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Unión Europea
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"Una manera de hacer Europa"



AGENCIA
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Quantum simulation and Quantum Phase Transitions of an extended Agassi model

10th International workshop on Quantum Phase Transitions in Nuclei and
Many-Body Systems

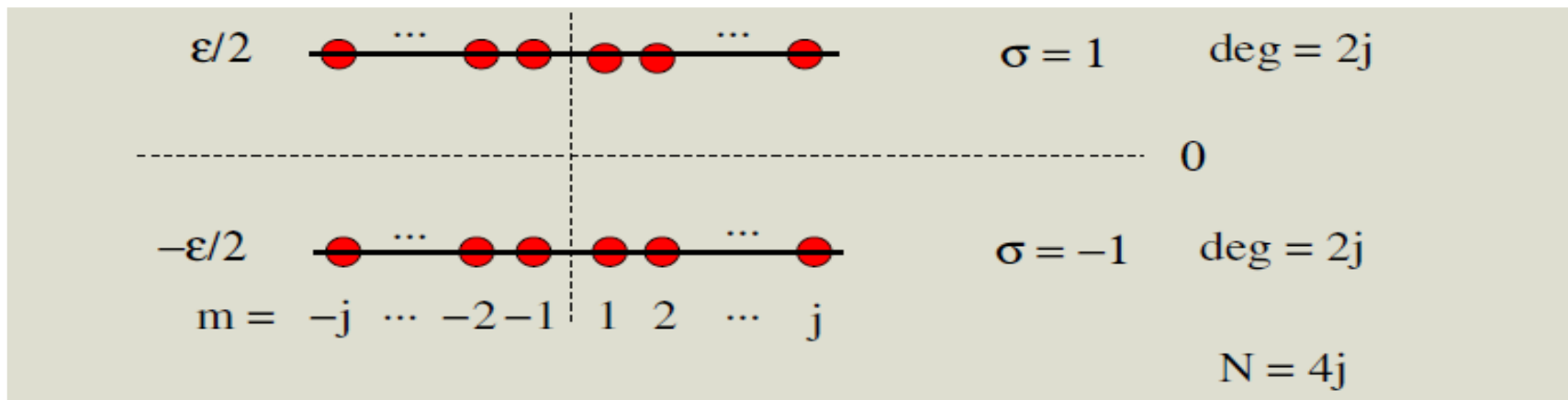
Pedro Pérez-Fernández

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José Miguel Arias, Lucas Lamata



The extended Agassi Model. Why?

- It is a solvable many-body model that allows to **mimic the main characteristics of the pairing-plus-quadrupole model**.
- It can be **exactly solved** even in the case of large systems.
- It is used to **benchmark many-body approximations** because of its great flexibility and simplicity to be solved for large systems.
- The model owns a very **rich phase diagram** and even presents **shape coexistence**.
- The model is, somehow, an extension of the **two-level Lipkin-Meshkov-Glick model** that **incorporates pairing interaction**.
- It is a model slightly **more complex than the used ones in Quantum Information Science** (e.g. Lipkin, Dicke, Tavis-Cumming or Hubbard models) and, therefore, of great interest.



The extended Agassi Model

$$H = \varepsilon J^0 - g \sum_{\sigma, \sigma'} A_{\sigma}^{\dagger} A_{\sigma'} - \frac{V}{2} [(J^+)^2 + (J^-)^2] - 2\hbar A_0^{\dagger} A_0$$

$$J^+ = \sum_{m=-j}^j c_{1,m}^{\dagger} c_{-1,m}$$

$$J^0 = \frac{1}{2} \sum_{m=-j}^j (c_{1,m}^{\dagger} c_{1,m} - c_{-1,m}^{\dagger} c_{-1,m})$$

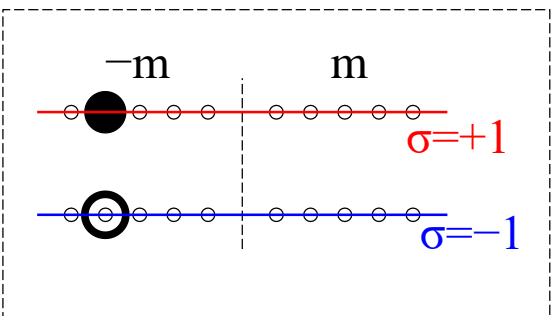
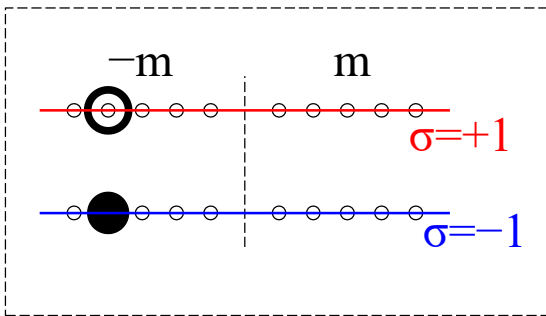
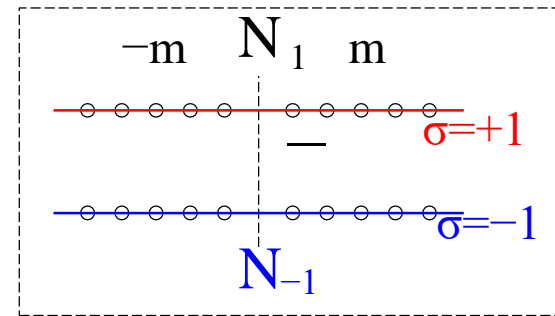
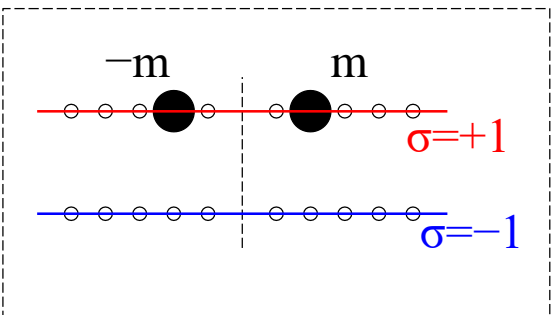
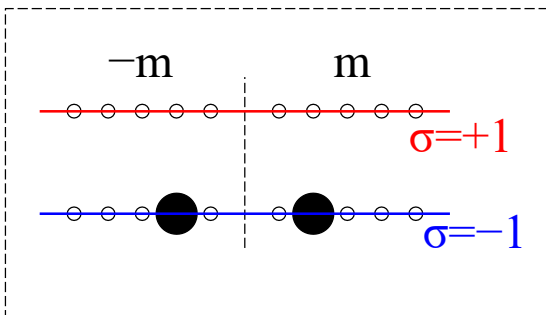
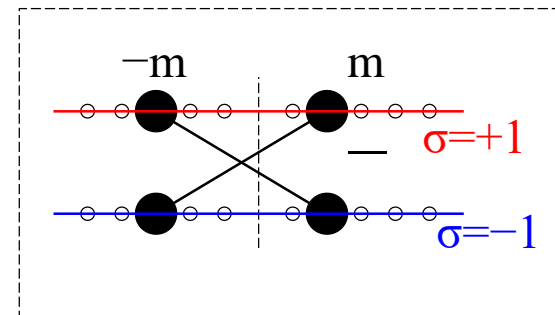
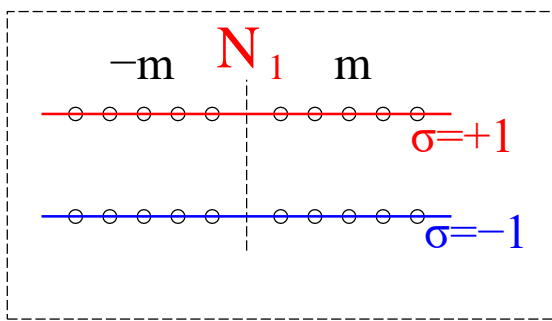
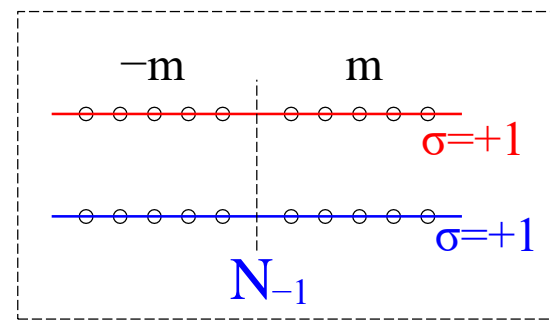
$$A_1^{\dagger} = \sum_{m=1}^j c_{1,m}^{\dagger} c_{1,-m}^{\dagger}$$

$$A_{-1}^{\dagger} = \sum_{m=1}^j c_{-1,m}^{\dagger} c_{-1,-m}^{\dagger}$$

**Spectrum Generator
Algebra $O(5)$**

$$A_0^{\dagger} = \frac{1}{2} \sum_{m=1}^j (c_{-1,m}^{\dagger} c_{1,-m}^{\dagger} - c_{-1,-m}^{\dagger} c_{1,m}^{\dagger})$$

$$N_{\sigma} = \sum_{m=-j}^j c_{\sigma,m}^{\dagger} c_{\sigma,m} \quad N = N_1 + N_{-1}$$

