

Shape Coexistence and Quantum Phase Transitions in Sr isotopes

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What is shape coexistence?

Shape coexistence: It appears in quantum systems where eigenstates with very different density distribution coexist.



Shape of the nucleus

(Implicit geometric interpretation)

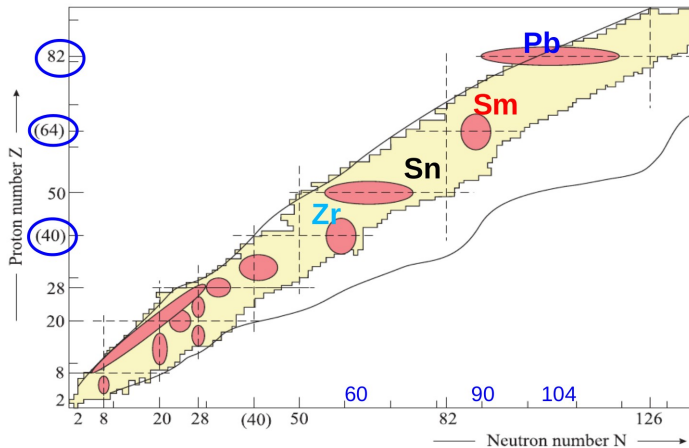


- ▶ **Stabilizing effect:** closed shell
- ▶ **Deformed tendency:** pairing and quadrupole force



Regions around closed shells with **spherical shapes** and near mid-shell are **well deformed**

Regions of interest



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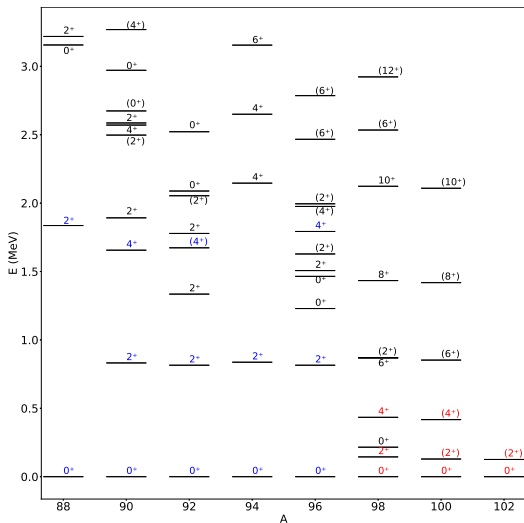
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Experimental data in the even-even nuclei



Blue labels are for spherical states while red labels correspond to the deformed ones.

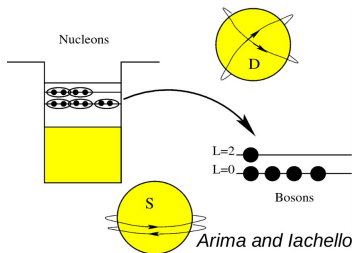
Interacting boson Model. IBM

Nucleons couple preferably in pairs with angular momentum either equal to 0 (S) or equal to 2 (D).

$$s^\dagger, d_m^\dagger (m = 0, \pm 1, \pm 2)$$

$$s, d_m (m = 0, \pm 1, \pm 2)$$

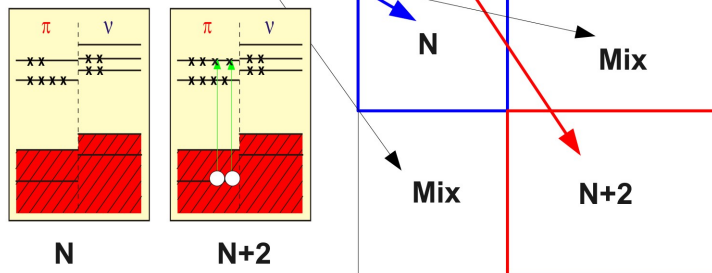
$$\hat{H}_{ECQF} = \epsilon \hat{n}_d + \kappa \hat{Q} \cdot \hat{Q} + \kappa' \hat{L} \cdot \hat{L}$$



- ▶ Model based on a **u(6) spectrum generator algebra**. It is especially suited for **medium and heavy-mass** nuclei.
- ▶ The number of bosons, **N**, corresponds the **number of nucleons pairs**, regardless its proton, neutron, particle or hole nature.

