

Quantum Simulation of the Agassi Model in Trapped Ions (part I)

José-Enrique García-Ramos¹,
J.M. Arias², L. Lamata², P. Pérez-Fernández³, and A. Sáiz³

¹Departamento de Ciencias Integradas y Centro de Estudios Avanzados en Física, Matemática y Computación, Universidad de Huelva, Spain

²Dpto. de Física Atómica, Molecular y Nuclear, Facultad de Física, Universidad de Sevilla, Spain

³Dpto. de Física Aplicada III, Escuela Técnica Superior de Ingeniería, Universidad de Sevilla, Spain



10th International workshop on quantum phase transitions in nuclei and many-body systems
Dubrovnik (Croatia), July 11-15 2022

- 1 Motivation
- 2 The Agassi model
- 3 The phase diagram and the different shapes
- 4 Quantum Computing: the mapping
- 5 Quantum Computing: the numerics
- 6 Quantum Computing: the shape
- 7 Conclusions

Why Quantum Computing in Nuclear Physics?

- The U.S. Department of Energy in its document Energy Research Opportunity II (2019) establishes “A broad theory program should be supported, which can, e.g., develop methods to address problems in NP using digital quantum computers and quantum simulators, utilize QIS concepts to better understand nuclear phenomena (such as the nuclear many-body problem and hadronization), and develop new QIS applications of importance to nuclear physics”.
- In the future, quantum computers will allow to outperform present computational (classical) capabilities.
- The applications of Quantum Computing in Nuclear Physics are still scarce.
- At present is a rather active area, but still it is only dealing with small systems and schematic models: Variational Quantum Eigensolver or Quantum Equation of Motion applied to the Lipkin model up to 4 particles.
 - ▶ “Lipkin model on a quantum computer” PRC **104**, 024305 (2021)
 - ▶ “Cloud Quantum Computing of an Atomic Nucleus” PRL **120**, 210501 (2018)
 - ▶ “Simulating excited states of the Lipkin model on a quantum computer” arXiv:2203.01478 (2022)
 - ▶ “Quantum computing of the 6 Li nucleus via ordered unitary coupled cluster” arXiv:2205.00864 (2022)
 - ▶ “QCSH: a Full Quantum Computer Nuclear Shell-Model Package” arXiv:2205.12087 (2022)

Why the Agassi model?

- It is a solvable many-body model that allows to **mimic the main characteristics of the pairing-plus-quadrupole model**.
- It can be exactly solved even in the case of large systems.
- Nowadays, it is used to **benchmark many-body approximations** because of its great flexibility and simplicity to be solved for large systems.
- The model (and in particular its extension) owns a very **rich phase diagram and even presents shape coexistence**.
- The model is, somehow, an extension of the **two-level Lipkin-Meshkov-Glick model that incorporates pairing interaction**.
- It is a model slightly **more complex than the used ones in Quantum Information Science** (e.g, Lipkin, Dicke, Tavis-Cumming or Hubbard models) and, therefore, of great interest.

What a (digital) Quantum Computer is?

A device composed of:

- m **2-level** quantum systems (qubits),
- a set of quantum gates (acting as unitary operators),
- a set of measurement operators (measuring the state of defined subset of qubits),
- a classical control unit which determines which gate should be applied.

I. M. Georgescu, S. Ashhab, and Franco Nori, *Rev. Mod. Phys.* **86**, 153 (2014).

What a (digital) Quantum Computer is?

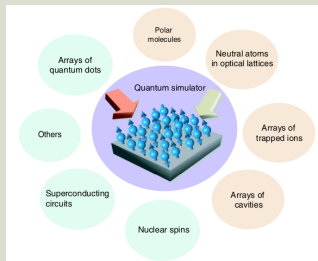
A device composed of:

- m **2-level** quantum systems (qubits),
- a set of quantum gates (acting as unitary operators),
- a set of measurement operators (measuring the state of defined subset of qubits),
- a classical control unit which determines which gate should be applied.

I. M. Georgescu, S. Ashhab, and Franco Nori, *Rev. Mod. Phys.* **86**, 153 (2014).

Implementations

- **Trapped ions.**
- Superconducting circuits.
- Nuclear spins (NMR).
- Photons.
- Neutral atoms.
- Cavity arrays.