J^+  J^-  J^0  A_1^\dagger  A_{-1}^\dagger  A_0^\dagger  N_1  N_{-1} 

Quantum Simulation of the model

The Jordan-Wigner Transformation

- It is a non-local transformation that maps the fermion creation/annihilation operators into spinless fermions
- It is usual to relabel the fermion index, i.e., $\sigma, m \rightarrow i$

$$\begin{cases} c_{\sigma, m}^{\dagger} \rightarrow c_i^{\dagger} = I_1 \otimes \cdots \otimes I_{i-1} \otimes \sigma_i^+ \otimes \sigma_{i+1}^z \otimes \cdots \otimes \sigma_N^z \\ c_{\sigma, m} \rightarrow c_i = I_1 \otimes \cdots \otimes I_{i-1} \otimes \sigma_i^- \otimes \sigma_{i+1}^z \otimes \cdots \otimes \sigma_N^z \end{cases}$$

$$\sigma^+ = \frac{\sigma^x + i\sigma^y}{2}$$

$$\sigma^- = \frac{\sigma^x - i\sigma^y}{2}$$

Pauli Matrices

$$\sigma^x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\sigma^y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\sigma^z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Quantum Simulation of the model

J=2: the case of 8 sites

$$\begin{array}{ll} c_{1,2} \rightarrow c_1 & c_{-1,2} \rightarrow c_5 \\ c_{1,1} \rightarrow c_2 & c_{-1,1} \rightarrow c_6 \\ c_{1,-1} \rightarrow c_3 & c_{-1,-1} \rightarrow c_7 \\ c_{1,-2} \rightarrow c_4 & c_{-1,-2} \rightarrow c_8 \end{array}$$

For example:

$$A_1 = \sigma_2^- \otimes \sigma_3^- + \sigma_1^- \otimes \sigma_2^Z \otimes \sigma_3^Z \otimes \sigma_4^-$$

The Hamiltonian

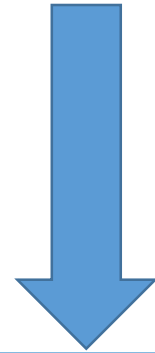
$$H = H_1 + H_2 + H_3 + H_4 + H_5 + H_6$$

$$[H_i, H_j] \neq 0 \quad i \neq j = 1 \dots 6$$

Quantum Simulation of the model

The evolution operator

$$U(t) = e^{-itH}$$



Digital quantum simulation

LIE-TROTTER-SUZUKI (TROTTER) EXPANSION

$$U(t, n_T) = \left(\prod_{k=1}^6 e^{-itH_k/n_T} \right)^{n_T} + \mathcal{O}\left(\frac{t^2}{n_T}\right)$$

Trotter Steps

How good is the Trotter approach?

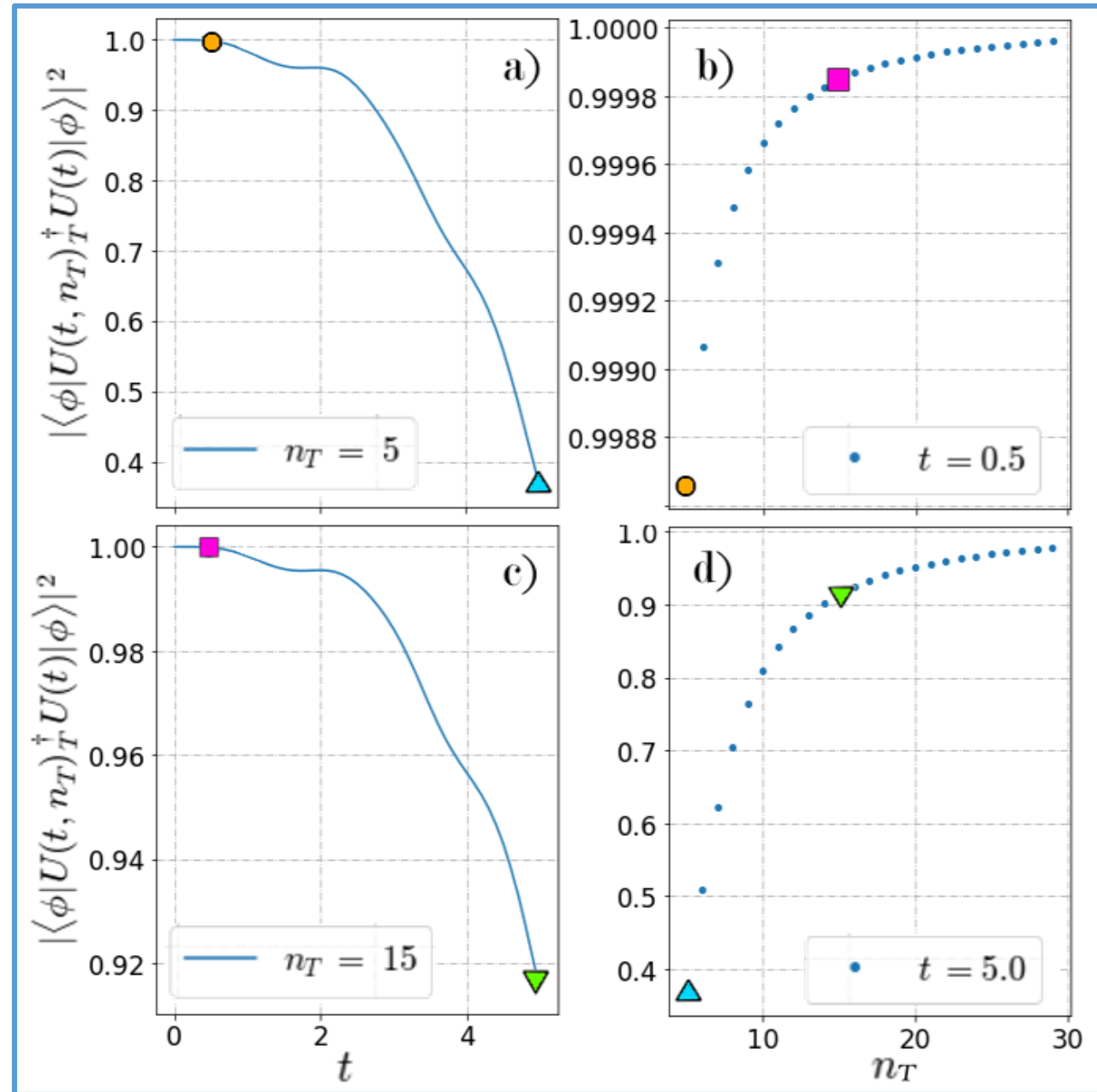


Check the Fidelity

$$F(t, n_T) = |\langle \phi | U(t, n_T)^\dagger U(t) | \phi \rangle|^2$$

Initial state: $|\phi\rangle = |\downarrow_1 \downarrow_2 \downarrow_3 \downarrow_4 \uparrow_5 \uparrow_6 \uparrow_7 \uparrow_8\rangle$
(With minimum value of $\langle J^0 \rangle = -2$)

