

# *Intertwined QPTs in the Zr isotopes*

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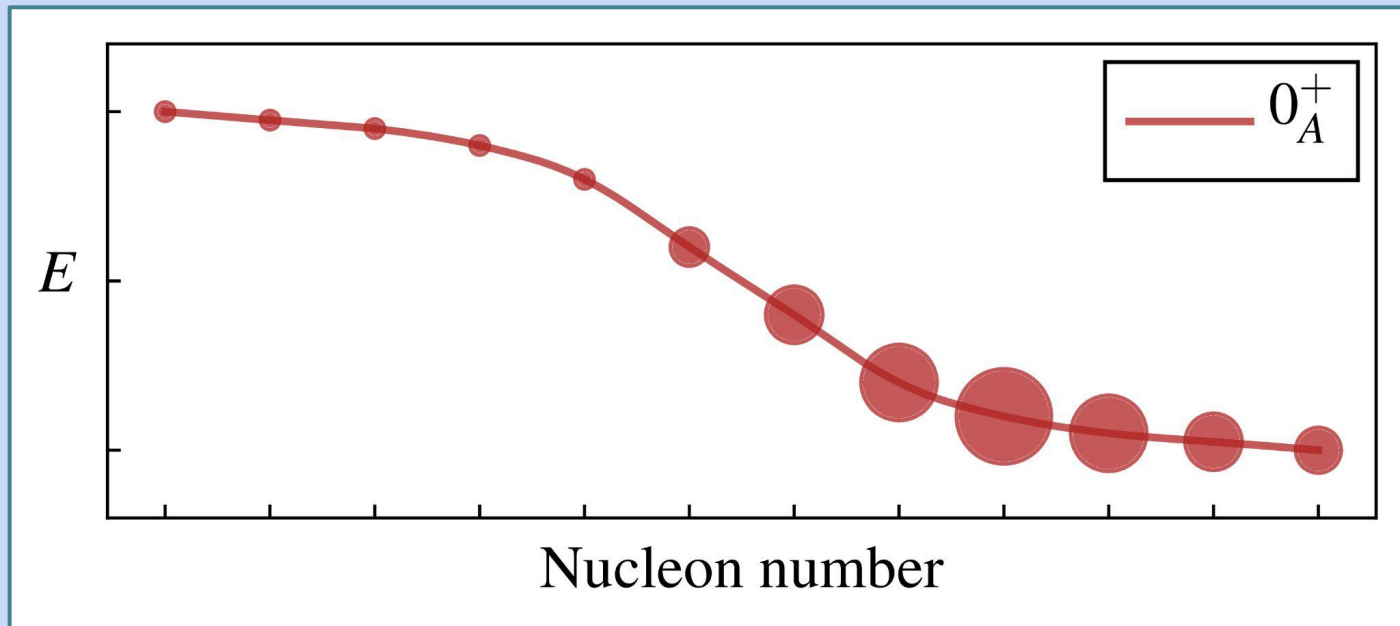
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*with A. Leviatan and F. Iachello*

*Dubrovnik 2022*

# Introduction

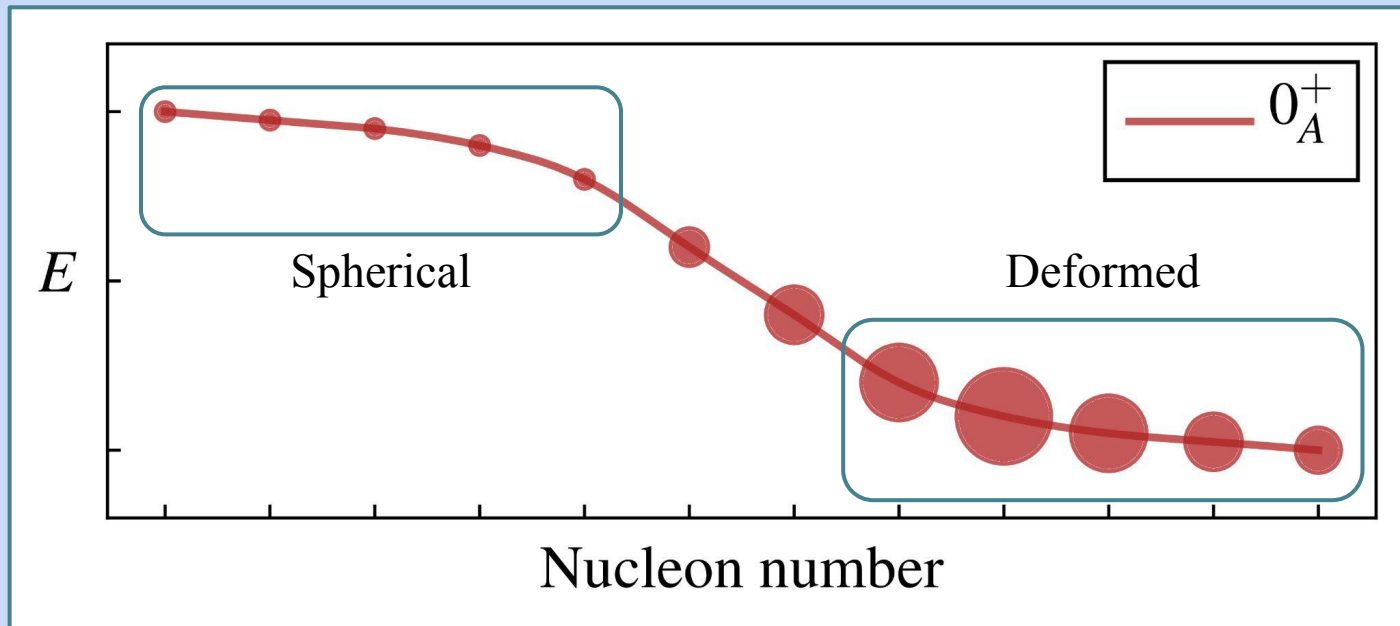
Type I QPT: shape evolution (single configuration)



$$\hat{H} = (1-\xi)\hat{H}_1 + \xi\hat{H}_2; \quad 0 \leq \xi \leq 1$$

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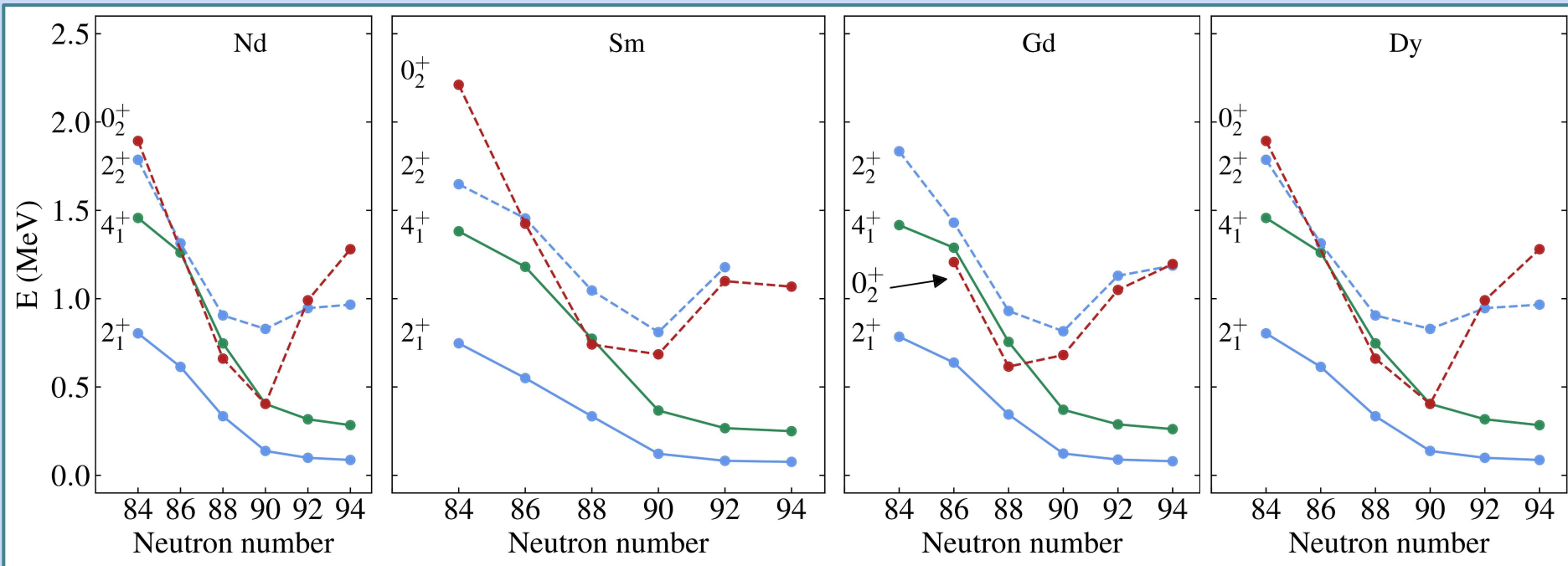
## Example

