Beyond mean field approaches for nuclear neutrinoless double beta decay

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TECHNIS

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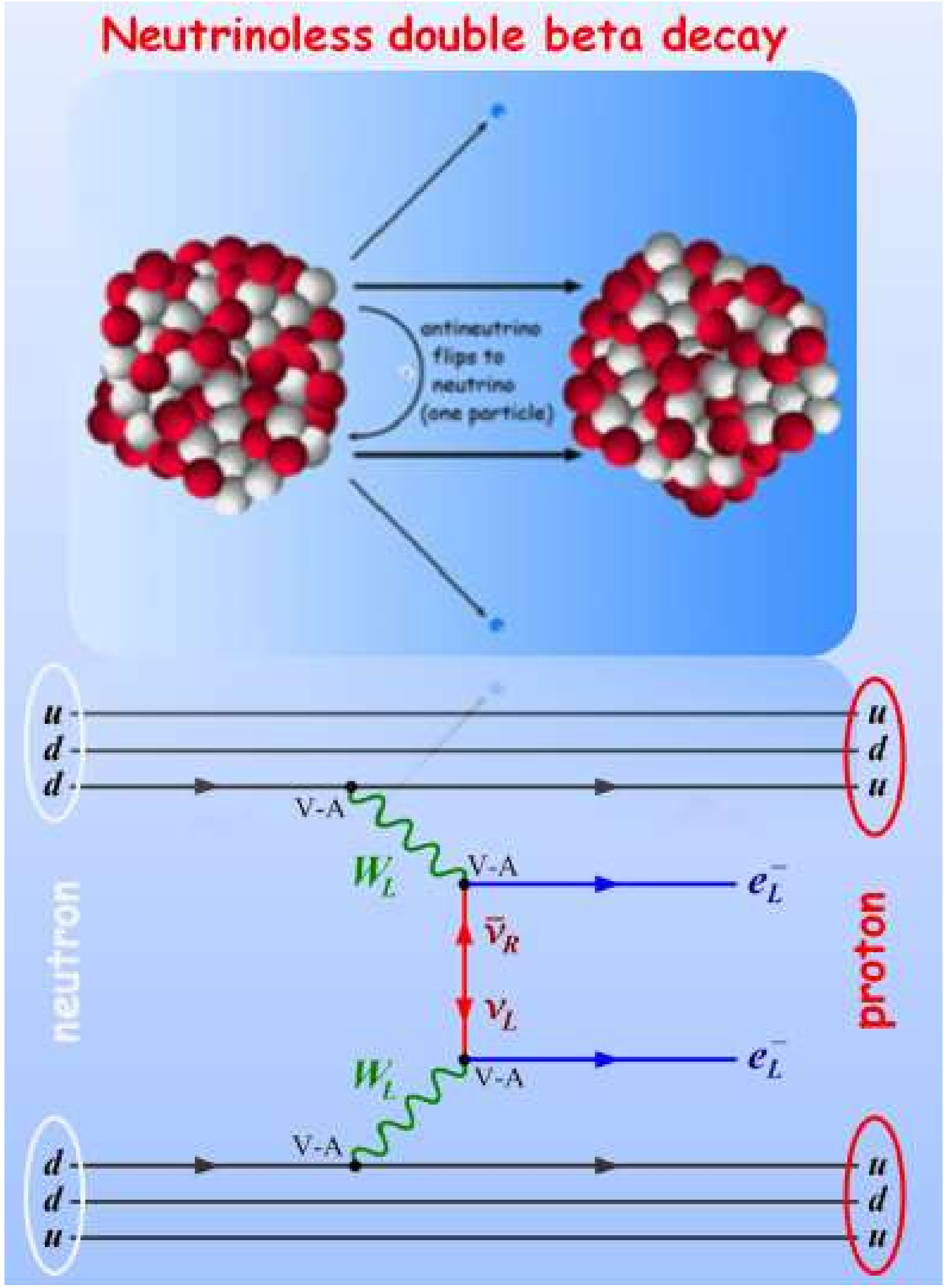


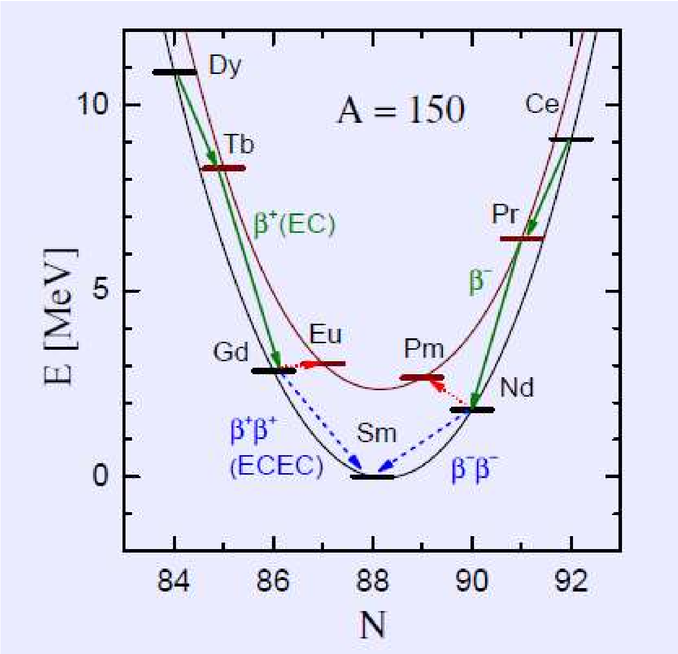
Neutrinoless double beta decay
antineutrino
flips to
neutrino
(one particle)
u
d
d
u
d
u
V-A
W_L
V-A
e_L^-
$\bar{\nu}_R$
ν_L
W_L
V-A
e_L^-
V-A
d
d
u
u
d
u
neutron
proton

Introduction

In ββ-decay the nucleus (A,Z) decays:

$(A\,,Z) \rightarrow (A\,,Z \pm 2) + 2e^{\mp}$ + light particles

emitting 2 electrons (positrons) and, usually, additional light particles.



It can be observed in some even–even nuclei, where single beta-decay is energetically forbidden, as for instance in the nucleus ^{150}Nd.

for $\beta^-\beta^-$ we have:

2v-ββ: $(A,Z) \rightarrow (A,Z+2) + 2e^- + 2\bar{v}$

0v-ββ: $(A,Z) \rightarrow (A,Z+2) + 2e^-$

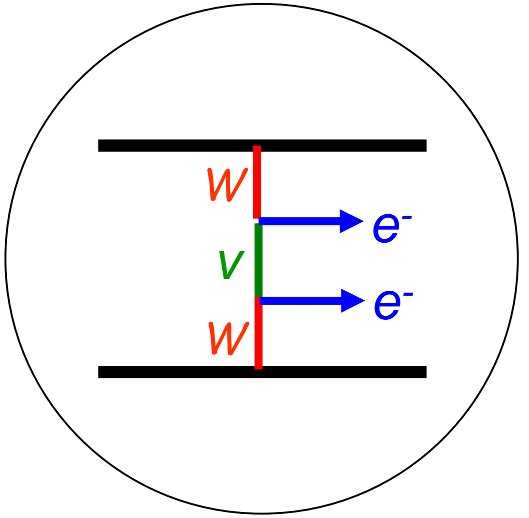
others exotic modes

Neutrino-less double beta-decay is not observed yet in experiment
Lepton number is violated.
Its observation would prove that the neutrino is a Majorana particle

Half live of 0νββ decay

Assuming the light neutrino decay mechanism, we find the decay rate:

$$\left[T_{1/2}^{0\nu}\right]^{-1} = G_{0\nu}\, g_A^4 \frac{\langle m_\nu \rangle^2}{m_e^2} |M^{0\nu}|^2$$



g_A : axial vector coupling constant

m_e : electron mass

$G_{0\nu}$: kinematic phase space factor

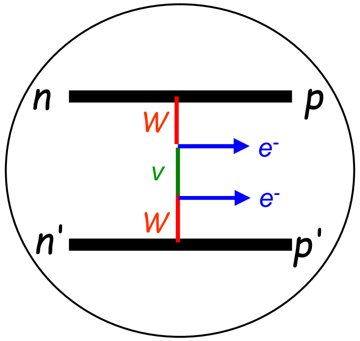
$\langle m_\nu \rangle$: effective neutrino mass: $\langle m_\nu \rangle = \sum_k U_{ek}^2 m_k \xi_k$

$M^{0\nu}$: nuclear matrix element (NME)

Kotila 2012: PRC 85, 034016
Bilenky 1987: RMP 59, 671

The observation of 0νββ-decay
would teach us the nature of the neutrino.
and the neutrino mass (provided that the NME is known)

Nuclear matrix element NME:



$$M^{0\nu} = \langle \Psi_F(Z+2) | \mathcal{O}^{0\nu} | \Psi_I(Z) \rangle$$

depends on the nuclear wave functions $|\Psi_I\rangle$ and $|\Psi_F\rangle$
and $\mathcal{O}^{0\nu}$ is an effective 2-body transition operator

Various non-relativistic models have been used in the literature:

- ✓ Quasiparticle random phase approximation (QRPA)
  Simkovic 1999, PRC 60, 055502; Simkovic 2008, PRC 77, 045503; Fang 2011, PRC 83, 034320; Kortelainen 2007, PRC 75, 051303(R); Mustonen 2013, PRC 87, 064302; ...
- ✓ Interacting shell model (ISM)
  Caurier 2008, PRL 100, 052503; Menéndez 2009, NPA 818, 139; Neacsu 2012, PRC 86, 067304; ...
- ✓ Interacting boson model (IBM)
  Barea 2009, PRC 79, 044301; Barea 2013, PRC 87, 014315
- ✓ Projected Hartree-Fock-Bogoliubov (PHFB)
  Rath 2010, PRC 82, 064310; Rath 2013, PRC 88, 064322; ...
- ✓ Energy density functional theory (EDF)
  Rodrígez 2010, PRL 105, 252503; Rodrígez 2011, PPNP 66, 436; Rodrígez 2013, PLB 719, 174; Vaquero 2013, PRL 111, 142501; ...

