What Energy Density Functionals can tell us on Partial Dynamical Symmetries in Nuclei

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Dynamical Symmetry

$$G_{\rm dyn} \supset G \supset \cdots \supset G_{\rm sym} \qquad |[N]\langle\Sigma\rangle\Lambda\rangle \qquad \hat{H} = \sum_G a_G \hat{C}_G$$

- Complete solvability
- Good quantum numbers for all states

$$E = E_{[N]\langle\Sigma\rangle\ldots\Lambda}$$

$U(6) \supset U(5) \supset SO(5) \supset SO(3)$	$\vert [N]\ n_d\ \tau\ n_\Delta\ L \rangle$	Spherical vibrator
$U(6) \supset SU(3) \supset SO(3)$	$\vert [N]\ (\lambda, \mu)\ K\ L \rangle$	Axial rotor
$U(6) \supset SO(6) \supset SO(5) \supset SO(3)$	$\vert [N]\ \sigma\ \tau\ n_\Delta\ L \rangle$	γ-unstable rotor

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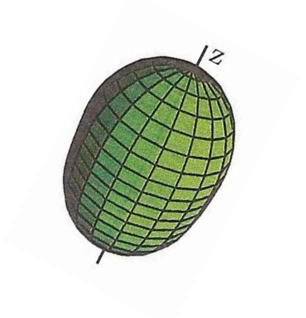
- Geometry

$$|\beta,\gamma;N\rangle = (N!)^{-1/2}(b_c^\dagger)^N|0\rangle$$

$$b_c^\dagger = (1+\beta^2)^{-1/2}\left[\beta\cos\gamma\, d_0^\dagger + \beta\sin\gamma\frac{1}{\sqrt{2}}\left(d_2^\dagger + d_{-2}^\dagger\right) + s^\dagger\right]$$



Energy surface $\qquad E_N(\beta,\gamma) = \langle\beta,\gamma;N|\hat{H}|\beta,\gamma;N\rangle$

Global min: equilibrium shape (β_0,γ_0)

Intrinsic state ground band $|\beta_0,\gamma_0;N\rangle$; Lowest weight state in a particular G-irrep $\langle\Sigma_0\rangle$

U(5)	$\beta_0 = 0$	$n_d = 0$
SU(3)	$(\beta_0 = \sqrt{2}, \gamma_0 = 0)$	$(\lambda,\mu) = (2N,0)$
SO(6)	$(\beta_0 = 1, \gamma_0$ arbitrary$)$	$\sigma = N$

Dynamical Symmetry

$$G_{\rm dyn} \supset \; G \; \supset \cdots \supset G_{\rm sym}$$

- Complete solvability
- Good quantum numbers for all states

$$E = E_{[N]\langle\Sigma\rangle\ldots\Lambda}$$

$$|[N]\langle\Sigma\rangle\Lambda\rangle$$

Exact DS provides considerable insight, however, it is **broken** in most nuclei

Partial Dynamical Symmetry

- Some states solvable and/or with good quantum numbers

Leviatan, Prog. Part. Nucl. Phys. **66**, 93 (2011)

Construction of Hamiltonians with PDS

$$G_{\text{dyn}} \supset G \supset \cdots \supset G_{\text{sym}}$$

$[N]$ $\langle\Sigma\rangle$ Λ

n-particle annihilation operator

$$\hat{T}_{[n]\langle\sigma\rangle\lambda}\,|\,[N]\,\langle\Sigma_0\rangle\,\Lambda\rangle = 0$$

for **all** possible Λ contained in the irrep $\langle\Sigma_0\rangle$ of G

Equivalently:

$$\hat{T}_{[n]\langle\sigma\rangle\lambda}\,|\,[N]\,\langle\Sigma_0\rangle\,\rangle = 0$$

| **Lowest weight state** 〉

- Condition is satisfied if $\langle\sigma\rangle\otimes\langle\Sigma_0\rangle \notin [N-n]$

$$\hat{H} = \sum_{\alpha,\beta} u_{\alpha\beta}\hat{T}^{\dagger}_{\alpha}\hat{T}_{\beta}$$

DS is **broken** but
solvability of states with $\langle\Sigma\rangle = \langle\Sigma_0\rangle$ Is **preserved**

Garcia-Ramos, Leviatan, Van Isacker, PRL **102**, 112502 (2009)

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- PDS Hamiltonian $H_{\rm PDS} = \hat{H} + \hat{H}_{\rm c}$ Intrinsic collective resolution

Intrinsic part: $H \,|\, [N]\, \langle\Sigma_0\rangle\, \Lambda\rangle = 0$

Collective part: H_c composed of Casimir operators of conserved $G_i \subset G$ in the chain

SU(3) PDS

U(6) ⊃ SU(3) ⊃ SO(3)