Z = 60

Z = 62

Z = 64

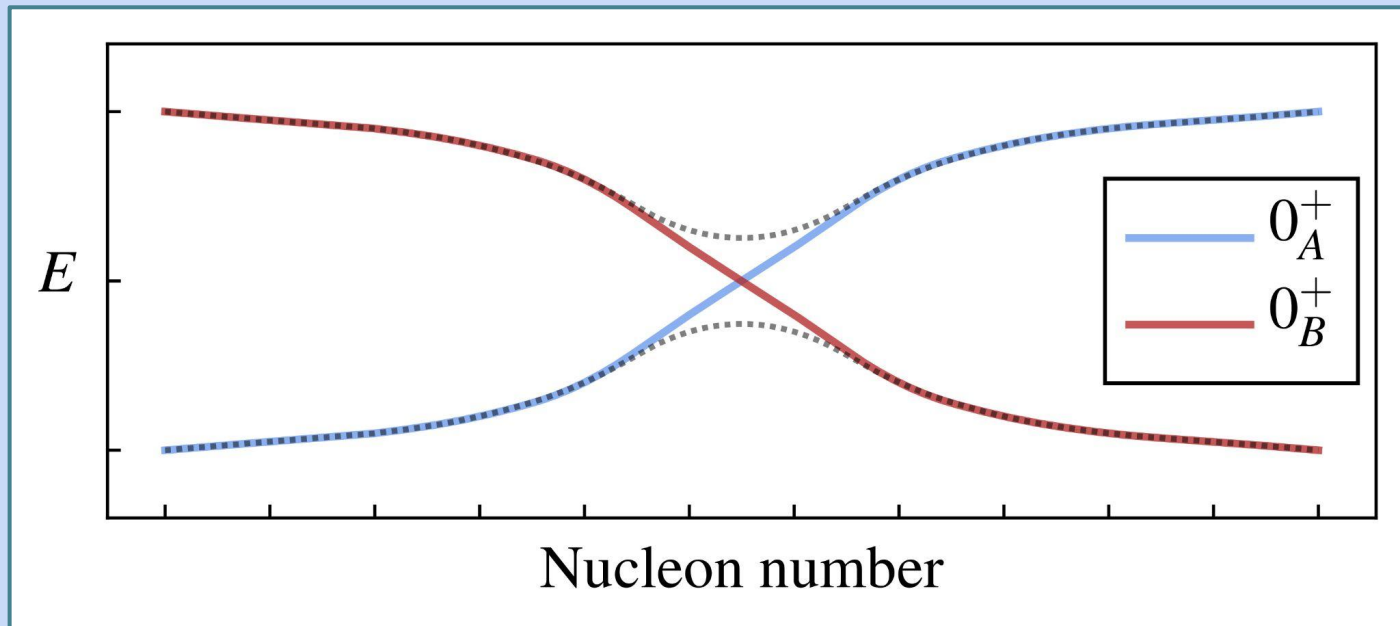
Z = 66



Spherical [U(5)]  $\rightarrow$  Deformed [SU(3)]

# Introduction

Type II QPT: configuration crossing (multiple configurations)

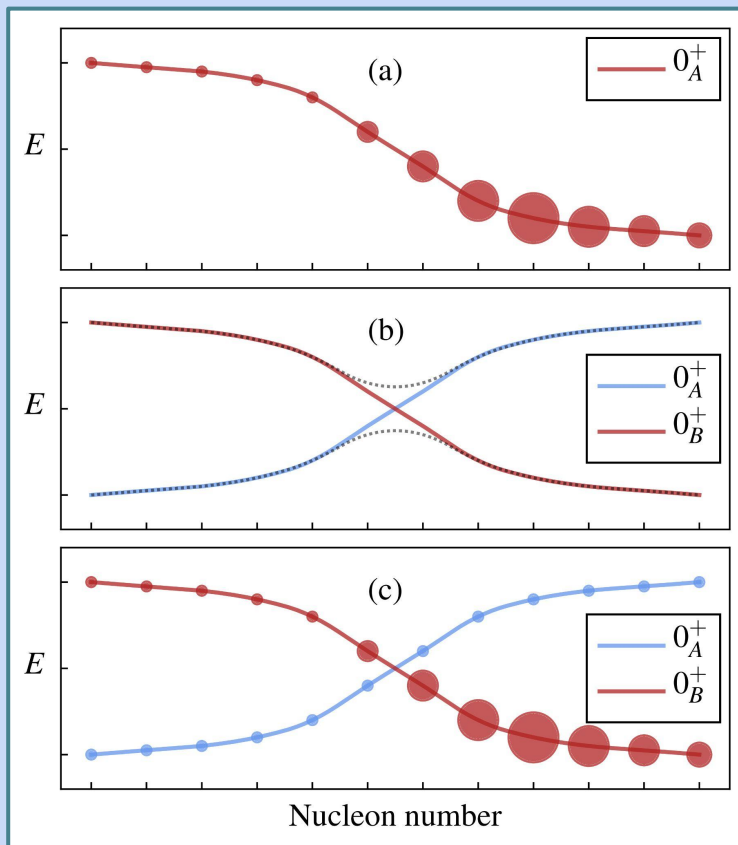


$$\hat{H} = \begin{bmatrix} \hat{H}_A(\xi_A) & \hat{W}(\omega) \\ \hat{W}(\omega) & \hat{H}_B(\xi_B) \end{bmatrix}$$

