Theory of dynamical phase transitions driven by excited-state quantum phase transitions

Ángel L. Corps[1,*]

[1]Instituto de Estructura de la Materia-Consejo Superior de Investigaciones Científicas, IEM-CSIC, Madrid, Spain

[*]corps.angel.l@gmail.com

w/ Relaño

PRL **127**, 130602 (2021)

arXiv:2205.03443

arXiv:2205.11199

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AGENCIA ESTATAL DE INVESTIGACIÓN
"la Caixa" Foundation

Reviewing some types of phase transitions...

(1) System coupled to **bath** at temperature $\beta = 1/k_B T$

$$f = -\frac{k_B T}{N} \ln \mathcal{Z}, \quad \mathcal{Z} = \mathrm{Tr}\, e^{-\beta \hat{\mathcal{H}}}$$

Thermal phase transition

$f(T = T_c)$ non-analytic

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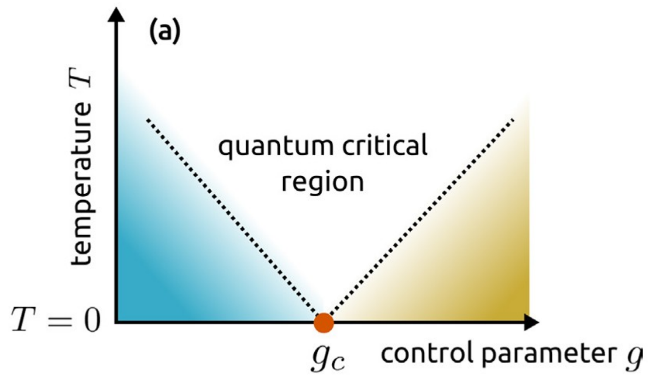
(2) **Closed** system at temperature $T = 0$

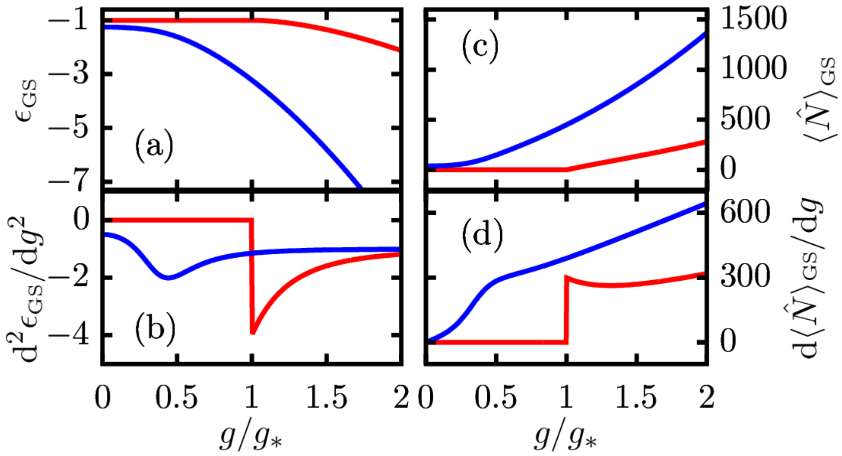
Dynamics governed by Hamiltonian, $\hat{\mathcal{H}}(\lambda)$

Non-analytic behavior in **observables** at critical λ_c

Quantum phase transition



Rep. Prog. Phys. **81**, 054001 (2018)

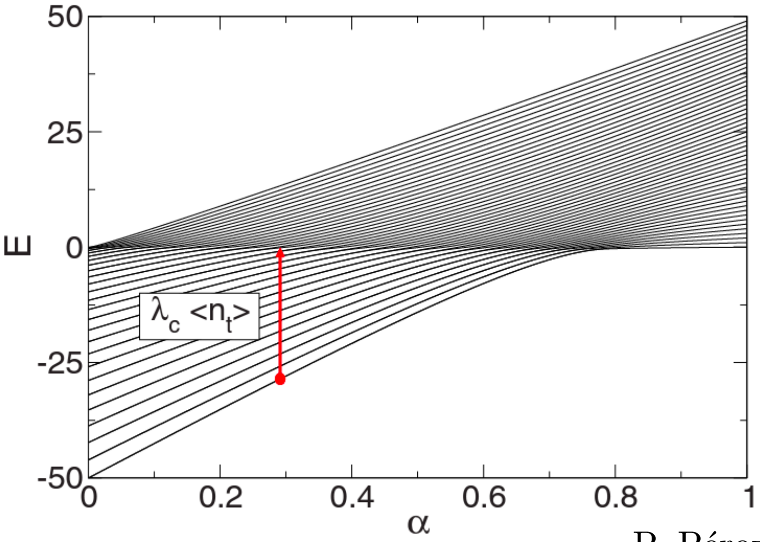


ALC & Relaño, PRA **105**, 052204 (2022)

Reviewing some types of phase transitions...

