Evolution of nuclear structure in and around semi-magic nuclei

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Outline

- **Seniority isomers**
- **Isomers in Sn isotopes**
- **Generalized seniority on going from Cd to Sn to Te isotopes**
- **Conclusion**

Nuclear Isomers

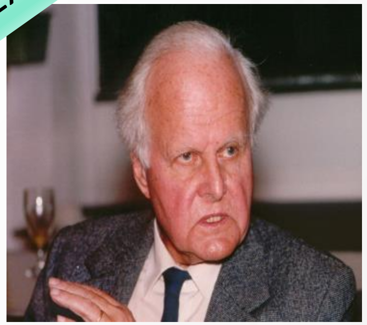
Predicted

F. Soddy

Observed

O. Hahn

Explained



C. F. von Weizsäcker

2015: 2469 isomers & Cutoff date: 15th Aug 2015

2022: 2620 isomers (S. Garg, B. Maheshwari, B. Singh, Y. Sun, A. Goel and A. K. Jain) & Cutoff date: 31st April 2022

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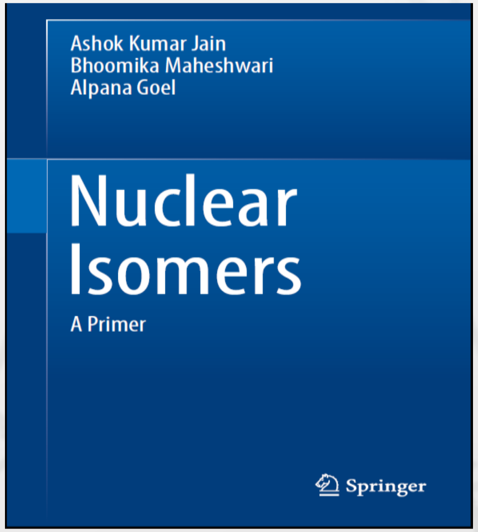
100 years of nuclear isomers—then and now

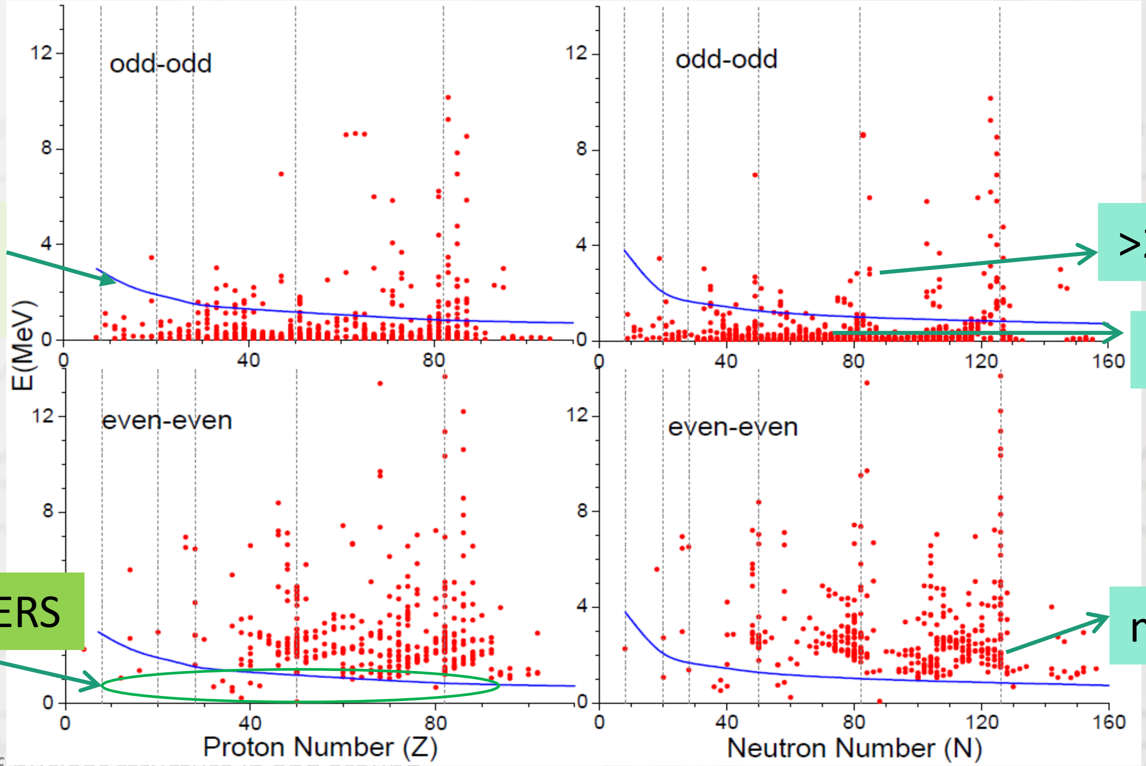
Philip Walker[1] and Zsolt Podolyák
Department of Physics, University of Surrey, Guildford GU2 7XH, United Kingdom



Even-A isomers

Pairing Gap line

E0 ISOMERS



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Nuclear Data Sheets 128 (2015) 1–130
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ATLAS OF NUCLEAR ISOMERS

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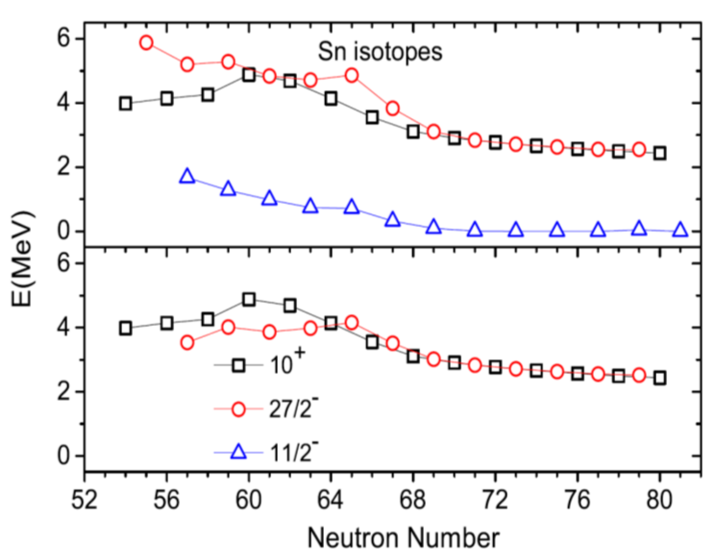
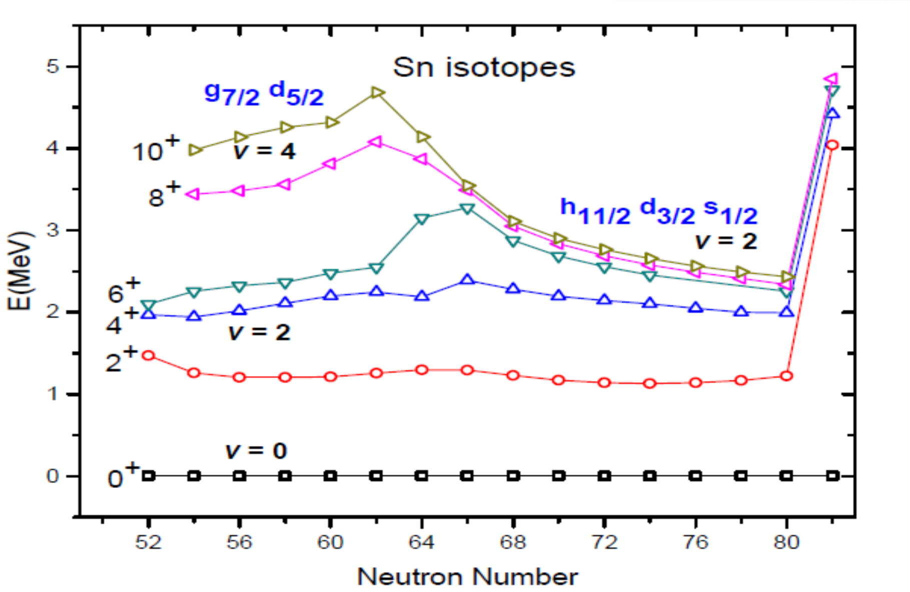
Sn isotopes

