The symmetry structure of octupole phonons in nuclei

P. Van Isacker, GANIL, France
Collaborator: M. Rejmund

Motivation

A schematic model of octupole phonons in doubly-magic nuclei

Comparison with a `realistic' Hamiltonian

Extension to odd-mass nuclei

2PTN-10, Dubrovnik, July 2022

An experiment on ^{207}Pb

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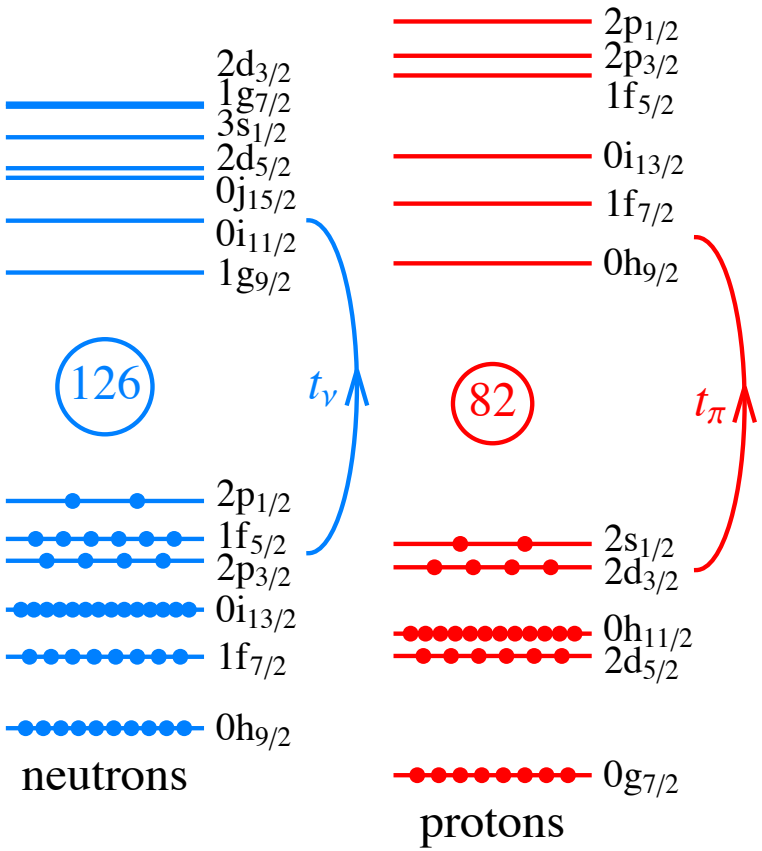
Evidence of octupole-phonons at high spin in ^{207}Pb

D. Ralet [a,b], E. Clément [b,*], G. Georgiev [a], A.E. Stuchbery [c], M. Rejmund [b], P. Van Isacker [b], G. de France [b], A. Lemasson [b], J. Ljungvall [a], C. Michelagnoli [b], A. Navin [b], D.L. Balabanski [d], L. Atanasova [u], A. Blazhev [g], G. Bocchi [e,ab], R. Carroll [f], J. Dudouet [h], E. Dupont [a], B. Fornal [i], S. Franchoo [aa], C. Fransen [g], C. Müller-Gatermann [g], A. Goasduff [k], A. Gadea [j], P.R. John [t,k], D. Kocheva [v], T. Konstantinopoulos [a], A. Korichi [a], A. Kusoglu [l], S.M. Lenzi [k], S. Leoni [e,ab], R. Lozeva [a,m], A. Maj [i], R. Perez [j], N. Pietralla [t], C. Shand [f], O. Stezowski [h], D. Wilmsen [b], D. Yordanov [aa], D. Barrientos [o], P. Bednarczyk [i], B. Birkenbach [g], A.J. Boston [p], H.C. Boston [p], I. Burrows [q], B. Cederwall [n], M. Ciemala [i], J. Collado [r], F. Crespi [e], D. Cullen [s], H.J. Eberth [g], J. Goupil [b], L. Harkness [p], H. Hess [g], A. Jungclaus [x], W. Korten [w], M. Labiche [q], R. Menegazzo [k], D. Mengoni [k], B. Million [e], J. Nyberg [y], Zs. Podolyák [f], A. Pullia [e], B. Quintana Arnés [z], F. Recchia [k], P. Reiter [g], F. Saillant [b], M.D. Salsac [w], E. Sanchis [r], C. Theisen [w], J.J. Valiente Dobon [o], O. Wieland [e]

Octupole phonons in ^{208}Pb



Empirical single-particle energies.

A `realistic' interaction with about 35000 TBMEs.

1p-1h space: $t_\nu + t_\pi \leq 1$

M. Rejmund *et al.*, Phys. Rev. C **59** (1999) 2520
B.A. Brown, Phys. Rev. Lett. **85** (2000) 5300

Structure of a 3^- phonon

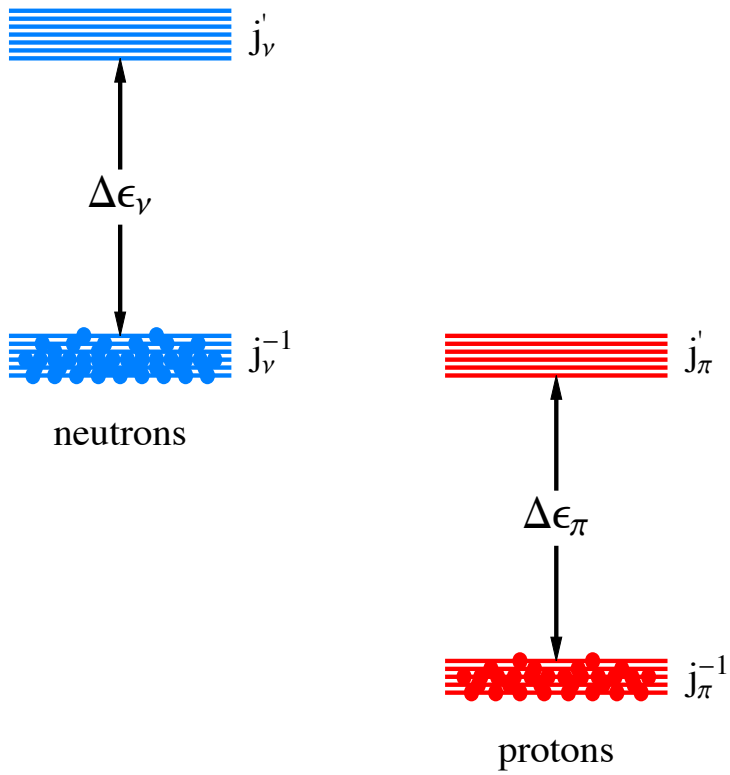
$$
\begin{array}{c}
\nu\nu^{-1} \\
\\
\pi\pi^{-1}
\end{array}
\left[\begin{array}{c:c}
V_{\nu\nu} & V_{\nu\pi} \\
\hdashline
V_{\nu\pi} & V_{\pi\pi}
\end{array}\right]
\begin{array}{c}
\\
\nu\nu^{-1} \quad \pi\pi^{-1}
\end{array}
$$

Particle-hole basis.

Diagonalisation of Hamiltonian matrix leads to a collective low-energy eigenstate

$$
\left|3^-_c\right\rangle = \sum_{k'k} c^{\nu}_{k'k} \left| j_{\nu k'} j^{-1}_{\nu k}; 3^- \right\rangle + \sum_{l'l} c^{\pi}_{l'l} \left| j_{\pi l'} j^{-1}_{\pi l}; 3^- \right\rangle
$$

A schematic model



Degenerate single-particle levels below and above shell gaps.

Surface delta interaction (SDI):

- $a_{1\nu}$: $\nu\nu$ interaction
- $a_{1\pi}$: $\pi\pi$ interaction
- a_0: $T=0$ $\nu\pi$ interaction
- a_1: $T=1$ $\nu\pi$ interaction

Properties of the SDI

Particle-hole matrix elements are written as

$$\left\langle j_{\rho k'} j_{\rho k}^{-1}; J \middle| \hat{V}_{\rho\rho}^{\mathrm{SDI}} \middle| j_{\rho l'} j_{\rho l}^{-1}; J \right\rangle = \frac{a_{1\rho}}{2}\left(f_{k'k}^{\rho} f_{l'l}^{\rho} - g_{k'k}^{\rho} g_{l'l}^{\rho} \right)$$

$$\left\langle j_{\nu k'} j_{\nu k}^{-1}; J \middle| \hat{V}_{\nu\pi}^{\mathrm{SDI}} \middle| j_{\pi l'} j_{\pi l}^{-1}; J \right\rangle = \frac{a_1 - a_0}{4} f_{k'k}^{\nu} f_{l'l}^{\pi} - \frac{3a_0 + a_1}{4} g_{k'k}^{\nu} g_{l'l}^{\pi}$$

with

$$f_{k'k}^{\rho} = (-)^{\ell_{\rho k}} \sqrt{(2j_{\rho k'}+1)(2j_{\rho k}+1)} \begin{pmatrix} j_{\rho k'} & j_{\rho k} & J \\ 1/2 & 1/2 & -1 \end{pmatrix}$$

$$g_{k'k}^{\rho} = (-)^{j_{\rho k} - 1/2} \sqrt{(2j_{\rho k'}+1)(2j_{\rho k}+1)} \begin{pmatrix} j_{\rho k'} & j_{\rho k} & J \\ 1/2 & -1/2 & 0 \end{pmatrix}$$

Solution of the schematic model

Exact solution: 4 x 4 matrix

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}_{\nu\nu} & \mathbf{H}_{\nu\pi} \\ \mathbf{H}_{\pi\nu} & \mathbf{H}_{\pi\pi} \end{bmatrix}$$

with

$$\mathbf{H}_{\rho\rho} = \begin{bmatrix} \Delta\varepsilon_\rho R_\rho + \frac{a_{1\rho}}{2}\left(R_\rho R_\rho - T_\rho T_\rho\right) & \Delta\varepsilon_\rho T_\rho + \frac{a_{1\rho}}{2}\left(R_\rho T_\rho - S_\rho T_\rho\right) \\ \Delta\varepsilon_\rho T_\rho + \frac{a_{1\rho}}{2}\left(R_\rho T_\rho - S_\rho T_\rho\right) & \Delta\varepsilon_\rho S_\rho + \frac{a_{1\rho}}{2}\left(T_\rho T_\rho - S_\rho S_\rho\right) \end{bmatrix}$$

$$\mathbf{H}_{\rho\dot{\rho}} = \begin{bmatrix} \frac{a_1 - a_0}{4} R_\rho R_{\dot{\rho}} - \frac{a_1 + 3a_0}{4} T_\rho T_{\dot{\rho}} & \frac{a_1 - a_0}{4} R_\rho T_{\dot{\rho}} - \frac{a_1 + 3a_0}{4} T_\rho S_{\dot{\rho}} \\ \frac{a_1 - a_0}{4} T_\rho R_{\dot{\rho}} - \frac{a_1 + 3a_0}{4} S_\rho T_{\dot{\rho}} & \frac{a_1 - a_0}{4} T_\rho T_{\dot{\rho}} - \frac{a_1 + 3a_0}{4} S_\rho S_{\dot{\rho}} \end{bmatrix}$$

$$R_\rho = \sum_{k'k} f^\rho_{k'k} f^\rho_{k'k}, \quad S_\rho = \sum_{k'k} g^\rho_{k'k} g^\rho_{k'k}, \quad T_\rho = \sum_{k'k} f^\rho_{k'k} g^\rho_{k'k}.$$

Solution of the schematic model

Exact solution: 4 x 4 matrix

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}_{\nu\nu} & \mathbf{H}_{\nu\pi} \\ \mathbf{H}_{\pi\nu} & \mathbf{H}_{\pi\pi} \end{bmatrix}$$

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$$\mathbf{H}_{\rho\dot{\rho}} = \begin{bmatrix} \frac{a_1 - a_0}{4} R_\rho R_{\dot{\rho}} - \frac{a_1 + 3a_0}{4} T_\rho T_{\dot{\rho}} & \frac{a_1 - a_0}{4} R_\rho T_{\dot{\rho}} - \frac{a_1 + 3a_0}{4} T_\rho S_{\dot{\rho}} \\ \frac{a_1 - a_0}{4} T_\rho R_{\dot{\rho}} - \frac{a_1 + 3a_0}{4} S_\rho T_{\dot{\rho}} & \frac{a_1 - a_0}{4} T_\rho T_{\dot{\rho}} - \frac{a_1 + 3a_0}{4} S_\rho S_{\dot{\rho}} \end{bmatrix}$$

$$R_\rho = \sum_{k'k} f^\rho_{k'k} f^\rho_{k'k}, \quad S_\rho = \sum_{k'k} g^\rho_{k'k} g^\rho_{k'k}, \quad T_\rho = \sum_{k'k} f^\rho_{k'k} g^\rho_{k'k} \approx 0.$$

Solution of the schematic model

Exact solution: 4 x 4 matrix

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}_{\nu\nu} & \mathbf{H}_{\nu\pi} \\ \mathbf{H}_{\pi\nu} & \mathbf{H}_{\pi\pi} \end{bmatrix}$$

with

$$\mathbf{H}_{\rho\rho} = \begin{bmatrix} \Delta\varepsilon_\rho R_\rho + \frac{a_{1\rho}}{2}\left(R_\rho R_\rho - T_\rho T_\rho\right) & \Delta\varepsilon_\rho T_\rho + \frac{a_{1\rho}}{2}\left(R_\rho T_\rho - S_\rho T_\rho\right) \\ \Delta\varepsilon_\rho T_\rho + \frac{a_{1\rho}}{2}\left(R_\rho T_\rho - S_\rho T_\rho\right) & \Delta\varepsilon_\rho S_\rho + \frac{a_{1\rho}}{2}\left(T_\rho T_\rho - S_\rho S_\rho\right) \end{bmatrix}$$

$$\mathbf{H}_{\rho\dot{\rho}} = \begin{bmatrix} \frac{a_1 - a_0}{4} R_\rho R_{\dot{\rho}} - \frac{a_1 + 3a_0}{4} T_\rho T_{\dot{\rho}} & \frac{a_1 - a_0}{4} R_\rho T_{\dot{\rho}} - \frac{a_1 + 3a_0}{4} T_\rho S_{\dot{\rho}} \\ \frac{a_1 - a_0}{4} T_\rho R_{\dot{\rho}} - \frac{a_1 + 3a_0}{4} S_\rho T_{\dot{\rho}} & \frac{a_1 - a_0}{4} T_\rho T_{\dot{\rho}} - \frac{a_1 + 3a_0}{4} S_\rho S_{\dot{\rho}} \end{bmatrix}$$

$$R_\rho = \sum_{k'k} f^\rho_{k'k} f^\rho_{k'k}, \quad S_\rho = \sum_{k'k} g^\rho_{k'k} g^\rho_{k'k}, \quad T_\rho = \sum_{k'k} f^\rho_{k'k} g^\rho_{k'k} = 0.$$

