

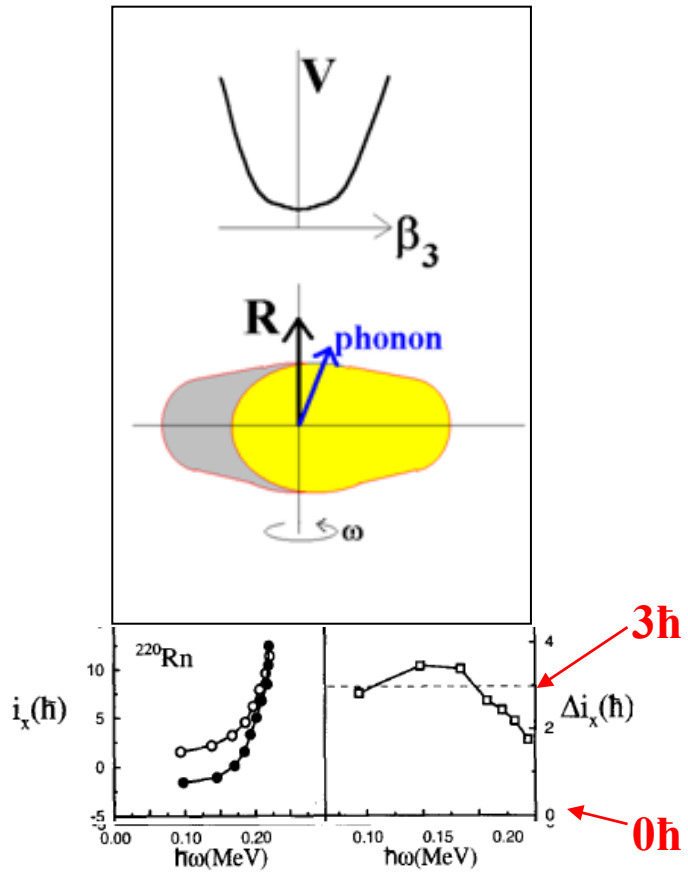
# Studies of Pear-Shaped Nuclei

*Peter Butler*

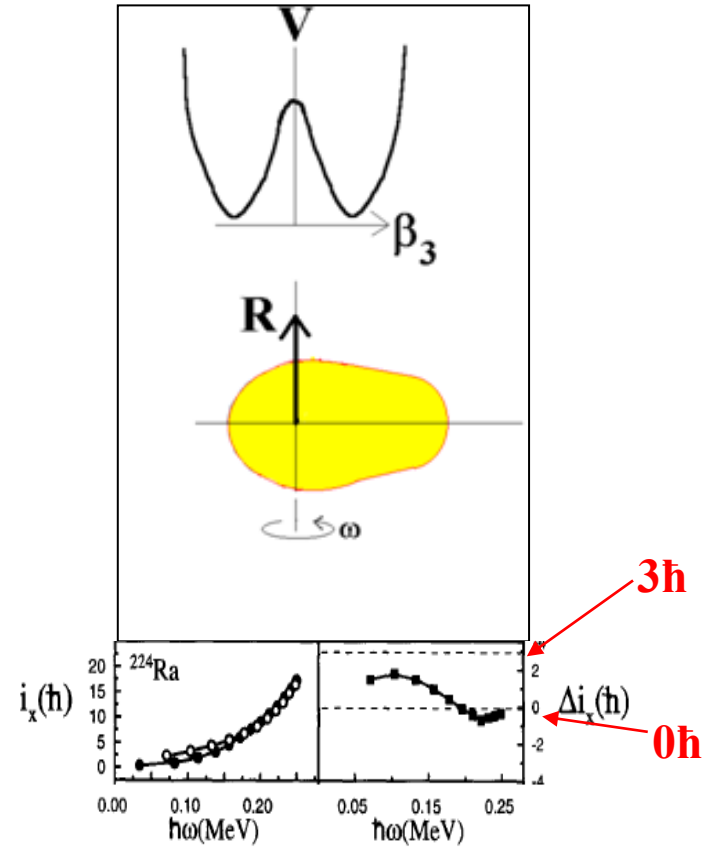
*University of Liverpool*

**Introduction**  
**Rotations**  
 **$E\lambda$  moments**  
**Future prospects**

# Rotating octupole shapes



**Octupole vibrational**

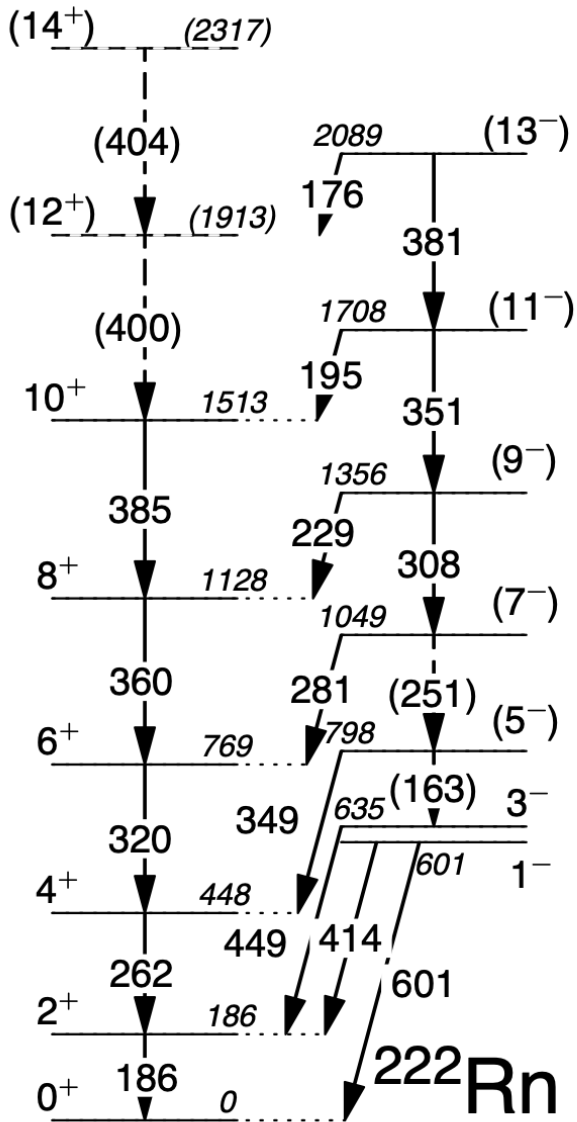


**Octupole deformed**

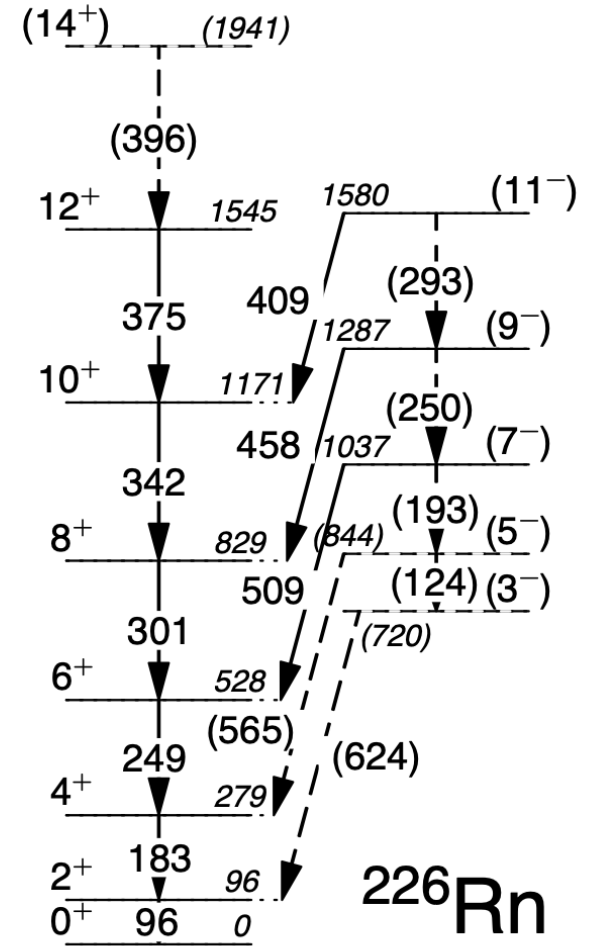
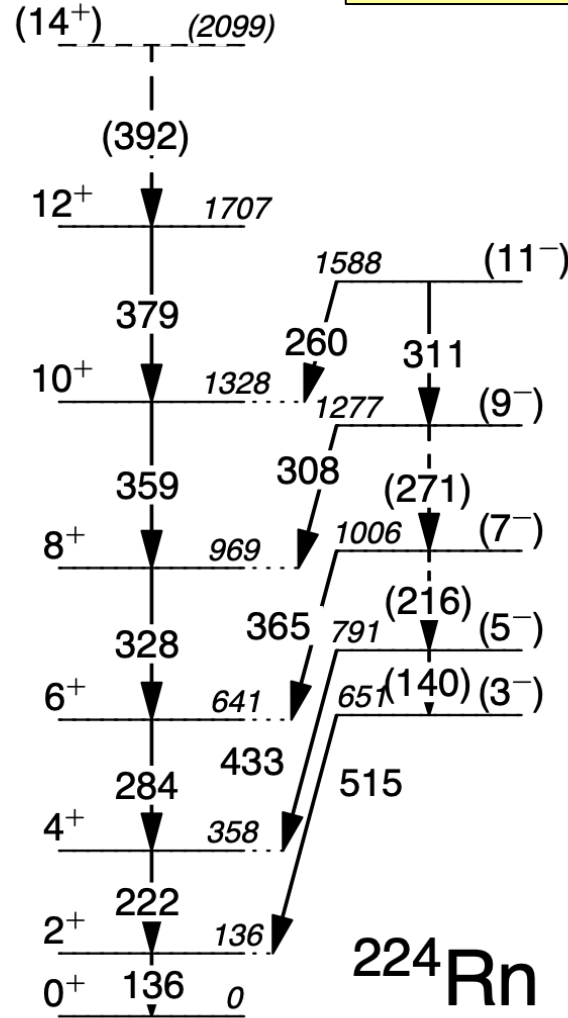