10$^+$ in even-A Sn and 27/2$^-$ states in odd-A Sn follow each other

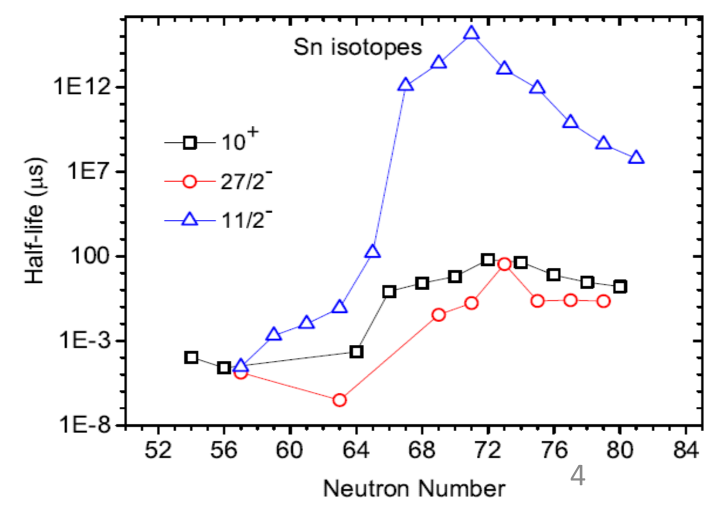


- Longest isotopic chain
- Two doubly-closed shells and beyond

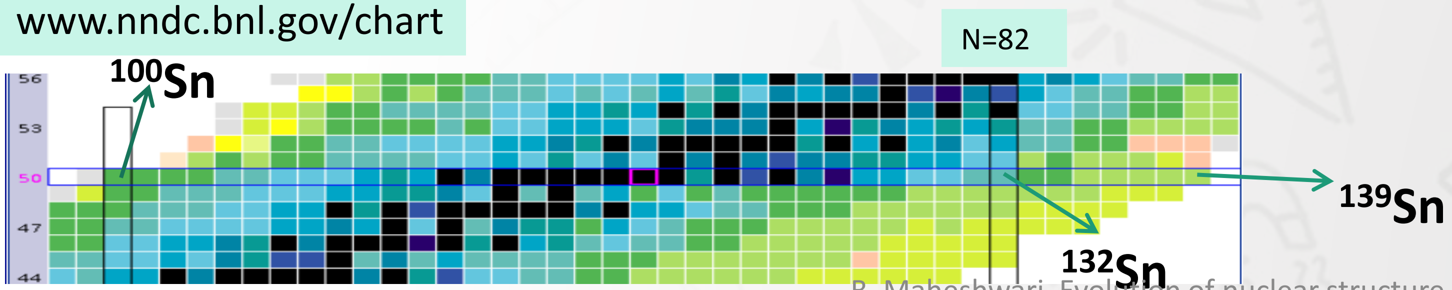


The key features of seniority seem to persist in Sn isotopes throughout the full valence space up to j=11/2, where seniority mixing may take place. Generalized Seniority is hence needed.

Half-life peaks at N=72 for 10$^+$ and N=73 for 27/2$^-$ states



www.nndc.bnl.gov/chart



Seniority: Let us recap

G. Racah, Phys. Rev. 63, 367 (1943); ibid. 76, 1352 (1949)

- Introduced by Racah (1943) in the atomic context.
- To distinguish states having same L, S and J values but arising from different number of unpaired particles.
- Adopted for nuclei in a similar fashion in the 1950s.
- Seniority quantum number (*v*) may be loosely defined as the number of unpaired nucleons to generate any given state.
    - For even-even and odd-odd nuclei, seniority is even.
    - For odd-even, or even-odd nuclei, seniority is odd.

*In Racah's words: "these `saturated' pairs, being coupled to zero angular momentum, possess particular symmetry properties and either do not contribute at all or contribute in a very simple way to many characteristic properties of a state, like angular momentum and multipole moments. A number of saturated pairs can, therefore, be added to any given system of particles without changing these properties very much. Thus, **we can consider a state as consisting of a number, v, of non-saturated particles which determines most of the properties of the state, plus a number of saturated pairs**. We can construct a series of states for systems containing v, v + 2, v + 4, etc. particles, each differing from the others only by the number of additional saturated pairs and all having very similar properties. The number v of non-saturated particles is called the seniority number because it gives the smallest number of particles needed for building a state with a given set of properties and therefore specifies the simplest configuration which contains such a state".*

Basic algebra & notations

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$$A^\dagger(jj;00) = \frac{1}{\sqrt{2}} \sum_{m_1,m_2} \langle jm_1 jm_2 | jj00 \rangle a^\dagger_{jm_1} a^\dagger_{jm_2} = \frac{1}{\sqrt{2}} \sum_m \frac{(-1)^{j-m}}{\sqrt{2j+1}} a^\dagger_{jm} a^\dagger_{j,-m}$$

Pair creation operator

$$A(jj:00) = \frac{1}{\sqrt{2}} \sum_m \frac{(-1)^{j-m}}{\sqrt{2j+1}} a_{j,-m} a_{jm}$$

