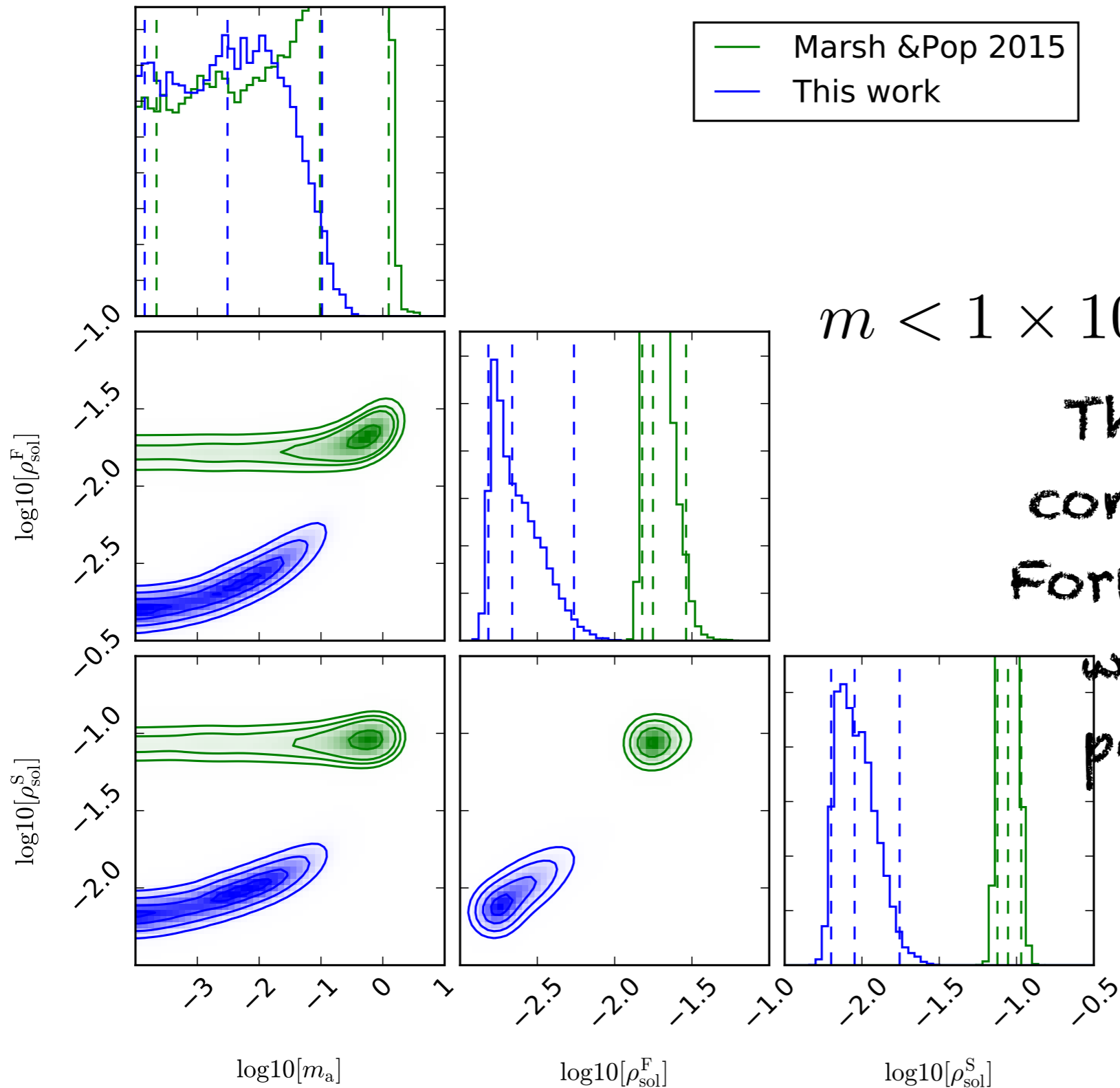
Again, we test it with synthetic data and use a different estimator



$$\langle \sigma_{\text{los}}^2(r_{\text{half}}) \rangle = \frac{\int_0^{\infty} \sigma_{\text{los}}^2(R') I(R') R' dR'}{\int_0^{\infty} I(R') R' dR'}$$

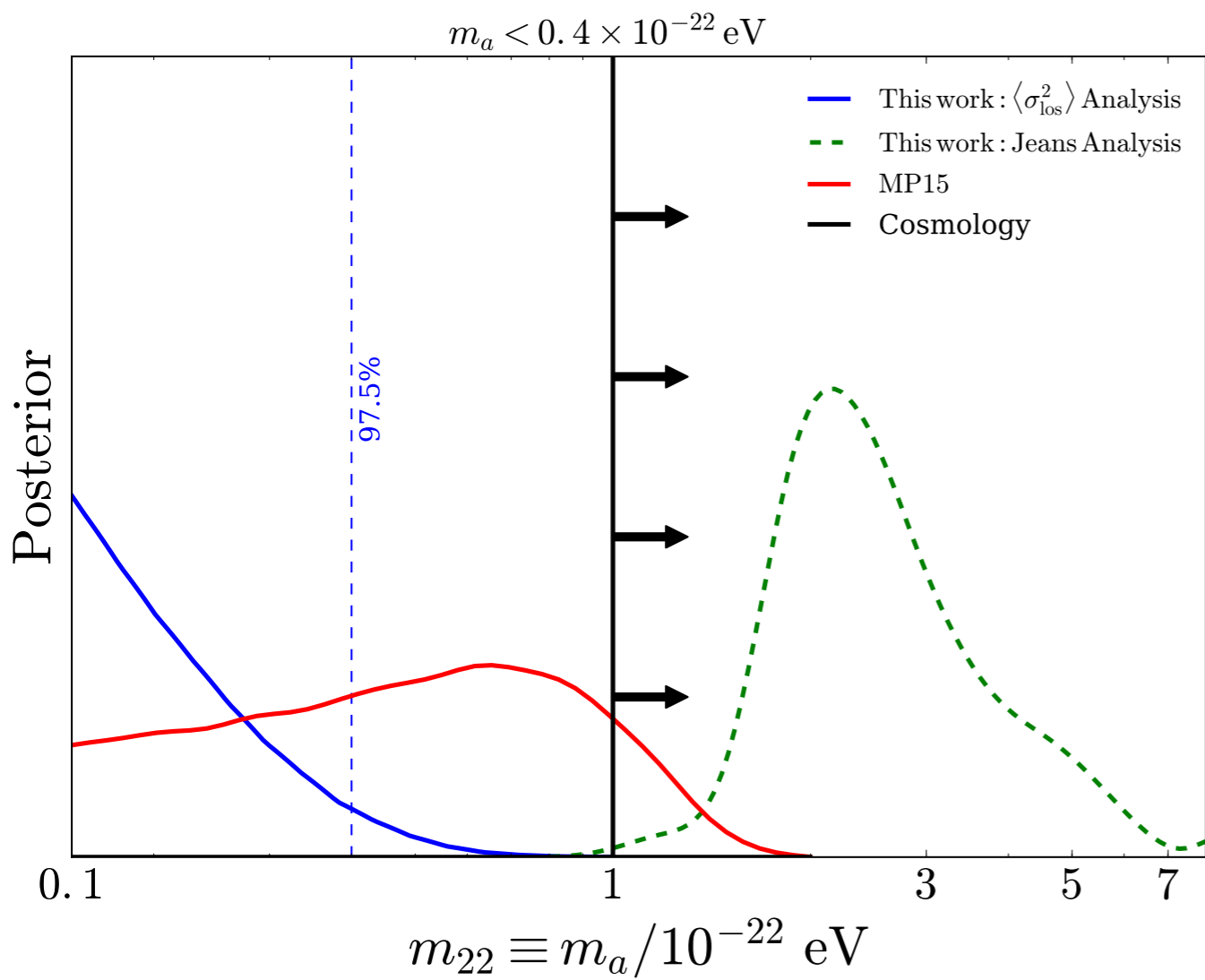


# Now applied to real data



$$m < 1 \times 10^{-23} \text{ eV}$$

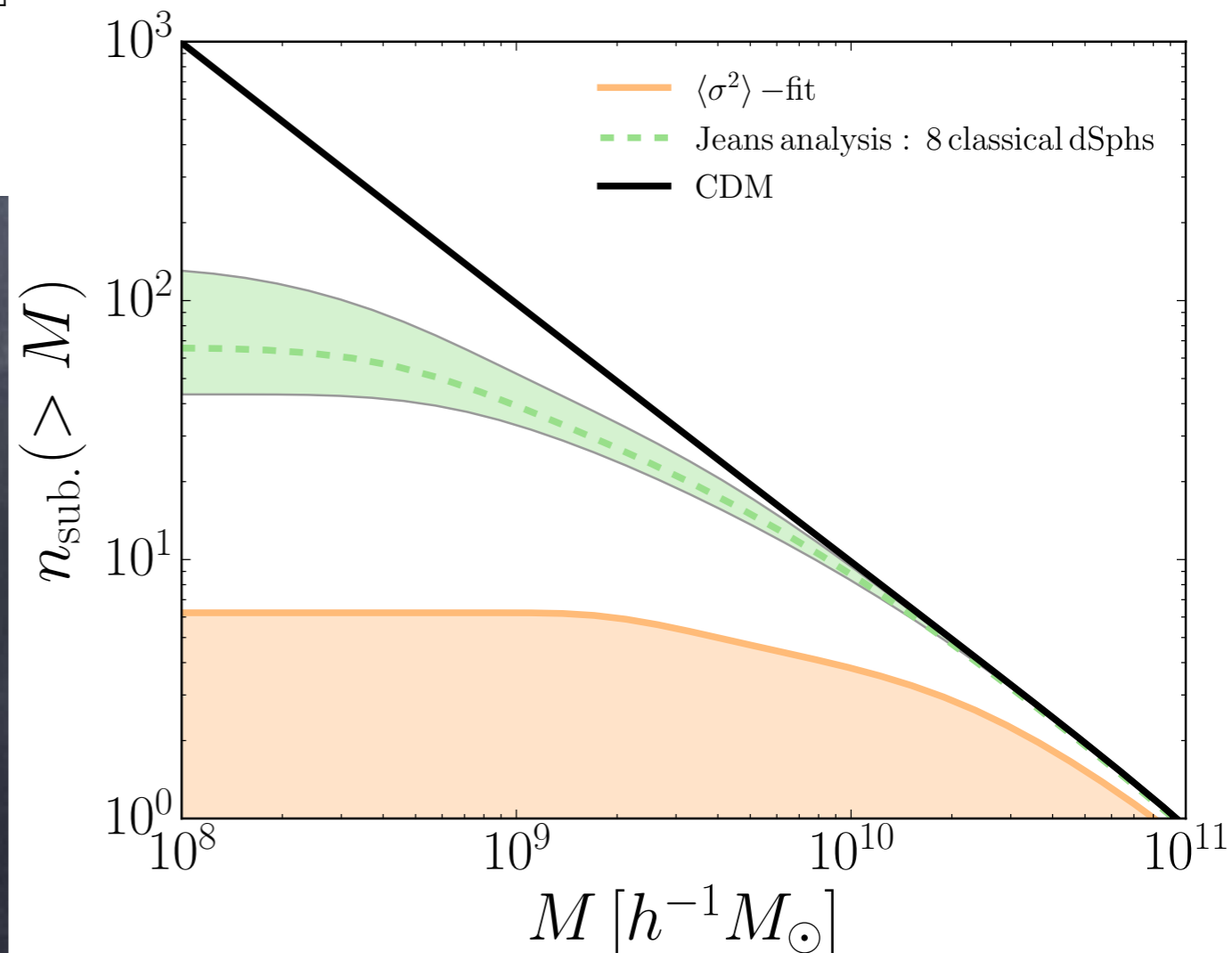
The axion mass is a common parameter to Fornax & Sculptor, and we fit the 2 stellar population in each galaxy.



# Final Results

New unbiased constraint  
in tension with previous  
analysis

A, VERY, simple  
calculation of the number  
of substructures shows  
that with this mass, the  
ULA-DM suffers a  
catch 22 problem



# Work in progress

Study the dSph's kinematics with other techniques.