Present work:

- We use:

  Beyond mean field covariant density functional theory

- It is based on a unified density functional

  no parameters,
  full space

- Correlations are taken into account

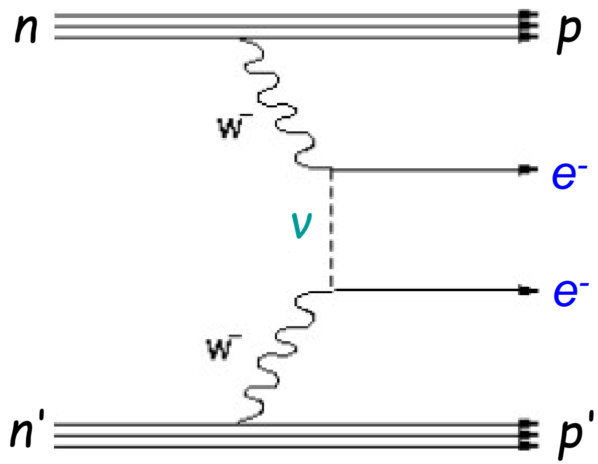
  by deformed and superfluid intrinsic wave functions,
  by superposition of deformed wave functions (GCM),
  by projection and the restoration of the broken symmetries

- Systematic investigations over a large number of nuclei

- We study:

  Influence of relativistic effects
  Influence of deformations
  Influence of pairing correlations

0νßß - matrix elements:



weak interaction:

$$\mathcal{H}_{\text{weak}}(x) = \frac{G_F \cos\theta_C}{\sqrt{2}} j^\mu(x) J_\mu^\dagger(x) + h.c.$$

leptonic current (V-A):

$$j^\mu(x) = \bar{e}(x)\gamma^\mu(1-\gamma_5)\nu_e(x)$$

hadronic current:

$$J_\mu^\dagger(x) = \bar{\psi}_p(x)\left[g_V(q^2)\gamma_\mu - i g_M(q^2)\frac{\sigma_{\mu\nu}}{2m_p}q^\nu - g_A(q^2)\gamma_\mu\gamma_5 + g_P(q^2)\gamma_5 q_\mu\right]\tau_-\psi_n(x)$$

Second order perturbation theory and integration over leptonic sector:

$$\mathcal{O}^{0\nu} = \frac{4\pi R}{g_A^2}\int \frac{d^3q}{(2\pi)^2}\frac{e^{i\mathbf{q}(\mathbf{x}_1-\mathbf{x}_2)}}{q}\sum_m \frac{J_\mu^\dagger(\mathbf{x}_1)|m\rangle\langle m|J^{\mu\dagger}(\mathbf{x}_2)}{q+E_m-E_0-Q_{\beta\beta}/2}$$

$E_m - E_0 - Q_{\beta\beta}/2 \to E_d$ and closure approximation: $\sum_m |m\rangle\langle m| \to 1$

Basic assumptions:

- Closure approximation
- Higher order currents are fully incorporated
- The tensorial part is included automatically
- Finite nuclear size corrections are taken into accout by form factors $g(q^2)$ (from Simkovic et al, PRC 2008)
- Short range correlations are neglected
- $g_A(0) = 1.254$ (no renormalization)

Nuclear wave functions:

- Intrinsic state:
  self-consistent constained RMF+BCS calculations: $|\beta\rangle = |\Phi(\beta)\rangle$
- Projected state: $|JZN, \beta\rangle = \hat{P}^J \hat{P}^Z \hat{P}^N |\beta\rangle$
- Generator coordinate method (GCM): shape mixing

$$|\Psi^{JZN}\rangle = \int d\beta f(\beta) |JZN, \beta\rangle$$

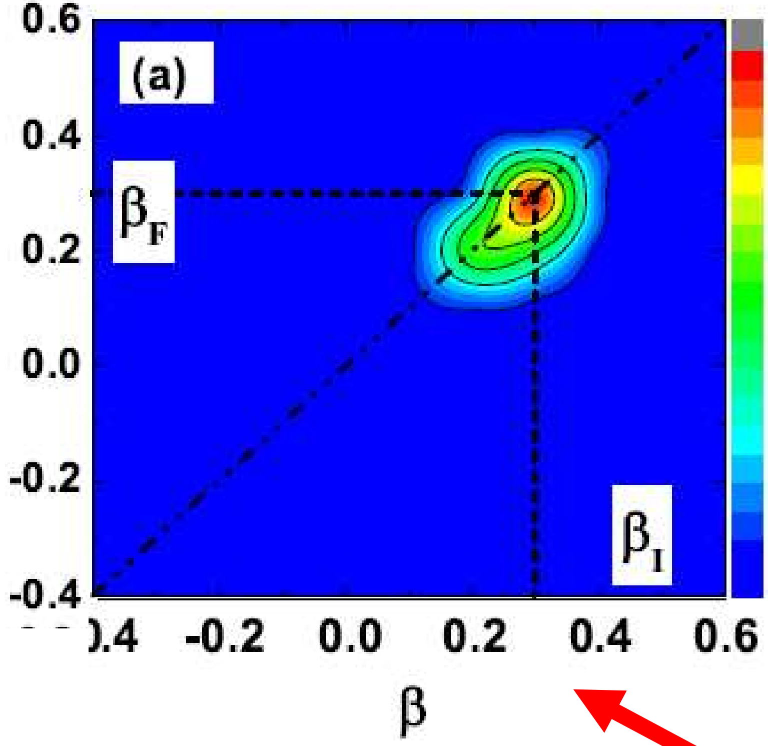
- Transition matrix element:

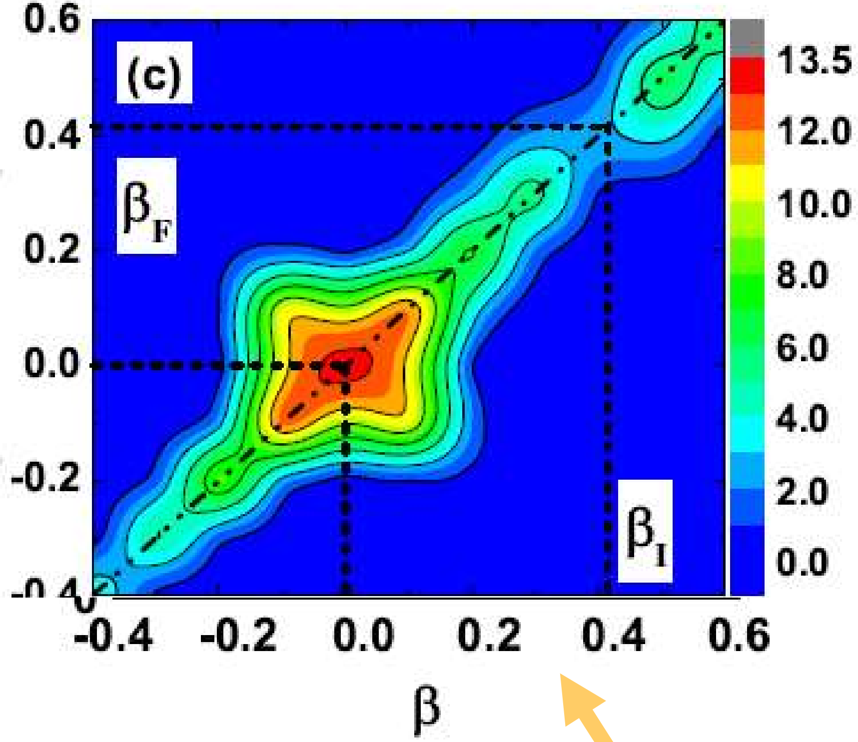
$$M^{0\nu} = \int\int d\beta_F d\beta_I \, f^*(\beta_F) f(\beta_I) \, M^{0\nu}(\beta_F, \beta_I)$$

$$M^{0\nu}(\beta_F, \beta_I) = \sum_{pp'nn'} \langle pp'|\mathcal{O}|nn'\rangle \langle \beta_F | c_p^\dagger c_p^\dagger c_n c_n | IZN, \beta_I\rangle$$