IBM with configuration mixing

$$\hat{H} = \hat{P}_N^\dagger \hat{H}_{ECQF}^N \hat{P}_N + \hat{P}_{N+2}^\dagger (\hat{H}_{ECQF}^{N+2} + \Delta^{N+2}) \hat{P}_{N+2} + \hat{V}_{mix}^{N,N+2}$$



A different Hamiltonian, \hat{H}_{ECQF}^N and \hat{H}_{ECQF}^{N+2} , acts on the regular $[N]$ and intruder $[N+2]$ sectors, separately

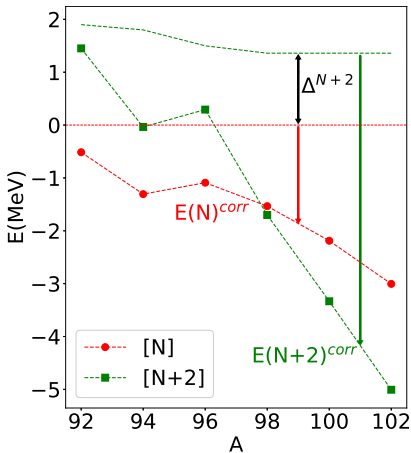
Fitting procedure

Least squares fit to the experimental data, including
excitation energies and absolute B(E2) transitions



$$\chi^2 = \frac{1}{N_{data} - N_{par}} \sum_{i=1}^{N_{data}} \frac{(X_i(data) - X_i(IBM))^2}{\sigma_i^2}$$