Solution of the schematic model

Approximate solution: 2 x 2 Hamiltonian matrix

$$\mathbf{H} = \begin{bmatrix} \Delta\varepsilon_\nu - \frac{a_{1\nu}}{2} S_\nu & -\frac{3a_0+a_1}{4}\sqrt{S_\nu S_\pi} \\ -\frac{3a_0+a_1}{4}\sqrt{S_\nu S_\pi} & \Delta\varepsilon_\pi - \frac{a_{1\pi}}{2} S_\pi \end{bmatrix}$$

$$S_\rho \approx \frac{\left(N_\rho - 1\right) N_\rho \left(N_\rho + 1\right)\left(4N_\rho + 7\right)}{\left(2N_\rho + 1\right)\left(2N_\rho + 3\right)}$$

Solution of the schematic model

A generic expression for the octupole phonon:

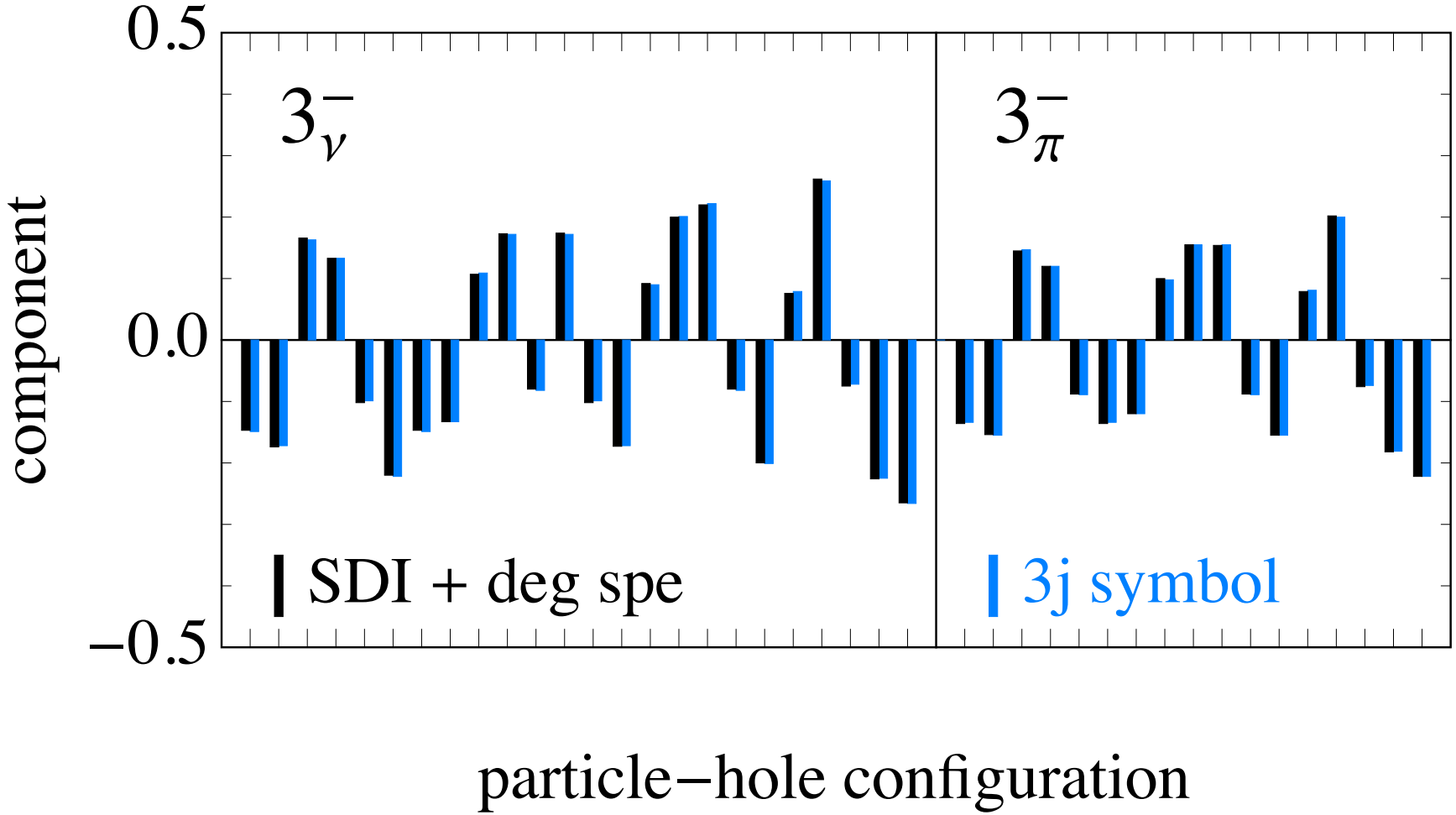
$$\left|3_{\mathrm{c}}^{-}\right\rangle \approx \sum_{k'k} c_{k'k}^{\nu} \left| j_{\nu k'} j_{\nu k}^{-1}; 3^{-} \right\rangle + \sum_{k'k} c_{k'k}^{\pi} \left| j_{\pi k'} j_{\pi k}^{-1}; 3^{-} \right\rangle$$

with

$$c_{k'k}^{\rho} = \alpha_{\rho} g_{k'k}^{\rho}$$

$$= \alpha_{\rho} (-)^{j_{\rho k} - 1/2} \sqrt{\frac{\left(2 j_{\rho k'} + 1\right)\left(2 j_{\rho k} + 1\right)}{S_{\rho}}} \begin{pmatrix} j_{\rho k'} & j_{\rho k} & 3 \\ \frac{1}{2} & -\frac{1}{2} & 0 \end{pmatrix}$$

The octupole phonon in ^{208}Pb



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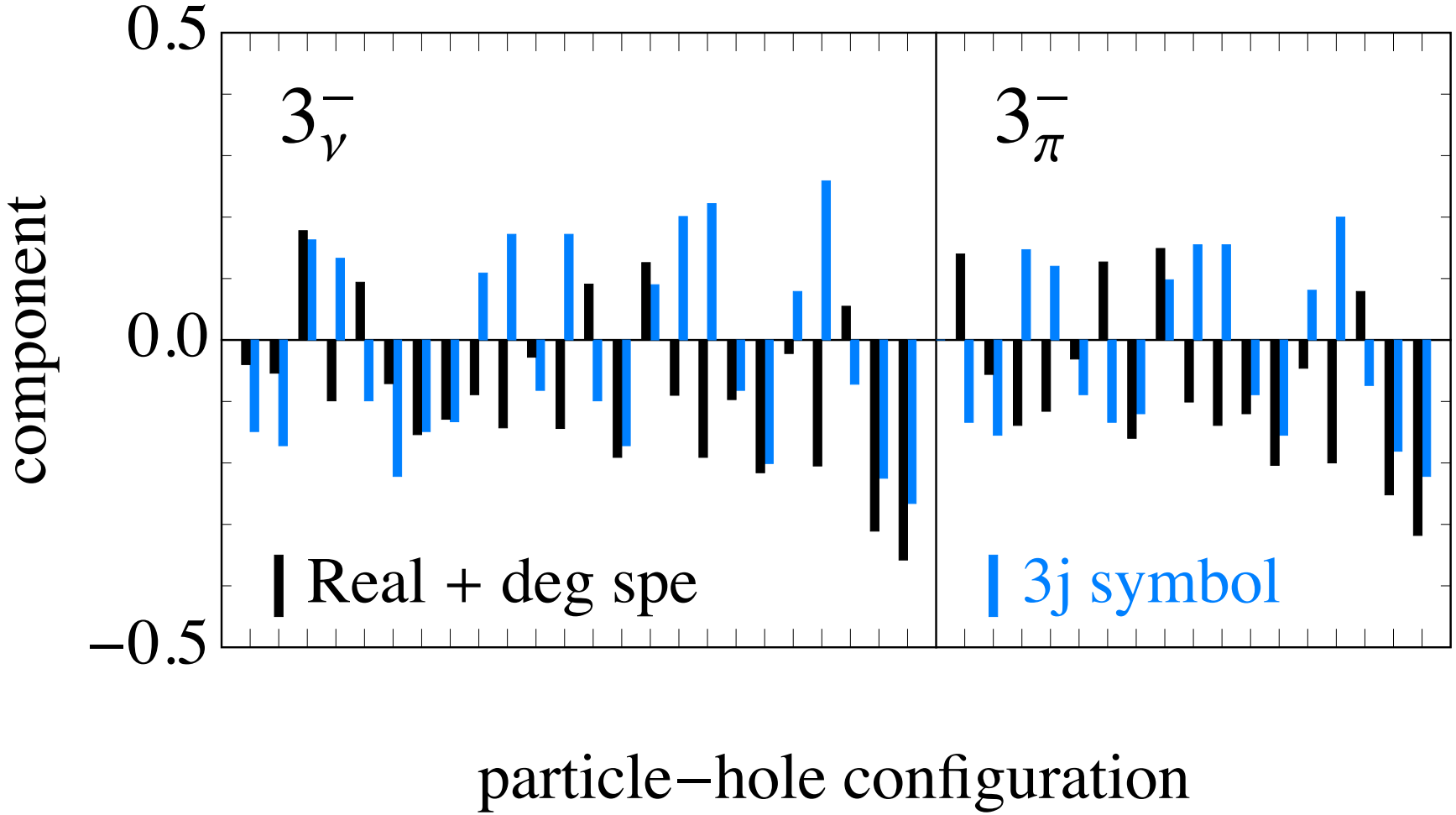

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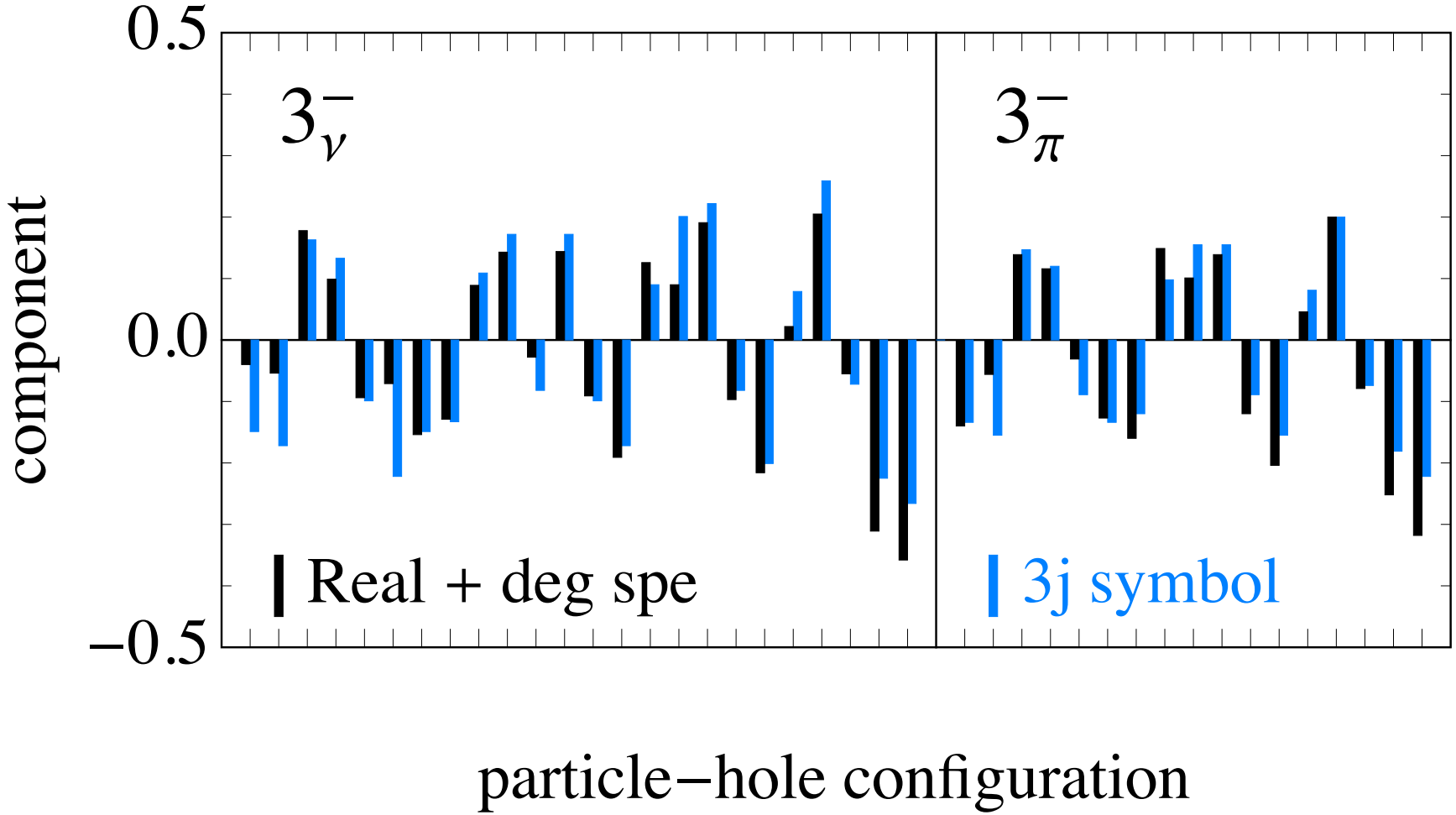