Pair annihilation operator

$$A^\dagger(jj;00) = \sqrt{\frac{1}{\Omega}} \sum_{m>0} (-1)^{j-m} a^\dagger_{jm} a^\dagger_{j,-m}$$

$$A(jj:00) = \sqrt{\frac{1}{\Omega}} \sum_{m>0} (-1)^{j-m} a_{j,-m} a_{jm}$$

With pair degeneracy Ω

$$S_j^+ = \sqrt{\Omega} A^\dagger = \sum_{m>0} (-1)^{j-m} a^\dagger{}_{jm} a^\dagger{}_{j,-m}$$

$$S_j^- = \sqrt{\Omega} A = \sum_{m>0} (-1)^{j-m} a_{j,-m} a_{jm}$$

Quasi-spin operators
SU(2) Lie algebra

$S^0 = (n-\Omega)/2$, n being number operator

Kerman, Ann. Phys. 12, 300 (1961); Helmers, NP23, 594 (1961)

$$\hat{H}_{pair} = -G S_j^+ S_j^-$$

Pairing Hamiltonian

$$\hat{H}_{pair} = -G \sum_{m,m'>0} (-1)^{2j-m-m'} a^\dagger{}_{jm} a^\dagger{}_{j,-m} a_{j,-m'} a_{jm'}$$

$$\langle j^2; JM | \hat{H}_{pair} | j^2; JM \rangle = \frac{-G}{2} (2j+1) \delta_{J0} \delta_{M0}$$

Pairing energy

where quasi-spin s = ½ (Ω-v); v being the seniority

Key features

Particle number independent energy of a state having same J^π and v

Specific EM properties for single-j orbital emerge:

- A parabolic behavior of electric quadrupole transition probabilities, and
- A particle number independent variation of magnetic moments

Any interaction between identical fermions in single-j shell exactly conserves seniority if j≤7/2, since there is no chance of seniority mixing for such states.

Electromagnetic properties (single-j)

The reduced transition probability

$$B(EL) = \frac{1}{2J_i+1}\left|\left(J_f \left\| \sum_i r_i^L Y^{(L)}(\theta_{i,}\phi_i \right\| J_i \right)\right|^2$$

For L even, κ=0 component of quasi-spin vector: electric transitions

For Δν=0, seniority preserving transitions

$$\left\langle j^n v l J_f \left\| \sum_i r_i^L Y^{(L)}(\theta_{i,}\phi_i) \right\| j^n v l' J_i \right\rangle = \left(\frac{\Omega-n}{\Omega-v}\right)\left\langle j^v v l J_f \left\| \sum_i r_i^L Y^{(L)}(\theta_{i,}\phi_i) \right\| j^v v l' J_i \right\rangle$$

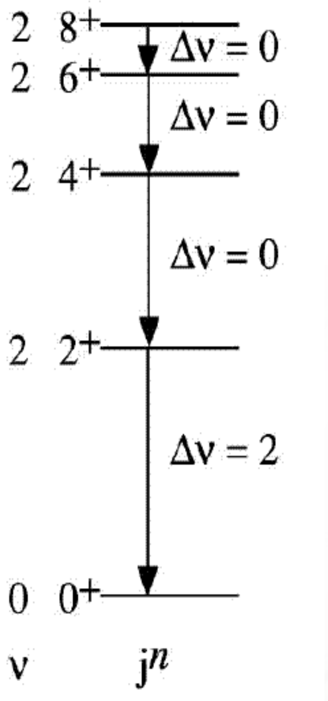
For Δν=2, seniority changing transitions

$$\left\langle j^n v l J_f \left\| \sum_i r_i^L Y^{(L)}(\theta_{i,}\phi_i) \right\| j^n, v\mp 2, l' J_i \right\rangle = \sqrt{\frac{(n-v+2)(2\Omega+2-n-v)}{2(2\Omega+2-2v)}}\left\langle j^v v l J_f \left\| \sum_i r_i^L Y^{(L)}(\theta_{i,}\phi_i) \right\| j^v, v\mp 2,' l' J_i \right\rangle$$

For L odd, quasi-spin scalar, only Δν=0: magnetic transitions

$$\langle j^n v J_f \| O(ML) \| j^n v J_i \rangle = \langle j^v v J_f \| O(ML) \| j^v v J_i \rangle$$

8^+ isomers in N=50 isotones



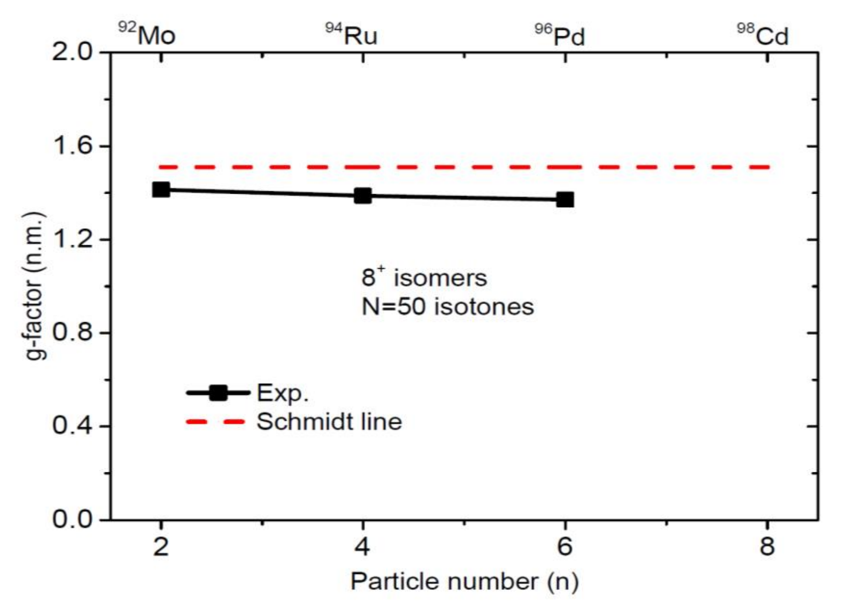
j=9/2

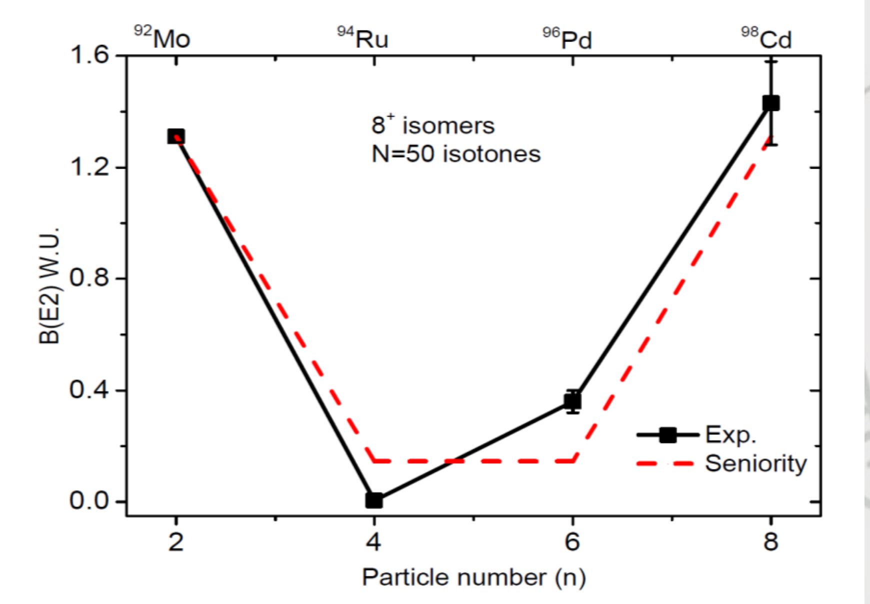
Magnetic Moment and Seniority

$$\langle j^n|\hat{\mu}|j^n\rangle = \langle j^v|\hat{\mu}|j^v\rangle$$

$$\vec{\mu} = \sum_i^n g\vec{j}_i = g\sum_i^n \vec{j}_i = g\vec{J}$$

Dominance of $\pi g_{9/2}$ orbital
Single-j seniority works well
Schmidt line explains the magnetic moments