Correlation energy in the configuration mixing approach

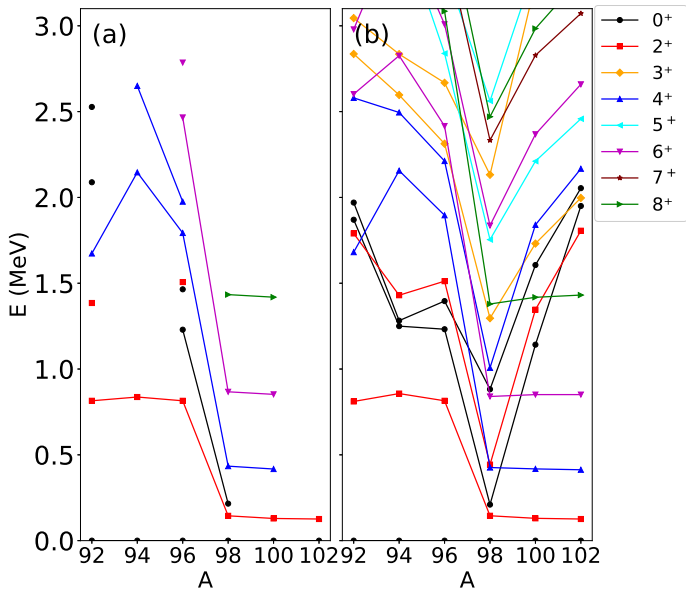


Absolute energy of the lowest unperturbed regular and intruder 0_1^+ states

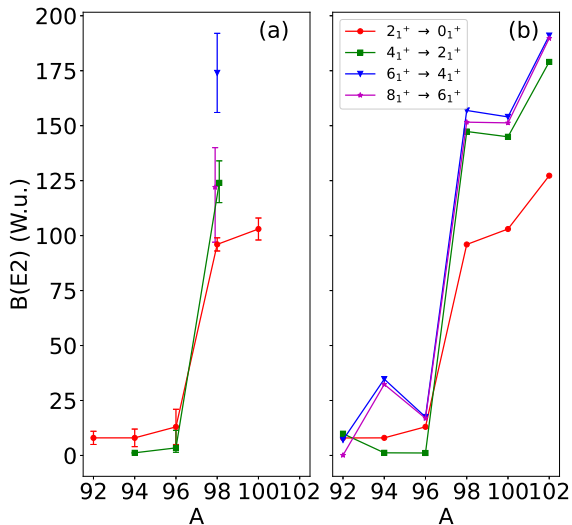


$$V_{mix} = 0$$

Excitation energies

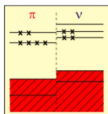


B(E2) transition probabilities-Yrast Band

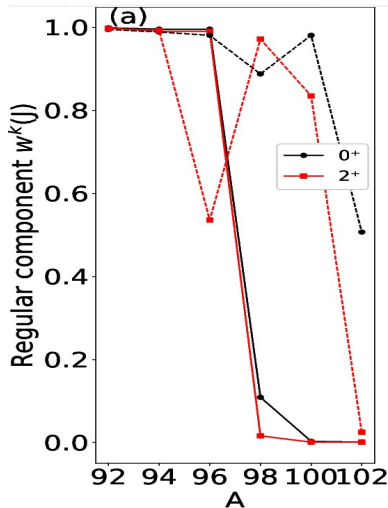
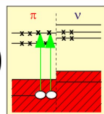


Wave function: Regular component

$$\phi(J, M) = a(J, M)$$



$$+ b(J, M)$$

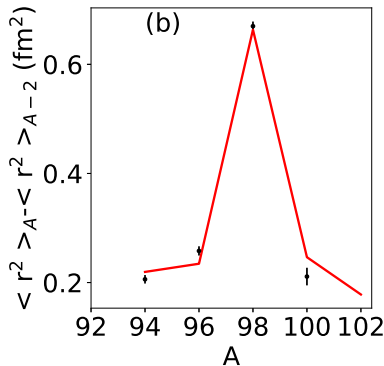
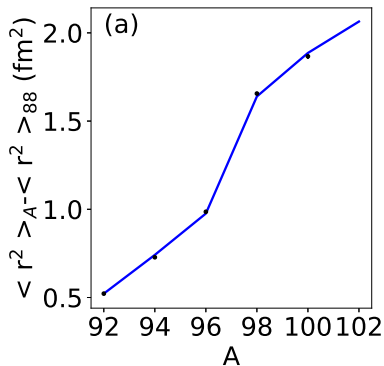


Wave function within the
regular sector



$$w^k(J) \equiv \sum_i |a_i^k(J)|^2$$

$$r^2 = r_c^2 + \hat{P}_N^\dagger (\gamma_N \hat{N} + \beta_N \hat{n}_d) \hat{P}_N + \hat{P}_{N+2}^\dagger (\gamma_{N+2} \hat{N} + \beta_{N+2} \hat{n}_d) \hat{P}_{N+2}$$