- 1 Motivation
- 2 The Agassi model**
- 3 The phase diagram and the different shapes
- 4 Quantum Computing: the mapping
- 5 Quantum Computing: the numerics
- 6 Quantum Computing: the shape
- 7 Conclusions

Agassi model

The first appearance

“Validity of the BCS and RPA approximations in the pairing-plus-monopole solvable model”, Dan Agassi, Nuclear Physics A **116**, 49 (1968).

The original Hamiltonian

$$H = \frac{1}{2}\epsilon \sum_{m\sigma} \sigma a_{m\sigma}^\dagger a_{m\sigma} + \frac{1}{2}V \sum_{mm'\sigma} a_{m\sigma}^\dagger a_{m'\sigma}^\dagger a_{m'-\sigma} a_{m-\sigma} - \frac{1}{4}g \sum_{mm'\sigma\sigma'} a_{m\sigma}^\dagger a_{-m\sigma}^\dagger a_{-m'-\sigma'} a_{m'\sigma'}$$

$\sigma = +1, -1$ and $m = -j, \dots, -2, -1, 1, 2, \dots, j$. Degeneracy $\Omega = 2j$

The O(5) as the spectrum generator algebra

$$J^+ = \sum_{m=-j}^j c_{1m}^\dagger c_{-1m} = (J^-)^\dagger; \quad J^0 = \frac{1}{2} \sum_{m=-j}^j (c_{1m}^\dagger c_{1m} - c_{-1m}^\dagger c_{-1m})$$

$$A_1^\dagger = \sum_{m=1}^j c_{1m}^\dagger c_{1,-m}^\dagger; \quad A_{-1}^\dagger = \sum_{m=1}^j c_{-1m}^\dagger c_{-1,-m}^\dagger; \quad A_0^\dagger = \sum_{m=1}^j (c_{-1m}^\dagger c_{1,-m}^\dagger - c_{-1-m}^\dagger c_{1,m}^\dagger)$$

$$N_\sigma = \sum_{m=-j}^j c_{\sigma m}^\dagger c_{\sigma m}, \quad N = N_1 + N_{-1}$$

The O(5) as the spectrum generator algebra

$$\begin{aligned}
 J^+ &= \sum_{m=-j}^j c_{1m}^\dagger c_{-1m} = (J^-)^\dagger; & J^0 &= \frac{1}{2} \sum_{m=-j}^j (c_{1m}^\dagger c_{1m} - c_{-1m}^\dagger c_{-1m}) \\
 A_1^\dagger &= \sum_{m=1}^j c_{1m}^\dagger c_{1,-m}^\dagger; & A_{-1}^\dagger &= \sum_{m=1}^j c_{-1m}^\dagger c_{-1,-m}^\dagger; & A_0^\dagger &= \sum_{m=1}^j (c_{-1m}^\dagger c_{1,-m}^\dagger - c_{-1-m}^\dagger c_{1,m}^\dagger) \\
 N_\sigma &= \sum_{m=-j}^j c_{\sigma m}^\dagger c_{\sigma m}, & N &= N_1 + N_{-1}
 \end{aligned}$$

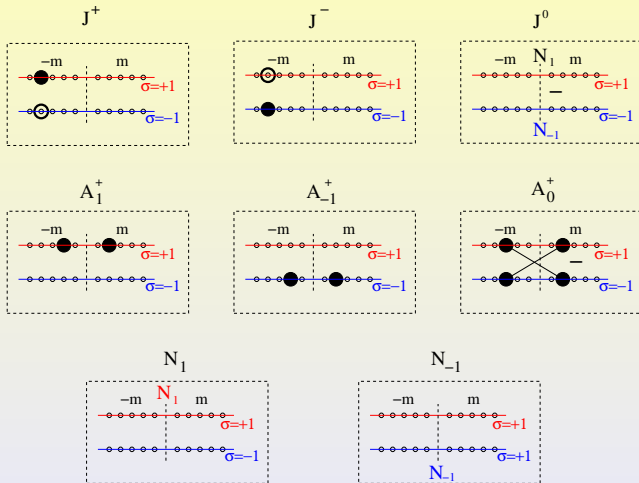
The Hamiltonian

$$H = \varepsilon J^0 - g \sum_{\sigma\sigma'} A_\sigma^\dagger A_{\sigma'} - \frac{V}{2} \left[(J^+)^2 + (J^-)^2 \right] - 2h A_0^\dagger A_0$$

