Shape coexistence and mixing within the Bohr model

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CONTENT

- Brief presentation of the model
- Description of the shape coexistence and mixing phenomena
- Applications of the model to the experimental data
- Conclusions

Bohr Hamiltonian with a sextic oscillator potential

Radial-like differential equation for the β variable for prolate deformed nuclei is:

F. Iachello, Phys. Rev. Lett. 87 (2001) 052502

$$\left[-\frac{1}{\beta^4}\frac{\partial}{\partial\beta}\beta^4\frac{\partial}{\partial\beta}+\frac{L(L+1)}{3\beta^2}+v(\beta)\right]\Psi(\beta)=\epsilon^{\beta}\Psi(\beta).$$

A general sextic potential is used, which can have simultaneously a spherical minimum and a deformed one:

$$v(\beta)=a\beta^2+b\beta^4+c\beta^6$$

R. Budaca, P. Buganu, A. I. Budaca, Phys. Lett. B 776 (2018) 26
R. Budaca, A. I. Budaca, EPL 123 (2018) 42001.

Making the change of function $\Psi(\beta)=\beta^{-2}\psi(\beta)$ and using a scaling property of the parameters for the polynomial potentials, one obtains a new form for the β equation:

$$\left[-\frac{\partial^2}{\partial\beta^2}+\frac{L(L+1)}{3\beta^2}+v_{eff}(\beta)\right]\psi(\beta)=\varepsilon^{\beta}\psi(\beta) \qquad v_{eff}(\beta)=\frac{2}{\beta^2}+\beta^2+a\beta^4+b\beta^6$$

The energies and the wave functions are obtained by numerical diagonalization using as a basis the solutions of the same equation but for an infinite square well potential:

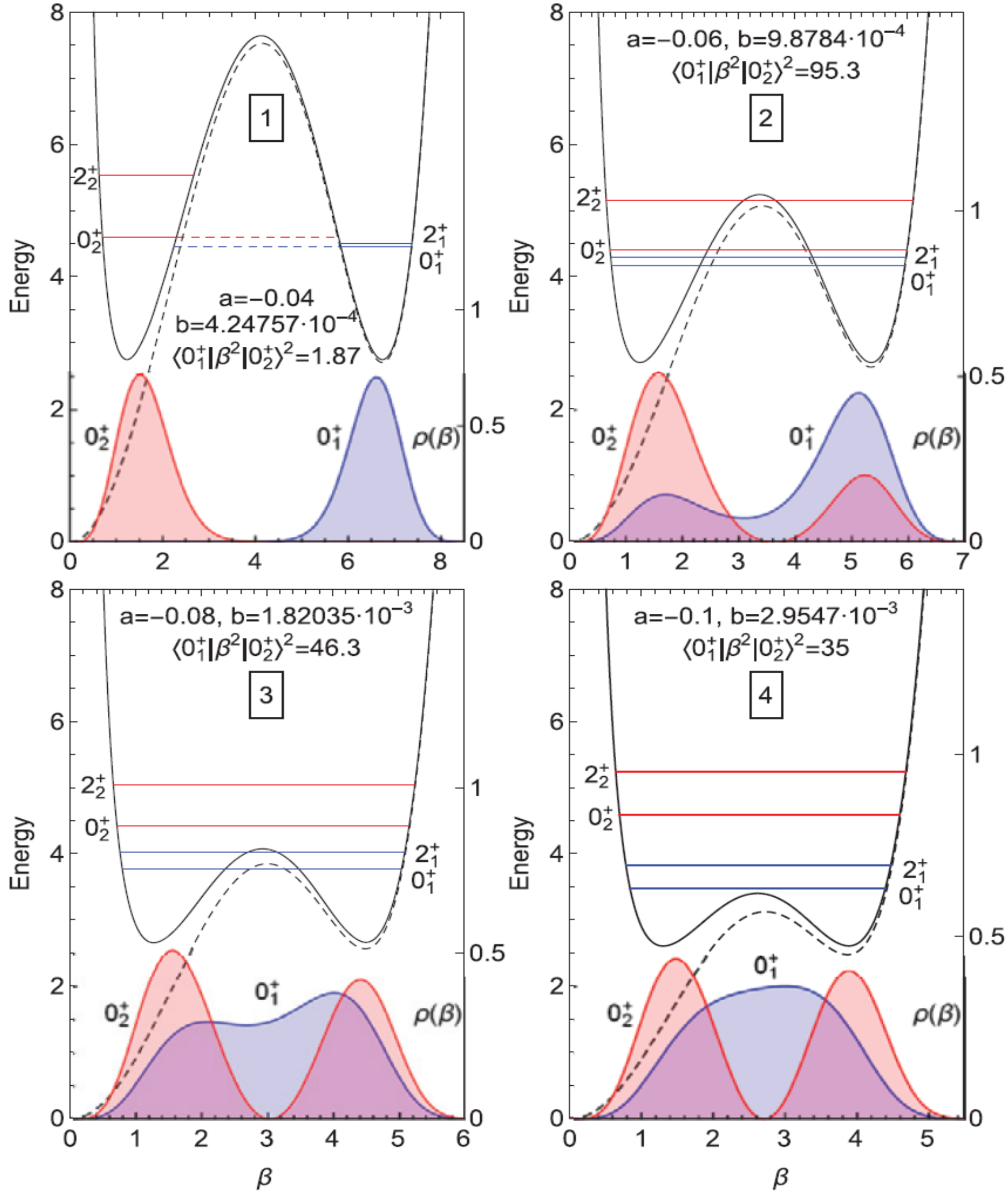
$$\tilde{\Psi}_{\nu n}(\beta)=\frac{\sqrt{2}\beta^{-\frac{3}{2}}J_{\nu}\left(\frac{\alpha_n\beta}{\beta_W}\right)}{\beta_W J_{\nu+1}(\alpha_n)},\quad \nu=\sqrt{\frac{L(L+1)}{3}+\frac{9}{4}},\ \text{for } v(\beta)=\begin{cases}0, & \beta<\beta_W\\ \infty, & \beta>\beta_W\end{cases}$$