Light Pb-Hg isotopes

# Introduction

## Intertwined quantum phase transitions (IQPT)

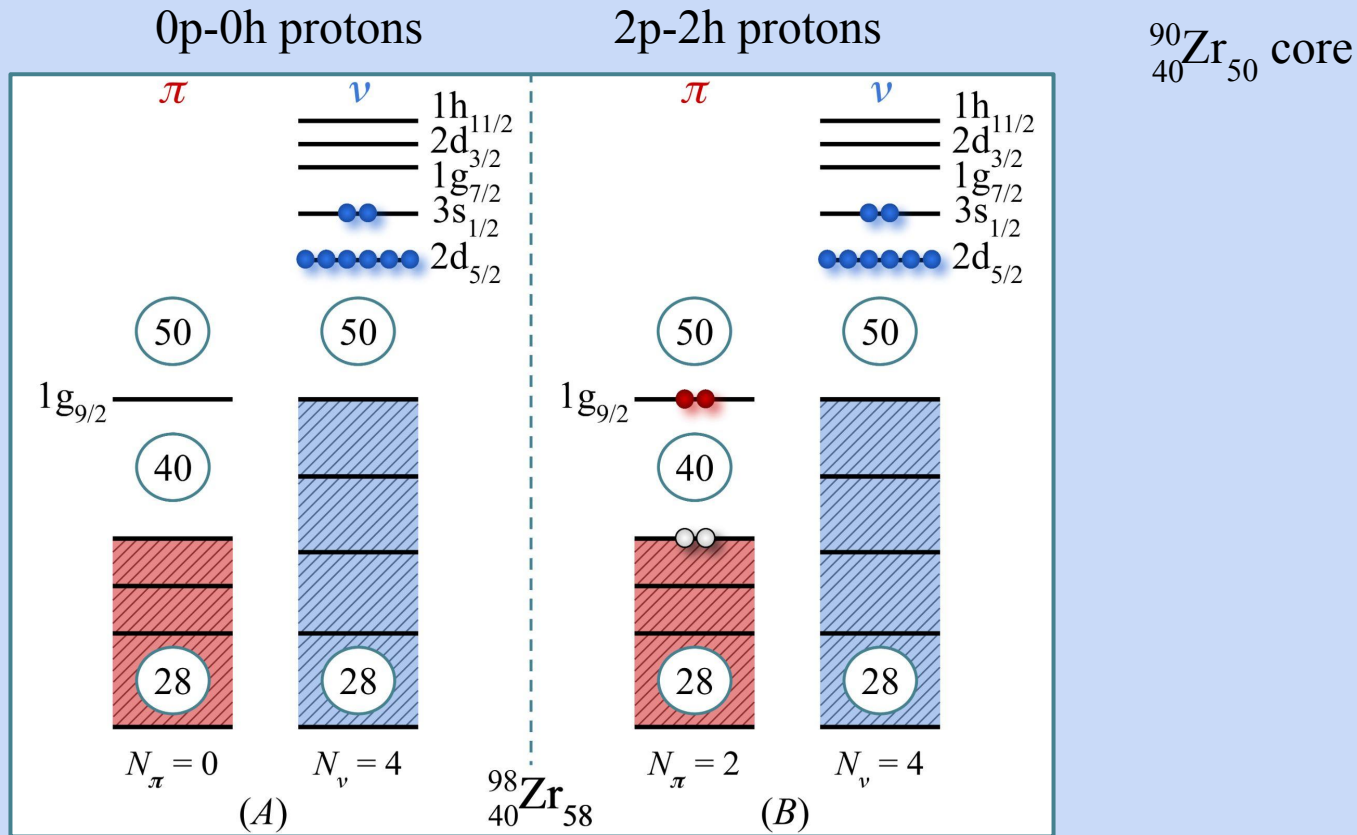


***IQPT:***  
Type I and Type II  
simultaneously  
(Type II with weak mixing)

N. Gavrielov, A. Leviathan and F. Iachello,  
Phys. Rev. C **99**, 064324 (2019)  
Phys. Scr. **95**, 024001 (2020)  
Phys. Rev. C **105**, 014305 (2022)

# Introduction

Boson counting:  $^{98}\text{Zr}$



Configuration A:  
Seniority-like neutron excitations.  
P. Federman and S. Pittel,  
Phys. Rev. C **20**, 820 (1979)

$$\hat{H} = \hat{H}_A^{(N)} + \hat{H}_B^{(N+2)} + \hat{W}^{(N,N+2)}$$

*Normal configuration (0p-0h):*

$$\hat{H}_A = \varepsilon_d^{(A)} \hat{n}_d + \kappa^{(A)} Q \cdot Q \quad [N] \text{ irrep.}$$

*Intruder configuration (2p-2h):*

$$\hat{H}_B = \varepsilon_d^{(B)} \hat{n}_d + \kappa^{(B)} Q \cdot Q + \kappa'^{(B)} L \cdot L + \Delta_p \quad [N+2] \text{ irrep.}$$

*Coupling:*

$$\hat{W}^{(N,N+2)} = \omega [(d^\dagger d^\dagger)^{(0)} + (s^\dagger)^2] + h.c. \quad [N] \oplus [N+2] \text{ irrep.}$$

# Introduction

## Wave function structure and decomposition

$$|\psi; L\rangle = a |\psi; [N], L\rangle + b |\psi; [N+2], L\rangle; \quad a^2 + b^2 = 1$$

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$$|\psi; [N], L\rangle = \sum_{(\lambda, \mu), K} C_{(\lambda, \mu), K}^{(N,L)} |N, (\lambda, \mu), K, L\rangle \quad \text{SU(3)}$$

$$|\psi; [N], L\rangle = \sum_{\sigma, \tau, n_\Delta} C_{\sigma, \tau, n_\Delta}^{(N,L)} |N, \sigma, \tau, n_\Delta, L\rangle \quad \text{SO(6)}$$

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$$|\psi; [N], L\rangle = \sum_{n_d, \tau, n_\Delta} C_{n_d, \tau, n_\Delta}^{(N,L)} |N, n_d, \tau, n_\Delta, L\rangle \rightarrow P_{n_d}^{(N,L)} = \sum_{\tau, n_\Delta} [C_{n_d, \tau, n_\Delta}^{(N,L)}]^2 \quad \text{U(5) decomposition}$$

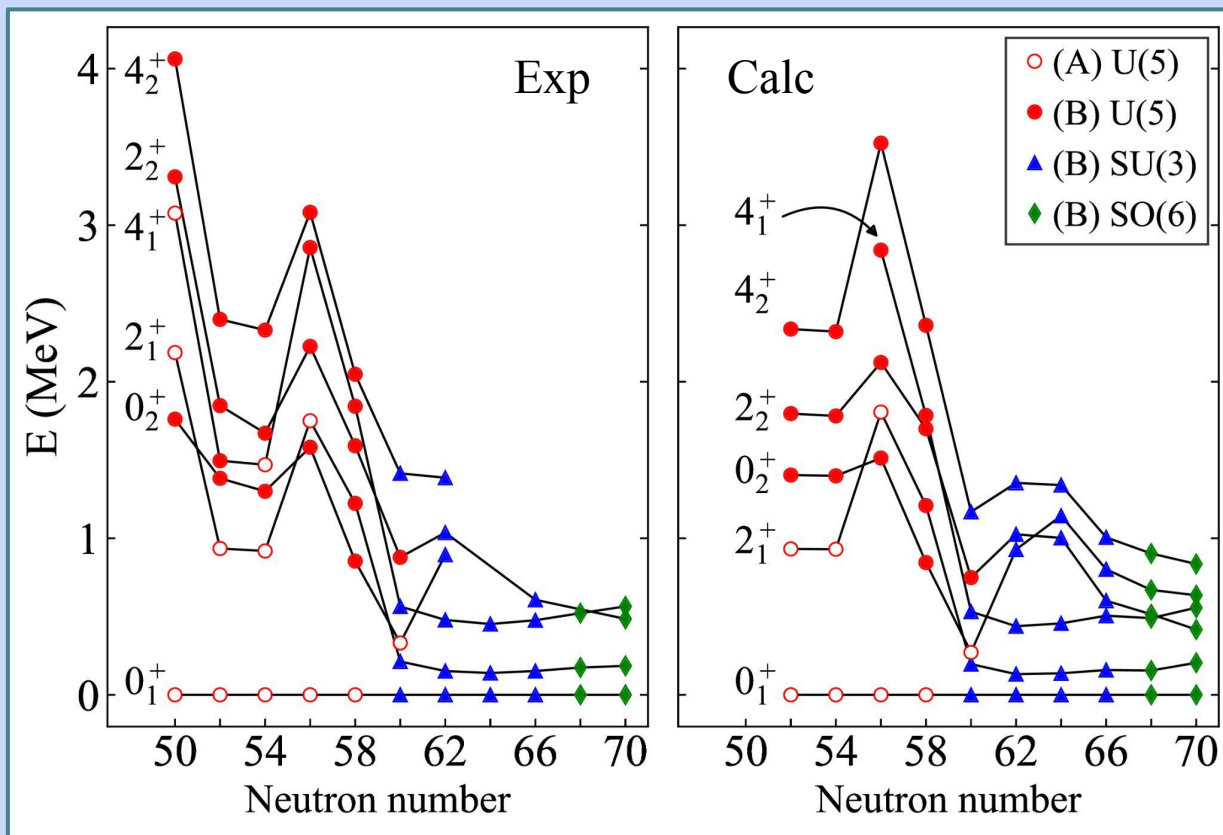
$$|\psi; [N], L\rangle = \sum_{(\lambda, \mu), K} C_{(\lambda, \mu), K}^{(N,L)} |N, (\lambda, \mu), K, L\rangle \rightarrow P_{(\lambda, \mu)}^{(N,L)} = \sum_K [C_{(\lambda, \mu), K}^{(N,L)}]^2 \quad \text{SU(3) decomposition}$$

$$|\psi; [N], L\rangle = \sum_{\sigma, \tau, n_\Delta} C_{\sigma, \tau, n_\Delta}^{(N,L)} |N, \sigma, \tau, n_\Delta, L\rangle \rightarrow P_{\sigma}^{(N,L)} = \sum_{\tau, n_\Delta} [C_{\sigma, \tau, n_\Delta}^{(N,L)}]^2 \quad \text{SO(6) decomposition}$$

$$\rightarrow P_{\tau}^{(N,L)} = \sum_{n_d, n_\Delta} [C_{n_d, \tau, n_\Delta}^{(N,L)}]^2 \quad \text{SO(5) decomposition}$$