Contributions to NME

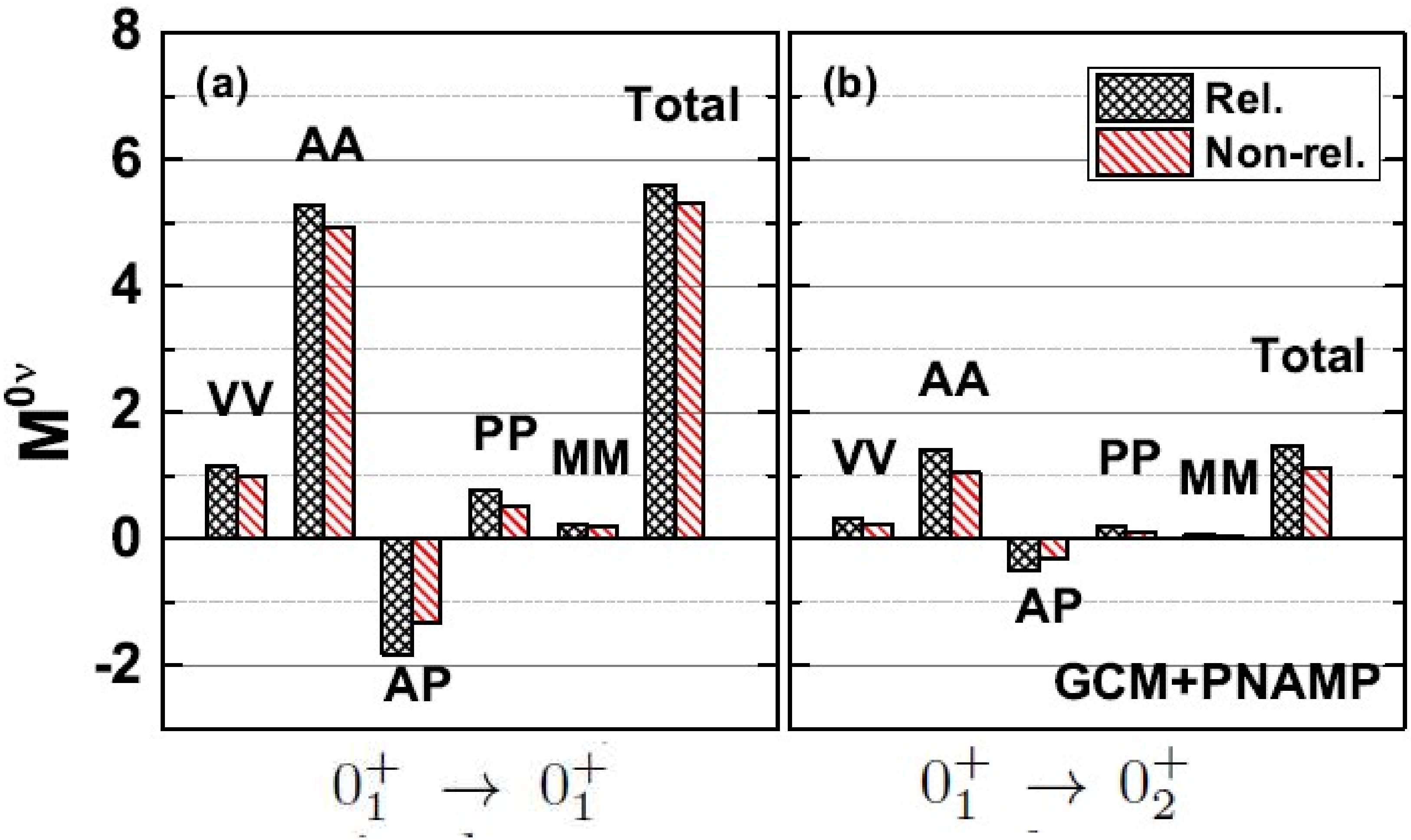
NME at fixed deformations

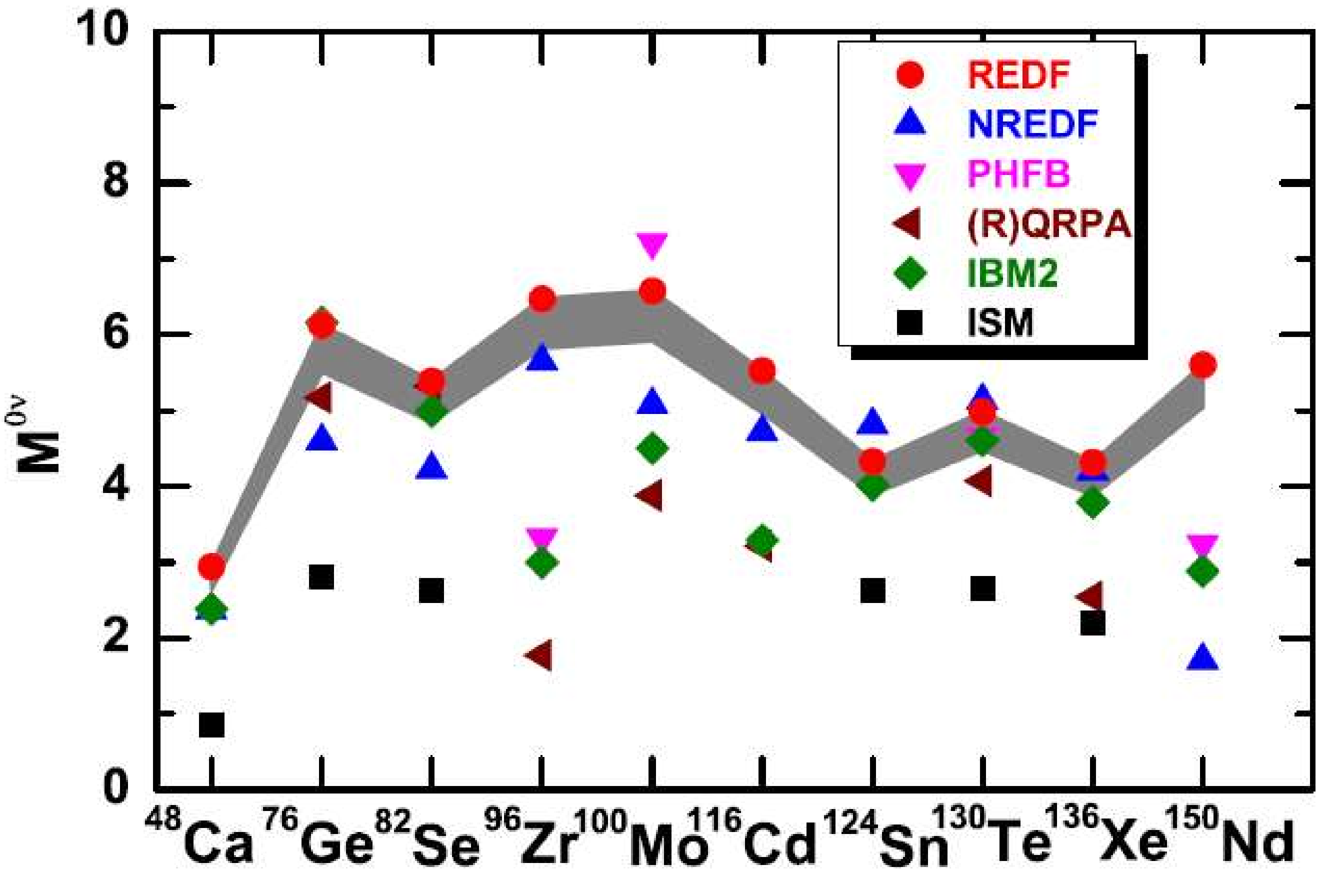




$$M^{0\nu} = \int\int d\beta_F d\beta_I \, f^*(\beta_F) f(\beta_I) M^{0\nu}(\beta_F, \beta_I)$$