Parameters: $\varepsilon = 1, g = 0.25,$
 $V = 0.25, h = 0.25$

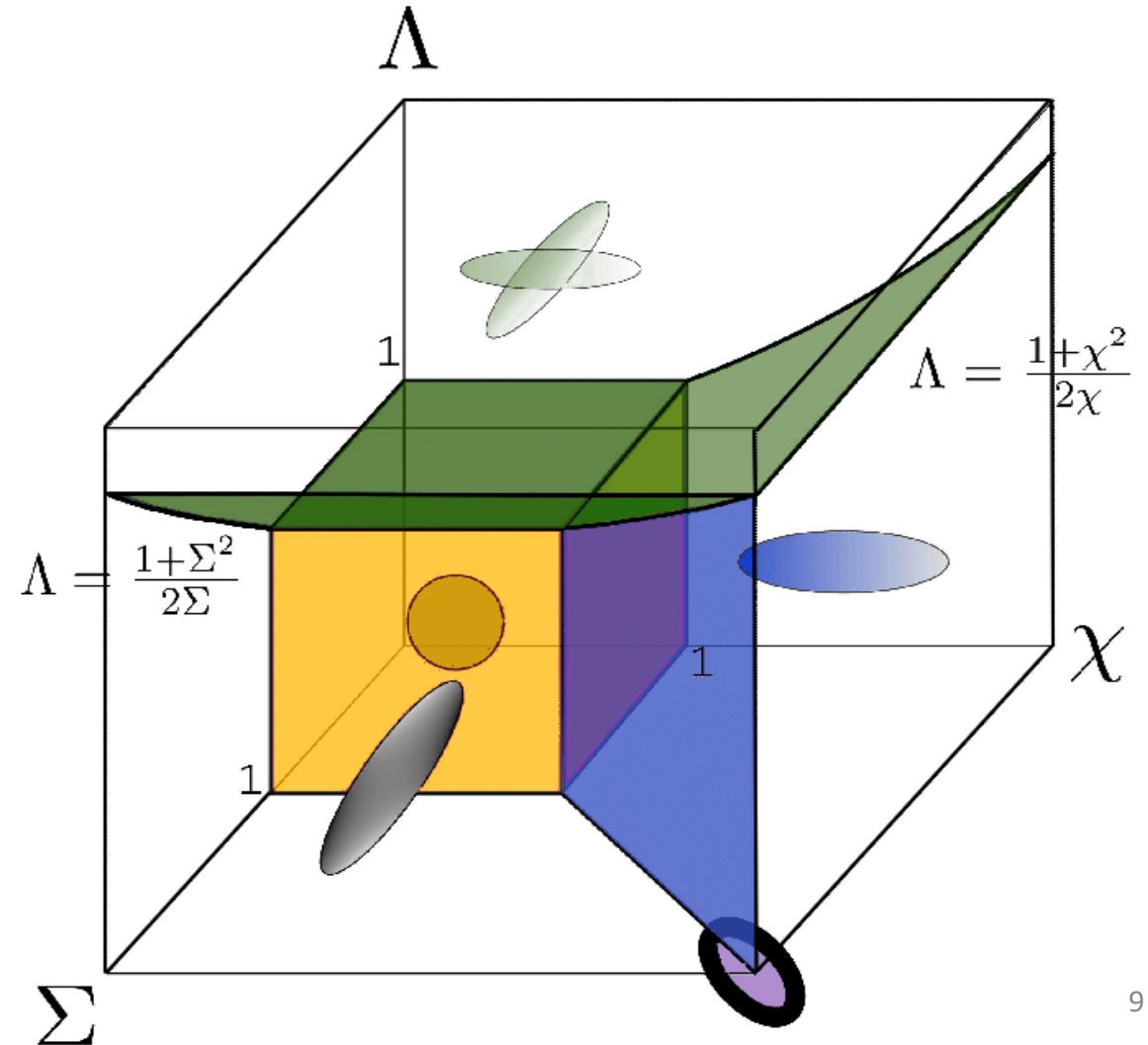


Exploring Quantum Phase Transitions (QPTs)

Quantum Phase Diagram

- **Symmetric phase**
- **Hartree-Fock phase (HF)**
- **Bardeen-Cooper-Schrieffer phase (BCS)**
- **Combined HF-BCS phase**
- **Closed Valley solution**

$$g = \frac{\varepsilon \Sigma}{2j-1} \quad V = \frac{\varepsilon \chi}{2j-1} \quad h = \frac{\varepsilon \lambda}{2j-1}$$



Exploring Quantum Phase Transitions (QPTs)

Correlation Functions

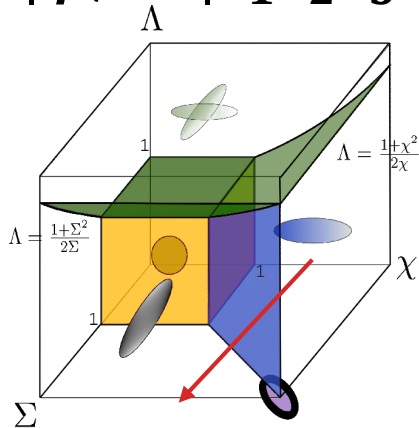
$$C_{\alpha,\beta}(i,j) = \langle \sigma_i^\alpha \otimes \sigma_j^\beta \rangle - \langle \sigma_i^\alpha \rangle \langle \sigma_j^\beta \rangle$$



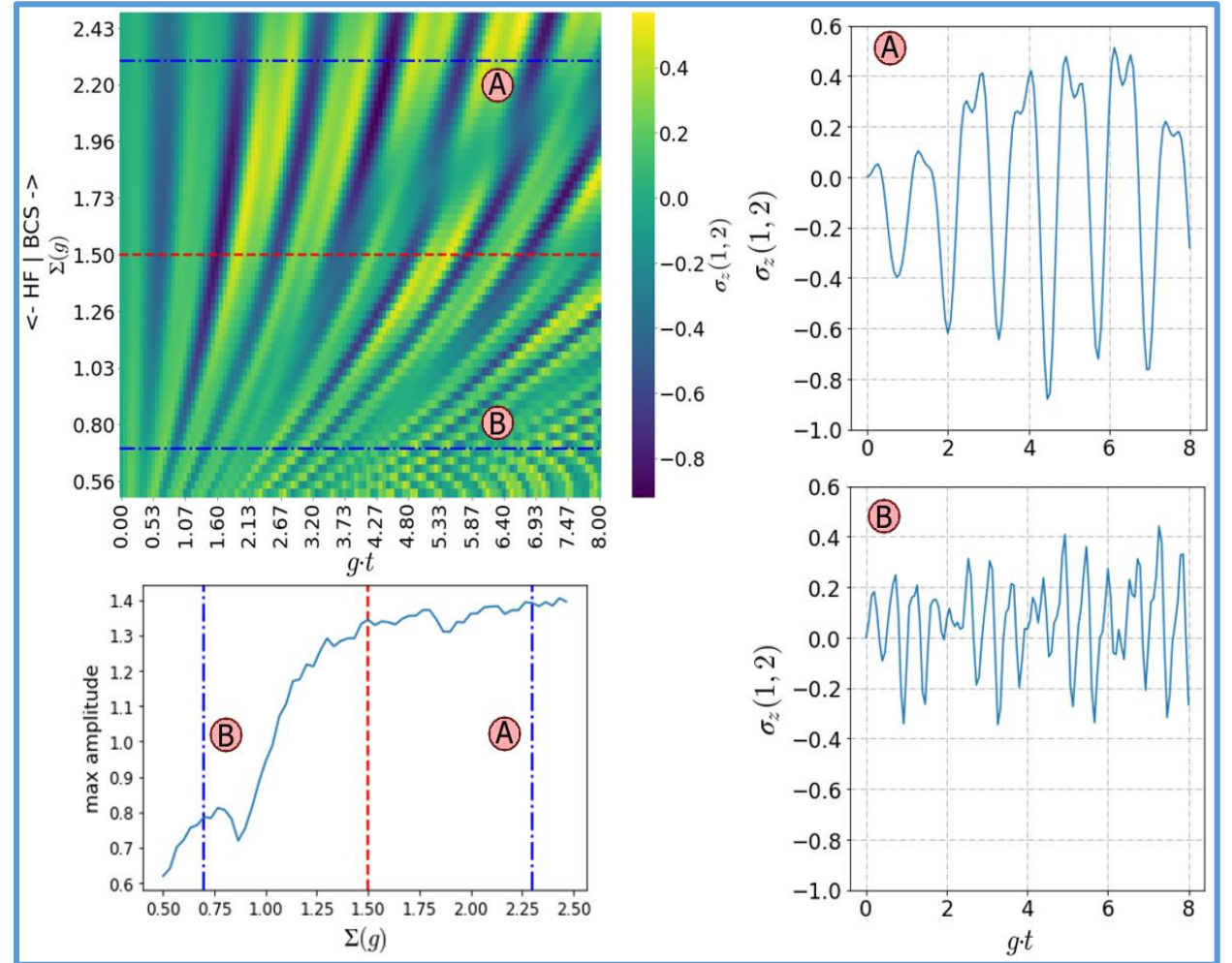
Particular case

$$C_z(1,2) = \langle \sigma_1^z \otimes \sigma_2^z \rangle - \langle \sigma_1^z \rangle \langle \sigma_2^z \rangle$$

Initial state: $|\phi\rangle = |\downarrow_1 \downarrow_2 \downarrow_3 \downarrow_4 \uparrow_5 \uparrow_6 \uparrow_7 \uparrow_8\rangle$



From HF phase to BCS phase



Machine Learning - How to train your AI

We know how a dog looks like, but,
how do we explain that to a computer?