[N] **(λ,μ)** **K** **L**

$$\hat{T}^{\dagger}_{[n](\lambda,\mu)\ell m} \qquad \left.\begin{aligned} P^{\dagger}_{0} &= d^{\dagger}\cdot d^{\dagger} - 2\,(s^{\dagger})^{2} \\ P^{\dagger}_{2,\mu} &= 2\,s^{\dagger}d^{\dagger}_{\mu} + \sqrt{7}(d^{\dagger}d^{\dagger})^{(2)}_{\mu} \end{aligned}\right\}$$

(λ,μ) = (0,2)

(2,0) ⊗ (2N,0) ∉ [N-2]

$$P_{\ell,\mu}|[N](2N,0)L\rangle \;=\; 0$$

(λ,μ) = (2N,0) **K**=0

$$P_0|[N](2N-4k,2k),K=2k,L\rangle = 0 \qquad k=1,2,\ldots$$

(λ,μ) = (2N-4k,2k,0) **K**=2k

$$H \;=\; h_0 P^{\dagger}_0 P_0 + h_2 P^{\dagger}_2\cdot\tilde{P}_2$$

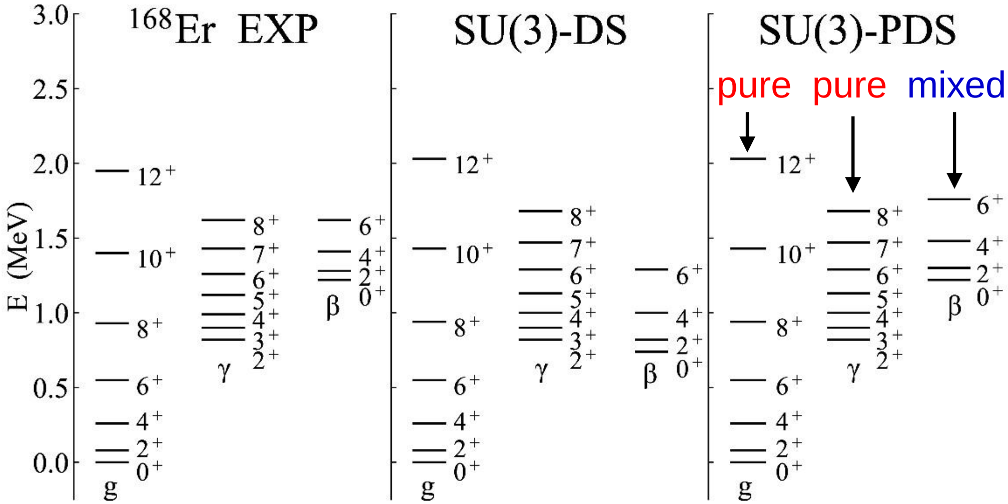
(λ,μ) = (0,0)⊕(2,2)

$$H(h_0=h_2) = \left[-\hat{C}_{SU(3)} + 2\hat{N}(2\hat{N}+3)\right]$$

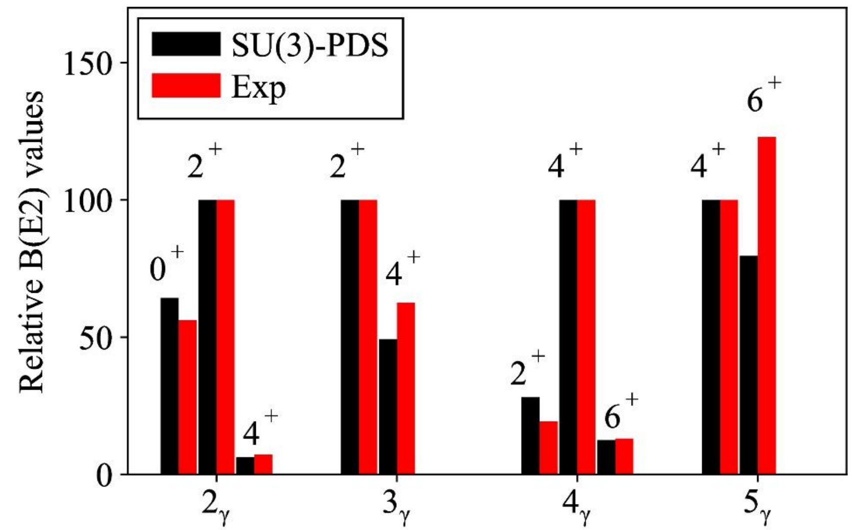
SU(3) PDS $H_{PDS} = h_0\,P^{\dagger}_0 P_0 + h_2\,P^{\dagger}_2\cdot\tilde{P}_2 + \rho\,\hat{C}_2[\mathrm{SO}(3)]$