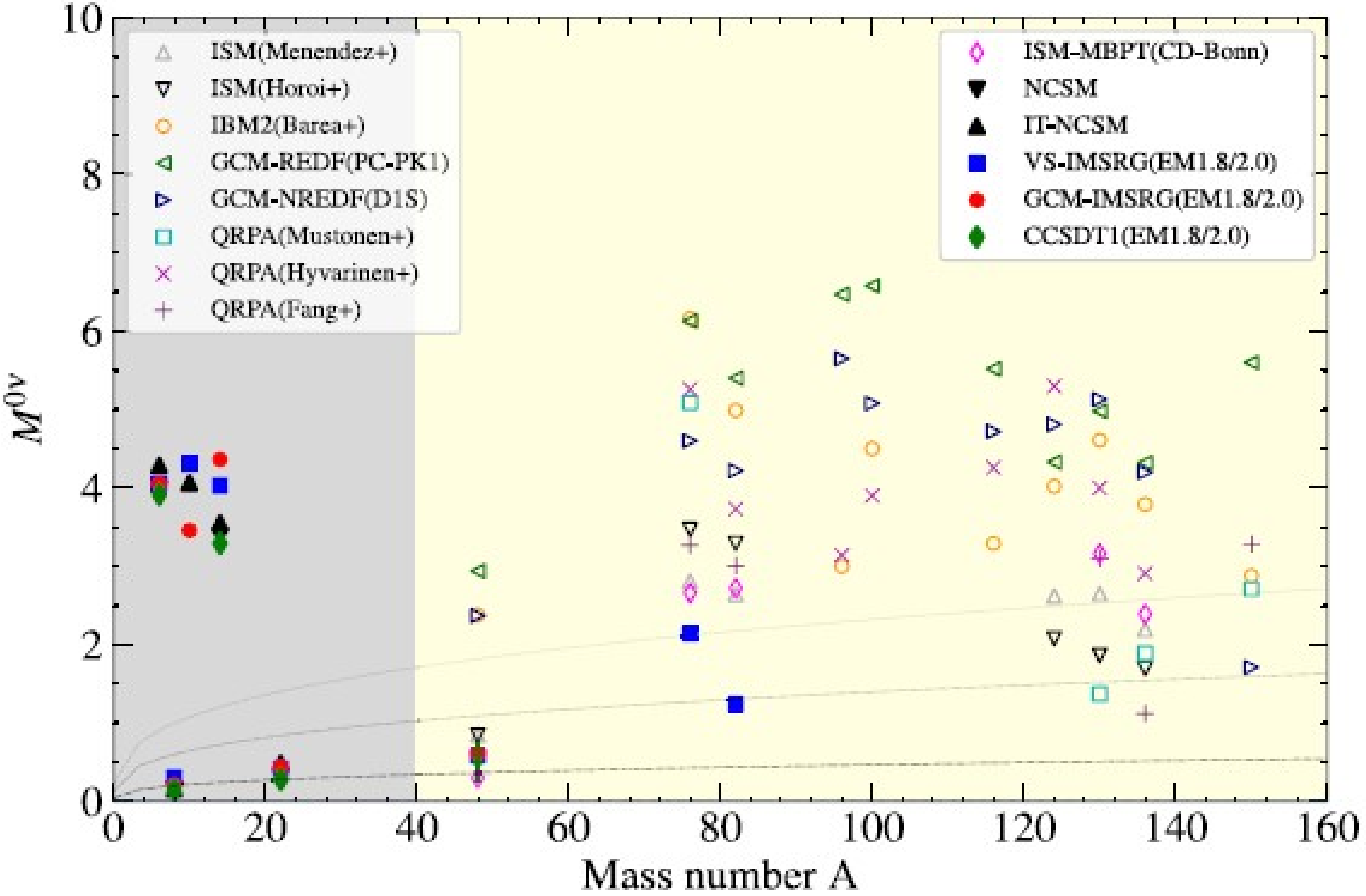
Transition ^{150}Nd → ^{150}Sm:
Matrix element of 0νββ decay and its contributions:





- The matrix elements differ by a factor 2 to 3
- Density functionals are at the upper end
- Not much sensitivity to the EDF (except for ^{150}Nd)
- Relativistic effects and tensor terms are with 10 %

J.M. Yao, L.S. Song, K.Hagino, P.R., J.Meng, PRC 91, 24316 (2015)



J.M. Yao, J.Meng, Y.F Niu, P.R. PPNP 126, 103965 (2022)

Upper limits for neutrino masses:

$$\left[T_{1/2}^{0\nu}\right]^{-1} = G_{0\nu}\, g_A^4 \frac{\langle m_\nu\rangle^2}{m_e^2}|M^{0\nu}|^2$$

Upper limits of the effective neutrino mass $\langle m_\nu \rangle$ (eV) based on:

a) the nuclear matrix elements $M^{0\nu}$ from this work

b) the lower limits of the half lives $T_{1/2}^{0\nu}(\times 10^{24}\text{yr})$ for the $0\nu\beta\beta$-decay from recent measurements

c) the phase space factors $G_{0\nu}(\times 10^{-15}\text{yr}^{-1})$

	^{48}Ca	^{76}Ge	^{82}Se	^{100}Mo	^{130}Te	^{136}Xe	^{150}Nd
$\langle m_{\beta\beta}\rangle$	$\leqslant 2.92$	$\leqslant 0.20$	$\leqslant 1.00$	$\leqslant 0.38$	$\leqslant 0.33$	$\leqslant 0.11$	$\leqslant 1.72$
$T_{1/2}^{0\nu}$	$\geqslant 0.058$	$\geqslant 30$	$\geqslant 0.36$	$\geqslant 1.1$	$\geqslant 2.8$	$\geqslant 34$	$\geqslant 0.018$
$G_{0\nu}$	24.81	2.363	10.16	15.92	14.22	14.58	63.03

Summary:

- Calculation of $0\nu\beta\beta$ decay using covariant DFT with GCM method
- First investigation of relativistic effects
- The influence of deformation and pairing in agreement with other models
- Our values are in general larger than earlier investigations based on non-rel. GCM, QRPA, IBM-2 and PHFB

Outlook:

- Additional degrees of freedom: triaxial, pairing-fluctuations, pn-pairing
- Other density functionals, ab-initio density functionals
- Collective hamiltionan (5DCH) simplifies the calculations for heavier nuclei
- Inclusion of short range correlations in the matrix elements

- Ab-initio derivation of density functional theory

  first attempts of ab-initio go back to the fifties:

  Brueckner theory:

  based on mean field concept

  effective density-dep. interaction: G[ρ]

  mother of density functional theory
- Non-relativistic BHF fails: Three-body forces
- 1980: Relativistic BHF: no NNN necessary

problems:

a) no exact solution of RBHF in nuclear matter

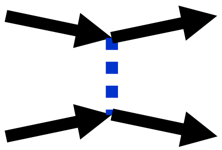
many different approximations

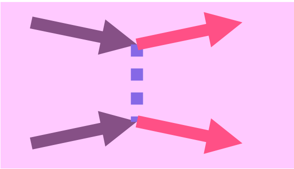
b) no solution of RBHF in finite nuclei (tensor?)

Brueckner theory (1958):

Brueckner,Gammel, Phys. Rev. 109, 1023 (1958)

- The nucleons in the interior of the nuclear medium do not feel the same bare force V, as the nucleons feel in free space.



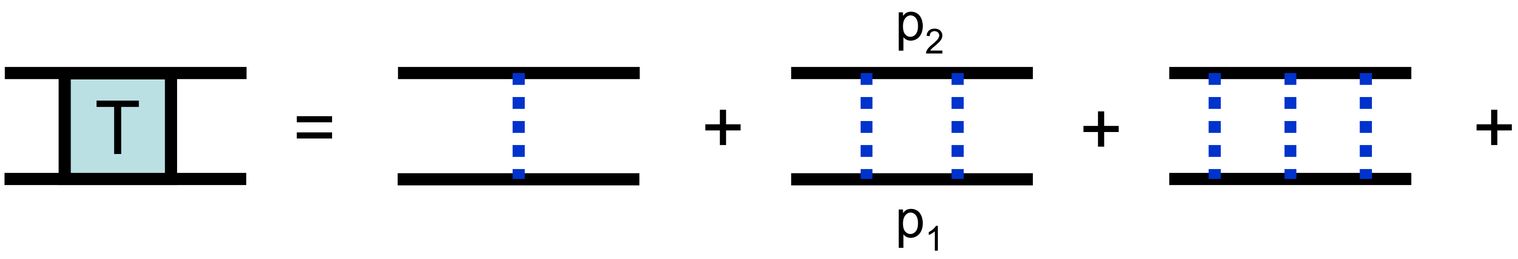


- They feel an effective force G.
- The Pauli principle prohibits the scattering into states, which are already occupied in the medium.
- Therefore this force G(ρ) depends on the density
- This force G is much weaker than bare force V.
- Nucleons move nearly free in the nuclear medium and feel only a strong attraction at the surface (shell model)

Free nucleon-nucleon scattering:

Lippmann-Schwinger-Eq.