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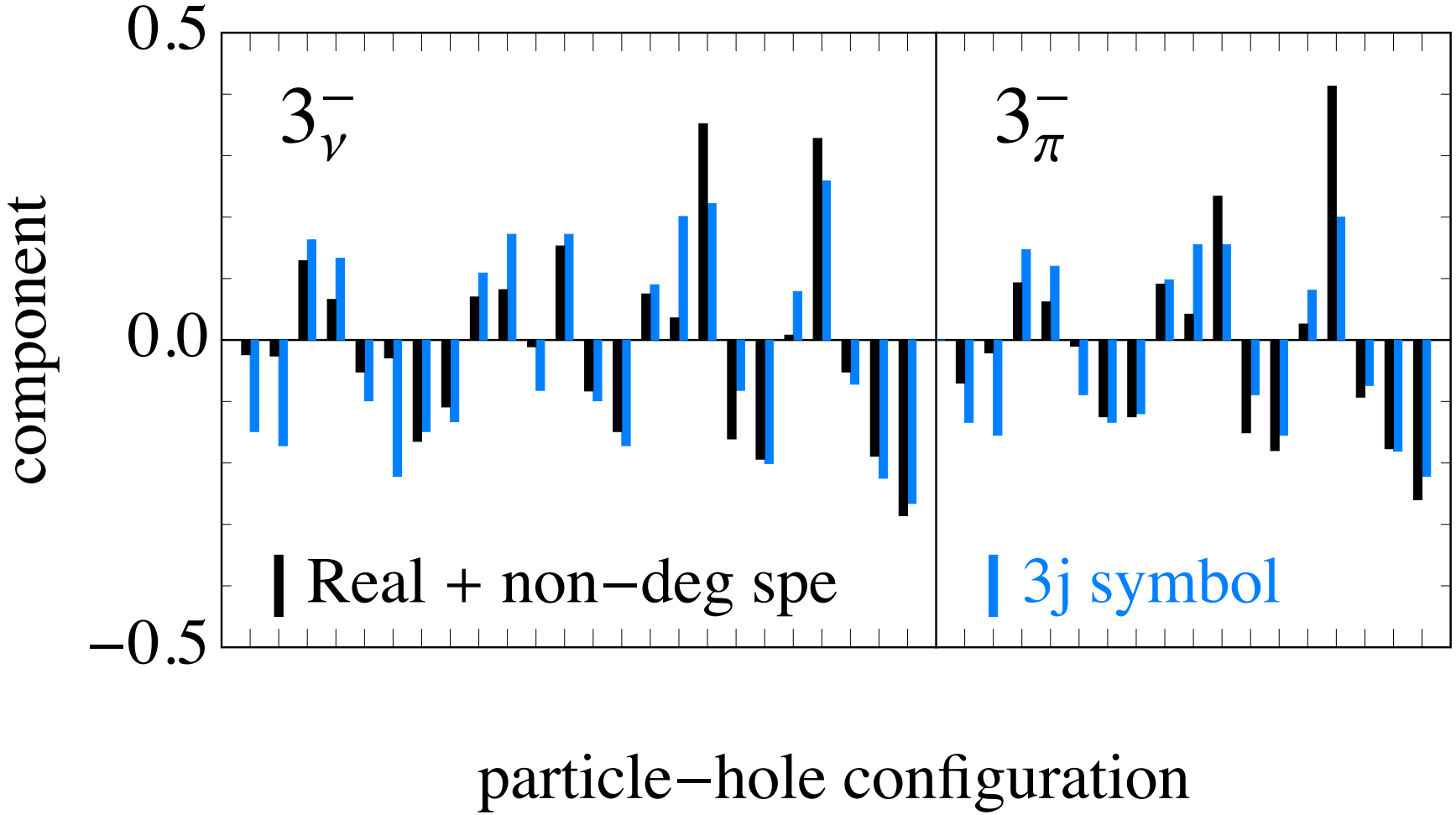
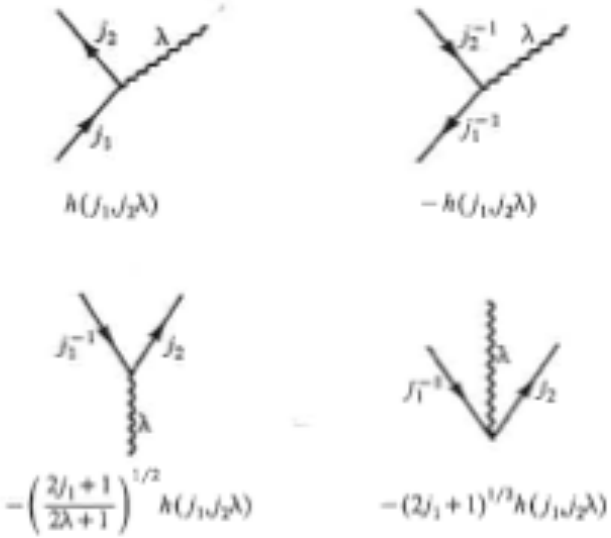


Extension to odd-mass nuclei

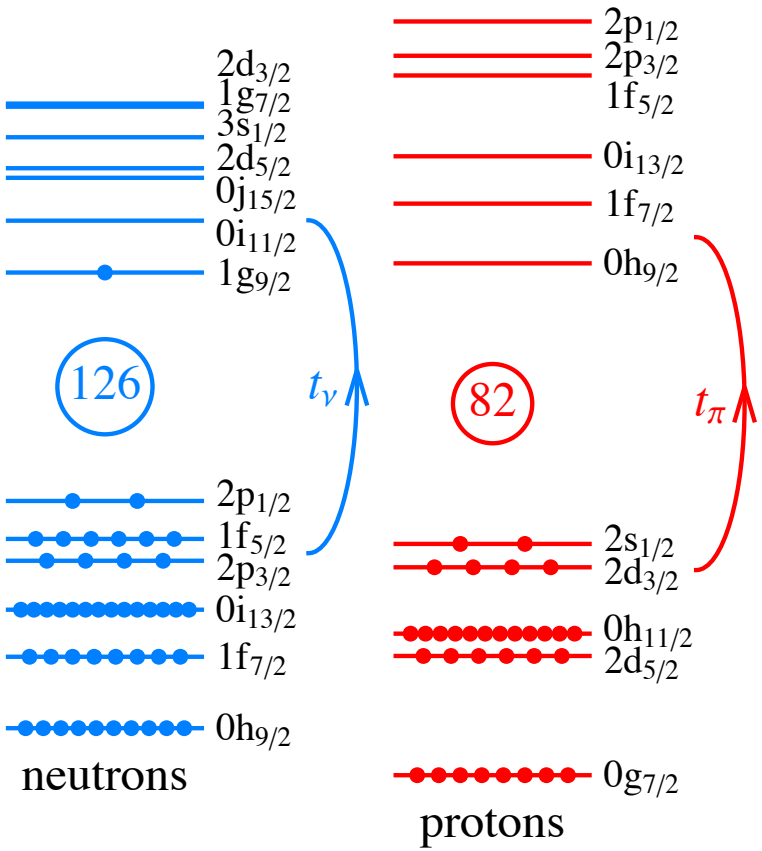
How to couple a particle or a hole to a phonon?

Read *Nuclear Structure. II Nuclear Deformations* by A. Bohr and B.R. Mottelson.

§ 6-5 PARTICLE-VIBRATION COUPLING



Octupole phonons in ^{209}Pb



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M. Rejmund *et al.*, Phys. Rev. C **59** (1999) 2520
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Particle/octupole states in ^{209}Pb

The $9/2^+$ states:

$$\left|9/2_1^+\right\rangle = \alpha_9 \left|1g_{9/2}\right\rangle + \beta_9 \left|3_c^- \times 0j_{15/2}; 9/2\right\rangle$$

$$\left|9/2_2^+\right\rangle = \beta_9 \left|1g_{9/2}\right\rangle - \alpha_9 \left|3_c^- \times 0j_{15/2}; 9/2\right\rangle$$

The $15/2^-$ states:

$$\left|15/2_1^-\right\rangle = \alpha_{15} \left|0j_{15/2}\right\rangle + \beta_{15} \left|3_c^- \times 1g_{9/2}; 15/2\right\rangle$$

$$\left|15/2_2^-\right\rangle = \beta_{15} \left|0j_{15/2}\right\rangle - \alpha_{15} \left|3_c^- \times 1g_{9/2}; 15/2\right\rangle$$

The particle-vibration coupling model (Bohr & Mottelson) estimates the mixing coefficients α and β.

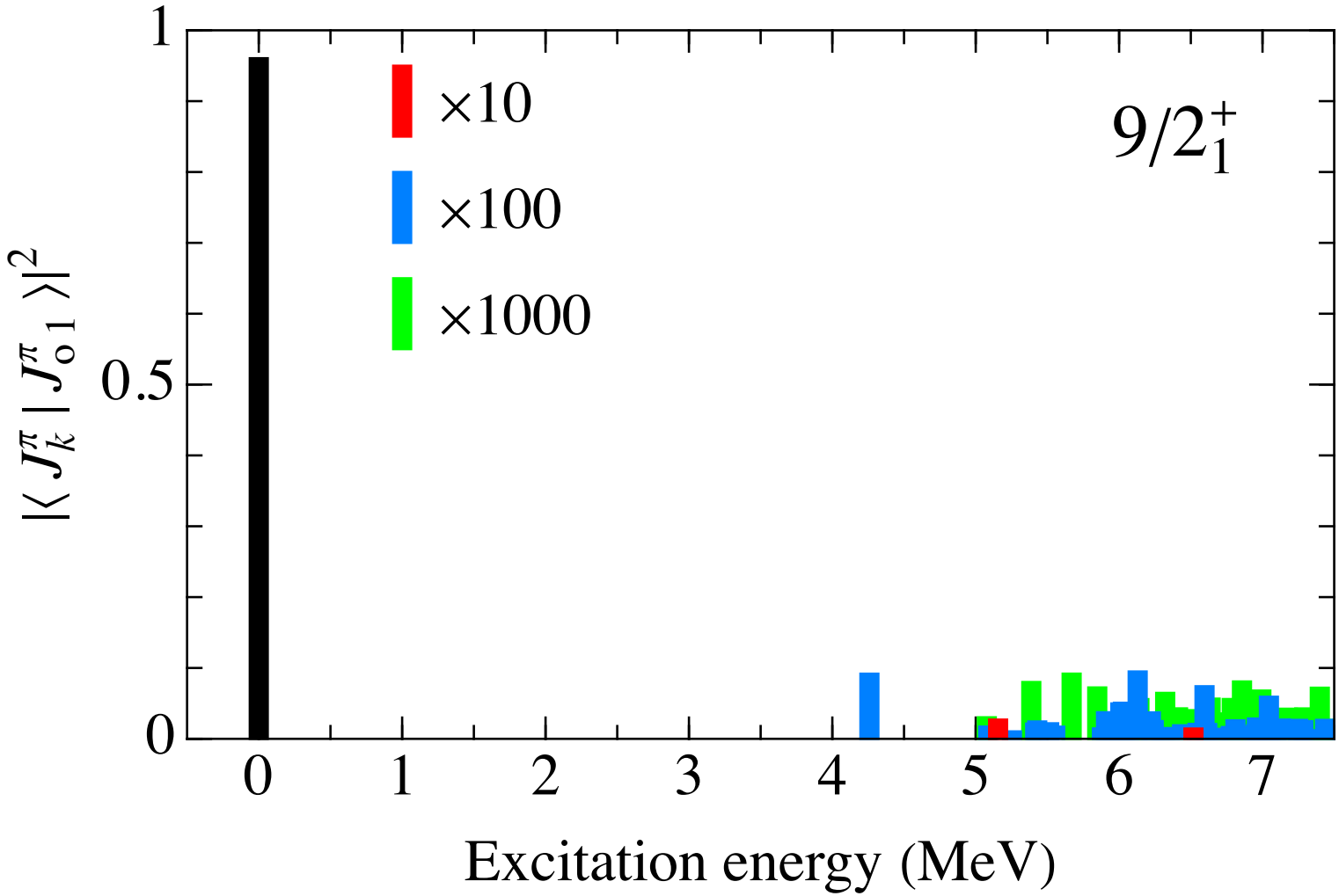
Octupole model for odd-mass nuclei

Assume that states in the odd-mass nucleus are either of single-particle nature or a single particle coupled to the octupole phonon.

Calculate shell-model matrix elements in this one- or two-dimensional basis.

Test: Take overlap with the `full' shell-model calculation in 1p-0h + 2p-1h basis.

Shell-model analysis of ^{209}Pb



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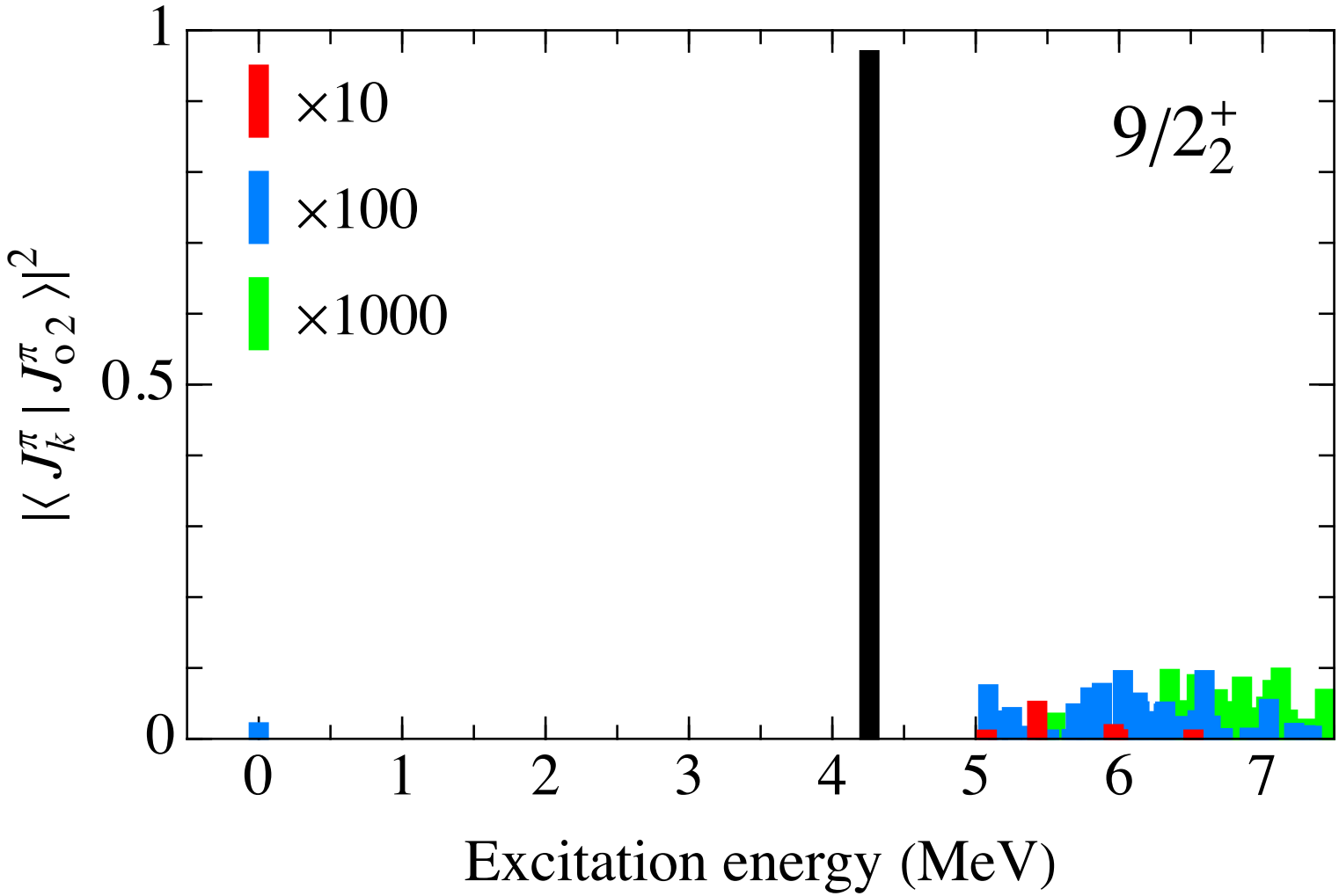