WE DON'T

DOG



On its side, but
definitely a dog



Clearly a cute
dog with a
cute party hat



This is not a dog. This is a cat

Machine Learning - How to train your AI

A lot of different methods

- Regression
- Clustering
- Decision Trees
- Reinforced Learning
- Genetic Algorithms
- **Neural Networks**

Supervised **VS** Unsupervised

Exploration **VS** Exploitation

Accuracy **VS** Efficiency

Underfitting **VS** Overfitting

Training **VS** Testing

“No Free Lunch” theorem

Machine Learning - How to train your AI

AI in two steps - Step #1: Train



AI in two steps - Step #2: Predict

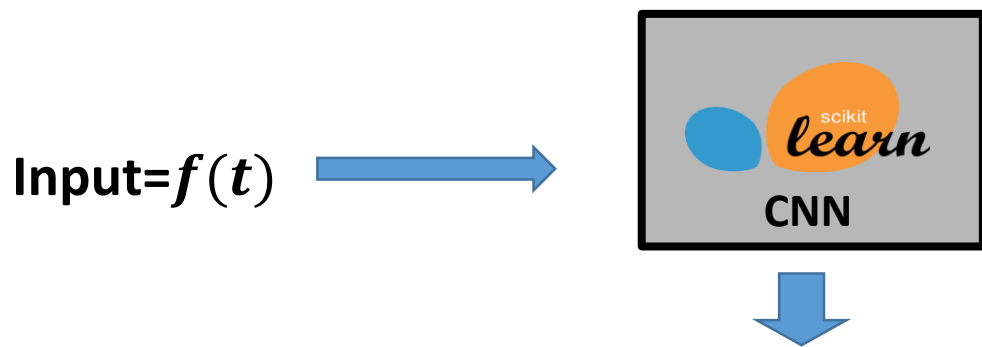


Deep Learning - Results

$$C_z(\mathbf{1}, \mathbf{2}) = \langle \sigma_1^z \otimes \sigma_2^z \rangle - \langle \sigma_1^z \rangle \langle \sigma_2^z \rangle$$

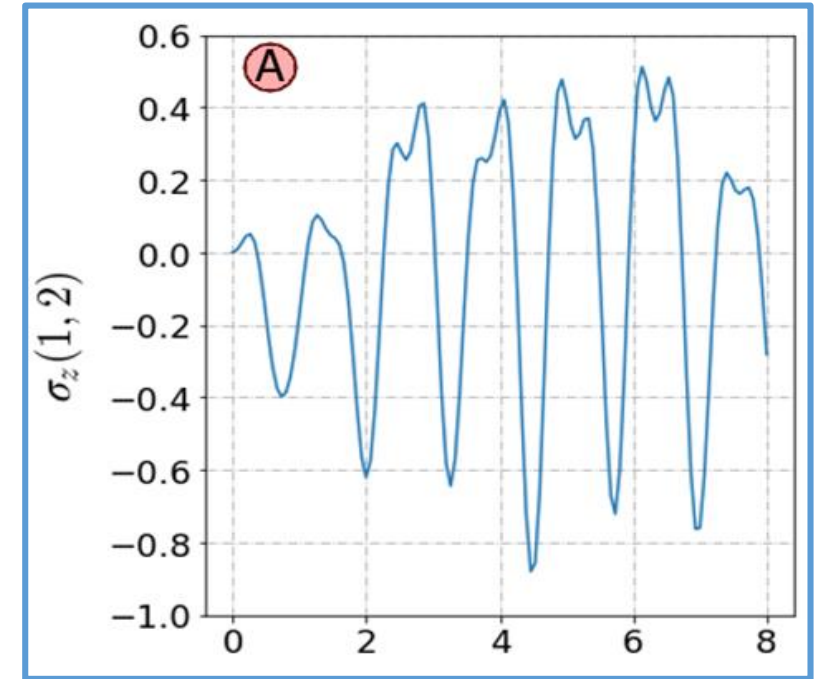
$$U(t) = e^{-itH} \approx \left(\prod_{k=1}^6 e^{-itH_k/n_T} \right)^{n_T}$$

$$f(t) = \langle \phi(t) | \sigma_1^z \otimes \sigma_2^z | \phi(t) \rangle - \langle \phi(t) | \sigma_1^z | \phi(t) \rangle \langle \phi(t) | \sigma_2^z | \phi(t) \rangle$$



Output= $[P(y_1), P(y_2), P(y_3), P(y_4), P(y_5)]$

Input:

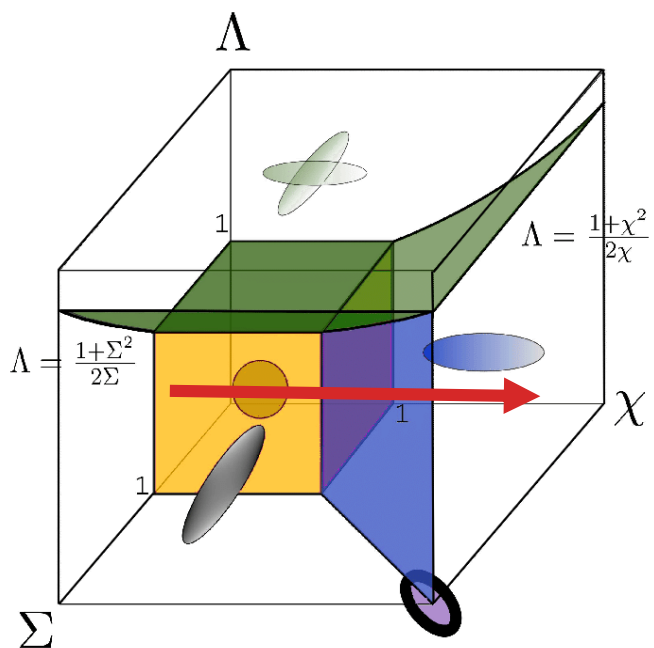


Output:

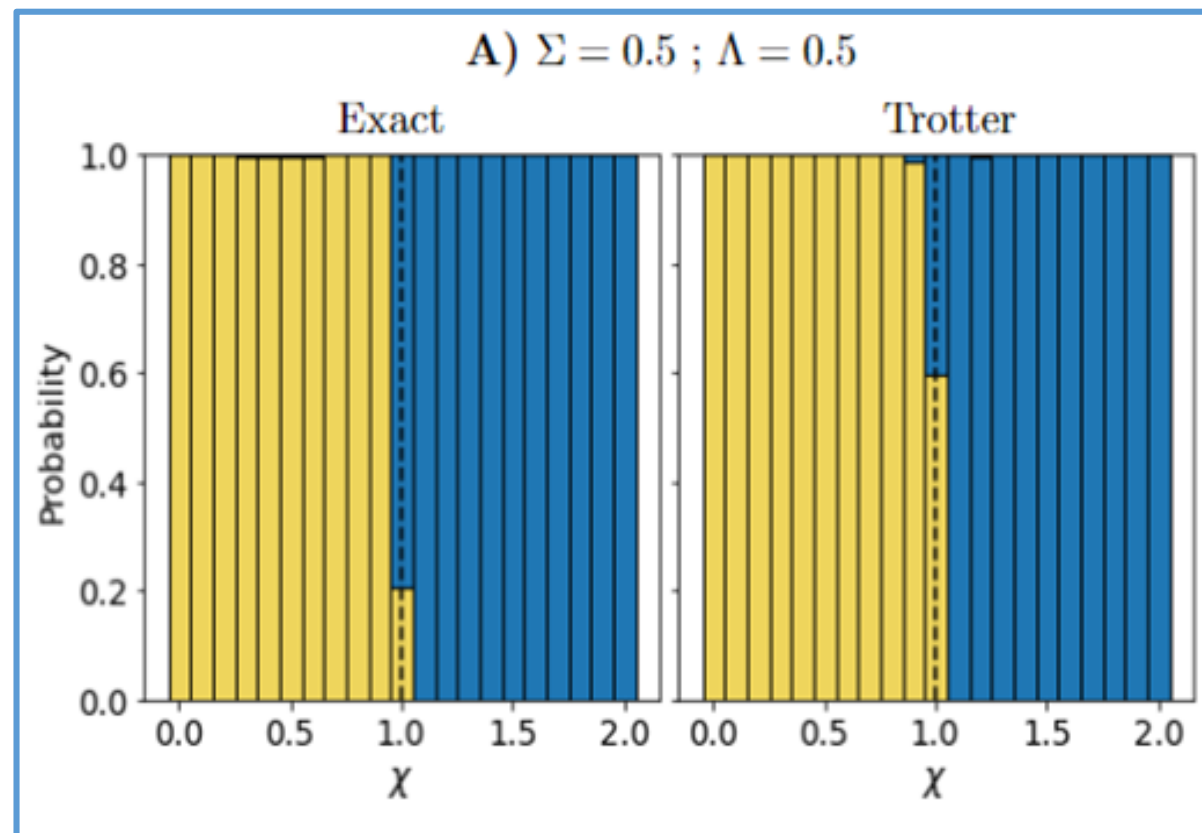
- Symmetric= 0.00
- HF= 0.04
- **BCS= 0.94**
- Hf-BCS= 0.01
- Closed Valley= 0.01

Convolutional Neural Network (CNN)- Results

98.35%
Categorical
Accuracy

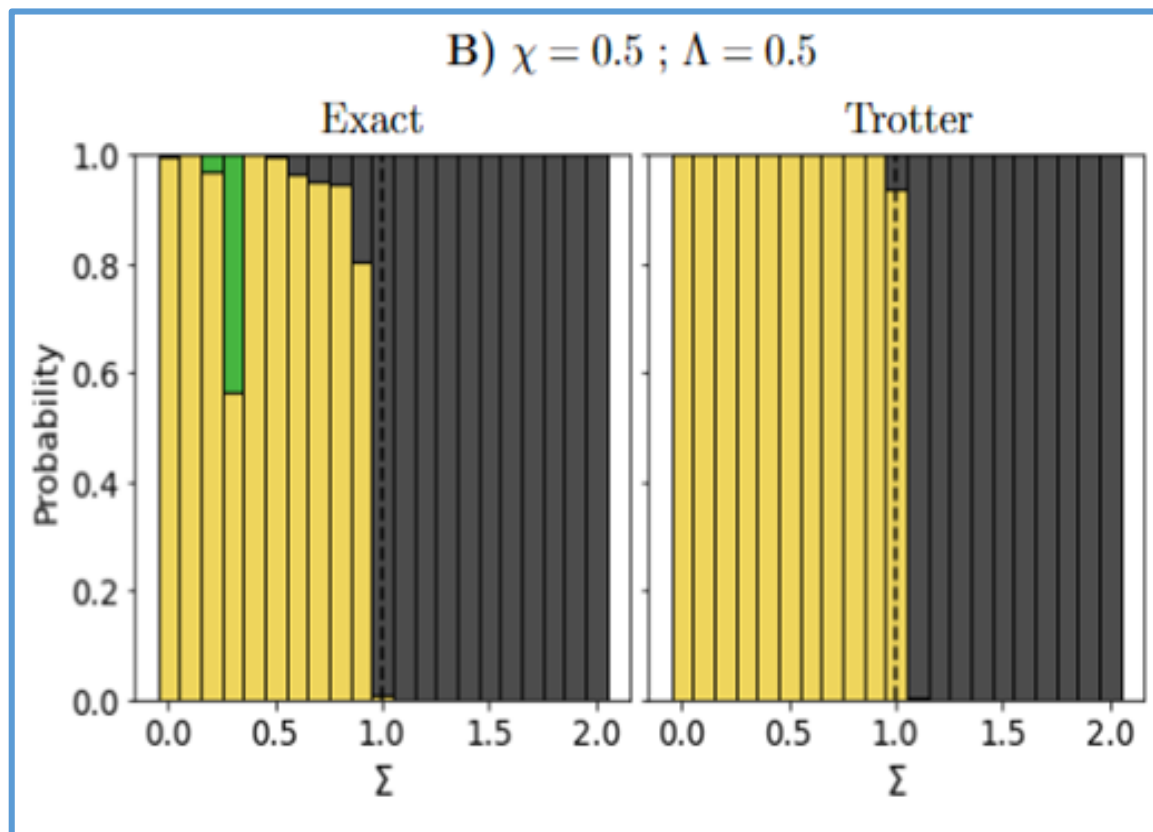
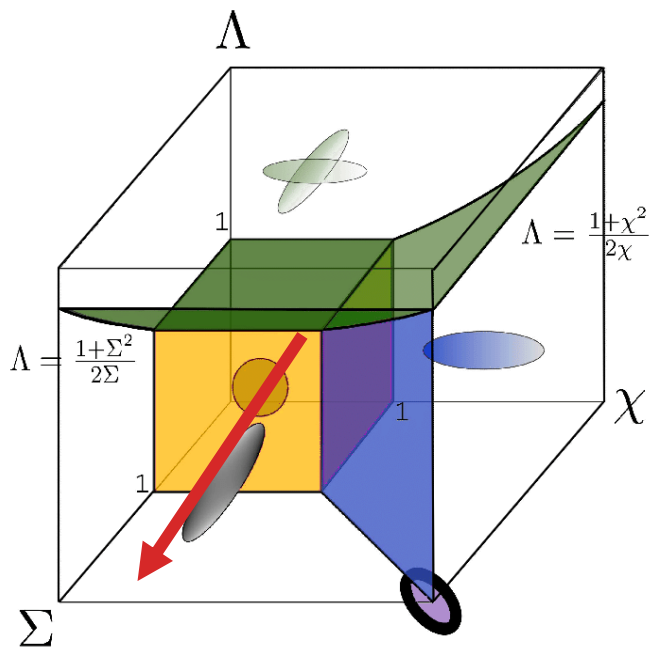


Legend: Symmetric (Yellow), HF (Blue), BCS (Grey), HF-BCS (Green), Valley (Purple)



