

# MICROSCOPIC DESCRIPTION OF OCTUPOLE DEFORMATIONS IN EVEN- EVEN Xe AND Ba ISOTOPES

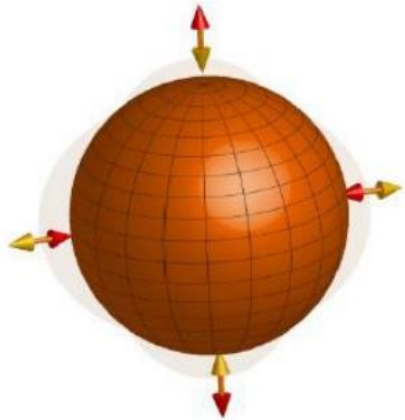
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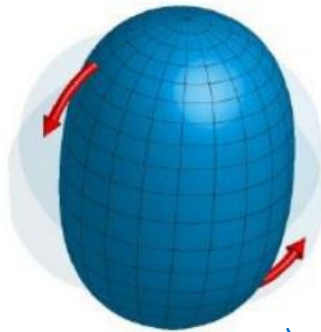
+ 11th of July, 2022

+Introduction:

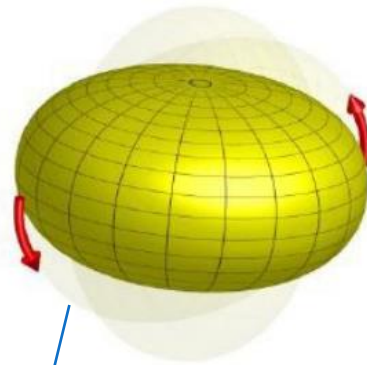
- Octupole deformations: N or Z near 34, 56, 88, 134



a) Spherical nucleus

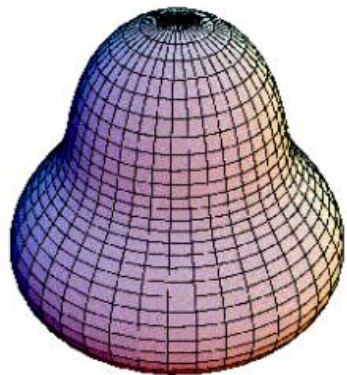


b) Prolate



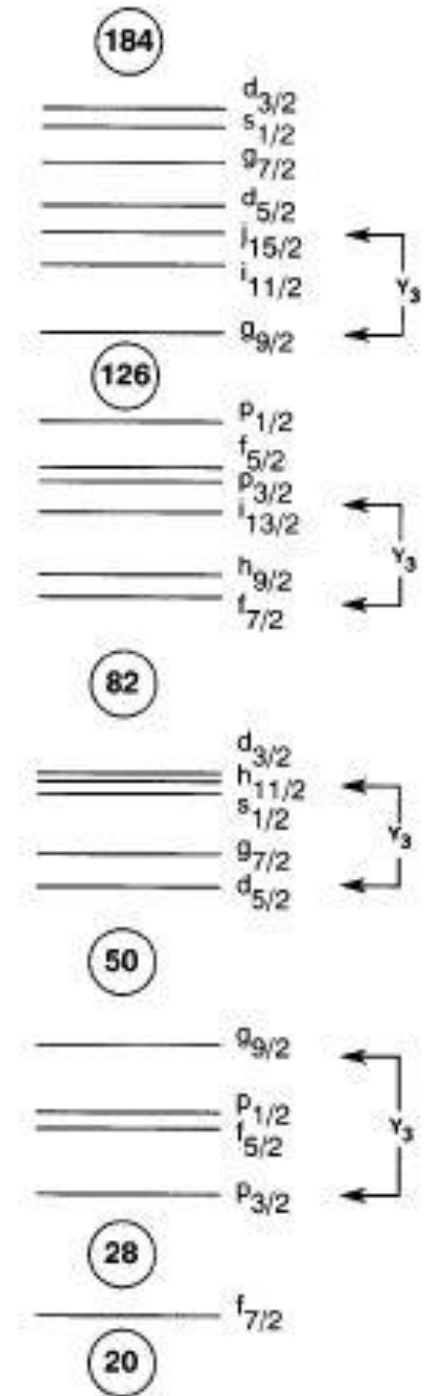
c) Oblate

Quadrupole deformations



Octupole deformations

Pear-shaped



## + Introduction:

- The goal: exploration of octupole correlations and intrinsic octupole shapes for even-even Xe ( $Z=54$ ) and Ba ( $Z=56$ ) isotopes in the region  $N \approx 56$  and  $N \approx 88$ , calculation of low-lying energy spectra and transition strengths  $B(E1)$ ,  $B(E2)$ ,  $B(E3)$
- Octupole collectivity in the region  $N \approx Z \approx 56$  has not been as extensively studied as in actinides ( $N \approx 134$ ,  $Z \approx 88$ ) and lanthanides ( $N \approx 88$ ,  $Z \approx 56$ )
- Calculations: SCMF calculations with the DD-PC1 functional
- Extension of SCMF: Quadrupole-octupole collective Hamiltonian (QOCH)