Study the fuzzy DM in more detail. Go back to cosmology first... Next slides

Axion DM  
cosmology

# Background Cosmology

$$T_{\nu}^{\mu} = g^{\mu\alpha} \phi_{\alpha} \phi_{,\nu} - \delta_{\nu}^{\mu} (V(\phi) + 1/2 g^{\kappa\lambda} \phi_{\kappa} \phi_{,\lambda})$$

$$H^2 = \frac{\kappa^2}{3} \left( \sum_I \rho_I + \rho_{\phi} \right) \quad \dot{H} = -\frac{\kappa^2}{2} \left[ \sum_I (\rho_I + p_I) + (\rho_{\phi} + p_{\phi}) \right]$$

$$\dot{\rho}_I = -3H(\rho_I + p_I), \quad \ddot{\phi} = -3H\dot{\phi} - \frac{dV(\phi)}{d\phi} \quad \text{Klein-Gordon equation}$$

For the SF we can identify:

$$\kappa^2 = 8\pi G$$

$$\rho_{\phi} = \frac{1}{2} \dot{\phi}^2 + V(\phi), \quad p_{\phi} = \frac{1}{2} \dot{\phi}^2 - V(\phi)$$

Not zero in general

We'll talk about two cases

$$V(\phi) = (1/2)m^2 \phi^2$$

Free scalar field

$$V(\phi) = m^2 f^2 \left[ 1 + \cos \left( \frac{\phi}{f} \right) \right]$$

Axion field or self interacting  $\lambda = 3/\kappa^2 f^2$

Some convenient variable transformation to write KG equation.

$$\Omega_\phi^{1/2} \sin(\theta/2) \equiv \frac{\kappa \dot{\phi}}{\sqrt{6}H}, \quad \Omega_\phi^{1/2} \cos(\theta/2) \equiv \frac{\kappa V^{1/2}}{\sqrt{3}H}, \quad y_1 \equiv -\frac{2\sqrt{2}}{H} \partial_\phi V^{1/2}$$

$$\theta' = -3 \sin \theta + y_1$$

$$y_1' = \frac{3}{2} (1 + w_{tot}) y_1 + \frac{\lambda}{2} \Omega_\phi \sin \theta$$

$$\Omega_\phi' = 3(w_{tot} - w_\phi) \Omega_\phi$$

The free case is recovered for

$$\lambda \rightarrow 0$$

$$w_\phi \equiv \frac{p_\phi}{\rho_\phi} = \frac{x^2 - y^2}{x^2 + y^2} = -\cos \theta$$

$$\Omega_I \equiv \frac{\kappa^2 \rho_I}{3H^2}, \quad w_{tot} \equiv \frac{p_{tot}}{\rho_{tot}} = \sum_I \Omega_I w_I + \Omega_\phi w_\phi$$

Initial conditions set at radiation  
domination epoch (Background cosmology)

$$\theta' \simeq -3\theta + y_1, \quad y_1' \simeq 2y_1, \quad \Omega_\phi' \simeq 4\Omega_\phi \quad \text{To first order}$$

$$\theta = (1/5)y_1 + C(a/a_i)^{-3}$$

$$y_1 = y_{1i}(a/a_i)^2$$

$$\Omega_\phi = \Omega_{\phi i}(a/a_i)^4$$

The scalar field behaves as DM if it is oscillating  
around the minimum of the potential.

$$a_{osc} < a_{eq}$$

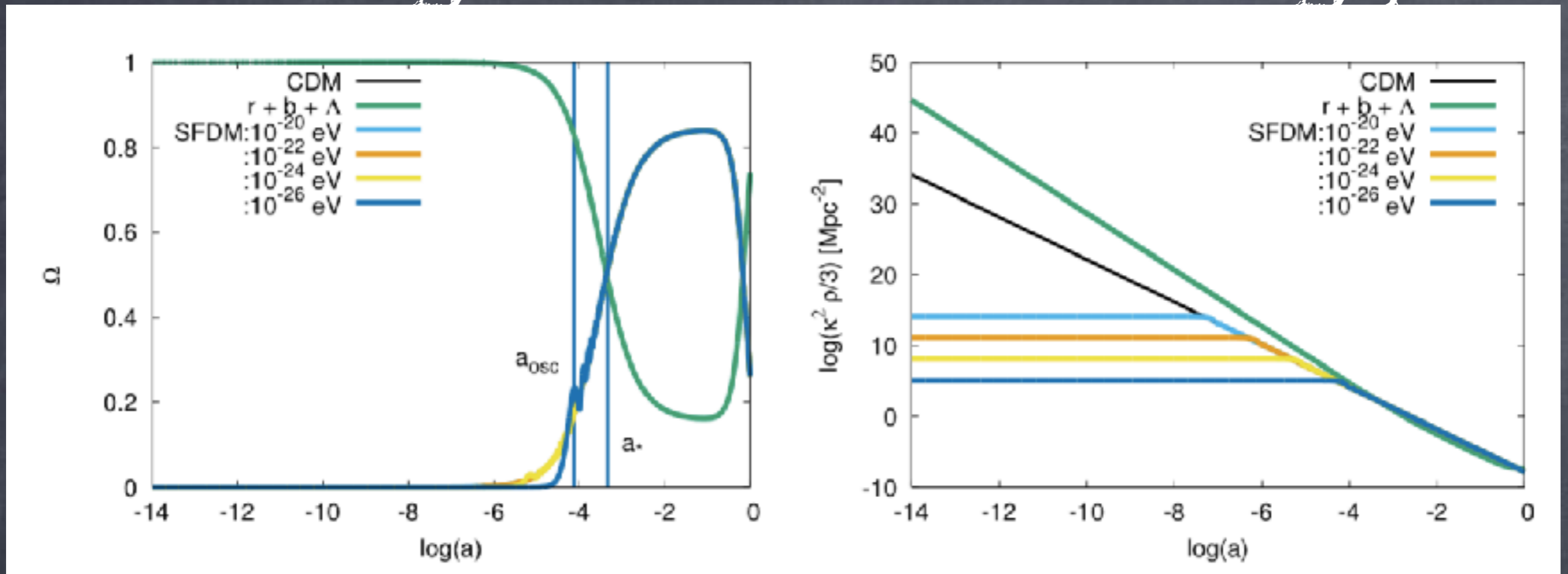
$$H^{-1} > m^{-1}$$

$$a_{osc}^2 \left( 1 + \frac{\lambda}{72} \frac{\Omega_{\phi 0}}{\Omega_{r0}} a_{osc} \right) = \frac{\pi \theta_i^{-1} a_i^2}{2\sqrt{1 + \pi^2/36}}$$

$$4 \frac{m^2}{H_i^2} = y_{1i}^2 + 4\lambda \Omega_{\phi i} \quad y_{1i} = 5\theta_i \left( 1 + \frac{\lambda}{40} \Omega_{\phi i} \right), \quad \Omega_{\phi i} = \frac{a_i^4}{a_{osc}^3} \frac{\Omega_{\phi 0}}{\Omega_{r0}}$$

We solve this inside CLASS code with a shooting parameter

# Background Cosmology



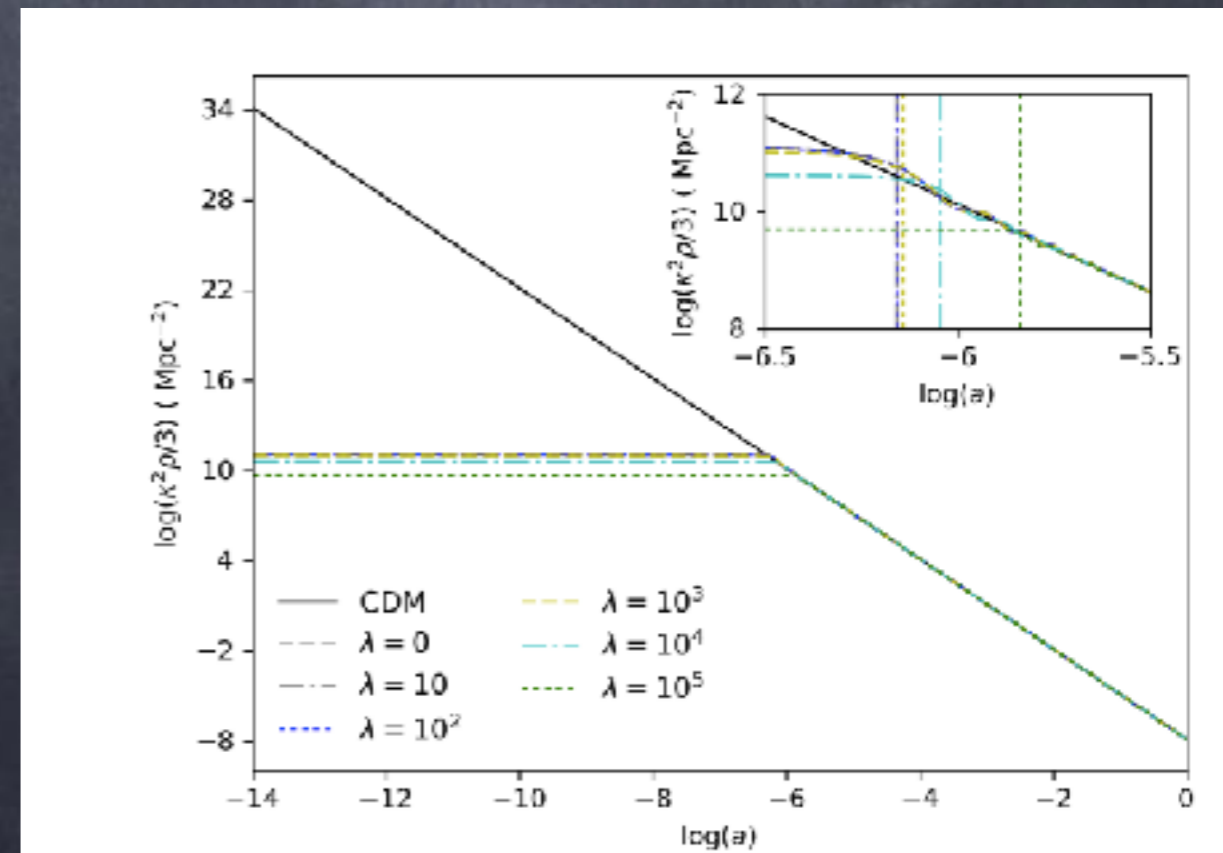
\*Technical complication about the oscillations. So we have to cut them at some point.

$$\{\cos_\star \gamma, \sin_\star \gamma\} \equiv$$

$$(1/2) [1 - \tanh(\gamma^2 - \gamma_\star^2)] \{\cos \gamma, \sin \gamma\}$$

$$\{\cos_\star \gamma, \sin_\star \gamma\} \rightarrow 0$$

$$\gamma > \gamma_\star$$





# Linear Perturbation Theory

$$ds^2 = -dt^2 + a^2(t)(\delta_{ij} + h_{ij})dx^i dx^j$$

Synchronous gauge

$$\ddot{\varphi} = -3H\dot{\varphi} - \left( \frac{k^2}{a^2} + \frac{\partial^2 V(\phi)}{\partial \phi^2} \right) \varphi - \frac{1}{2} \dot{\phi} \ddot{h}$$

Perturbed KG eq. to linear order, and for a k-mode

$$\phi(x, t) = \phi(t) + \varphi(x, t)$$

$$\delta_\phi = \frac{\dot{\phi}\dot{\varphi} + \partial_\phi V \varphi}{\dot{\phi}^2/2 + V(\phi)}, \quad \delta_{\delta p_\phi} = \frac{\dot{\phi}\dot{\varphi} - \partial_\phi V \varphi}{\dot{\phi}^2/2 + V(\phi)}, \quad (\rho_\phi + p_\phi)\theta_\phi = (k^2/a)\dot{\phi}\varphi$$

$$\delta_\phi \equiv \delta\rho_\phi/\rho_\phi$$

$$\delta_{p_\phi} \equiv \delta p_\phi/p_\phi$$

Various approaches to solve this. Ours tries to keep information about the oscillations, both in the background and the perturbations for as long possible (numerical stiffness)

After some variable changes, as we did  
with the background...

$$\delta'_0 = \left[ -3 \sin \theta - \frac{k^2}{k_J^2} (1 - \cos \theta) \right] \delta_1 + \frac{k^2}{k_J^2} \sin \theta \delta_0 - \frac{\bar{h}'}{2} (1 - \cos \theta)$$

$$\delta'_1 = \left[ -3 \cos \theta - \frac{k^2}{k_J^2} \sin \theta + \Omega_\phi^{1/2} \sin \left( \frac{\theta}{2} \right) \frac{y_2}{y_1} \right] \delta_1 + \left[ \frac{k^2}{k_J^2} (1 + \cos \theta) - \Omega_\phi^{1/2} \cos \left( \frac{\theta}{2} \right) \frac{y_2}{y_1} \right] \delta_0$$

$$k_J^2 = H^2 a^2 y_1 \quad \delta_\phi = \delta_0, \quad \delta_{p_\phi} = \sin \theta \delta_1 - \cos \theta \delta_0, \quad -\frac{\bar{h}'}{2} \sin \theta$$

$$(\rho_\phi + p_\phi) \theta_\phi = \frac{k^2}{2am} \rho_\phi [(1 + \omega_\phi) \delta_1 - \sin \theta \delta_0]$$

For the axion potential

$$\delta'_0 = \left[ -3 \sin \theta - \frac{k^2}{k_J^2} (1 - \cos \theta) \right] \delta_1 + \frac{k^2}{k_J^2} \sin \theta \delta_0 - \frac{\bar{h}'}{2} (1 - \cos \theta)$$

$$\delta'_1 = \left[ -3 \cos \theta - \frac{k_{eff}^2}{k_J^2} \sin \theta \right] \delta_1 + \frac{k_{eff}^2}{k_J^2} (1 + \cos \theta) \delta_0 - \frac{\bar{h}'}{2} \sin \theta$$

$$k_{eff}^2 \equiv k^2 - \lambda a^2 H^2 \Omega_\phi / 2$$

$$k_J = a \sqrt{2Hm}$$

The free case is  
recovered for

can be positive or negative

$$\lambda \rightarrow 0$$

For  $\lambda > 0$  interesting thing happens.