Generalized Seniority:
Arima and Ichimura (1966)
Talmi (1971)
Shlomo and Talmi (1972)

Multi-j Generalized Seniority

Multi-j pair creation operator: $S^+ = \sum_j (-1)^{l_j} S_j^+$ as proposed by Arvieu and Moszokowski (1966)

pair creation operator: $S_j^+ = \sum_m (-1)^{j-m} a_{jm}^+ a_{j,-m}$

Effective multi-j: $\tilde{j} = j \otimes j' \otimes \ldots.$ $H = 2S^+S^-$:Pairing Hamiltonian

Pairing Energy Solution: $$2s(s+1) - \frac{1}{2}(\Omega - n)(\Omega + 2 - n) = \frac{n-v}{2}(2\Omega + 2 - n - v)$$

Quasi spin: $s = \frac{1}{2}(\Omega - v)$ $s = \sum_j s_j$

Pair degeneracy: $\Omega = \sum_j \frac{2j+1}{2} = \frac{2\tilde{j}+1}{2}$ $n = \sum_j n_j$:Total number of nucleons

Generalized Seniority: $v = \sum_j v_j$

Electromagnetic transitions and moments

$$[S^+, T_\kappa^{(k)}] = \frac{1}{\sqrt{2k+1}} \sum_{j<j'} \left[1+(-1)^k\right] \left(j \left\| T^{(k)} \right\| j'\right) \left(a_j^+ \times a_{j'}^+\right)_\kappa^{(k)}$$

For k odd, the tensor behaves as a quasi-spin scalar.

For k even, the tensor behaves as a κ =0 component of the quasi-spin vector.

$l+l'+L$ always remains even, for any *L* value (odd or even) in electric transitions. l and l' controls the parity of the initial and final states of the transition. *L* specifies the nature of the transition.

For *L* odd, $l+l'$ = odd.
For *L* even, $l+l'$= even.

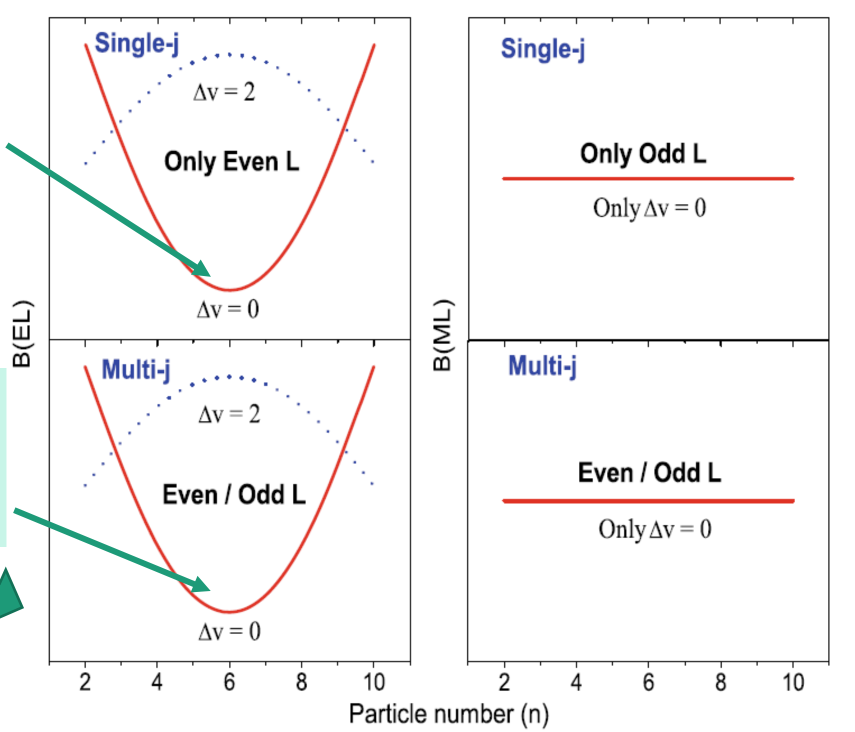
Hence, electric transitions, for both the odd and even L, now behave as the κ=0 component of the quasi-spin vector.

The magnetic transitions, on the other hand, for both even and odd L, behave as quasi-spin scalar due to $l+l'+L$ =odd.

Seniority and generalized seniority

E2 seniority isomers

E1,E2,….. seniority isomers



(a) For generalized seniority conserving ($\Delta v = 0$) transitions

$$\langle \tilde{j}^n v l J || \sum_i r_i^2 Y^2(\theta_i, \phi_i) || \tilde{j}^n v l J \rangle = \left[\frac{\Omega - n}{\Omega - v} \right] \langle \tilde{j}^v v l J || \sum_i r_i^2 Y^2(\theta_i, \phi_i) || \tilde{j}^v v l J \rangle$$

(b) For generalized seniority changing ($\Delta v = 2$) transitions

$$\langle \tilde{j}^n v l J || \sum_i r_i^2 Y^2(\theta_i, \phi_i) || \tilde{j}^n v \pm 2 l J \rangle = \left[\sqrt{\frac{(n - v + 2)(2\Omega + 2 - n - v)}{4(\Omega + 1 - v)}} \right] \langle \tilde{j}^v v l J || \sum_i r_i^2 Y^2(\theta_i, \phi_i) || \tilde{j}^v v \pm 2 l J \rangle$$

Single-j shell

Seniority: Odd L → quasi-spin scalar → Δ*v* = 0 Even L → quasi-spin vector → Δ*v* = 0, 2

EM decay: Odd L → only magnetic transitions Even L → only electric transitions

Multi-j shell

Generalized seniority (GS):

Odd/Even L → Magnetic transitions → quasi-spin scalar → Δ*v* = 0

Odd/Even L → Electric transitions → quasi-spin vector → Δ*v* = 0, 2

EM decay selection rules become redundant

Odd/Even L → both electric/ magnetic transitions.