# Results

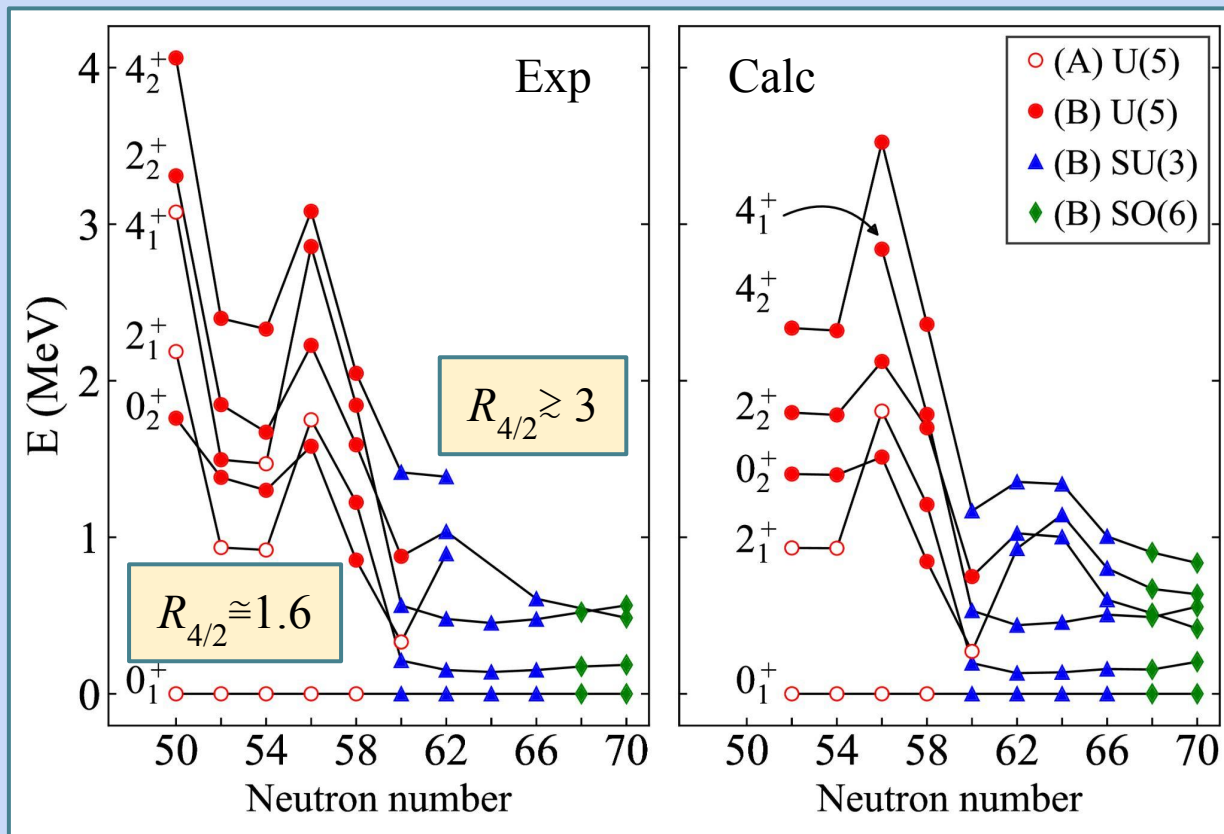
## Energy levels



# Results

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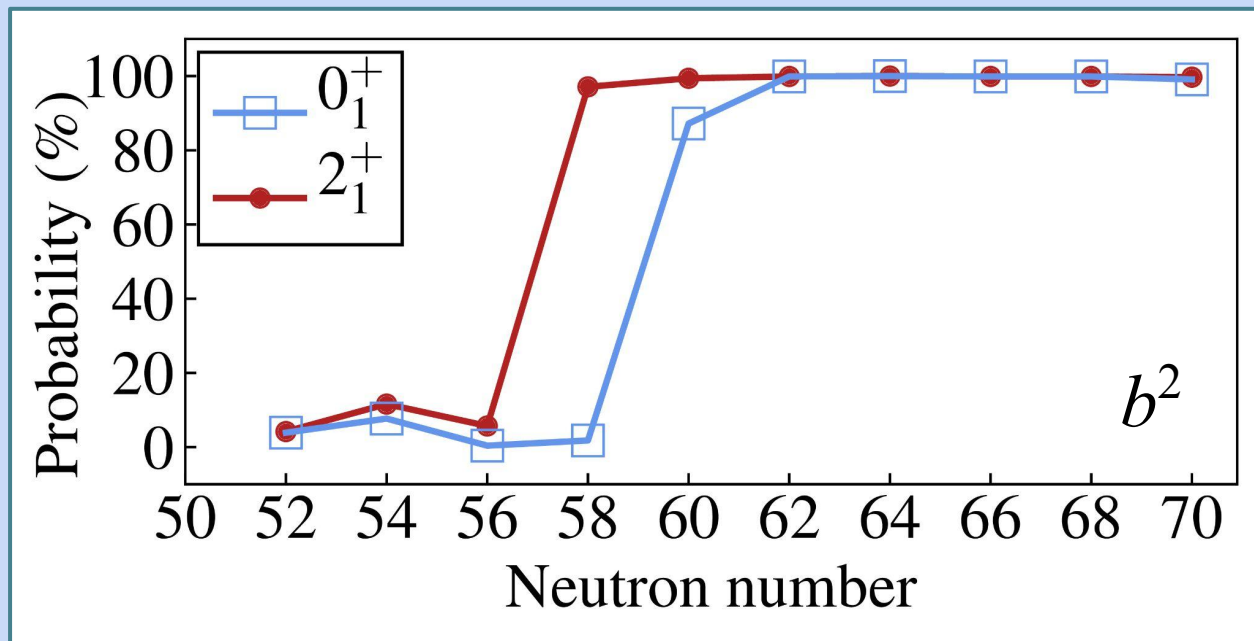
$$R_{4/2} = E(4_1^+) / E(2_1^+)$$