- The perturbations can grow larger for some scales (wavenumbers).

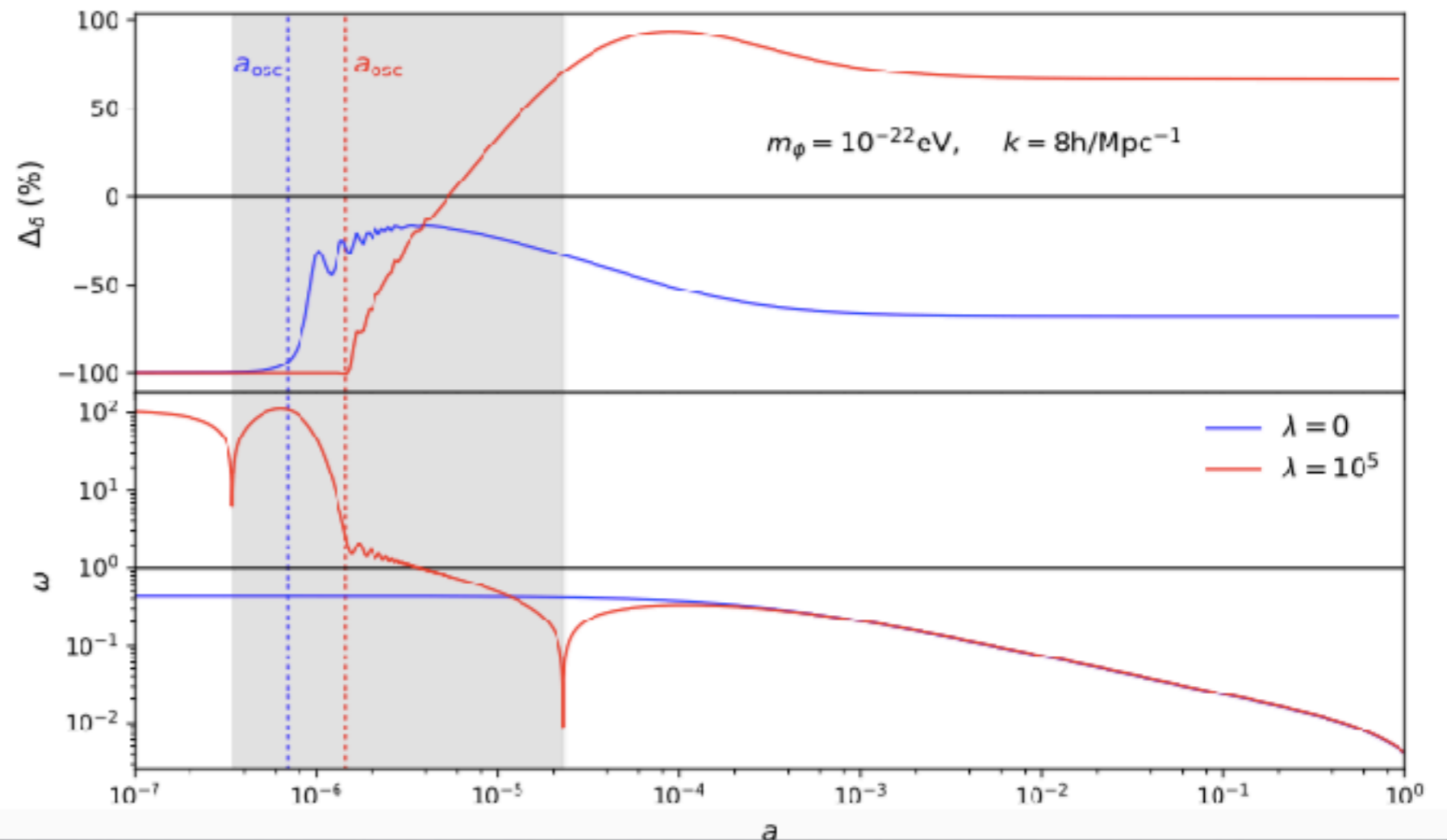
Qualitatively, once oscillations started:

$$\delta'_0 = -\frac{k^2}{k_J^2} \delta_1 - \frac{\bar{h}'}{2}, \quad \delta'_1 = \frac{k_{eff}^2}{k_J^2} \delta_0. \quad \delta''_0 + \omega^2 \delta_0 = -\frac{\bar{h}''}{2}, \quad \omega^2 \equiv \frac{k^2 k_{eff}^2}{k_J^4}$$

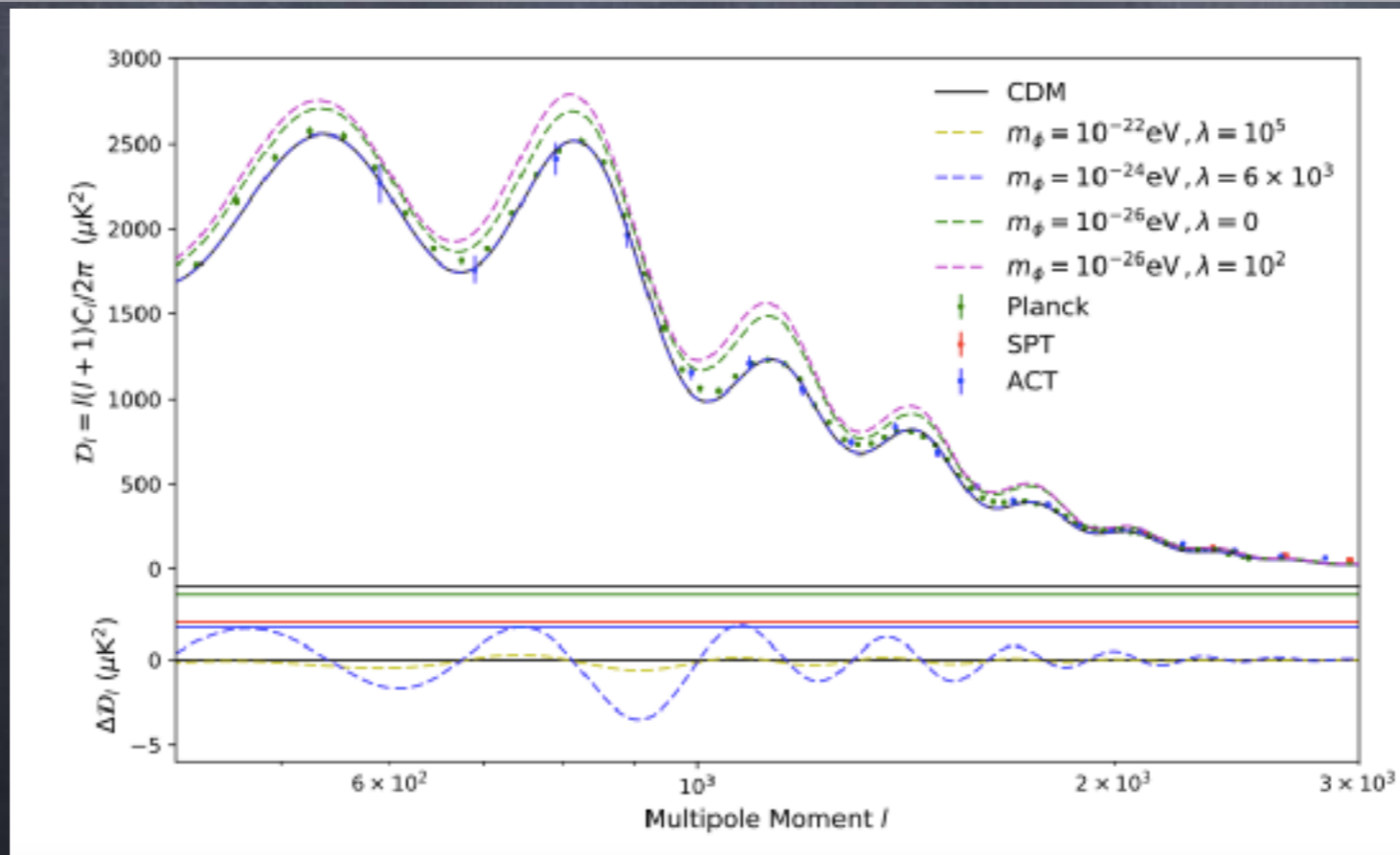
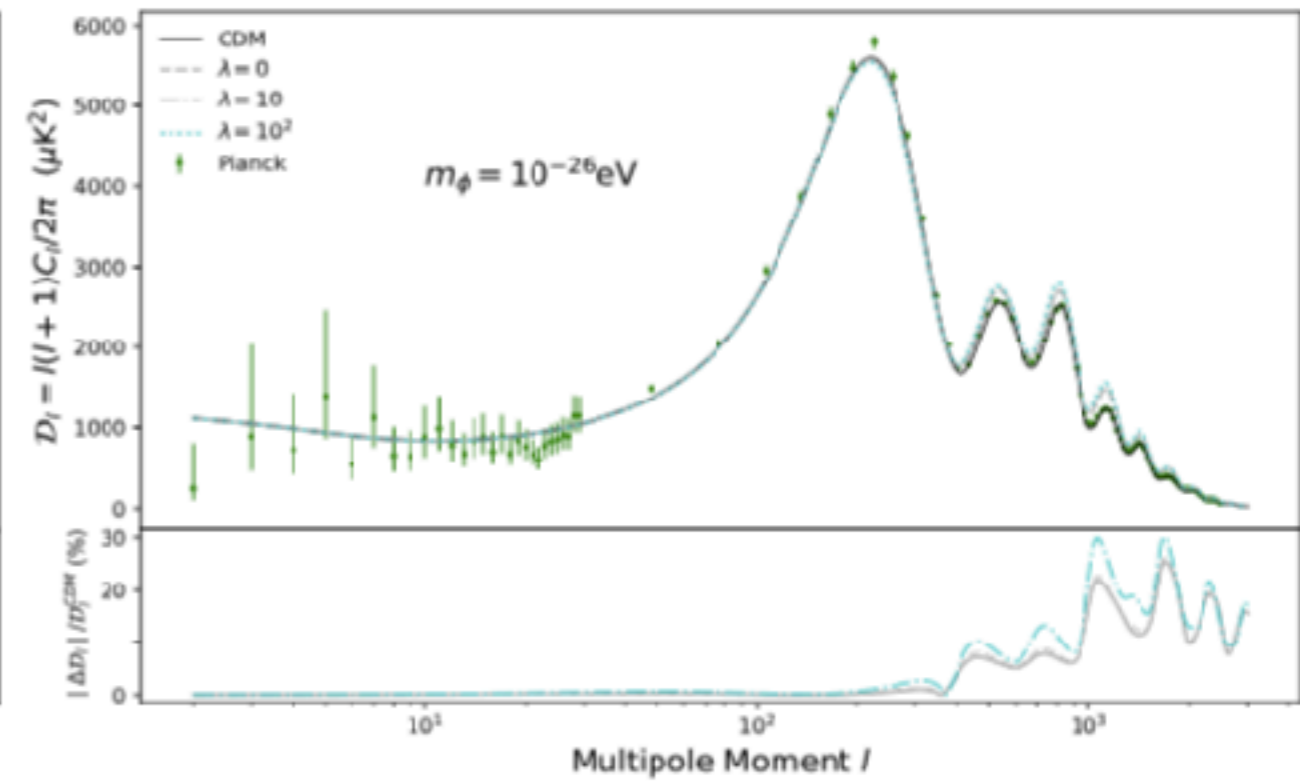
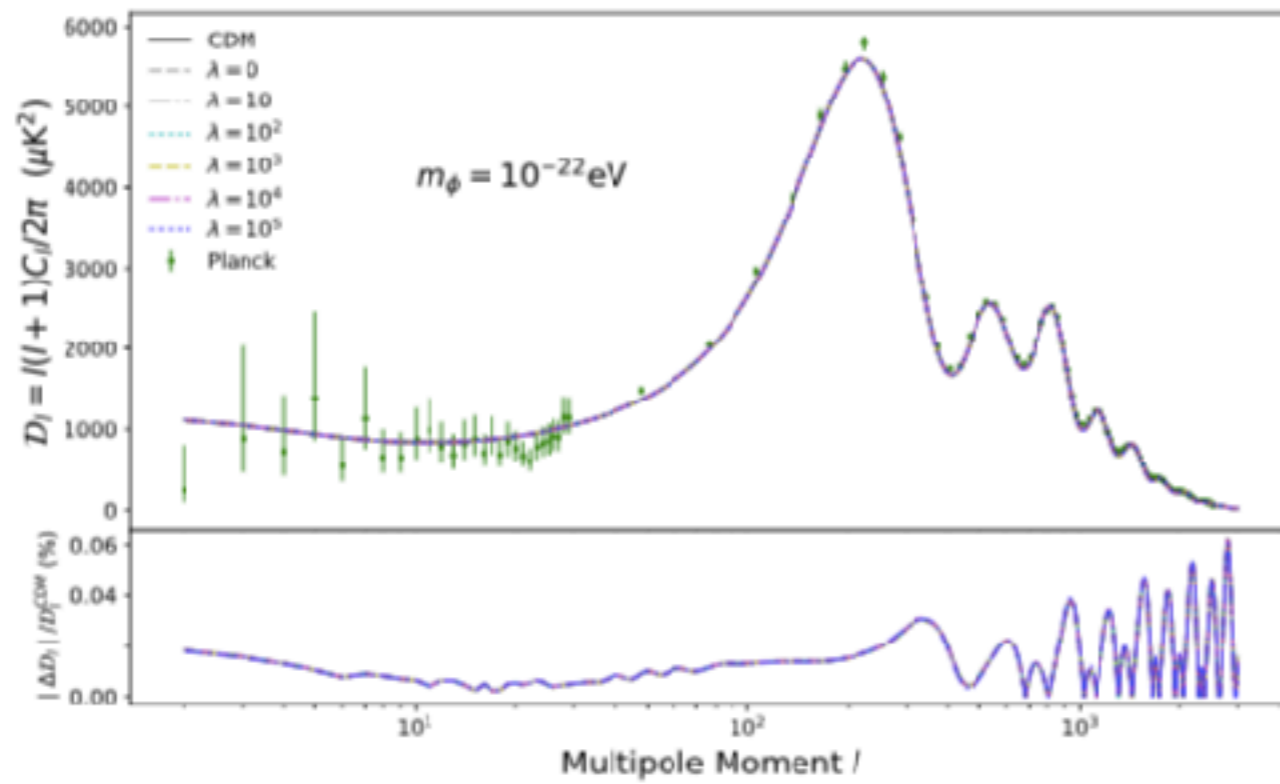
$$\Delta_\delta \equiv (\delta_\phi - \delta_{CDM}) / \delta_{CDM}$$