For convenience

$$V = \frac{\varepsilon\chi}{2j-1}, \quad g = \frac{\varepsilon\Sigma}{2j-1}, \quad h = \frac{\varepsilon\Lambda}{2j-1}$$

A pictorial view



Outline

- 1 Motivation
- 2 The Agassi model
- 3 The phase diagram and the different shapes**
- 4 Quantum Computing: the mapping
- 5 Quantum Computing: the numerics
- 6 Quantum Computing: the shape
- 7 Conclusions

The phases of the system

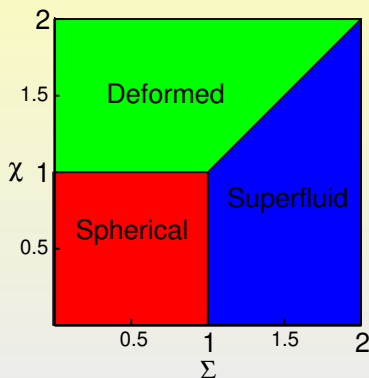
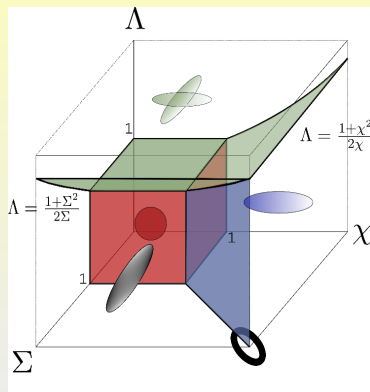
- The spherical phase: $\varphi = 0$ and $\beta = 0$.
- The Hartree-Fock deformed phase: $\varphi \neq 0$ and $\beta = 0$.
- The BCS deformed phase: $\varphi = 0$ and $\beta \neq 0$.
- The Hartree-Fock plus BCS deformed phase: $\varphi \neq 0$ and $\beta \neq 0$.

The phases of the system

- The spherical phase: $\varphi = 0$ and $\beta = 0$.
- The Hartree-Fock deformed phase: $\varphi \neq 0$ and $\beta = 0$.
- The BCS deformed phase: $\varphi = 0$ and $\beta \neq 0$.
- The Hartree-Fock plus BCS deformed phase: $\varphi \neq 0$ and $\beta \neq 0$.

In the original formulation of the Agassi model only the three first phases were present, but in the extended version of the model the four basis can be found and, moreover, **there is coexistence of some of the phases.**

The phase diagram



Phase transition for the extended and simple Agassi model

(JEGR, J. Dukelsky, P. Pérez-Fernández, and J. M. Arias, PRC **97**, 054303 (2018))

Numerical calculations

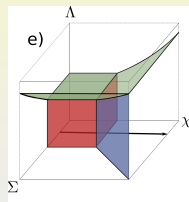
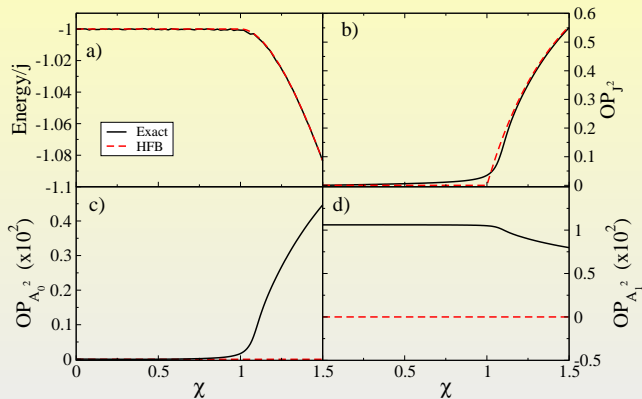


Figure: Comparison of HFB and exact results. $j = 100$ and Hamiltonian parameters $\Sigma = 0.5$, $\Lambda = 0$.