# Results: Type II QPT

Evolution of configuration content:  $b^2$

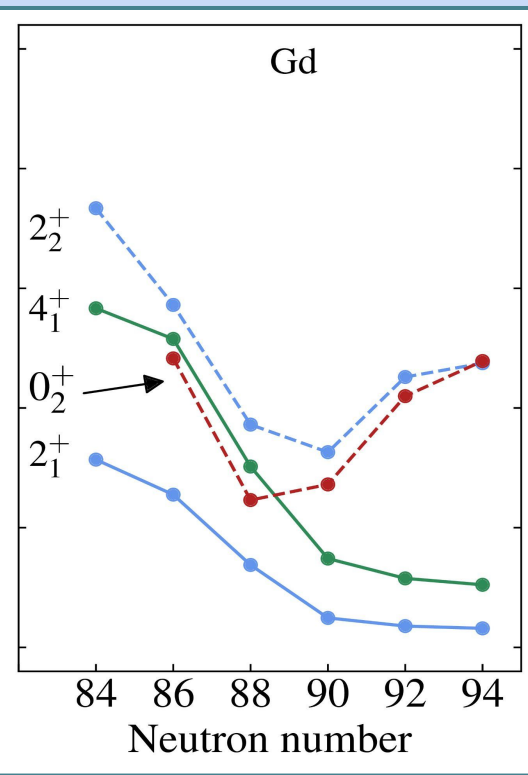
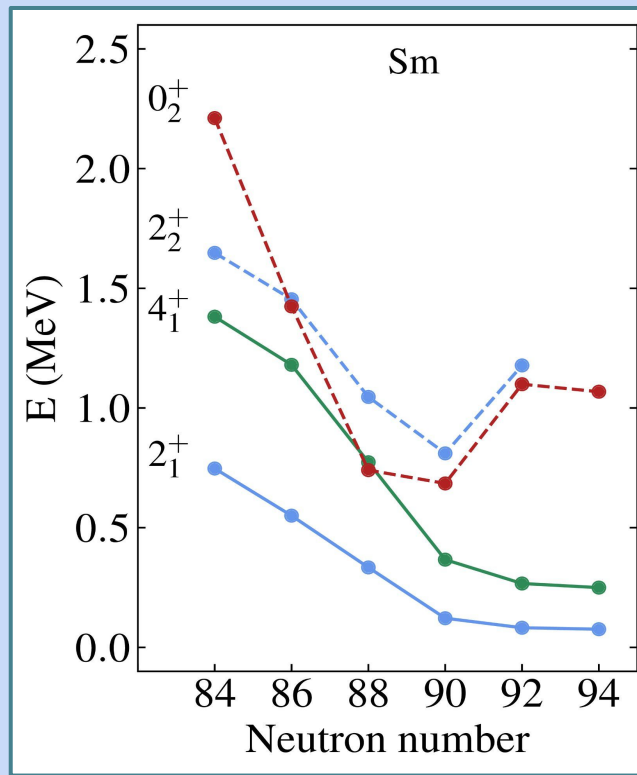
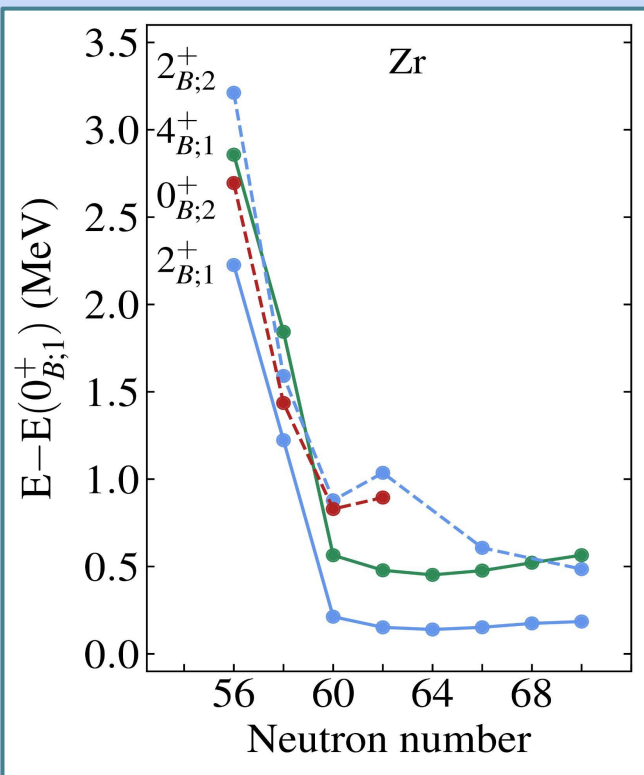
$$|\psi; L\rangle = a|\psi; [N], L\rangle + b|\psi; [N+2], L\rangle$$



$2^+$  changes at 58, as suggested by W. Witt *et al.*, Phys. Rev. C **98**, 041302(R) (2018)