Jv - Bessel functions of the first kind, α_n - zeros, β_w is fixed such that to achieve a satisfactory convergence (10^{-7}), n – dimension of basis (n=20).

0_2^+

—— $v_{eff}(\beta) = \frac{2}{\beta^2} + \beta^2 + a\beta^4 + b\beta^6$

- - - - $v(\beta) = \beta^2 + a\beta^4 + b\beta^6$



R. Budaca, A. I. Budaca, EPL 123 (2018) 42001..

Applications of the model to the experimental data

Mixing and coexistence between an approximately spherical shape and a prolate one

^{238}Pb, ^{152}Nd, ^{170}Hf: R. Budaca, P. Buganu, A. I. Budaca, Phys. Lett. B 776 (2018) 26.

^{76}Kr: R. Budaca, A. I. Budaca, EPL 123 (2018) 42001.

^{72,74,76}Se: R. Budaca, P. Buganu, A. I. Budaca, Nucl. Phys. A 990 (2019) 137.

Mixing and coexistence between an approximately spherical shape and a γ-unstable one

^{96,98,100}Mo: R. Budaca, A. I. Budaca, P. Buganu, J. Phys. G: Nucl. Part. Phys. 46 (2019) 125102.

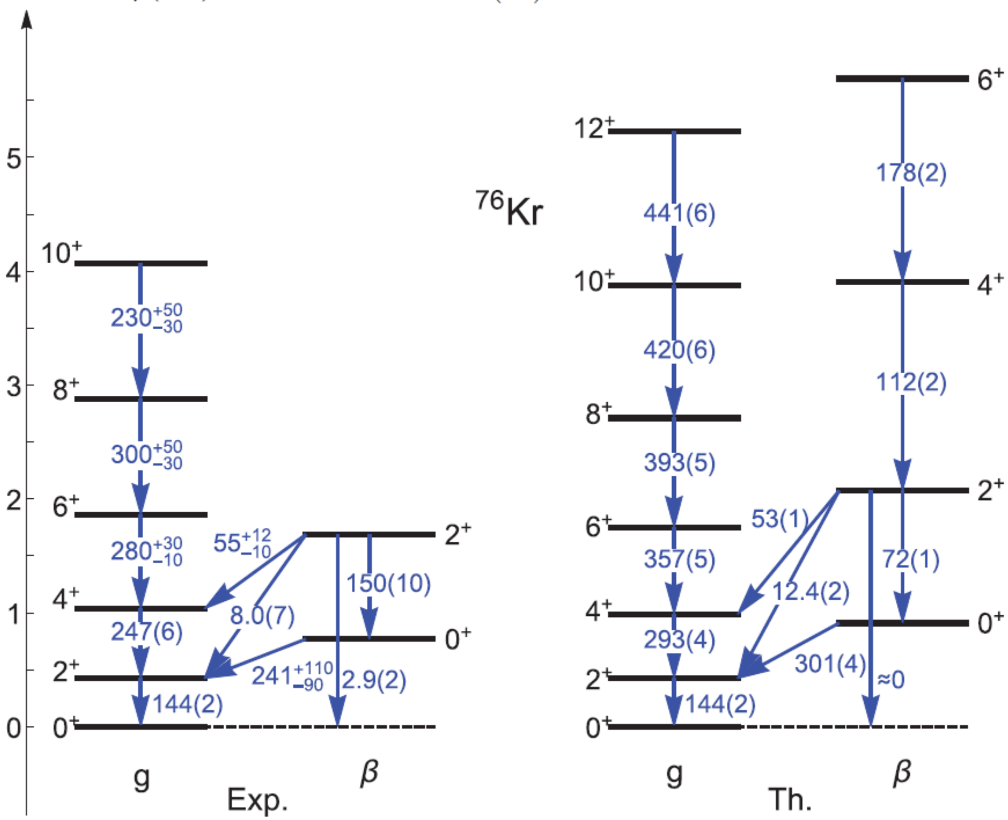
^{74}Ge, ^{74}Kr: A. Ait Ben Mennana, R. Benjedi, R. Budaca, P. Buganu, Y. El Bassem, A. Lahbas, M. Oulne, Phys. Scr. 96 (2021) 125306.

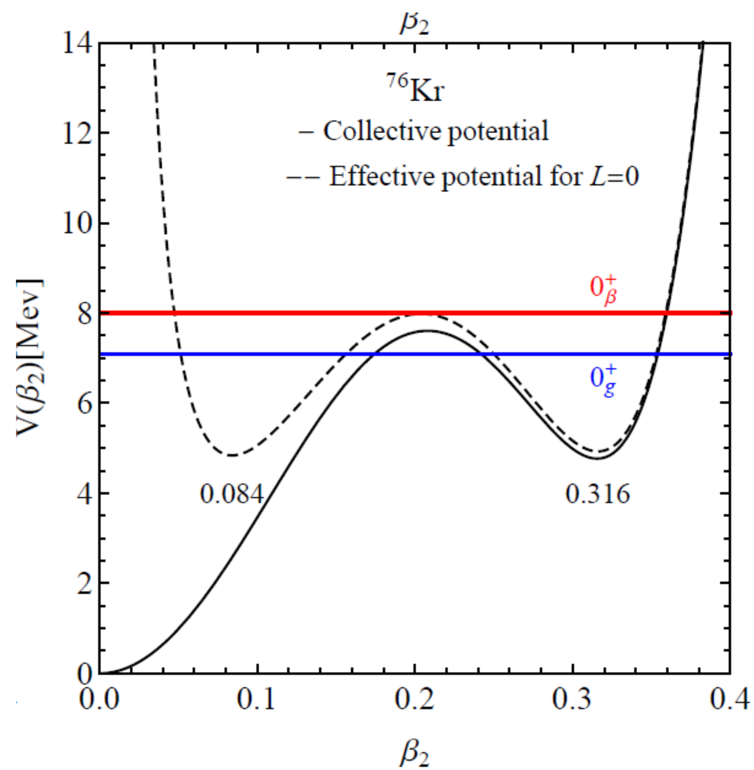
^{80}Ge: A. Ait Ben Mennana, R. Benjedi, R. Budaca, P. Buganu, Y. El Bassem, A. Lahbas, M. Oulne, Phys. Rev. C 105 (2022) 034347.

Applications of the model to the experimental data

Th. Exp.