- **Solvable** bands: g(**K**=0) , γ^k(**K**=2k) **good SU(3) symmetry (2N-4k,2k)**
- Other bands: **mixed** e.g. β(**K**=0)

SU(3) PDS in ^{168}Er



B(E2) branching ratios from states in the γ band



- Solvable g and γ bands: SU(3)-pure 100 %
- Non-solvable states: strongly mixed ~ 13%
- Extensive empirical tests of SU(3)-PDS

  Casten, Cakirli, Blaum, Couture (2014)

→γg ratios: **parameter-free** predictions

$$T(E2) = \alpha\,\hat{Q} + \theta\,(d^\dagger s + \tilde{d} s)$$

$H_{PDS}(h_0,h_2,\rho)$

h_0, h_2, ρ, determined from a fit to $E(0_2)$, $E(2_2)$, $E(2_1)$

*Leviatan, PRL **66**, 818 (1996)*

Various types of PDSs have been identified in nuclei

- Phenomenological grounds

  - Introduce H_{PDS}
  - Determine its parameters from a fit
  - Compare PDS predictions (often parameter-free) with empirical data

Emergent Symmetry: does not arise from invariance properties of the Hamiltonian

- Microscopic justification

Light nuclei:
Emergent Sp(3,R) DS from realistic nn interactions (Dytrich, Launey, Caprio, Draayer 2020)
Unmixing symmetries SU(3) QDS $\rightarrow$ DS from SRG sd shell model interaction (Johnson 2020)

Heavy nuclei:
SU(3) PDS from energy density functionals (This talk)

Algebraic framework

$$\hat{H} = h_0 P_0^\dagger(\beta_0) P_0(\beta_0) + h_2\, P_2^\dagger(\beta_0)\cdot \tilde{P}_2(\beta_0) + \rho\, \hat{L}\cdot\hat{L}.$$

$$P_0^\dagger(\beta_0) = d^\dagger\cdot d^\dagger - \beta_0^2 (s^\dagger)^2$$

$$P_{2m}^\dagger(\beta_0) = \beta_0\sqrt{2}\, d_m^\dagger s^\dagger + \sqrt{7}\,(d^\dagger\, d^\dagger)_m^{(2)}$$

$$E_{\mathrm{IBM}}(\tilde{\beta},\gamma) = N(N-1)(1+\tilde{\beta}^2)^{-2}[\,h_0(\tilde{\beta}^2-\beta_0^2)^2 + 2h_2\tilde{\beta}^2(\tilde{\beta}^2 - 2\beta_0\tilde{\beta}\cos 3\gamma + \beta_0^2)\,]$$

Global min: (β_0,γ_0=0) prolate deformed shape

{h_0, h_2, β_0} affect $E_{\mathrm{IBM}}(\tilde{\beta},\gamma)$ and band structure

ρ affects rotational splitting

$$\epsilon_\beta = 2N\beta_0^2(2h_0 + h_2)$$

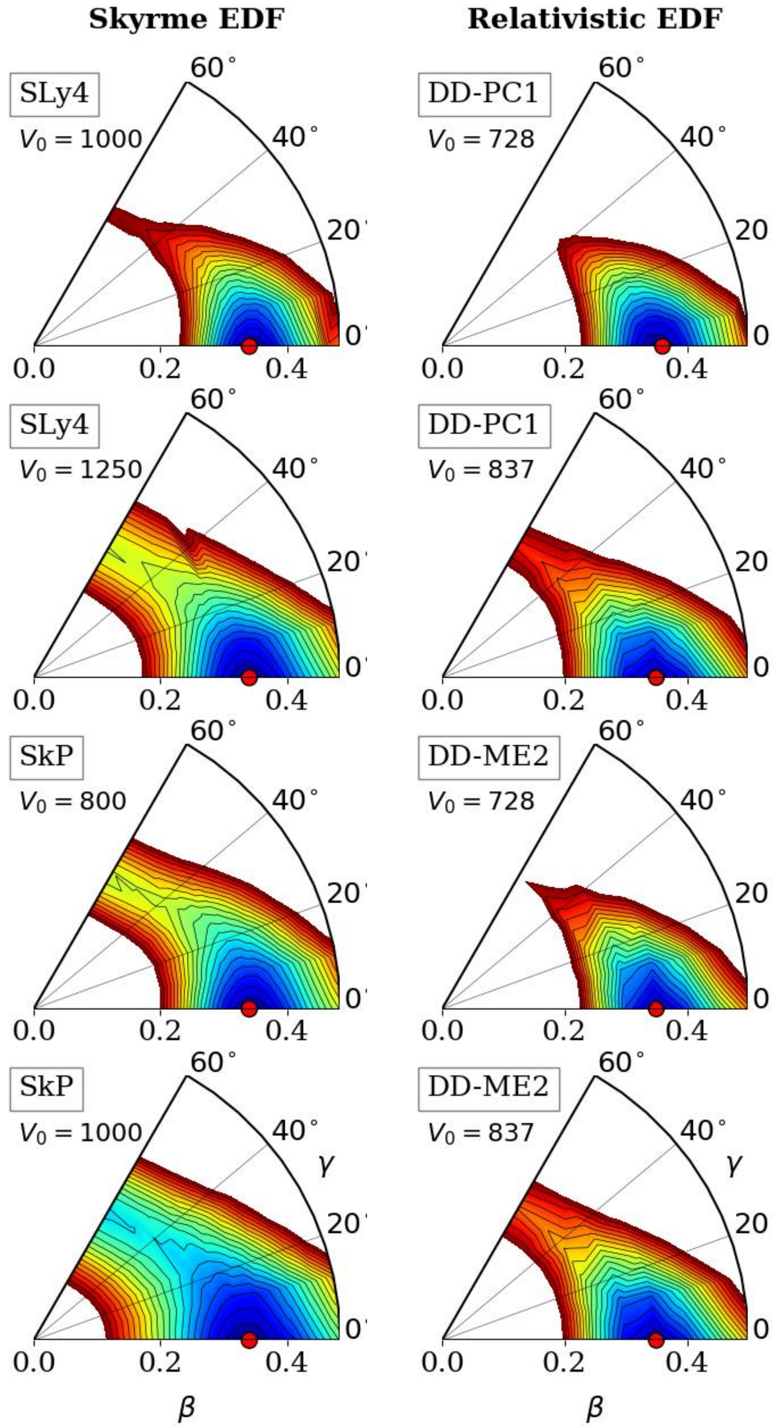
$$\epsilon_\gamma = 18Nh_2\beta_0^2(1+\beta_0^2)^{-1}$$