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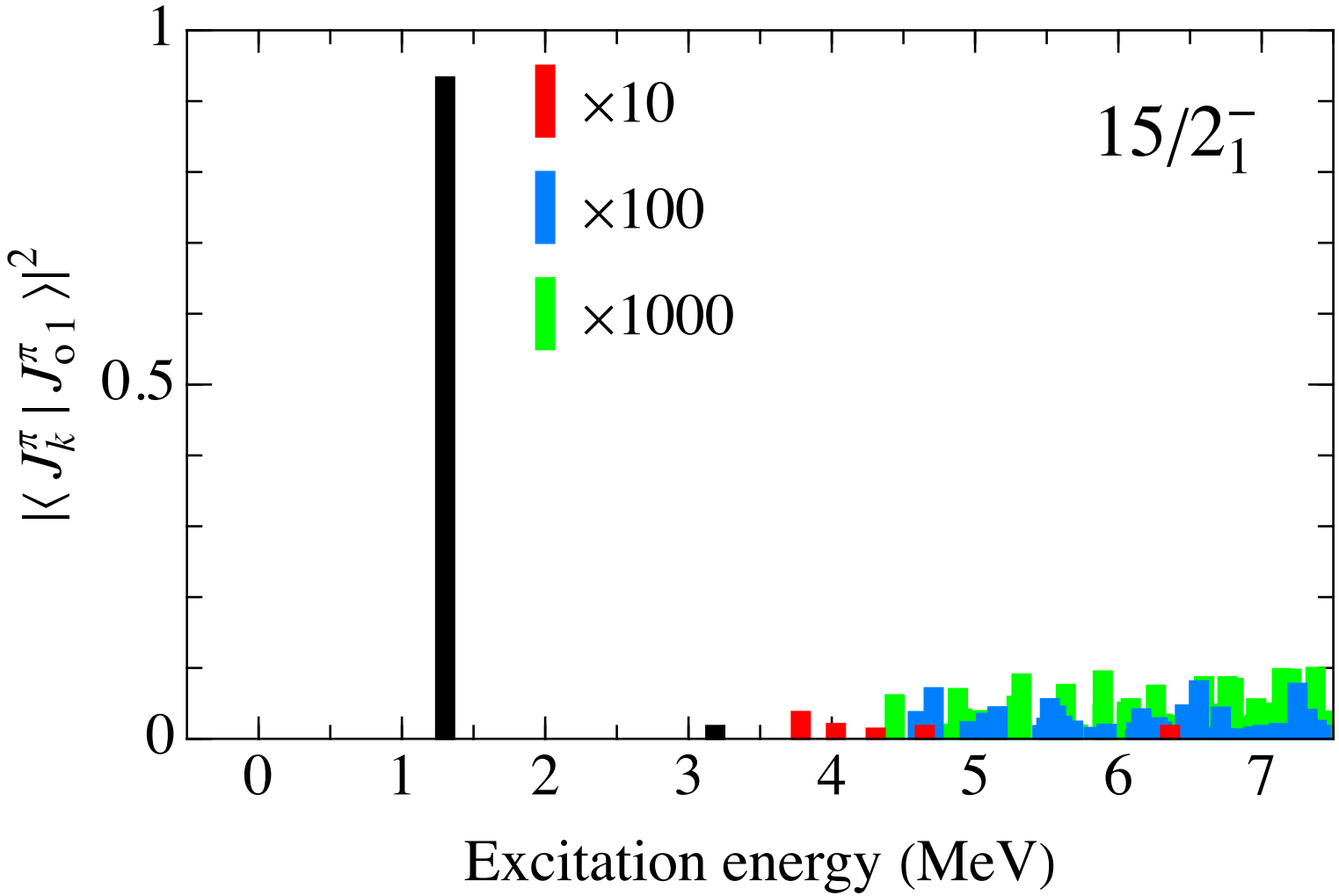

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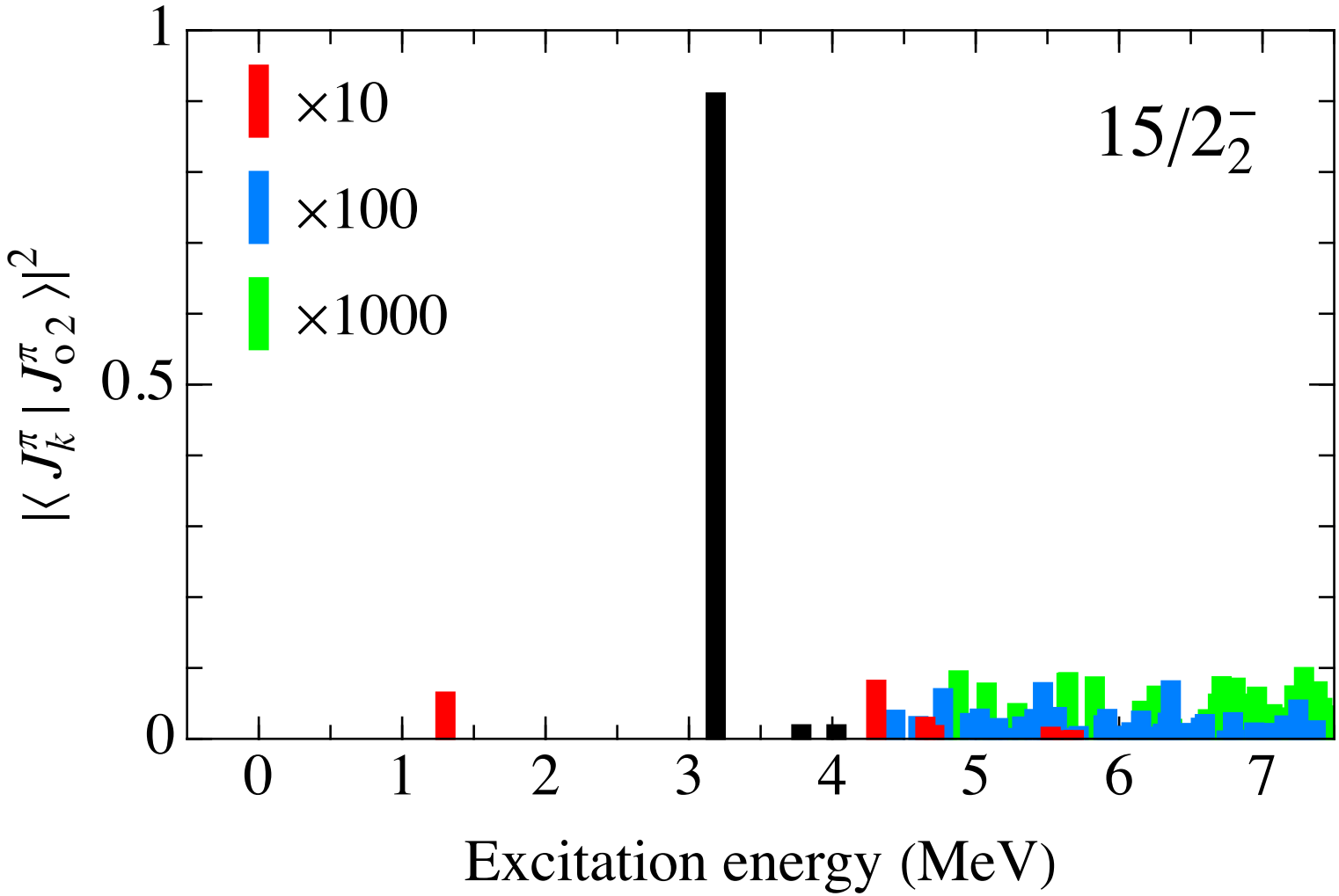
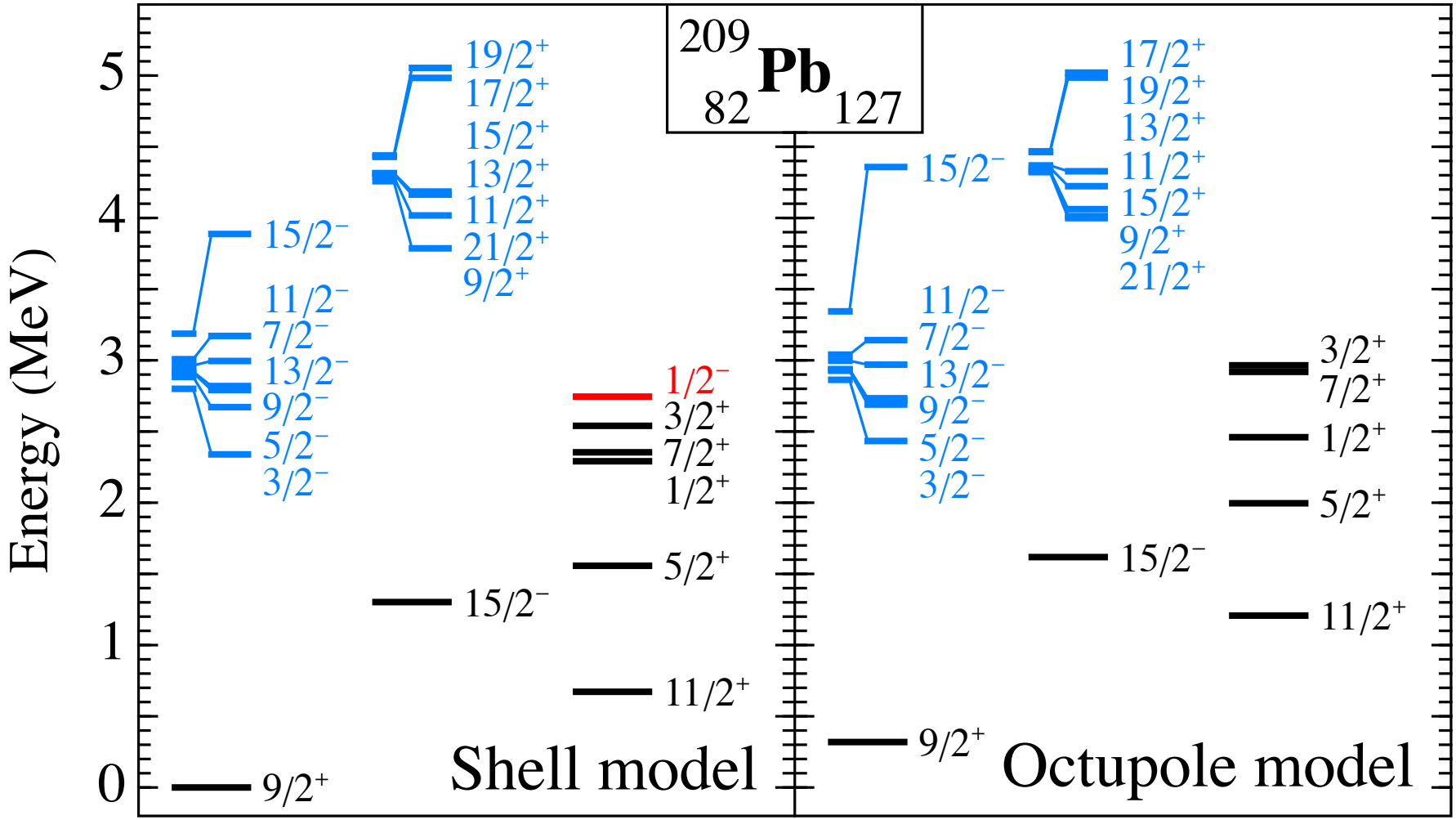


Spectrum of ^{209}Pb



Conclusions and outlook

Properties of the schematic model are found approximately in a realistic calculation.

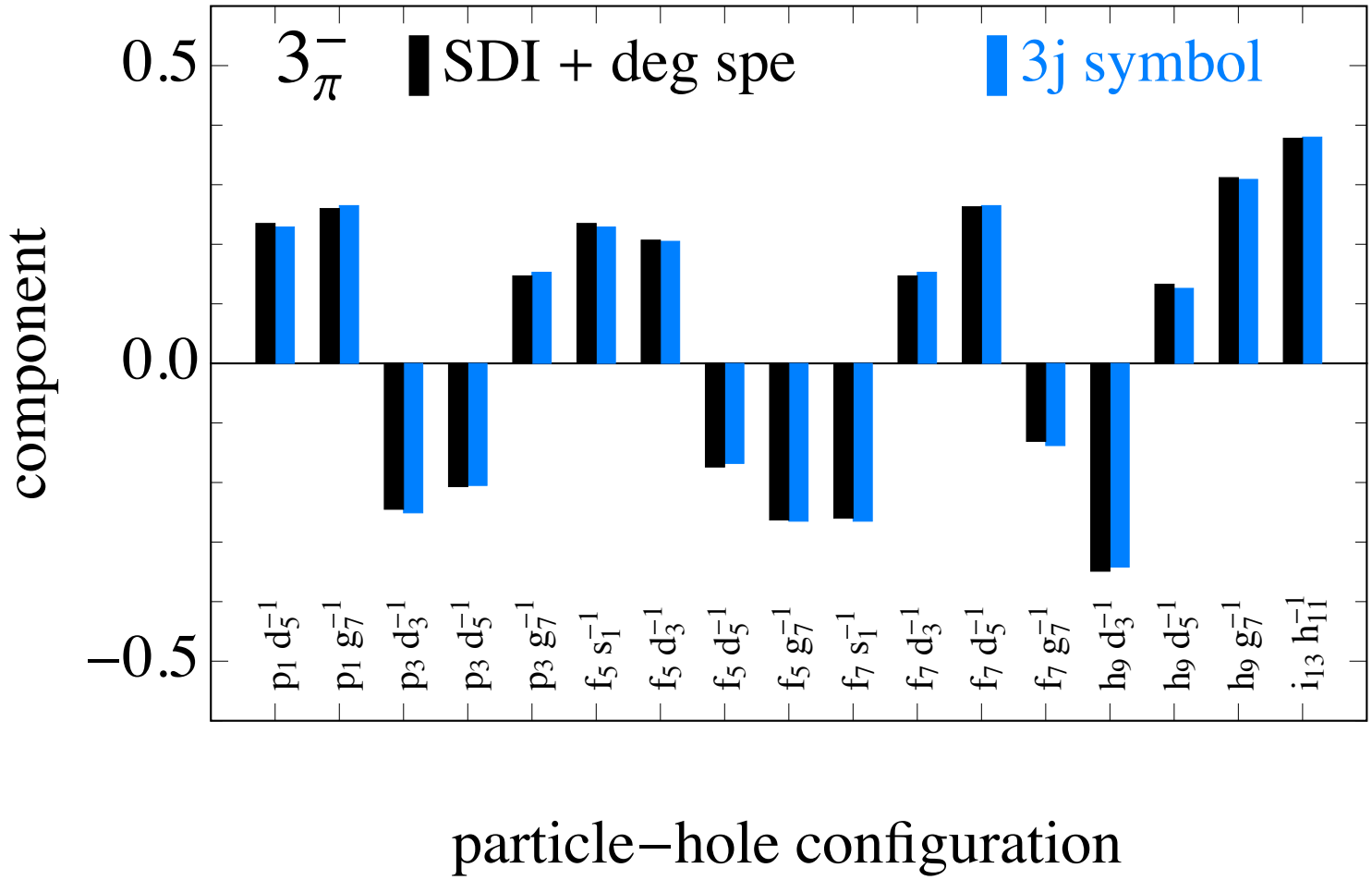
The separate neutron and proton phonons exist by virtue of the neutron-proton interaction, which, besides generating their collective structure, also couples them.

The octupole phonon retains its collective structure when coupled to a particle or hole.