+SCMF calculations:

- Relativistic Hartree - Bogoliubov (RHB) method:
- DD-PC1 functional + the separable pairing force of finite range
- Axially symmetric quadrupole and octupole moments:

$$\hat{Q}_{20} = 2z^2 - x^2 - y^2$$
$$\hat{Q}_{30} = 2z^3 - 3z(x^2 + y^2)$$

- Quadrupole and octupole deformation parameters:

$$\beta_2 = \frac{\sqrt{5\pi}}{3r_0^2 A^{5/3}} \langle \hat{Q}_{20} \rangle$$

$$\beta_3 = \frac{\sqrt{7\pi}}{3r_0^3 A^2} \langle \hat{Q}_{30} \rangle$$

RHB: D. Vretenar, A. V. Afanasjev,  
G. A. Lalazissis, and P. Ring, Phys.  
Rep. 409, 101 (2005)

DD-PC1: T. Nikšić, D. Vretenar, and  
P. Ring, Phys. Rev. C 78, 034318  
(2008).

+QOCH calculations:

- Hamiltonian:

$$\hat{H}_{coll} = T_{vib} + T_{rot} + V_{coll}$$

- Vibrational kinetic energy:

$$T_{vib} = \frac{1}{2} B_{22} \dot{\beta}_2^2 + B_{23} \dot{\beta}_2 \dot{\beta}_3 + \frac{1}{2} B_{33} \dot{\beta}_3^2$$

- Rotational kinetic energy:

$$T_{rot} = \frac{1}{2} \sum_{k=1}^3 \mathcal{I}_k \omega_k^2$$

S. Y. Xia, H. Tao, Y. Lu, Z. P. Li, T. Nikšić, and  
D. Vretnar, Phys. Rev. C 96, 054303  
(2017).

+ QOCH calculations:

- Collective potential: includes zero-point energy (ZPE) corrections

- Collective Hamiltonian after quantization:

$$\hat{H}_{coll} = -\frac{\hbar^2}{2\sqrt{\omega I}} \left[ \frac{\partial}{\partial \beta_2} \sqrt{\frac{\mathfrak{I}}{\omega}} B_{33} \frac{\partial}{\partial \beta_2} - \frac{\partial}{\partial \beta_2} \sqrt{\frac{\mathfrak{I}}{\omega}} B_{23} \frac{\partial}{\partial \beta_3} - \frac{\partial}{\partial \beta_3} \sqrt{\frac{\mathfrak{I}}{\omega}} B_{23} \frac{\partial}{\partial \beta_2} + \frac{\partial}{\partial \beta_3} \sqrt{\frac{\mathfrak{I}}{\omega}} B_{22} \frac{\partial}{\partial \beta_3} \right] + \frac{\hat{j}^2}{2\mathfrak{I}} + V_{coll}(\beta_2, \beta_3)$$

- Mass parameters are defined as the inverse of mass tensors:  $B_{ij}(\mathbf{q}) = \mathcal{M}_{ij}^{-1}(\mathbf{q})$

- Cranking approximation:

$$\mathcal{M}^{Cp} = \hbar^2 M_{(1)}^{-1} M_{(3)} M_{(1)}^{-1}$$

$$[M_{(k)}]_{ij} = \sum_{\mu\nu} \frac{\langle 0 | \hat{Q}_i | \mu\nu \rangle \langle \mu\nu | \hat{Q}_j | 0 \rangle}{(E_\mu + E_\nu)^k}$$

+ QOCH calculations:

- ZPE energy:

$$E_{ZPE} = \frac{1}{4} \text{Tr} [M_{(2)}^{-1} M_{(1)}]$$

- The basis functions:

$$|n_2 n_3 IMK\rangle = (\omega \mathfrak{I})^{-\frac{1}{4}} \phi_{n_2}(\beta_2) \phi_{n_3}(\beta_3) |IMK\rangle$$

1D harmonic oscillator wave functions

- Collective wave function:

$$\Psi_{\alpha}^{IM\pi}(\beta_2, \beta_3, \Omega) = \psi_{\alpha}^{I\pi}(\beta_2, \beta_3) |IM0\rangle$$

Euler angles

## + QOCH calculations:

- The probability density distribution:

$$\rho_{\alpha}^{I\pi}(\beta_2, \beta_3) = \sqrt{\omega \mathfrak{I}} |\psi_{\alpha}^{I\pi}(\beta_2, \beta_3)|^2$$

- Transition probabilities:

$$B(E\lambda; I_i \rightarrow I_f) = |\langle I_i 0 \lambda 0 | I_f 0 \rangle|^2 \left| \int d\beta_2 d\beta_3 \sqrt{\omega \mathfrak{I}} \psi_i \mathcal{M}_{E\lambda}(\beta_2, \beta_3) \psi_f^* \right|^2$$