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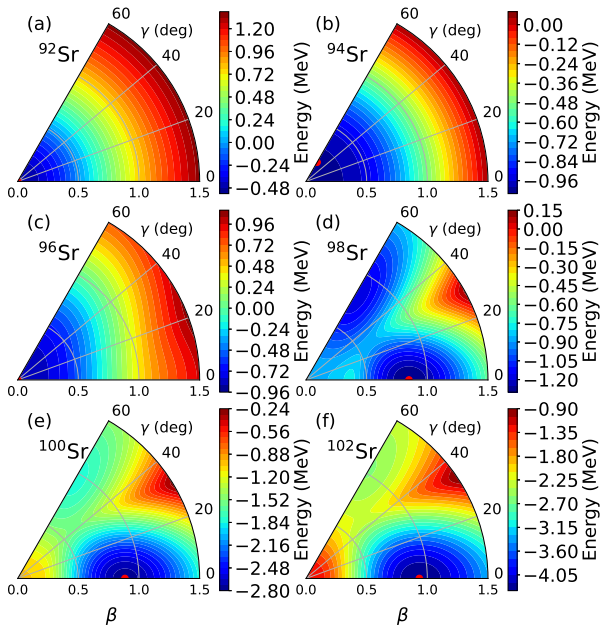
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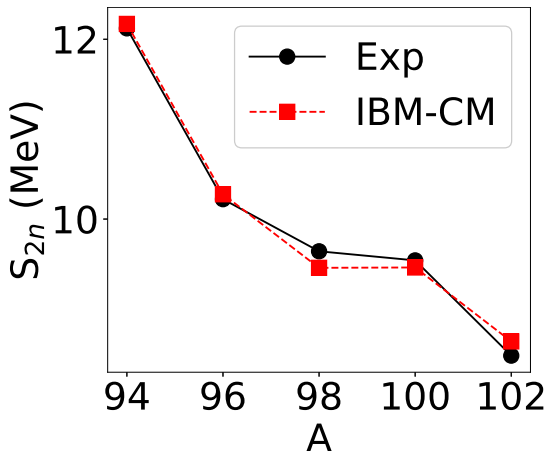
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Quantum Phase Transitions in Sr isotopes

Some hints points towards a Quantum Phase Transition in the Sr region

- ▶ Two neutron separation energies



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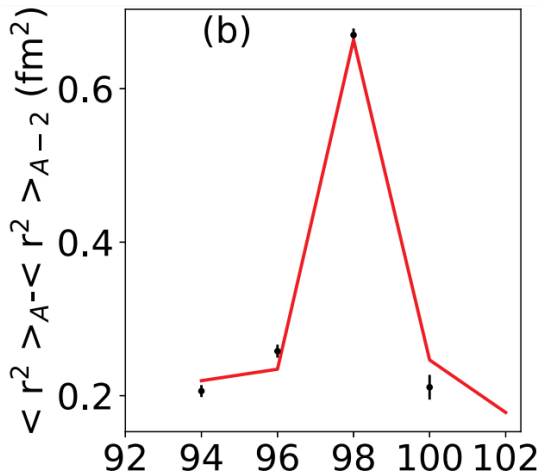
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Some hints points towards a Quantum Phase Transition
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- Isotopic shift



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Conclusions

- ▶ In the Sr isotopes studied we have notice a clear change in structure, with an evolution from spherical shapes to more deformed ones, with a clear change in ^{98}Sr .
- ▶ We have developed a phenomenological study in order to obtain the spectrum from experimental energy spectra and $B(E2)$.
- ▶ The IBM-CM provides an accurate description of the observed changes and of the different shapes in the spectrum.
- ▶ We have described the theoretical S_{2n} and radii in comparison with the experimental results, as a method to the reliability of our calculations and as posible hints pointing to a QPT.

THANK YOU

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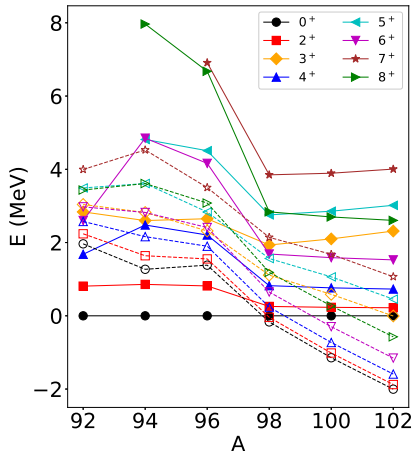
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Nucleus	N	ε_N	κ_N	χ_N	κ'_N	ε_{N+2}	κ_{N+2}	χ_{N+2}	κ'_{N+2}	ω	Δ^{N+2}	e_N	e_{N+2}
^{92}Sr	3	838	-32.01	0.00	-7.84	347.2	-15.00	-0.88	0.00	15	1900	1.53	1.53 ^a
^{94}Sr	4	365	-50.00	0.00	75.01	451.7	-41.81	0.00	0.00	15	1800	1.16	1.53
^{96}Sr	5	620	-35.00	0.64	53.43	242.7	-20.00	-0.79	9.84	15	1500	1.33	0.86
^{98}Sr	6	526	-28.19	1.88	18.59	279.1	-34.96	-0.72	0.23	15	1360	0.78	2.22
^{100}Sr	7	526 ^b	-28.19 ^b	1.88 ^b	18.59 ^b	387.3	-43.16	-0.77	-2.99	15	1360	0.78 ^b	1.93
^{102}Sr	8	526 ^b	-28.19 ^b	1.88 ^b	18.59 ^b	387.3	-46.41	-0.77	-2.99	15	1360	0.78 ^b	1.93 ^c