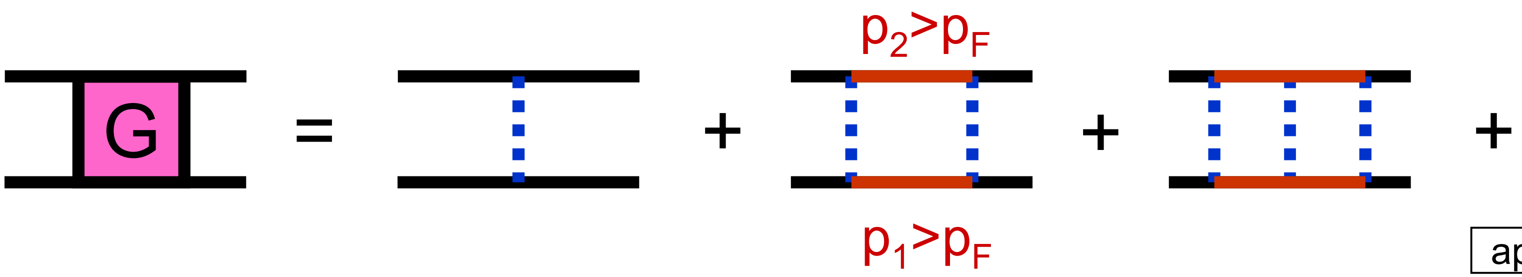
$$\langle \mathbf{k}_1\mathbf{k}_2|T|\mathbf{k}_1'\mathbf{k}_2'\rangle = \langle \mathbf{k}_1\mathbf{k}_2|V|\mathbf{k}_1'\mathbf{k}_2'\rangle + \sum_{\mathbf{p}_1\mathbf{p}_2} \langle \mathbf{k}_1\mathbf{k}_2|V|\mathbf{p}_1\mathbf{p}_2\rangle \frac{1}{E - \frac{\mathbf{p}_1^2}{2m} - \frac{\mathbf{p}_2^2}{2m} + i\eta} \langle \mathbf{p}_1\mathbf{p}_2|T|\mathbf{k}_1'\mathbf{k}_2'\rangle$$



exact

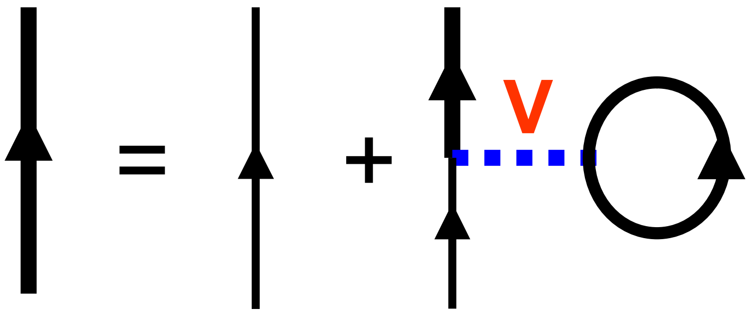
Scattering in the nuclear medium:

Bethe-Goldstone-Eq.

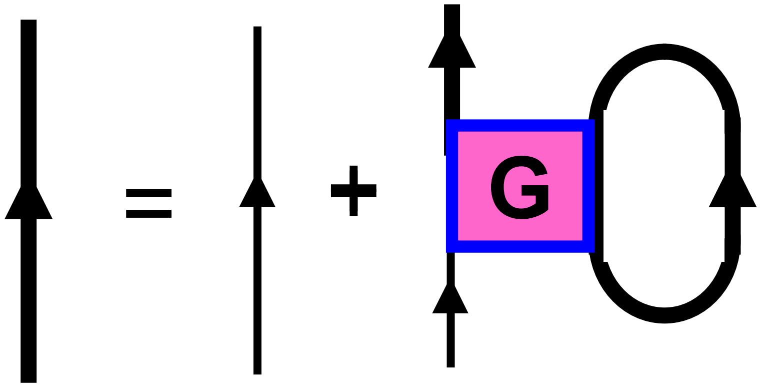


approximation

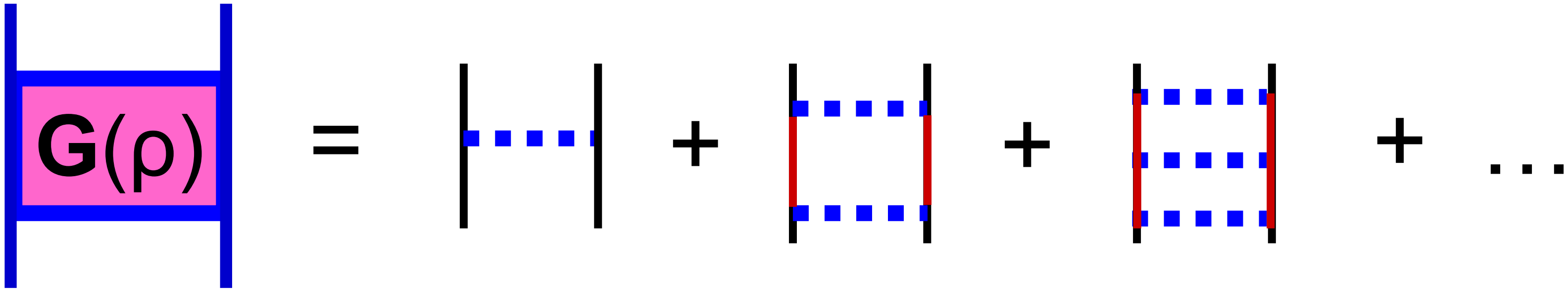
Ab-initio: Relativistic Brueckner Hartree-Fock:



Hartree-Fock

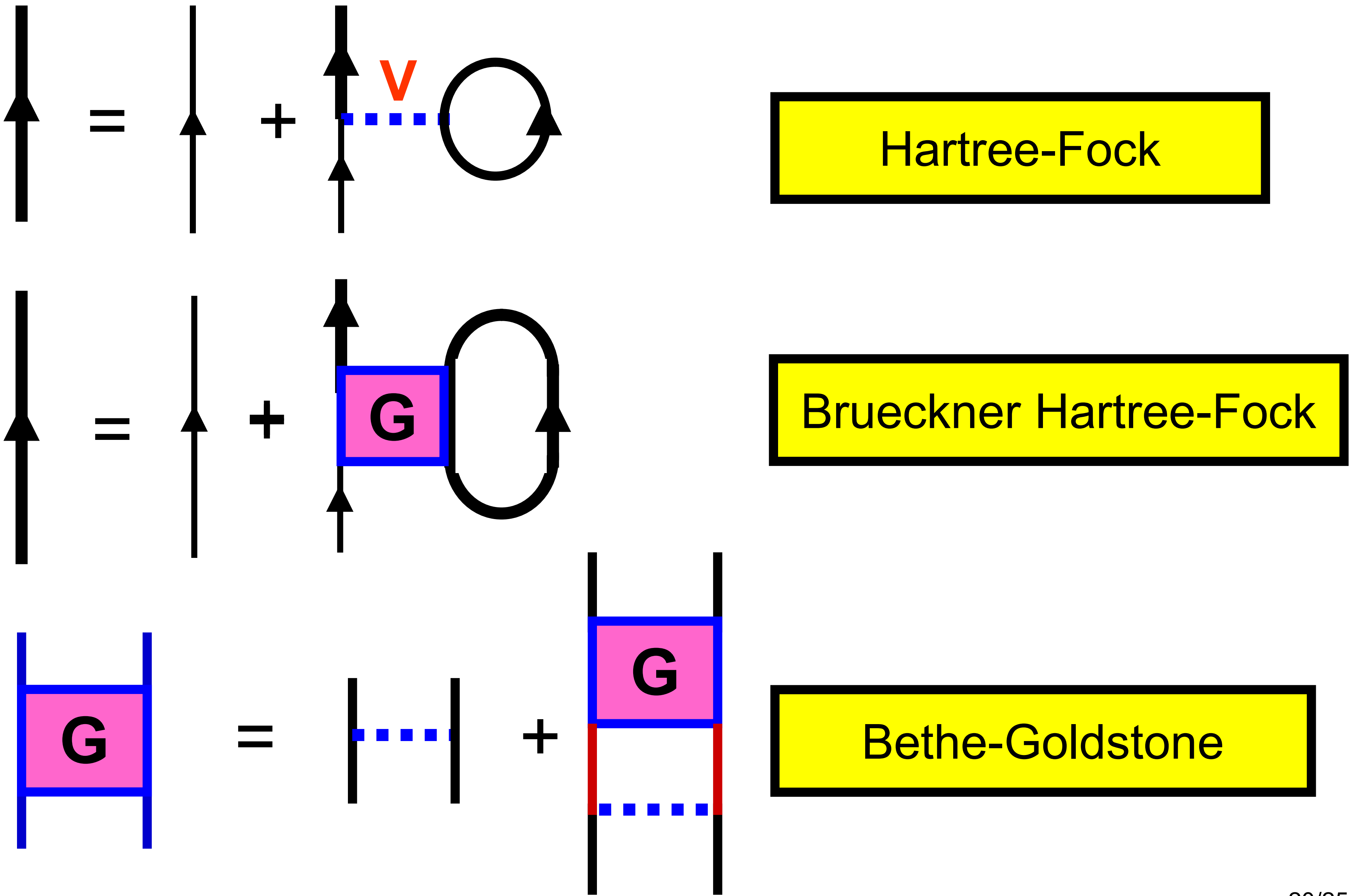


Brueckner Hartree-Fock



Summing up all ladder diagramms

Ab-initio: Relativistic Brueckner Hartree-Fock:

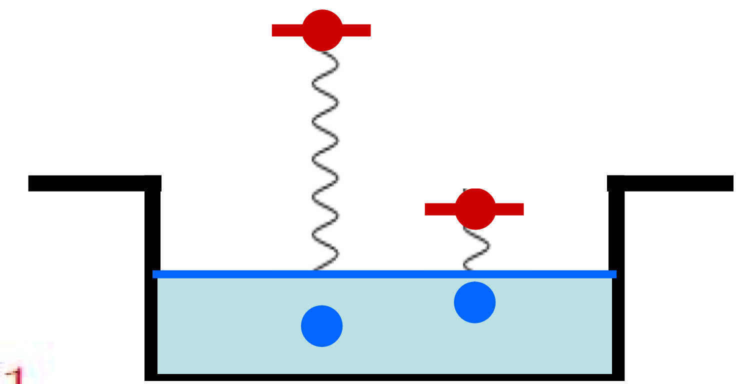


Bethe-Goldstone equation:

- ω is the starting energy
- V is realistic interaction
- Q_F is the Pauli operator

$$G(\omega) = V + VQ_F \frac{1}{\omega - H_{HF}} Q_F G(\omega)$$

$$G(\omega) = V + VP_F(\omega)G(\omega)$$

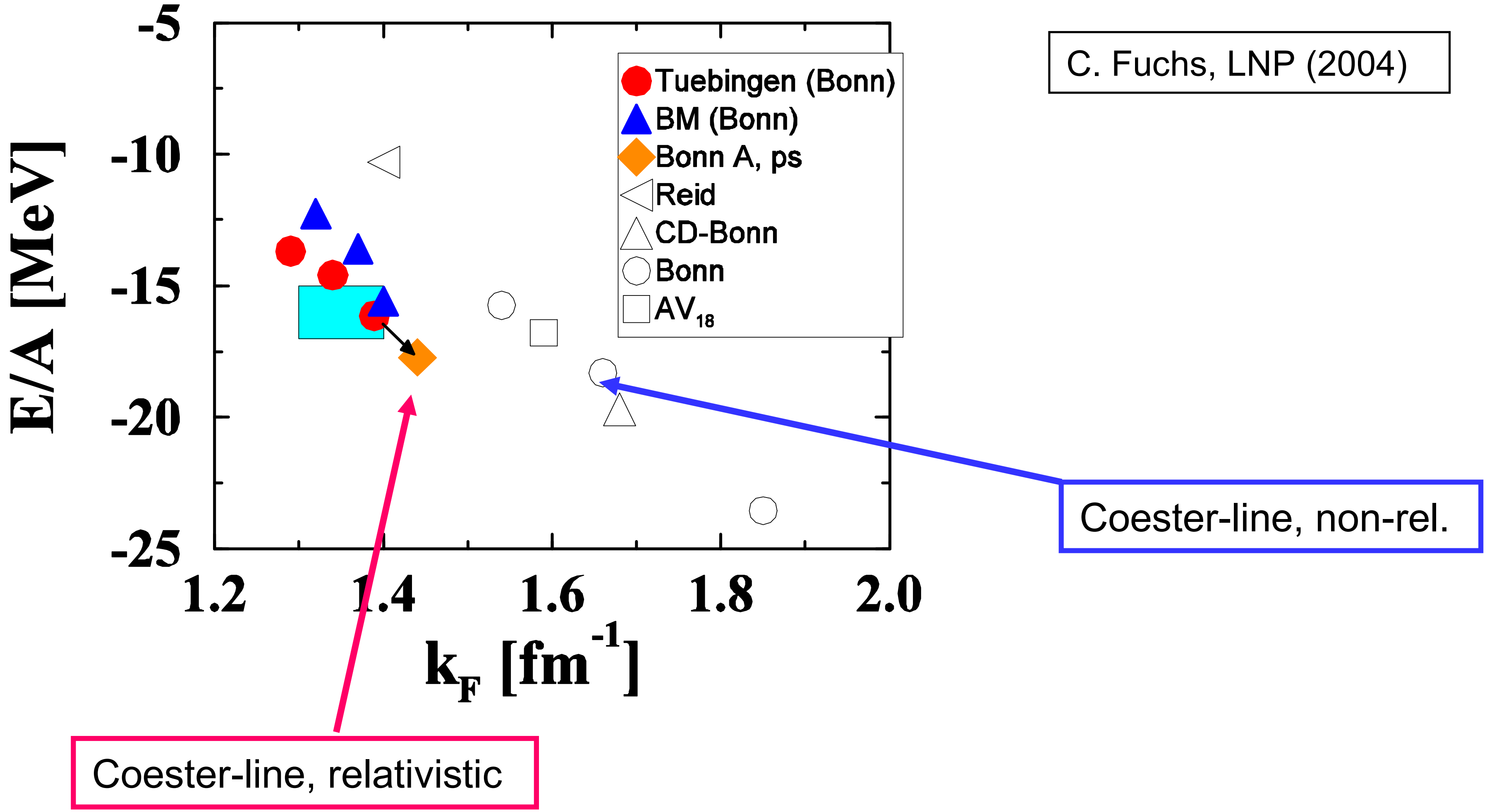


$$P_F(\omega) = \sum_{m_1 m_2 > \varepsilon_F} |m_1 m_2\rangle \frac{1}{\omega - \varepsilon_{m_1} - \varepsilon_{m_2}} \langle m_1 m_2|$$

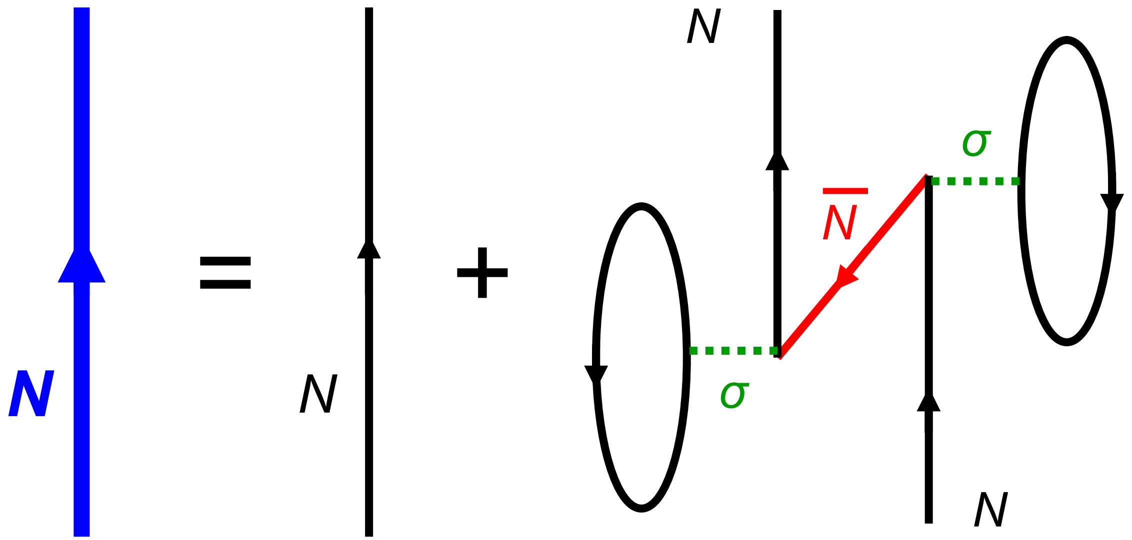
$$G(\omega) = \frac{1}{1 - VP_F(\omega)} V$$

Is solved in each step of the iteration

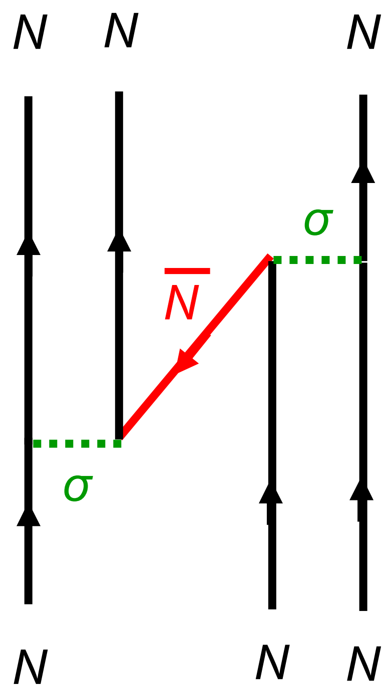
Dirac-Brueckner-Hartree-Fock in nuclear matter