- Operators  $\widehat{\mathcal{M}}_{E\lambda}$ :

$$D_1 = \sqrt{\frac{3}{4\pi}} e \left( \frac{N}{A} z_p - \frac{Z}{A} z_n \right)$$

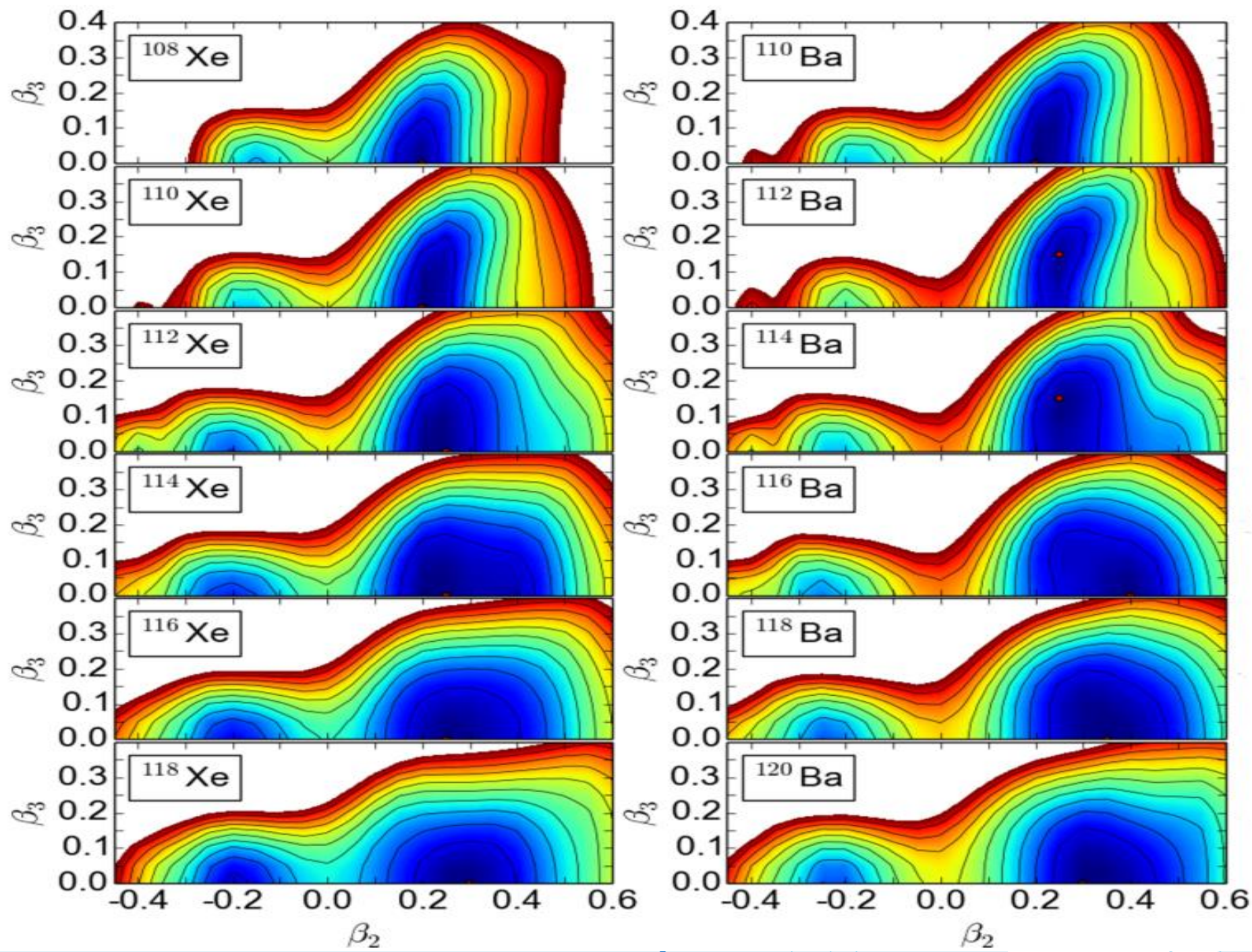
$$Q_2^p = \sqrt{\frac{5}{16\pi}} e (2z_p^2 - x_p^2 - y_p^2)$$

$$Q_3^p = \sqrt{\frac{7}{16\pi}} e (2z_p^3 - 3z_p(x_p^2 + y_p^2))$$

+ Results:

- Neutron deficient Xe and Ba isotopes - potential energy surface (PES)