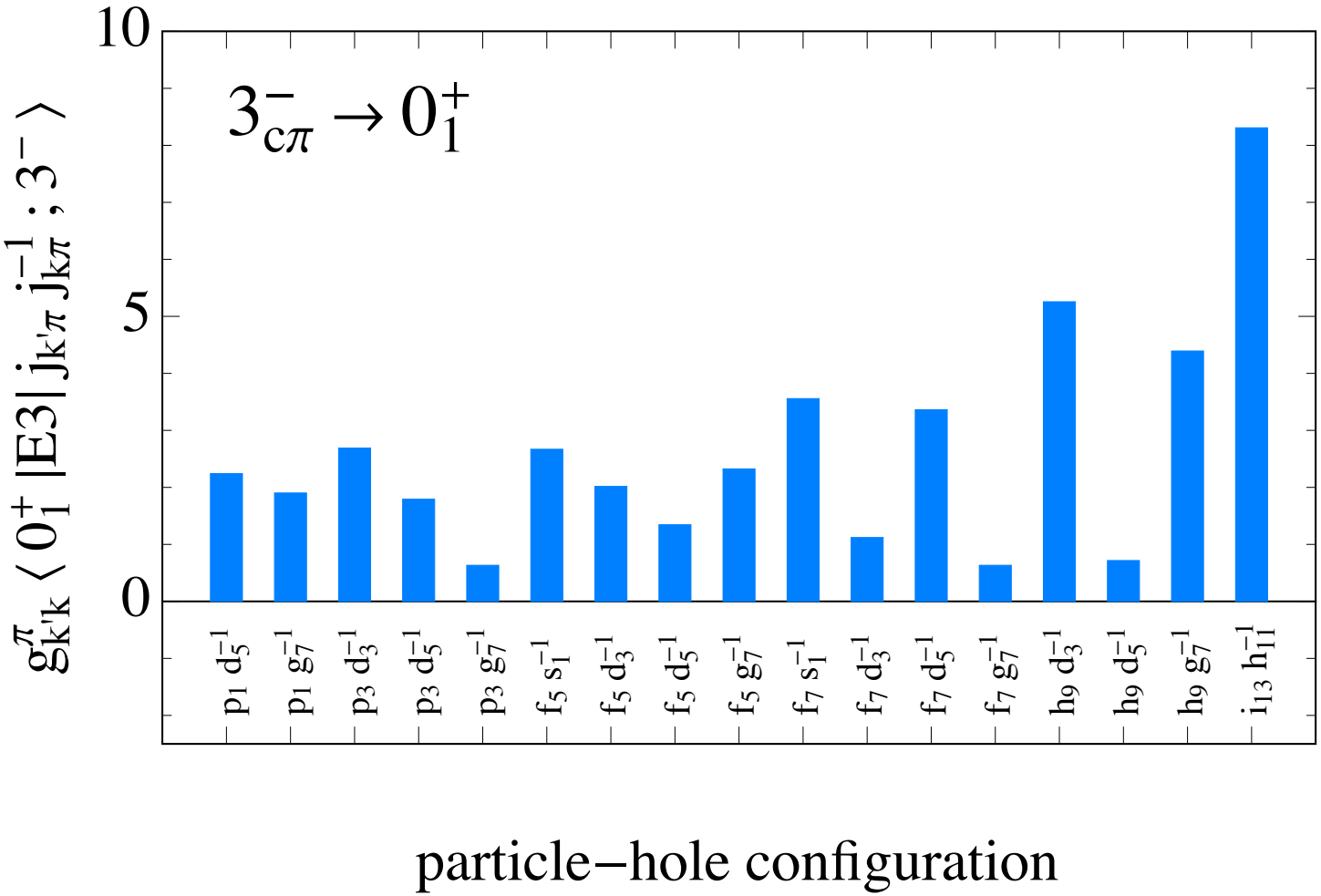
To do: Schematic model of two-octupole-phonon states.