$$R = \frac{\epsilon_\beta}{\epsilon_\gamma} = \frac{1}{9}(1+\beta_0^2)(1+2\frac{h_0}{h_2})$$

$h_0 = h_2$, $\beta_0 = \sqrt{2}$ SU(3) DS

$h_0 \neq h_2$, $\beta_0 = \sqrt{2}$ SU(3) PDS

- Goal: determine $\hat{H}(h_0, h_2, \beta_0, \rho)$ from microscopic considerations and see if it complies with SU(3)-PDS
- Method: SCMF → IBM mapping Nomura, Shimizu, Otsuka PRL **101**, 142501 (2008)



Self Consistent Mean Field framework

SCMF calculations:
energy density functional (EDF)
pairing interaction (strength V_0)

Constraints:
mass quadrupole moments → energy surface $E_{SCMF}(\beta,\gamma)$

Non-relativistic EDFs (Skyrme)
Hartree-Fock plus BCS
SLy4 parametrization (V_0 = 1000, 1250 MeVfm^3)
SkP parametrization (V_0 = 800, 1000 MeVfm^3)
Density-dependent pairing delta force (cut-off 5 MeV)

Relativistic EDFs
Relativistic Hartree-Bogoliubov (RHB)
DD-PC1 density-depend. point coupling (V_0 =728, 837)
DD-ME2 meson exchange (V_0 = 728, 837 MeVfm^3)
Separable pairing force finite range (~ Gogny D1S)

$E_{SCMF}(\beta,\gamma)$ for ^{168}Er

Global minimum ($\beta \approx 0.35$, $\gamma = 0$)
Minimum less steep for larger pairing strength V_0

EDF-based IBM Hamiltonian

- SCMF-to-IBM mapping

Nomura, Shimizu, Otsuka PRL **101**, 142501 (2008)