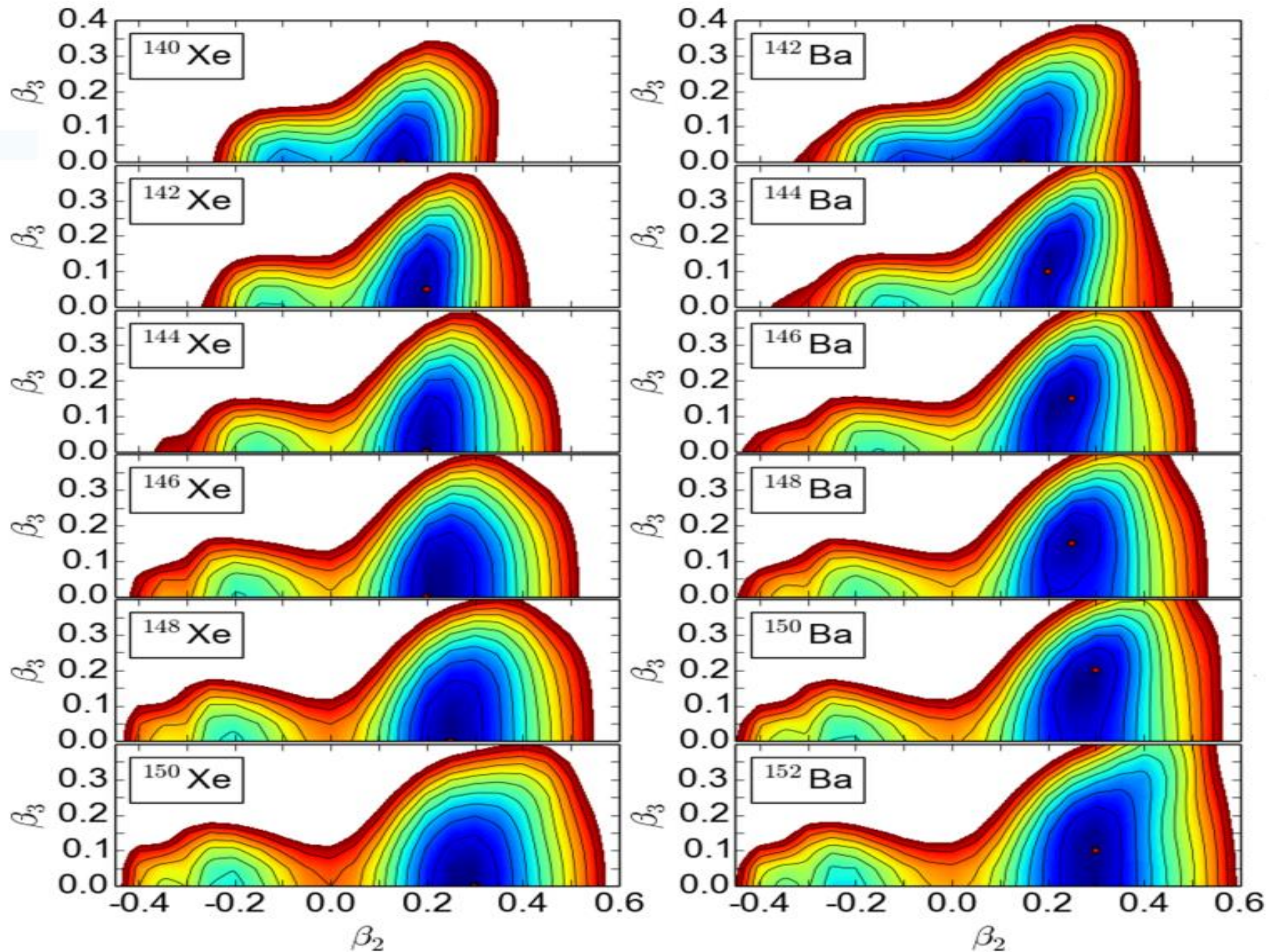
- Intrinsic octupole deformations:  $^{112}\text{Ba}$  (N=56),  $^{114}\text{Ba}$  (N=58)



+ Results:

- Neutron rich Xe and Ba - PES:

- Intrinsic octupole deformations:  $^{142}\text{Xe}$  (N=88),  $^{144-152}\text{Ba}$  (N=88 - 96)