$\rho(E0) = 87.14 \cdot 10^{-3}$ $79(11) \cdot 10^{-3}$





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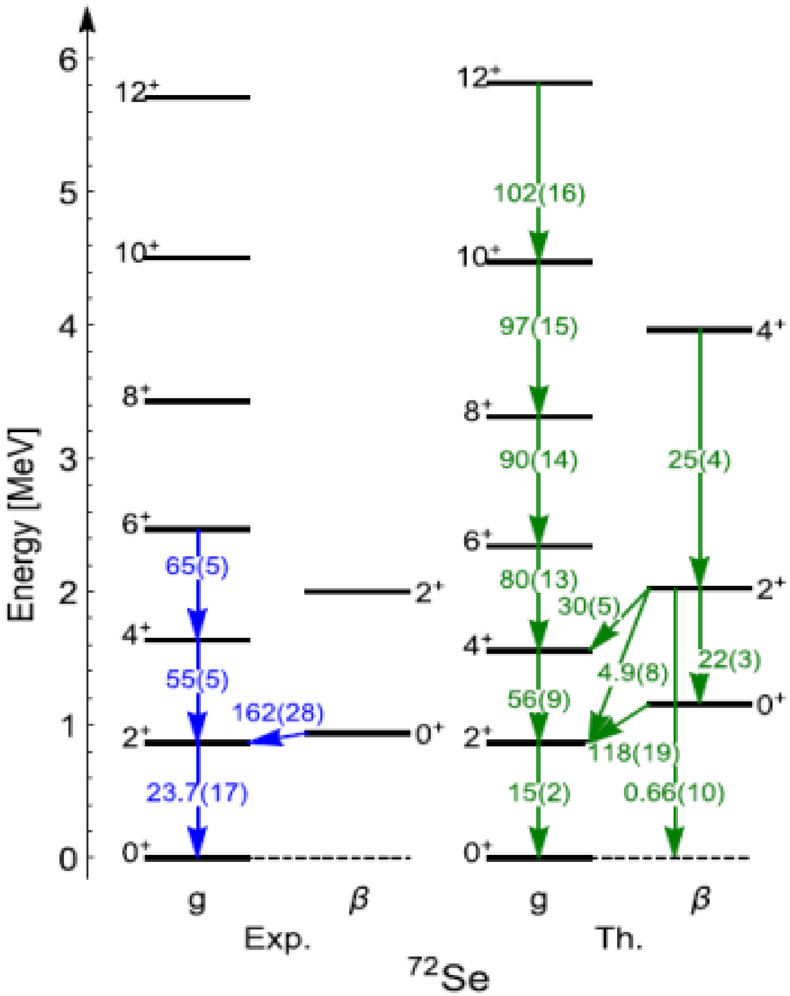
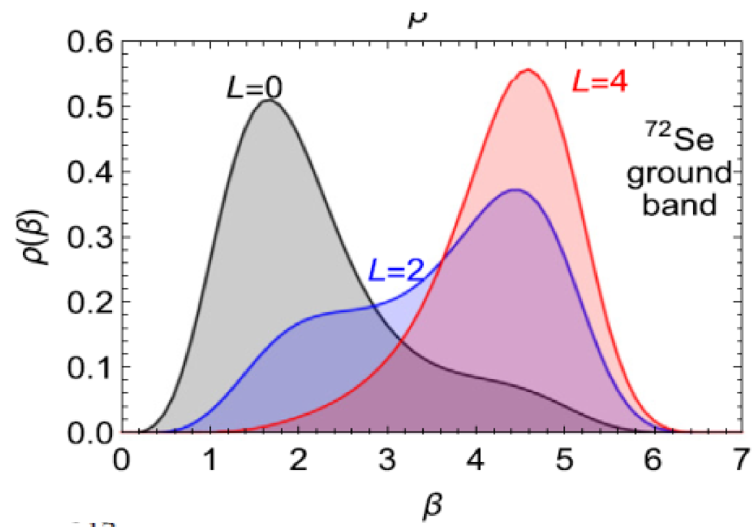
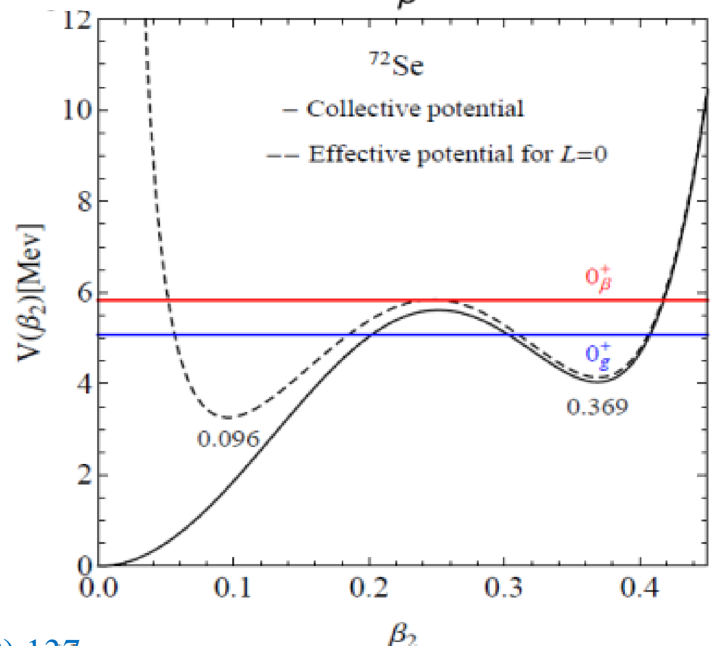
Nuclear Theory Vol. 38 (2019),
eds. M. Gaidarov, N. Minkov, Heron Press, Sofia.