Non-rel. 3-body-forces and relativistic effects



eff. 3-body force

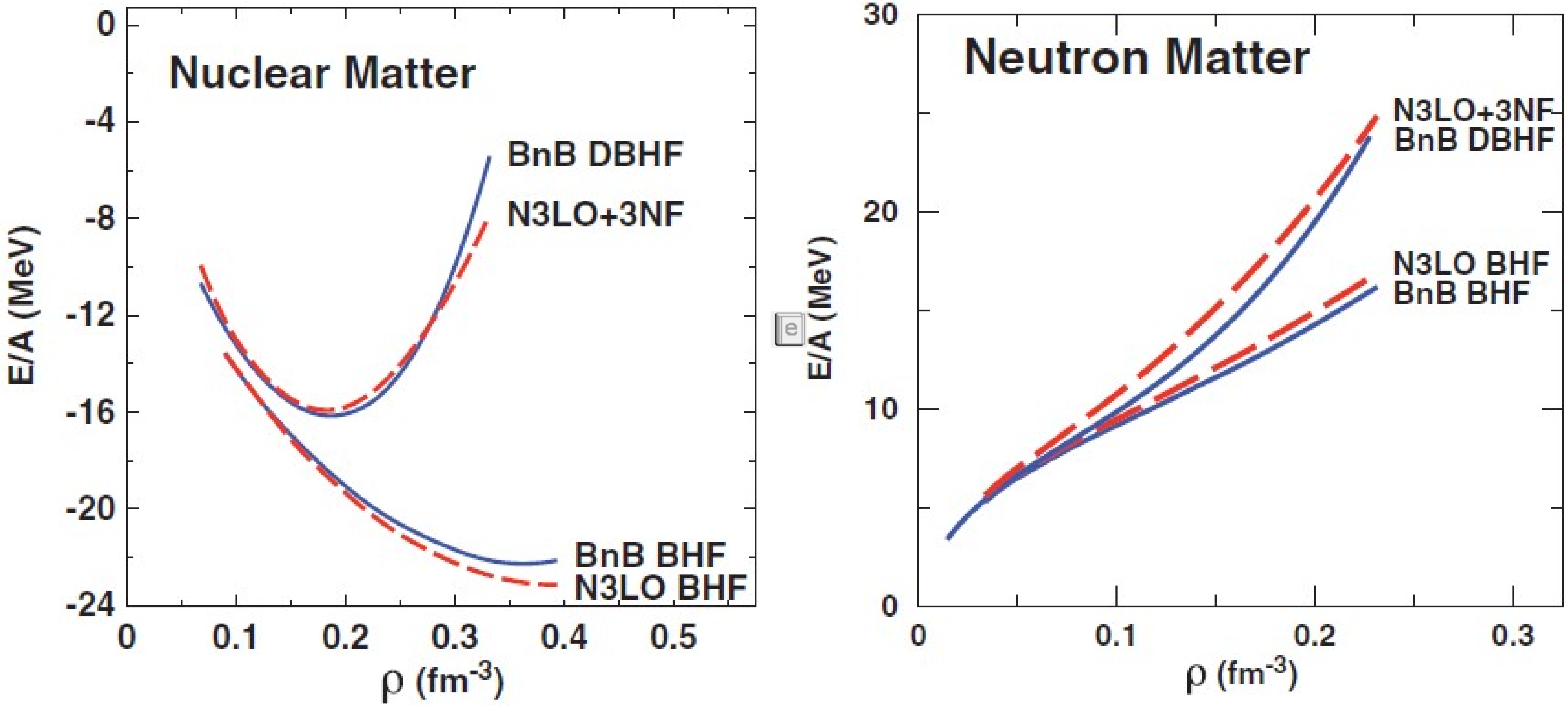


$$|\boldsymbol{u}(\boldsymbol{k},\lambda,m^*)\rangle = \alpha(m^*)|u(\boldsymbol{k},\lambda,m)\rangle + \beta(m^*)|\boldsymbol{v}(-\boldsymbol{k},-\lambda,m)\rangle$$

Dressed spinor — free spinor E>0 — free spinor E<0

$$\rightarrow \quad \frac{\Delta E}{A} \approx 4.2 \text{ MeV} \left(\frac{\rho}{\rho_0}\right)^{\frac{8}{3}}$$

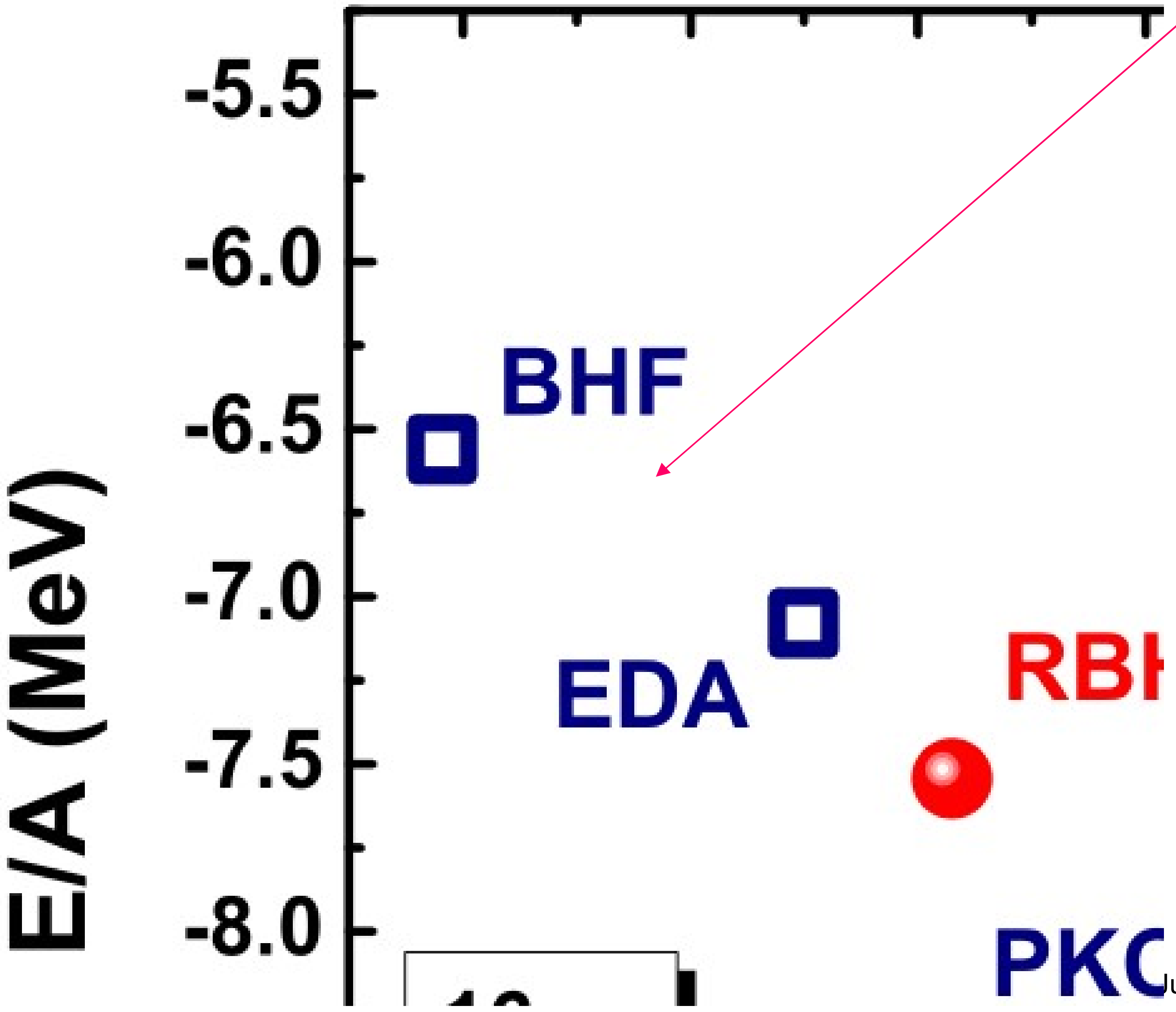
Anastasio et al., PRC **23**, 2606 (1981) non-relativistic calculations with 3-body-forces vs. relativistic calculations without 3-body forces



Sammarruca, Chen, Coraggio, Itaco, Machleidt, PRC 86 , 054317 (2012)

Relativistic BHF for finite nuclei:

S.H. Shen et al (2017).