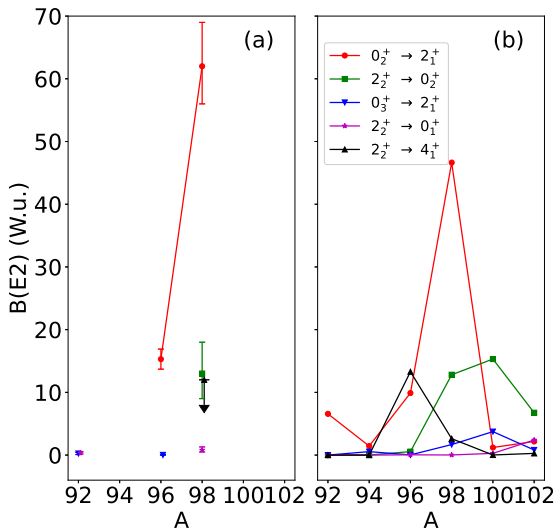
Correlation energy in the configuration mixing approach



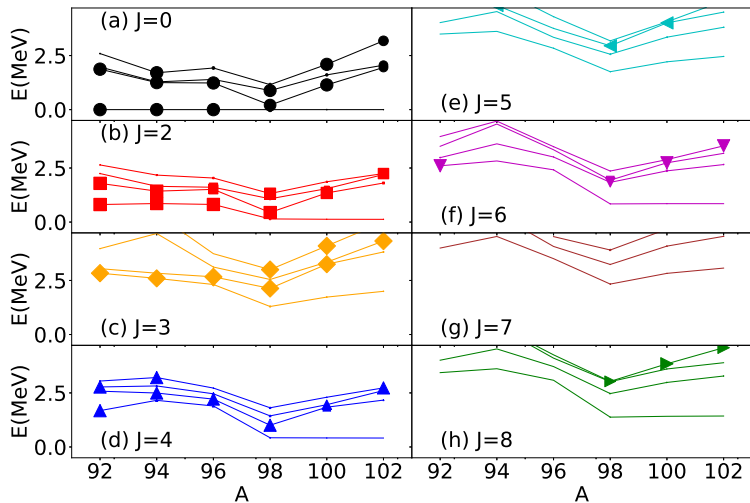
Energy spectra for the IBM-CM Hamiltonian obtained

$$\Downarrow$$
$$V_{mix} = 0$$

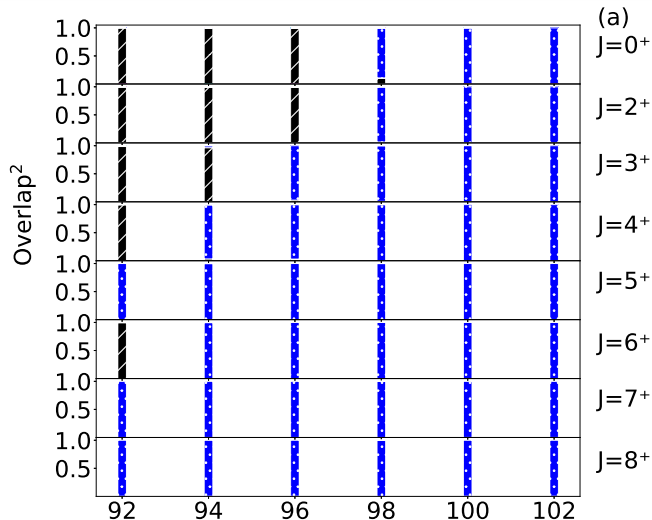
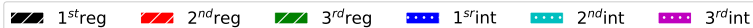
B(E2) transition probabilities-Non Yrast Band