Numerical calculations

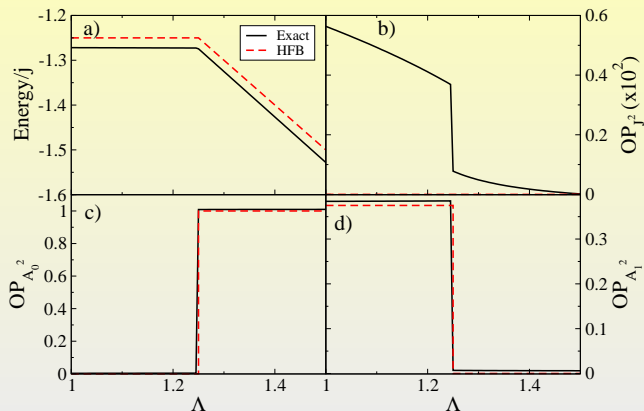


Figure: Comparison of HFB and exact results. $j = 100$ and Hamiltonian parameters $\chi = 1.5$, $\Sigma = 2$.

Outline

- 1 Motivation
- 2 The Agassi model
- 3 The phase diagram and the different shapes
- 4 Quantum Computing: the mapping**
- 5 Quantum Computing: the numerics
- 6 Quantum Computing: the shape
- 7 Conclusions

The Jordan-Wigner transformation

- It is a non-local transformation that maps the fermion creation/annihilation operators into Pauli matrices.
- It is usual to relabel the fermion index, i.e., $\sigma, m \rightarrow i$.

The transformation

$$\begin{aligned}c_i^\dagger &= I_1 \otimes \dots \otimes I_{i-1} \otimes \sigma_i^+ \otimes \sigma_{i+1}^z \otimes \dots \otimes \sigma_N^z, \\c_i &= I_1 \otimes \dots \otimes I_{i-1} \otimes \sigma_i^- \otimes \sigma_{i+1}^z \otimes \dots \otimes \sigma_N^z,\end{aligned}$$

with

$$\sigma^+ = \frac{\sigma^x + i\sigma^y}{2}, \sigma^- = \frac{\sigma^x - i\sigma^y}{2},$$

and

$$\sigma^x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma^y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma^z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},$$

The case of 4 sites, $j = 1$

The mapping of the building blocks

$$C_{1,1} \rightarrow C_1,$$

$$C_{1,-1} \rightarrow C_2,$$

$$C_{-1,1} \rightarrow C_3,$$

$$C_{-1,-1} \rightarrow C_4.$$

$$J^+ = -\sigma_2^+ \otimes \sigma_3^z \otimes \sigma_4^- - \sigma_1^+ \otimes \sigma_2^z \otimes \sigma_3^-,$$

$$J^0 = (1/4)(\sigma_1^z + \sigma_2^z - \sigma_3^z - \sigma_4^z),$$

$$J^- = (J^+)^{\dagger} = -\sigma_2^- \otimes \sigma_3^z \otimes \sigma_4^+ - \sigma_1^- \otimes \sigma_2^z \otimes \sigma_3^+,$$

$$A_1^{\dagger} = \sigma_1^+ \otimes \sigma_2^+, \quad A_{-1}^{\dagger} = \sigma_3^+ \otimes \sigma_4^+,$$

$$A_1 = \sigma_1^- \otimes \sigma_2^-, \quad A_{-1} = \sigma_3^- \otimes \sigma_4^-.$$

The case of 4 sites, $j = 1$

The mapping of the building blocks

$$\begin{aligned} C_{1,1} &\rightarrow C_1, & J^+ &= -\sigma_2^+ \otimes \sigma_3^z \otimes \sigma_4^- - \sigma_1^+ \otimes \sigma_2^z \otimes \sigma_3^-, \\ C_{1,-1} &\rightarrow C_2, & J^0 &= (1/4)(\sigma_1^z + \sigma_2^z - \sigma_3^z - \sigma_4^z), \\ C_{-1,1} &\rightarrow C_3, & J^- = (J^+)^{\dagger} &= -\sigma_2^- \otimes \sigma_3^z \otimes \sigma_4^+ - \sigma_1^- \otimes \sigma_2^z \otimes \sigma_3^+, \\ C_{-1,-1} &\rightarrow C_4. & A_1^{\dagger} &= \sigma_1^+ \otimes \sigma_2^+, \quad A_{-1}^{\dagger} = \sigma_3^+ \otimes \sigma_4^+, \\ & & A_1 &= \sigma_1^- \otimes \sigma_2^-, \quad A_{-1} = \sigma_3^- \otimes \sigma_4^-. \end{aligned}$$