(3) **Excited-state quantum phase transition**: Non-analiticity in the high-lying states

Most common signature: divergence in the density of states (or in derivatives)



P. Pérez-Fernández *et al*, PRA **78**, 060102(R) (2008)

Impact on:

Expectation value of observables

Decoherence

Recent review by Cejnar *et al*: J. Phys. A **54** (2021) 133001

(4) **Closed** system evolving unitarily after a **quench**

$\hat{\mathcal{H}} = \hat{\mathcal{H}}(\lambda)$ Time-independent Hamiltonian

$$\lambda_i \longrightarrow \lambda_f$$

Initial state $|\Psi_0(\lambda_i)\rangle \longrightarrow |\Psi_t(\lambda_f)\rangle = e^{-i\hat{\mathcal{H}}(\lambda_f)t}|\Psi_0(\lambda_i)\rangle$

Time-evolved wave function

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Time-evolved wave function

Non-analytic **times** in the return amplitude

$$G(t) \equiv \langle\Psi_0(\lambda_i)|\, e^{-i\hat{\mathcal{H}}(\lambda_f)t}\,|\Psi_0(\lambda_i)\rangle$$

Phys. Rev. Lett. **110**, 135704 (2013)

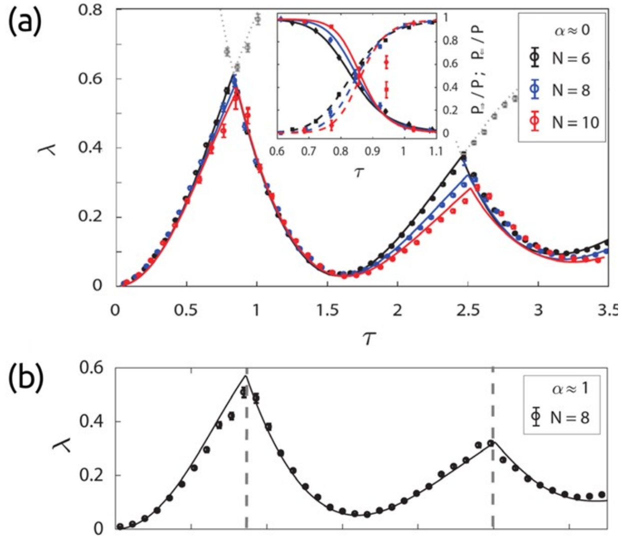
(*) Formally reminiscent of partition function at imaginary temperature $z = it$

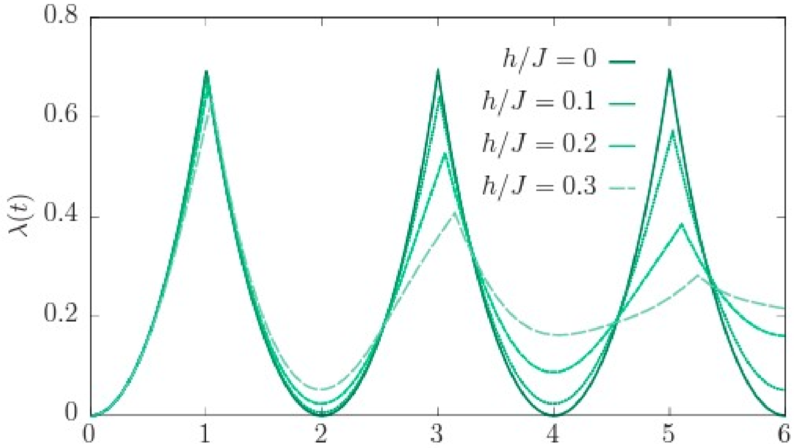
$$G(t) = \langle e^{-z\hat{\mathcal{H}}}\rangle \longleftrightarrow \mathcal{Z} = \mathrm{Tr}\, e^{-\beta\hat{\mathcal{H}}}$$

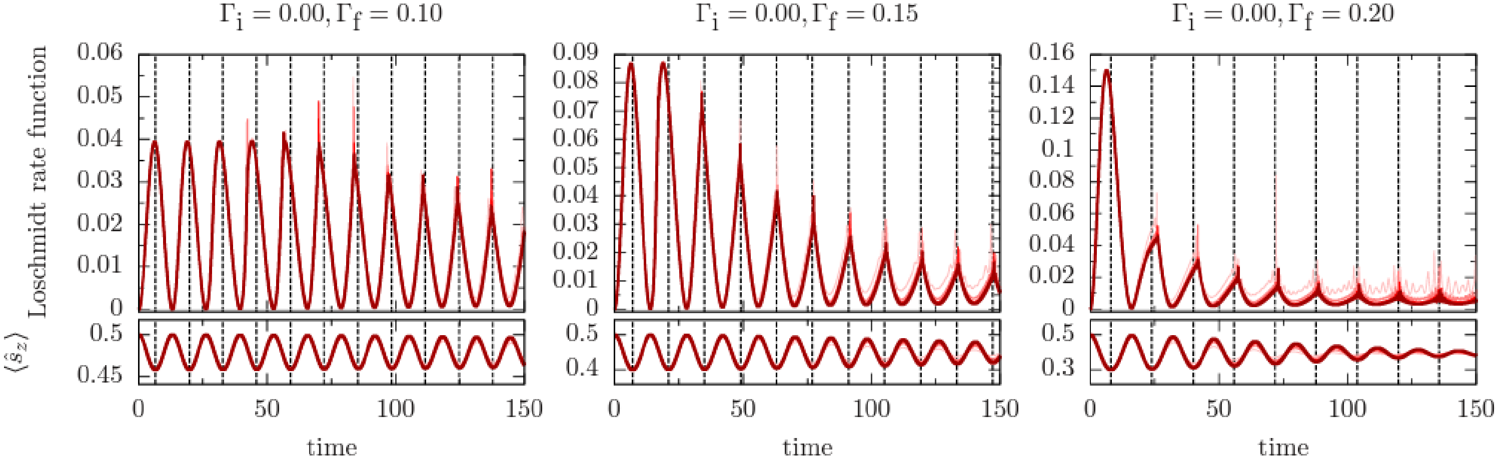
Dynamical phase transition

Non-analytic **times** in return probabilities

Rep. Prog. Phys. **81**, 054001 (2018)
Phys. Rev. Lett. **110**, 135704 (2013)
Phys. Rev. B **96**, 104436 (2017)







Q: Is there any **connection** between DPTs and ESQPTs?