*JFC Cocks et al*  
*PRL 78 (1997) 2920*

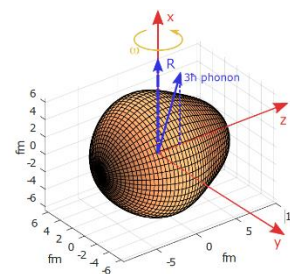
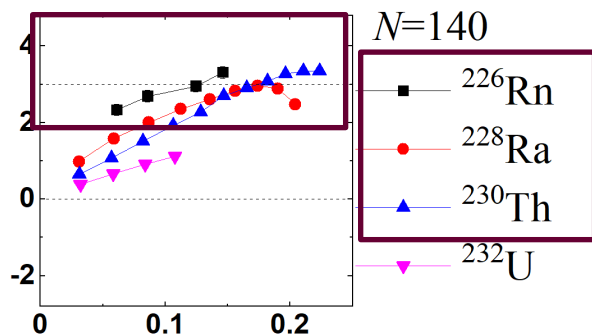
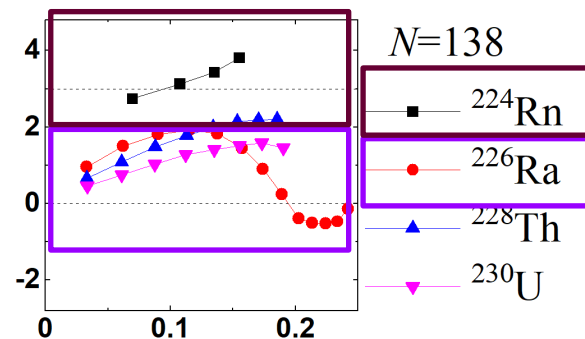
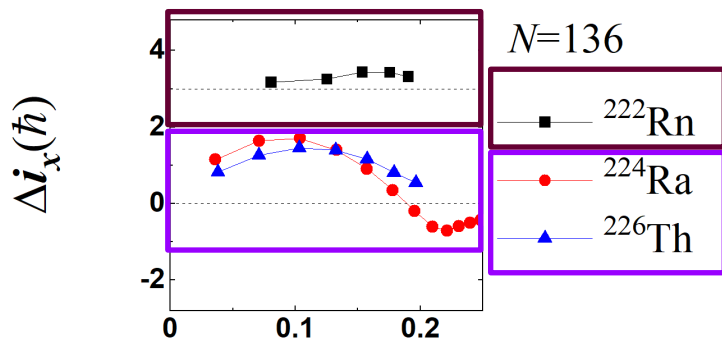
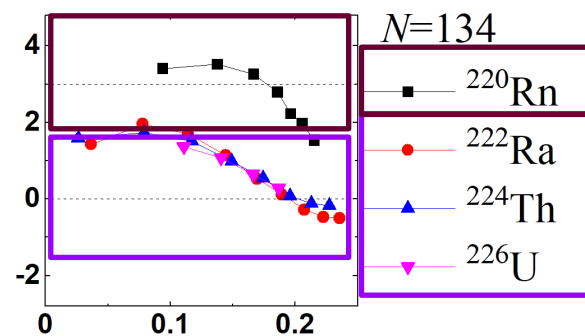
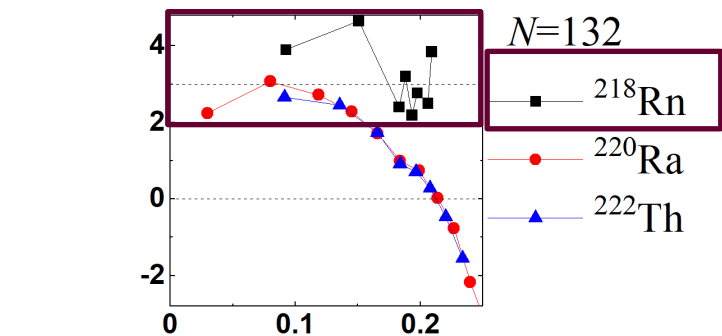
# Level schemes Rn



*PA Butler et al.*  
*Nat. Comm. 10 (2019) 2473,*  
*Nat Comm. Add. 11 (2020) 3560*



# Rotating pear shapes



octupole vibration

rigid pear shape

$\hbar\omega$  (MeV)

# E3 moments

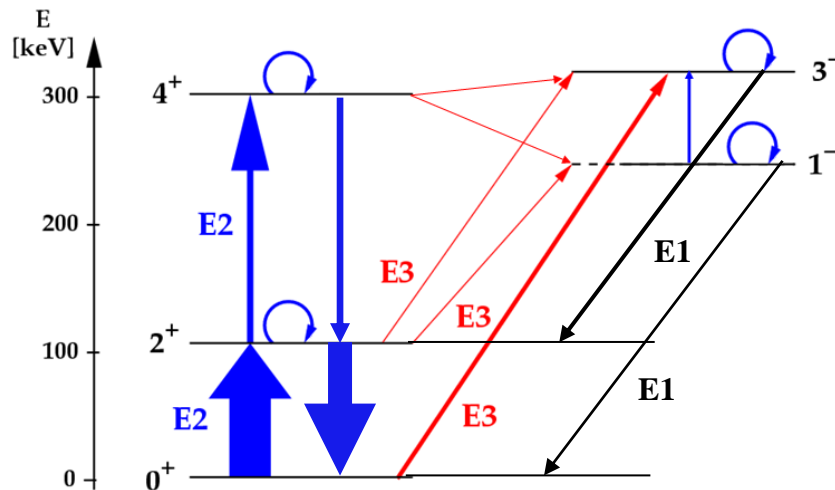
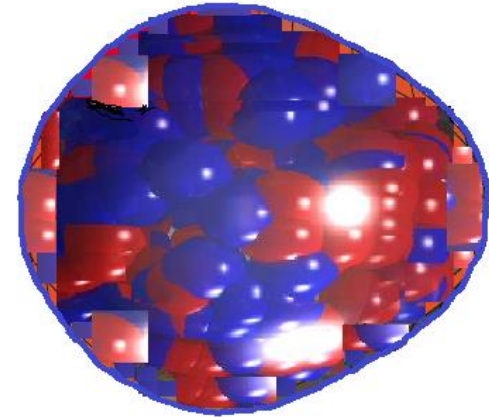
$$Q_3 = \frac{3}{\sqrt{7\pi}} ZeR_0^3 \bar{\beta}_3$$

$$\bar{\beta}_3 = f(\beta_3, \dots)$$

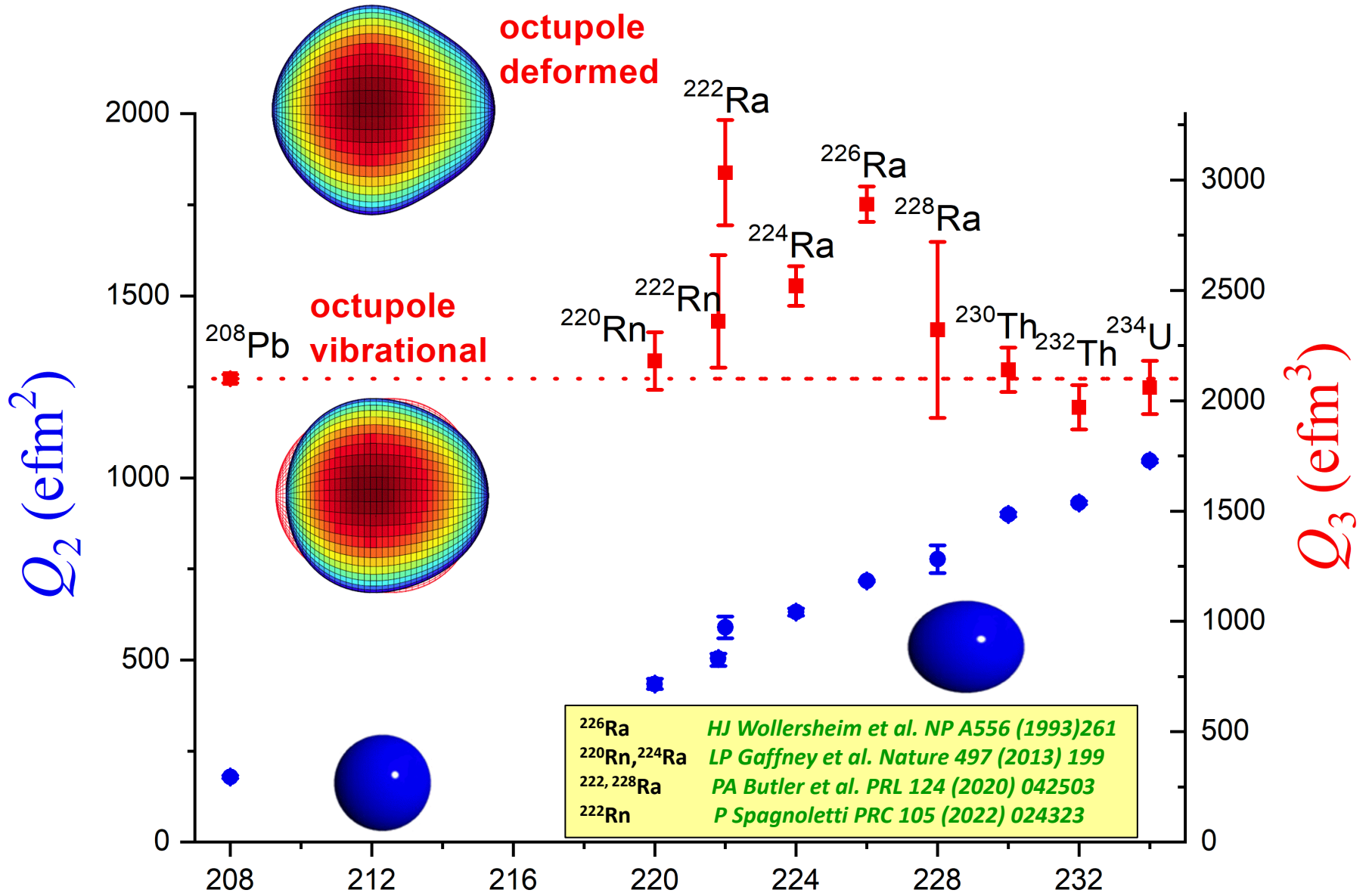
$$B(E3; I_i \rightarrow I_f) = \frac{7}{16\pi} \langle I_i 030 | I_f 0 \rangle^2 Q_3^2$$