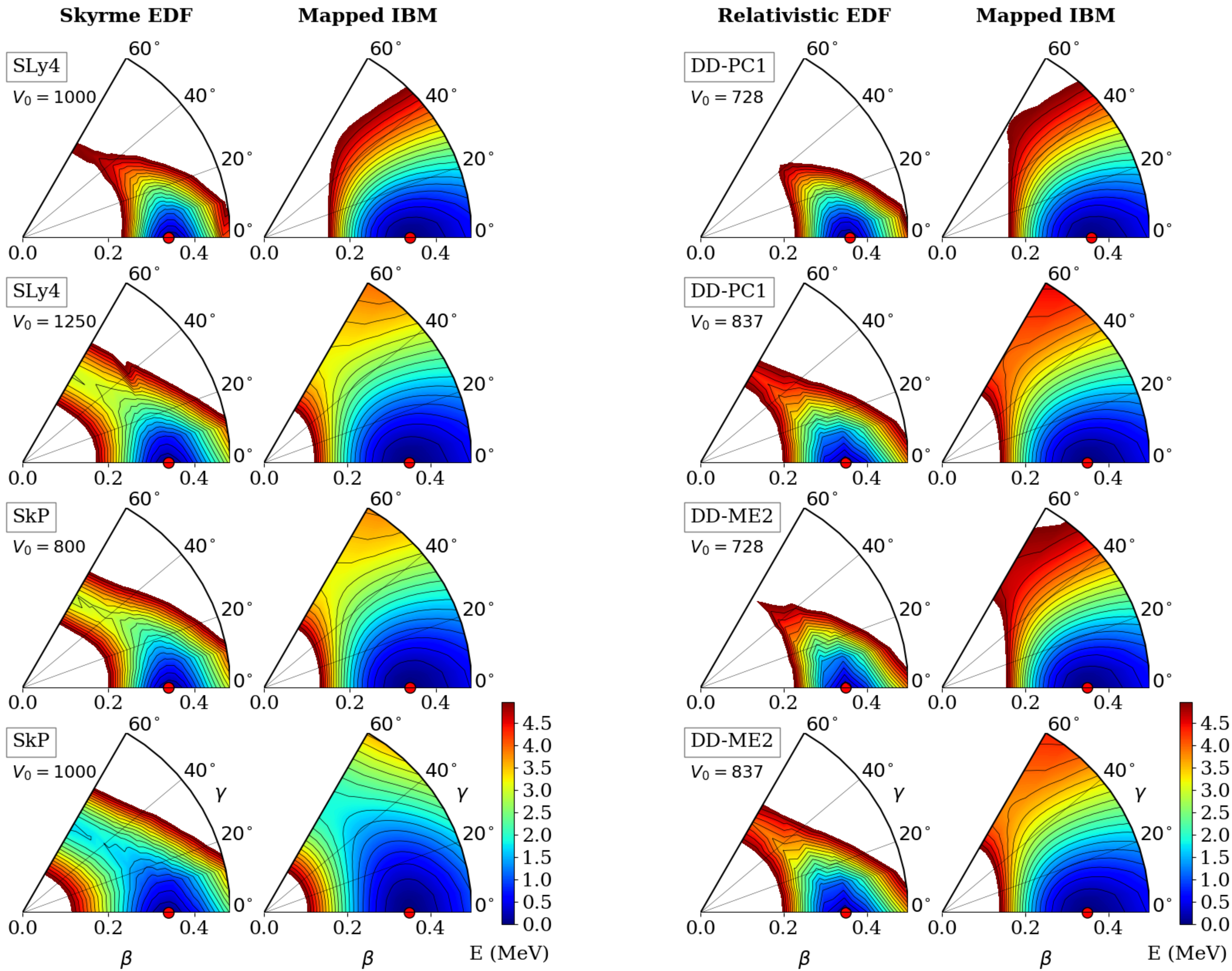
$E_{SCMF}(\beta,\gamma) \approx E_{IBM}(\beta,\gamma)$ neighborhood of global minimum

IBM Hamiltonian $\hat{H}(h_0, h_2, \beta_0, \rho)$

$\{h_0, h_2, \beta_0\}$ determined by the above condition

ρ determined by equating IBM cranking moment of inertia to the Thouless-Valatin value

^{168}Er: IBM and SCMF surfaces share common essential features near and up to a few MeV above the global minimum



Skyrme EDF
Mapped IBM
Relativistic EDF
Mapped IBM
SLy4
$V_0 = 1000$
SLy4
$V_0 = 1250$
SkP
$V_0 = 800$
SkP
$V_0 = 1000$
DD-PC1
$V_0 = 728$
DD-PC1
$V_0 = 837$
DD-ME2
$V_0 = 728$
DD-ME2
$V_0 = 837$
60°
40°
20°
0°
γ
0.0
0.2
0.4
β
4.5
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
E (MeV)

Parameters of EDF-based IBM Hamiltonians and calculated bandhead energies (keV)