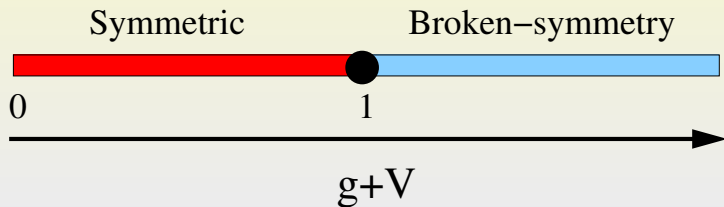
The Hamiltonian

$$\begin{aligned} H &= H_1 + H_2 + H_3, \\ H_1 &= \frac{\epsilon - g}{4}(\sigma_1^z + \sigma_2^z) - \frac{\epsilon + g}{4}(\sigma_3^z + \sigma_4^z), \\ H_2 &= -\frac{g}{4}(\sigma_1^z \otimes \sigma_2^z + \sigma_3^z \otimes \sigma_4^z), \\ H_3 &= -(g + V)(\sigma_1^+ \otimes \sigma_2^+ \otimes \sigma_3^- \otimes \sigma_4^- + \sigma_1^- \otimes \sigma_2^- \otimes \sigma_3^+ \otimes \sigma_4^+). \end{aligned}$$

$$[H_1, H_2] = 0, \quad [H_2, H_3] = 0, \quad [H_1, H_3] \neq 0.$$

The phase diagram (1D) for 4 sites, $j = 1$

- For $j = 1 \Rightarrow N = 4$ sites: $g = \Sigma$ and $V = \chi$
- The hamiltonian only depends on $g + V$
- Two phases: symmetric $g + V < 1$ and broken symmetry $g + V > 1$



Outline

- 1 Motivation
- 2 The Agassi model
- 3 The phase diagram and the different shapes
- 4 Quantum Computing: the mapping
- 5 Quantum Computing: the numerics**
- 6 Quantum Computing: the shape
- 7 Conclusions

What do we measure?

The evolution operator

$$U(t) = \exp(-i H t)$$

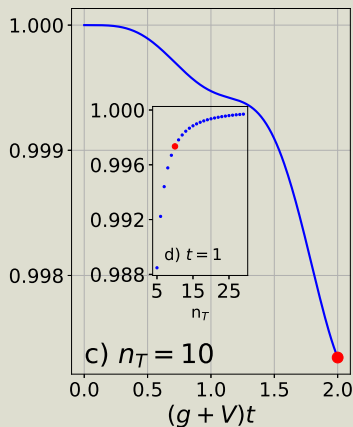
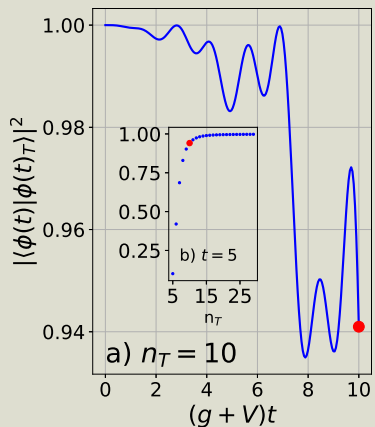
Experimentally it is implemented through the **Lie-Trotter-Suzuki decomposition** (Trotter in short)

$$U(t) \simeq \{\exp[-i(H_1 + H_2)(t/n_T)] \exp[-iH_3(t/n_T)]\}^{n_T},$$

where the error produced will depend on the commutator $[(H_1 + H_2), H_3]$ and scale as $1/n_T$, where n_T denotes the number of Trotter steps.

How good is the Trotter approach?

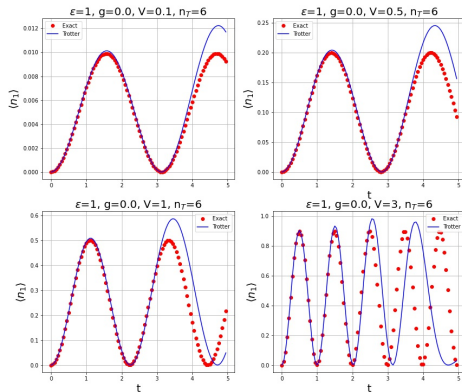
The fidelity



The initial state is $|\downarrow_1 \otimes \downarrow_2 \otimes \uparrow_3 \otimes \uparrow_4\rangle$ (with minimum value of $\langle \mathcal{J}^0 \rangle = -1$). The parameters of the Hamiltonian are $\epsilon = 1$ and $g = V = 1$.