Tachyonic instability.

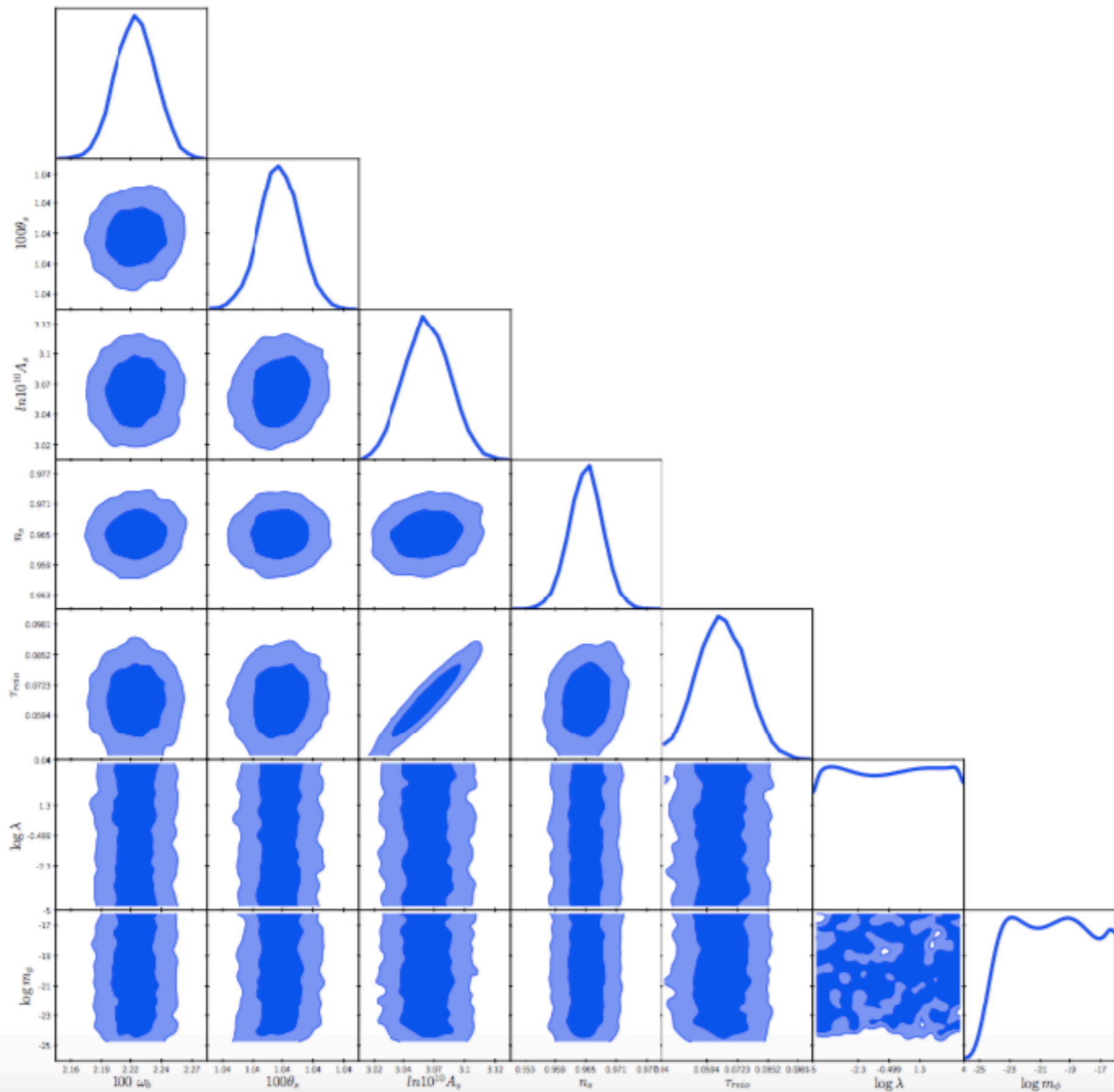
It is being called the extreme axion by other authors