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Exp.: Phys. Rev. Lett. 68, 11 (1992), Phys. Rev. C 62, 057305 (2000)

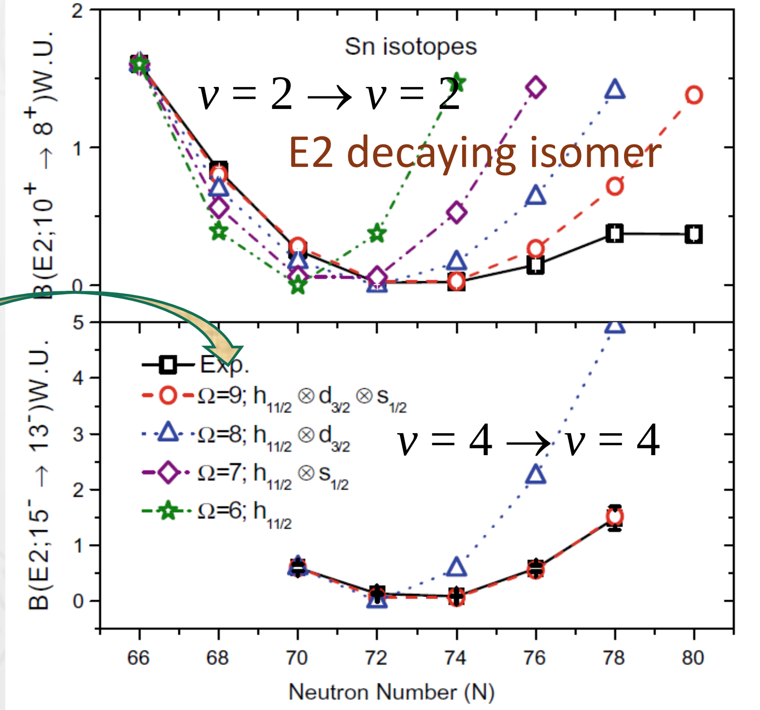
Seniority isomers

Established new type of isomer and the new selection rules

B. Maheshwari and A. K. Jain, Physics Letters B 753, 122 (2016)

N=64 is assumed to be core by freezing $g_{7/2}$ and $d_{5/2}$.

Active orbitals
$h_{11/2}$
$d_{3/2}$
$s_{1/2}$



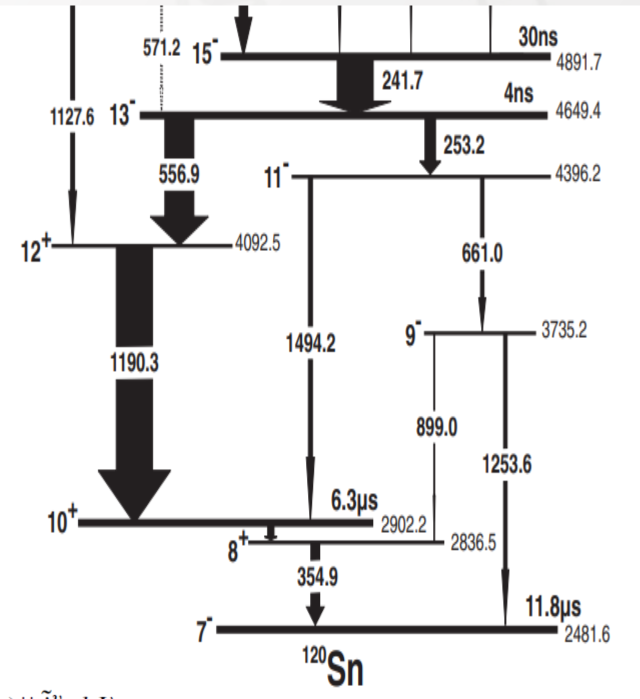
Exp.: Iskra et al., Phys. Rev. C 89, 044324 (2014)

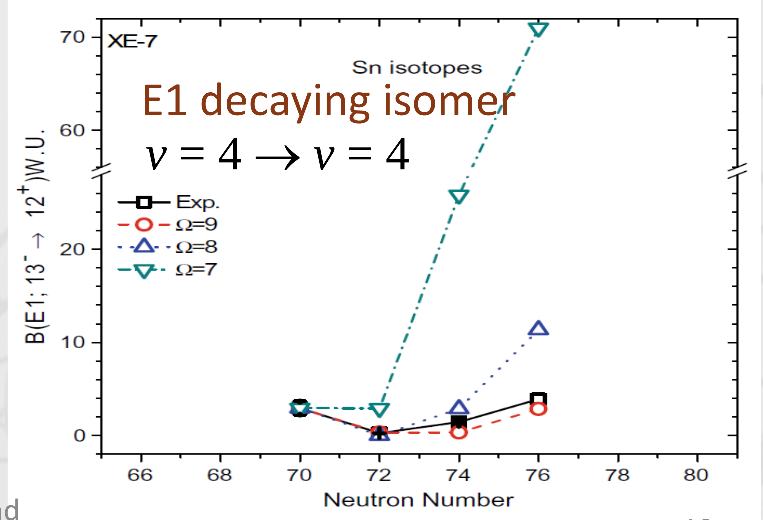
10^+ isomers: well known case of seniority v=2 isomers but the role of configuration mixing is found to be crucial

15^- isomers: seniority v=4 isomers, E2 decaying to the 13^- states

13^- isomers: seniority v=4 isomers, E1 decaying to the 12^+ states

Phys. Rev. C 89, 044324 (2014)





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Generalized Seniority Schmidt Model (GSSM)

Magnetic moments in generalized seniority

$$\langle \tilde{j}^n | \hat{\mu} | \tilde{j}^n \rangle = \langle \tilde{j}^v | \hat{\mu} | \tilde{j}^v \rangle$$