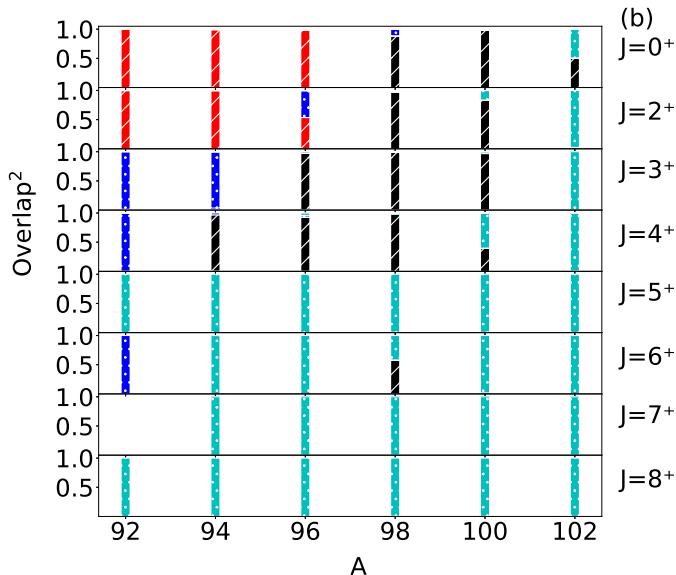
Wave function: energy systematics



Overlap of the wave function



Overlap of the wave function

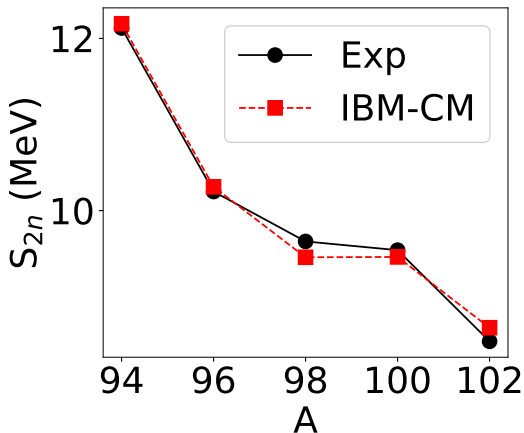


Two-neutron separation energies

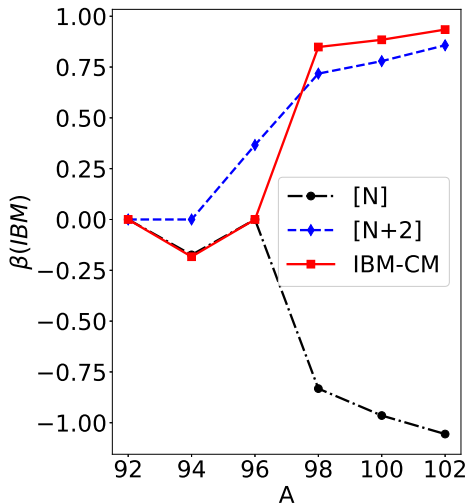
$$S_{2n}(A) = \mathcal{A} + \mathcal{B}A + BE^{lo}(A) - BE^{lo}(A - 2)$$

↓
Because of the influence of intruder states

$$S_{2n}(A) = \mathcal{A} + \mathcal{B}(A + 2(1 - w)) + BE^{lo}(A) - BE^{lo}(A - 1)$$