P. Van Isacker & M. Rejmund, Phys. Rev. Research **4** (2022) L022031

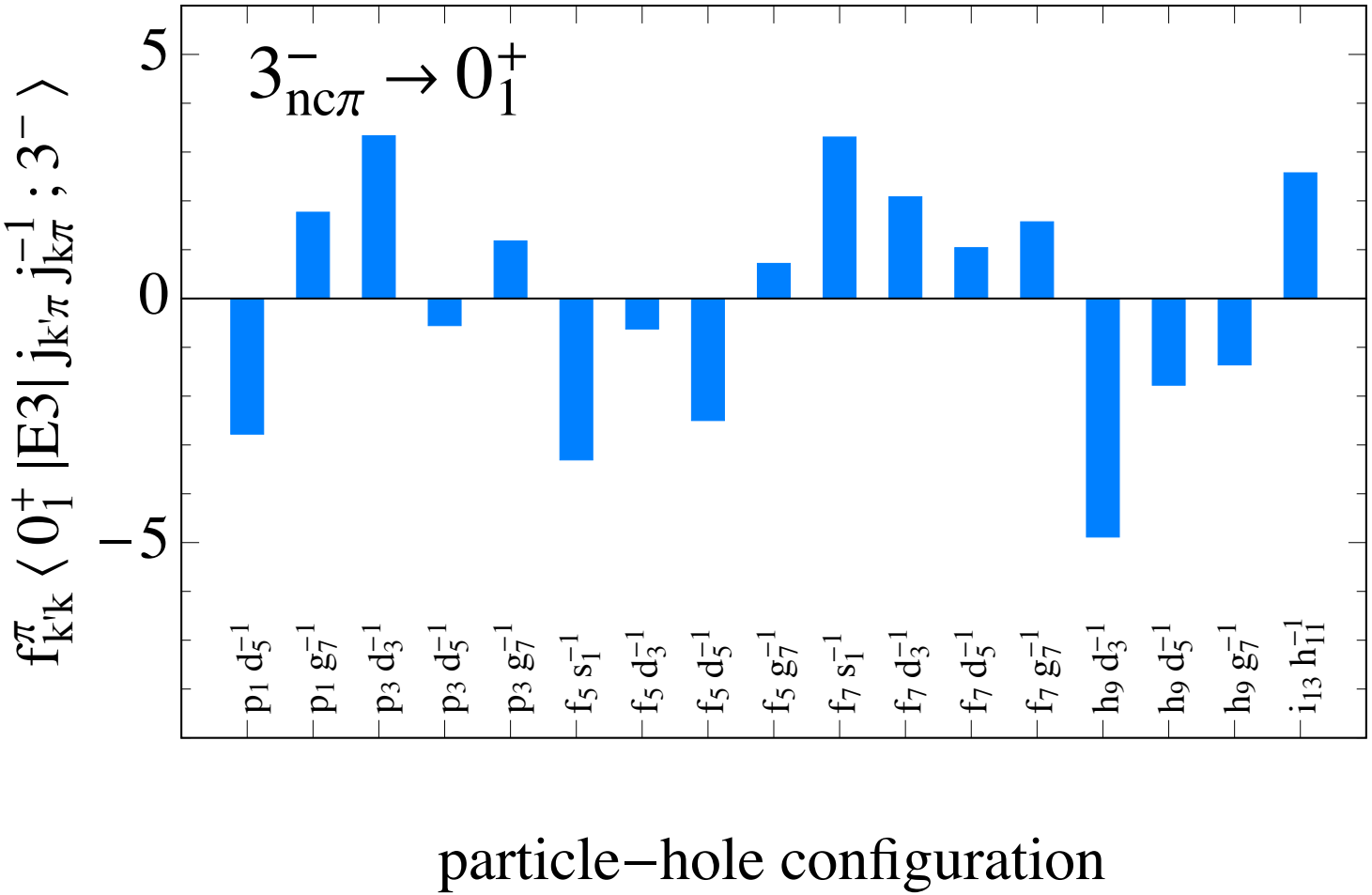
Proton octupole phonon



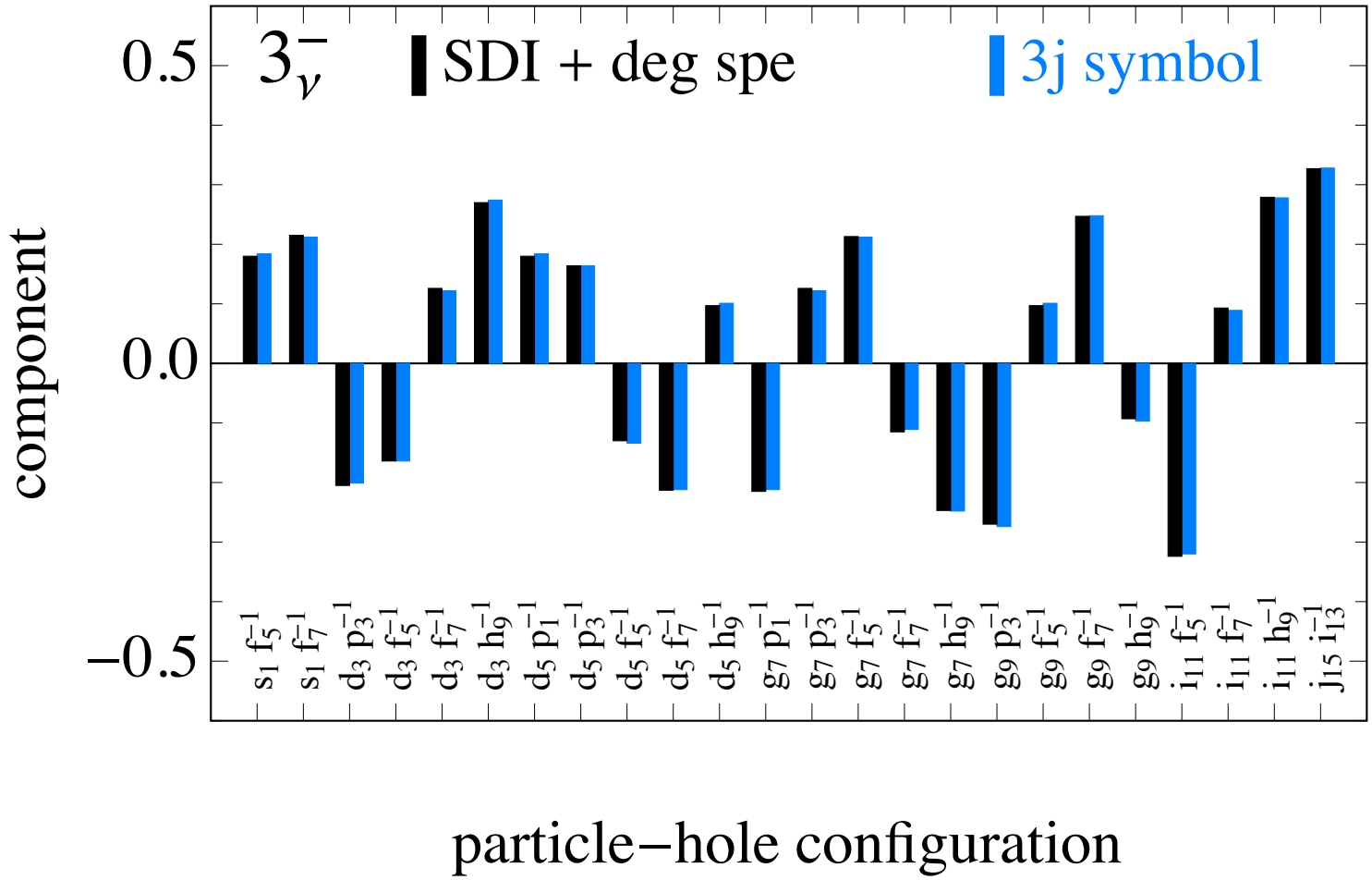
A collective E3 transition



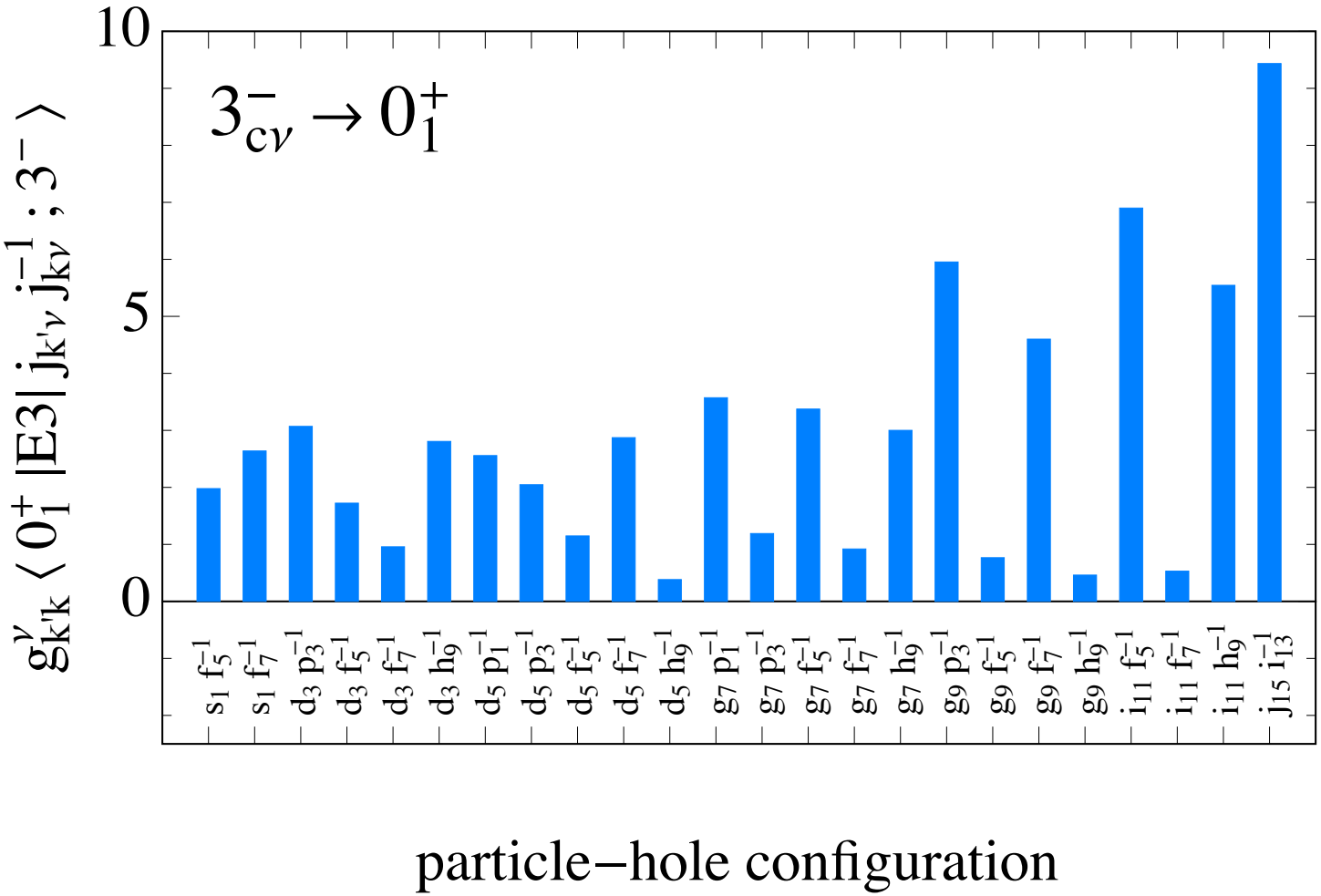
An anti-collective E3 transition



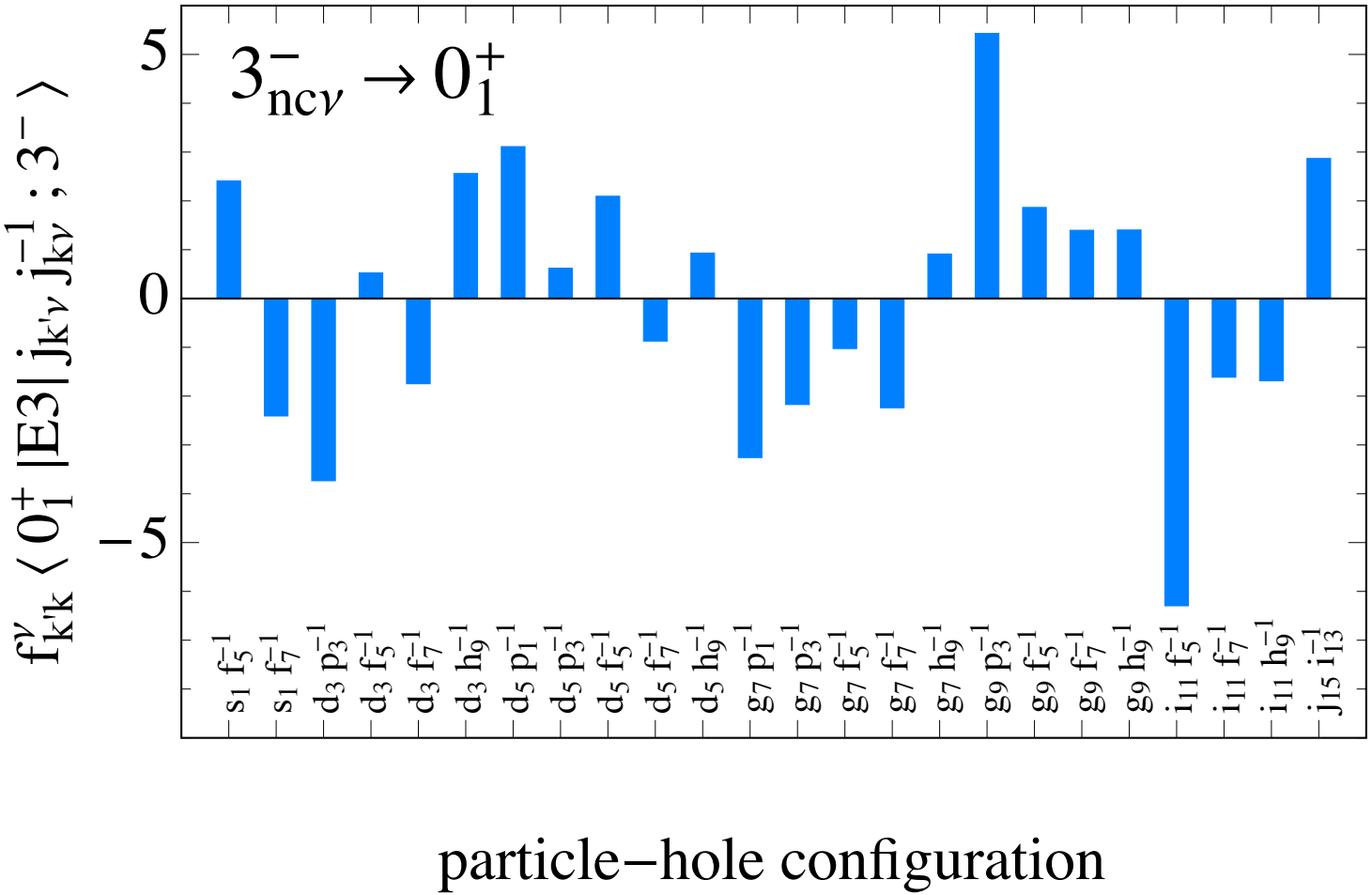
Neutron octupole phonon



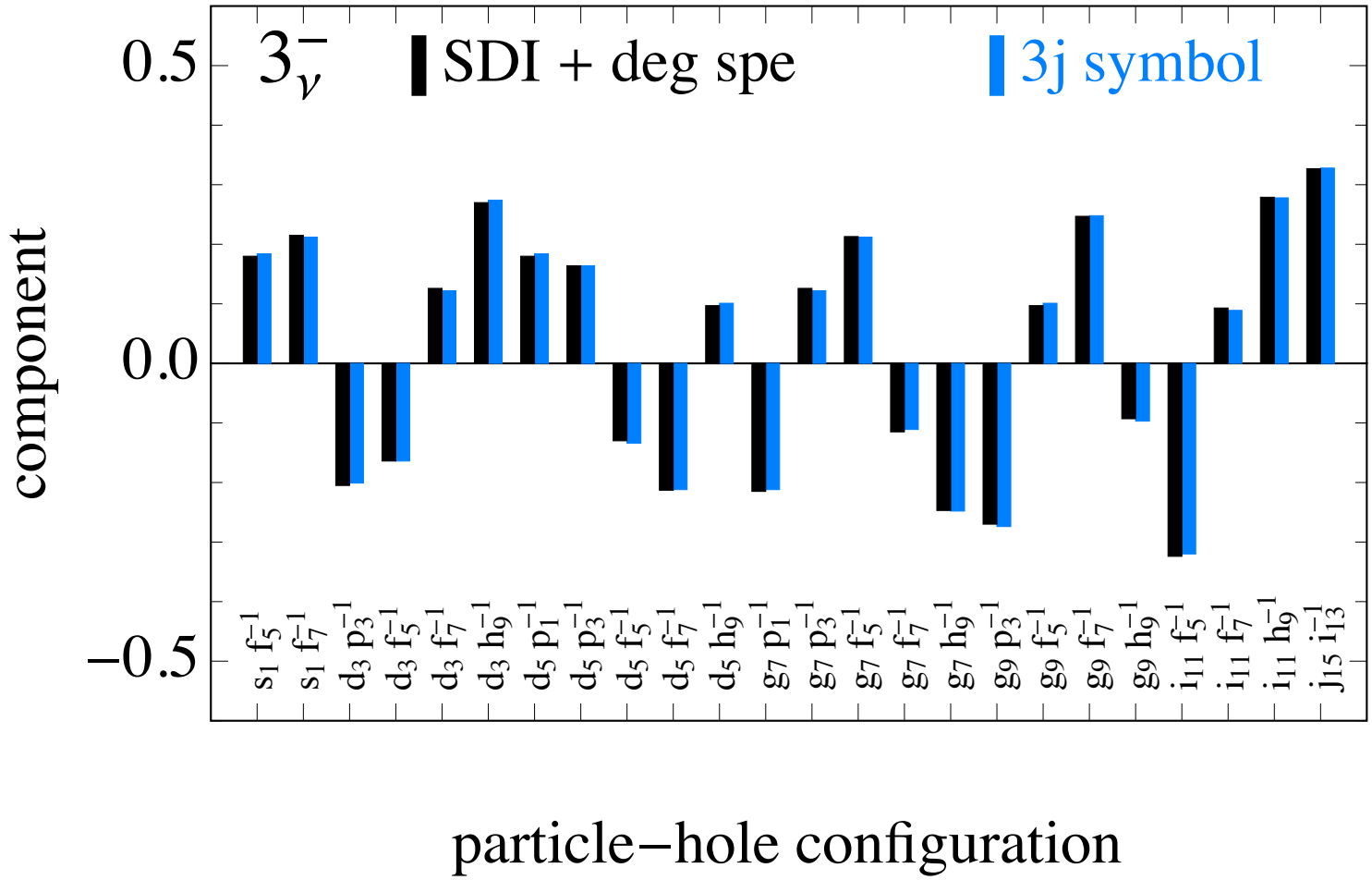
A collective E3 transition



An anti-collective E3 transition



Deconstructing the neutron phonon



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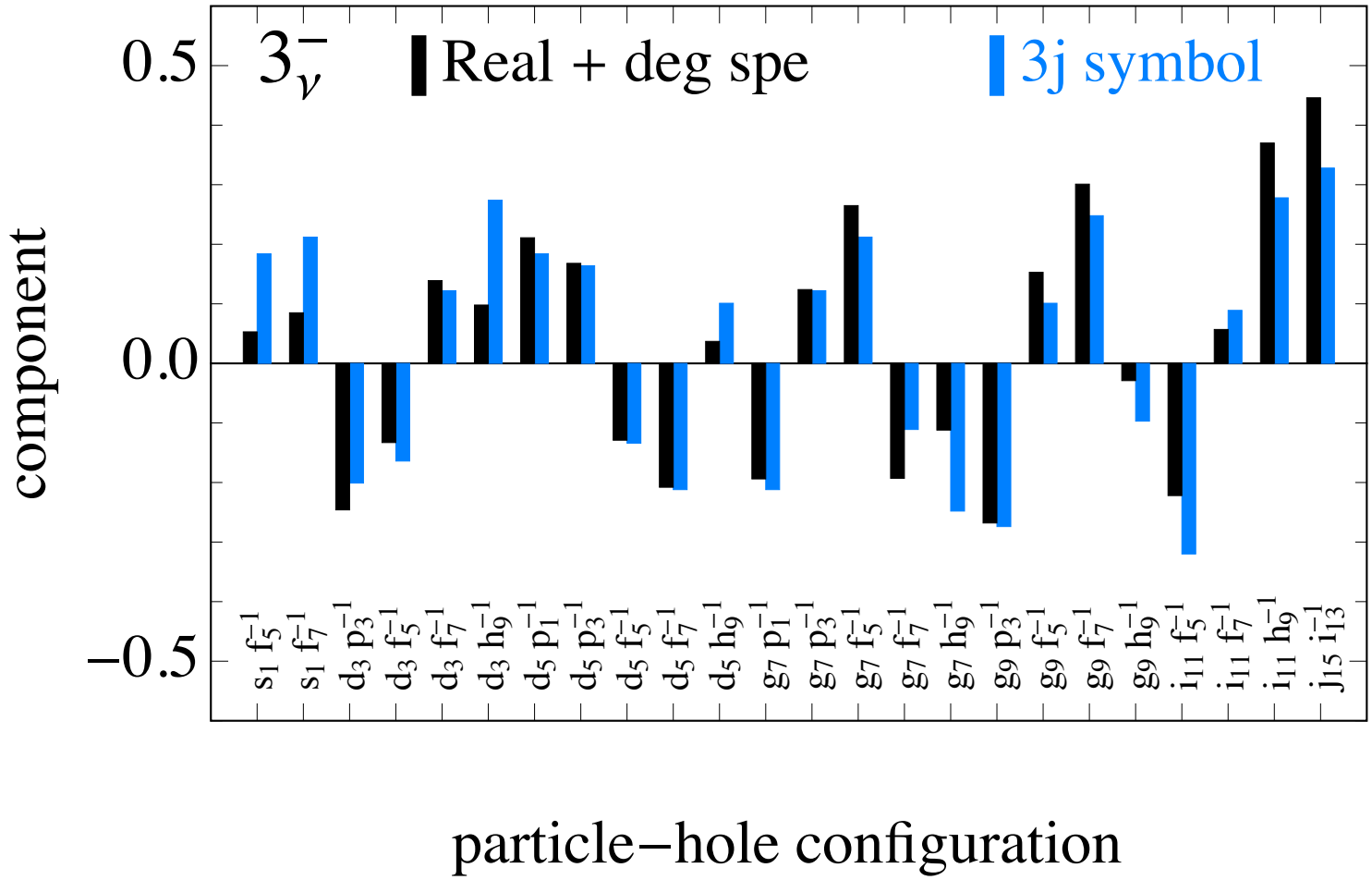

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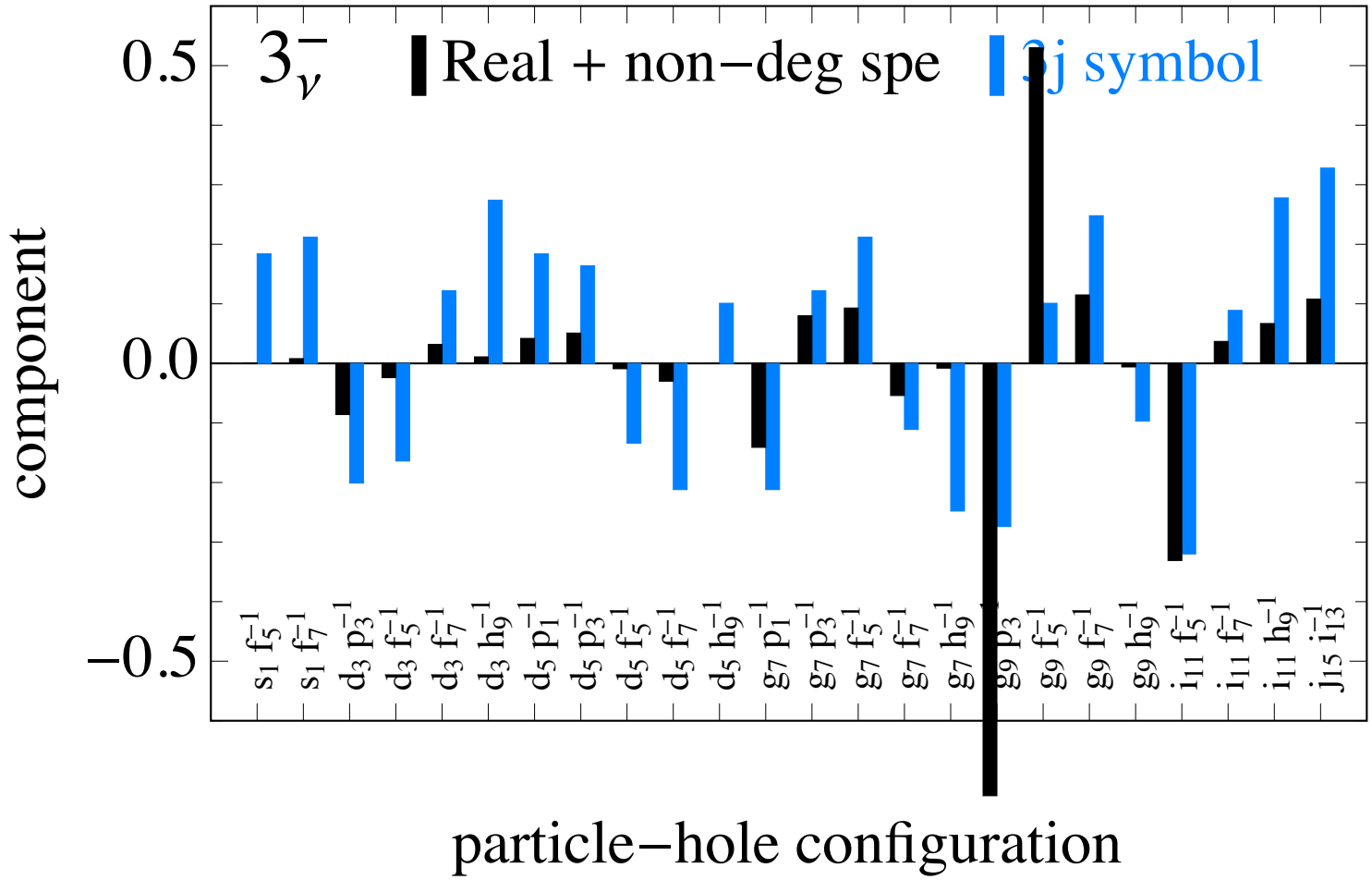
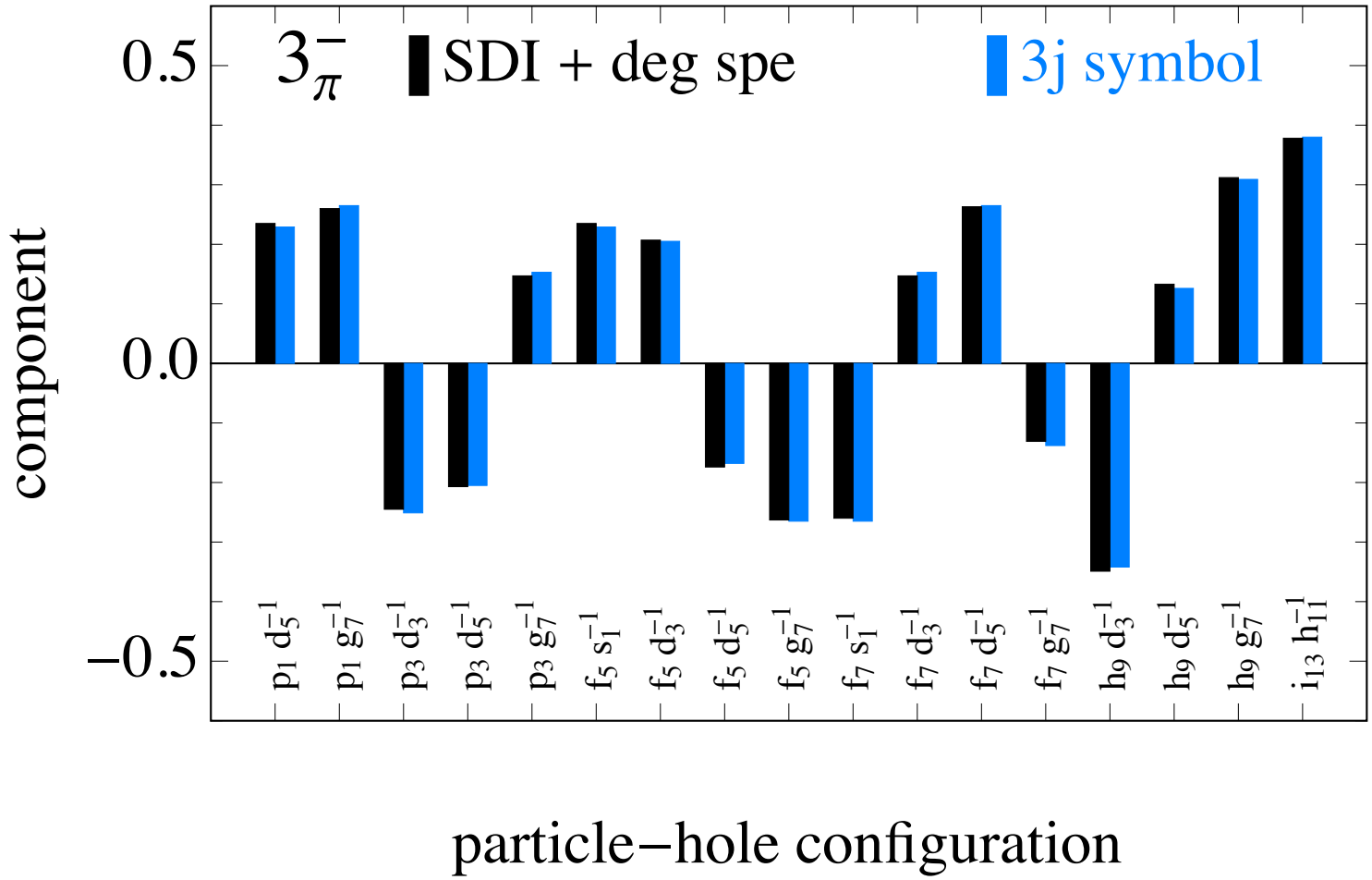