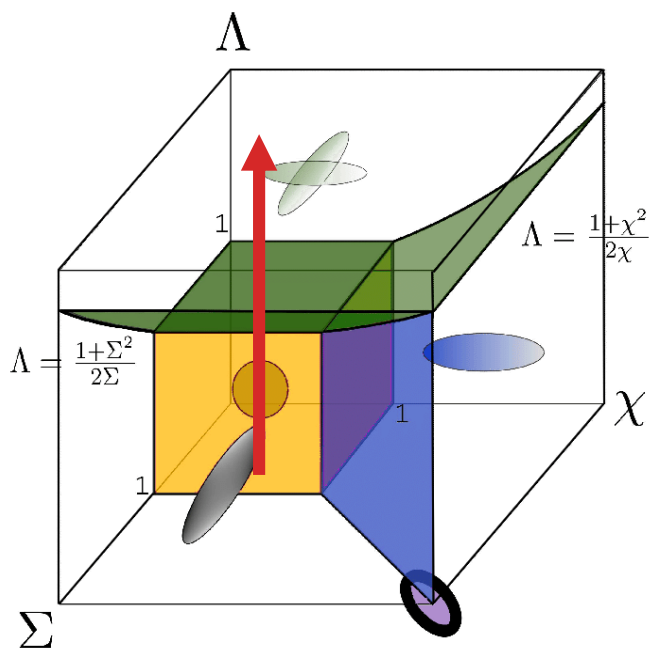
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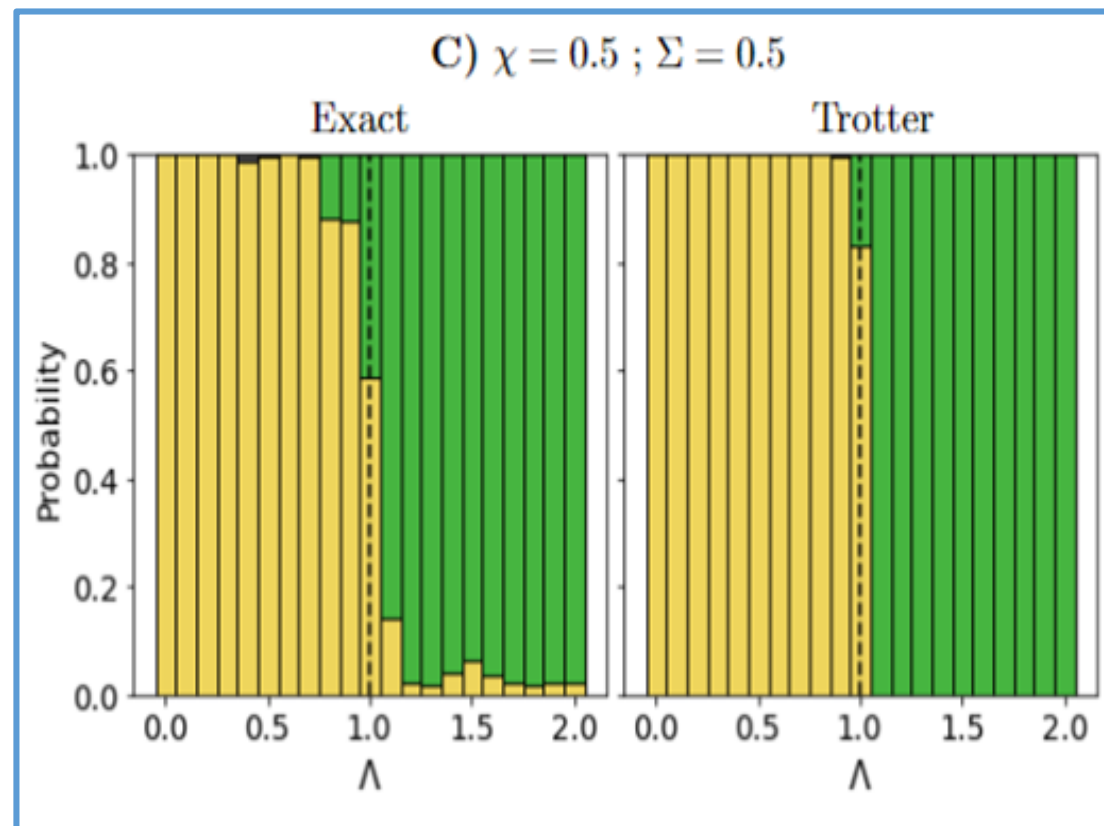


Convolutional Neural Network (CNN)- Results

98.35%
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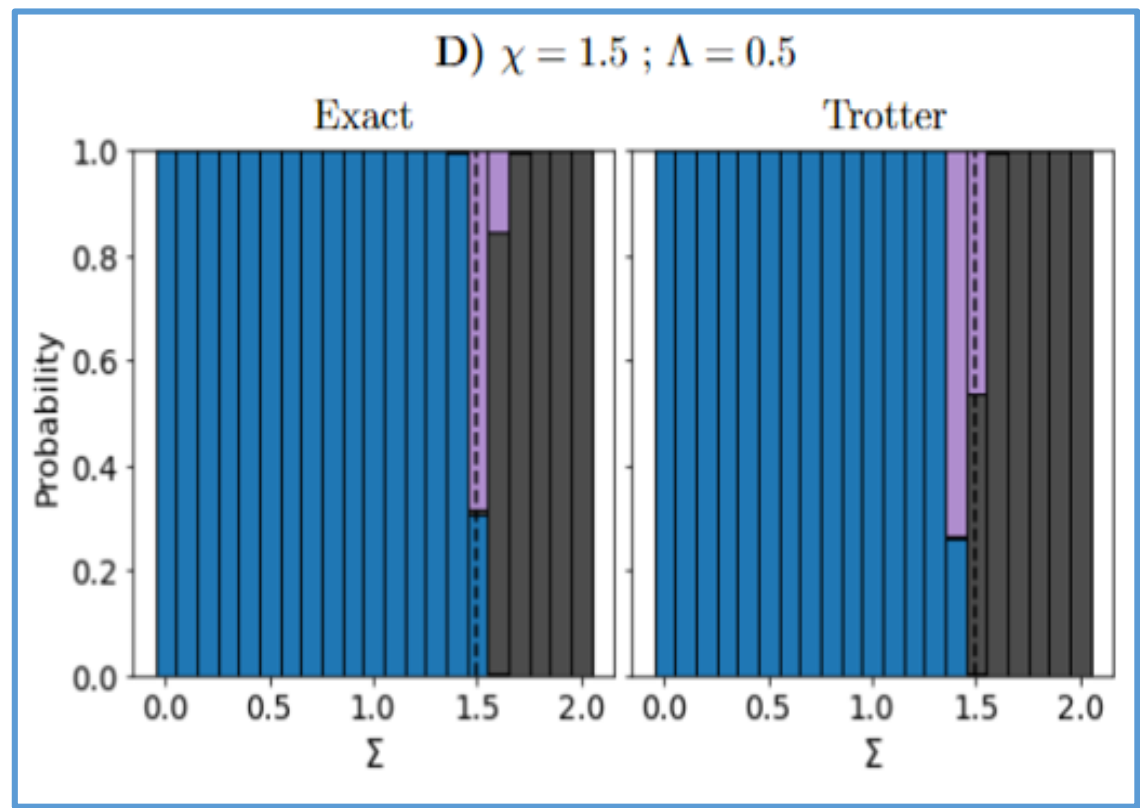
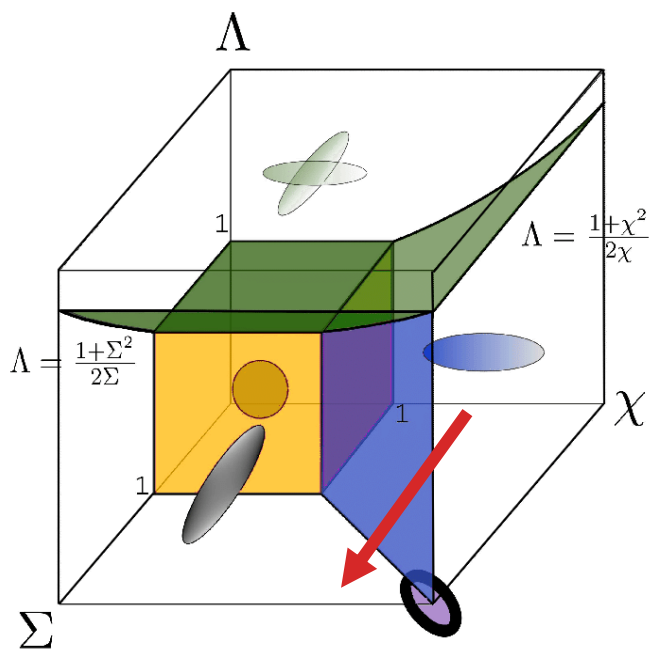


Legend: Symmetric (yellow), HF (blue), BCS (grey), HF-BCS (green), Valley (purple)