$B(E3; 0^+ \rightarrow 3^-) \sim 30 - 50$  single particle units for  $\beta_3 \sim 0.1$

$B(E1) \sim 10^{-(2-3)}$  single particle units



# E2 and E3 moments



$$\langle 0^+ || \mathcal{M}(E2) || 2^+ \rangle = \sqrt{5/16\pi} Q_2$$

A

$$\langle 0^+ || \mathcal{M}(E3) || 3^- \rangle = \sqrt{7/16\pi} Q_3$$

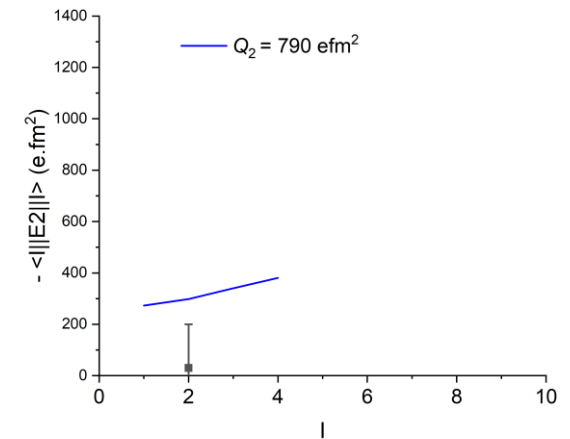
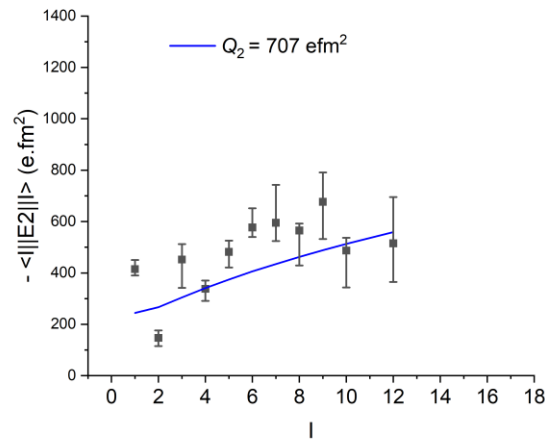
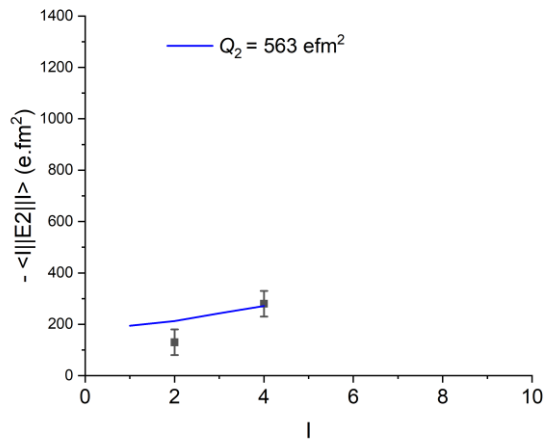
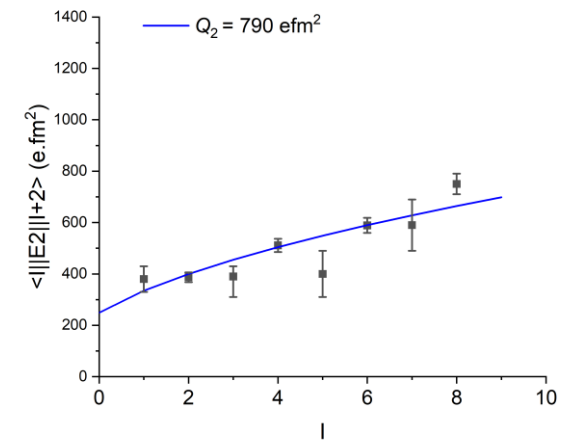
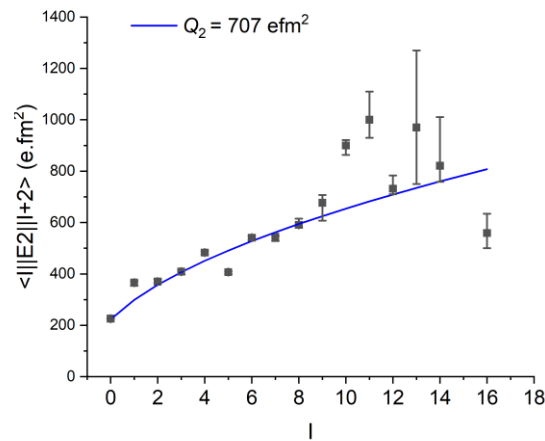
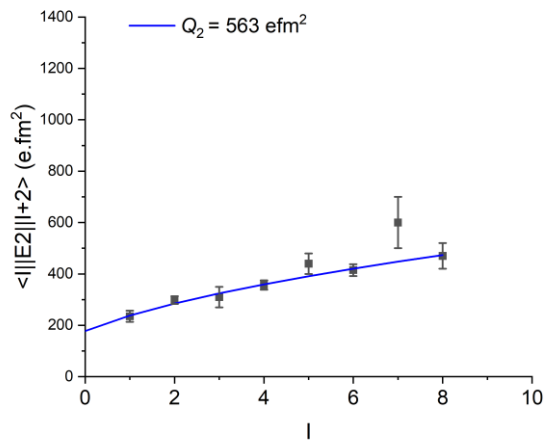
# Behaviour of $Q_2$ with $I_i, I_f$

See also  $^{220,222}\text{Rn}, ^{224}\text{Ra}$

$^{222}\text{Ra}$

$^{226}\text{Ra}$

$^{228}\text{Ra}$



$$\langle I_i || \mathcal{M}(E2) || I_f \rangle = \sqrt{5/16\pi} (2I_i + 1) (I_i 020 | I_f 0) Q_2$$

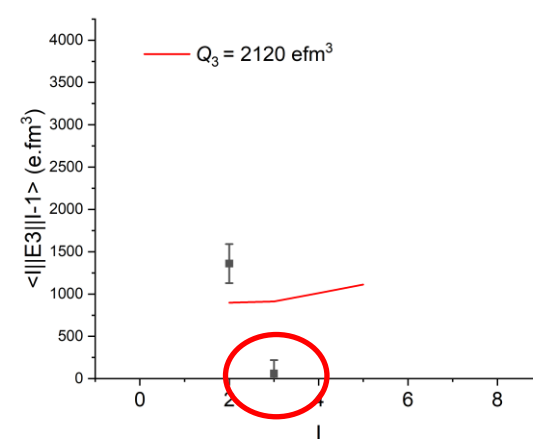
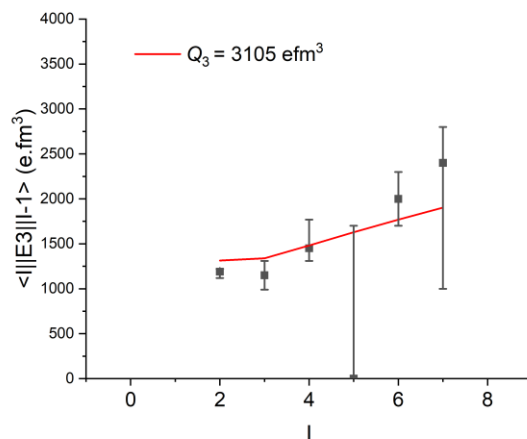
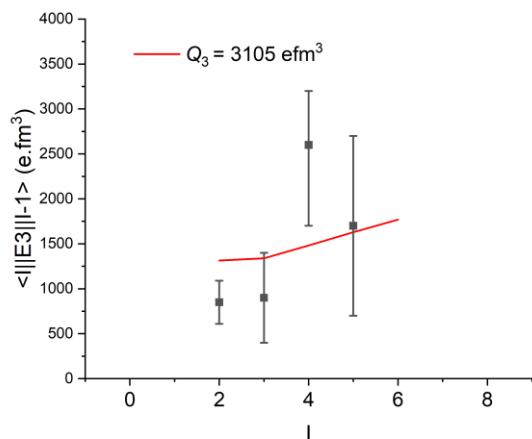
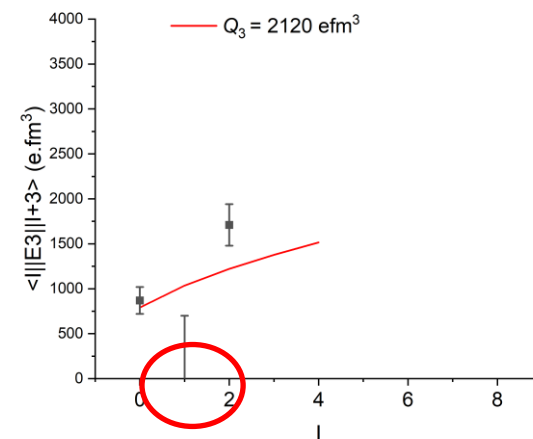
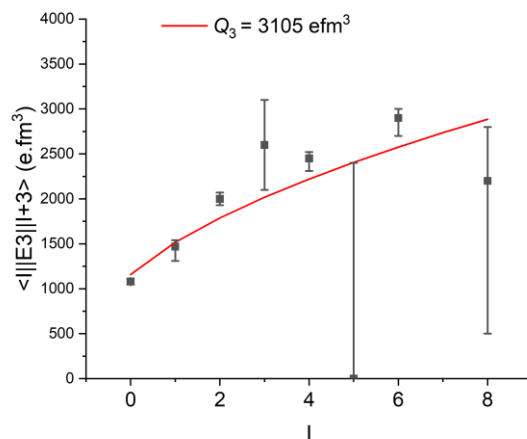
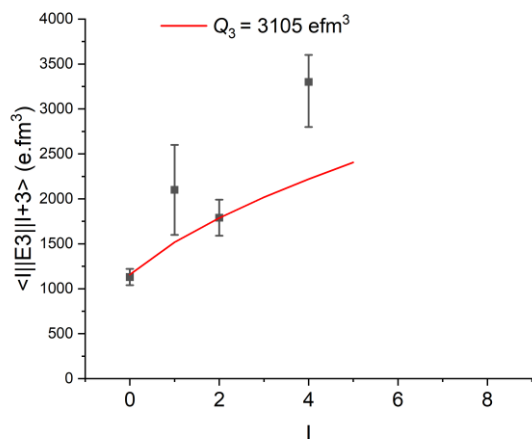
# Behaviour of $Q_3$ with $I_i, I_f$