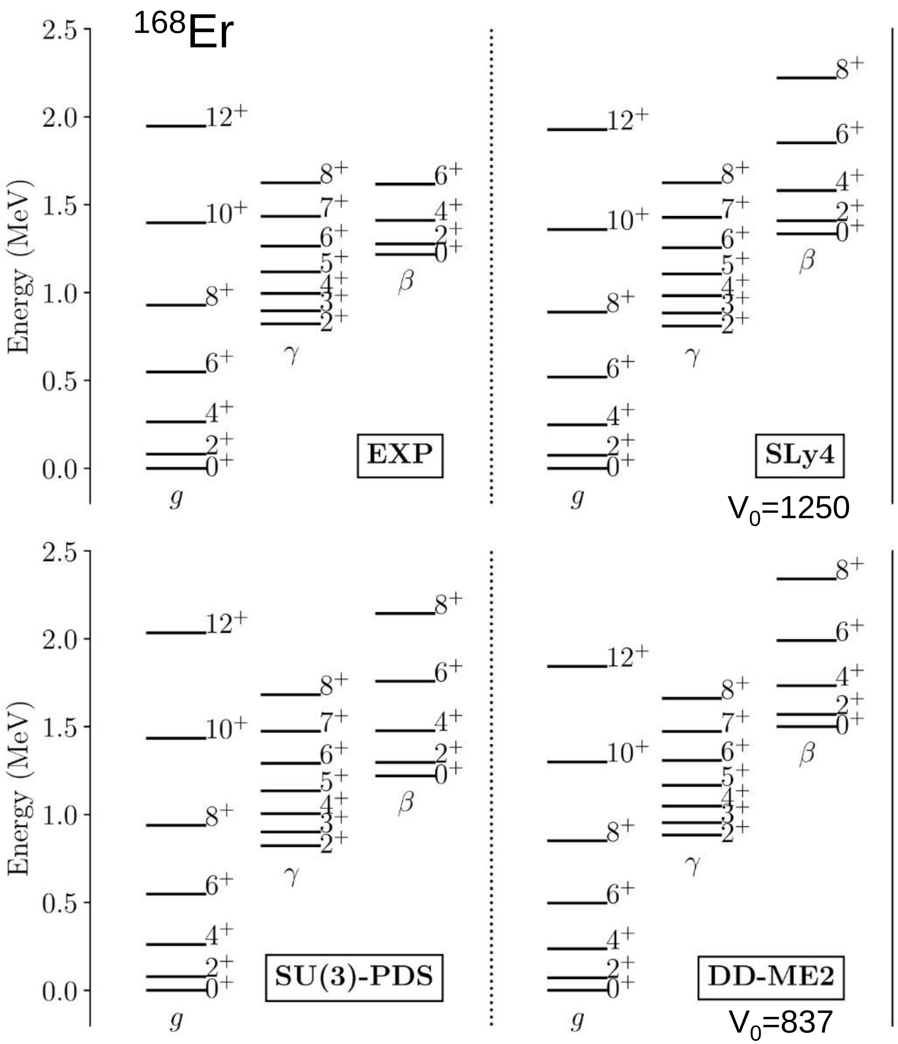
EDF	V_0	h_0	h_2	ρ	h_0/h_2	β_0	$E(2_2)$	$E(0_2)$	R = $E(0_2)/E(2_2)$
SLy4	1000	10	5.3	11.8	1.89	1.59	1132	1911	1.68
	1250	10.4	4.0	12.3	2.60	1.39	809	1334	1.65
SkP	800	10.5	3.7	12.6	2.84	1.45	776	1306	1.68
	1000	30.6	4.4	12.2	6.95	0.99	672	1087	1.62
DD-PC1	728	10.5	5.1	11.74	2.06	1.59	1092	1889	1.73
	837	9.8	4.4	11.73	2.23	1.51	925	1564	1.69
DD-ME2	728	10.4	4.8	11.74	2.17	1.59	1032	1794	1.74
	837	9.9	4.2	11.73	2.36	1.50	883	1499	1.70
SU(3)-PDS		8.0	4.0	13.0	2	$\sqrt{2}$	822	1220	1.48 ^{168}Er

^{168}Er fit: $h_0/h_2 = 2$; In most EDFs $1.9 < h_0/h_2 < 2.8$

Derived β_0 close or slightly larger than SU(3)-PDS $\beta_0 = \sqrt{2} \approx 1.4142$

Notable exception: (SkP EDF V_0 =1000 MeVfm^3) $h_0/h_2 = 6.95$, $\beta_0 = 0.99$

For any EDF, larger pairing strength $\Rightarrow$ larger (smaller) h_0/h_2 (β_0)