Applications of the model to the experimental data







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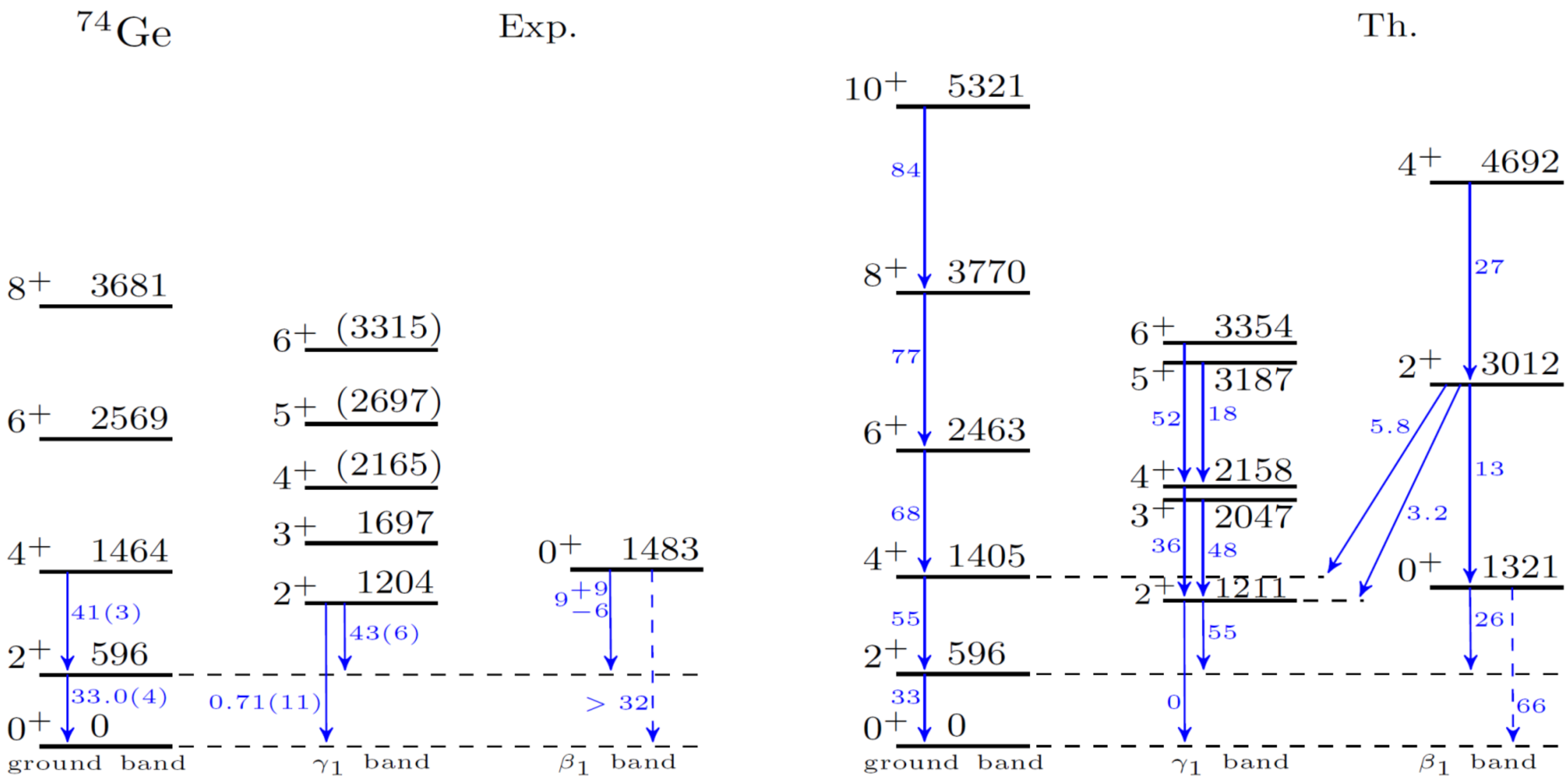