See also  $^{220,222}\text{Rn}$ ,  $^{224}\text{Ra}$ ; and  $Q_1$

$^{222}\text{Ra}$

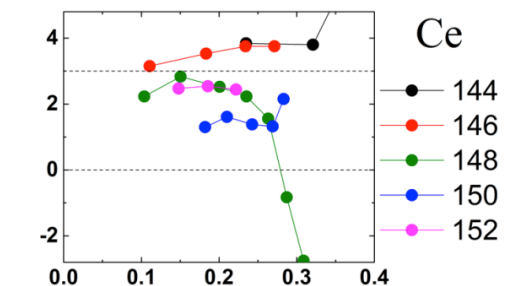
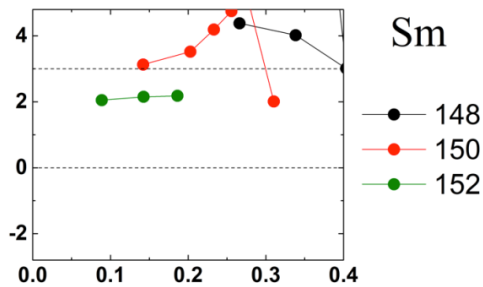
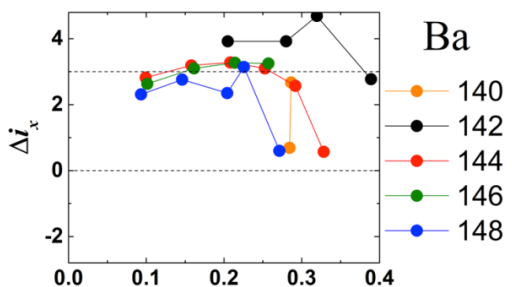
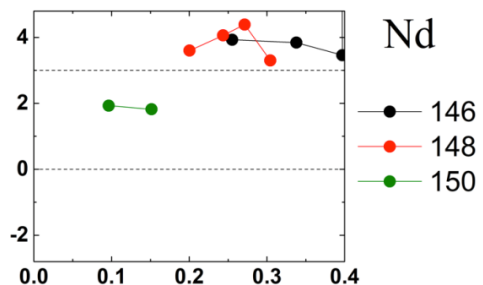
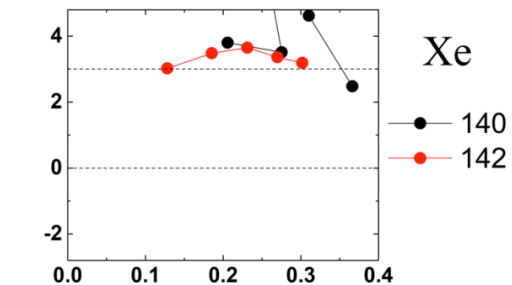
$^{226}\text{Ra}$

$^{228}\text{Ra}$

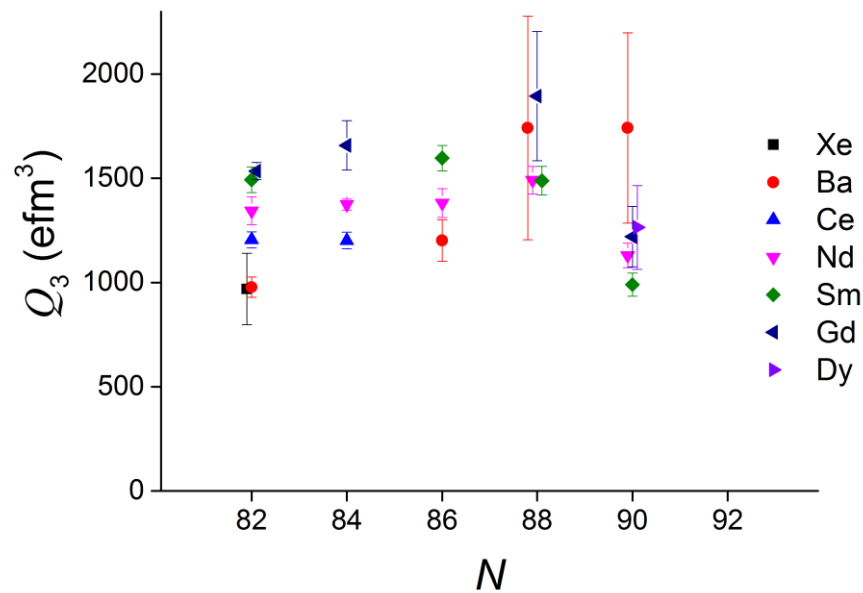
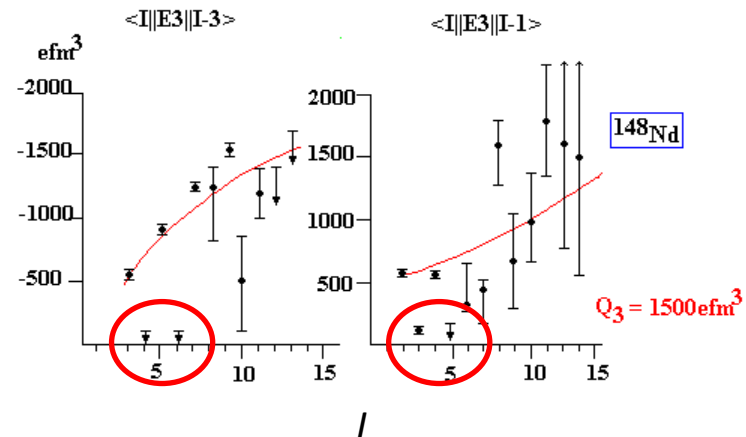


$$\langle I_i || \mathcal{M}(E3) || I_f \rangle = \sqrt{7/16\pi} (2I_i + 1) (I_i 030 | I_f 0) Q_3$$

# lanthanides

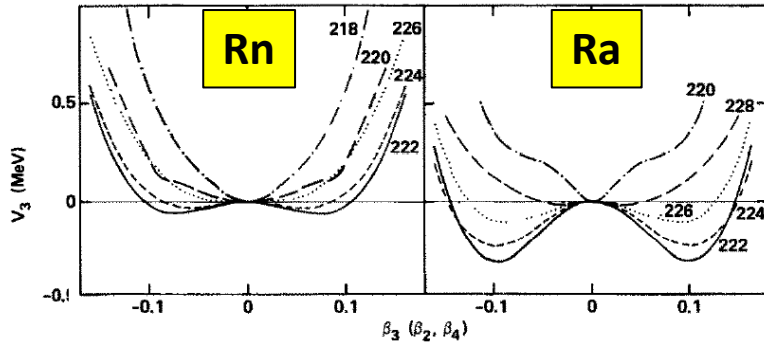


$h\omega$  (MeV)

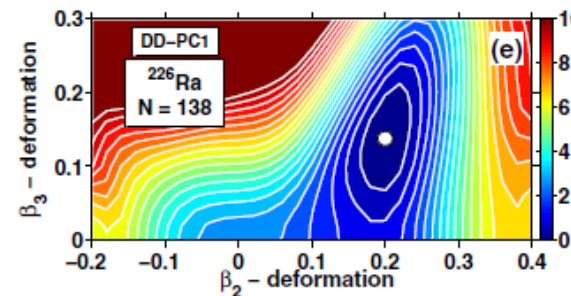
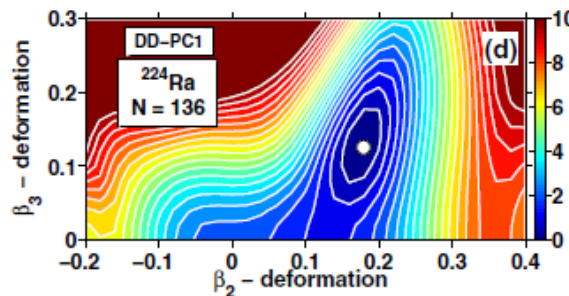
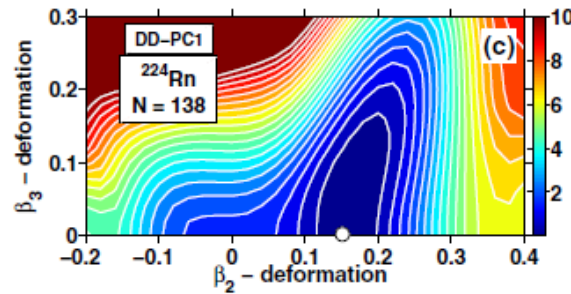
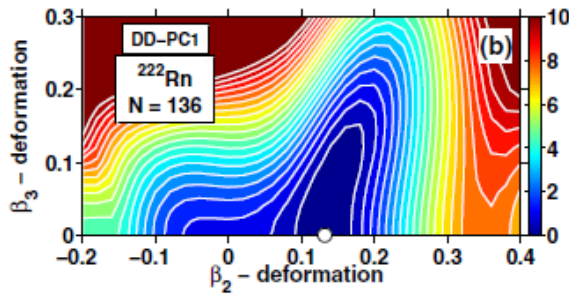
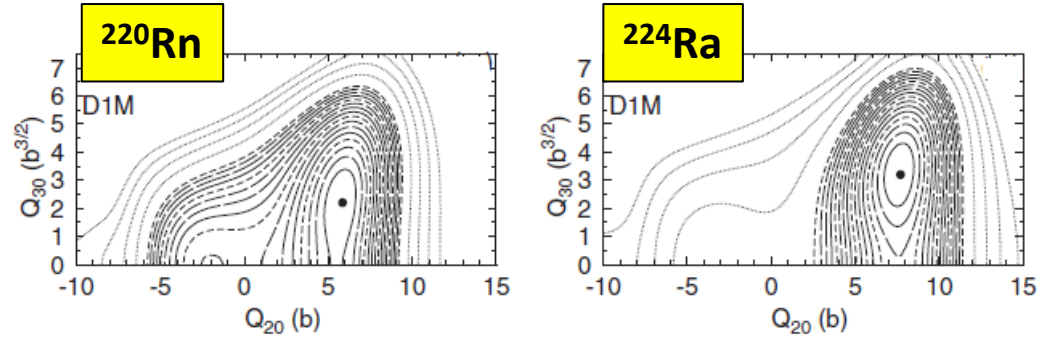