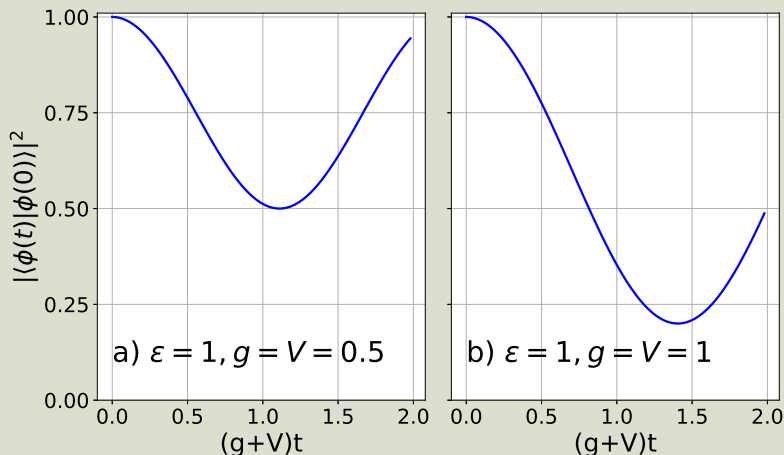
How good is the Trotter approach?

The value of $\langle n_1 \rangle$



The initial state is $|\downarrow_1 \otimes \downarrow_2 \otimes \uparrow_3 \otimes \uparrow_4\rangle$ (with minimum value of $\langle \mathcal{J}^0 \rangle = -1$).

The survival probability



The initial state is $|\downarrow_1 \otimes \downarrow_2 \otimes \uparrow_3 \otimes \uparrow_4\rangle$ (with minimum value of $\langle J^0 \rangle = -1$).

Feasibility

- $\exp(-iH_1 t)$: single-qubit gates with fidelities often above 99.99% (in trapped ions).
- $\exp(-iH_2 t)$: two two-qubit gates carried out via Mølmer-Sørensen gates with fidelities above 99.9%, plus single-qubit gates to rotate the basis from X to Z .
- $\exp(-iH_3 t)$: two Mølmer-Sørensen gates and a local gate, plus single qubit gates to rotate the bases. All the terms of H_3 are implemented with a single Trotter step.

Feasibility

- $\exp(-iH_1 t)$: single-qubit gates with fidelities often above 99.99% (in trapped ions).
- $\exp(-iH_2 t)$: two two-qubit gates carried out via Mølmer-Sørensen gates with fidelities above 99.9%, plus single-qubit gates to rotate the basis from X to Z .
- $\exp(-iH_3 t)$: two Mølmer-Sørensen gates and a local gate, plus single qubit gates to rotate the bases. All the terms of H_3 are implemented with a single Trotter step.

- **The scaling of our protocol is efficient**: the number of elementary gates is polynomial in the number of interacting fermions, N .
- **With a classical computer the scaling would be inefficient***: the Hilbert space dimension would grow exponentially in N .
- 4-qubit proposal: 52 single-qubit gates and 50 two-qubits gates. Assuming gate errors of 0.0001 for the single-qubit and 0.001 for the two-qubit one, the total gate error, assuming $n_T = 5$, with be $E_G \simeq 0.28$ (fidelity above 70%).

Outline

- 1 Motivation
- 2 The Agassi model
- 3 The phase diagram and the different shapes
- 4 Quantum Computing: the mapping
- 5 Quantum Computing: the numerics
- 6 Quantum Computing: the shape**
- 7 Conclusions

Is it possible to determine the shape/phase of the system?

The obvious things

- Shape is not really an observable.
- The shape of the system is a property of its ground state (it is true that it can be also defined for a excited state).
- It is well defined at the mean-field level.

Is it possible to determine the shape/phase of the system?

The obvious things

- Shape is not really an observable.
- The shape of the system is a property of its ground state (it is true that it can be also defined for a excited state).
- It is well defined at the mean-field level.

A different view

- The shape of the system characterizes its spectrum.