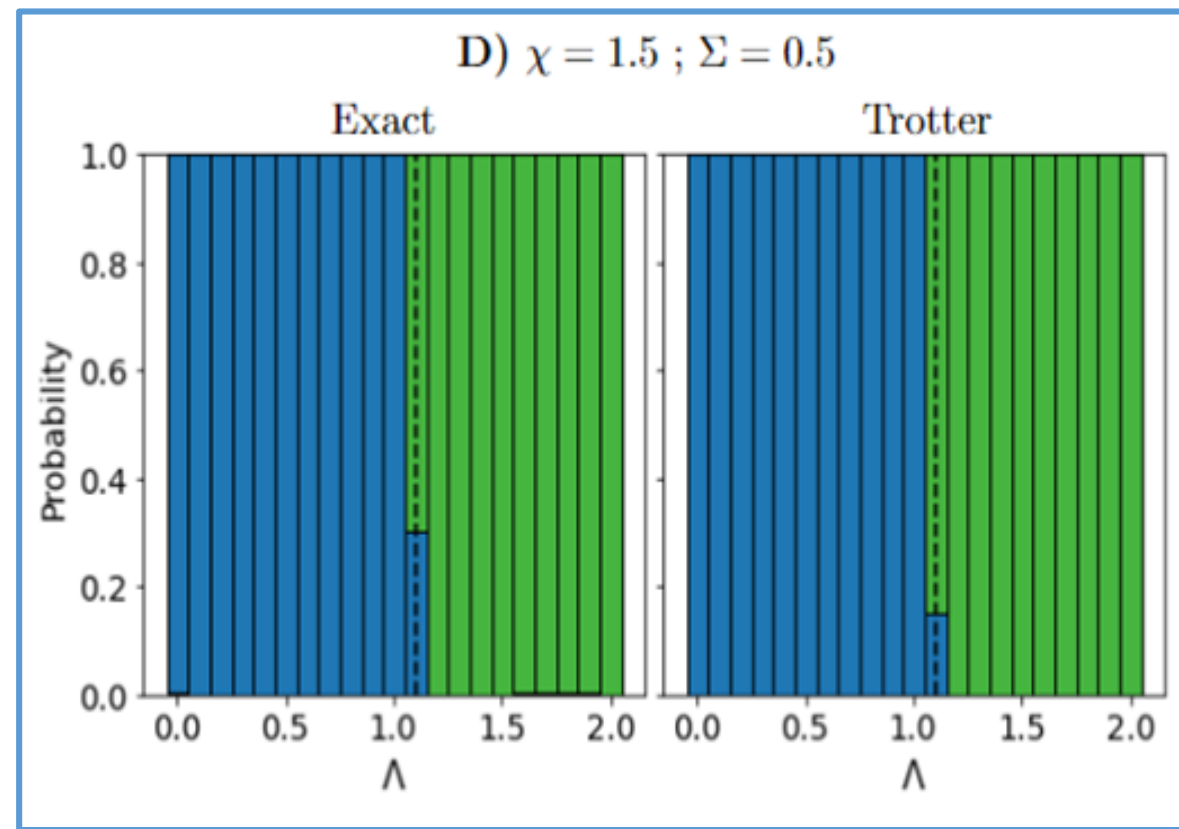
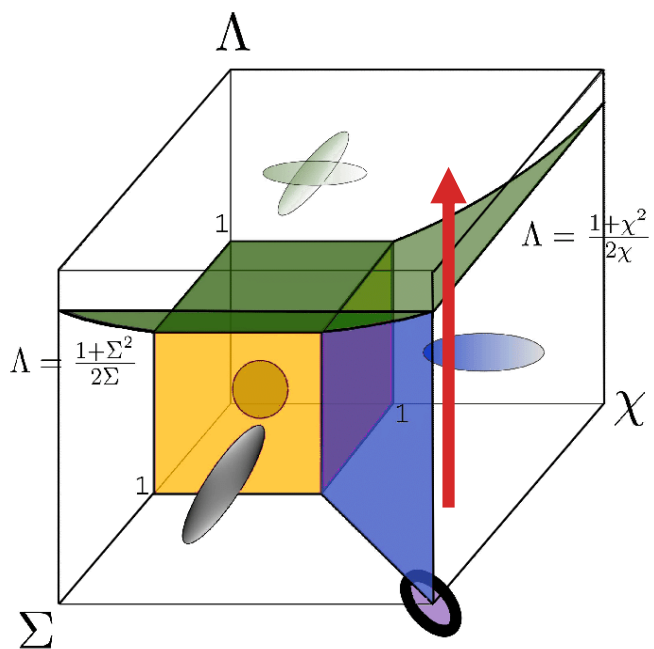
Convolutional Neural Network (CNN)- Results

98.35%
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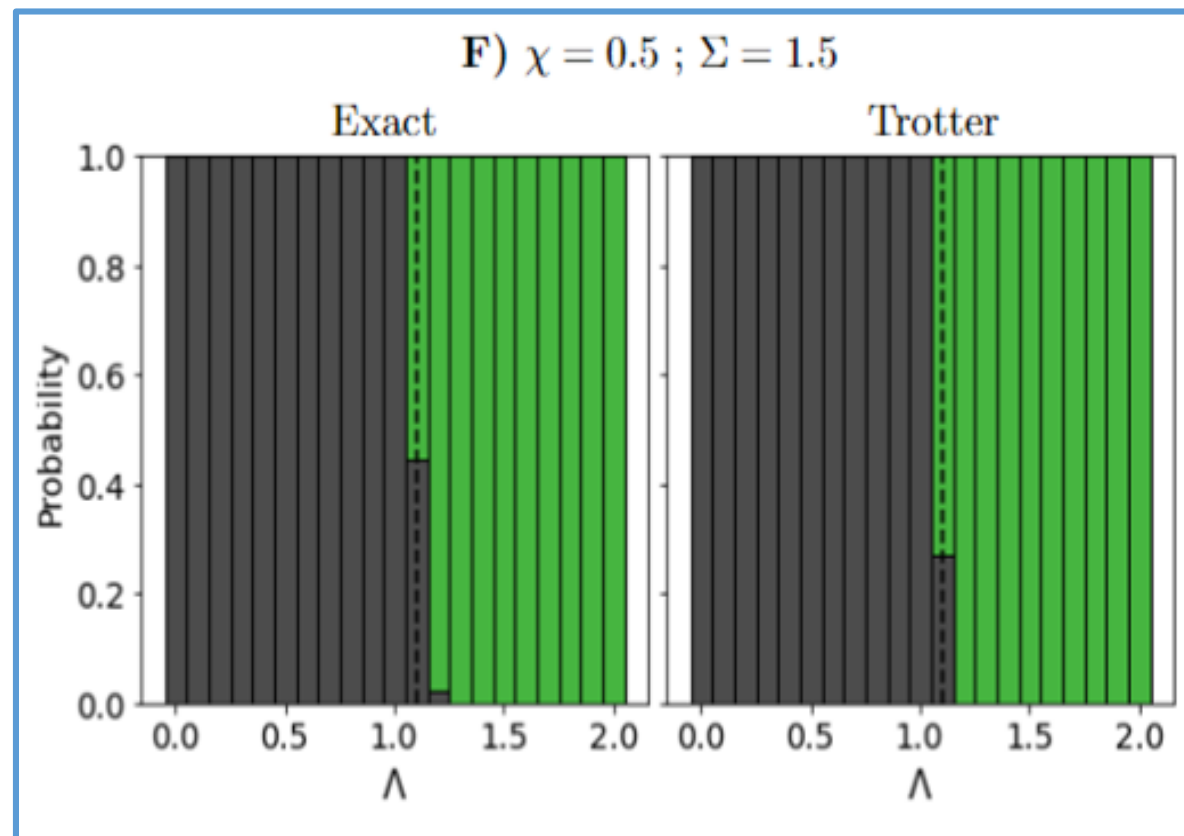
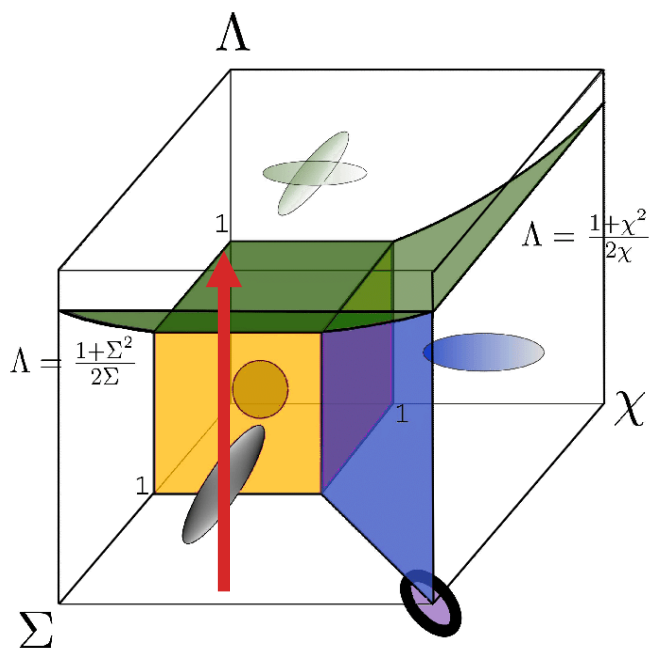
Convolutional Neural Network (CNN)- Results