# Calculated nuclear shapes: radon & radium

Nazarewicz et al. *Strut. WS* (MaMi)  
*NP A429 (1984) 269*



Robledo & Butler *HFB Gogny* (D1M2d)  
*PRC 88 (2013) 051302*

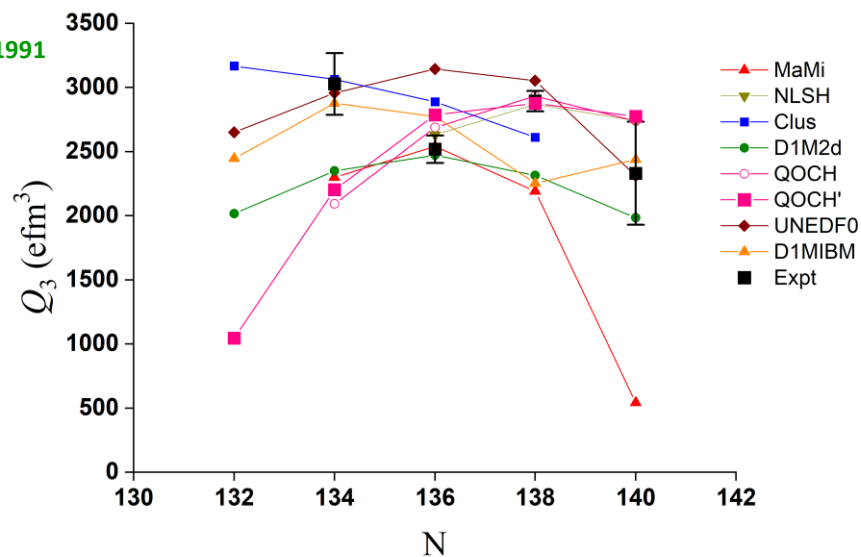
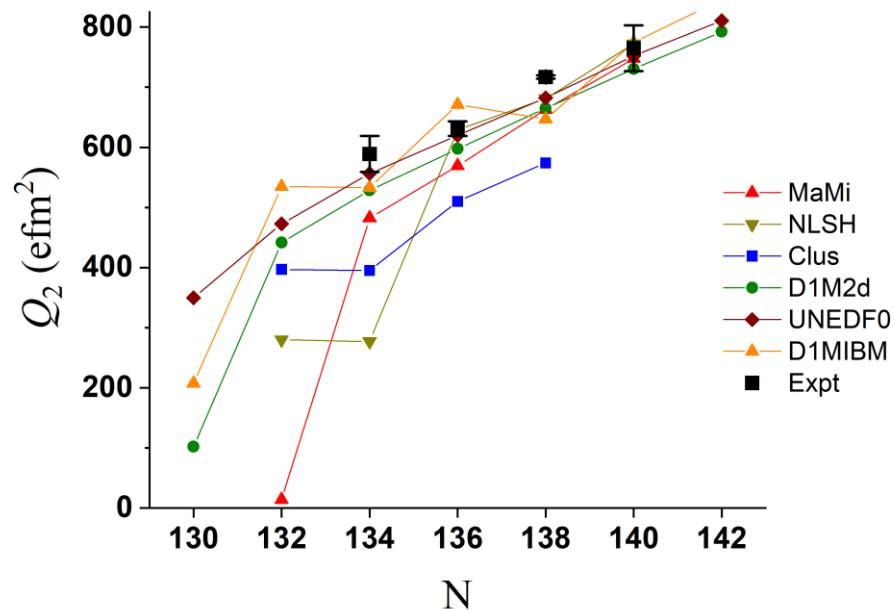
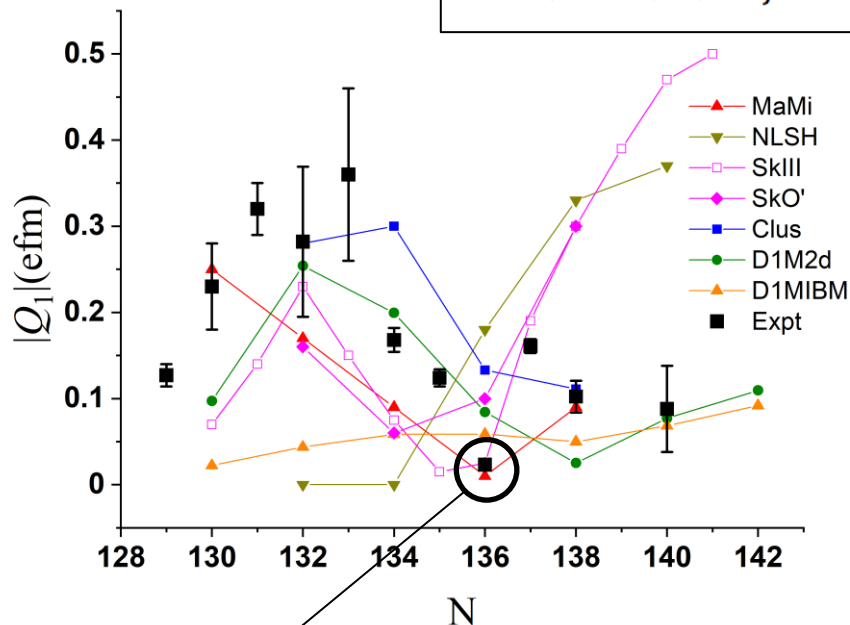


Agbemava et al. *Rel. HFB*  
*PRC 93 (2016) 044304*

Radium predicted to have a more rigid pear shape than radon

# Expt v. theory for electric moments: radium

$$\langle I_i \parallel \mathcal{M}(E\lambda) \parallel I_f \rangle = (2I_i + 1)^{\frac{1}{2}} (I_i 0 \lambda 0 | I_f) Q_\lambda a_\lambda$$



Very small E1 observed for  $^{224}\text{Ra}$

From E1/E2 branching ratios:

*Poynter et al. PLB 232 (1989) 447*

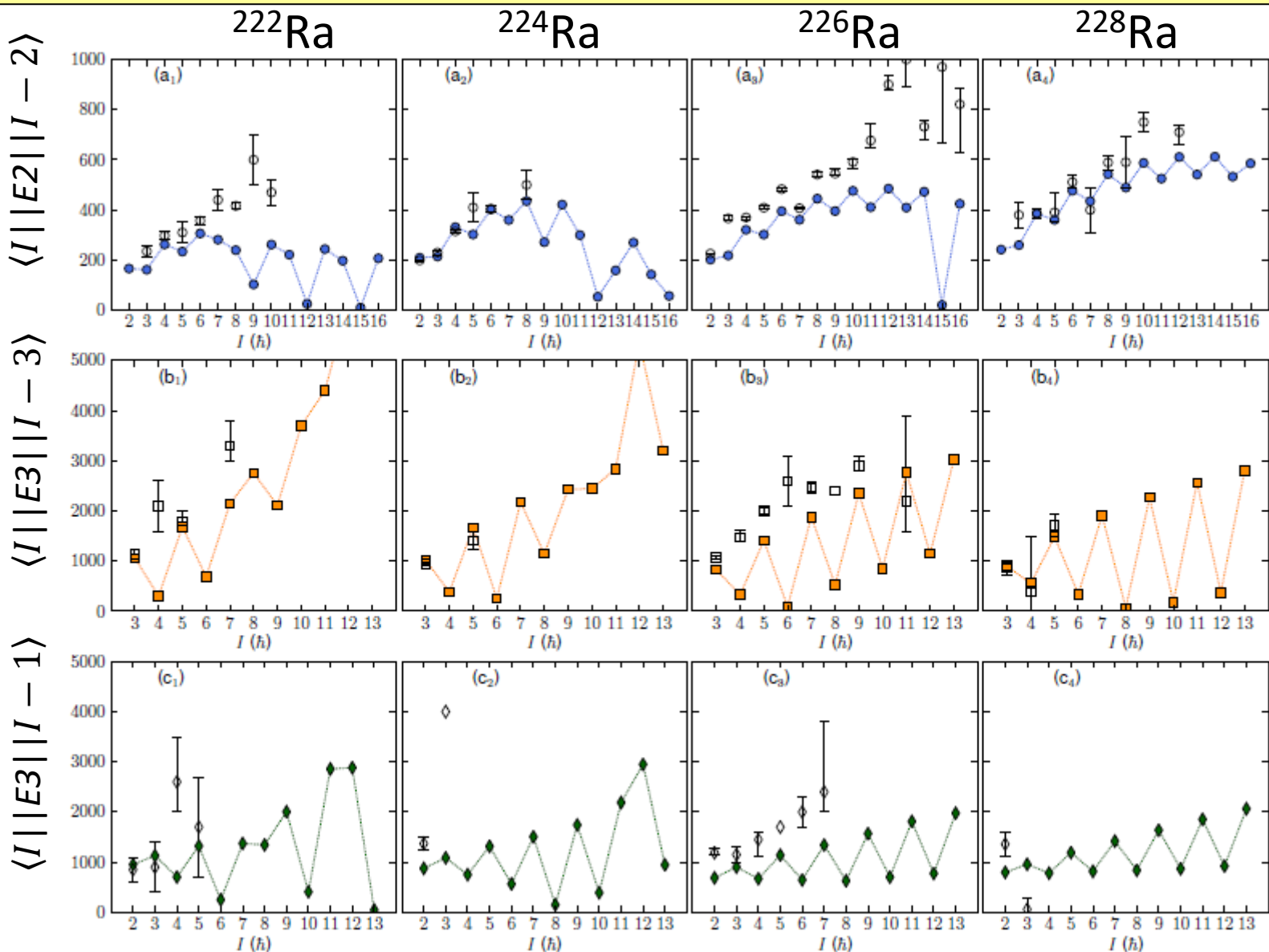
*Cocks et al. NPA 645 (1999) 61*

Absolute measurement of E1

*LP Gaffney et al. Nature 497 (2013) 199*

MaMi Nazarewicz 1984,1991  
 NLSH Rutz 1993  
 SkIII Tsvetlov 2002  
 SkO' Engel 2003  
 Clus Shneidman 2003  
 D1M2d Robledo 2013  
 QOCH Sun 2019  
 QOCH' Zhang 2019  
 UNEDF0 Cao 2020  
 D1MIBM Nomura 2020