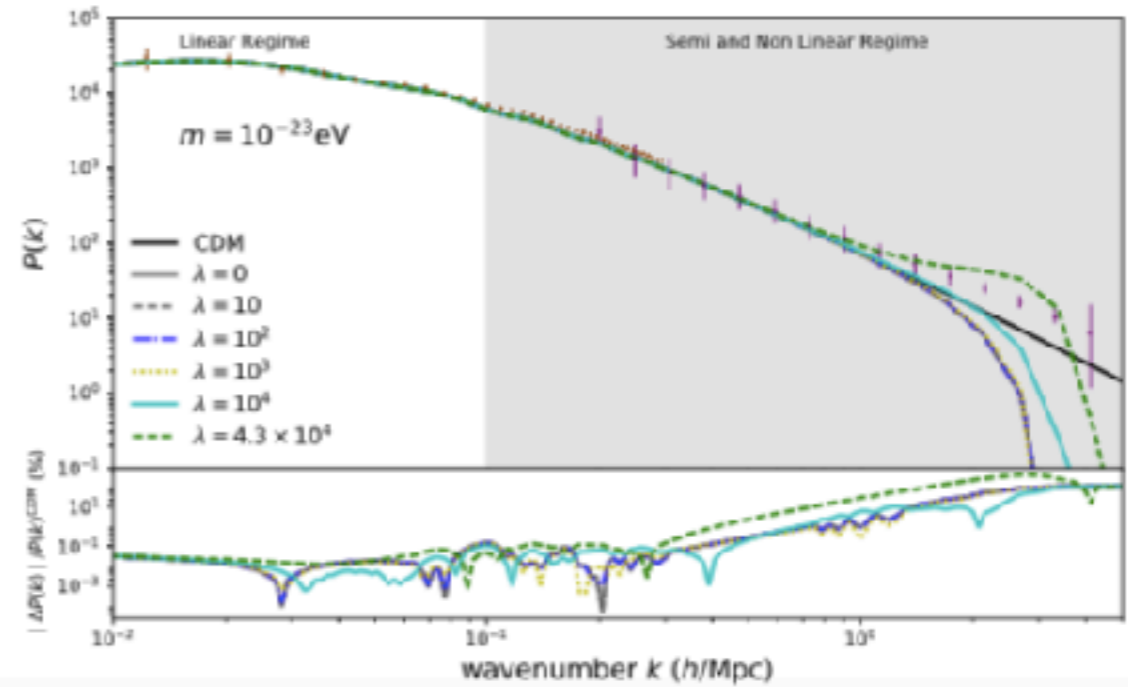
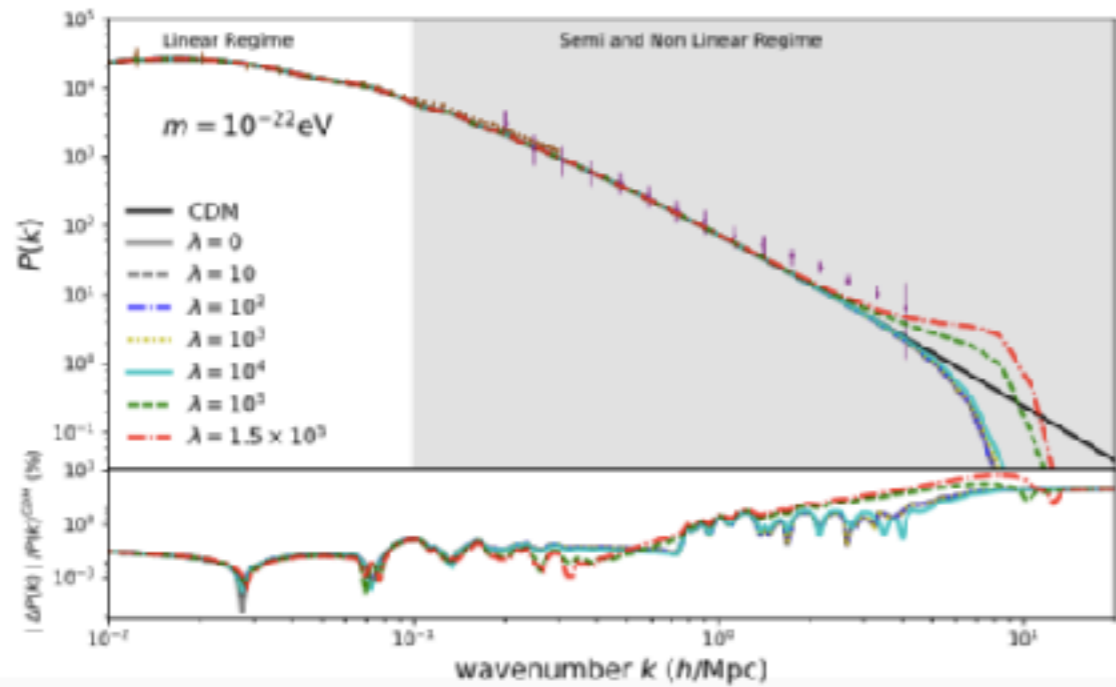
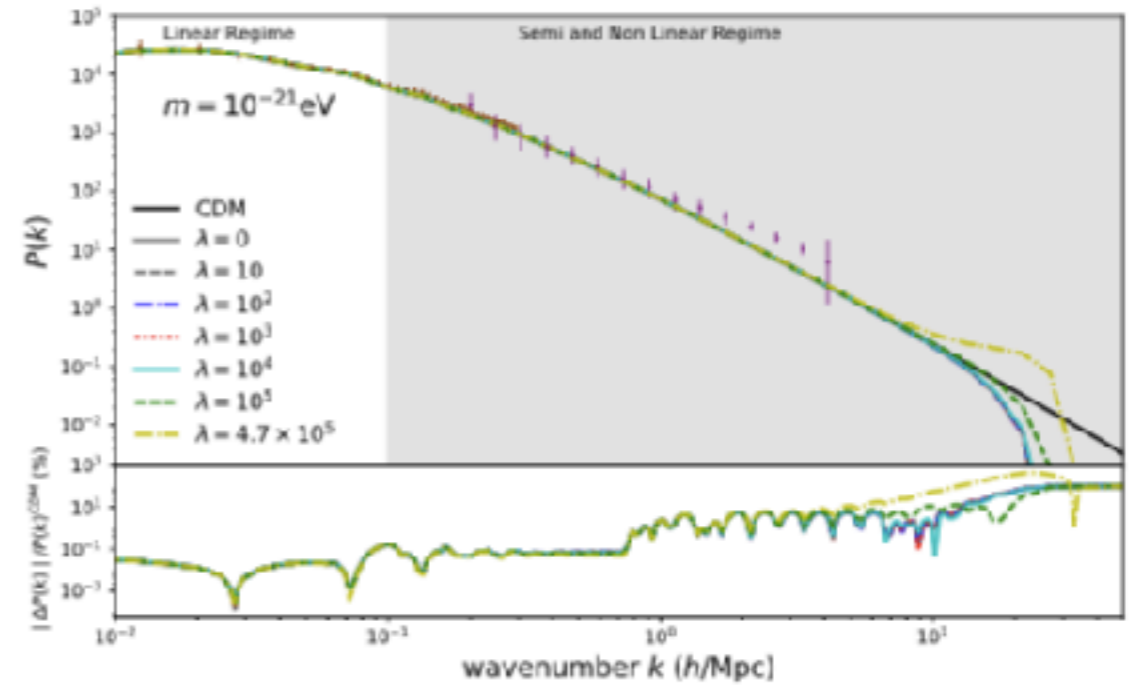
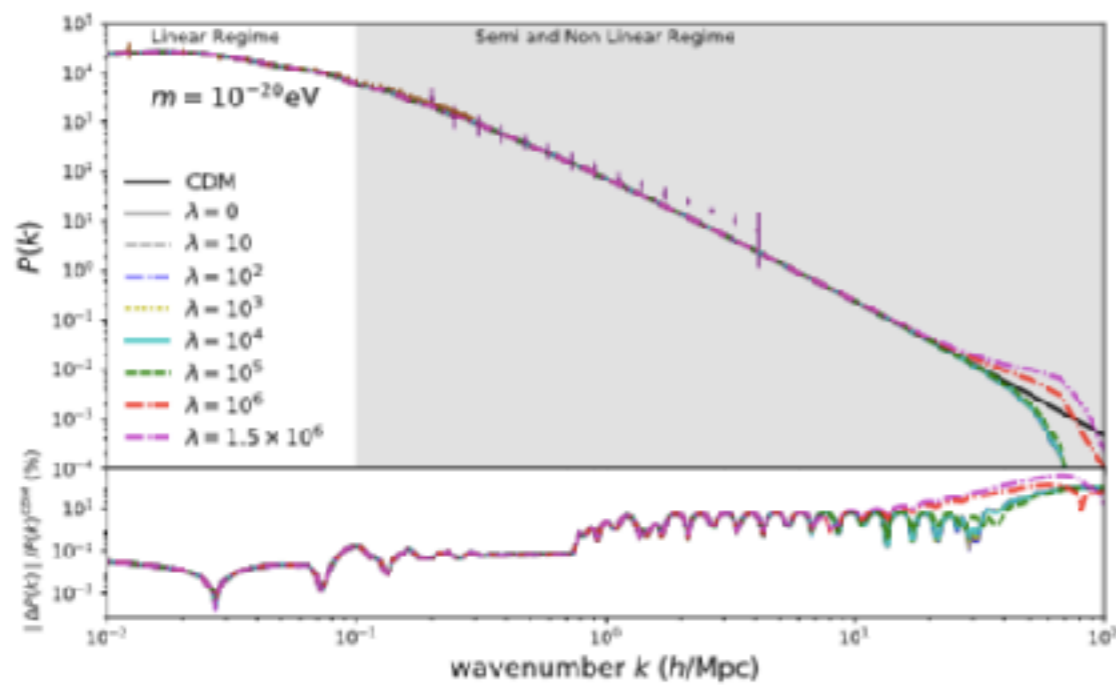
# Observables, CMB



# CMB Constraints

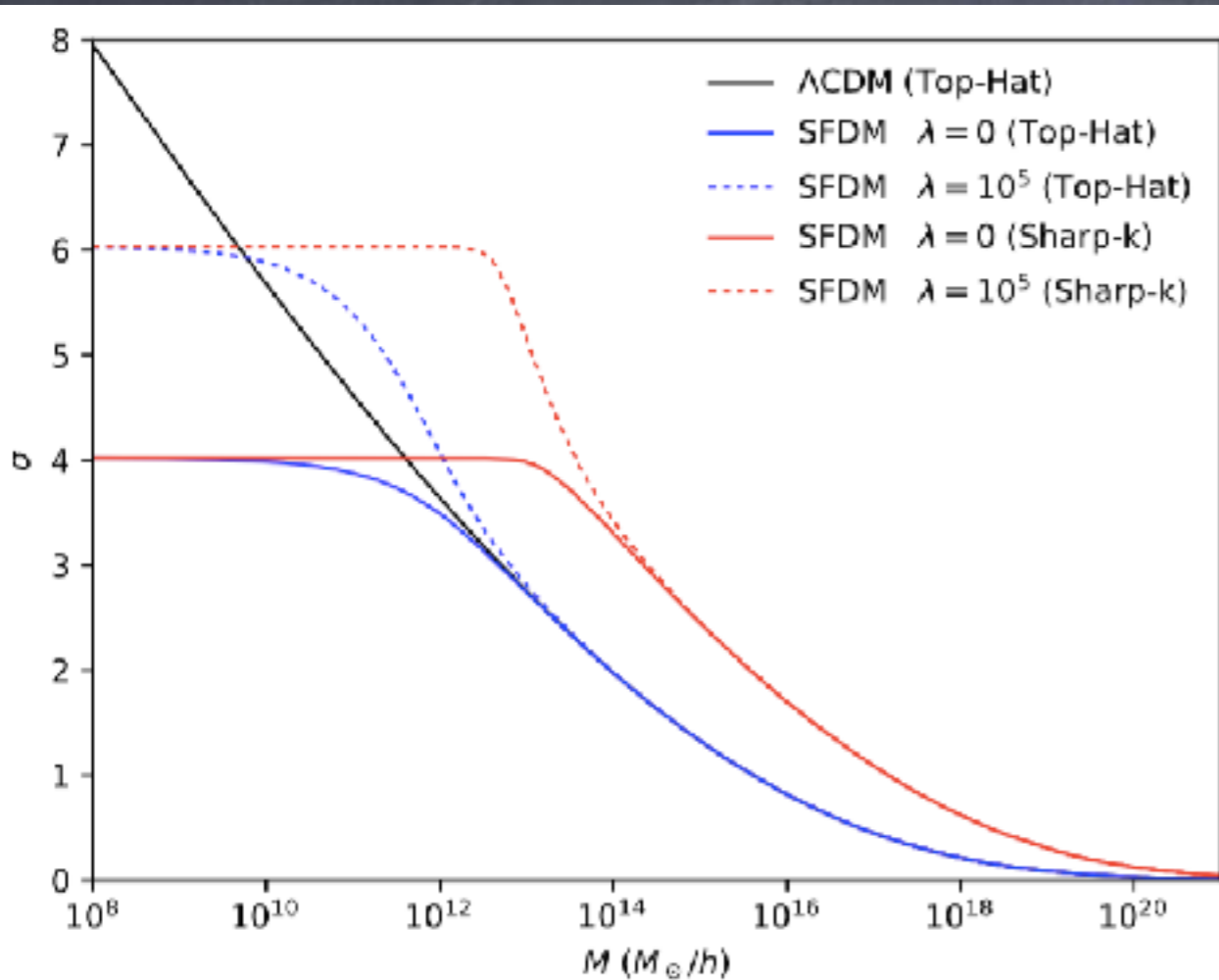


# Mass Power Spectrum



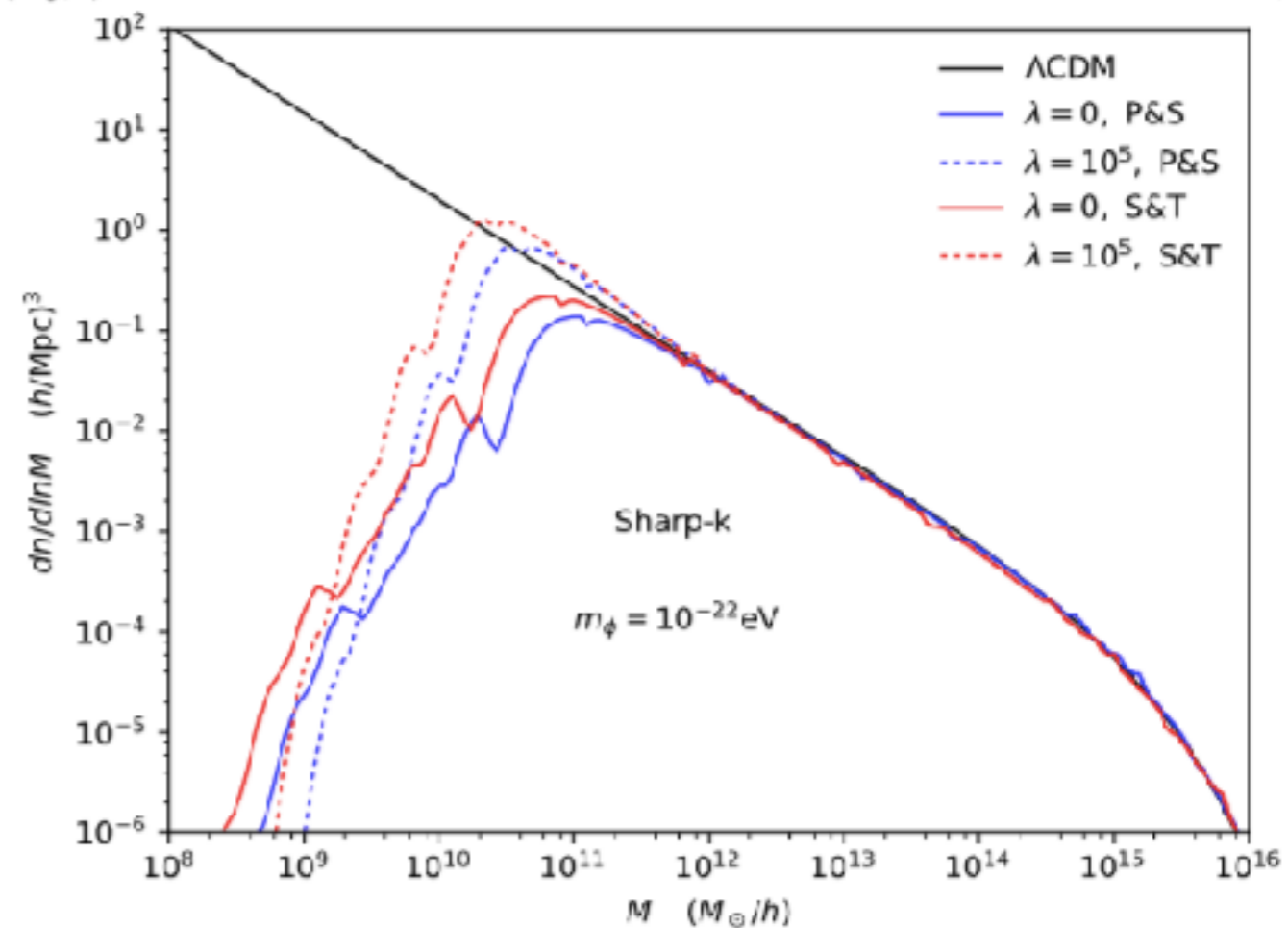
Lya PS data is only illustrative of the scales.