Is it possible to determine the shape/phase of the system?

The obvious things

- Shape is not really an observable.
- The shape of the system is a property of its ground state (it is true that it can be also defined for a excited state).
- It is well defined at the mean-field level.

A different view

- The shape of the system characterizes its spectrum.
- **An observable depending on the spectrum could encode the shape of the system. That, in general, will happen for the time evolution of the matrix element of a non-eigenstate.**

Is it possible to determine the shape/phase of the system?

The obvious things

- Shape is not really an observable.
- The shape of the system is a property of its ground state (it is true that it can be also defined for a excited state).
- It is well defined at the mean-field level.

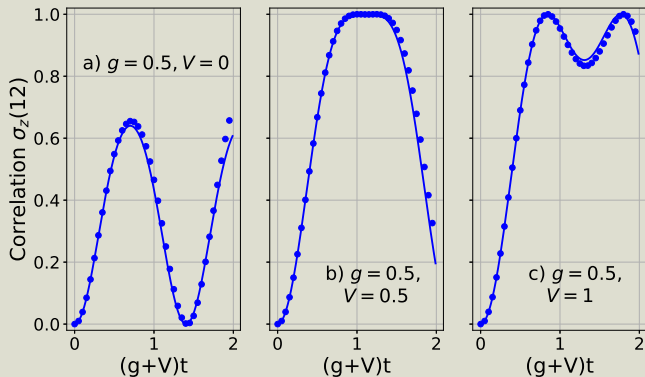
A different view

- The shape of the system characterizes its spectrum.
- **An observable depending on the spectrum could encode the shape of the system. That, in general, will happen for the time evolution of the matrix element of a non-eigenstate.**
- Most probably the results will depend of the state and on the used operator. Difficult to determine a priori the best state and operator.
- These types of measurements are the easiest ones in Quantum Computing.

Is it possible to determine the phase of the system?

The correlation function

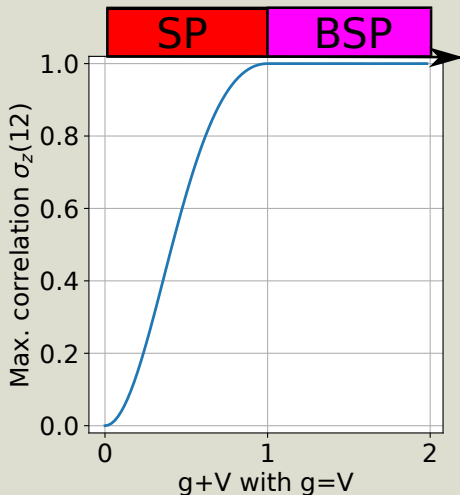
$$\sigma_z(1,2) \equiv \langle \sigma_1^z \sigma_2^z \rangle - \langle \sigma_1^z \rangle \langle \sigma_2^z \rangle$$



The initial state is $|\downarrow_1 \otimes \downarrow_2 \otimes \uparrow_3 \otimes \uparrow_4\rangle$ (exact and Trotter results).

Is it possible to determine the phase of the system?

The maximum value of the correlation function as an “order parameter”



Outline

- 1 Motivation
- 2 The Agassi model
- 3 The phase diagram and the different shapes
- 4 Quantum Computing: the mapping
- 5 Quantum Computing: the numerics
- 6 Quantum Computing: the shape
- 7 Conclusions**

Conclusions and outlook

- It has been proposed and analyzed the quantum simulation of the Agassi model for a size $N = 4$.
- Numerical simulations and analytical estimations show that this protocol is feasible with current technology.
- **Time dynamics of a quantum correlation function allows to determine the different quantum phases of the model.**
- In a further work, the analysis has been extended up to $N = 8$ and to the full fledged Agassi model (see **Pedro Pérez-Fernández talk**).

Further reading

- P. Pérez-Fernández, J.M. Arias, J.E. García-Ramos, and L. Lamata, “A digital quantum simulation of the Agassi model”, *Physics Letters B* **829**, 137133 (2022).
- A. Sáiz, J.E. García-Ramos, J.M. Arias, L. Lamata, and P. Pérez-Fernández, “Quantum Simulations of an Extended Agassi Model in Trapped Ions using Machine Learning”, arXiv:2205.15122.

Thank you for your attention