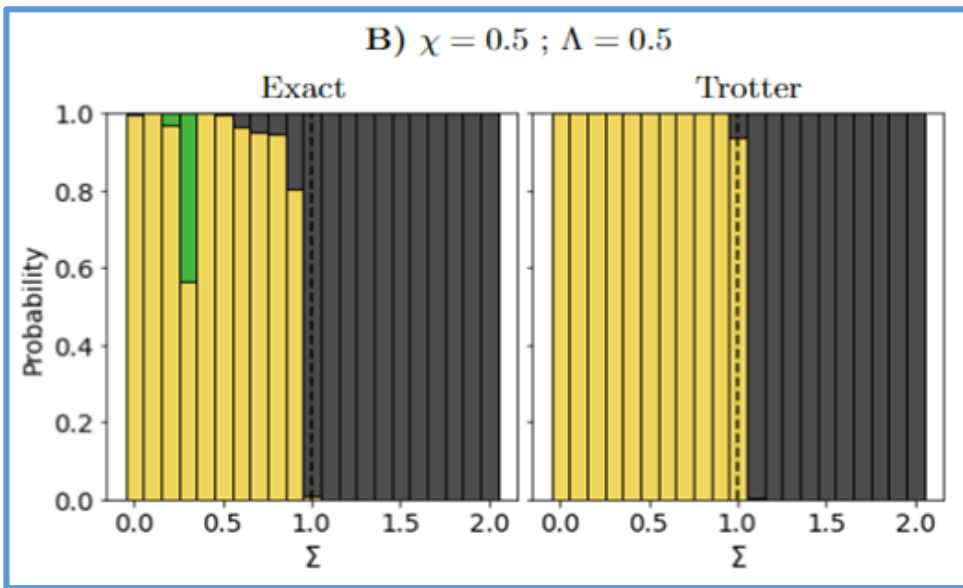
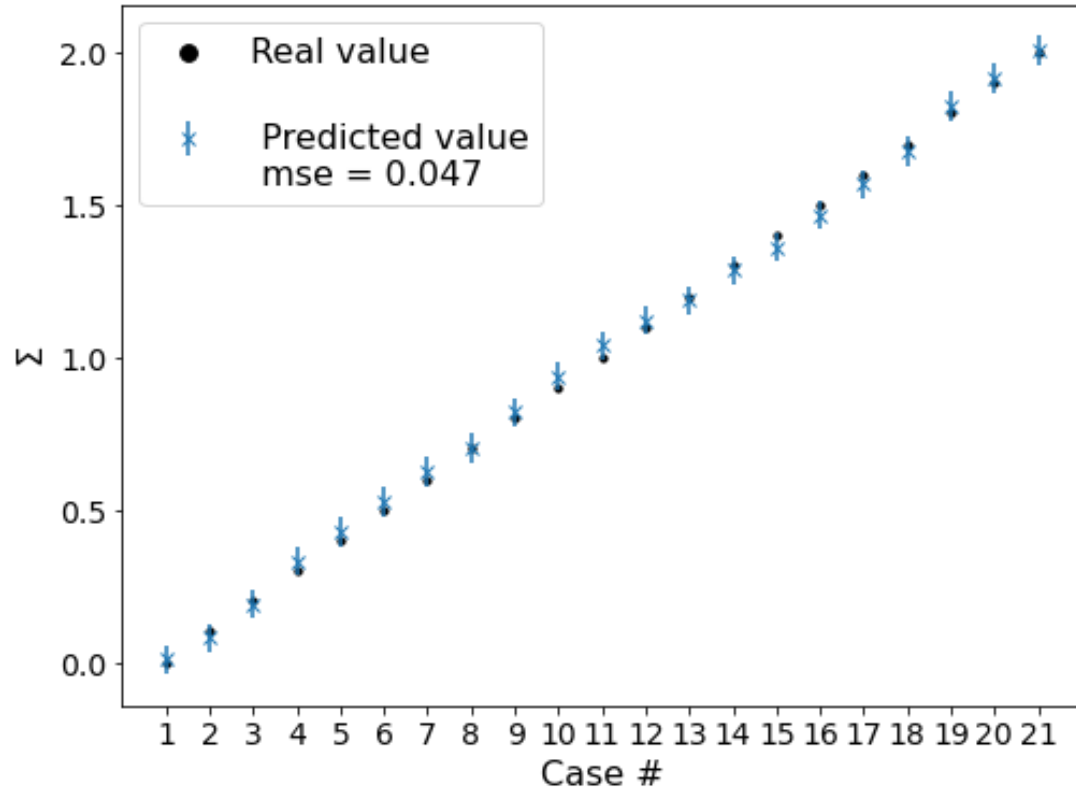
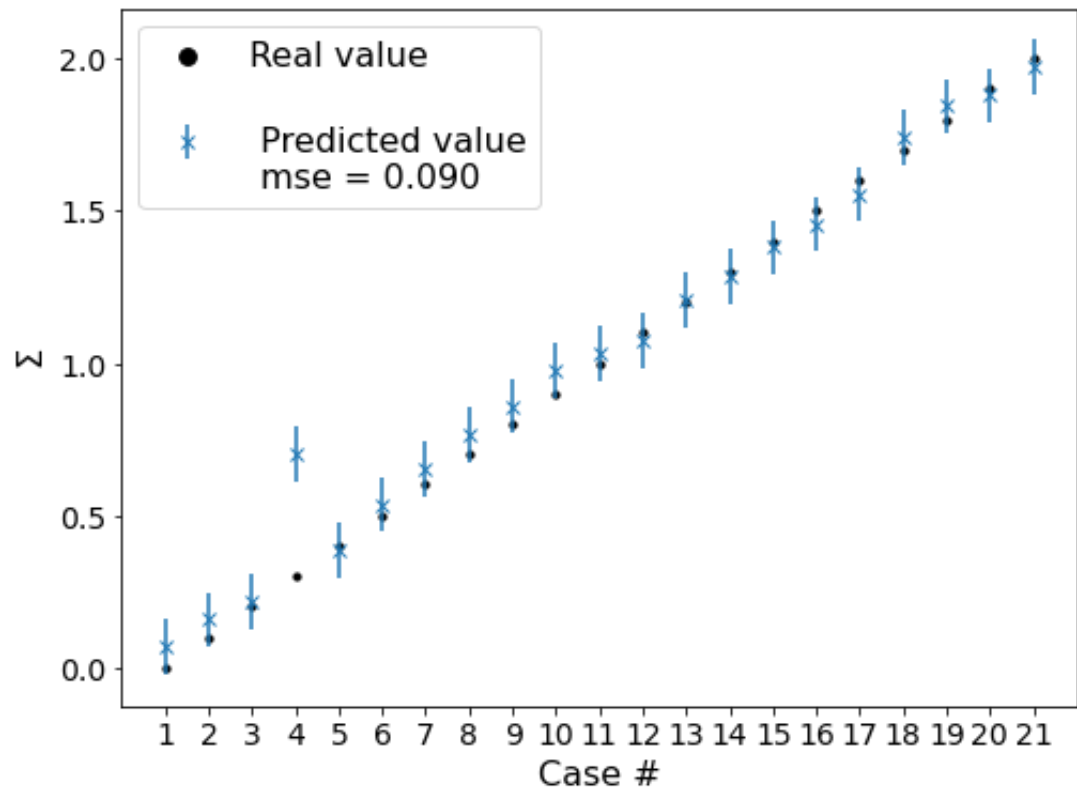
98.35%
Categorical
Accuracy



Convolutional Neural Network (CNN)- Results

98.35%
Categorical
Accuracy





Mean error smaller than the step size (0.1)

Conclusions

- The quantum simulation is feasible with polynomial resources.
- Observables can be measured with this experimental setup.
- These observables can provide information about the phase/shape of the system.
- Machine Learning methods can make use of this information to extract the Quantum Phases of the system.
- These methods are robust against introduced errors.
- It's fast and very accurate. Easily tailored to specific cases and different Hamiltonians, as long as there is data.