A. Ait Ben Mennana, R. Benjedi, R. Budaca, P. Buganu, Y. El Bassem, A. Lahbas, M. Oulne, Phys. Scr. 96 (2021) 125306.

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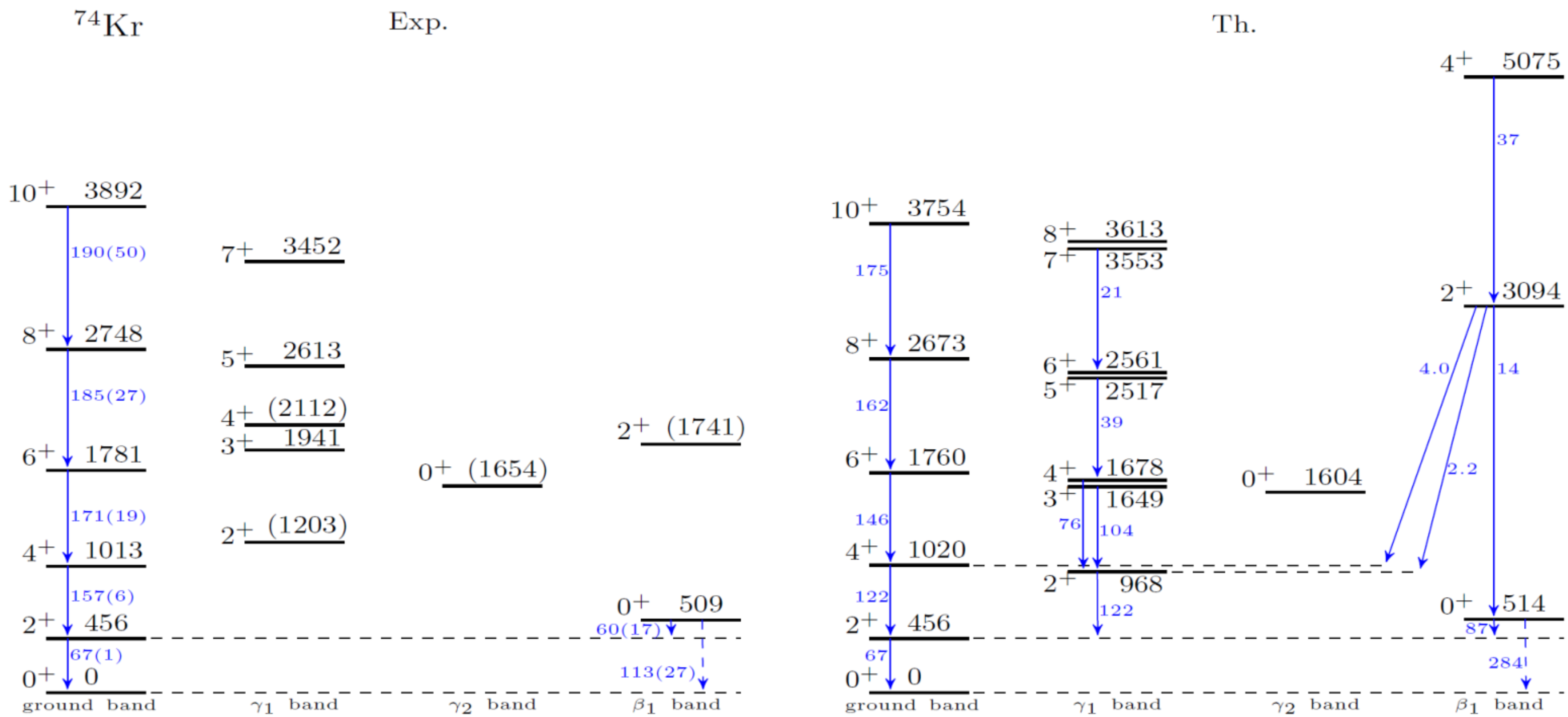