Spectra of EDF-based Hamiltonians

EDF-based spectra conform with SU(3)-PDS and experimental spectra

Description for ground and γ bands stable β-band more case sensitive

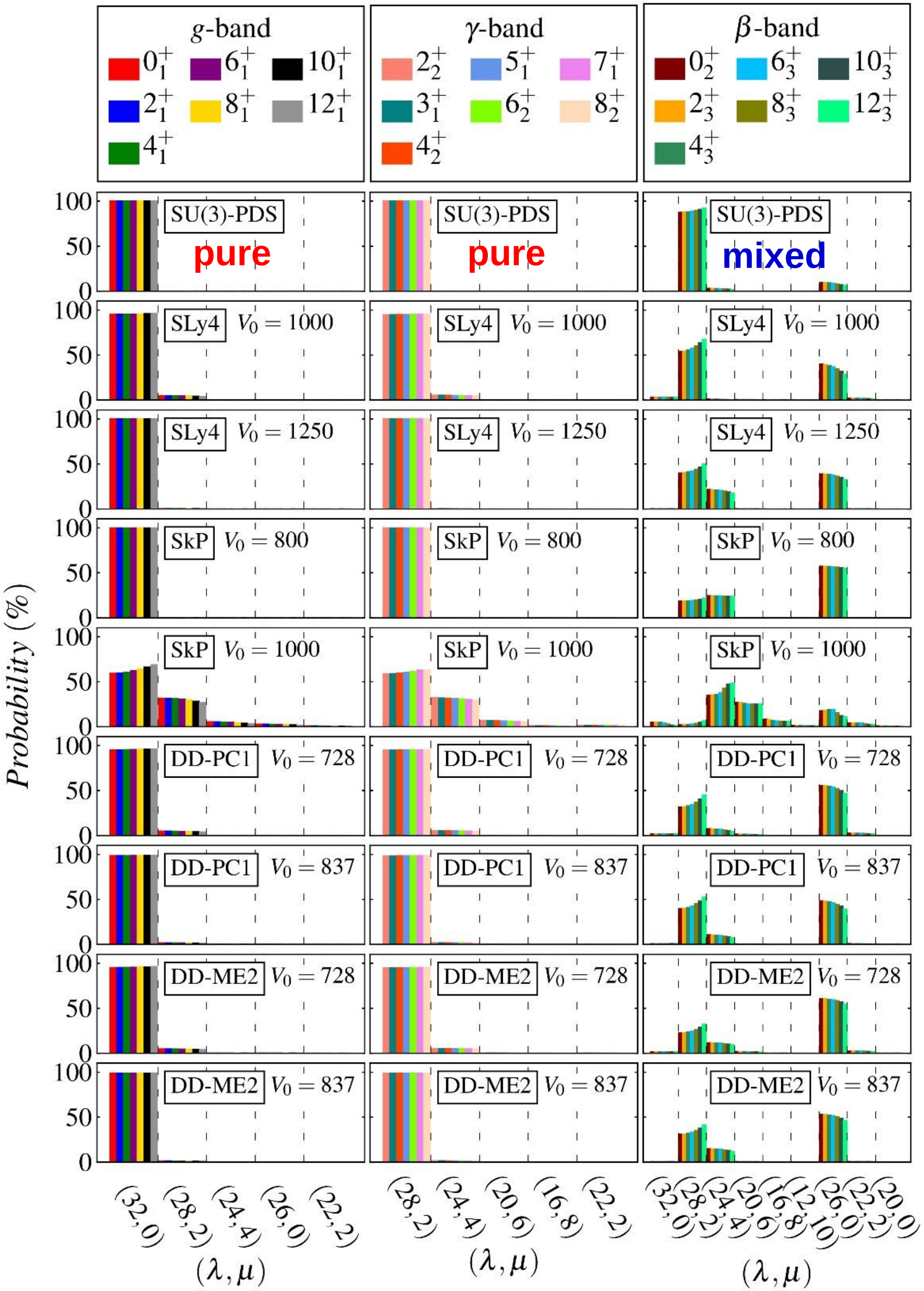
Calculated $E(0_2)$ above exp and SU(3)-PDS values

Relativistic EDFs generally result in higher β-band energies than Skyrme EDFs

Increase in pairing strength systematically decreases β-band energies

SkP EDF V_0 =1000, only case where both $E(2_2)$ and $E(0_2)$ below exp and SU(3)-DS values