# Expt v. theory for spin dependence of E2 and E3s



*K Nomura et al. HFB Gogny + sdf IBM (D1MIBM)  
PRC 102, 064326 (2020)*

See Kosuke's talk Wed am

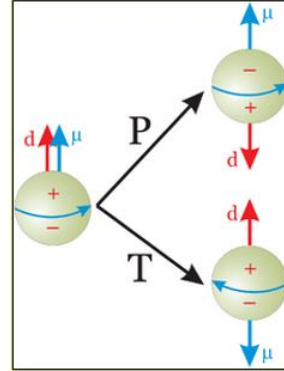
# Octupole enhanced atomic EDM moment

Schiff moment:

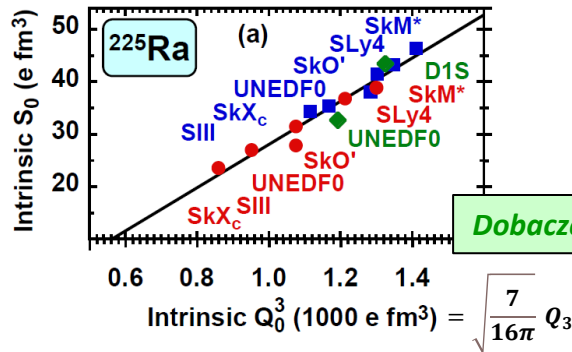
$$S = -2 \frac{J}{J+1} \beta_2 \beta_3^2 Z A^{2/3} \frac{\langle \hat{V}_{PT} \rangle}{\Delta E}$$

energy splitting of parity doublet in odd-A nucleus

CP violation in nuclear force



Measured in  $^{220,222}\text{Rn}$ ,  $^{222-228}\text{Ra}$   
(but not odd-A nuclei)



Dobaczewski et al. PRL 121 (2018) 232501

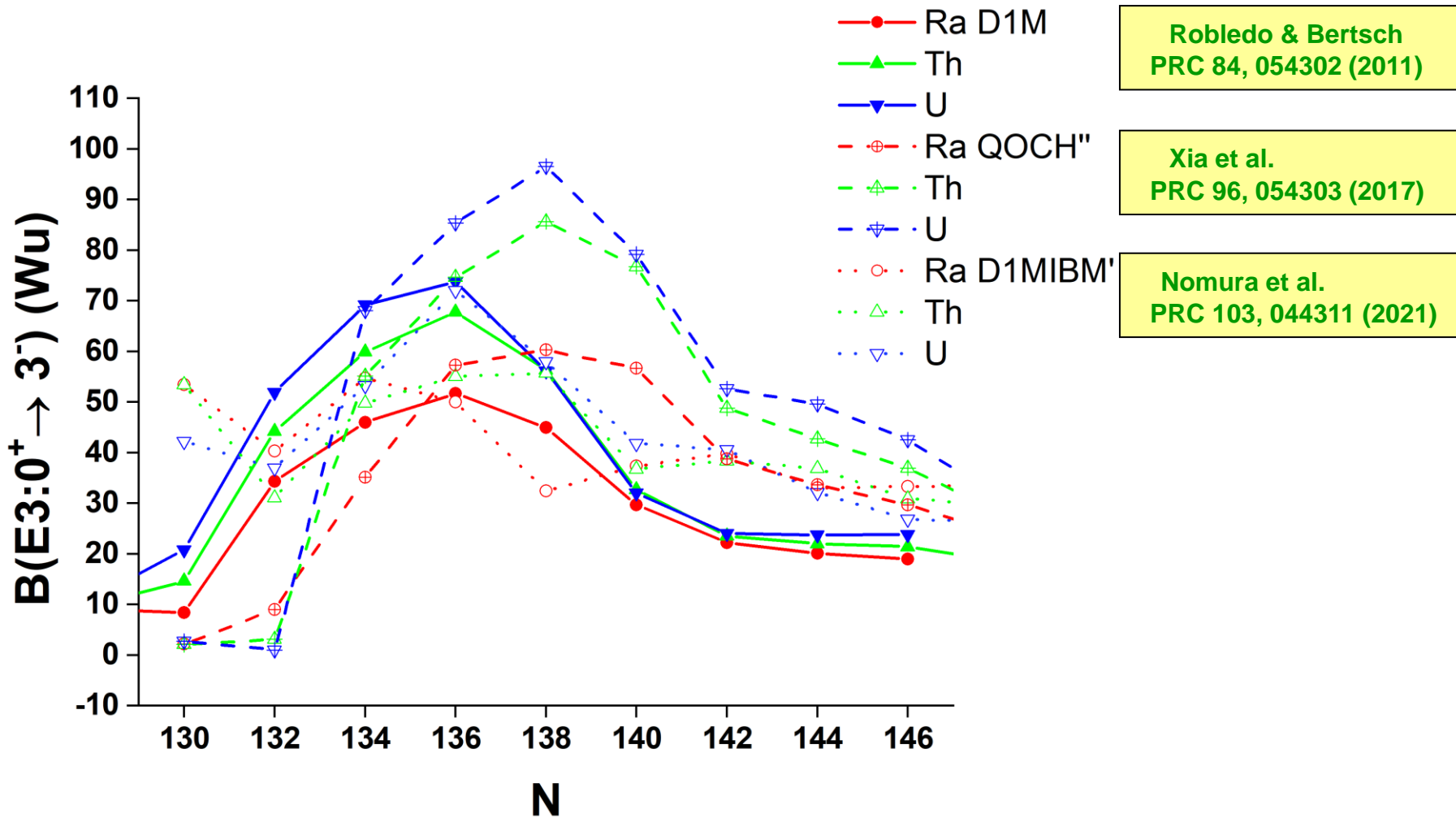
Measured in  $^{223,225}\text{Ra}$  (~ 50 keV)  
 $\Delta E$  smaller in Ra than in Rn

EDM ( $^{225}\text{Ra}$ ) ~ 100-1000 x EDM ( $^{199}\text{Hg}$ )

EDM ( $^{229}\text{Pa}$ ) ~ 1000 x EDM ( $^{225}\text{Ra}$ ) ???

Latest (ANL/MSU)  $^{225}\text{Ra}$  limit =  $10^6$  x  $^{199}\text{Hg}$  limit

# Future prospects



Estimated re-accelerated intensity of  $^{228}\text{U}$  from FRIB:  $3.5 \cdot 10^5$  /s (LoI endorsed)  
 Possibility to extract  $\text{UF}_x$  from ISOLDE (LoI endorsed)

# Summary

We can classify the rotational behaviour of Rn-U isotopes with  $N=132-138$  as octupole vibrational or as having static octupole deformation.

This is consistent with the observed behaviour of the E3 moments, suggesting that  $^{222,224,226}\text{Ra}$  are pear-shaped.

Expect higher  $B(E3)$  values for U isotopes.