Deconstructing the proton phonon



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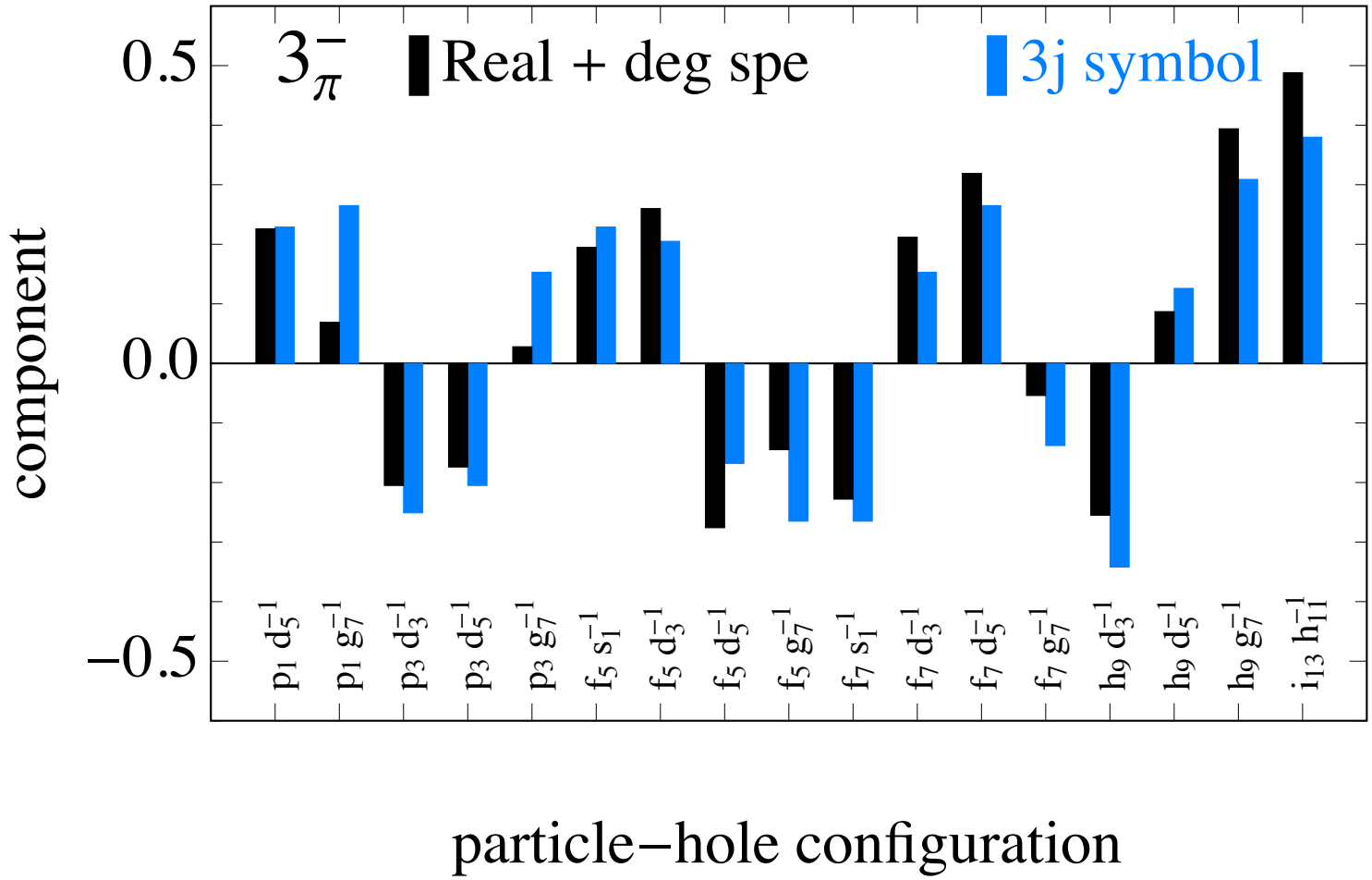

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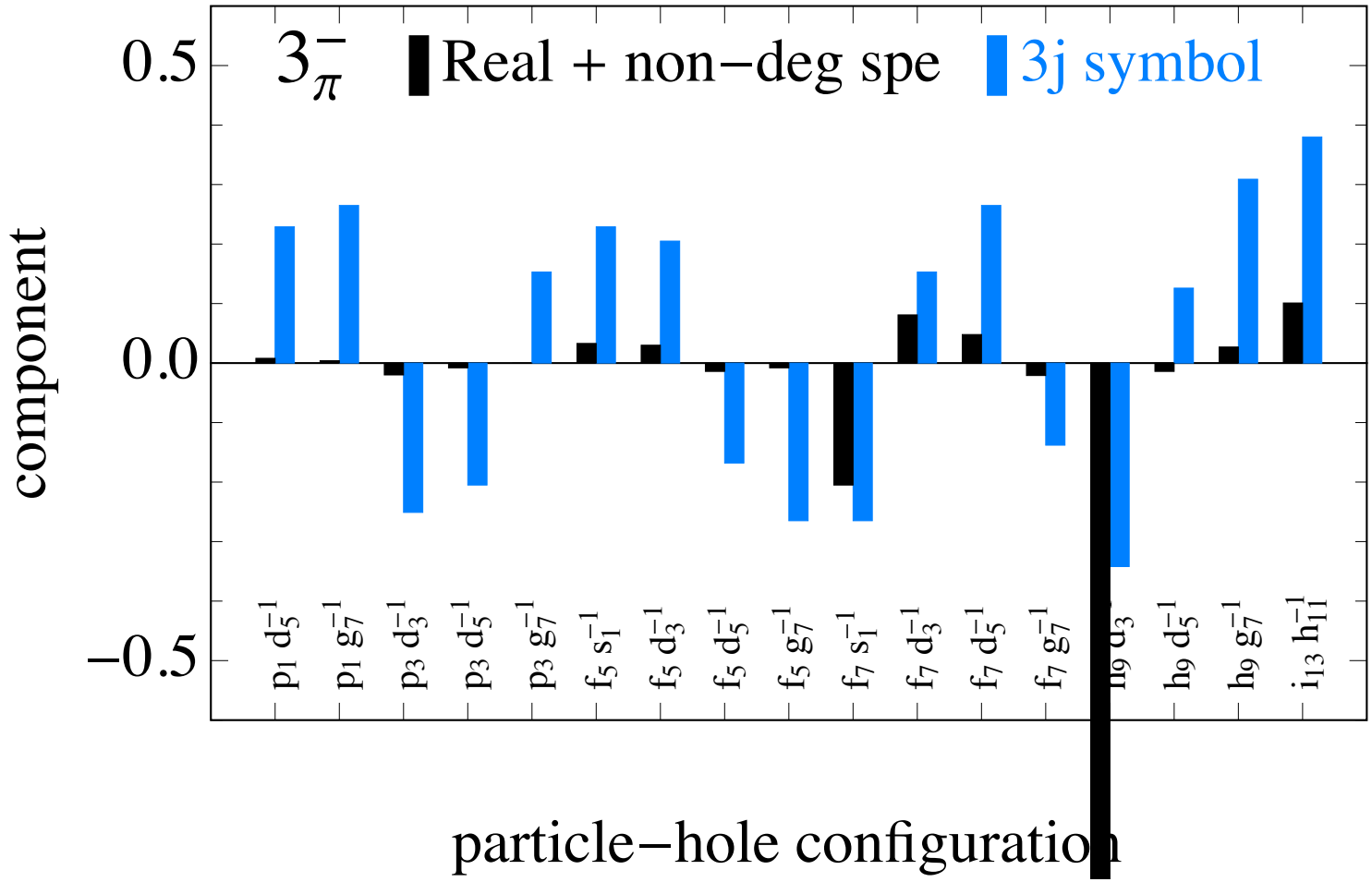
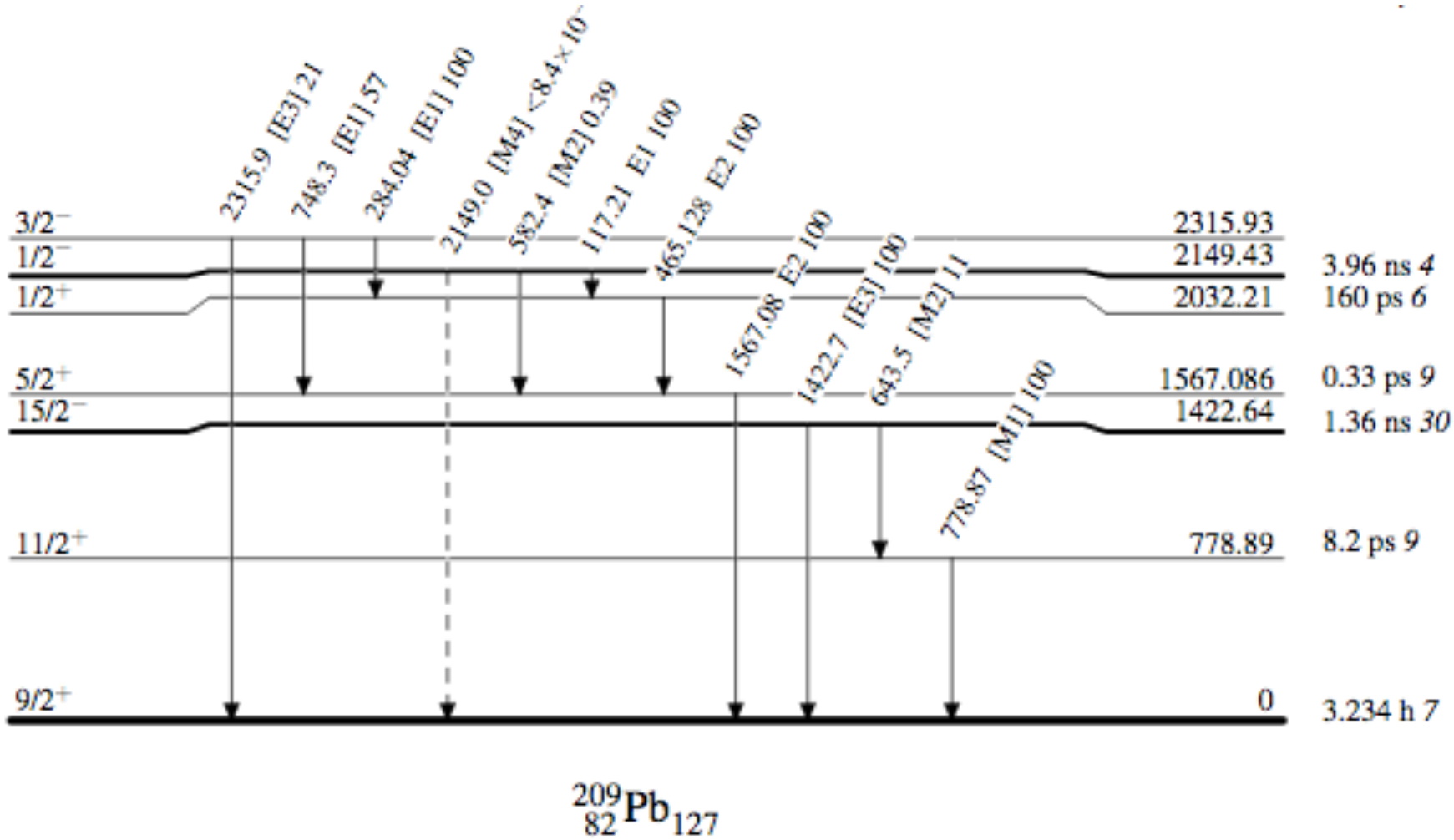
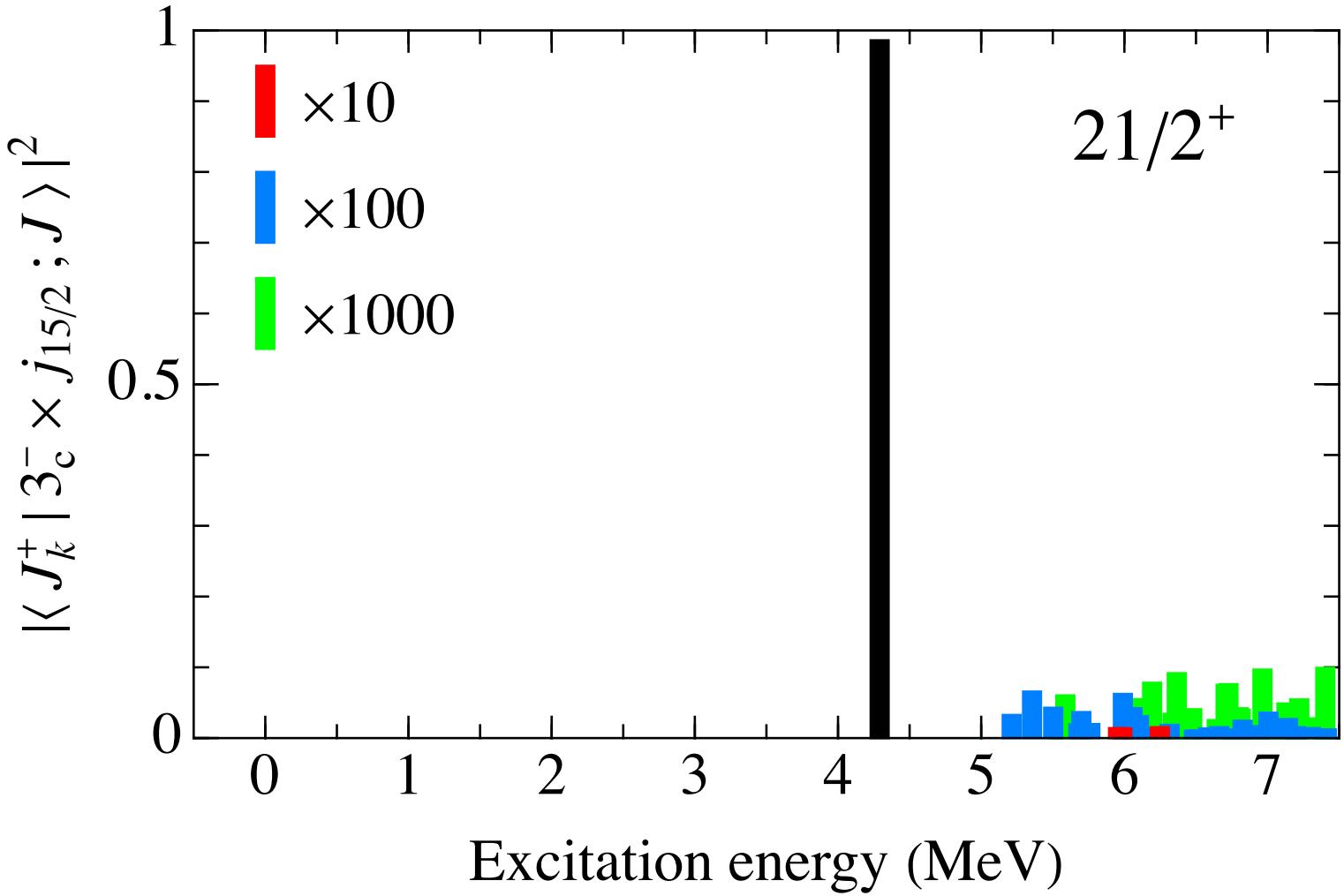