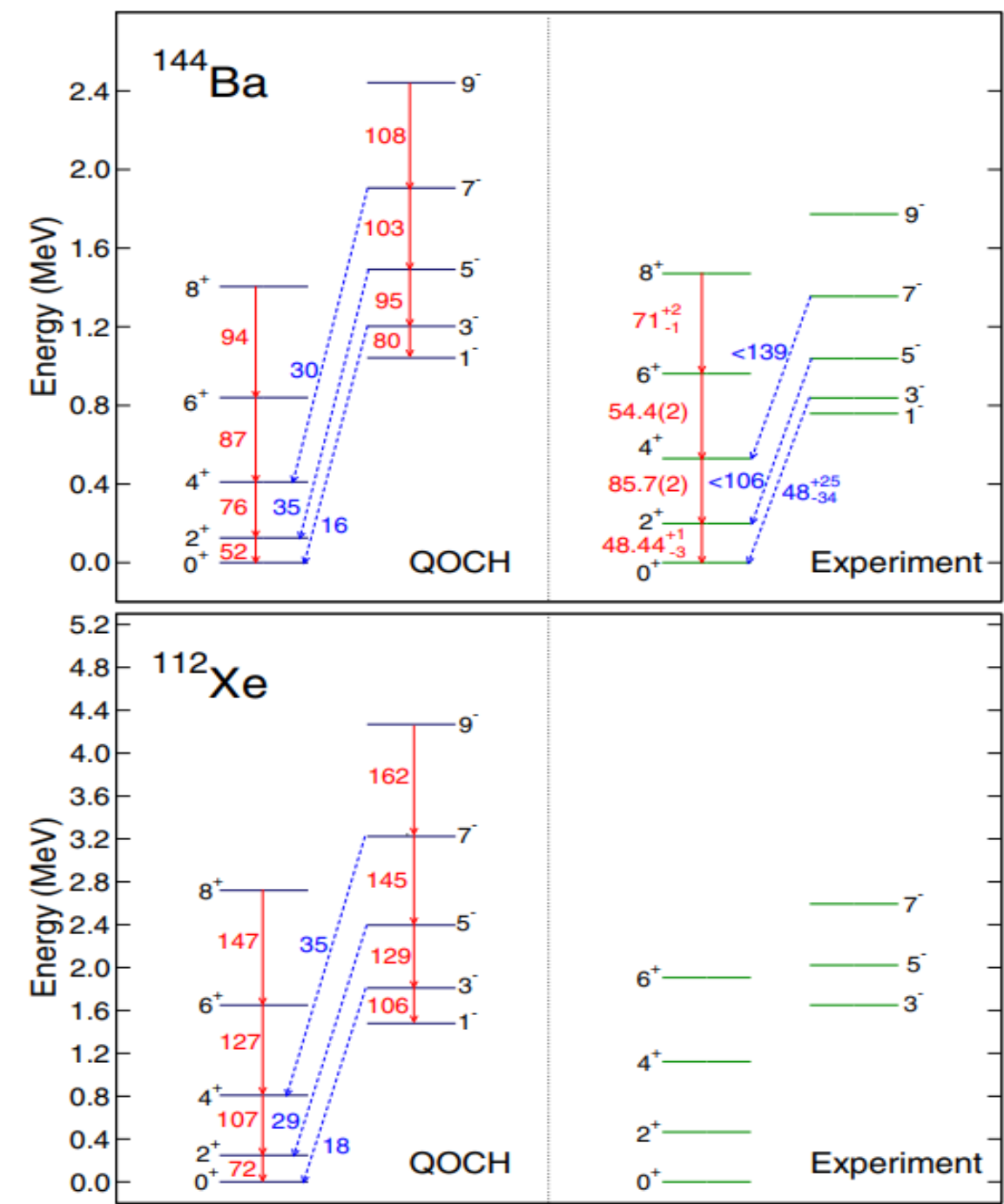
+ Results:

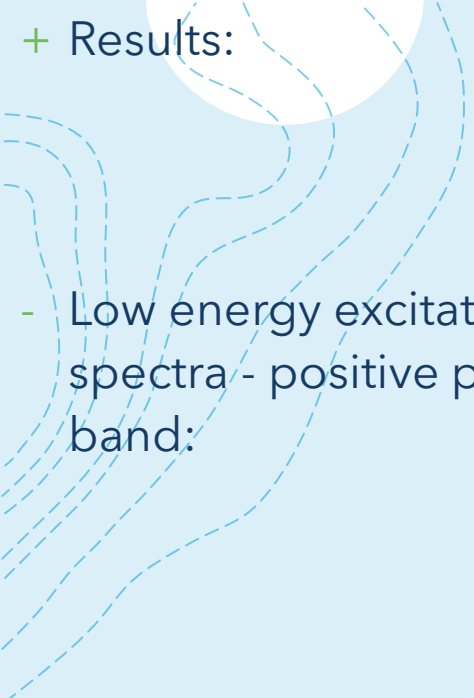
- Energy spectrum: examples  $^{144}\text{Ba}$  and  $^{112}\text{Xe}$ :

- Numbers along the arrows stand for  $B(E2)$  (red) and  $B(E3)$  (blue) transition strengths in Weisskopf units

- Positive-parity band energies and transition strengths in good agreement with experimental data