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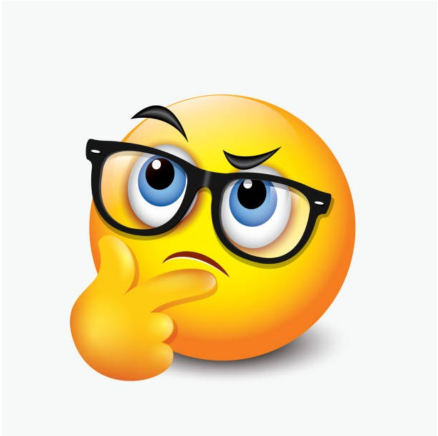
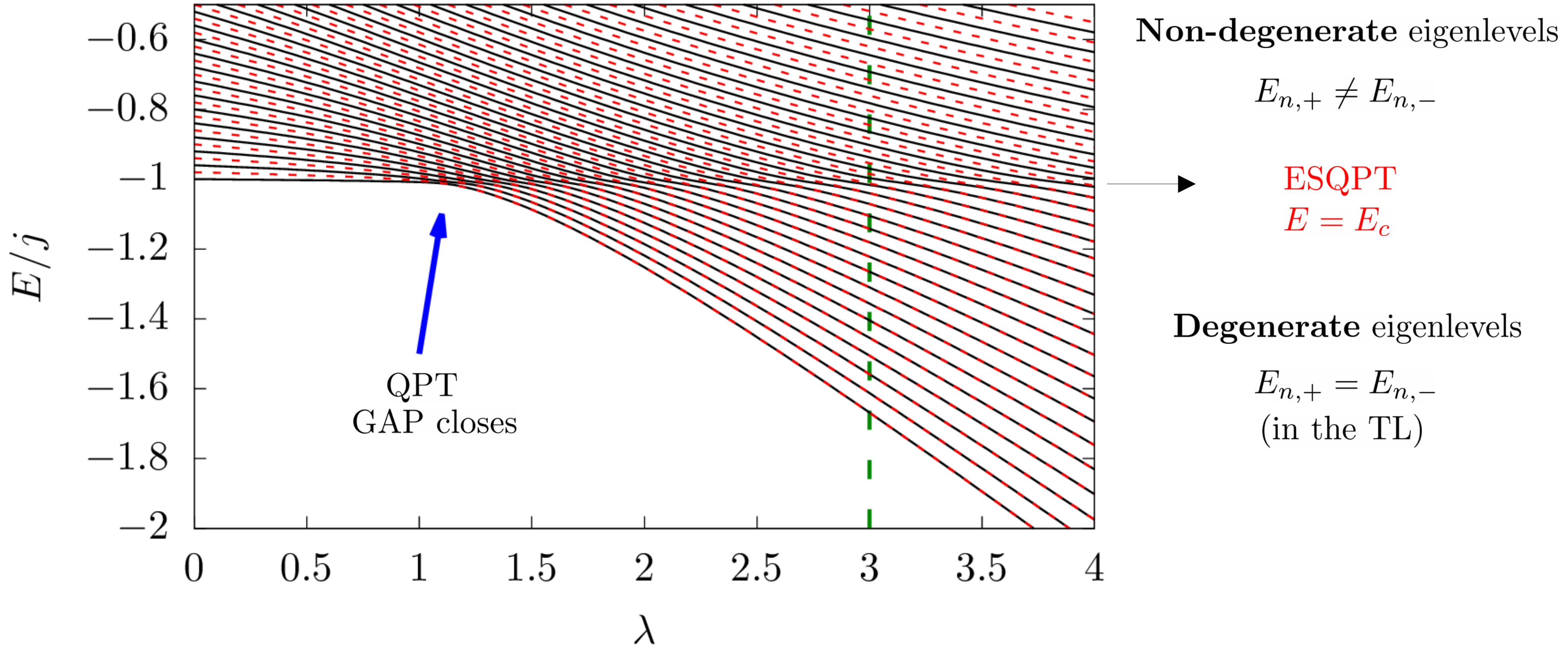
A: I'm here talking. Better hope there is...

Maybe look at the **spectrum**?

$$|\Psi_t(\lambda_f)\rangle = e^{-i\hat{\mathcal{H}}(\lambda_f)t}\,|\Psi_0(\lambda_i)\rangle = \sum_n c_n e^{-i{\color{red}E_n(\lambda_f)}t}\,|E_n(\lambda_f)\rangle$$



$$\hat{\mathcal{H}}(\lambda) = -\frac{\lambda}{4N}\sum_{i,j=1}^{N}\hat{\sigma}_i^x\hat{\sigma}_j^x + \frac{h}{2}\sum_{i=1}^{N}\hat{\sigma}_i^z = -\frac{\lambda}{N}\hat{J}_x^2 + h\hat{J}_z$$ Infinite-range transverse-field Ising model