University of Liverpool

CERN

U. of the West of Scotland

University of the Western Cape

TRIUMF

University of Lund

University of Michigan

INFN Legnaro

KU Leuven

University of Guelph

University of Cologne

TU Darmstadt

HIL, University of Warsaw

University of Jyvaskyla

Helsinki Institute of Physics

University of Oslo

University of York

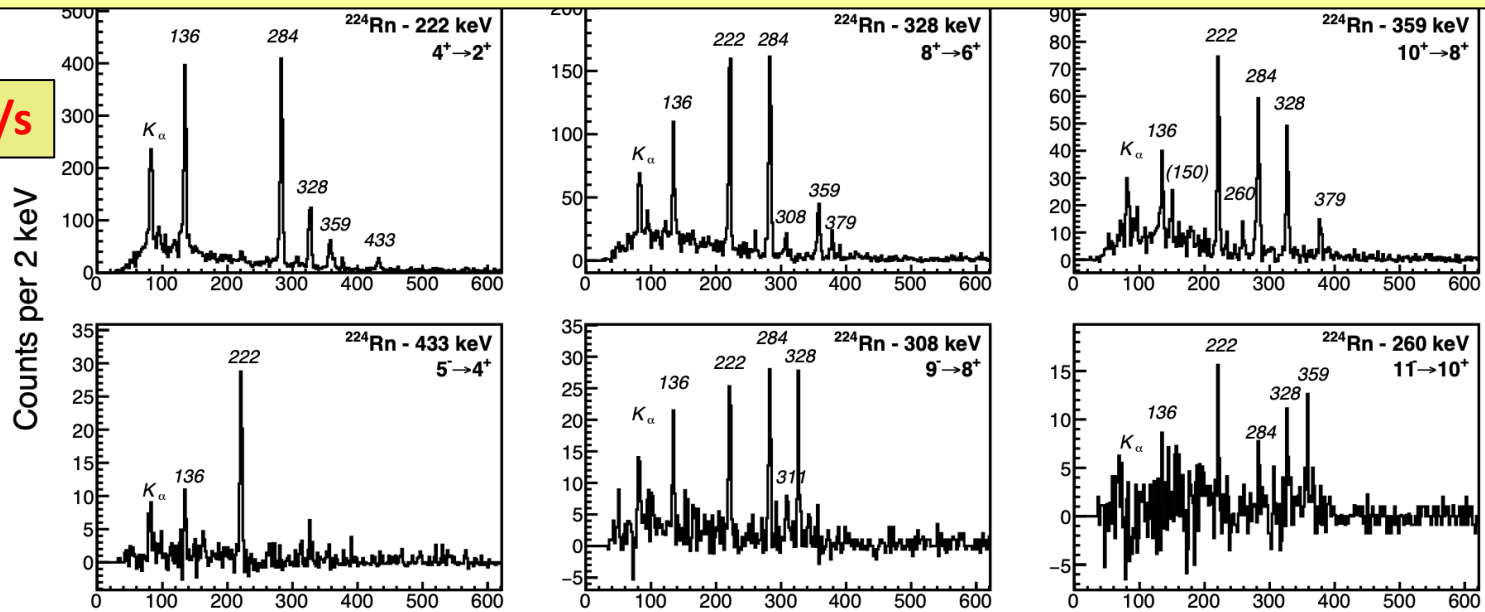
JINR Dubna

CSIC Madrid

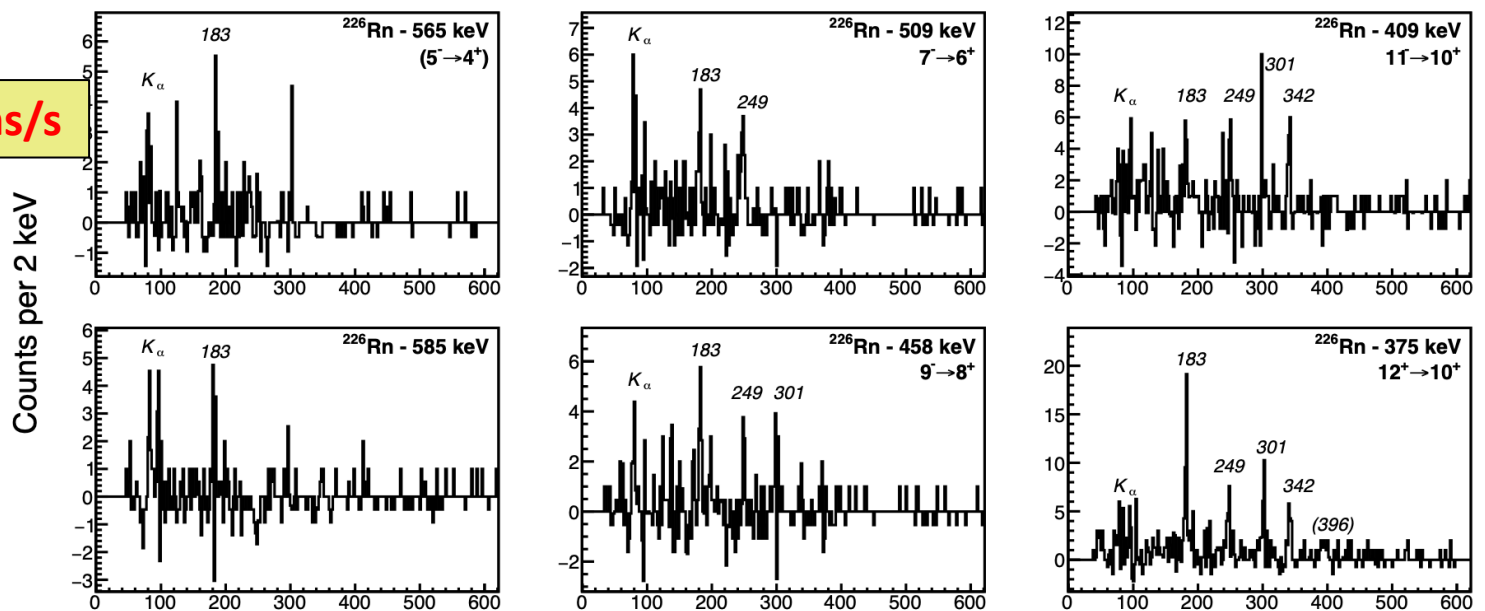
CEA Saclay

# $\gamma$ - $\gamma$ spectra: gates on transitions in $^{224,226}\text{Rn}$

$10^5$   $^{224}\text{Rn}$  ions/s



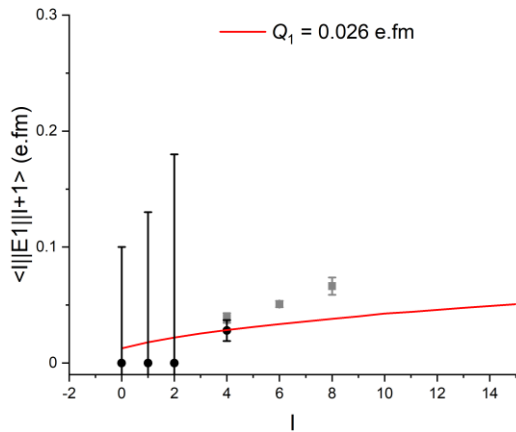
$2 \times 10^3$   $^{226}\text{Rn}$  ions/s



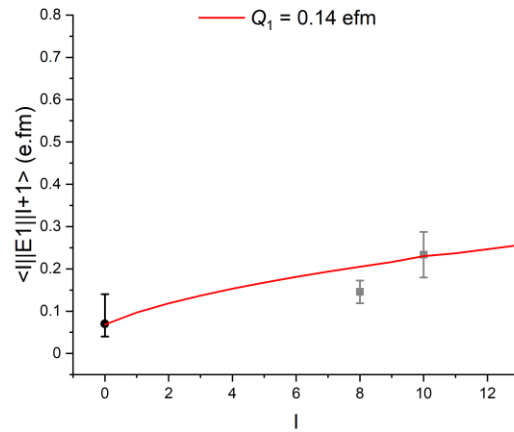
Energy (keV)