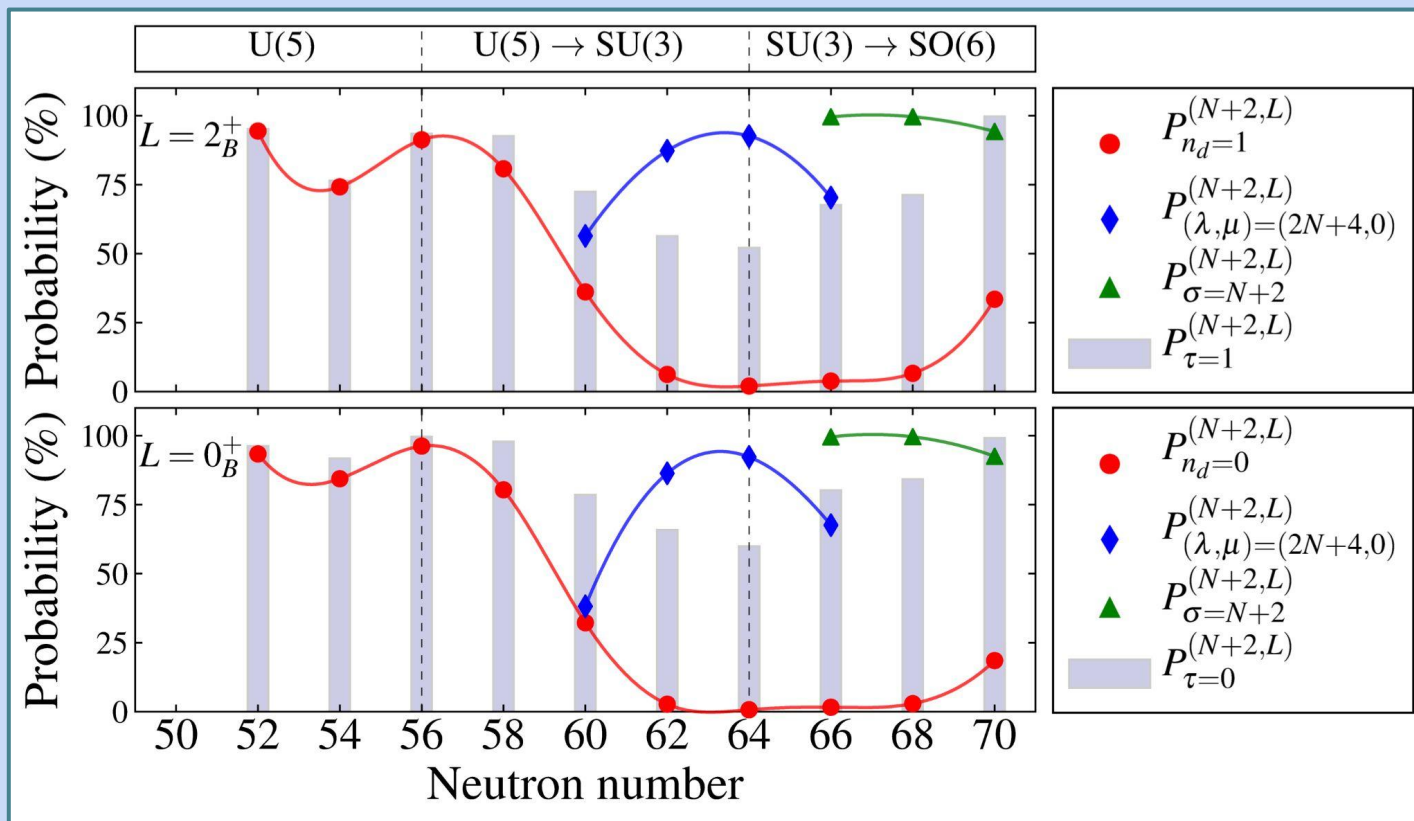
# Results

Comparison with Sm and Gd isotopes



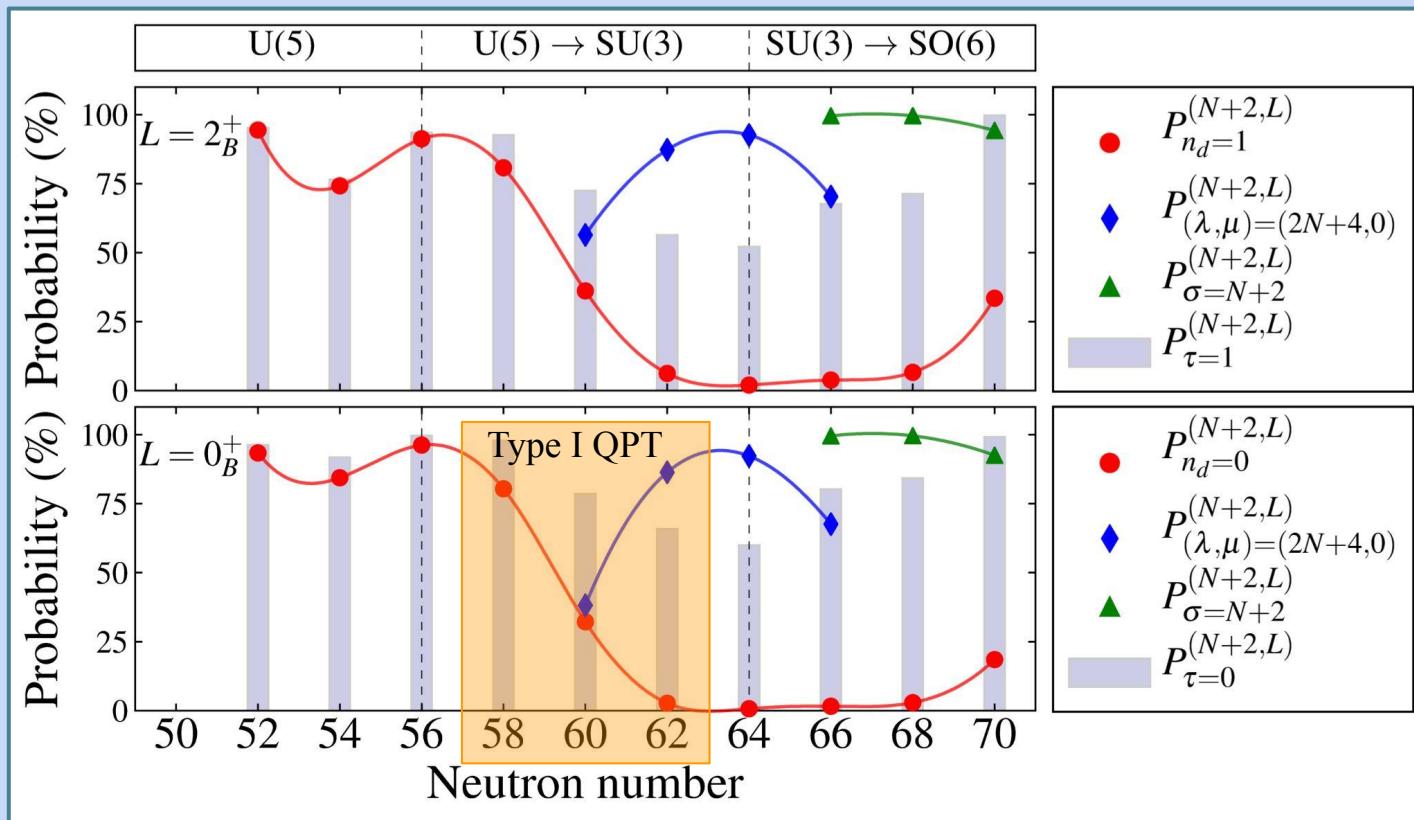
# Results: Type I QPT

Evolution of symmetry content



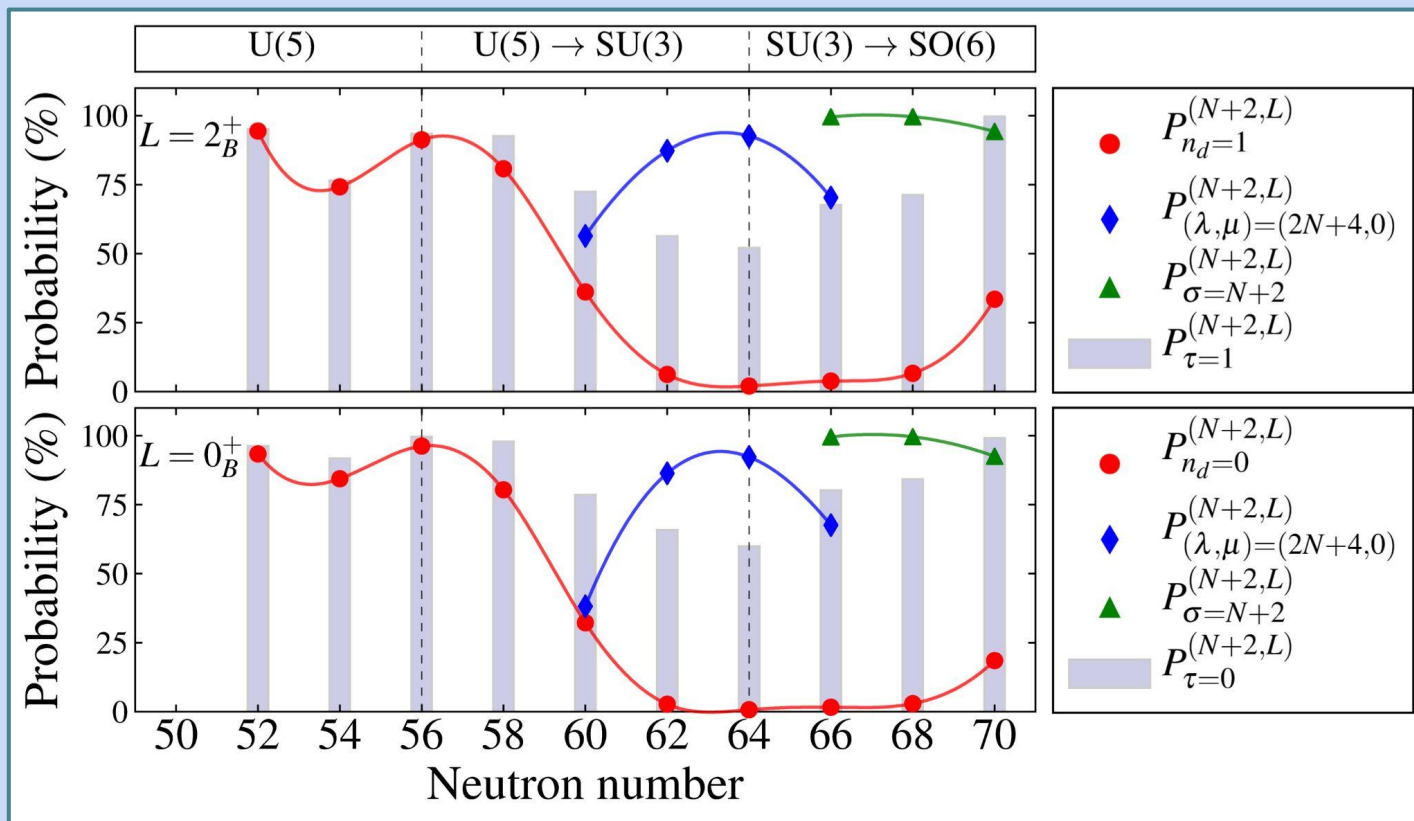
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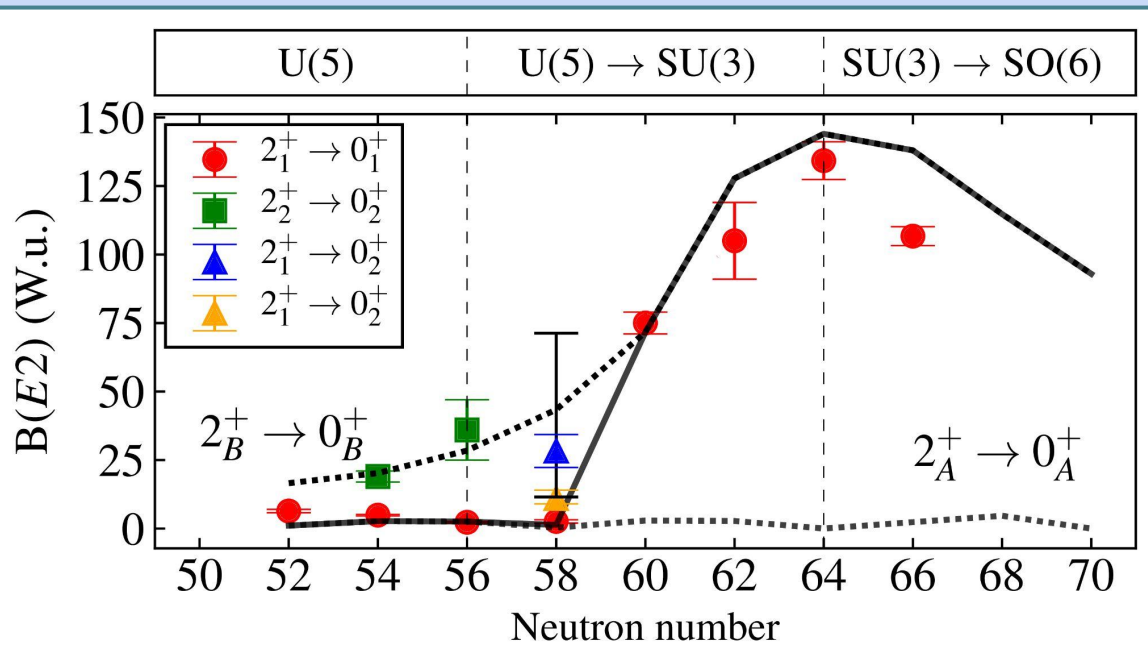
Evolution of symmetry content