# Approximate Mass Function



$$\sigma^2(r) = \int \frac{d^3 \vec{k}}{(2\pi)^3} P(k) W^2(kr)$$

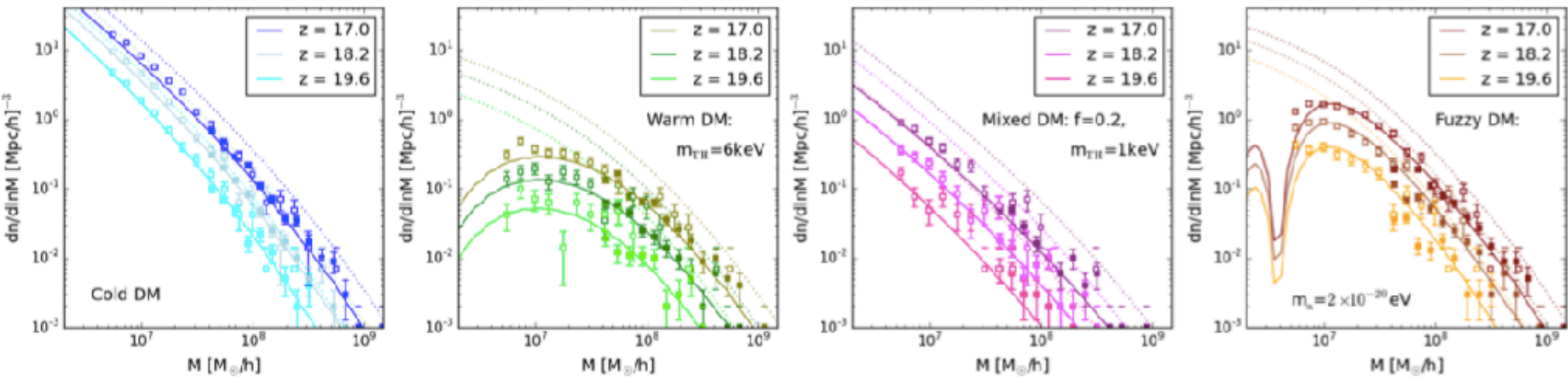
$$\frac{dn}{d \ln M} = -\frac{1}{2} \frac{\bar{\rho}}{M} f(\nu) \frac{d \ln \sigma^2}{d \ln M}$$



$$W_{TH}(kr) = \frac{3}{(kr)^3} [\sin(kr) - kr \cos(kr)]$$

$$W_{SK}(kr) = \Theta(2\pi - kr)$$

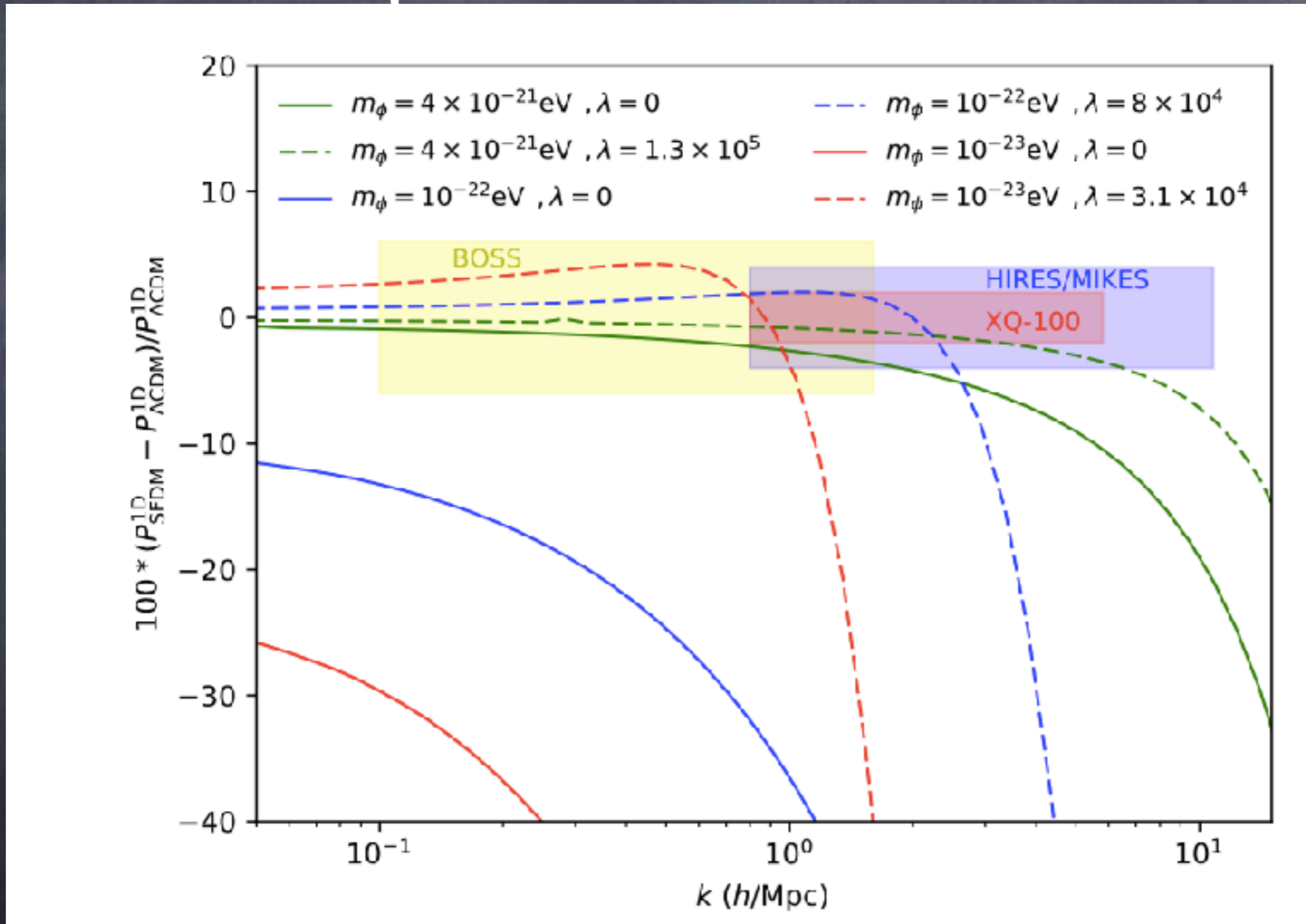
Sharp-K WF seems to work better for PK's with cut-off



Schneider 2018 & 2015



# Some naive prospects for Lyman-alpha constraints

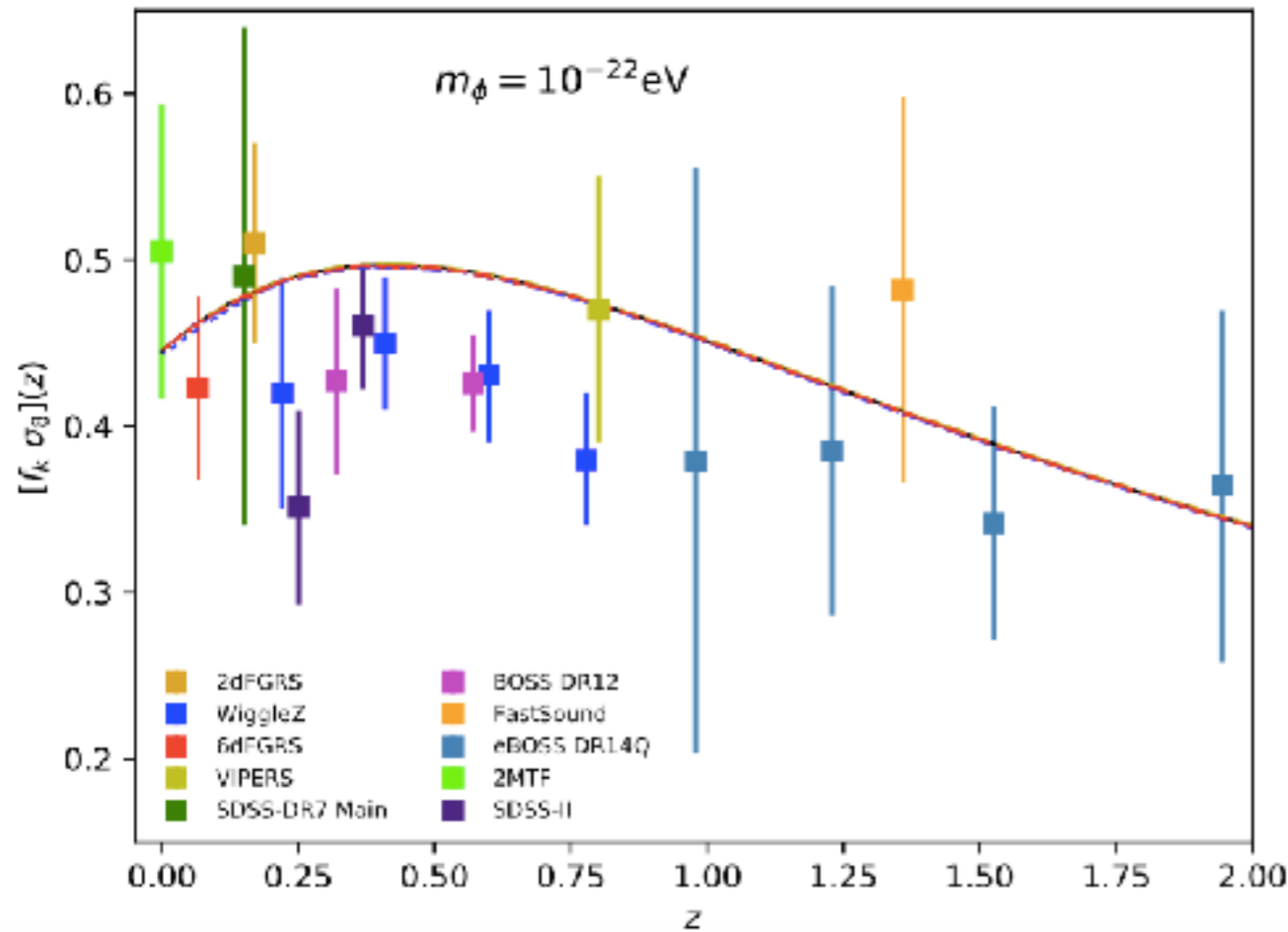


For  $\lambda = 0$ , strong constraints have been set.