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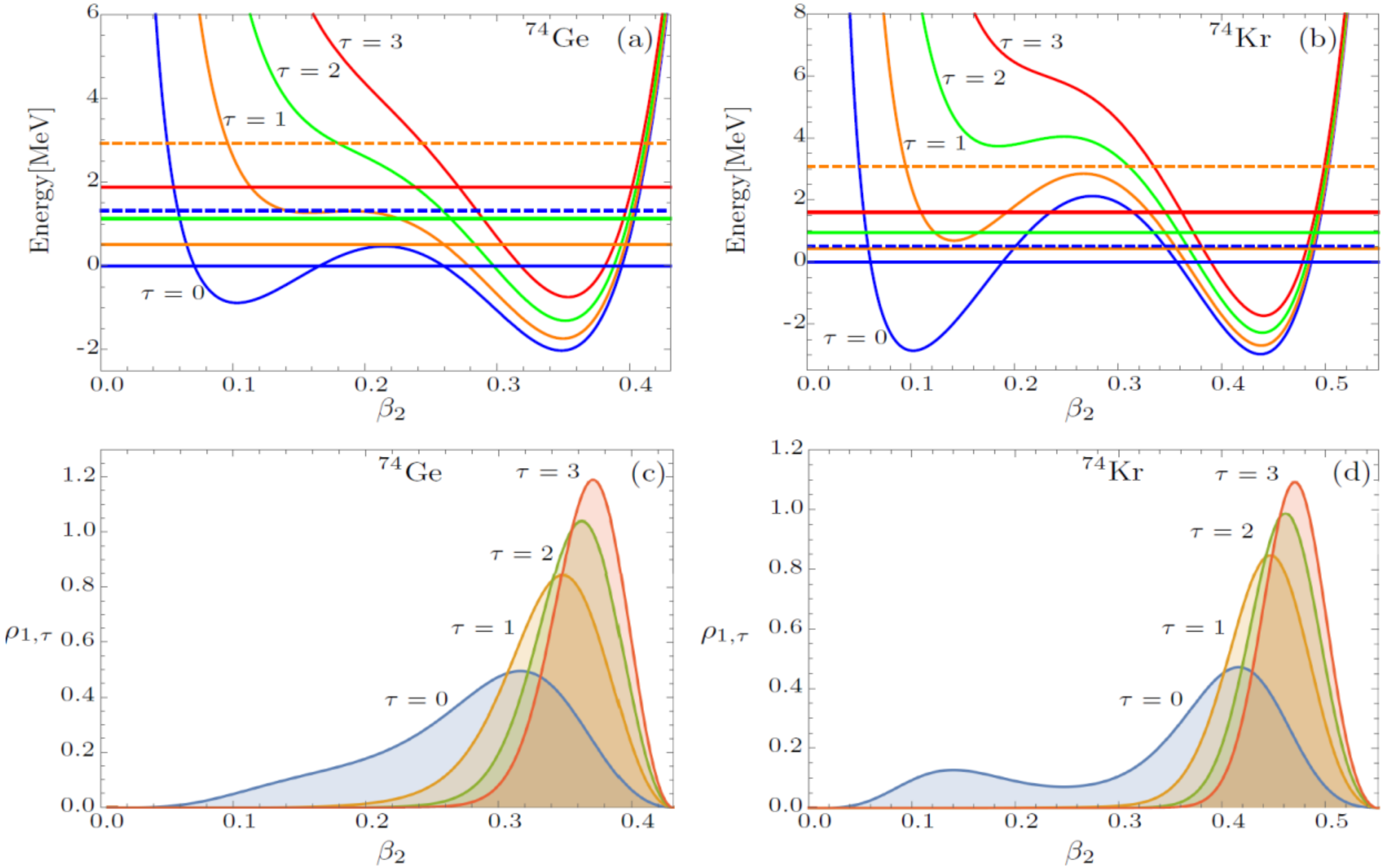

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Conclusions

- The Bohr Hamiltonian was numerically solved for a sextic potential in the β variable using as a basis the eigenstates of the X(5) and E(5) models.

- The model is able to describe nuclear shape phase transitions and their critical points, respectively shape coexistence and mixing phenomena as a function of the height of the barrier (maximum) which separates the two minima of the potential, a spherical and a deformed one.

- Some preliminary applications of the model to the experimental data, evidenced the ability of the model to describe such phenomena, as well as to describe some features in the level structure of these nuclei.

- The results obtained until now are very promising for future applications and possible development of the model.