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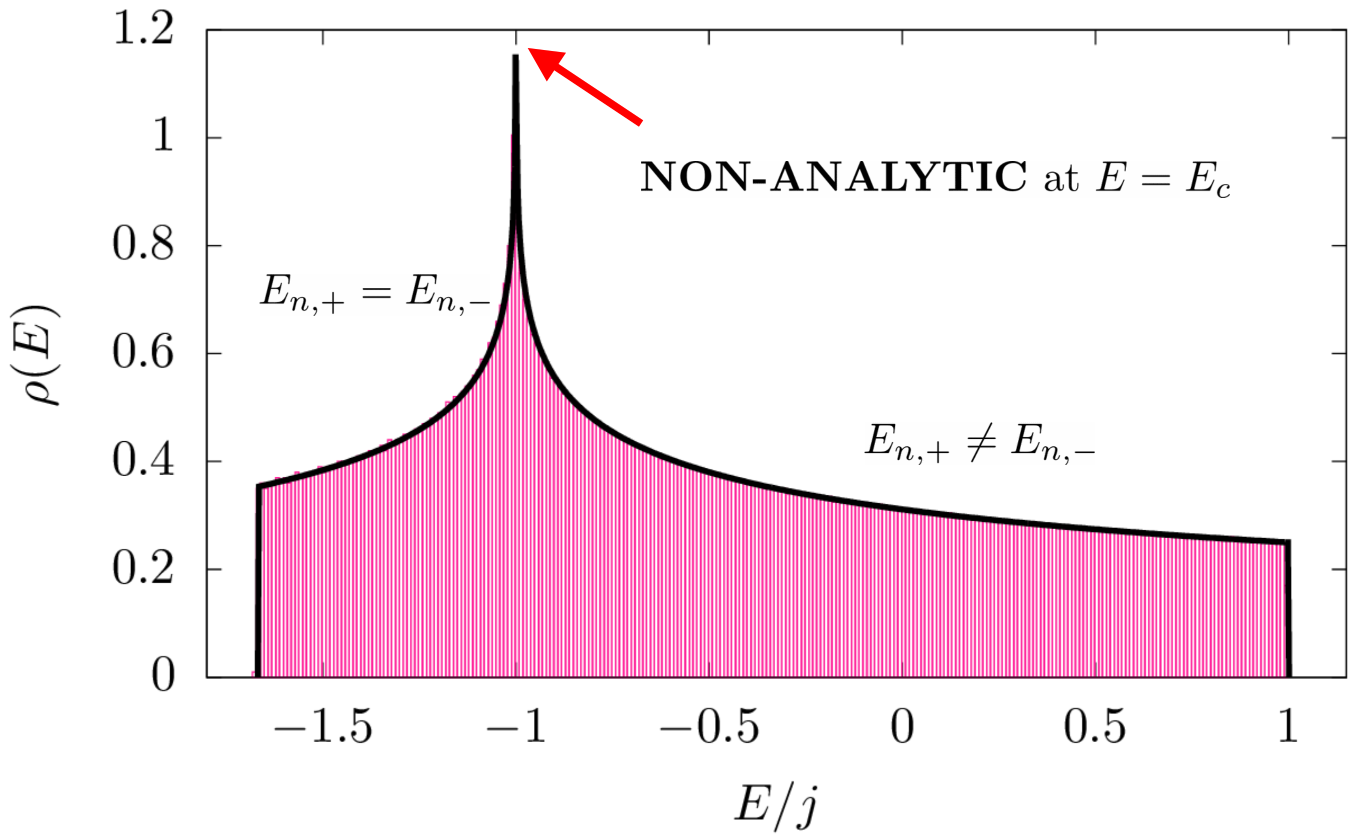
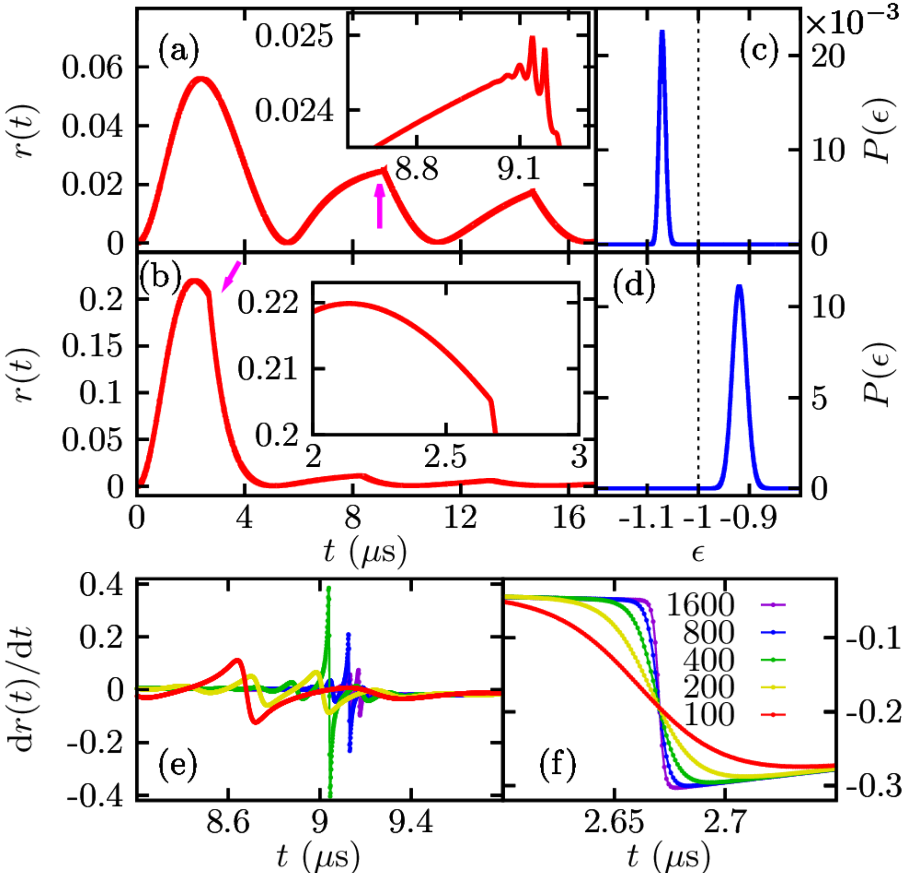