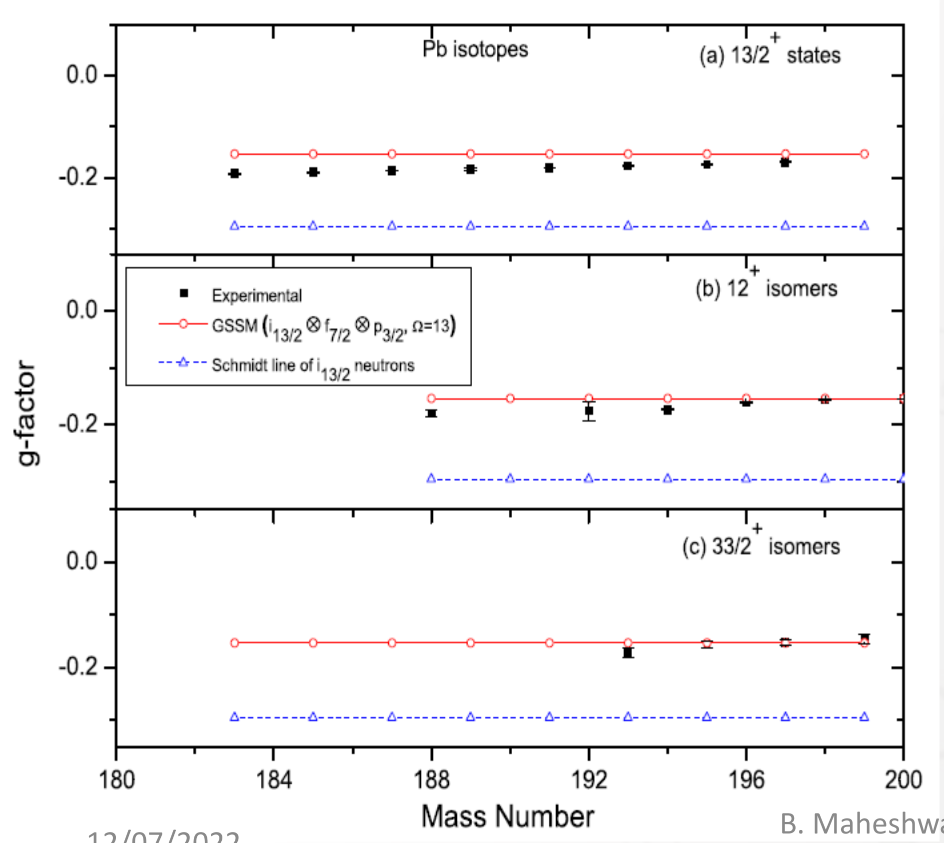
Multi-j →

$$\vec{\mu} = g \sum_i^n \vec{\tilde{j}}_i = g \vec{J}$$

Multi-j Clubbed with Schmidt model →

$$g = \frac{1}{\tilde{j}} \left[\frac{1}{2} g_s + (\tilde{j} - \frac{1}{2}) g_l \right]; \tilde{j} = \tilde{l} + \frac{1}{2}$$

$$= \frac{1}{\tilde{j}+1} \left[-\frac{1}{2} g_s + (\tilde{j} + \frac{3}{2}) g_l \right]; \tilde{j} = \tilde{l} - \frac{1}{2}$$



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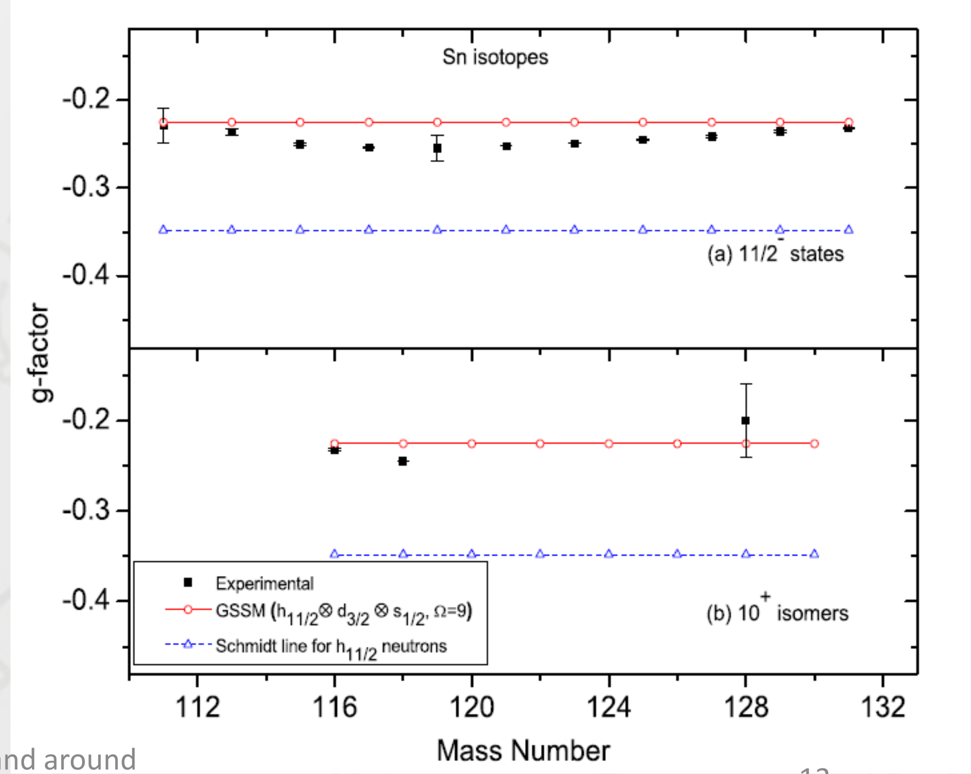
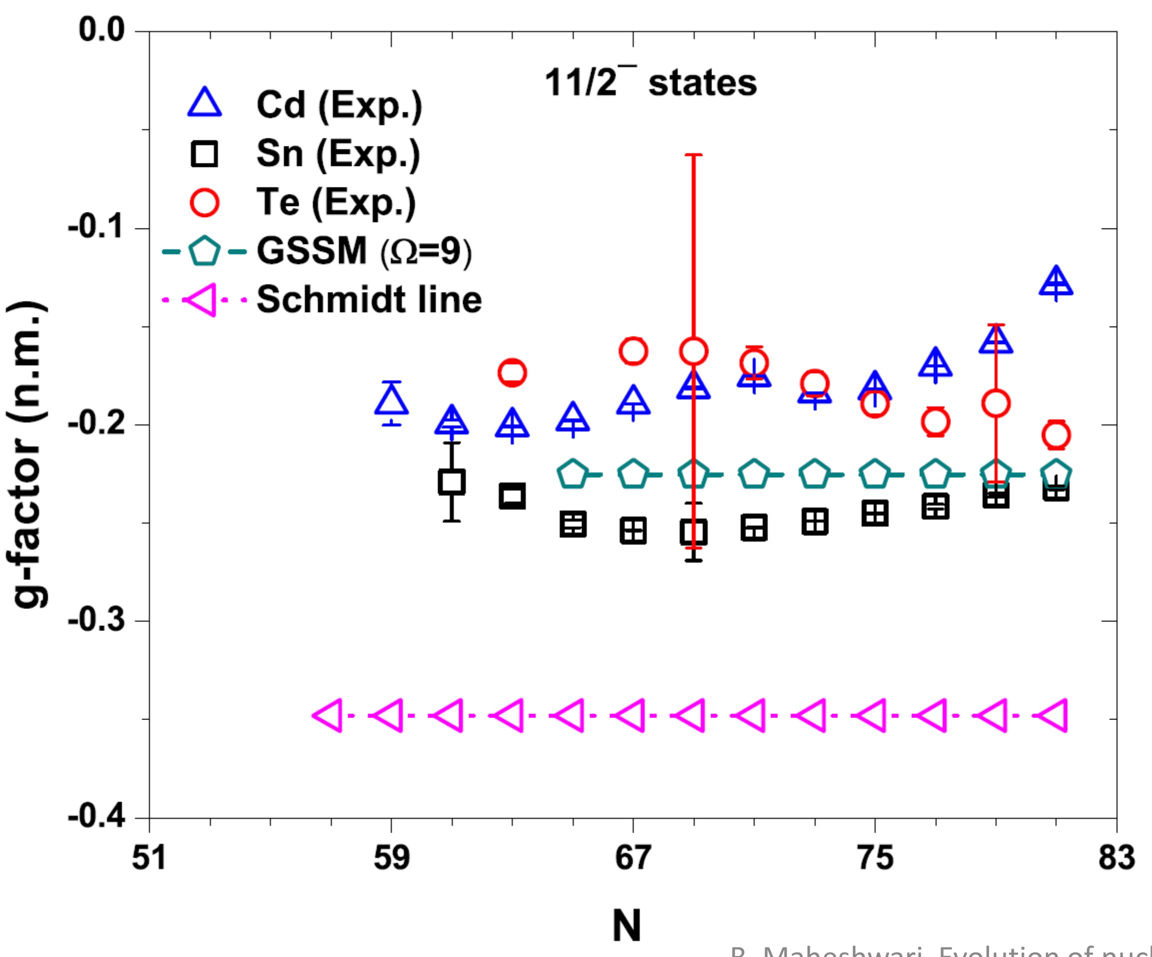
B. Maheshwari, A.K. Jain / Nuclear Physics A 986 (2019) 232–244