K. Numura, N. Gavrielov, A. Leviatan, Phys. Rev. C **104**, 044317 (2021)



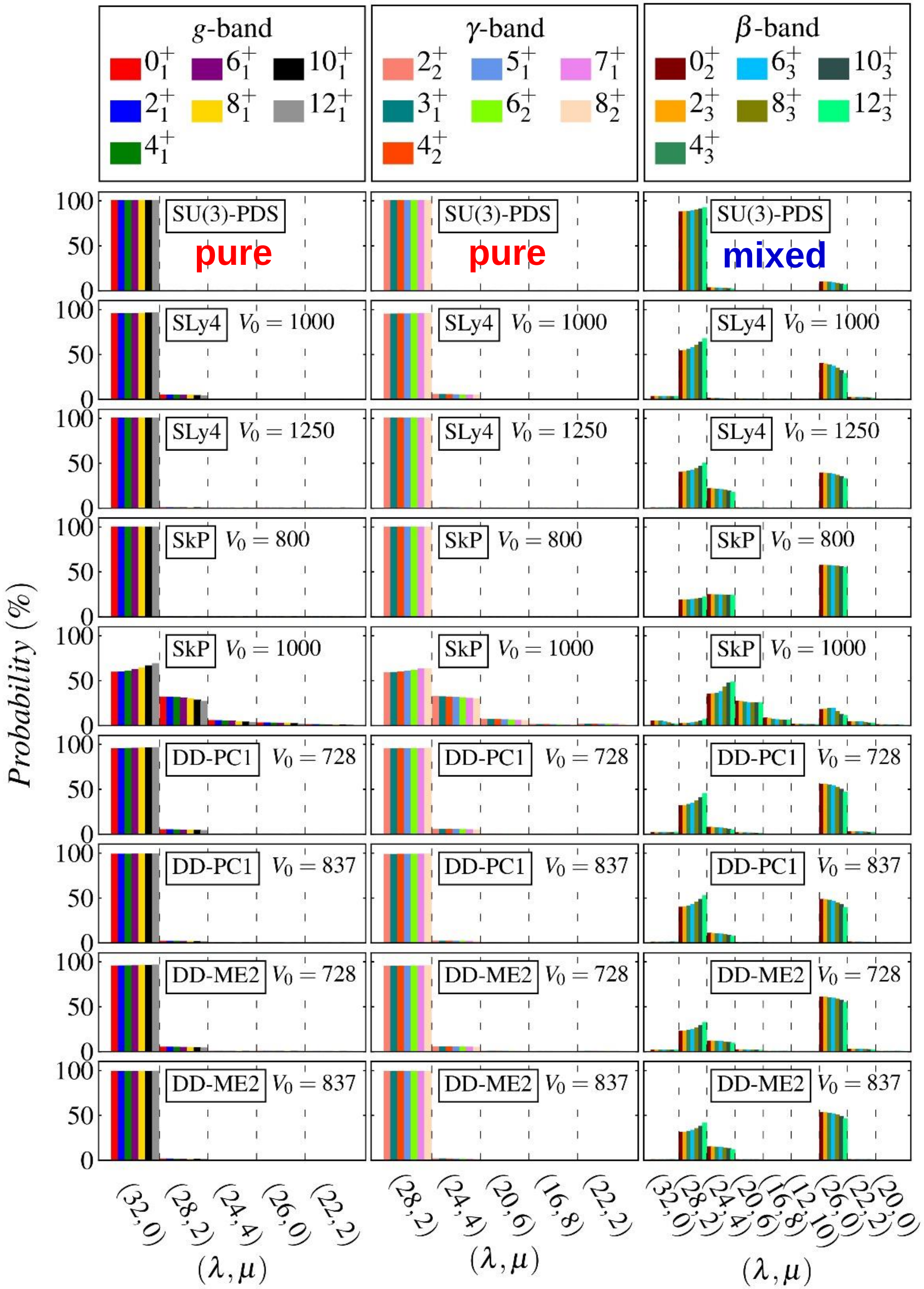
Symmetry analysis

SU(3) (λ,μ) decomposition

SU(3)-PDS:
ground (2N,0) and γ (2N-4,2) **pure** bands
β-band **mixed** (2N-4,2) 87.5 %, (2N-6,0) 9.6%, (2N-8,4) 2.9%

All EDFs reproduce well SU(3)-PDS prediction of SU(3) purity for ground and γ bands with probability > 95 %

$\Rightarrow$ PDS is robust with microscopic roots



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⇒ PDS is robust with microscopic roots

β-band more sensitive to the choice of EDF
Dominant SU(3) irreps observed but relative weights differ from SU(3)-PDS
Additional d.o.f. may contribute to β-band

Notable exception: SkP EDF V_0 =1000 exhibiting large fragmentation of SU(3) irreps

From all EDFs considered, SLy4 V_0 = 1250 and SkP V_0= 800 closest to SU(3)-PDS predictions for ^{168}Er (g & γ purity > 99.8%)

Concluding remarks

Microscopic justification for Partial Dynamical Symmetry (PDS) in nuclei

- Tools: SCMF methods with universal EDFs in combination with algebraic models
- ^{168}Er: IBM Hamiltonian derived from known EDFs complies with SU(3)-PDS

$\Rightarrow$ Linkage between microscopic EDFs and algebraic PDS notion

Algebraic, PDS, Symmetries $\leftrightarrow$ Shapes, EDF, Microscopic

- Results valid for both non-relativistic and relativistic EDFs
  with several choices of pairing strengths

$\Rightarrow$ PDS notion is robust and founded on microscopic grounds

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Outlook

- Explore the microscopic origin of other types of PDSs, e.g., SO(6)-PDS in γ-soft nuclei
- When a PDS is found to be manifested empirically in certain nuclei, it can be used to constrain and optimize (e.g., choice of paring strength) a given EDF in that region
- Exploit the demonstrated EDF-PDS linkage to predict unchartered regions of exotic nuclei where partial symmetries can play a role

K. Numura, N. Gavrielov, A. Leviatan, Phys. Rev. C **104**, 044317 (2021)

Thank you