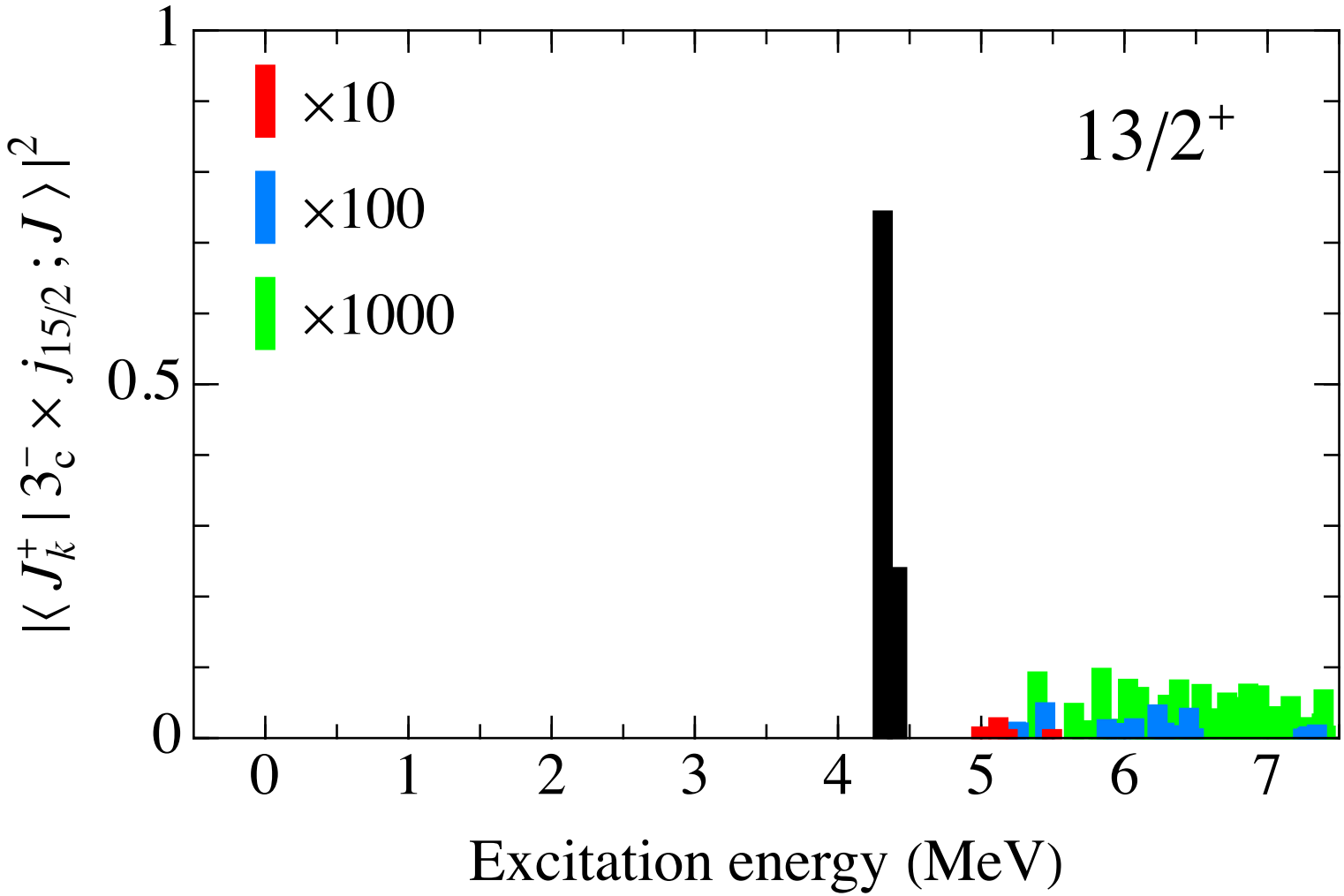
Octupole phonons in ^{209}Pb



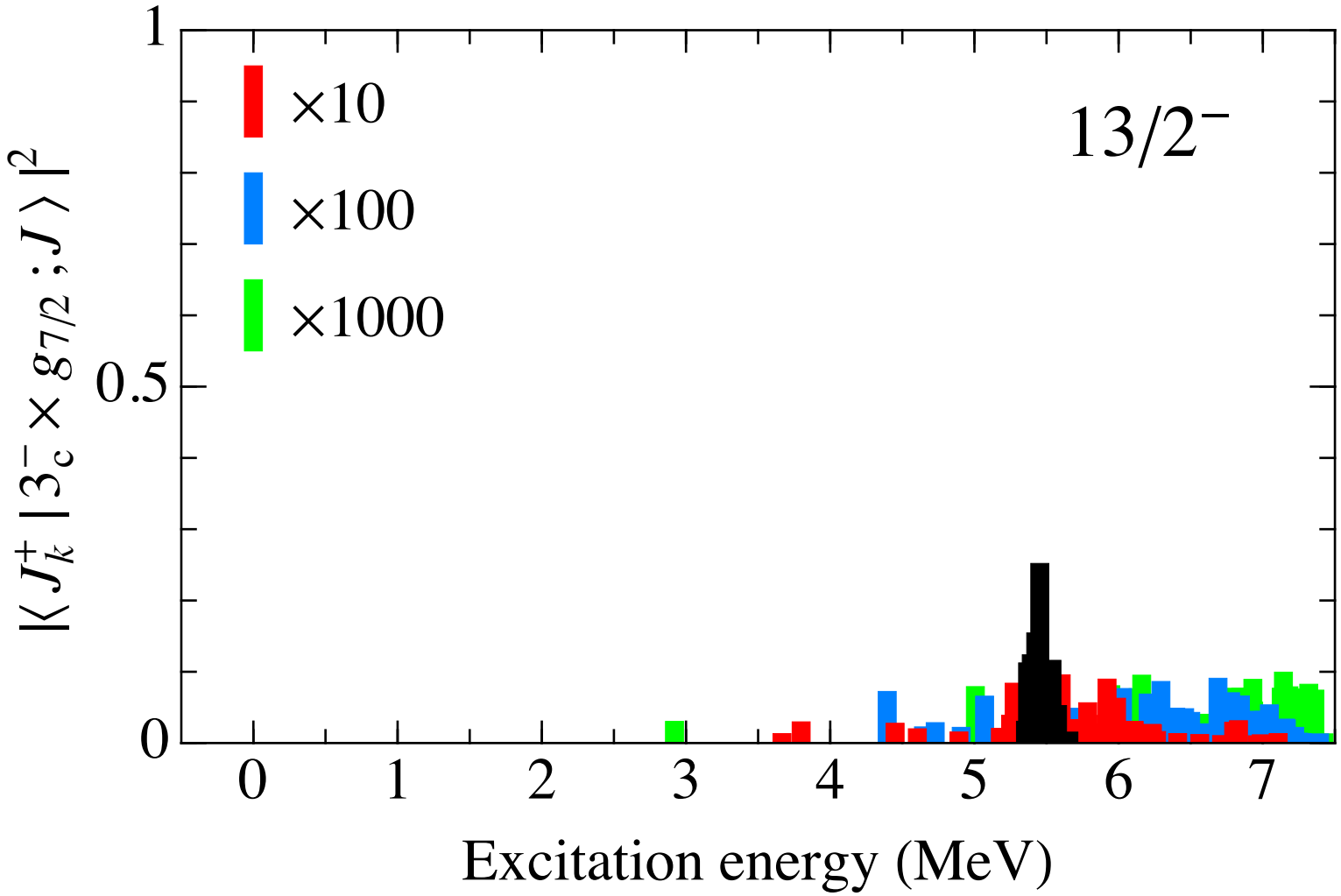
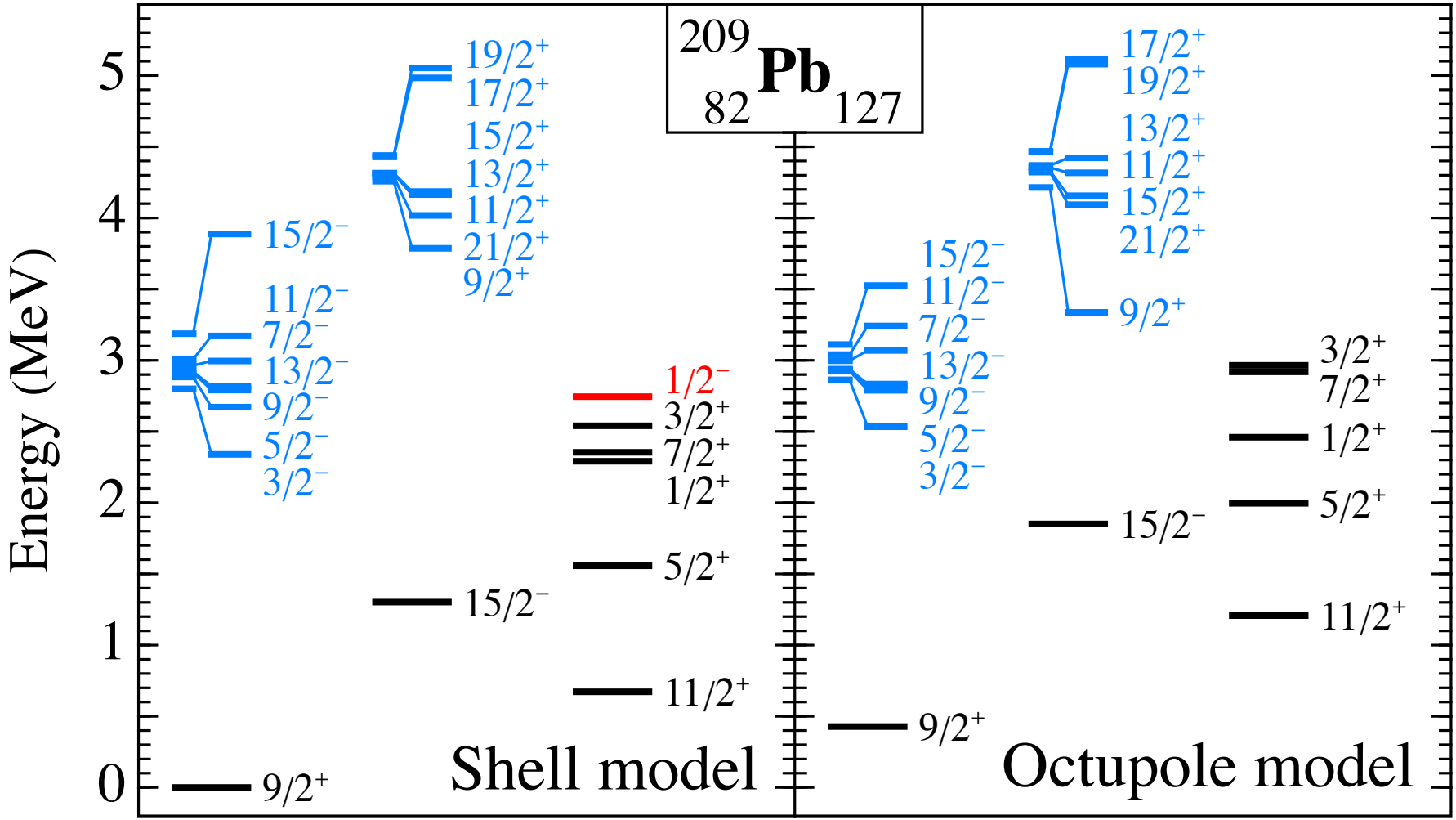
Shell-model analysis of ^{209}Pb



Shell-model analysis of ^{209}Pb



Shell-model analysis of ^{209}Pb



Spectrum of ^{209}Pb