GSSM in Cd, Sn and Te isotopes

B. Maheshwari et al., NPA 992 (2019)

Exp. Data taken from Stone's table

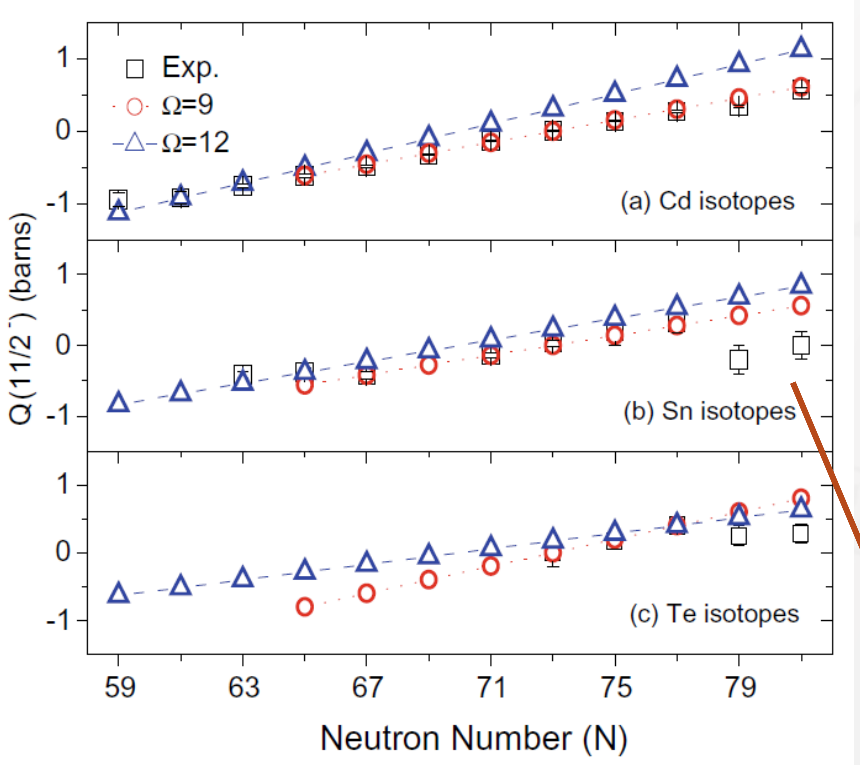


- GSSM comes closer to the experimental data for the g-factors of 11/2- states in all the three Cd, Sn and Te isotopic chains.
- Schmidt line stay far from the measured data in all the three chains.
- This is due to the involved configuration mixing corresponding to Ω=9 pair degeneracy, eventually leading to spin quenching as provided by any first-order perturbation theory.
- Interestingly, GSSM results do not need any kind of tuning to estimate the amount of spin quenching for explaining the experimental data.
- In GSSM, the spin quenching is governed by the multi- j configuration as suggested by generalized seniority, which consistently explains other nuclear properties also.

Moments in Cd, Sn and Te isotopes

Cd [Exp.]: Yordanov et al., Phys. Rev. C 98,011303(R) (2018)

Exp. Data taken from Stone's table



B. Maheshwari et al., NPA 986 (2019); NPA 992 (2019)

- $11/2^-$, 10^+ and $27/2^-$ isomers
- Active valence space:

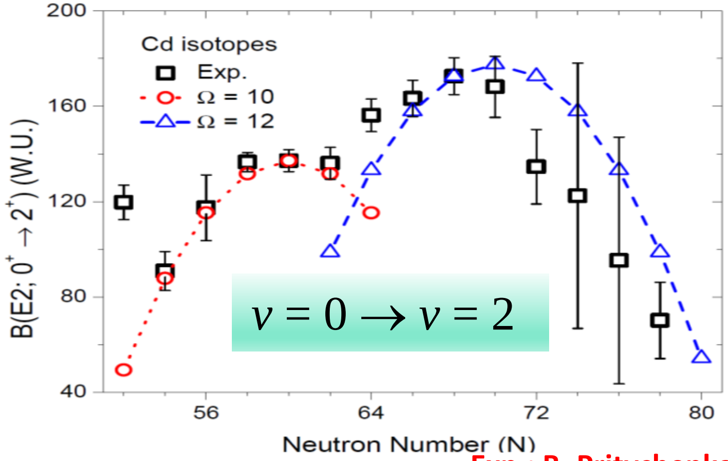
$$g_{7/2}\ d_{5/2}\ d_{3/2}\ s_{1/2}\ h_{11/2}$$

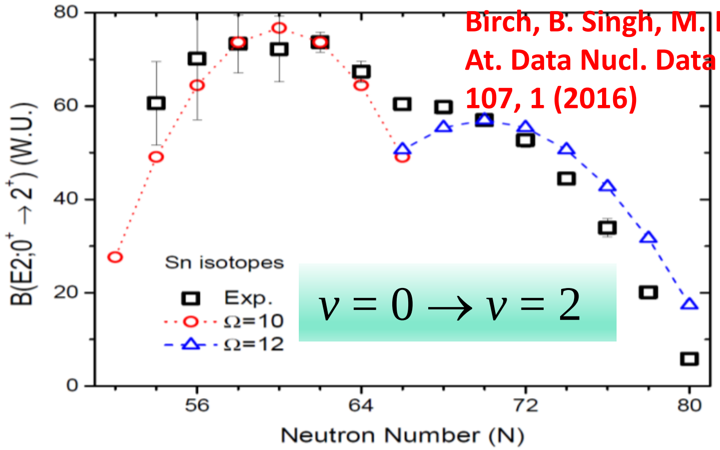
Here, the used configuration mixings (as in the case of 10^+ Sn-isomers):

- $\Omega = 9$: $d_{3/2} \otimes s_{1/2} \otimes h_{11/2}$
- $\Omega = 12$: $d_{5/2} \otimes d_{3/2} \otimes s_{1/2} \otimes h_{11/2}$