Problems of RBHF in finite nuclei:

1. Limitation to light spherical nuclei (^{16}O, Ca, …)
   limitation in memory
   limitation in time (no parallelization for inversion)

2. Future goal: Softening of the bare relativistic force
   relativistic V_{lowk} (derived in nuclear matter)

3. Problem (since 40 years):
   There is no full solution of RBHF in nuclear matter !

Relativistic Hartree-Fock in nucl. matter

$$H = H_0 + \Sigma = \beta M + \vec{\alpha}\vec{k} + \Sigma$$

Self-energy Σ in the Walecka model:

$$\Sigma = \beta S + V_0 + \vec{\alpha}\vec{V} = \begin{pmatrix} S + V_0 & \vec{\sigma}\vec{V} \\ \vec{\sigma}\vec{V} & -S + V_0 \end{pmatrix}$$

Self-energy Σ in BHF:

$$\Sigma_{12} = \sum_{34} G[\rho]_{1324}\rho_{43} = \begin{pmatrix} \Sigma^{++} & \Sigma^{+-} \\ \Sigma^{-+} & \Sigma^{--} \end{pmatrix}$$

Conventional solution of RBHF in nucl. Matter:

Thompson-equation: (3D reduction of the Bethe-Salpeter Equation)

$$T^{++++}(E) = V^{++++} + V^{++++} \frac{1}{E - E_{kin}} T^{++++}(E)$$

Bethe-Goldstone equation

$$G^{++++}(W) = V^{++++} + V^{++++} \frac{Q}{W - E_{56}} G^{++++}(W)$$

Self energy:

$$\Sigma_{12}^{++} = \sum_{34} G_{1324}^{++++} \rho_{43}^{++} \qquad \Sigma^{-+} = ???, \qquad \Sigma^{--} = ???$$

Approximations for Σ^{-+}, Σ^{--} …

Perturbation theory: Anastasio et al, PRC 23 (1981)

Projection onto Lorentz invariants: Horowitz et al NPA 464 (1987)

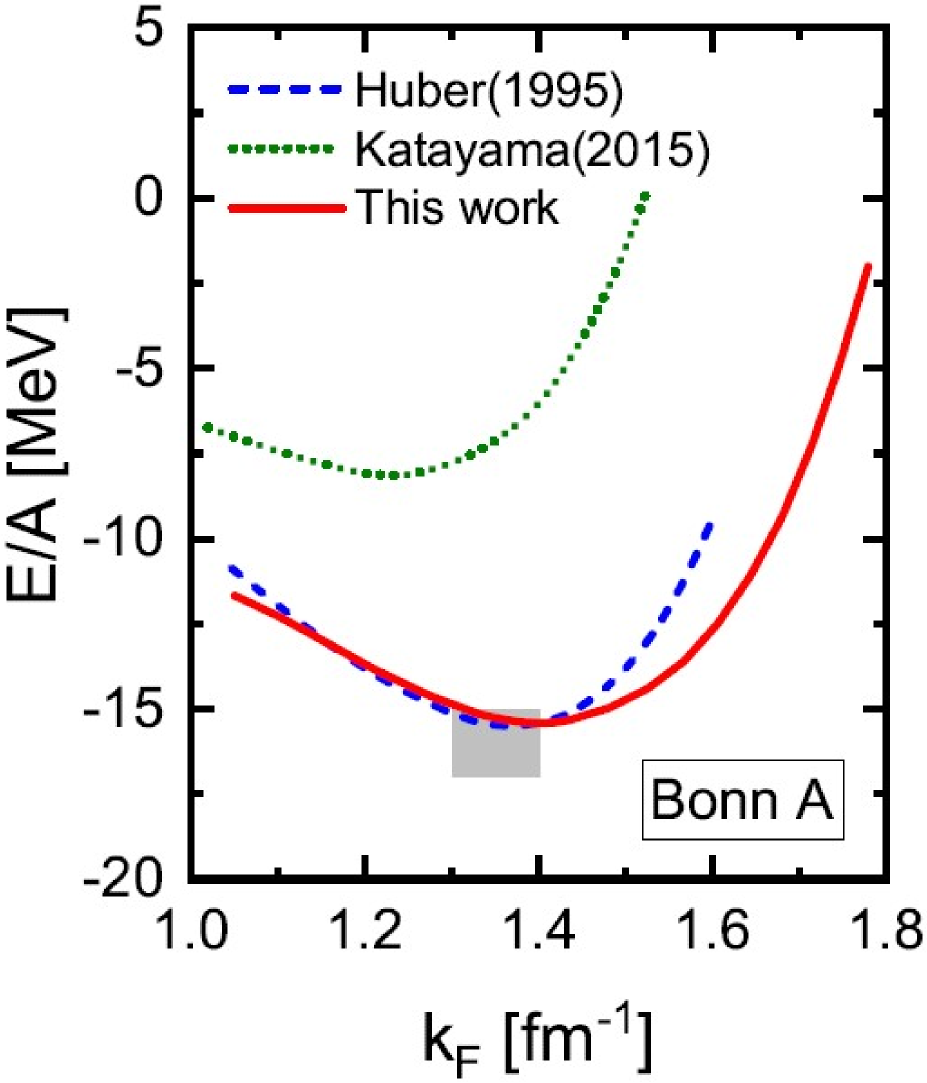
Greens-function techniques: Weigel et al, PRC 38 (1988)

Momentum dependence of $\Sigma^{++}(p)$ is used to determine S and V_0
Brockmann et al, PRC 42 (1990)

Effective DBHF-method, Schiller et al, EPJA 11 (2001)

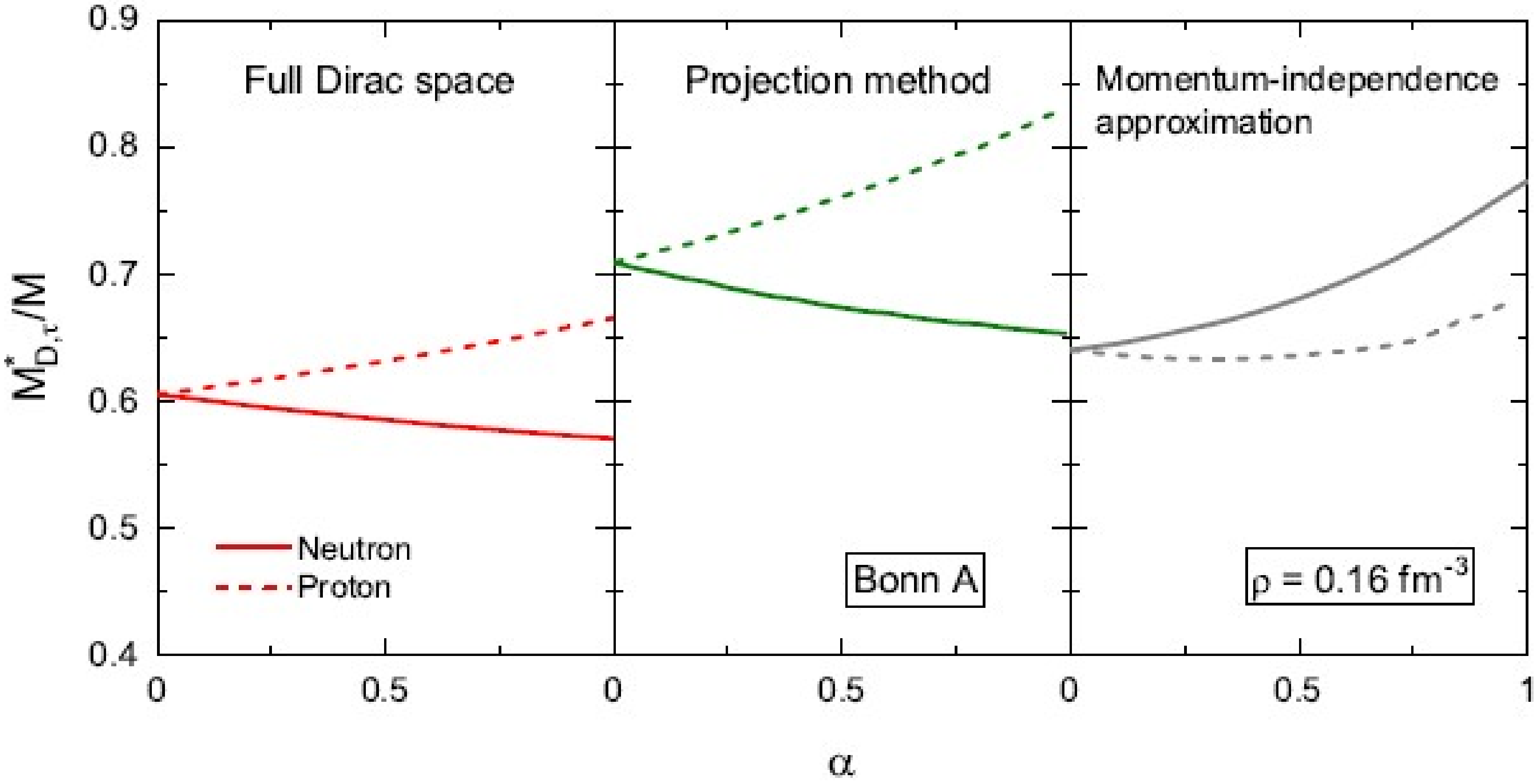
Full solution ????, Katayama et al, PLB 747 (2015)

Results for symmetric nuclear matter:



Sibo Wang, Ziang Zhao, P.R. and Jie Meng, PRC 103, 054301 (2021)

Isospin dependence of the effective Dirac mass:



asymmetry parameter: $\alpha = (\rho_n - \rho_p)/\rho$ at saturation

Thank you !