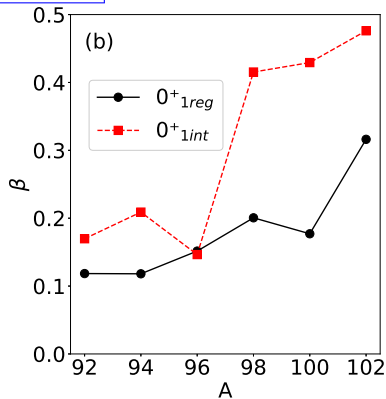
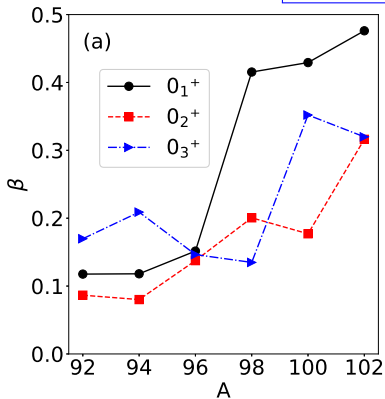
β from IBM



β from the quadrupole shape invariants

$$q_{2,i} = \sqrt{5} \langle 0_i^+ | [\hat{Q} \times \hat{Q}]^{(0)} | 0_i^+ \rangle$$

$$\beta = \frac{4\pi\sqrt{q_2}}{3Ze_0^2 A^{2/3}}$$



Interacting Boson Model

$$\hat{H}_{\text{ecqf}}^i = \varepsilon_i \hat{n}_d + \kappa'_i \hat{L} \cdot \hat{L} + \kappa_i \hat{Q}(\chi_i) \cdot \hat{Q}(\chi_i)$$

$$\hat{L}_\mu = \left[d^\dagger \times \tilde{d} \right]_\mu^{(1)}$$

$$\hat{Q}_\mu(\chi_i) = \left[s^\dagger \times \tilde{d} + d^\dagger \times s \right]_\mu^{(2)} + \chi_i \left[d^\dagger \times \tilde{d} \right]_\mu^{(2)}$$

$$\hat{V}_{\text{mix}}^{N,N+2} = \omega_0^{N,N+2} (s^\dagger \times s^\dagger + s \times s) + \omega_2^{N,N+2} (d^\dagger \times d^\dagger + \tilde{d} \times \tilde{d})^{(0)}$$

$$\hat{T}(E2)_\mu = \sum_{i=N,N+2} e_i \hat{P}_i^\dagger \hat{Q}_\mu(\chi_i) \hat{P}_i$$

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We have considered the coherent state:

$$|N, \alpha_m\rangle = \left(s^\dagger + \sum_m \alpha_m d_m^\dagger \right)^N |0\rangle$$

Where the relation with the collective parameters:

$$\alpha_0 = \beta \cos \gamma, \quad \alpha_{\pm 1} = 0, \quad \alpha_{\pm 2} = \frac{\beta}{\sqrt{2}} \cos \gamma$$

$$|N; \beta, \gamma\rangle = \left\{ s^\dagger + \beta \left[\cos \gamma d_0^\dagger + 1/\sqrt{2} \sin \gamma (d_{+2}^\dagger + d_{-2}^\dagger) \right] \right\}^N |0\rangle$$

$$E(N; \beta, \gamma) = \frac{\langle N; \beta, \gamma | H | N; \beta, \gamma \rangle}{\langle N; \beta, \gamma | N; \beta, \gamma \rangle}$$

One Weisskopf unit of $B(E\lambda)$ is equal to

$$B(E\lambda) = \frac{(1.2)^{2\lambda}}{4\pi} \left(\frac{3}{\lambda + 2} \right)^2 A^{2\lambda/3} \quad \text{in unit of } e^2(fm)^\lambda$$

Transition probability

$$T(E2) = 1.223 \times 10^9 E_\gamma^5 B(E2) [1/\text{sec}]$$

E_γ is in MeV.

B(E2) in $e^2(fm)^4$