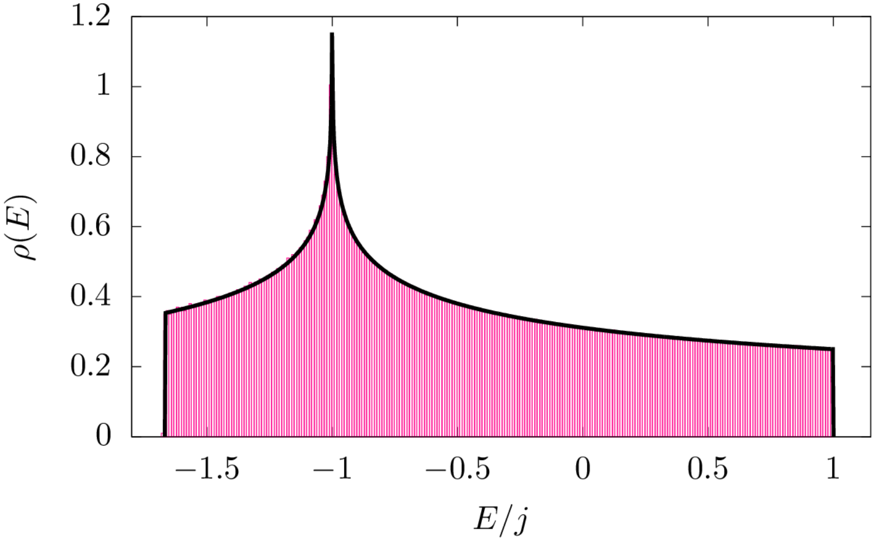
Clear scaling of a phase transition ONLY if $E > E_c$

ALC & Relaño, arXiv:2205.03443

$$\mathcal{L}(t) = \left|\langle E_{0,+}(\lambda_i)|e^{-i\hat{\mathcal{H}}(\lambda_f)t}|\Psi_0(\lambda_i)\rangle\right|^2 + \left|\langle E_{0,-}(\lambda_i)|e^{-i\hat{\mathcal{H}}(\lambda_f)t}|\Psi_0(\lambda_i)\rangle\right|^2 \equiv \mathcal{L}_+(t) + \mathcal{L}_-(t)$$



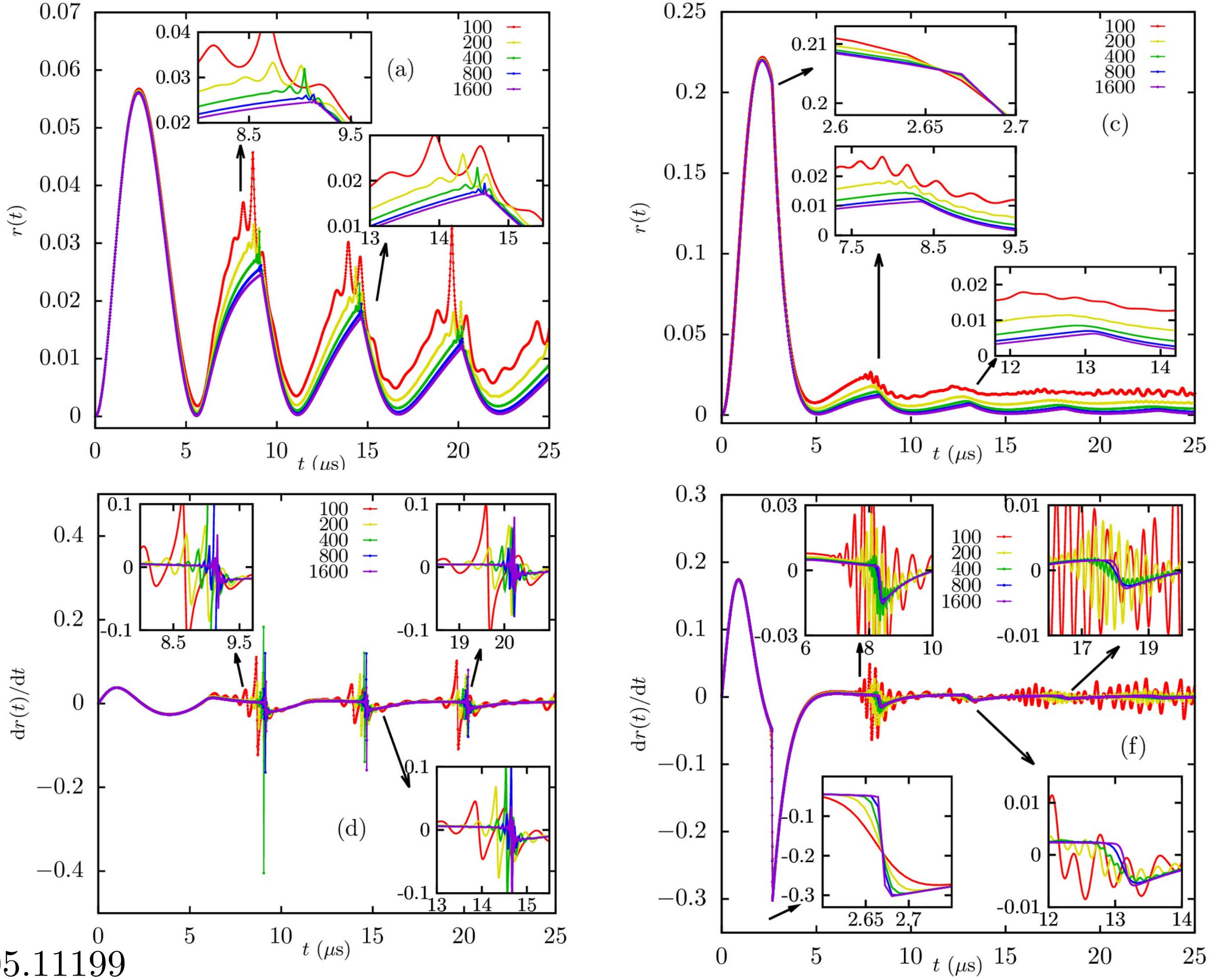
Rate function: $r_N(t) = -\frac{1}{N}\ln \mathcal{L}(t)$