# Results

## $E2$ transitions

$$T(E2) = e^{(A)} Q_x^{(N)} + e^{(B)} Q_x^{(N+2)}$$



$$e^{(A)} = 0.9 \sqrt{\text{W.u.}}$$

$$e^{(B)} = 2.24 \sqrt{\text{W.u.}}$$

$$52-58: 2_A^+ \rightarrow 0_A^+ = 2_1^+ \rightarrow 0_1^+$$

$$54-56: 2_B^+ \rightarrow 0_B^+ = 2_2^+ \rightarrow 0_2^+$$

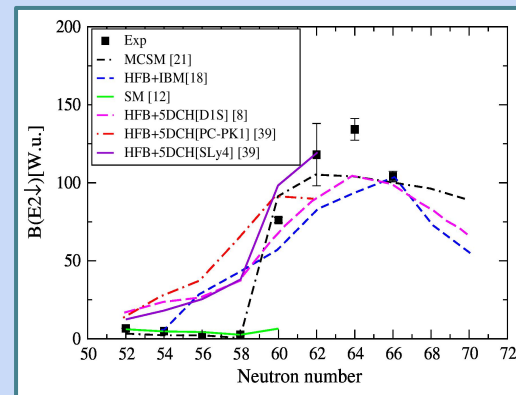
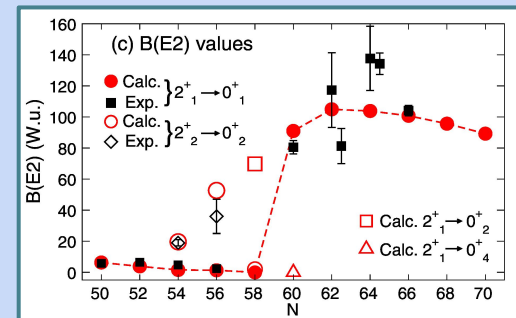
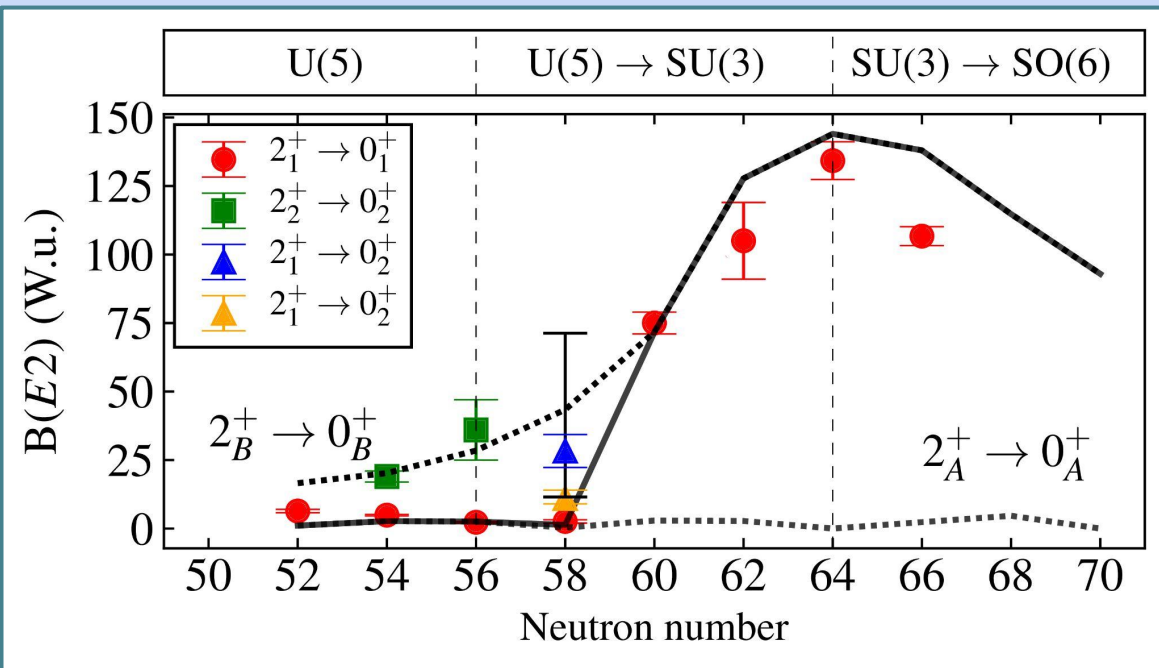
$$58: 2_B^+ \rightarrow 0_B^+ = 2_1^+ \rightarrow 0_2^+$$

$$60-70: 2_B^+ \rightarrow 0_B^+ = 2_1^+ \rightarrow 0_1^+$$

# Results

## $E2$ transitions

T. Togashi *et al.*, Phys. Rev. Lett. **117**, 172502 (2016)

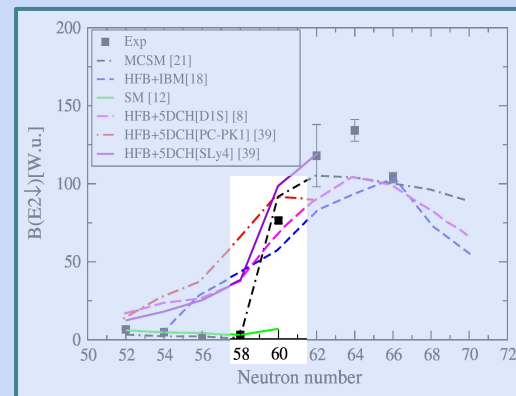
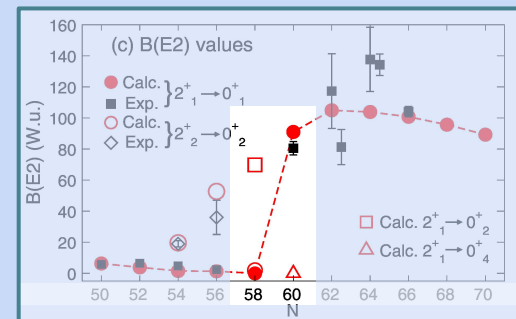
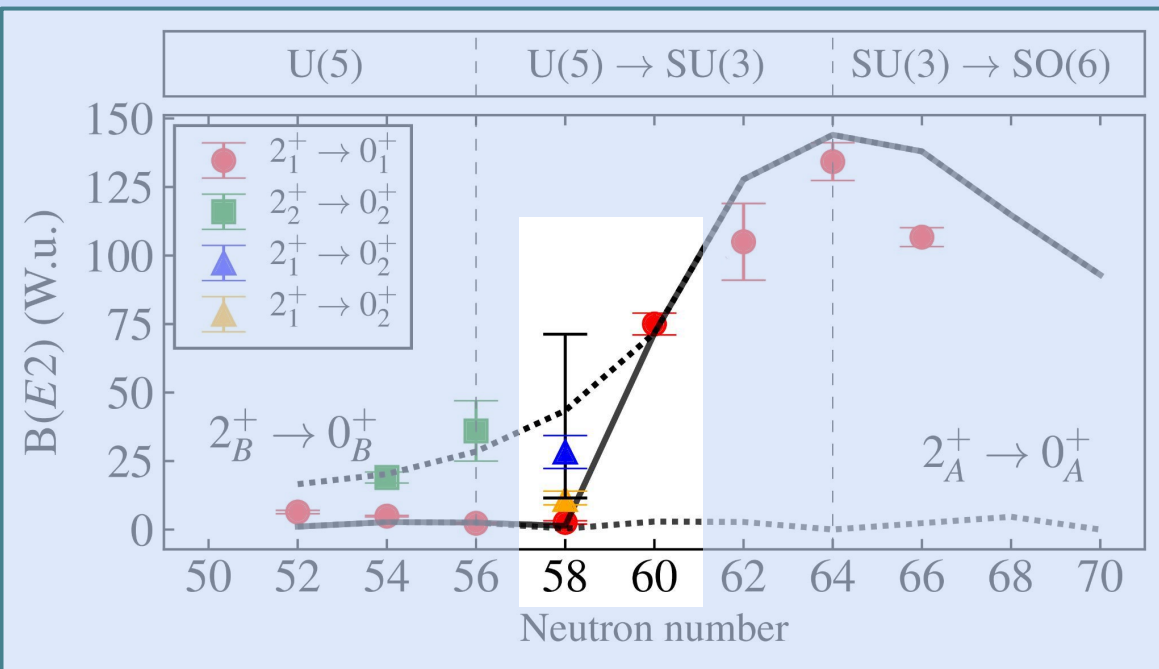