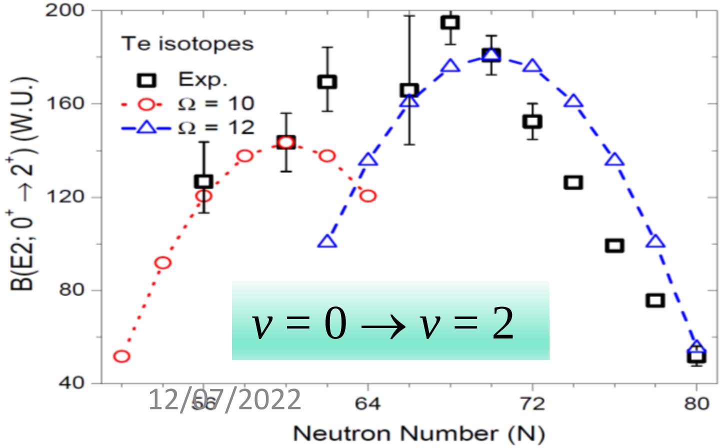
$$Q \propto \left(\frac{\Omega - n}{\Omega - v}\right)$$

New data available for these Sn isotopes;
Yordanov et al., Nature Comm. Phys. 3, 107 (2020)
Revisit to the calculations would be interesting.





Exp.: B. Pritychenko, M. Birch, B. Singh, M. Horoi, At. Data Nucl. Data Tables 107, 1 (2016)



Cd, Sn and Te isotopes

- First 2^+ and 3^- states are important to understand the low-lying excitations and nuclear configurations.
- Active valence space:

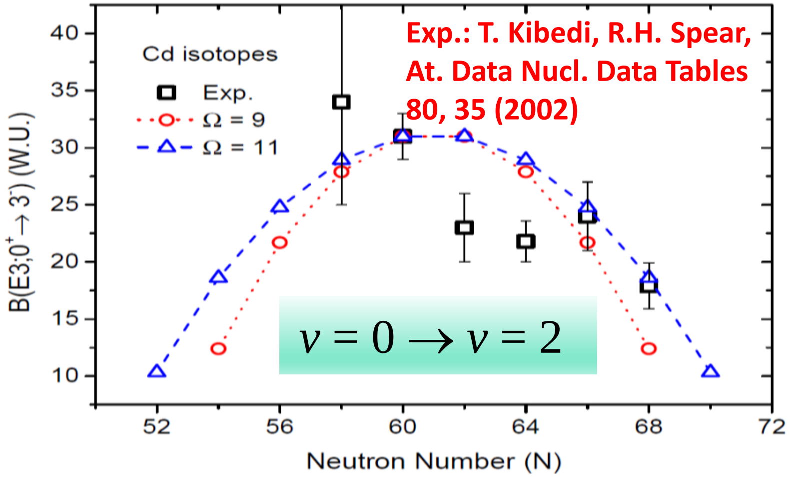
$g_{7/2}\ d_{5/2}\ d_{3/2}\ s_{1/2}\ h_{11/2}$

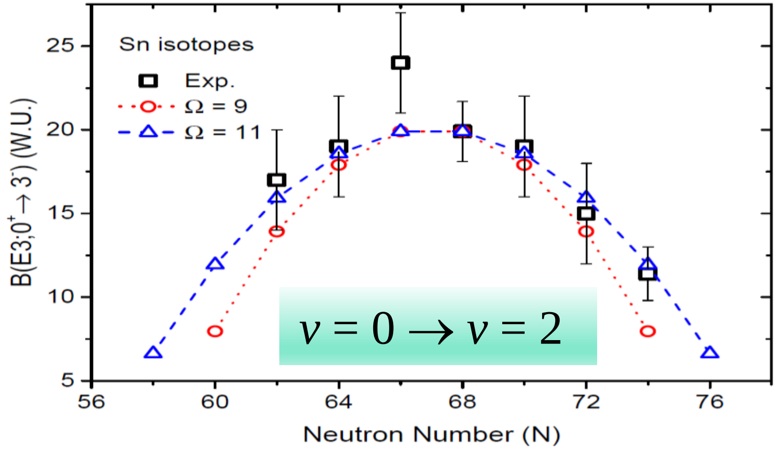
Here, the used configuration mixings are:

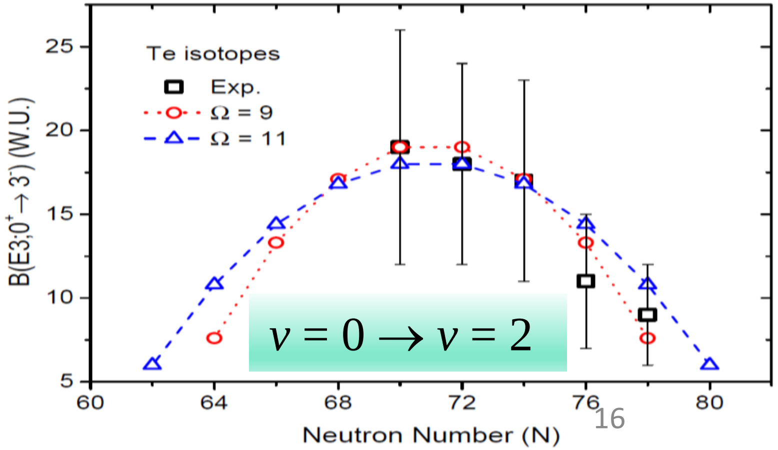
- $\Omega = 10$: $g_{7/2} \otimes d_{5/2} \otimes d_{3/2} \otimes s_{1/2}$
- $\Omega = 12$: $d_{5/2} \otimes d_{3/2} \otimes s_{1/2} \otimes h_{11/2}$
- $\Omega = 9$: $d_{5/2} \otimes h_{11/2}$
- $\Omega = 11$: $d_{5/2} \otimes d_{3/2} \otimes h_{11/2}$

B. Maheshwari, European Physical Journal Special Topics 229, 2485 (2020)

Exp.: T. Kibedi, R.H. Spear, At. Data Nucl. Data Tables 80, 35 (2002)







Conclusion & outlook

- For 100 years, isomers have been enriching the concepts of nuclear structure, for both the single-particle and collective motions and continue to be pivotal in contemporary nuclear research providing chance to explore the unknown territory of nuclear chart.
- Generalized Seniority is a simple but an amazing tool to understand the seniority isomerism as well as low-lying excitations in and around semi-magic nuclei.
- Similarities in various mass regions have been seen due to these generalized seniority symmetries; though the involved valence spaces are quite different.
- Role of configuration mixing is found to be crucial.
- Predictions can be made.
- Understanding isomerism and moments for the nuclei situated at the boundary between the generalized seniority and collective excitations would be interesting.

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Prof. P. M. Walker, Uni of Surrey, UK

Prof. B. K. Agrawal, SINP, India

Dr. S. Garg, SJTU, China

Prof. R. Palit, TIFR, India

Prof. V. K. B. Kota, PRL, India

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………….

HRZZ Hrvatska zaklada za znanost

ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

Thank you
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