$\mathbf{E < E_c}$ $\mathbf{E > E_c}$

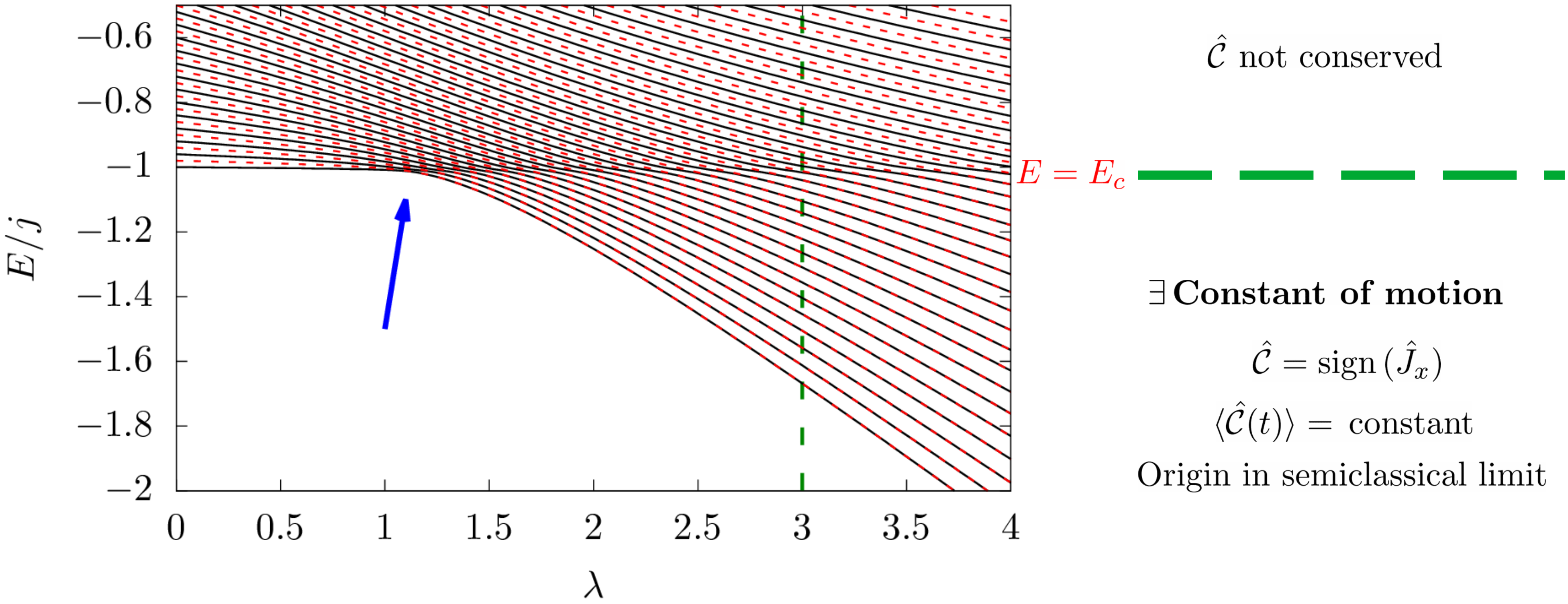
Rate functions

1st derivatives



ALC & Relaño, arXiv:2205.11199

Main points of our theory



$\hat{\mathcal{C}}$ not conserved

$\exists$ Constant of motion

$$\hat{\mathcal{C}} = \text{sign}\,(\hat{J}_x)$$

$$\langle \hat{\mathcal{C}}(t) \rangle = \text{constant}$$

Origin in semiclassical limit

ESQPT phases are characterized by $\hat{\mathcal{C}}$

PRL **127**, 130602 (2021)

We can show analytically that the main mechanism for DPTs is **forbidden** in the degenerate phase

It is assumed that $\mathcal{L}_{\pm}(t) = e^{-N\Omega_{\pm}(t)} \Longrightarrow$ Non-analyticities at t^*, $\Omega_+(t^*) = \Omega_-(t^*)$

Phys. Rev. Lett. **113**, 205701 (2014)

(OUR) CLAIM: No such intersections are possible if $E < E_c$

It may be shown that $\dfrac{\mathcal{L}_+(t)}{\mathcal{L}_-(t)} =$ constant, $\forall t \Rightarrow \Omega_+(t), \Omega_-(t)$ can never intersect

This is a **consequence of the constancy** of $\hat{\mathcal{C}}$

PROOF SKETCH:

(1) Initial state: $|\Psi_0(\lambda_i)\rangle = \sqrt{\alpha}\,|E_{0,+}(\lambda_i)\rangle + e^{i\phi}\sqrt{1-\alpha}\,|E_{0,-}(\lambda_i)\rangle \qquad \alpha \in [0,1],\ \phi \in [0,2\pi)$

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(2) Expand in final eigenstates after the quench:

$$|\Psi_t(\lambda_f)\rangle = \sqrt{\alpha}\sum_n c_{n,+}e^{-iE_{n,+}(\lambda_f)t}|E_{n,+}(\lambda_f)\rangle + e^{i\phi}\sqrt{1-\alpha}\sum_n c_{n,-}e^{-iE_{n,-}(\lambda_f)t}|E_{n,-}(\lambda_f)\rangle$$

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(3) PPRP is $\mathcal{L}(t) = \mathcal{L}_+(t) + \mathcal{L}_-(t),\ \mathcal{L}_+(t) = \alpha|f_+(t)|^2,\ \mathcal{L}_-(t) = (1-\alpha)|f_-(t)|^2,$

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(5) $\mathcal{L}_+(t) = \alpha|f(t)|^2,\ \mathcal{L}_-(t) = (1-\alpha)|f(t)|^2 \Rightarrow \dfrac{\mathcal{L}_+(t)}{\mathcal{L}_-(t)} = \dfrac{\alpha}{1-\alpha},\ \forall t \Rightarrow \Omega_+(t),\ \Omega_-(t)$ can never intersect

ALC & Relaño, arXiv:2205.11199

Are the PPRP $\mathcal{L}(t)$ and the (standard) survival probability $\mathrm{SP}(t)$ connected in any way?

$$\mathcal{L}(t) = \left|\langle E_{0,+}(\lambda_i)|e^{-i\hat{\mathcal{H}}(\lambda_f)t}|\Psi_0(\lambda_i)\rangle\right|^2 + \left|\langle E_{0,-}(\lambda_i)|e^{-i\hat{\mathcal{H}}(\lambda_f)t}|\Psi_0(\lambda_i)\rangle\right|^2 \equiv \mathcal{L}_+(t) + \mathcal{L}_-(t)$$

$$\mathrm{SP}(t) = |\langle\Psi_0(\lambda_i)|\Psi_t(\lambda_f)\rangle|^2$$

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$$\mathrm{SP}(t) = |\langle\Psi_0(\lambda_i)|\Psi_t(\lambda_f)\rangle|^2$$

$$\mathcal{L}(t) = \mathcal{L}_+(t) + \mathcal{L}_-(t) = \alpha|f_+(t)|^2 + (1-\alpha)|f_-(t)|^2$$

$$\mathrm{SP}(t) = \alpha^2|f_+(t)|^2 + (1-\alpha)^2|f_-(t)|^2 + \alpha(1-\alpha)\left[f_+(t)f_-^*(t) + f_+^*(t)f_-(t)\right]$$

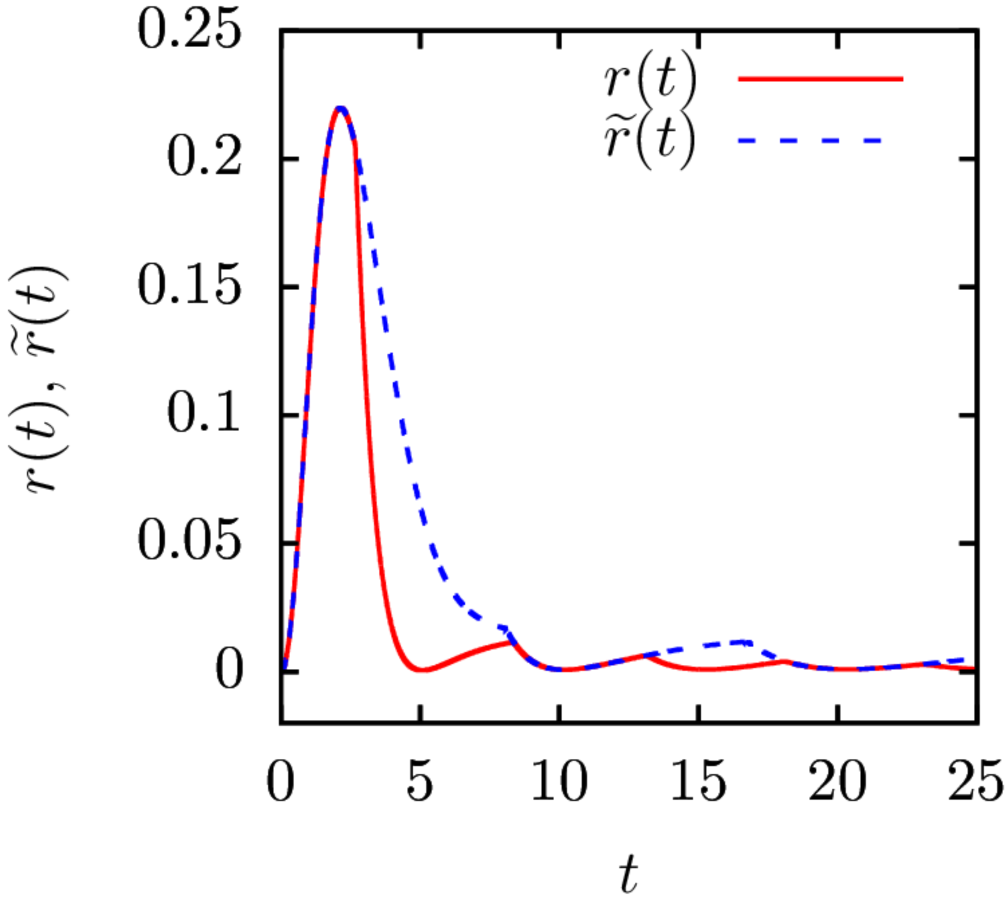
NOTE:

If $E < E_c \Rightarrow f_+(t) = f_-(t) \Rightarrow \mathcal{L}(t) = \mathrm{SP}(t)$

If $E > E_c \Rightarrow f_+(t) \neq f_-(t) \Rightarrow \mathcal{L}(t) \neq \mathrm{SP}(t)$

ALC & Relaño, arXiv:2205.03443 **ALC** & Relaño, arXiv:2205.11199

$\mathbf{E > E_c}$



ALC & Relaño, arXiv:2205.11199

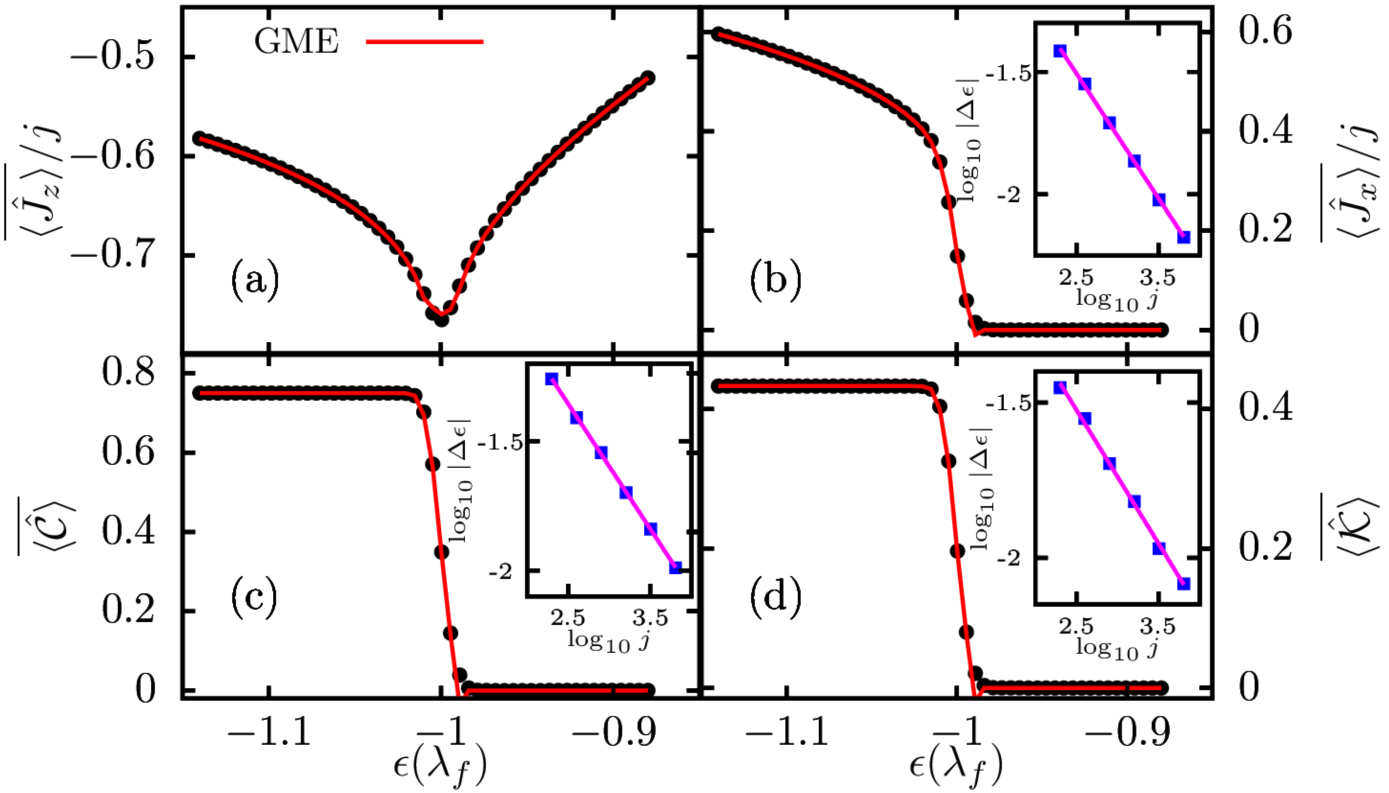
$\mathcal{L}(t) \neq \mathrm{SP}(t)$

• Non-analytical times in $\mathcal{L}(t)$ when $\mathcal{L}(t)$ and $\mathrm{SP}(t)$ **separate** or **come together**

$$r_N(t) = -\frac{1}{N} \ln \mathcal{L}(t)$$

$$\widetilde{r}_N(t) = -\frac{1}{N} \ln \mathrm{SP}(t)$$

BONUS: There is another kind of DPT



$$\overline{\langle \hat{\mathcal{O}} \rangle}$$

vs.

$$\langle \hat{\mathcal{O}} \rangle_{\mathrm{GME}} = \mathrm{Tr}\left[\rho_{\mathrm{GME}} \hat{\mathcal{O}}\right]$$

ALC & Relaño, arXiv:2205.03443

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Excellent agreement!

Thank **you** for your attention!

w/ Relaño

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