- Moments of inertia of negative parity bands are smaller than the measured ones

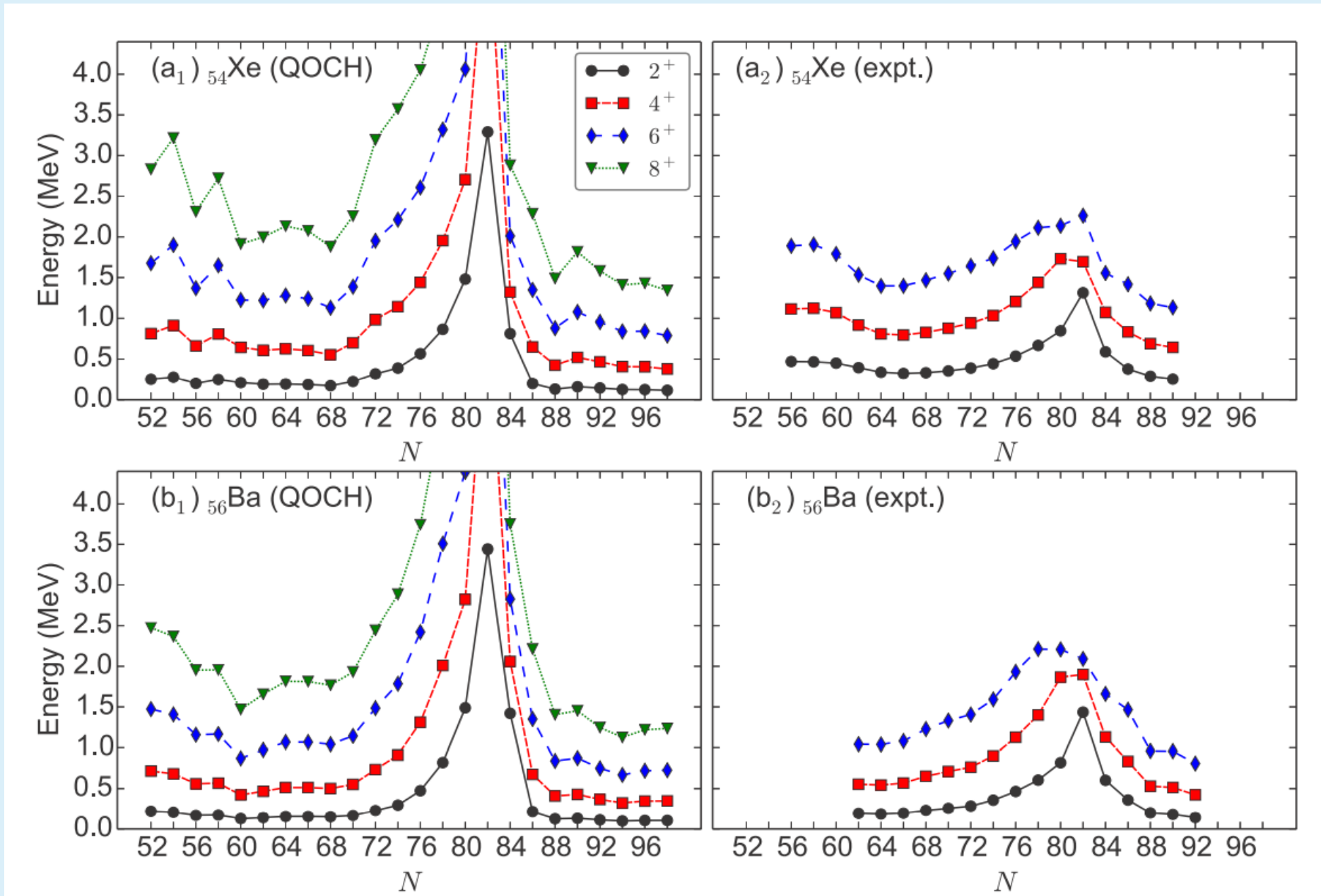




+ Results:

- Low energy excitation spectra - positive parity band:

- Good agreement with experimental data, except around the magic number  $N=82$

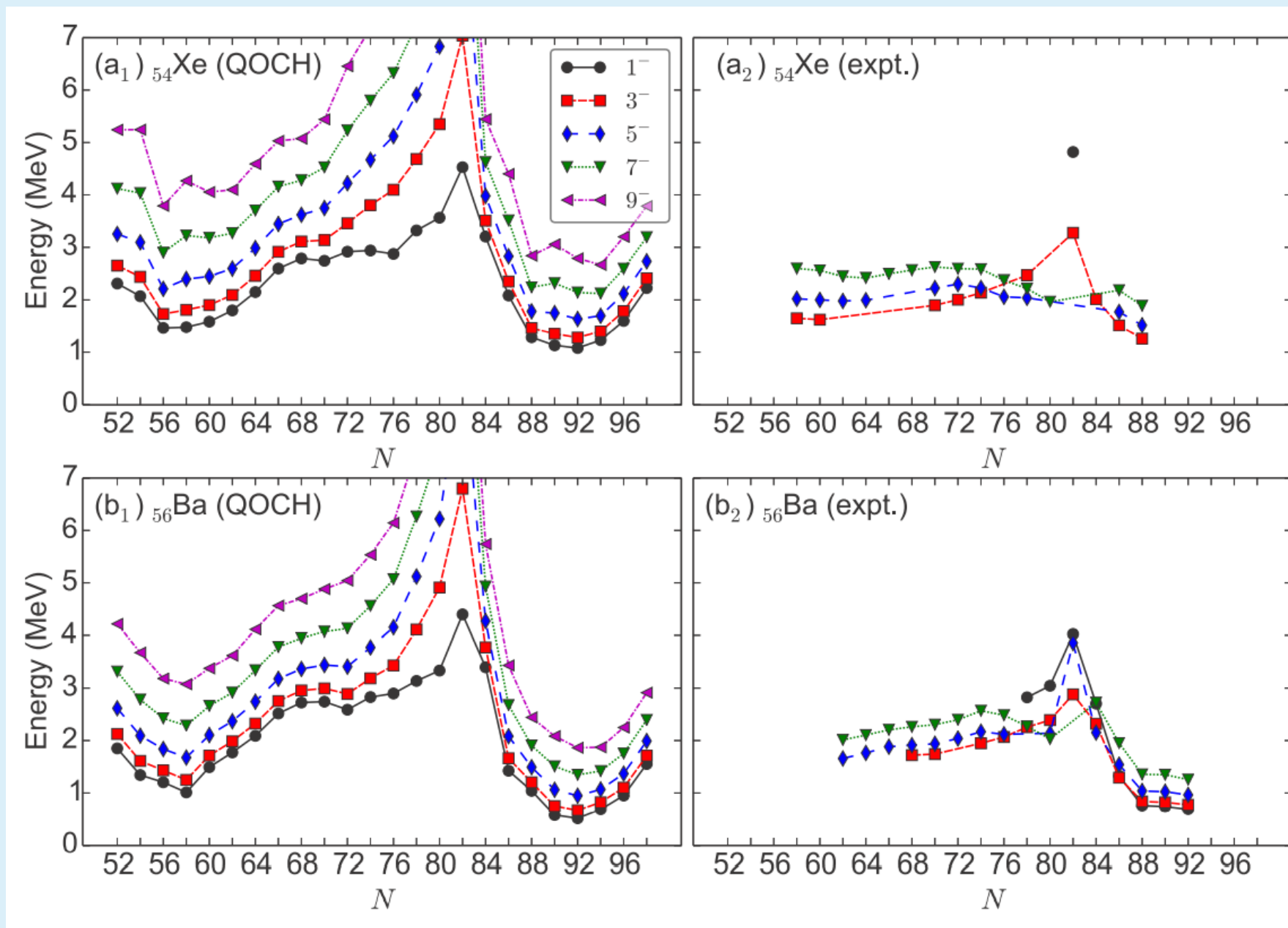


+ Results:

- Low energy excitation spectra - negative parity bands:

- Parabolic behaviour of negative-parity band energies with respect to „octupole magic numbers“  $N=56$ ,  $N=88$ ; signature of octupole collectivity