# Behaviour of $Q_1$ with $I_i, I_f$

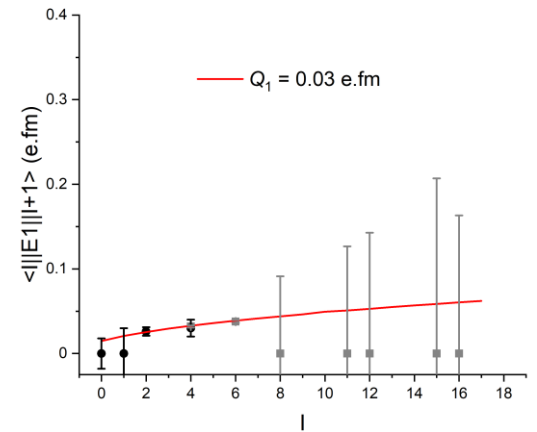
$^{220}\text{Rn}$



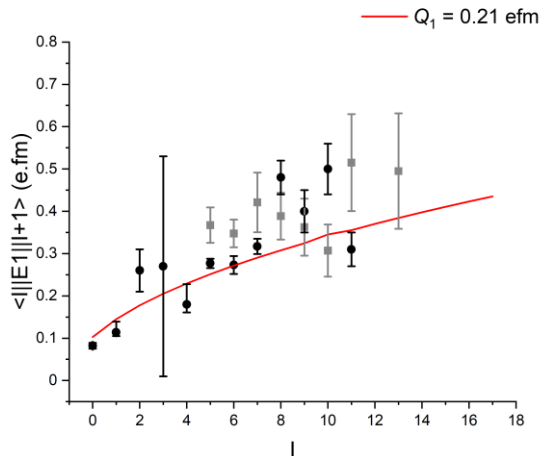
$^{222}\text{Rn}$



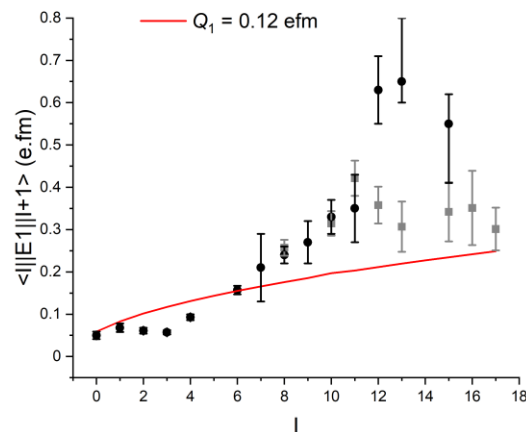
$^{224}\text{Ra}$



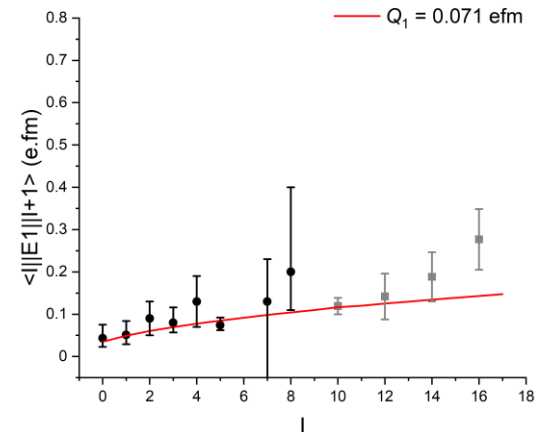
$^{222}\text{Ra}$



$^{226}\text{Ra}$



$^{228}\text{Ra}$



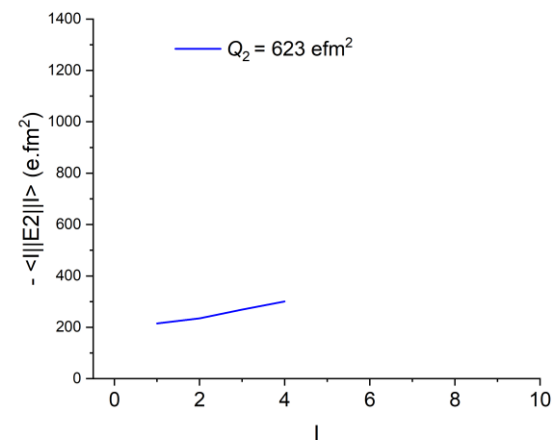
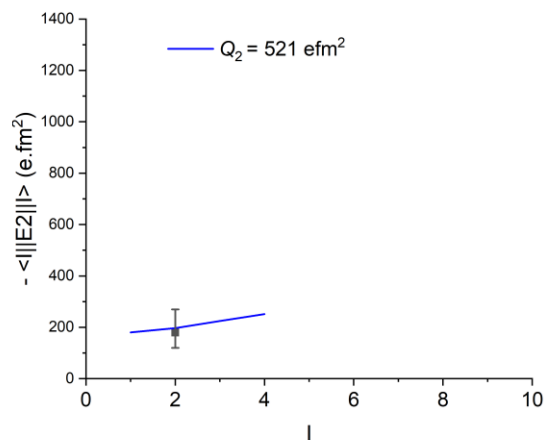
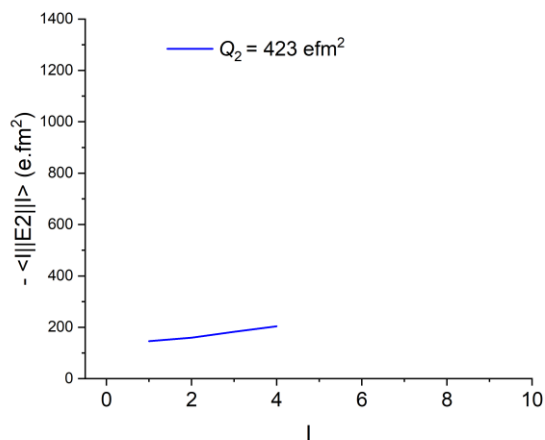
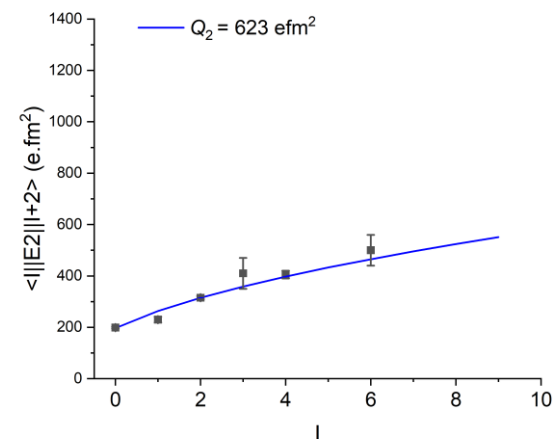
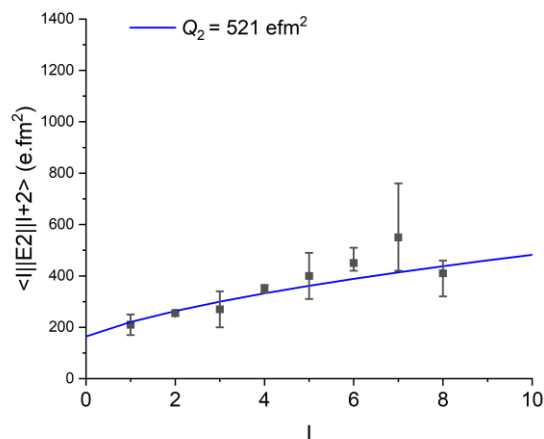
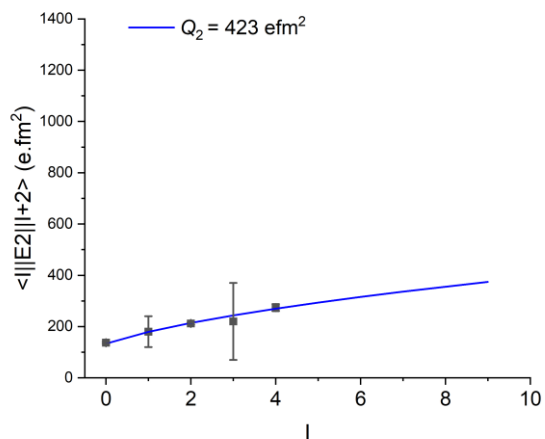
$$\langle I_i || \mathcal{M}(E1) || I_f \rangle = \sqrt{3/4\pi (2I_i + 1) (I_i 010 | I_f 0) Q_1}$$

# Behaviour of $Q_2$ with $I_i, I_f$

$^{220}\text{Rn}$

$^{222}\text{Rn}$

$^{224}\text{Ra}$



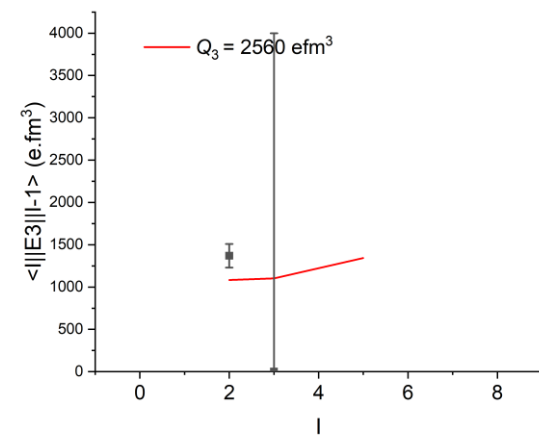
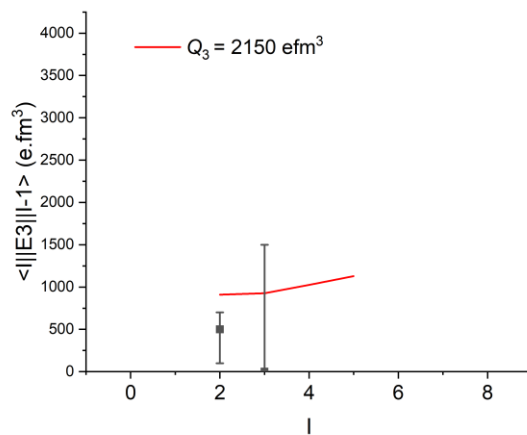
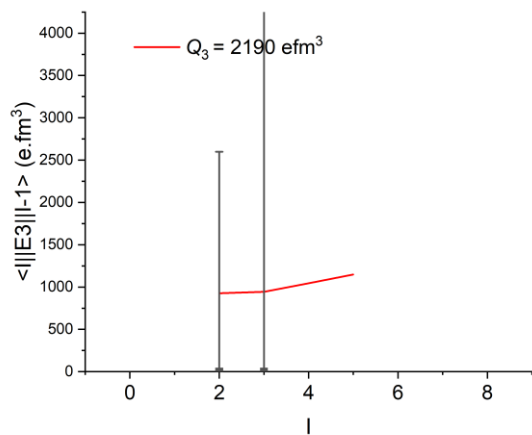
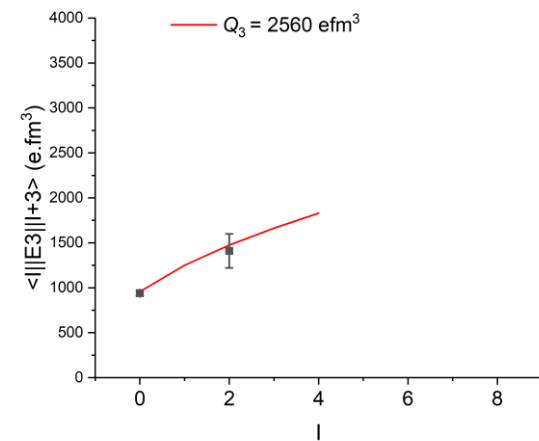
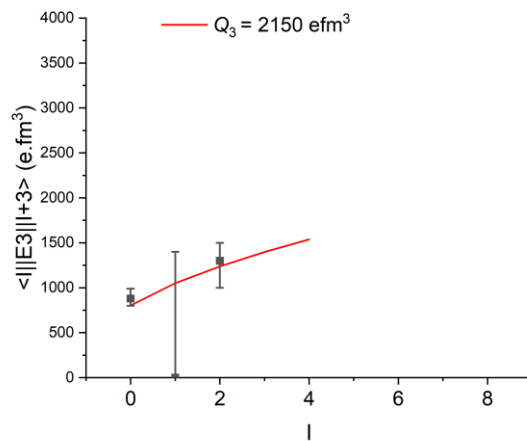
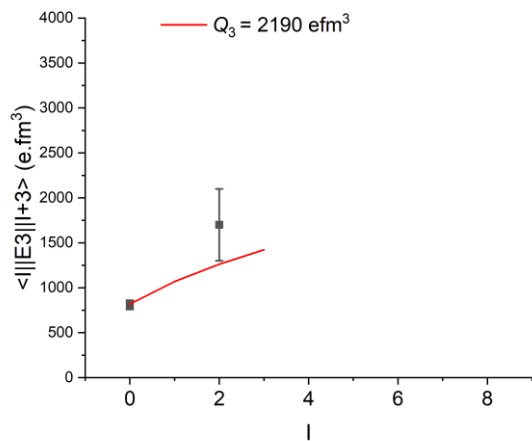
$$\langle I_i || \mathcal{M}(E2) || I_f \rangle = \sqrt{5/16\pi (2I_i + 1)} (I_i 020 | I_f 0) Q_2$$

# Behaviour of $Q_3$ with $I_i, I_f$

$^{220}\text{Rn}$

$^{222}\text{Rn}$

$^{224}\text{Ra}$



$$\langle I_i || \mathcal{M}(E3) || I_f \rangle = \sqrt{7/16\pi (2I_i + 1)} (I_i 030 | I_f 0) Q_3$$