Armengaud et. al 2017 & Iršič et. al 2017

$$m > 3 \times 10^{-21} \text{ eV}$$

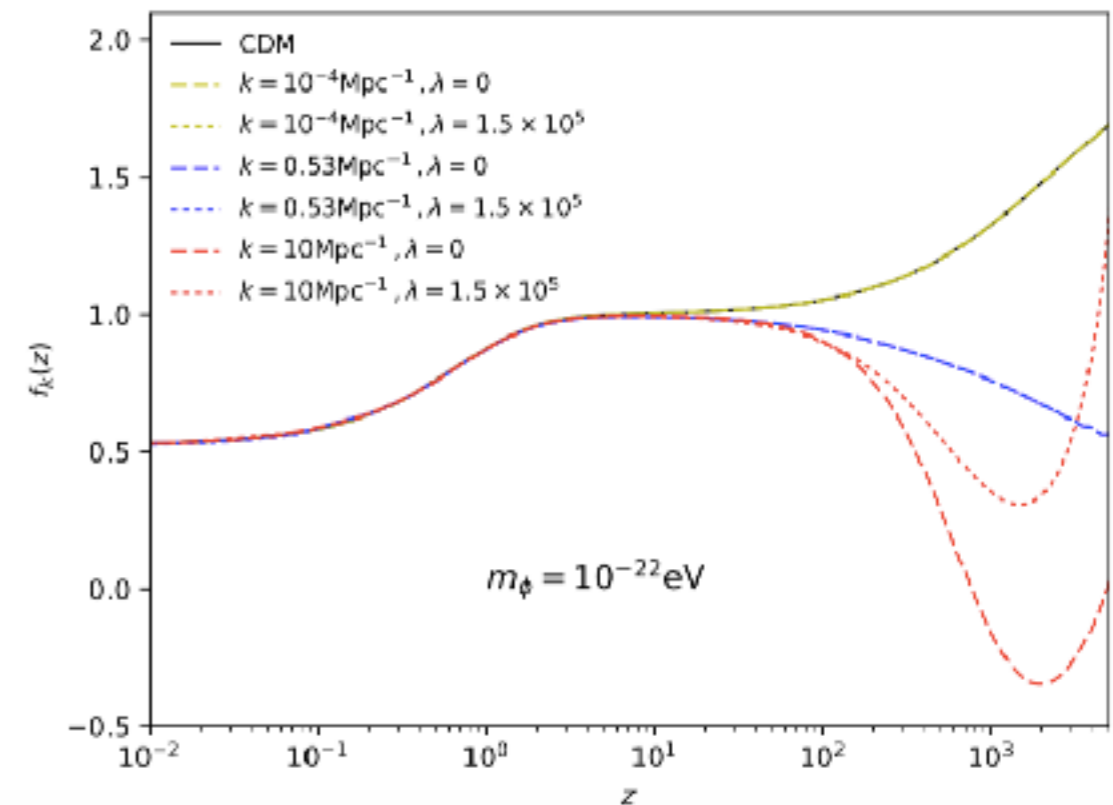
$f\sigma_8$  Observable at low redshift is unaffected



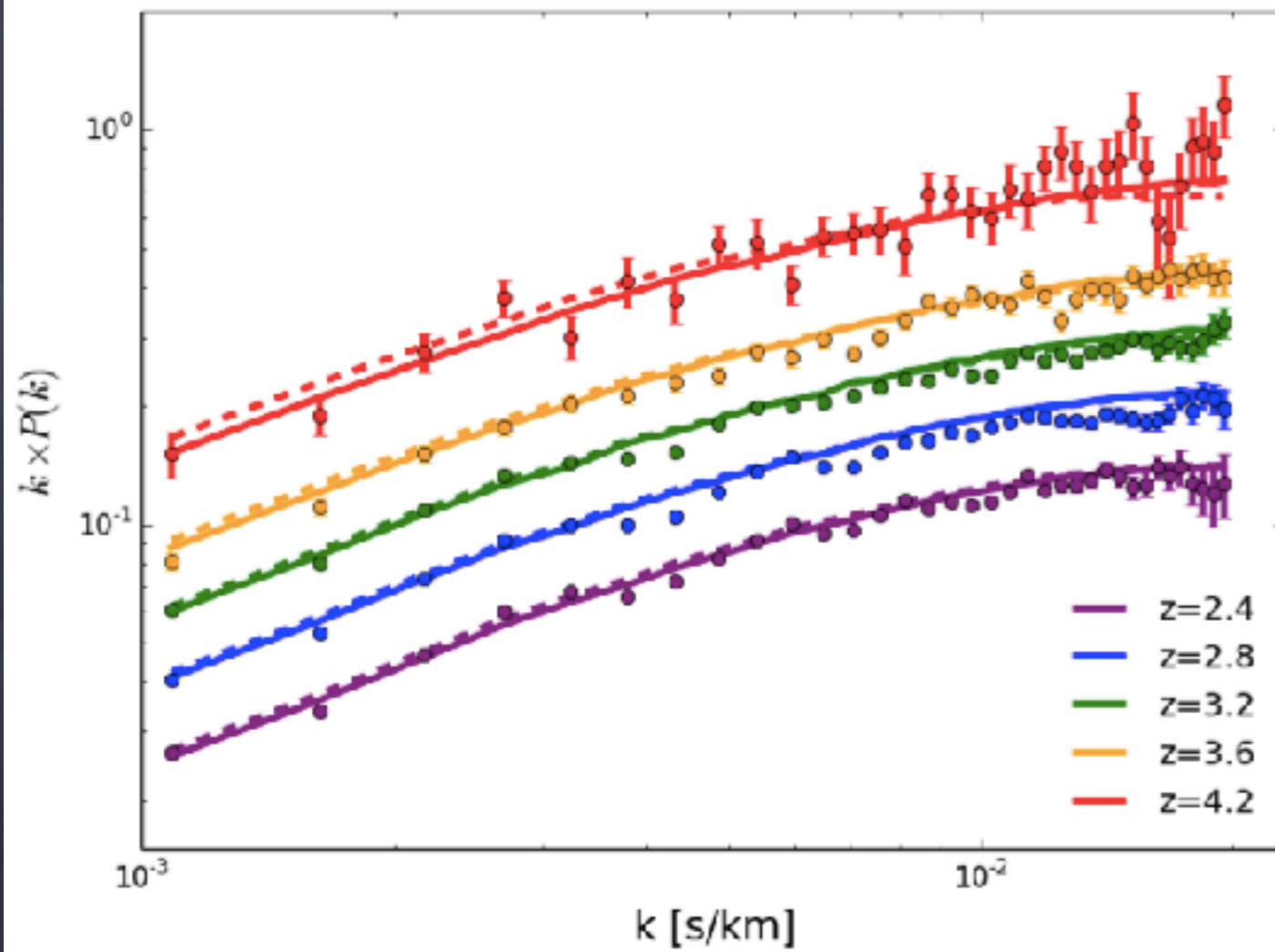
\*might be obvious...

$$D_k(N) \equiv \frac{\delta_0(N, k)}{\delta_0(N_{\text{late}}, k)}$$

$$f_k(N) = \frac{d \log D_k(N)}{dN} = \frac{\delta'_0(N, k)}{\delta_0(N, k)}$$



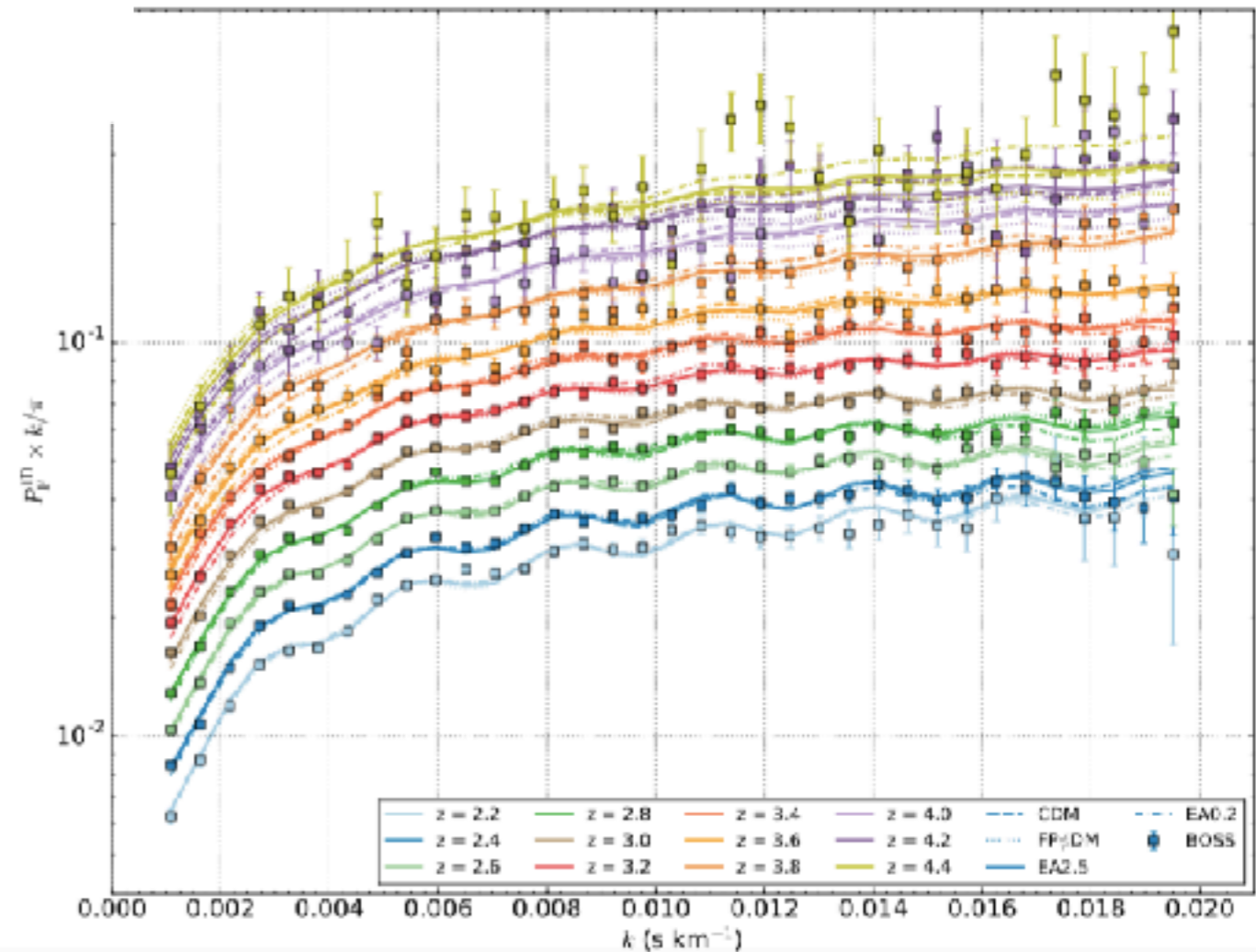
Armengaud et.al 2017



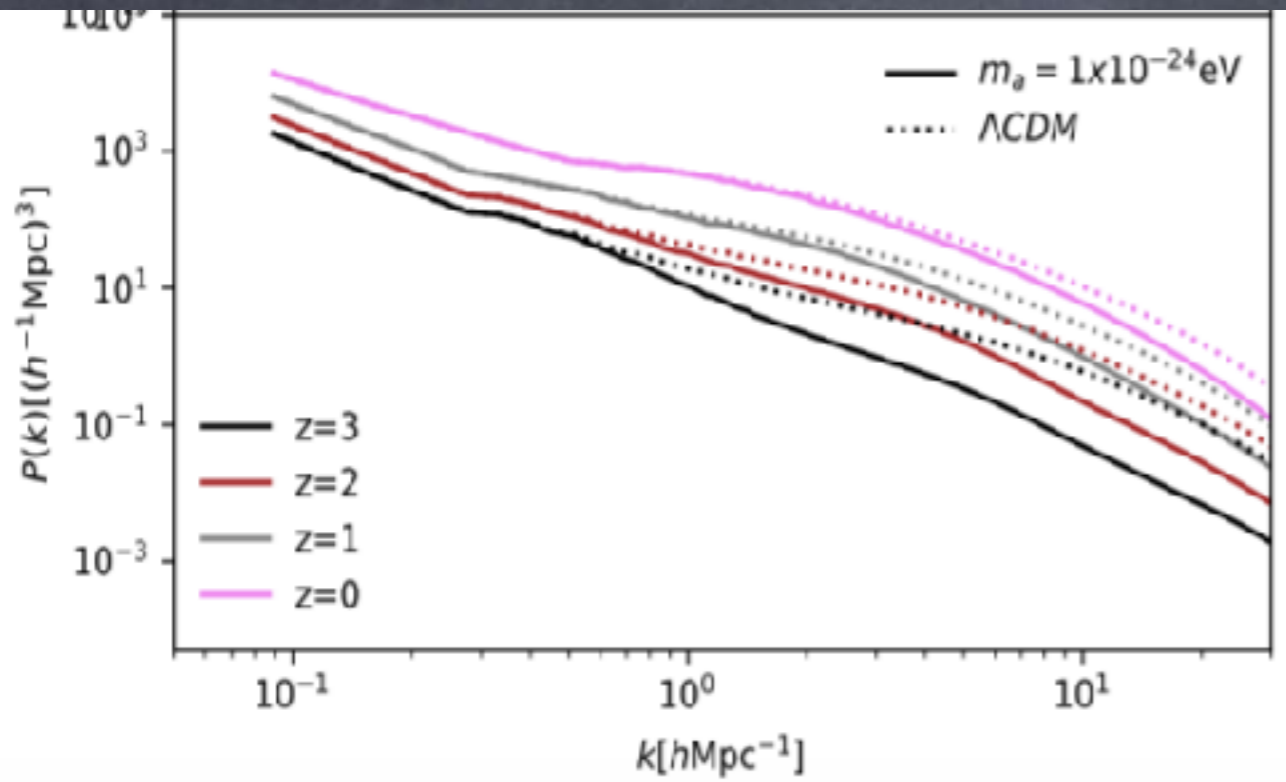
Ka-Hou Leong, 2018

$\lambda > 0$  Seems to fit slightly better BOSS data. However thermal history is affected.