- Good agreement with experimental data around  $N=56$  and  $N=88$



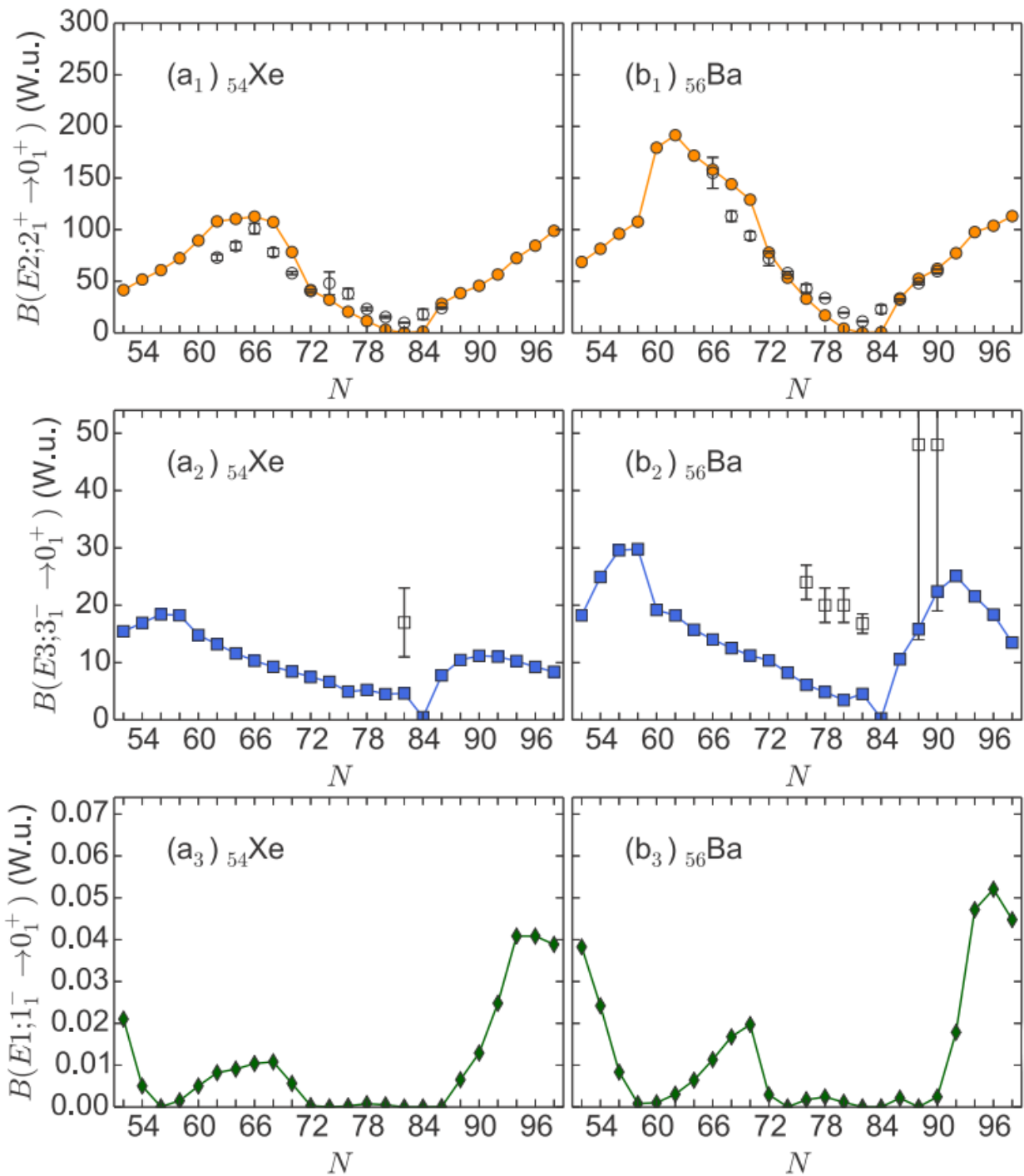
+ Results:

- Transition strengths  $B(E1)$ ,  $B(E2)$ ,  $B(E3)$ :

- Good agreement with experimental data for  $B(E2)$  transition strengths

-  $B(E3)$  values significantly lower compared to measured values

- No experimental data for  $B(E1)$  transition strengths



## + Conclusion:

- QOCH calculations mostly in good agreement with experimental data
- Intrinsic octupole deformations calculated for one Xe and seven Ba isotopes, two Ba isotopes ( $^{112}\text{Ba}$ ,  $^{114}\text{Ba}$ ) in the ( $N \approx Z \approx 56$ ) region
- Positive parity band energies and  $B(E2)$  transition strengths well reproduced (exception around  $N=82$ )
- Negative parity band energies well reproduced in the regions  $N \approx 56$  and  $N \approx 88$
- A limitation of the model: restricted to axially-symmetric shapes; inclusion of triaxial shape degrees of freedom might lead to improved results

+ Data taken from:

K. Nomura, L. Lotina, T. Nikšić, and D. Vretenar: Microscopic description of octupole collective excitations near  $N=56$  and  $N=88$ , Phys. Rev. C **103**, (2021)

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## Microscopic description of octupole collective excitations near $N = 56$ and $N = 88$

K. Nomura, L. Lotina, T. Nikšić, and D. Vretenar  
Phys. Rev. C **103**, 054301 – Published 3 May 2021

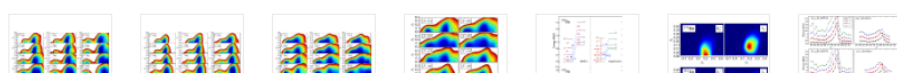
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### ABSTRACT

Octupole deformations and related collective excitations are analyzed using the framework of nuclear density functional theory. Axially symmetric quadrupole-octupole constrained self-consistent mean-field (SCMF) calculations with a choice of universal energy density functional and a pairing interaction are performed for Xe, Ba, and Ce isotopes from proton-rich to neutron-rich regions, and neutron-rich Se, Kr, and Sr isotopes, in which enhanced octupole correlations are expected to occur. Low-energy positive- and negative-parity spectra and transition strengths are computed by solving the quadrupole-octupole collective Hamiltonian, with the inertia parameters and collective potential determined by the constrained SCMF calculations. Octupole-deformed equilibrium states are found in the potential energy surfaces of the Ba and Ce isotopes with  $N \approx 56$  and 88. The evolution of spectroscopic properties indicates enhanced octupole correlations in the regions corresponding to  $N \approx Z \approx 56$ ,  $Z \approx 88$  and  $Z \approx 56$ , and  $N \approx 56$  and  $Z \approx 34$ . The average  $\beta_{30}$  deformation parameter and its fluctuation exhibit signatures of octupole shape-phase transition around  $N = 56$  and 88.



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