P. Singh *et al.*, Phys. Rev. Lett. **121**, 192501 (2018)

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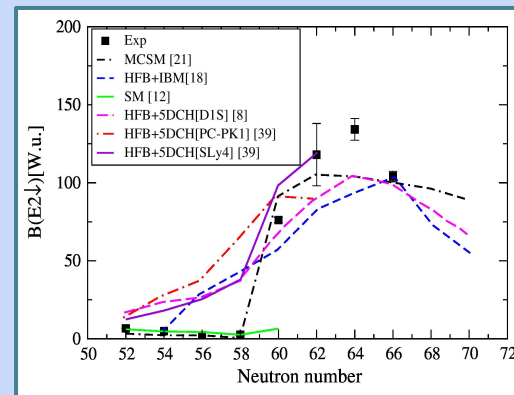
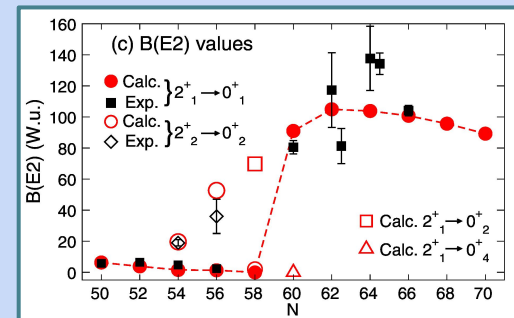
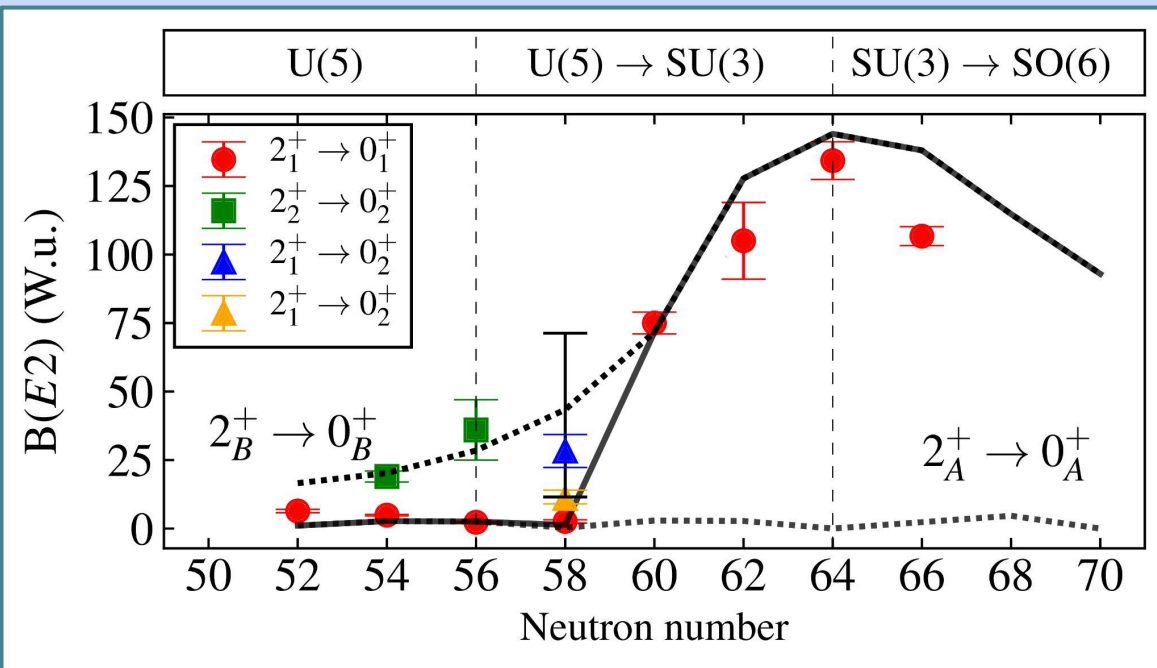


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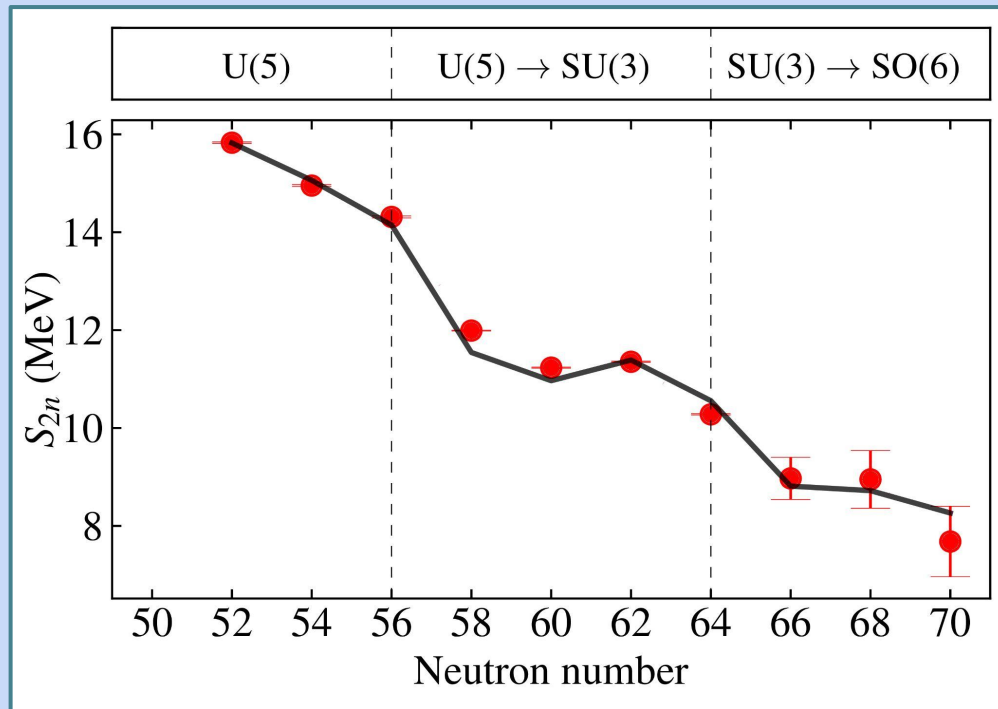


P. Singh *et al.*, Phys. Rev. Lett. **121**, 192501 (2018)

# Results

## Two neutron separation energy

$$S_{2n} = -\tilde{A} - \tilde{B}N_v \pm S_{2n}^{\text{def}} - \Delta_n$$



$$\tilde{A} = -16.5 \text{ MeV}$$
$$\tilde{B} = -0.758 \text{ MeV}$$

$$S_{2n}^{\text{def}} = \langle H \rangle_{GS}$$

+ for particles  
- for holes

$$\Delta_n = \begin{cases} 0 \text{ MeV} & 50-56 \\ 2 \text{ MeV} & 58-70 \end{cases}$$

- Quantum analysis of the evolution of energy levels and other observables (two-neutron separation energies,  $E2$  and  $E0$  transition rates, isotope shifts and magnetic moments).
- Calculated change in the configuration content and symmetry-content of wave functions.
- Classical analysis based on coherent states, examined individual shapes and their evolution with neutron number.

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*All point toward the occurrence of IQPTs:*

- Type II QPT between two configurations (normal and intruder) at  $N=60$ .
- Type I QPT [U(5)-SU(3)] of the intruder B configuration for  $N=58-62$ .
- Crossover [SU(3)-SO(6)] of the intruder B configuration for  $N=66-70$ .

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*Other IQPTs?*

*Thank you*