Work in progress with Armengaud et. al.

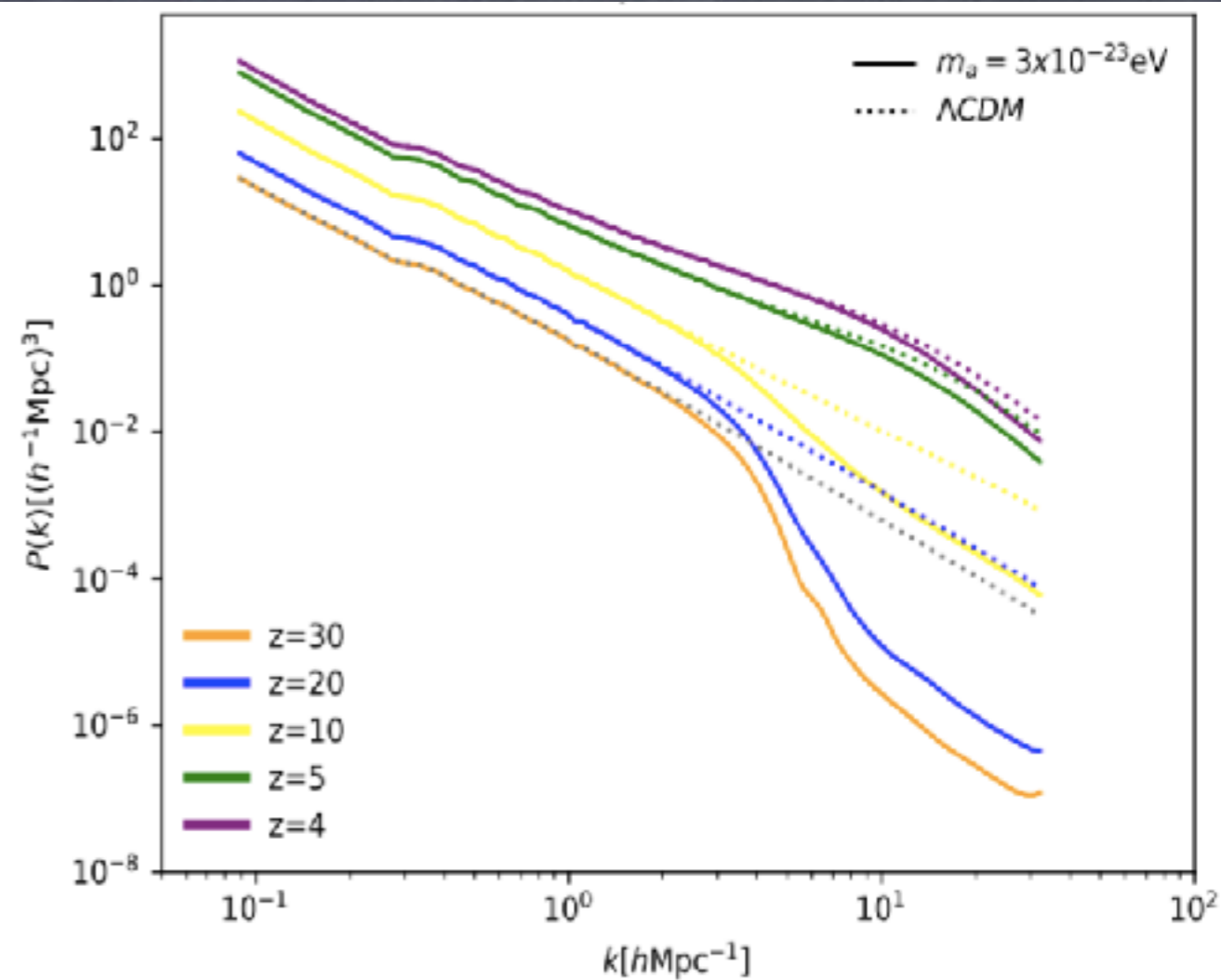


# Work in progress. Cosmological simulations using COLA like methods



$$m = 1 \times 10^{-24} \text{eV}$$

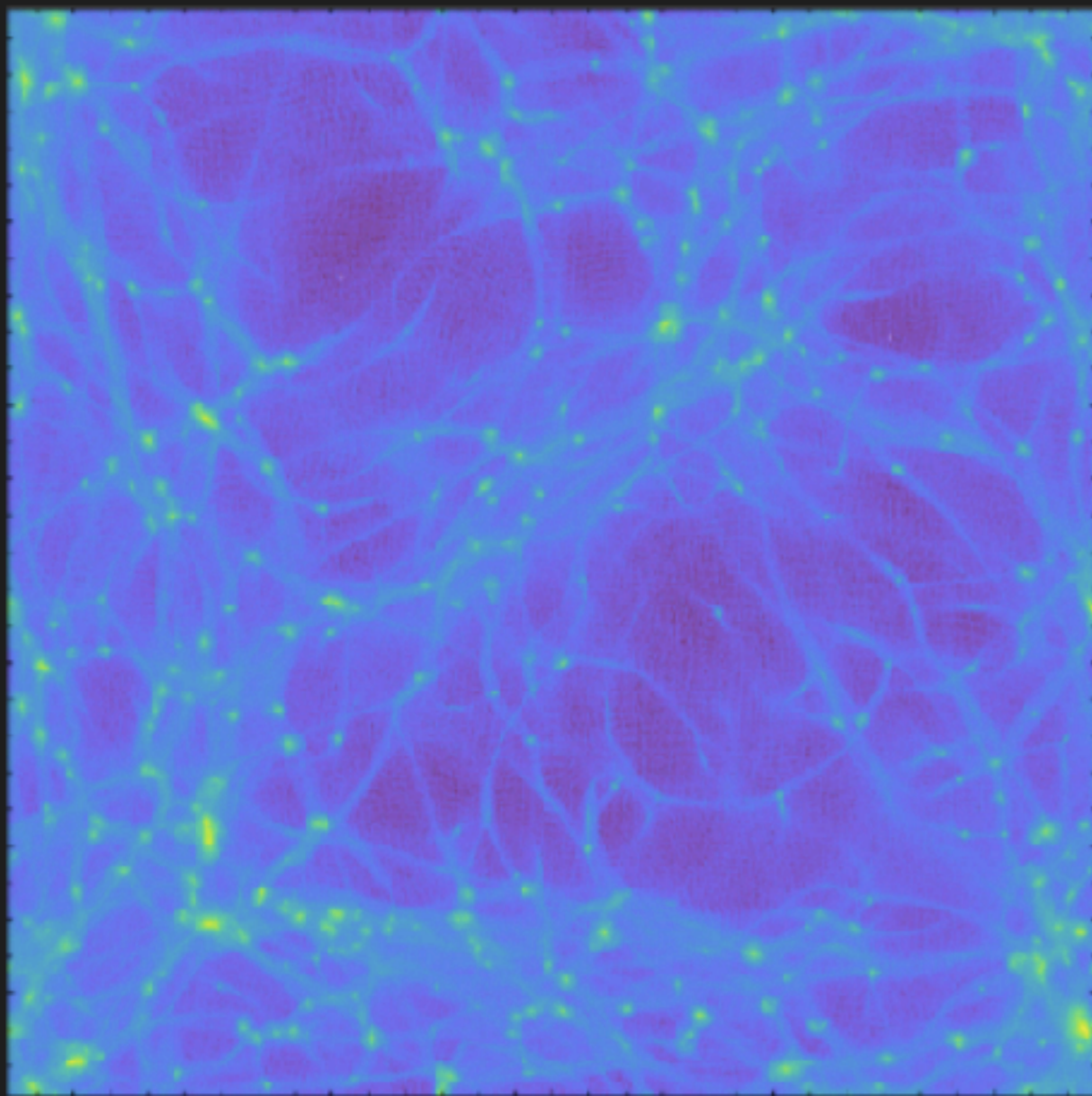
$N_{\text{part}}=1024$ ,  $L_{\text{box}}=100 \text{Mpc}$   $\sim 200$  steps.



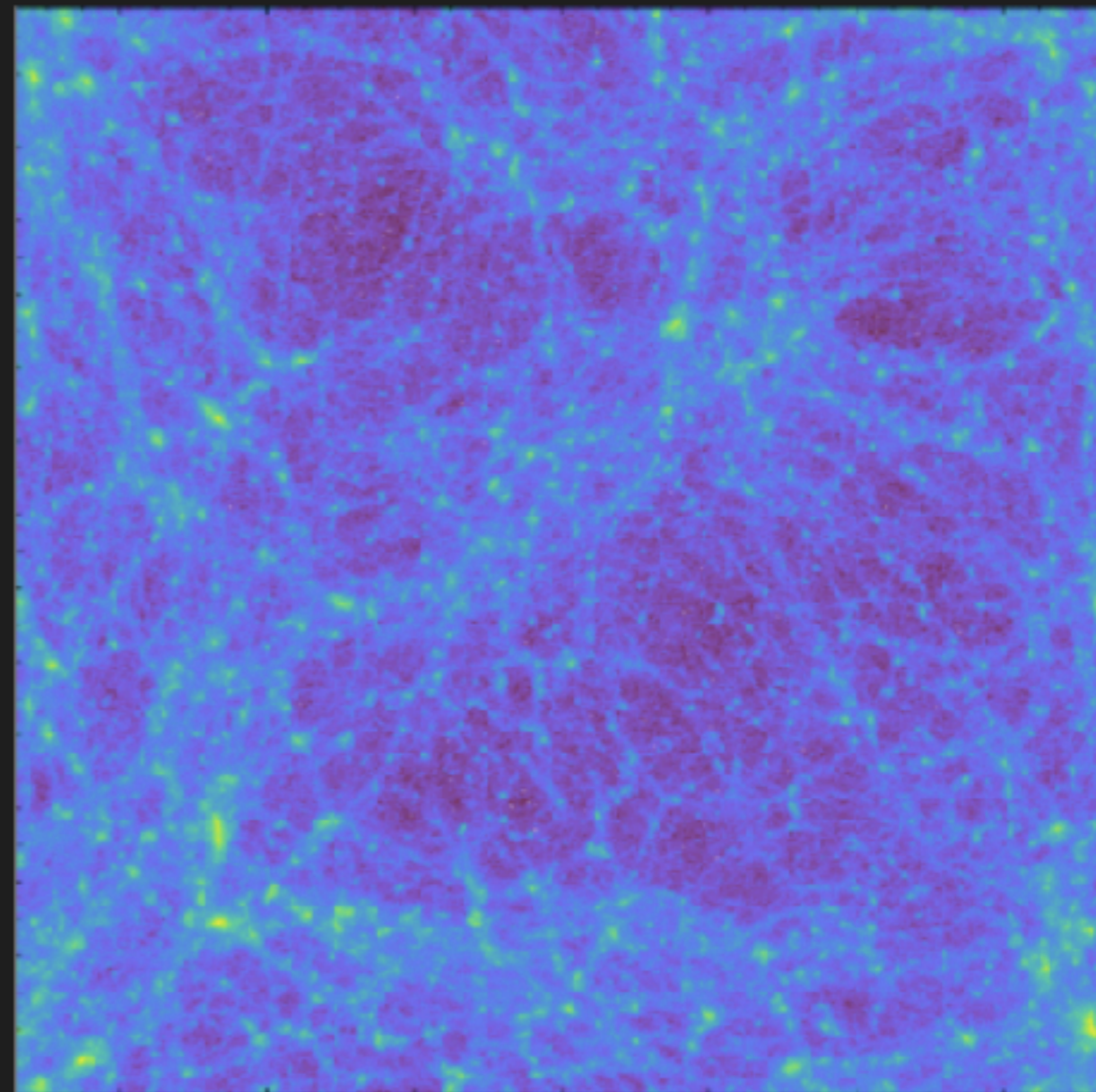
$$m = 3 \times 10^{-23} \text{eV}$$

# COLA LIKE SIMULATIONS OF SFDM

FDM



$\Lambda$ -CDM



# Conclusion

- We are in a good track to get to the non-linear regime.
- Lots of things to do, and observables to compare with in the near future.

FIG FESTIVAL, León, November

Thanks